Nonleptonic charmless B_c decays and their search at LHCb

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We discuss the decay of B_c mesons into two light mesons (π, $K^{(*)}$, $\eta^{(\prime)}$, ρ, ω, ϕ). All these decay channels come from a single type of diagram, namely, tree annihilation. This allows us to derive extremely simple $SU(3)$ relations among these processes. The size of annihilation contributions is an important issue in B physics, and we provide two different estimates in the case of nonleptonic charmless B_c decays, either a comparison with annihilation decays of heavy-light mesons or a perturbative model inspired by QCD factorization. We finally discuss a possible search for these channels at LHCb.

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I. INTRODUCTION

The investigation of the properties of the B_c meson started in 1998 when the CDF Collaboration observed 20.4 events containing a B_c in the channel $B_c \to J/\psi l \nu$ [1]. Since then, its mass and width have been measured [2], and bounds on some nonleptonic channels have been set (J/ψ with one or three pions, $D^{*+}\bar{D}^0$ ...). From the theoretical point of view, the B_c meson shares many features with the better known quarkonia, with the significant difference that its decays are not mediated through strong interaction but weak interaction due to its flavor quantum numbers $B = -C = \pm 1$. Theoretical investigations have been carried out on the properties of the B_c meson, such as its lifetime, its decay constant, and some of its form factors [3], based on operator product expansion [4,5], potential models [6,7], nonrelativistic QCD and perturbative methods [8–11], sum rules [12–15], or lattice gauge simulations [16–19]. The properties of the B_c meson will be further scrutinized by the LHC experiments; the high luminosity of the LHC machine opens the possibility to observe many B_c decay channels beyond the discovery one, in particular, at LHCb.

This article is focused on the two-body nonleptonic charmless B_c decays. Indeed, the charmless B_c decays with two light mesons (π, $K^{(*)}$, $\eta^{(\prime)}$, ρ, ω, ϕ) in the final states come from a single diagram: the initial b and c quarks annihilate into a charged weak boson that decays into a pair formed of a u and a d/s quark, which hadronize into the two light mesons. This picture is rather different from processes such as $B_c \to J/\psi \pi$ for which the initial c quark behaves as a spectator. The recent high-precision measurements of the $B_{u,d,s}$ and D_s decays indicate that such annihilation processes can be significant, contrary to the theoretical expectation of their suppression in the heavy-quark limit [20–23]. Indeed, fits of the data not taking into account annihilation processes are generally of poor quality (see the examples in $B \to PV$ and $B \to VV$ decay channels [24–27]).

But the understanding of these contributions remains limited. The theoretical computation of annihilation diagrams is very difficult, so that the annihilation contributions are often considered as a free parameter in these decays. On the other hand, in many $B_{u,d,s}$ decays, these annihilation contributions come from several different operators (tree and penguin), and they interfere with many different other (nonannihilation) diagrams, making it difficult to obtain an accurate value of annihilation by fitting experimental data. For this reason, the processes such as $B_d \to K^+K^-$, $B_d \to D_s^- K^+$, $B_u \to D_s^- K^0$, where only the annihilation diagram contributes, have been intensively worked out while the current experimental measurements are still of limited accuracy.

The nonleptonic charmless B_c decays which we discuss in this article can have an important impact on this issue. We have 32 decay channels which come from annihilation only, as mentioned above. Moreover, these decays involve a single tree operator, which allows us to derive extremely simple relations among the different decay channels. Finally, when LHCb starts running and observes the pattern of (or at least, provides bounds for) the branching fractions of these decays channels, it will certainly help us to further improve our understanding of the annihilation contributions.

The remainder of the article is organized as follows. In Sec. II, we introduce the decay processes which we consider in this article. In Sec. III, we exploit $SU(3)$ flavor symmetry to derive relations among amplitudes for nonleptonic charmless B_c decays. In Sec. IV, we discuss theoretical issues related to this annihilation diagram and attempt to estimate its size. In Sec. V, we discuss the prospects of searching nonleptonic charmless B_c decays at LHCb, and we conclude in Sec. VI. Two appendixes are devoted to relating our work to results from factorization approaches.

II. NONLEPTONIC CHARMLESS B_c DECAYS AS PURE ANNIHILATION PROCESSES

The diagram for the nonleptonic charmless B_c decays is shown in Fig. 1 (the case of singlet states will be discussed below). The initial $\bar{b}$ and c quarks annihilate into u and $\bar{d}$ or $\bar{s}$ quarks, which form two light mesons by hadronizing with a pair of $q\bar{q}$ ($q = u$, d, s) emitted from a gluon. There are 32 decay channels of this kind if we consider only the lightest pseudoscalar and vector mesons. In the case of two outgoing pseudoscalar mesons (PP), there are 4 modes with strangeness one,

$$K^+\pi^0,\ K^+\eta,\ K^+\eta',\ K^0\pi^+,$$

and 4 modes with strangeness zero,

$$\pi^+\pi^0,\ \pi^+\eta,\ \pi^+\eta',\ K^+\bar{K}^0.$$

The same applies for two vectors (VV) up to the obvious changes:

$$K^{*+}\rho^0,\ K^{*+}\phi,\ K^{*+}\omega,\ K^{*0}\rho^+,\ \rho^+\rho^0,\ \rho^+\phi,\ \rho^+\omega,\ K^{*+}\bar{K}^{*0}.$$

In the case of VP decays, one can get two decay modes from one in PP decays, depending on the pseudoscalar meson which is turned into a vector one, yielding 8 strange decay modes ($\Delta S = 1$ processes),

$$K^{*+}\pi^0,\ K^{*+}\eta,\ K^{*+}\eta',\ K^{*0}\pi^+,\ \rho^0K^+,\ \phi K^+,\ \omega K^+,\ \rho^+K^0,$$

and 8 nonstrange decay modes ($\Delta S = 0$ processes),

$$\rho^+\pi^0,\ \rho^+\eta,\ \rho^+\eta',\ K^{*+}\bar{K}^0,\ \rho^0\pi^+,\ \phi\pi^+,\ \omega\pi^+,\ \bar{K}^{*0}K^+.$$

In the next section, we describe these decay channels in terms of a few reduced amplitudes using $SU(3)$ flavor symmetry. Similar expressions have been obtained for the charmless $B_{u,d}$ decays, which have been very useful to disentangle the rather complicated decay amplitudes of these decays containing many different contributions (tree, penguin, emission, annihilation, etc. ...) [28–30]. Compared to the case of $B_{u,d}$ decays, the $SU(3)$ relations for the B_c decays are extremely simple, as they come from only a single tree-annihilation diagram as mentioned earlier.

The theoretical computation of the process shown in Fig. 1 amounts to determining the matrix element:

$$\langle h_1h_2|\mathcal{H}_{\text{eff}}|B_c\rangle, \tag{1}$$

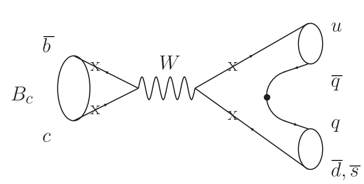

FIG. 1. Generic diagram for the nonleptonic charmless B_c decays.

where $\mathcal{H}_{\text{eff}}$ is the effective Hamiltonian which we discuss later on. This matrix element contains contributions coming from the $q\bar{q}$ state (i) produced perturbatively from one-gluon exchange linking the dot and one of the crosses in Fig. 1 and (ii) produced through strong interaction in the nonperturbative regime. Which type of these two contributions dominates this matrix element is an important issue in the theoretical computation of the hadronic B decays. In many approaches to nonleptonic B decays [26,31–34], it has been pointed out that annihilation diagrams may be sizable, with a large imaginary part, so that they have an important impact on the phenomenology of CP violation in B decays. Indeed, their contributions seem to be needed to bring agreement between theoretical computations and experimental results. There might be a significant difference in the annihilation contributions for B and B_c decays since the B_c is likely to be considered as a heavy-heavy system rather than a heavy-light one. We will discuss the theoretical estimation of the annihilation diagram in more detail in Sec. IV.

III. RELATIONS FROM $SU(3)$ FLAVOR SYMMETRY

In this section, we derive relations among the decay channels relying on the $SU(3)$ flavor symmetry between u, d, and s quarks. Following [35], we first write down the charmless B_c decays in terms of the reduced amplitudes using the Wigner-Eckart theorem.

Let us first see the possible $SU(3)$ representation of the external states. The initial state, B_c^+, is a singlet under $SU(3)$, whereas the outgoing state is the product of two mesons, which can be either both in the octet representation or in one octet and one singlet representations. We have therefore outgoing states which transform as

$$8\times 8 = 1 + 8_S + 8_A + 10 + 10^*, \qquad 1\times 8 = 8_I, \tag{2}$$

where the subscripts allow one to distinguish between the three different octet representations involved. We sandwich the operators induced by the weak interaction Hamiltonian between these external states to obtain the amplitudes for the B_c decays. The weak Hamiltonian for such transitions is given by

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}}[V_{ud}V_{cb}^*\mathcal{O}^{\Delta S=0} + V_{us}V_{cb}^*\mathcal{O}^{\Delta S=1}], \tag{3}$$

where the operators are

$$\mathcal{O}^{\Delta S=0} = \bar{u}\gamma_\mu(1-\gamma_5)d\bar{c}\gamma^\mu(1-\gamma_5)b, \tag{4}$$

$$\mathcal{O}^{\Delta S=1} = \bar{u}\gamma_\mu(1-\gamma_5)s\bar{c}\gamma^\mu(1-\gamma_5)b. \tag{5}$$

These two operators are both $SU(3)$ octets and have the following $SU(3)$ tensor structures:

$$\mathcal{O}^{\Delta S=0}:\ (Y, I, I_3) = (0, 1, 1), \tag{6}$$

$$\mathcal{O}^{\Delta S=1}\colon (Y, I, I_3) = (0, 1/2, 1/2), \tag{7}$$

where (Y, I, I_3) denotes hypercharge, isospin, and isospin projection, respectively. Since B_c charmless decays involve only operators in an octet representation, one can use the Wigner-Eckart theorem to express all the decay amplitudes in terms of three reduced matrix elements:

(i) A reduced amplitude $S = \langle 8_S \| \mathcal{O}^8 \| 1 \rangle$ from the symmetric product of the two incoming octet mesons.

(ii) A reduced amplitude $A = \langle 8_A \| \mathcal{O}^8 \| 1 \rangle$ from the antisymmetric product of the same representations.

(iii) A reduced amplitude $I = \langle 8_I \| \mathcal{O}^8 \| 1 \rangle$ from the product of an octet and a singlet meson.

The operator $\mathcal{O}$ can be $\mathcal{O}^{\Delta S=0}$ or $\mathcal{O}^{\Delta S=1}$. Note that the values of the reduced quantities S, A, I are in principle different for the PP, VP, or VV final states. The Wigner-Eckart theorem requires one to compute the Clebsch-Gordan coefficients describing the projection of a given 8×8 and 8×1 final state onto the two octet operators of interest. These coefficients can be easily determined by combining the usual $SU(2)$ Clebsch coefficients with the so-called isoscalar coefficients given in Ref. [36].

Finally, we must consider the different symmetry properties of the outgoing states (P and V) as discussed in Ref. [37]. For PP decays, where the wave function of the final state is symmetric, only S contributes, apart from the case of final states containing η or η' where both S and I are present. For VP decays, the amplitude gets contributions from S and A (and I for final states containing η, η', ω, or ϕ). For VV decays, there are three amplitudes corresponding to the three possible polarizations (or equivalently partial waves) allowed for the outgoing state. The wave function is symmetric for S and D waves and antisymmetric for the P wave, so that the matrix element S contributes to S and D waves, whereas A contributes to P waves. I contributes only to S and D waves of outgoing states containing ϕ, ω mesons.

A comment is in order on the mixing of the mesons containing $SU(3)$ singlet states. The η, η', ω, ϕ mesons are mixtures of the $SU(3)$ octet (η^8 or ω^8) and singlet (η^0 or ω^0) flavor states

$$|\eta(\omega)\rangle = \cos\theta_{p(v)}|\eta^8(\omega^8)\rangle + \sin\theta_{p(v)}|\eta^0(\omega^0)\rangle, \tag{8}$$

$$|\eta'(\phi)\rangle = -\sin\theta_{p(v)}|\eta^8(\omega^8)\rangle + \cos\theta_{p(v)}|\eta^0(\omega^0)\rangle, \tag{9}$$

where $|\eta^8\rangle$ and $|\omega^8\rangle$ have the flavor composition $|u\bar{u} + d\bar{d} - 2s\bar{s}\rangle/\sqrt{6}$, and $|\eta^0\rangle$ and $|\omega^0\rangle$ are $|u\bar{u} + d\bar{d} + s\bar{s}\rangle/\sqrt{3}$. The determination of the mixing angles $\theta_{p,v}$ is an important phenomenological issue in understanding the nature of these particles. Since we do not aim at a high accuracy in our $SU(3)$ analysis, we will adopt the following values for the mixing angles which are not very far from the phenomenological determinations:

$$\tan\theta_p = \frac{1}{2\sqrt{2}}, \qquad \tan\theta_v = \sqrt{2}. \tag{10}$$

These angles correspond to the ideal mixing for the vector sector,

$$\omega = (u\bar{u} + d\bar{d})/\sqrt{2}, \qquad \phi = s\bar{s}, \tag{11}$$

and also yield a simple expression of the pseudoscalar mesons:

$$\eta = (u\bar{u} + d\bar{d} - s\bar{s})/\sqrt{3}, \qquad \eta' = (u\bar{u} + d\bar{d} + 2s\bar{s})/\sqrt{6}. \tag{12}$$

The nonideal mixing of the (η, η') mesons is linked to the $U(1)_A$ anomaly (see, e.g., Refs. [38,39]). This value of the pseudoscalar mixing angle $\theta_p = \arctan(2\sqrt{2})^{-1} \simeq -19.5°$ is close to phenomenological determination, e.g. from the J/ψ radiative decays, $\theta_p \simeq -22°$ [40]. Let us stress that for the light mesons, we take the same phase conventions as in Ref. [26], so that some amplitudes have a minus sign with respect to those obtained from Ref [36].[1]

A. PP modes

Taking into account the Clebsch-Gordan coefficients together with the issue of octet-singlet mixing, we obtain the following amplitudes for the PP modes:

Mode	Amplitude	Mode	Amplitude
$K^+\pi^0$	$\sqrt{\frac{3}{10}}S^{PP}$	$\pi^+\pi^0$	0
$K^0\pi^+$	$\sqrt{\frac{3}{5}}S^{PP}$	$K^+\bar{K}^0$	$\sqrt{\frac{3}{5}}S^{PP}$
$K^+\eta$	$-\frac{2}{3\sqrt{5}}S^{PP} + \frac{\sqrt{2}}{3}I^{PP}$	$\pi^+\eta$	$\frac{4}{3\sqrt{5}}S^{PP} + \frac{\sqrt{2}}{3}I^{PP}$
$K^+\eta'$	$\frac{1}{3\sqrt{10}}S^{PP} + \frac{4}{3}I^{PP}$	$\pi^+\eta'$	$-\frac{1}{3}\sqrt{\frac{2}{5}}S^{PP} + \frac{4}{3}I^{PP}$

Here and in the following tables, these amplitudes must be multiplied by $G_F/\sqrt{2}$ and also the appropriate Cabibbo-Kobayashi-Maskawa (CKM) factor $V_{uD}V_{cb}^*$ with $D = d$ or s. We notice the relations

$$\begin{aligned} A(B_c^+ \to K^0\pi^+) &= \sqrt{2}A(B_c^+ \to K^+\pi^0) \\ &= \hat{\lambda}A(B_c^+ \to K^+\bar{K}^0) \end{aligned} \tag{13}$$

[1] In detail, $(-\bar{u}, \bar{d}, \bar{s})$ transform as an antitriplet [37], which means that there is a (-1) phase between the conventions of Refs. [26,36] for the pseudoscalar mesons π^-, π^0, K^-, η, η' and the vector mesons ρ^-, ρ^0, K^{*-}, ϕ, ω. We have multiplied all the amplitudes by a further (-1) factor, so that the differences between our results and those obtained using Ref. [36] are limited to a (-1) factor for the decay amplitudes for $K^0\pi^+$, $K^+\bar{K}^0$ and their vector counterparts.

with the Cabibbo-suppressing factor $\hat{\lambda} = V_{us}/V_{ud}$. The above relations are valid in the exact $SU(3)$ limit (for instance, we have $S^{PP} = S^{K^+\pi^0} = S^{K^0\pi^+} = S^{K^+\bar{K}^0}$). Obviously, these relations have some interest only if the size of $SU(3)$ breaking remains limited—we will discuss this issue in Sec. IV.

B. *VP* modes

For the VP modes, we have for the strange modes

Mode	Amplitude	Mode	Amplitude
$K^{+*}\pi^0$	$\frac{1}{2}\sqrt{\frac{3}{5}}S^{VP} + \frac{1}{2\sqrt{3}}A^{VP}$	ωK^+	$-\frac{1}{2\sqrt{15}}S^{VP} - \frac{1}{2\sqrt{3}}A^{VP} + \sqrt{\frac{2}{3}}I^{VP}$
$\rho^0 K^+$	$\frac{1}{2}\sqrt{\frac{3}{5}}S^{VP} - \frac{1}{2\sqrt{3}}A^{VP}$	ϕK^+	$\frac{1}{\sqrt{30}}S^{VP} + \frac{1}{\sqrt{6}}A^{VP} + \frac{1}{\sqrt{3}}I^{VP}$
$K^{*+}\eta$	$-\frac{1}{3}\sqrt{\frac{2}{5}}S^{VP} + \frac{\sqrt{2}}{3}A^{VP} + \frac{1}{3}I^{VP}$	$\rho^+ K^0$	$\sqrt{\frac{3}{10}}S^{VP} - \frac{1}{\sqrt{6}}A^{VP}$
$K^{*+}\eta'$	$\frac{1}{6\sqrt{5}}S^{VP} - \frac{1}{6}A^{VP} + \frac{2\sqrt{2}}{3}I^{VP}$	$K^{*0}\pi^+$	$\sqrt{\frac{3}{10}}S^{VP} + \frac{1}{\sqrt{6}}A^{VP}$

and for the nonstrange modes

Mode	Amplitude	Mode	Amplitude
$\rho^+\pi^0$	$\frac{1}{\sqrt{3}}A^{VP}$	$\omega\pi^+$	$\frac{1}{\sqrt{15}}S^{VP} + \sqrt{\frac{2}{3}}I^{VP}$
$\rho^0\pi^+$	$-\frac{1}{\sqrt{3}}A^{VP}$	$\phi\pi^+$	$-\sqrt{\frac{2}{15}}S^{VP} + \frac{1}{\sqrt{3}}I^{VP}$
$\rho^+\eta$	$\frac{2}{3}\sqrt{\frac{2}{5}}S^{VP} + \frac{1}{3}I^{VP}$	$K^{*+}\bar{K}^0$	$\sqrt{\frac{3}{10}}S^{VP} - \frac{1}{\sqrt{6}}A^{VP}$
$\rho^+\eta'$	$-\frac{1}{3\sqrt{5}}S^{VP} + \frac{2\sqrt{2}}{3}I^{VP}$	$\bar{K}^{*0}K^+$	$\sqrt{\frac{3}{10}}S^{VP} + \frac{1}{\sqrt{6}}A^{VP}$

providing the simple relations

$$\begin{aligned} A(B_c^+ \to K^{*0}\pi^+) &= \sqrt{2}A(B_c^+ \to K^{*+}\pi^0) \\ &= \hat{\lambda}A(B_c^+ \to \bar{K}^{*0}K^+), \end{aligned} \tag{14}$$

$$\begin{aligned} A(B_c^+ \to \rho^+ K^0) &= \sqrt{2}A(B_c^+ \to \rho^0 K^+) \\ &= \hat{\lambda}A(B_c^+ \to K^{*+}\bar{K}^0). \end{aligned} \tag{15}$$

It should be noted that the amplitude I^{VP} can be significantly different for the processes involving the vector singlet (ϕ, ω) and the pseudoscalar singlet (η, η') since it is known that the latter should receive a contribution from the anomaly diagram. This could induce a significant breaking of the above relations for channels involving η, η'.

C. *VV* modes

For the VV modes, we have three different configurations for the outgoing mesons, labeled by their (common) helicity. The left-handedness of weak interactions and the fact that QCD conserves helicity at high energies suggest that the longitudinal amplitude should dominate over the transverse ones (corresponding to helicities equal to ± 1).These helicity amplitudes can be combined linearly into S, P, and D wave amplitudes, and in particular, the longitudinal amplitude is a linear combination of only S and D waves:

Mode	S, D Amplitudes	P Amplitude
$K^{*+}\rho^0$	$\sqrt{\frac{3}{10}}S_{0,2}^{VV}$	$\frac{1}{\sqrt{6}}A_1^{VV}$
$K^{*+}\omega$	$-\frac{1}{\sqrt{30}}S_{0,2}^{VV} + \frac{2}{\sqrt{3}}I_{0,2}^{VV}$	$-\frac{1}{\sqrt{6}}A_1^{VV}$
$K^{*+}\phi$	$\sqrt{\frac{1}{15}}S_{0,2}^{VV} + \sqrt{\frac{2}{3}}I_{0,2}^{VV}$	$\sqrt{\frac{2}{3}}A_1^{VV}$
$K^{*0}\rho^+$	$\sqrt{\frac{3}{5}}S_{0,2}^{VV}$	$\frac{1}{\sqrt{3}}A_1^{VV}$
$\rho^+\rho^0$	0	$\sqrt{\frac{2}{3}}A_1^{VV}$
$\rho^+\omega$	$\sqrt{\frac{2}{15}}S_{0,2}^{VV} + \frac{2}{\sqrt{3}}I_{0,2}^{VV}$	0
$\rho^+\phi$	$-\frac{2}{\sqrt{15}}S_{0,2}^{VV} + \sqrt{\frac{2}{3}}I_{0,2}^{VV}$	0
$K^{*+}\bar{K}^{*0}$	$\sqrt{\frac{3}{5}}S_{0,2}^{VV}$	$-\frac{1}{\sqrt{3}}A_1^{VV}$

where the subscript denotes the partial wave under consideration $l = 0, 1, 2$. In particular, we have the interesting relations

$$A(B_c^+ \to K^{*0}\rho^+) = \sqrt{2}A(B_c^+ \to K^{*+}\rho^0), \tag{16}$$

$$\hat{\lambda}A(B_c^+ \to K^{*+}\bar{K}^0) = \sqrt{2}(-1)^l A(B_c^+ \to K^{*+}\rho^0). \tag{17}$$

D. Zweig rule

In the above expressions, the normalizations between S, A, and I amplitudes are different: the first is related to the 8×8 representation and the last to 1×8. One may relate these two amplitudes by means of the Zweig rule for the $\Delta S = 0$ processes involving ϕ. At the level of quark diagrams, one can see that the $B_c^+ \to \phi\pi^+(\rho^+)$ process cannot come from the diagram in Fig. 1—since ϕ is a pure $s\bar{s}$ state. The only production process comes from nonplanar diagrams similar to Fig. 1, but with a different combination of quarks into the outgoing mesons: the $u\bar{d}$ quarks produced from the W go into $\pi^+(\rho^+)$, whereas the ϕ meson is made of a $s\bar{s}$ pair produced from vacuum. Such a nonplanar diagram is expected to be suppressed, especially for vector mesons such as ϕ (at least three gluons are needed perturbatively, and it is $1/N_c$ suppressed in the limit of a large number of colors).

Assuming that the amplitudes for $B_c^+ \to \phi\pi^+$ and $B_c^+ \to \phi\rho^+$ vanish, one obtains the following relations,

$$I^{VP} = \sqrt{\frac{2}{5}} S^{VP}, \qquad I_{0,2}^{VV} = \sqrt{\frac{2}{5}} S_{0,2}^{VV}, \tag{18}$$

providing simpler expressions for the following VP decay amplitudes

Mode	Amplitude	Mode	Amplitude
$K^{*+}\eta$	$\frac{\sqrt{2}}{3}A^{VP}$	$\rho^+\eta$	$\sqrt{\frac{2}{5}}S^{VP}$
$K^{*+}\eta'$	$\frac{3}{2\sqrt{5}}S^{VP} - \frac{1}{6}A^{VP}$	$\rho^+\eta'$	$\frac{1}{\sqrt{5}}S^{VP}$
ωK^+	$\frac{1}{2}\sqrt{\frac{3}{5}}S^{VP} - \frac{1}{2\sqrt{3}}A^{VP}$	$\omega\pi^+$	$\sqrt{\frac{3}{5}}S^{VP}$
ϕK^+	$\sqrt{\frac{3}{10}}S^{VP} + \frac{1}{\sqrt{6}}A^{VP}$	$\phi\pi^+$	0

so that

$$A(B_c^+ \to \rho^+\eta) = \sqrt{2} A(B_c^+ \to \rho^+\eta'). \tag{19}$$

Although we assume here that the I^{VP} amplitude is the same for the processes involving the vector singlet (ϕ, ω) and the pseudoscalar singlet (η, η'), as required by $SU(3)$ symmetry, this assumption is broken for the pseudoscalar singlets due to the anomaly (seen for instance in the mass difference between η and η'). At the quark level, the detached diagram for the pseudoscalar singlet states (η, η') cannot be neglected since there is the well-known anomaly contribution modifying the previous relation:

$$\begin{aligned} &A(B_c^+ \to \rho^+\eta) - \frac{1}{3}\Delta I^{VP} \\ &= \sqrt{2}\left[A(B_c^+ \to \rho^+\eta') - \frac{2\sqrt{2}}{3}\Delta I^{VP}\right], \end{aligned} \tag{20}$$

where ΔI^{VP} denotes a potentially large $1/N_c$-suppressed anomaly contribution.

Keeping the same caveat in mind, we can simplify some VV amplitudes

Mode	S, D Amplitudes	P Amplitude
$K^{*+}\omega$	$\sqrt{\frac{3}{10}}S_{0,2}^{VV}$	$-\frac{1}{\sqrt{6}}A_1^{VV}$
$K^{*+}\phi$	$\sqrt{\frac{3}{5}}S_{0,2}^{VV}$	$\sqrt{\frac{2}{3}}A_1^{VV}$
$\rho^+\omega$	$\sqrt{\frac{6}{5}}S_{0,2}^{VV}$	0
$\rho^+\phi$	0	0

with the obvious relations among partial waves.

IV. ESTIMATING THE BRANCHING RATIOS

As mentioned in Sec. II, a precise estimate of the matrix element for the annihilation diagram is an important theoretical issue in B physics. Although the domination of the one-gluon exchange diagram has been argued in various theoretical frameworks for $B_{u,d,s}$ decays [26,31], it has not been investigated whether one-gluon exchange or other (nonperturbative) contributions dominate in B_c decays. In this section, we provide branching ratio estimates for the nonleptonic charmless B_c decays in two ways, by using experimental data on pure annihilation B decays and by relying on the one-gluon picture *à la* QCD factorization. The branching ratios can then be readily obtained by the usual formulas

$$\begin{aligned} &\mathrm{Br}(B_c \to h_1 h_2) \\ &= \frac{\sqrt{[M_{B_c}^2 - (m_1+m_2)^2][M_{B_c}^2 - (m_1-m_2)^2]}}{\Gamma_{B_c}^{\mathrm{tot}} 16\pi M_{B_c}^3} \\ &\quad \times |\langle h_1 h_2|\mathcal{H}_{\mathrm{eff}}|B_c\rangle|^2, \end{aligned} \tag{21}$$

and the expression obtained in Sec. III in terms of the reduced amplitudes is related to the matrix element through

$$|\langle h_1 h_2|\mathcal{H}_{\mathrm{eff}}|B_c\rangle| = G_F/\sqrt{2}|V_{ud(s)}V_{cb}^*| \times |R(h_1, h_2)|, \tag{22}$$

where the reduced amplitude $R(h_1, h_2)$ involves the amplitudes listed in Sec. III, expressed in terms of (S, A, I).

A. Estimate from B_d annihilation process

There are two pure annihilation processes observed in heavy-light B decays [41–43]:

$$\mathrm{Br}(B^0 \to K^+K^-) = (0.15^{+0.11}_{-0.10}) \times 10^{-6}, \tag{23}$$

$$\mathrm{Br}(B^0 \to D_s^- K^+) = (3.9 \pm 2.2) \times 10^{-5}. \tag{24}$$

Although the large experimental errors do not allow us to draw any firm conclusion, these data seem to indicate that the annihilation contribution is not negligible. We may attempt to use these decay channels to very roughly estimate the size of the nonleptonic charmless B_c decays. Since we are interested in charmless final states, let us compare the $B_c \to K^+\bar{K}^0$ and the $B^0 \to K^+K^-$ processes.

Assuming naive factorization between initial and final states, the final-state contribution cancels out when taking the ratio of the amplitudes. As a result, we find

$$\frac{\mathrm{Br}(B_c\to K^+\bar{K}^0)}{\mathrm{Br}(B^0\to K^+K^-)}\simeq \underbrace{\left(\frac{V_{cb}}{V_{ub}}\right)^2}_{\sim 100}\underbrace{\left(\frac{f_{B_c}}{f_B}\right)^2}_{\sim 4}\underbrace{\frac{\tau_{B_c}}{\tau_{B_d}}}_{\sim 0.3}\frac{1}{\xi^2}. \tag{25}$$

The factor ξ represents the difference due to the fact that the $B^0\to K^+K^-$ process comes from a diagram similar to Fig. 1 but the W boson propagates in the t channel.[2] In the one-gluon picture, $\xi = C_1/C_2 \simeq 4$, whereas it might be smaller once nonperturbative effects are included. Indeed these effects would add new contributions to both decays, but the B_c decay would be more affected relatively, since its Wilson coefficient in the one-gluon picture is smaller. This very naive argument leads to the relation between these two branching ratios:

$$\begin{aligned}\mathrm{Br}(B_c\to K^+\bar{K}^0)&\simeq \mathrm{Br}(B^0\to K^+K^-)\times\frac{1.2\times 10^2}{\xi^2}\\ &\gtrsim \mathrm{Br}(B^0\to K^+K^-)\times 7.5.\end{aligned} \tag{26}$$

The estimate for $\mathrm{Br}(B_c\to K^+\bar{K}^0)$ using this relation depends on the result of $\mathrm{Br}(B^0\to K^+K^-)$, which should be improved in the near future. Taking the current central value of $\mathrm{Br}(B^0\to K^+K^-)$, we find a lower limit of $\mathrm{Br}(B_c\to K^+\bar{K}^0)$ at the order of 10^{-6}. Using this result, one can estimate the Wigner-Eckart reduced matrix elements S^{PP},

$$S^{PP}\gtrsim 0.085\ \mathrm{GeV}^3 \qquad [(PP)=(\bar{K}^0K^+)], \tag{27}$$

where we used the following CKM central values: $V_{ub} = 0.0035$, $V_{cb} = 0.041$ [44].

As mentioned before, there is no good reason to assume that S^{PP}, S^{PV}, and S^{VV} should be related. Since we are only looking for order of magnitudes, we will assume as a dimensional estimate that $|S^{PP}|\simeq\sqrt{2}|S^{PV}|\simeq|S_0^{VV}|$. We emphasize that these relations have no strong theoretical supports and are just meant as a way to extract order of magnitudes for the branching ratios. In addition, we assume the Zweig rule to determine the singlet contributions I, and we neglect the antisymmetric contributions A, as well as transverse VV amplitudes. This provides, for instance, the following branching ratios of interest:

$$\begin{aligned}[B_d\ \text{annihil}]\qquad &\mathrm{BR}(B_c\to\phi K^+)\simeq\mathcal{O}(10^{-7}-10^{-8}),\\ &\mathrm{BR}(B_c\to\bar{K}^{*0}K^+)\simeq\mathcal{O}(10^{-6}),\\ &\mathrm{BR}(B_c\to\bar{K}^{0}K^+)\simeq\mathcal{O}(10^{-6}),\\ &\mathrm{BR}(B_c\to\bar{K}^{*0}K^{*+})\simeq\mathcal{O}(10^{-6}).\end{aligned} \tag{28}$$

The suppression of the ϕK^+ channel is due to the small CKM factor for the $\Delta S = 1$ processes, Cabibbo suppressed compared to $\Delta S = 0$.

B. Estimate from one-gluon exchange model

A second method consists of a model based on one-gluon exchange, in close relation with the model proposed in QCD factorization to estimate annihilation contributions for the decays of heavy-light mesons. In this method, described in more detail in Appendix A, the matrix element in Eq. (1) can be given as

$$\langle h_1h_2|\mathcal{H}_{\mathrm{eff}}|B_c\rangle = i\frac{G_F}{\sqrt{2}}V_{cb}^*V_{ud(s)}N_{h_1h_2}b_2(h_1,h_2), \tag{29}$$

where

$$N_{h_1h_2}=f_{B_c}f_{h_1}f_{h_2},\qquad b_2(h_1,h_2)=\frac{C_F}{N_C^2}C_2A_i^1(h_1h_2). \tag{30}$$

The function $A_i^1(h_1h_2)$ is estimated as the convolution of the kernel given by one-gluon exchange diagrams and the distribution amplitudes of the initial and final-state mesons. While the detailed computation of this function can be found in Appendix A, we would like to emphasize a few differences in B_c decays compared to the $B_{u,d}$ decays that we found: (i) the B_c decays are much simpler, since the only operator contributing is O_2, and there is only one combination of CKM factors ($V_{cb}^*V_{uD}$ where $D = d$, s depending on the strangeness of the outgoing state), and (ii) the long-distance divergences, which prevent us from estimating the annihilation contribution in $B_{u,d}$ decays, do not appear in B_c annihilation.

We can give the numerical results for the following channels: $B_c\to\phi K^+$, $B_c\to K^{*0}K^+$, $B_c\to\bar{K}^0K^+$, $B_c\to\bar{K}^{0*}K^{*+}$. We start with the function $b_2(h_1h_2)$ which turns out to be quite $SU(3)$ invariant:

$$b_2(\phi K^+)=1.5,\qquad b_2(K^{*0}K^+)=1.4, \tag{31}$$

$$b_2(\bar{K}^0K^+)=1.6,\qquad b_2(\bar{K}^{0*}K^{*+})=1.0. \tag{32}$$

Notice that $b_2(VP)=b_2(PV)$, and thus $A^{VP}=0$. One can see that the $SU(3)$ breaking is rather small as argued in Appendix A, while the difference between PP and $VP(VV)$ modes can be as large as $+13(+38)\%$. We next list the normalization factors:

$$N_{\phi K^+}=0.014\ \mathrm{GeV}^3,\qquad N_{K^{*0}K^+}=0.014\ \mathrm{GeV}^3, \tag{33}$$

$$N_{\bar{K}^0K^+}=0.010\ \mathrm{GeV}^3,\qquad N_{\bar{K}^{0*}K^{*+}}=0.019\ \mathrm{GeV}^3. \tag{34}$$

For these particular decay channels (no exchange between π and K), the $SU(3)$ breaking is also small while the difference between PP and $VP(VV)$ modes can be as large

[2] Here we neglect the small penguin diagram contribution to the $B^0\to K^+K^-$ decay.

as $-40(-90)\%$. Our numerical value for S, I, A amplitudes for the above processes are

$$S^{VP} = 0.036\ \mathrm{GeV}^3, \qquad [VP = \phi K^+, \bar{K}^{*0}K^+], \quad (35)$$

$$S^{PP} = 0.021\ \mathrm{GeV}^3, \qquad [PP = \bar{K}^0 K^+], \quad (36)$$

$$S_0^{VV} = 0.025\ \mathrm{GeV},^3 \qquad [VV = \bar{K}^{0*}K^{*+}], \quad (37)$$

where we have neglected transverse VV amplitudes. We present the relation between $N_{h_1h_2}$ and $b_2(h_1h_2)$ functions and the Wigner-Eckart reduced matrix elements S, I, A in Appendix B. Assuming the $SU(3)$-breaking effect is negligible in $b_2(h_1, h_2)$, this relation allows us to estimate all the other S, I, A amplitudes with the values given in Eq. (32) and the known values of decay constants for each final state.

We obtain finally the following values of branching ratios:

$$\begin{aligned} [\text{One-gluon}] \qquad & \mathrm{BR}(B_c \to \phi K^+) = 5 \times 10^{-9}, \\ & \mathrm{BR}(B_c \to \bar{K}^{*0}K^+) = 9.0 \times 10^{-8}, \\ & \mathrm{BR}(B_c \to \bar{K}^0 K^+) = 6.3 \times 10^{-8}, \\ & \mathrm{BR}(B_c \to \bar{K}^{*0}K^{*+}) = 9.1 \times 10^{-8}. \quad (38) \end{aligned}$$

The contributions to $\rho^0\pi^+$, $\phi\pi^+$, and $\rho^+\phi$ vanish in our approximations, which means that these power-suppressed decays must have significantly smaller branching ratios than the above ones. We do not quote any error bars on these results on purpose: we can easily estimate the uncertainties coming from our hadronic inputs, but certainly not the systematics coming from the hypothesis underlying our estimate (one-gluon approximation, asymptotic distribution amplitudes, neglect of $1/m_b$- and $1/m_c$-suppressed corrections, neglect of soft residual momentum of the heavy quarks in the B_c meson).

C. Comparison of the two methods

Our estimates of the branching ratio for e.g. the $B_c \to K^+\bar{K}^0$ in the above two ways are not consistent. This is clearly because two methods are conceptually different:

(i) The method based on B_d annihilation treats the charm quark as massless. It takes into account some of the nonperturbative long-distance effects expected to occur in B_d and B_c decays, but treats in a very naive way the relation between matrix elements of the operators O_1 and O_2. It relies also on extremely naive assumptions concerning the respective size of matrix elements for PP, VP, and VV modes.

(ii) The method based a perturbative one-gluon exchange treats the charm quark as heavy. It assumes the dominance from a specific set of diagrams computed in a perturbative way, but it provides a consistent framework to perform the estimation.

It is well-known that both kinds of estimates yield rather different results. This is illustrated by the fact that the estimate of $B_d \to K^+K^-$ in the annihilation models of QCD factorization [26] (around 10^{-8} with substantial uncertainties) and perturbative QCD [45] is 1 order of magnitude below the current experimental average. Therefore, it is not surprising that our two methods yield branching ratios differing by a similar amount. There are well-known cases where final-state interaction can increase significantly estimates based on factorization, for instance, $B \to K\chi_c$ [46,47] or $D_s^+ \to \rho^0\pi^+$ [48]. An observation of the nonleptonic charmless B_c decays will certainly constitute an important key to clarify such a controversy as well as further theoretical issues in computational methods for the annihilation diagram.

V. SEARCH PROSPECT AT LHCB

The LHC pp collider with the center of mass energy of 14 TeV has a large cross section for the $b\bar{b}$ hadroproduction, which can be followed by the production of not only B^0, B^+, and B_s^0 mesons but also other b hadrons such as Λ_b and B_c. The subtraction of the production of other known b hadrons [49]

$$\begin{aligned} b\bar{b} \to & (B_d{:}B_u{:}B_s{:}\Lambda_b' s) \\ & \simeq (42.2 \pm 0.9\%{:}42.2 \pm 0.9\%{:}10.5 \pm 0.9\%{:}9.1 \pm 1.5\%) \end{aligned} \quad (39)$$

leads to $b\bar{b} \to (B_c)$ to be less than 1%. The LHCb experiment [50], which is dedicated to B physics analyses with its optimized trigger scheme, allows one to detect the b-decay modes into hadronic final states.

The theoretical estimate of the B_c cross section is still under scrutiny. For the dominant gg-fusion process, there are two possible mechanisms: $gg \to b\bar{b}$ followed by the fragmentation, or $\bar{b} \to B_c b\bar{c}$. It is found that the latter dominates in the low-p_T region which corresponds to the LHCb coverage [51,52]. The $\mathcal{O}(\alpha_s^4)$ computation of the $\bar{b} \to B_c b\bar{c}$ process predicts $\sigma(pp \to B_c^+ X) \simeq 0.3$–$0.8\ \mu$b [53] where the error comes from the uncertainty in theoretical inputs such as the choice of the α_s scale and the B_c distribution function. Additional systematics could come from higher-twist and -radiative corrections. In the following, we follow the LHCb value for the cross section $\sigma(B_c) = 0.4\ \mu$b, but it must be noted that this value may be affected by a large uncertainty.

We can now estimate the expected sensitivity for a specific channel. First, let us discuss which channel has the best potential for the detection. The best trigger and reconstruction efficiencies with a large signal over background ratio can be achieved by the charged K and/or π tags (and by avoiding low-p_T neutral particles) at LHCb.

Since the initial B_c carries an electric charge, all two-body PP final states contain one neutral particle. The same remark applies for the VV channels when one considers the subsequent decays of the vector particles into pairs of pseudoscalars. In this respect, PV channels such as $B_c^+ \to \phi K^+$, $\bar{K}^{*0} K^+$, $\bar{K}^0 \pi^+$, $\rho^0 K^+$, $\rho^0 \pi^+$, $\phi \pi^+$ are the best candidates using the vector meson decays, $\phi \to K^+ K^-$, $\bar{K}^{*0} \to K^- \pi^+$, $\rho^0 \to \pi^+ \pi^-$, leading to three charged tracks. Among these subsequent decays, the small widths of ϕ and $\bar{K}^{*0}$ make the reconstructing particularly easy compared to e.g. ρ^0. On the theoretical side, our Zweig rule argument forbids the $B_c^+ \to \phi \pi^+$, whereas the $B_c^+ \to \rho^0 \pi^+$ channel comes only from the A (asymmetric) amplitude which is also subdominant. Finally, taking into account the fact that the $\Delta S = 1$ channels are Cabibbo suppressed, we draw the conclusion that the $B_c^+ \to \bar{K}^{*0} K^+$ channel might be the best candidate for the detection.

Since the selection criteria and trigger efficiencies are different for each channel, detailed simulations are necessary in order to estimate the expected sensitivity for different channels. For example, such a study has been done for $B_c \to J/\psi \pi^+$ [53,54]. From the expected branching ratio $\mathrm{Br}(B_c \to J/\psi \pi^+) \simeq 1\%$, it was deduced that over a thousand events are expected after the 1 yr run of LHCb. By scaling this observation to the processes of interest, we can very roughly estimate that an assumption of $\mathrm{Br}(B_c^+ \to \bar{K}^{*0} K^+) = 10^{-6}$ yields a few events per year at LHCb. The analysis of LHCb data will thus allow setting first experimental limits on the nonleptonic charmless B_c decays, and give hints on annihilation mechanisms in these decays.

VI. CONCLUSIONS

In this paper, we have discussed nonleptonic charmless B_c decays into two light pseudoscalar or vector mesons. It turns out that a single tree-annihilation diagram is responsible for all 32 processes, providing an interesting testing ground for annihilation. After discussing general aspects of the charmless B_c decays, we have shown that the very simple nature of these decays allows us to describe them in terms of a few reduced amplitudes by exploiting $SU(3)$ flavor symmetry to relate various PP, PV, and VV modes.

In order to discuss a possible search for charmless nonleptonic B_c decays at LHCb, we have proposed two different theoretical estimates of these reduced matrix elements, either by comparison with B_d annihilation processes or by a perturbative model based on the exchange of one gluon. The two models yield a rather wide range of branching ratio predictions, from 10^{-6} to 10^{-7}. The LHCb experiment has the potential to observe some of the decay channels (such as $B_c \to \phi K^+$, $\bar{K}^{*0} K^+$) if the branching ratio is at the larger side of these estimates.

From the theoretical point of view, a better understanding of annihilation diagrams is particularly important. They are often assumed to play a significant role in decays of heavy-light mesons, but they occur jointly with other kinds of diagrams, making it difficult to assess precisely their size. Furthermore, for the theoretical estimates of the $B_{u,d}$ annihilation diagram in the QCD factorization, there is an additional uncertainty caused by the infrared divergence occurring in its computation. It is worth mentioning that we found that such a divergence does not occur in the case of the B_c annihilation diagram, suggesting that predictions from models à la QCD factorization for the B_c decays should be more precise, and thus easier to confirm or reject.

On the other hand, it has been discussed that the annihilation diagrams may be enhanced by long-distance effects such as final-state interactions. Although only limited models of such effects have been proposed either for D or for B decays (the former likely more affected than the latter by such enhancements) [46–48], the observation of an unexpectedly large branching ratio for the B_c annihilation would call for a reassessment of such long-distance contributions. An observation of charmless nonleptonic B_c decays at LHCb will certainly provide substantial information on these models, in complement with the observation of other decays such as $B_d \to K^+ K^-$ or $B_s \to \pi^+ \pi^-$.

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APPENDIX A: SHORT-DISTANCE MODEL FOR WEAK ANNIHILATION

As highlighted in the introduction, weak annihilation plays a significant role in $B_{u,d,s}$ nonleptonic decays, but it is difficult to estimate it accurately. A model to estimate this contribution was provided in the framework of QCD factorization [26,55], relying on the following hypothesis:

(i) The diagrams are dominated by the exchange of a single gluon, whose off shellness is typically of order $O(\sqrt{\Lambda m_b})$.

(ii) Hadronization effects are taken into account through light-cone distribution amplitudes (generally taken in their asymptotic form).

(iii) Soft components are neglected.

Being power suppressed in the heavy-quark limit, the weak-annihilation contributions to $B_{u,d,s}$ nonleptonic decays cannot be factorized in short- and long-distance effects (in general). Their evaluation within this rough model exhibits endpoint divergences, which signals the presence of long- distance contributions not taken into account properly. The divergent integrals were regularized on the basis of dimensional analysis, which induces a significant uncertainty in the estimate of the annihilation contribution.

We can follow a similar method to estimate annihilation in the case of the B_c decay. Concerning the B_c meson, we work in the limit where both b and c quarks are heavy (keeping m_c/m_b fixed) and we set the momentum of the valence quarks to $p_b^\mu = m_b v^\mu$ and $p_c^\mu = m_c v^\mu$, neglecting the soft components of the heavy-quark momenta (and consistently setting $M_{B_c} = m_c + m_b$). Since we neglect the soft components of p_b and p_c, the integration over the B_c meson distribution amplitude is trivial and yields f_{B_c}. The diagrams to compute are not very difficult and correspond to a gluon emitted from the b antiquark or the c quark from the B_c meson and converted into a light-quark–antiquark pair. Following Ref. [26], we find in the case where $(M_1, M_2) = (P, P), (P, V), (V, V)$,

$$A_1^i(M_1M_2) = \pi\alpha_s \int dxdy\Big\{\phi_{M_1}(y)\phi_{M_2}(x) \times \Big[\frac{1}{y[(\bar{x}+y)z_b - \bar{x}y]} - \frac{1}{\bar{x}[(\bar{x}+y)z_c - \bar{x}y]}\Big] + r^{M_1}r^{M_2}\phi_{m_1}(y)\phi_{m_2}(x) \times \Big[\frac{2(1-z_b)}{(\bar{x}+y)z_b - \bar{x}y} - \frac{2(1-z_c)}{(\bar{x}+y)z_c - \bar{x}y}\Big]\Big\}. \tag{A1}$$

If $(M_1, M_2) = (V, P)$, one has to change the sign of the second (twist-4) term above. ϕ_M and ϕ_m are twist-2 and twist-3 two-particle distribution amplitudes of the meson M, and r^M is the normalization of the twist-3 distribution amplitude. In the case of pseudoscalar mesons, we have

$$r^\pi = \frac{2m_\pi^2}{m_b \times 2m_q}, \qquad r^K = \frac{2m_K^2}{m_b(m_q + m_s)}, \tag{A2}$$

responsible for the chiral enhancement of twist-4 contributions for pion and kaon outgoing states. In the case of vector mesons, we have

$$r^V = \frac{2m_V}{m_b}\frac{f_V^\perp}{f_V}. \tag{A3}$$

z_b and z_c denote the relative size of the b- and c-quark masses:

$$z_b = \frac{m_b}{m_b + m_c}, \qquad z_c = 1 - z_b = \frac{m_c}{m_b + m_c}. \tag{A4}$$

Their appearance allows one to distinguish the diagram of origin (corresponding to a gluon emitted from the b- or the c-quark line).

Equation (A1) is in agreement with the expressions obtained in Refs. [26,27] in the limit $z_b \to 1$ (and $z_c \to 0$). To further simplify the discussion, we take the asymptotic expression for the distribution amplitudes

$$\phi_P(x) = 6x(1-x), \qquad \phi_V(x) = 6x(1-x), \qquad \phi_p(x) = 1, \qquad \phi_v(x) = 3(2x-1). \tag{A5}$$

The structure of the singularities in the kernel is due to the propagator of the gluon and seems quite complicated. But if we take as an example

$$\int_0^1 d\bar{x}dy\frac{1}{(\bar{x}+y)z - \bar{x}y} = \int d\bar{x}\frac{1}{z-\bar{x}}\log\left|\frac{z - \bar{x} + \bar{x}z}{\bar{x}z}\right|, \tag{A6}$$

the function to be integrated is continuous at $x = z$ and has integrable singularities for $x = 0$ and $x = z/(1-z)$. The integration can therefore be performed without problem as long as z is different from 0, 1/2, and 1 (no coalescence of singularities). For the B_c meson, it means that the twist-2 and the twist-4 contributions have no endpoint singularities and yield finite integrals. Therefore, there is no need to introduce models to regularize the divergent integrals like in the case of heavy-light mesons.

The corresponding expressions for the four cases are slightly tedious, but they can be approximated to a very good accuracy through low-order polynomials in δ, where $z_b = 0.76 + \delta$ and $z_c = 0.24 - \delta$ (corresponding to $m_b = 4.2$ and $m_c = 1.3$ for $\delta = 0$):

$$A_1^i(PP) = \pi\alpha_s[(-22.83 + 4.84\delta + 808.3\delta^2 + 2507\delta^3 + 3425\delta^4) + r^{M_1}r^{M_2}(-8.23 - 3.65\delta + 73.6\delta^2 - 16.1\delta^3 + 3575\delta^4 + 16\,007\delta^5)], \tag{A7}$$

$$A_1^i(PV) = \pi\alpha_s[(-22.83 + 4.84\delta + 808.3\delta^2 + 2507\delta^3 + 3425\delta^4) + r^{M_1}r^{M_2}(-19.15 - 123.5\delta - 130\delta^2 + 58.7\delta^3 + 7982\delta^4 + 39\,778\delta^5)], \tag{A8}$$

$$A_1^i(VV) = \pi\alpha_s[(-22.83 + 4.84\delta + 808.3\delta^2 + 2507\delta^3 + 3425\delta^4) + r^{M_1}r^{M_2}(2.44 + 222.4\delta + 1565\delta^2 + 3386\delta^3 + 5824\delta^4 - 80\,831\delta^5 - 421\,927\delta^6)]. \tag{A9}$$

Within the set of approximations performed here, $A_1^i(PV)$ and $A_1^i(PV)$ are identical.

We use the above formulas to estimate a few branching ratios. We take our inputs for the vector decay constants and the Wilson coefficient $C_2(\sqrt{m_b\Lambda_h}) = -0.288$ from Ref. [56], and the rest of our inputs from Ref. [26]. We take the value of the B_c meson decay constant $f_{B_c} = 395$ MeV taking the central value from Ref. [3].

Let us comment on the $SU(3)$ breaking, which can be included in this QCD computation. We do not have the $SU(3)$-breaking effects coming from the distribution amplitude: for instance, a small m_s correction makes the K and $K^{(*)}$ distribution amplitudes slightly asymmetric. On the other hand, we have the breaking effect in the chiral enhancement parameter r^M. The $SU(3)$ breaking (e.g. comparison of r^π, r^K or r^ρ, r^{K^*}, r^ϕ) turns out to be relatively small. Another $SU(3)$ breaking arises from the following decay constants in the normalization factor $N_{h_1h_2}$ [49,56]:

$$\begin{aligned} f_\pi &= (130.4 \pm 0.2)\ \text{MeV}, & f_K &= (155.5 \pm 0.8)\ \text{MeV}, \\ f_\rho &= (216 \pm 3)\ \text{MeV}, & f_{K^*} &= (220 \pm 5)\ \text{MeV}, \\ f_\omega &= (187 \pm 5)\ \text{MeV}, & f_\phi &= (215 \pm 5)\ \text{MeV}. \end{aligned} \tag{A10}$$

The $SU(3)$ breaking in the decay constants for the vector mesons is rather small while there is a 24% difference in π and K decay constants.

APPENDIX B: IDENTIFICATION WITH RESULTS FROM QCD FACTORIZATION

The expressions for all the decay channels considered in Sec. III can be recovered from Refs. [26,27,57] if we identify between the Wigner-Eckart reduced matrix elements S, I, A and the O_2 reduced coefficients b_2. In these references, one must take the expressions for the decay amplitudes of B_u decays into the relevant final state, and pick up the O_2 contribution, which is the only remaining one once the related B_c decay is considered. If we perform this identification, we obtain

$$S^{PP} = \sqrt{\frac{5}{3}} N_{PP} b_2(PP), \tag{B1}$$

$$I^{PP} = \sqrt{\frac{2}{3}} N_{PP} b_2(PP), \tag{B2}$$

$$S^{VP} = \sqrt{\frac{5}{6}} N_{VP}(b_2(PV) + b_2(VP)), \tag{B3}$$

$$A^{VP} = \sqrt{\frac{3}{2}} N_{VP}(b_2(PV) - b_2(VP)), \tag{B4}$$

$$I^{VP} = \sqrt{\frac{1}{3}} N_{VP}(b_2(PV) + b_2(VP)), \tag{B5}$$

$$S^{VV}_{S,D} = \sqrt{\frac{5}{3}} N_{VV} b_2^{S,D}(VV), \tag{B6}$$

$$A^{VV}_P = 0, \tag{B7}$$

$$I^{VV}_{S,D} = \sqrt{\frac{2}{3}} N_{VV} b_2^{S,D}(VV). \tag{B8}$$

[1] F. Abe *et al.*, Phys. Rev. Lett. **81**, 2432 (1998).
[2] T. Aaltonen *et al.*, Phys. Rev. Lett. **100**, 182002 (2008).
[3] N. Brambilla *et al.*, CERN Yellow Report No. CERN-2005-005, 2005.
[4] Ikaros I. Y. Bigi, Phys. Lett. B **371**, 105 (1996).
[5] Martin Beneke and Gerhard Buchalla, Phys. Rev. D **53**, 4991 (1996).
[6] Pietro Colangelo and Fulvia De Fazio, Phys. Rev. D **61**, 034012 (2000).
[7] V. V. Kiselev, A. E. Kovalsky, and A. I. Onishchenko, Phys. Rev. D **64**, 054009 (2001).
[8] Chao-Hsi Chang, Yu-Qi Chen, and Robert J. Oakes, Phys. Rev. D **54**, 4344 (1996).
[9] Maurizio Lusignoli, M. Masetti, and S. Petrarca, Phys. Lett. B **266**, 142 (1991).
[10] Nora Brambilla and Antonio Vairo, Phys. Rev. D **62**, 094019 (2000).
[11] Nora Brambilla, Antonio Pineda, Joan Soto, and Antonio Vairo, Rev. Mod. Phys. **77**, 1423 (2005).
[12] V. V. Kiselev, J. Phys. G **30**, 1445 (2004).
[13] V. V. Kiselev, A. E. Kovalsky, and A. K. Likhoded, arXiv: hep-ph/0006104.
[14] V. V. Kiselev, A. E. Kovalsky, and A. K. Likhoded, Nucl. Phys. **B585**, 353 (2000).
[15] V. V. Kiselev, arXiv:hep-ph/0211021.
[16] C. T. H. Davies *et al.*, Phys. Lett. B **382**, 131 (1996).
[17] I. F. Allison *et al.*, Nucl. Phys. B, Proc. Suppl. **140**, 440 (2005).
[18] H. P. Shanahan, P. Boyle, C. T. H. Davies, and H. Newton, Phys. Lett. B **453**, 289 (1999).
[19] B. D. Jones and R. M. Woloshyn, Phys. Rev. D **60**, 014502 (1999).
[20] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **78**, 032005 (2008).
[21] A. Abulencia *et al.* (CDF Collaboration), Phys. Rev. Lett. **97**, 211802 (2006).
[22] R. Balest *et al.* (CLEO Collaboration), Phys. Rev. Lett. **79**, 1436 (1997).
[23] J. P. Alexander *et al.* (CLEO Collaboration), Phys. Rev. Lett. **100**, 161804 (2008).
[24] S. Mishima, Phys. Lett. B **521**, 252 (2001).
[25] A. L. Kagan, Phys. Lett. B **601**, 151 (2004).
[26] Martin Beneke and Matthias Neubert, Nucl. Phys. **B675**, 333 (2003).

[27] Martin Beneke, Johannes Rohrer, and Deshan Yang, Nucl. Phys. **B774**, 64 (2007).
[28] Cheng-Wei Chiang, Michael Gronau, and Jonathan L. Rosner, Phys. Lett. B **664**, 169 (2008).
[29] Cheng-Wei Chiang, Michael Gronau, Jonathan L. Rosner, and Denis A. Suprun, Phys. Rev. D **70**, 034020 (2004).
[30] Cheng-Wei Chiang, Michael Gronau, Zumin Luo, Jonathan L. Rosner, and Denis A. Suprun, Phys. Rev. D **69**, 034001 (2004).
[31] Yong-Yeon Keum, Hsiang-nan Li, and A. I. Sanda, Phys. Lett. B **504**, 6 (2001).
[32] Marco Ciuchini, E. Franco, G. Martinelli, M. Pierini, and L. Silvestrini, Phys. Lett. B **515**, 33 (2001).
[33] Christian W. Bauer, Dan Pirjol, Ira Z. Rothstein, and Iain W. Stewart, Phys. Rev. D **70**, 054015 (2004).
[34] Junegone Chay, Hsiang-nan Li, and Satoshi Mishima, Phys. Rev. D **78**, 034037 (2008).
[35] D. Zeppenfeld, Z. Phys. C **8**, 77 (1981).
[36] J. J. de Swart, Rev. Mod. Phys. **35**, 916 (1963).
[37] Michael Gronau and Jonathan L. Rosner, Phys. Lett. B **376**, 205 (1996).
[38] T. Feldmann, P. Kroll, and B. Stech, Phys. Rev. D **58**, 114006 (1998).
[39] Thorsten Feldmann, Int. J. Mod. Phys. A **15**, 159 (2000).
[40] J. M. Gerard and E. Kou, Phys. Lett. B **616**, 85 (2005).
[41] B. Aubert *et al.*, Phys. Rev. D **75**, 012008 (2007).
[42] S.-W. Lin *et al.* (Belle Collaboration), Phys. Rev. Lett. **98**, 181804 (2007).
[43] Michael Morello, Nucl. Phys. B, Proc. Suppl. **170**, 39 (2007).
[44] J. Charles *et al.*, Eur. Phys. J. C **41**, 1 (2005).
[45] Chuan-Hung Chen and Hsiang-nan Li, Phys. Rev. D **63**, 014003 (2000).
[46] T. N. Pham and Guo-huai Zhu, Phys. Lett. B **619**, 313 (2005).
[47] M. Beneke and L. Vernazza, Nucl. Phys. **B811**, 155 (2009).
[48] Svjetlana Fajfer, Anita Prapotnik, Paul Singer, and Jure Zupan, Phys. Rev. D **68**, 094012 (2003).
[49] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).
[50] A. Augusto Alves *et al.* (LHCb Collaboration), JINST **3**, S08005 (2008).
[51] I. P. Gouz, V. V. Kiselev, A. K. Likhoded, V. I. Romanovsky, and O. P. Yushchenko, Yad. Fiz. **67**, 1581 (2004) [Phys. At. Nucl. **67**, 1559 (2004)].
[52] K. Kolodziej, A. Leike, and R. Ruckl, Phys. Lett. B **355**, 337 (1995).
[53] O. P. Yushchenko, Report No. LHCb-2003-113.
[54] Y. Gao, J. He, and Z. Yang, Report No. LHCb-2008-077; CERN, Report No. CERN-LHCb-2008-077.
[55] M. Beneke, G. Buchalla, M. Neubert, and Christopher T. Sachrajda, Nucl. Phys. **B606**, 245 (2001).
[56] Patricia Ball, Gareth W. Jones, and Roman Zwicky, Phys. Rev. D **75**, 054004 (2007).
[57] Matthaus Bartsch, Gerhard Buchalla, and Christina Kraus, arXiv:0810.0249.