

Possible quantum numbers of the pentaquark $\Theta^+(1540)$ in QCD sum rulesPhilipp Gubler,^{1,*} Daisuke Jido,² Toru Kojo,³ Tetsuo Nishikawa,⁴ and Makoto Oka¹¹*Department of Physics, H-27, Tokyo Institute of Technology, Meguro, Tokyo 152-8551, Japan*²*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*³*RBRC, Brookhaven National Laboratory, Upton, New York 11973-5000, USA*⁴*Faculty of Health Science, Ryotokuji University, Urayasu, Chiba, 279-8567, Japan*

(Received 13 November 2009; published 31 December 2009)

The QCD sum rule technique is employed to investigate pentaquark states with strangeness $S = +1$ and $IJ^\pi = 0_2^\pm, 1_2^\pm, 0_2^{\pm\pm}, 1_2^{\pm\pm}$. Throughout the calculation, emphasis is laid on the establishment of a valid Borel window, which corresponds to a region of the Borel mass, where the operator product expansion converges and the presumed ground state pole dominates the sum rules. Such a Borel window is achieved by constructing the sum rules from the difference of two independent correlators and by calculating the operator product expansion up to dimension 14. Furthermore, we discuss the possibility of the contamination of the sum rules by possible KN scattering states. As a result, we conclude that the $0_2^{\pm\pm}$ state seems to be the most probable candidate for the experimentally observed $\Theta^+(1540)$, while we also obtain states with $0_2^\pm, 1_2^\pm, 1_2^{\pm\pm}$ at somewhat higher mass regions.

DOI: [10.1103/PhysRevD.80.114030](https://doi.org/10.1103/PhysRevD.80.114030)

PACS numbers: 14.20.Pt, 12.38.Lg

I. INTRODUCTION

$\Theta^+(1540)$ with strangeness $S = +1$ and baryon number $B = +1$ is evidently a flavor exotic state with minimal quark content $uudd\bar{s}$ [2]. The first announcement of its experimental detection was made in 2003 [2], and it has since confronted the hadron physics community with interesting novel phenomena and unanticipated problems that have not been solved until the present day.

Presently, the experimental situation of $\Theta^+(1540)$ seems to be rather unclear. After the CLAS collaboration has published several papers on their pentaquark search with high statistics [3–6], where no signal of Θ^+ could be found, many people now seem to believe that the pentaquark does not exist after all and that the whole story was just “a curious episode in the history of science” [7]. There are, however still experiments that claim to observe a signal of $\Theta^+(1540)$ [8,9] and therefore this issue should not be considered to be completely settled yet. Additional experimental results, which either unambiguously confirm the existence of $\Theta^+(1540)$ or otherwise can eliminate it completely, are eagerly waited for.

Theoretically, one not yet well understood property of $\Theta^+(1540)$ is its unnaturally narrow width, which was reported to be even less than 1 MeV [8] and which is very difficult to explain from our experience with ordinary baryons. Because $\Theta^+(1540)$ lies about 100 MeV above the KN threshold, one would expect the width to be much larger than the experimentally measured value. Of course, there have been many attempts to explain this narrow width of $\Theta^+(1540)$ [10–17], but none of these approaches has completely succeeded yet.

Another problem is the correct assignment of quantum numbers such as spin and parity to the $\Theta^+(1540)$ state. There are many studies, in which states with various quantum numbers were investigated using QCD sum rules [18–28] or lattice QCD [29–34], but no consistent understanding has yet emerged.

The main subject of the present paper is to determine the quantum numbers (isospin, spin, and parity) of $\Theta^+(1540)$ from a QCD sum rule approach. We therefore study and compare the sum rules of states with $IJ^\pi = 0_2^\pm, 1_2^\pm, 0_2^{\pm\pm}, 1_2^{\pm\pm}$. From this comparison, we aim to determine which of the investigated quantum numbers is the one that most likely has to be assigned to the $\Theta^+(1540)$ state. Furthermore, we also look for possible excited states below 2 GeV, that may be found in future experiments. For these purposes we use an improved version of the QCD sum rule method, which has first been proposed in [27]. The basic idea of improvement is to use the difference of two correlators to construct the sum rule. The continuum part of the spectral function is significantly suppressed by this procedure, which therefore helps to find a valid Borel window, whose existence is a necessary condition for obtaining reliable results within the QCD sum rule technique. Moreover, we calculate the operator product expansion (OPE) up to dimension 14, which is indispensable for a sufficient convergence of the expansion.

The paper is organized as follows. In Sec. II, the formalism of QCD sum rules is briefly reviewed. The details of our method, including the interpolating fields employed and the implications of the improvement mentioned in the last paragraph are then explained in Sec. III. In Sec. IV, the results of the analysis for the various quantum numbers are given in detail. These results are then discussed in Sec. V, and the conclusion is given in Sec. VI. Finally, the appen-

*phil@th.phys.titech.ac.jp

dix is devoted to the numerical results of the OPE and to the details of the establishment of the Borel window for the various sum rules.

II. FORMALISM

A. QCD sum rules

In the QCD sum rule method [35,36], we compute the two-point function of various operators. It is defined as

$$\begin{aligned}\Pi(q) &= i \int d^4x e^{iqx} \langle 0 | T[\eta(x) \bar{\eta}(0)] | 0 \rangle \\ &\equiv \Pi_1(q^2) \not{q} + \Pi_2(q^2),\end{aligned}\quad (1)$$

where $\eta(x)$ is a spin $\frac{1}{2}$ operator. $\Pi_1(q^2)$ is called the chiral-even, and $\Pi_2(q^2)$ the chiral-odd part.

Furthermore, the two-point function of a spin $\frac{3}{2}$ Rarita-Schwinger type operator $\eta_\mu(x)$ is defined as

$$\begin{aligned}\Pi(q) &= i \int d^4x e^{iqx} \langle 0 | T[\eta_\mu(x) \bar{\eta}_\nu(0)] | 0 \rangle \\ &\equiv -g_{\mu\nu} [\Pi_1(q^2) \not{q} + \Pi_2(q^2)] + \dots,\end{aligned}\quad (2)$$

where the dots stand for other Lorentz structures than $g_{\mu\nu}$. In the present study, we will only need the terms containing $g_{\mu\nu}$. We use the same notation as for the spin $\frac{1}{2}$ case and denote $\Pi_1(q^2)$ as the chiral-even, and $\Pi_2(q^2)$ as the chiral-odd part.

In the QCD sum rule approach, we use the analytic properties of these two-point functions to extract information of the physical states that couple to the operators η or η_μ . Concretely, the analyticity of Eqs. (1) and (2) allows one to write down the dispersion relation

$$\Pi_i(q^2) = \frac{1}{\pi} \int_0^\infty ds \frac{\text{Im}\Pi_i(s)}{s - q^2} + (\text{subtraction terms}), \quad (3)$$

for $i = 1, 2$. This equation has the same form for both the spin $\frac{1}{2}$ and spin $\frac{3}{2}$ case. To handle a possible divergence in the integral of the right-hand side, usually the subtracted dispersion relation is used. This means that subtraction terms have to be added to this equation, which contribute mainly to the high-energy part of the spectral function. The significance of these terms for the low-energy region is hence small. Moreover, the subtraction terms will disappear when the Borel transformation is applied as they are polynomials of q^2 . We will thus omit them in the following.

The imaginary part of the two-point function $\text{Im}\Pi_i(s)$, which corresponds to the spectral function of η or η_μ , satisfies the following spectral conditions:

$$\text{Im}\Pi_1(s) \geq 0, \quad \sqrt{s} \text{Im}\Pi_1(s) - \text{Im}\Pi_2(s) \geq 0. \quad (4)$$

Thus, the positivity condition only holds for the chiral-even part, while the spectral function obtained from the chiral-odd part is allowed to have negative values.

In this study, we employ the usual ‘‘pole + continuum’’ parametrization for the spectral function, which appears in the imaginary part of the correlator

$$\text{Im}\Pi_i(s) = \pi(\lambda_i)^2 \delta(s - m_{\Theta^+}^2) + \theta(s - s_{\text{th}}) \text{Im}\Pi_i^{\text{OPE}}(s). \quad (5)$$

The δ function for the ground state pole is justified by the experimental results, which show that the width of $\Theta^+(1540)$ is very narrow. The potential contribution of the KN scattering states, which are not included in the expression above will be discussed in Sec. III D.

Furthermore, to obtain consistent results, it is important to choose an appropriate operator in the two-point function, whose spectral function resembles that of Eq. (5) as much as possible. In other words, the chosen operator should couple strongly to the ground state pole (if it exists), leading to a large value of the residue (λ_i) and at the same time should only have a small overlap with the continuum states, which are included in the parametrization of Eq. (5) only above the threshold parameter s_{th} . In the present study, we will try to construct such an operator by considering linear combinations of two independent operators and then fixing the mixing angles so that the results are consistent with the ansatz of Eq. (5).

While the low-energy part of the spectral function below the threshold parameter s_{th} is phenomenologically parametrized as in Eq. (5), the left-hand side of Eq. (3) and the second term of Eq. (5) are calculated analytically using the OPE. The results of this calculation can be generally expressed as follows:

$$\begin{aligned}\Pi_1^{\text{OPE}}(q^2) &= \sum_{j=0}^5 C_{2j}(q^2)^{5-j} \log(-q^2) + \sum_{j=1}^{\infty} \frac{C_{10+2j}}{(q^2)^j}, \\ \Pi_2^{\text{OPE}}(q^2) &= \sum_{j=0}^5 C_{2j+1}(q^2)^{5-j} \log(-q^2) + \sum_{j=1}^{\infty} \frac{C_{11+2j}}{(q^2)^j}.\end{aligned}\quad (6)$$

Here, C_i contain various quark and gluon condensates and numerical factors.

The next step in the calculation is to apply the Borel transformation, which is defined as

$$L_M[\Pi_i(q^2)] \equiv \lim_{\substack{-q^2, n \rightarrow \infty \\ -q^2/n = M^2}} \frac{(-q^2)^{n+1}}{n!} \left(\frac{d}{dq^2}\right)^n \Pi_i(q^2), \quad (7)$$

where M is the so-called Borel mass. There are several reasons for using this transformation: firstly, the high-energy continuum part of the spectral function and the higher-order terms in the OPE are suppressed by the factors e^{-s/M^2} and $1/n!$, respectively. This considerably improves the accuracy of the sum rules. Secondly, as already mentioned above, the Borel transformation removes the subtraction terms in Eq. (3) and therefore eliminates possible ambiguities originating from these terms.

Substituting Eq. (5) into the dispersion relation of Eq. (3), and applying the Borel transformation, the following expressions can be obtained:

$$(\lambda_1)^2 e^{-m_{\Theta^+}^2/M^2} = - \int_0^{s_{\text{th}}} ds e^{-s/M^2} \sum_{j=0}^5 C_{2j} s^{5-j} + \sum_{j=1}^{\infty} \frac{(-1)^j C_{10+2j}}{\Gamma(j)(M^2)^{j-1}} \equiv f_1(M, s_{\text{th}}), \quad (8)$$

$$(\lambda_2)^2 e^{-m_{\Theta^+}^2/M^2} = - \int_0^{s_{\text{th}}} ds e^{-s/M^2} \sum_{j=0}^5 C_{2j+1} s^{5-j} + \sum_{j=1}^{\infty} \frac{(-1)^j C_{11+2j}}{\Gamma(j)(M^2)^{j-1}} \equiv f_2(M, s_{\text{th}}). \quad (9)$$

From these equations, the expressions for m_{Θ^+} and $(\lambda_i)^2$ can be extracted straightforwardly:

$$m_{\Theta^+}^2(M, s_{\text{th}}) = \frac{1}{f_i(M, s_{\text{th}})} \frac{\partial f_i(M, s_{\text{th}})}{\partial (-1/M^2)}, \quad (10)$$

$$(\lambda_i)^2 = f_i(M, s_{\text{th}}) e^{m_{\Theta^+}^2(M, s_{\text{th}})/M^2}. \quad (11)$$

Notice that m_{Θ^+} can be calculated independently either from the chiral-even term $f_1(M, s_{\text{th}})$ or from the chiral-odd term $f_2(M, s_{\text{th}})$. In this study we will mainly use $f_1(M, s_{\text{th}})$ to calculate m_{Θ^+} and refer to $f_2(M, s_{\text{th}})$ only for determining the parity of the investigated state.

Here, the Borel mass M and the threshold parameter s_{th} are variable parameters, which allow us to obtain distinct sum rules in Eqs. (10) and (11) for each chosen value of M and s_{th} . Comparing these different sum rules, it is possible to extract information on the shape of the investigated spectral function and on the physical states that contribute to the sum rules most strongly. We will discuss this issue in detail in the later sections.

B. Parity projection

The parity of the presumed ground state pole can not be determined from the sum rule of the chiral-even (or chiral-odd) part alone, as $\eta(x)$ or $\eta_\mu(x)$ couple to both states with positive and negative parity regardless of their own intrinsic parity. To this end, we use the parity-projected sum rules [37] to obtain information on the parity of the investigated state. In this method, instead of Eqs. (1) or (2), the ‘‘old fashioned’’ Green function is considered in the rest frame ($\vec{q} = 0$):

$$\begin{aligned} \Pi^{\text{of}}(q_0) &= i \int d^4x e^{iqx} \langle 0 | \theta(x_0) \eta(x) \bar{\eta}(0) | 0 \rangle_{\vec{q}=0} \\ &\equiv \Pi_1^{\text{of}}(q_0) \gamma^0 + \Pi_2^{\text{of}}(q_0). \end{aligned} \quad (12)$$

Here, only the spin $\frac{1}{2}$ case is shown. This expression leads then to two independent sum rules for states coupling to $\eta(x)$ with positive and negative parity, respectively. These

are given as

$$|\lambda_{\pm}|^2 e^{-(m_{\Theta^+}^{\pm})^2/M^2} = \frac{1}{\pi} \int_0^{q_0^{\text{th}}} dq_0 [\text{Im} \Pi_1^{\text{of}}(q_0) \pm \text{Im} \Pi_2^{\text{of}}(q_0)] e^{-q_0^2/M^2}, \quad (13)$$

where the intrinsic parity of the operator has been assumed to be positive. In the opposite case the signs of the right-hand side have to be switched. q_0^{th} is the threshold parameter corresponding to $s_{\text{th}}^{1/2}$ in Eq. (5).

We now have three sum rules, Eqs. (8), (9), and (13), which must in principle give the same results for m_{Θ^+} . However, the OPE of the chiral-odd part Π_2 has turned out to contain ambiguous terms in the first power of the strange quark mass m_s , related to an infrared divergence originating in the perturbative treatment of m_s . To circumvent this problem, we will use only the sum rule of the chiral-even part of Eq. (8) (where the divergencies do not occur) to calculate the mass of the ground state. Meanwhile, Eq. (13) will be applied in the chiral limit ($\frac{\langle \bar{s}s \rangle}{\langle \bar{q}q \rangle} = 1$, $m_s = 0$) in order to determine the parity of the ground state.

C. Borel window

It is well known since the QCD sum rule method has been formulated, that the condition of an existing Borel window provides an essential check of the accuracy of the method. We define the Borel window as the region of the Borel mass where the following two conditions are satisfied:

$$\frac{L_M[\Pi_{\text{highest order terms}}^{\text{OPE}}(q^2)]}{L_M[\Pi_{\text{all terms}}^{\text{OPE}}(q^2)]} \leq 0.1, \quad (14)$$

$$\frac{\int_0^{s_{\text{th}}} ds e^{-(s/M^2)} \text{Im} \Pi^{\text{OPE}}(s)}{\int_0^{\infty} ds e^{-(s/M^2)} \text{Im} \Pi^{\text{OPE}}(s)} \geq 0.5. \quad (15)$$

Equation (14) is a necessary condition for the OPE to converge. It gives a lower limit for the Borel mass because the higher-order terms are suppressed for larger M as can be seen in Eqs. (8) and (9). On the other hand, Eq. (15) ensures that the low-energy part of the spectral function dominates the sum rules, and that contributions from high-energy states do not deteriorate the result. It is therefore necessary that the high-energy part of the spectral function is sufficiently suppressed. As the suppression is stronger for smaller M , this condition gives an upper bound for the Borel mass.

One may wonder what the rationale for the numbers on the right-hand side of Eqs. (14) and (15) is. These numbers are in fact chosen quite reasonably, which can be understood from the following considerations. The main uncertainties in a QCD sum rule calculation in most cases originate from ambiguities of the vacuum condensates which often have error bars considerably larger than 10%. This justifies the usage of Eq. (14) as choosing any

much smaller number than 0.1 on the right-hand side of this condition would be meaningless. Moreover, in order for the low-energy states below s_{th} to contribute most strongly to the sum rules and that thus an inappropriate parametrization of the high-energy states do not introduce too large errors, the right-hand side of (15) is also a natural choice, as is known from experience with sum rules of other baryons and mesons [36]. Therefore, one can have some confidence that the errors coming from the neglected higher-order terms of the OPE and the possible inaccurate description of the spectral function above s_{th} are under control if both conditions of Eqs. (14) and (15) are satisfied.

Let us now discuss the difficulties of establishing a Borel window in pentaquark studies. As has been discussed above and also in [25,27,28], the existence of a valid Borel window is essential for obtaining reliable results within the QCD sum rule technique. Nevertheless almost all earlier studies investigating pentaquark states with QCD sum rules [18–24,26] did not consider this problem and therefore these results should not be seen as to be conclusive.

The reason why all these studies have ignored this issue, is that, in fact, it is very difficult (if not impossible) to establish a valid Borel window in the conventional QCD sum rules described so far in this paper. There are basically two difficulties. Firstly, the convergence of the OPE of the correlator of a five-quark operator is slower compared to the case of nonexotic baryons containing only three quarks. This can be understood from a simple argument: graphs containing quark loops are multiplied by a factor of $(\frac{1}{2\pi})^{2n}$ (n : number of quark loops), which comes from the integration of the momenta in each loop, and the graphs of higher-order terms, where some of the quark loops are cut, are thus enhanced [38]. Therefore, one can expect that the OPE starts to converge only after all the quark loops are being cut. For the pentaquark calculation, this happens only for terms of dimension 12 (or higher) and one therefore needs to calculate the OPE at least up to dimension 12 to make sure that all the terms with a possible large contribution are included. This problem can in principle be solved if one calculates the OPE up to high enough orders (which is a tedious, but straightforward task, if one uses the vacuum saturation approximation).

The second difficulty is more severe: because of the high dimensionality of the interpolating field of the pentaquark (15/2 compared to 9/2 for an ordinary three-quark baryon), the high-energy part of the spectral function well above the presumed Θ^+ resonance is enhanced and therefore in many cases dominates the behavior of the whole spectral function. This makes it very difficult to obtain a large enough pole contribution in Eq. (15) for establishing a valid Borel window. It has thus been a very hard task to make a reliable prediction on the resonance $\Theta^+(1540)$.

As discussed in the next section, this problem can be solved by a modification of the standard QCD sum rules technique, which consists of using, instead of a single correlator, the difference of two independent correlators to construct the sum rules. This will be our strategy in this paper.

III. DETAILS OF THE METHOD

A. Interpolating fields

To carry out QCD sum rule calculations, one first has to construct appropriate operators, which carry the desired quantum numbers. These operators should be chosen to couple strongly to the state that one wants to investigate, although this is not always a trivial task. The interpolating fields that we use in the present study are described in this section. Our general strategy is to assemble two independent operators for each quantum number and set up general interpolating fields by considering linear combinations of them.

All the operators used in this study are built from two ud diquarks and an \bar{s} antiquark, so that their KN component on the operator level is as small as possible. We therefore hope that these operators only have a small overlap with the KN scattering states while they should couple strongly to the possible pentaquark resonance. For orientation, the properties of the employed ud diquarks are given in Table I.

1. The $IJ^P = 0\frac{1}{2}^\pm$ and $1\frac{1}{2}^\pm$ states

For the isosinglet case with spin $\frac{1}{2}$, we use the following two operators. The same ones were used in [27], where the $IJ^P = 0\frac{1}{2}^\pm$ states were investigated with a similar strategy as in this paper.

$$\eta_1^{I=0}(x) = \epsilon_{cfg} [\epsilon_{abc} u_a^T(x) C \gamma_5 d_b(x)] \times [\epsilon_{def} u_d^T(x) C \gamma_\mu \gamma_5 d_e(x)] \gamma^\mu \gamma_5 C \bar{s}_g^T(x), \quad (16)$$

$$\eta_2^{I=0}(x) = \epsilon_{cfg} [\epsilon_{abc} u_a^T(x) C d_b(x)] \times [\epsilon_{def} u_d^T(x) C \gamma_\mu \gamma_5 d_e(x)] \gamma^\mu C \bar{s}_g^T(x). \quad (17)$$

Here, a, b, \dots are color indices, C is the charge conjugation matrix and T stands for the transposition operation. These fields both have positive intrinsic parity. They are con-

TABLE I. The quantum numbers of the ud diquarks used in this study. a, b, \dots are color indices and $C = i\gamma^2\gamma^0$ stands for the charge conjugation matrix.

Diquarks	I	J	π
$\epsilon_{abc}(u_a^T C \gamma_5 d_b)$	0	0	+
$\epsilon_{abc}(u_a^T C d_b)$	0	0	-
$\epsilon_{abc}(u_a^T C \gamma_\mu \gamma_5 d_b)$	0	1	-
$\epsilon_{abc}(u_a^T C \gamma_\mu d_b)$	1	1	+

constructed from a scalar diquark, a vector diquark and an antistrange quark operator in the case of η_1 and from a pseudoscalar diquark, a vector diquark, and an antistrange quark operator in the case of η_2 . To project out the spin $\frac{1}{2}$ component, both operators have been multiplied by γ^μ and η_1 is furthermore multiplied by γ_5 to get the correct intrinsic parity. It must be remembered that even though the intrinsic parity of these operators is positive, it can couple to both states with positive and negative parity.

By introducing a mixing angle $\theta_{1/2}^0$, a general operator can be constructed from $\eta_1^{I=0}(x)$ and $\eta_2^{I=0}(x)$:

$$\eta^{I=0}(x) = \cos\theta_{1/2}^0 \eta_1^{I=0}(x) + \sin\theta_{1/2}^0 \eta_2^{I=0}(x). \quad (18)$$

This is the operator that will be used in the actual calculation. Here, we are in principle allowed to choose any value for the mixing angle. The strategy for determining this free parameter will be given in the following subsection.

For the isotriplet, we employ the following two operators, which have a form similar to the isosinglet case

$$\eta_1^{I=1}(x) = \epsilon_{cfg} [\epsilon_{abc} u_a^T(x) C \gamma_5 d_b(x)] \times [\epsilon_{def} u_d^T(x) C \gamma_\mu d_e(x)] \gamma^\mu C \bar{s}_g^T(x), \quad (19)$$

$$\eta_2^{I=1}(x) = \epsilon_{cfg} [\epsilon_{abc} u_a^T(x) C d_b(x)] \times [\epsilon_{def} u_d^T(x) C \gamma_\mu d_e(x)] \gamma^\mu \gamma_5 C \bar{s}_g^T(x). \quad (20)$$

The notation is the same as before. The difference to the fields with $I = 0$ is that instead of a vector diquark, we here have an axial-vector diquark, which carries isospin $I = 1$. Moreover, note that both the operators are multiplied by $(-\gamma_5)$ to obtain positive intrinsic parity for $\eta_1^{I=1}(x)$ and $\eta_2^{I=1}(x)$. Analogously to the isosinglet case, we construct a general operator by introducing a mixing angle $\theta_{1/2}^1$:

$$\eta^{I=1}(x) = \cos\theta_{1/2}^1 \eta_1^{I=1}(x) + \sin\theta_{1/2}^1 \eta_2^{I=1}(x). \quad (21)$$

2. The $IJ^P = 0_{\frac{3}{2}}^{\pm}$ and $1_{\frac{3}{2}}^{\pm}$ states

The construction of the operators with spin $\frac{3}{2}$ can be done in a similar fashion as for spin $\frac{1}{2}$. There are, however, some additional steps arising from the properties of the spin $\frac{3}{2}$ Rarita-Schwinger type fields. These are, for instance, discussed in [28,39], and we do not repeat the details here. We only state the result of how the spin $\frac{3}{2}$ components can be extracted. In the case of Rarita-Schwinger fields, the two-point function of Eq. (2) generally contains various different tensor structures, with contributions from states with spin $\frac{1}{2}$ and $\frac{3}{2}$. It can be shown that the terms proportional to $g_{\mu\nu}$ receive only contributions from the spin $\frac{3}{2}$ states. Therefore, if one considers only the two terms

$$\Pi_{\mu\nu}(q) = -g_{\mu\nu} [\Pi_1(q^2) \not{q} + \Pi_2(q^2)] + \dots, \quad (22)$$

the spin $\frac{1}{2}$ contributions will automatically be eliminated.

Note that there is a minus sign on the right-hand side of Eq. (22), which is a consequence of the properties of the Rarita-Schwinger field.

To study the isosinglet states we employ the following interpolating fields:

$$\eta_{1,\mu}^{I=0}(x) = \epsilon_{cfg} [\epsilon_{abc} u_a^T(x) C \gamma_5 d_b(x)] \times [\epsilon_{def} u_d^T(x) C \gamma_\mu \gamma_5 d_e(x)] C \bar{s}_g^T(x), \quad (23)$$

$$\eta_{2,\mu}^{I=0}(x) = \epsilon_{cfg} [\epsilon_{abc} u_a^T(x) C d_b(x)] \times [\epsilon_{def} u_d^T(x) C \gamma_\mu \gamma_5 d_e(x)] \gamma_5 C \bar{s}_g^T(x). \quad (24)$$

These operators have the same structure as the ones with spin $\frac{1}{2}$ [Eqs. (16) and (17)]. The only difference is that $\gamma^\mu \gamma_5$ in front of $C \bar{s}^T$ has been omitted here, which allows the operators to couple to spin $\frac{3}{2}$ states and lets the intrinsic parity become positive. As above, a general operator is then constructed by a linear combination of $\eta_{1,\mu}^{I=0}$ and $\eta_{2,\mu}^{I=0}$:

$$\eta_\mu^{I=0}(x) = \cos\theta_{3/2}^0 \eta_{1,\mu}^{I=0}(x) + \sin\theta_{3/2}^0 \eta_{2,\mu}^{I=0}(x). \quad (25)$$

This is the same kind of operator that has been used in our previous work [28]. In this paper we will merely restate the results that have been obtained there in order to compare them with the results from the other quantum numbers.

Finally, for the isotriplet case, we will use the operators given below:

$$\eta_{1,\mu}^{I=1}(x) = \epsilon_{cfg} [\epsilon_{abc} u_a^T(x) C \gamma_5 d_b(x)] \times [\epsilon_{def} u_d^T(x) C \gamma_\mu d_e(x)] \gamma_5 C \bar{s}_g^T(x), \quad (26)$$

$$\eta_{2,\mu}^{I=1}(x) = \epsilon_{cfg} [\epsilon_{abc} u_a^T(x) C d_b(x)] \times [\epsilon_{def} u_d^T(x) C \gamma_\mu d_e(x)] C \bar{s}_g^T(x). \quad (27)$$

The structure of these operators is almost the same as the ones with quantum numbers $IJ^P = 1_{\frac{1}{2}}^{\pm}$. Here again, compared with Eqs. (19) and (20) the matrices $\gamma^\mu \gamma_5$ have been omitted in order to construct Rarita-Schwinger fields which couple to spin $\frac{3}{2}$ states and to adjust the intrinsic parity to be positive. As in all the cases above, a general operator $\eta_\mu^{I=1}$ is constructed from $\eta_{1,\mu}^{I=1}$ and $\eta_{2,\mu}^{I=1}$, which will then be used to formulate the sum rules

$$\eta_\mu^{I=1}(x) = \cos\theta_{3/2}^1 \eta_{1,\mu}^{I=1}(x) + \sin\theta_{3/2}^1 \eta_{2,\mu}^{I=1}(x). \quad (28)$$

B. Determination of the Borel mass, threshold parameter, and mixing angle

The Borel mass M appears in the formulation of QCD sum rules when the Borel transformation is applied in Eq. (7), the threshold parameter s_{th} in the ‘‘pole + continuum’’ ansatz of Eq. (5), and the mixing angles θ_j^I in the general expressions for the interpolating fields in Eqs. (18), (21), (25), and (28). In this subsection,

our strategy of determining these parameters will be explained.

Let us first discuss the question of how the Borel mass M has to be determined. As mentioned in the last section, it first has to be checked whether one can establish a valid Borel window from the sum rules. If not, the sum rules will not work and it will not be possible to obtain any reliable results from them. If one is able to find a valid Borel window, M has to be chosen within its boundaries. As will be discussed below, when the sum rules “work well,” the dependence of the results on M should be small and therefore it will not strongly depend on the exact position on M inside of the Borel window.

Next, our strategy of determining the threshold parameter s_{th} will be explained. Assuming that the low-energy part of the spectral function is dominated by a narrow resonance pole, the values of the resonance mass [given in Eq. (10)] and the residue [Eq. (11)] should not strongly depend on M and s_{th} . This is easily understood when one considers the (ideal) case, when the spectral function is given by a single δ function below s_{th} . Rewriting the right-hand side of Eq. (10), we obtain

$$\frac{\frac{\partial}{\partial(-1/M^2)} \int_0^{s_{\text{th}}} ds e^{-s/M^2} \text{Im}\Pi(s)}{\int_0^{s_{\text{th}}} ds e^{-s/M^2} \text{Im}\Pi(s)} = \frac{\int_0^{s_{\text{th}}} ds e^{-s/M^2} s \text{Im}\Pi(s)}{\int_0^{s_{\text{th}}} ds e^{-s/M^2} \text{Im}\Pi(s)}. \quad (29)$$

If $\text{Im}\Pi(s)$ is a simple δ function specified as $\text{Im}\Pi(s) = \pi(\lambda)^2 \delta(s - m^2) + \theta(s - s_{\text{th}}) \text{Im}\Pi'(s)$, then Eq. (29) gives m^2 and does not depend on M and s_{th} . On the other hand, if $\text{Im}\Pi(s)$ is described by some continuous positive curve, which corresponds to the scattering states, Eq. (29) should be a rising curve, because of the weight factor e^{-s/M^2} , which suppresses the part of the integral with large s values when M is small. Furthermore, Eq. (29) should have an increasing value when s_{th} is raised, as higher values of s will be included in the integral.

Following the arguments above, it can be understood that the threshold parameter s_{th} has to be chosen so that the dependence of the calculated resonance mass and its residue is smallest, because this corresponds to the case of the largest contribution of a narrow ground state pole to the spectral function. On the other hand, if no such value for s_{th} can be found, we can assume that the spectral function is dominated by the scattering states.

We therefore set up the following two conditions, by which we determine s_{th} (called in the following the conditions of pole domination):

- (1) A sufficiently wide Borel window exists.
- (2) $m_{\Theta^+}(M, s_{\text{th}})$ should only weakly depend on the Borel mass M and on the threshold parameter s_{th} .

Condition 1) is essential to obtain reliable results with the QCD sum rule method, while 2) follows from the discussion above. The problem that arises here, is how we should quantitatively define the “weak dependence” of condi-

tion 2). In other words, how “weak” should the dependence on M and s_{th} be that one can be unambiguously sure not just to observe scattering states? This important problem will be discussed in the part of the result section, which deals with KN scattering states.

Finally, the mixing angle θ_J^I has to be fixed. To do this, we repeat the analysis outlined above for various values of θ_J^I and at the end choose the one for which the conditions 1) and 2) are best satisfied. This concludes our discussion about the determination of the different parameters that appear in the sum rules.

C. Establishment of a valid Borel window

As was pointed out in Sec. II C, it has so far been very difficult to establish a valid Borel window in QCD sum rule studies of pentaquarks. We aim to solve this problem by a modification of the standard QCD sum rules technique [27]. The idea is to use, instead of a single correlator, the difference of two independent correlators to construct the sum rules. By this trick, it is hoped that we will achieve a large cancellation of the high-energy part of the spectral function, due to the restored chiral symmetry in this region. We will then be able to obtain a large pole contribution, which (if the OPE is calculated up to a sufficiently high dimension) will make it possible to establish a valid Borel window.

To illustrate this point more concretely, let us consider the difference of two independent correlators that have been constructed in the second part of this section. We take as an example the operators with quantum numbers $IJ^P = 0\frac{1}{2}^{\pm}$. In the same way as it was shown in [28] for operators of spin $\frac{3}{2}$, the operators of Eqs. (16) and (17) can be expressed with the help of the operators ξ_1 and ξ_2 , which belong to different chiral multiplets:

$$\begin{aligned} \xi_1 &= -2(u_R^T C d_R)[(u_L^T C \gamma_\mu d_R) - (u_R^T C \gamma_\mu d_L)] \gamma^\mu C \bar{s}_R^T \\ &\quad - 2(u_L^T C d_L)[(u_L^T C \gamma_\mu d_R) - (u_R^T C \gamma_\mu d_L)] \gamma^\mu C \bar{s}_L^T, \\ \xi_2 &= 2(u_R^T C d_R)[(u_L^T C \gamma_\mu d_R) - (u_R^T C \gamma_\mu d_L)] \gamma^\mu C \bar{s}_L^T \\ &\quad + 2(u_L^T C d_L)[(u_L^T C \gamma_\mu d_R) - (u_R^T C \gamma_\mu d_L)] \gamma^\mu C \bar{s}_R^T. \end{aligned} \quad (30)$$

Here, the color indices are omitted for simplicity. ξ_1 belongs to the $(\mathbf{3}, \overline{\mathbf{15}}) \oplus (\overline{\mathbf{15}}, \mathbf{3})$ multiplet of the chiral $SU(3)_L \otimes SU(3)_R$ group with 4(1) right-handed and 1(4) left-handed quarks, and ξ_2 to the $(\mathbf{8}, \mathbf{8})$ multiplet with 3(2) right-handed and 2(3) left-handed quarks.

Using these chiral operators, $\eta_1^{I=0}$ and $\eta_2^{I=0}$ are given as

$$\eta_1^{I=0} = \frac{1}{2}(\xi_1 + \xi_2), \quad \eta_2^{I=0} = \frac{1}{2}(\xi_1 - \xi_2). \quad (31)$$

Thus, the correlator of the general operator of Eq. (18), denoted as $\Pi_{1/2}^0(q^2, \theta_{1/2}^0) \equiv \langle \eta^{I=0} \bar{\eta}^{I=0} \rangle$, can be expressed as follows:

$$\begin{aligned} \Pi_{1/2}^0(q^2, \theta_{1/2}^0) &= \frac{1}{4}[\langle \xi_1 \bar{\xi}_1 \rangle + \langle \xi_2 \bar{\xi}_2 \rangle] \\ &+ \frac{1}{4} \cos(2\theta_{1/2}^0)[\langle \xi_1 \bar{\xi}_2 \rangle + \langle \xi_2 \bar{\xi}_1 \rangle] \\ &+ \frac{1}{4} \sin(2\theta_{1/2}^0)[\langle \xi_1 \bar{\xi}_1 \rangle - \langle \xi_2 \bar{\xi}_2 \rangle]. \end{aligned} \quad (32)$$

The first term of this expression does not depend on the mixing angle $\theta_{1/2}^0$, but is expected to couple strongly to the high-energy continuum states because this term can have perturbative parts. On the other hand, compared with the first term, the coupling to the high-energy states of the other two terms is expected to be smaller, which can be understood from the following arguments. The perturbative part of the second of Eq. (32) term vanishes because ξ_1 and ξ_2 belong to different chiral multiplets and therefore at least one nonperturbative quark condensate related to chiral symmetry breaking is needed to connect them. As the perturbative term largely couples to the high-energy states, their contributions will be suppressed in this term. Considering the third term, it is possible to cancel the leading perturbative terms with an appropriate normalization of ξ_1 and ξ_2 . Note that we here have implicitly used the positivity condition of the spectral function. As is seen in Eq. (4), this assumption is not necessarily valid for the chiral-odd part, but we employ in this paper only the sum rule of the chiral-even part and the parity-projected sum rules, where the positivity condition holds.

Therefore, by taking the difference of two correlators with different mixing angles $\theta_{1/2}^0$ and $\theta_{1/2}^{\prime}$, the first term in Eq. (32) will be eliminated, and a strong suppression of the high-energy continuum part can be obtained. It will thus become possible to establish a valid Borel window. Writing this difference down, we get

$$\begin{aligned} &\Pi_{1/2}^0(q^2, \theta_{1/2}^0) - \Pi_{1/2}^0(q^2, \theta_{1/2}^{\prime}) \\ &= \frac{1}{2} \sin(\theta_{1/2}^0 - \theta_{1/2}^{\prime}) \{ \cos(\theta_{1/2}^0 + \theta_{1/2}^{\prime}) [\langle \xi_1 \bar{\xi}_1 \rangle - \langle \xi_2 \bar{\xi}_2 \rangle] \\ &\quad - \sin(\theta_{1/2}^0 + \theta_{1/2}^{\prime}) [\langle \xi_1 \bar{\xi}_2 \rangle + \langle \xi_2 \bar{\xi}_1 \rangle] \}, \end{aligned} \quad (33)$$

$$\rho(s = q^2) = \frac{1}{\pi} |\lambda_{KN}|^2 \text{Im} \left\{ (-i) \int \frac{d^4 p}{(2\pi)^4} \frac{1}{(q-p)^2 - m^2 + i\epsilon} \gamma_5 \frac{\not{p} + M}{p^2 - M^2 + i\epsilon} (-\gamma_5) \right\}. \quad (34)$$

Here, λ_{KN} is the coupling of the used interpolating field to the KN scattering states. m and M are the masses of the kaon and the nucleon, respectively. The γ_5 's are necessary because of the P-wave coupling of our interpolating field to the nucleon and kaon fields.

Going to the rest frame ($\vec{q} = 0$), the integral can be easily calculated and one gets

$$\rho(q_0^2) = \frac{1}{4\pi^2} |\lambda_{KN}|^2 \frac{\sqrt{E_N^2 - M^2}}{4q_0} (\gamma_0 E_N - M), \quad (35)$$

where E_N is the energy of the nucleon, expressed as

which will be used to formulate the sum rules. It is understood that the factor $\sin(\theta_{1/2}^0 - \theta_{1/2}^{\prime})$ has no influence on the mass of the ground state, calculated in Eq. (10). We hence fix it at $\theta_{1/2}^0 - \theta_{1/2}^{\prime} = \frac{\pi}{2}$ and will only keep $\theta_{1/2}^0 + \theta_{1/2}^{\prime} \equiv \phi_{1/2}^0$ as a free parameter, which will have to be determined by the conditions stated in the last subsection.

D. Possible contribution of KN scattering states

We have in this paper several times mentioned the possible influence of the KN scattering states to the sum rules. Generally, if such scattering states have the same quantum numbers as the interpolating fields, they may always contribute to the sum rules to a certain extent, so we have to find a way to distinguish them from narrow pole states that we are really interested in. We have already mentioned in the discussion of Eq. (29), that in the ideal case when only one narrow pole is present in the spectral function, the results of the sum rules should not depend on the Borel mass M and the threshold parameter s_{th} . When only scattering states contribute to the spectral function, this behavior should change. The nature and extent of this change will be illustrated in this section.

Let us first consider how the contribution of the KN scattering states to the spectral function should look like. It is known that the KN interaction in the $S = +1$ channel is weak and slightly repulsive for $I = 0$, while the repulsion is stronger for $I = 1$ [40,41]. As an illustration, we will here use phase space as a first approximation of the KN spectral function, which thus corresponds more closely to the $I = 0$ case. Nevertheless, the qualitative behavior of the results of this section does not strongly depend on the detailed form of the spectral function and can therefore be considered to be quite general.

In the case of spin $\frac{1}{2}$ states, the contribution of KN phase space to the spectral function can be expressed as follows:

$$E_N = \frac{q_0^2 + M^2 - m^2}{2q_0}. \quad (36)$$

Therefore, the spectral function for the chiral-even part has the form

$$\frac{1}{\pi} \text{Im} \Pi_1^{KN}(q_0^2) = \frac{1}{4\pi^2} |\lambda_{KN}|^2 \frac{E_N \sqrt{E_N^2 - M^2}}{4q_0^2}, \quad (37)$$

which contains contributions from both positive and negative parity states.

For spin $\frac{3}{2}$ states, similar considerations can be applied, although there are some complications coming from pro-

jecting out the contributions of the spin $J = \frac{3}{2}$ states from the correlator. In this case, the spectral function is expressed as

$$\rho(s = q^2) = \frac{1}{\pi} \frac{|\lambda'_{KN}|^2}{m^2} \text{Im} \left\{ (-i) \int \frac{d^4 p}{(2\pi)^4} \frac{(q-p)_\mu (q-p)_\nu}{(q-p)^2 - m^2 + i\epsilon} \frac{\not{p} + M}{p^2 - M^2 + i\epsilon} \right\} \Big|_{J=3/2}, \quad (38)$$

and the $J = \frac{3}{2}$ projection is most easily done by applying the projection operator

$$P_{\mu\nu}(q) = g_{\mu\nu} - \frac{2q_\mu q_\nu}{3q^2} - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{1}{3q^2} (q_\mu \gamma_\nu - q_\nu \gamma_\mu) \not{q}. \quad (39)$$

Then, only considering the terms proportional to $g_{\mu\nu}$ and again going to the rest frame, one obtains

$$\rho(q_0^2) = \frac{1}{12\pi^2 m^2} |\lambda'_{KN}|^2 \frac{(E_N^2 - M^2)^{3/2}}{4q_0} g_{\mu\nu} (\gamma_0 E_N + M) + \dots, \quad (40)$$

from which finally the spectral function of the chiral-even part can be extracted:

$$\frac{1}{\pi} \text{Im} \Pi_1^{KN}(q_0^2) = \frac{1}{12\pi^2 m^2} |\lambda'_{KN}|^2 \frac{E_N (E_N^2 - M^2)^{3/2}}{4q_0^2}. \quad (41)$$

This expression again contains contributions from both positive and negative parity.

Next, we compute the results that would be obtained by the QCD sum rules if only the KN scattering states contribute to the spectral function.

This means that we calculate the quantity corresponding to Eq. (10) or Eq. (29), where for $\Pi(s)$, we now use the expressions obtained above. The results for spin $\frac{1}{2}$ and spin $\frac{3}{2}$ are given in Fig. 1.

It is clearly seen that while the dependence on the Borel mass M is relatively weak, the results depend strongly on the threshold parameter s_{th} . This can intuitively be understood from the fact that the spectral function containing only the phase space contribution is a fastly growing function with increasing energy. Therefore, the high-energy regions below the threshold parameter will dominate the integral of Eq. (29), which then leads to a behavior as seen in Fig. 1, with a strong dependence on s_{th} . Furthermore, this dominance of the high-energy states will make it difficult to obtain a large value for the pole contribution and to establish a valid Borel window.

Note that we have here assumed that λ_{KN} and λ'_{KN} to be constants with dimension $[\text{GeV}^5]$. They could in principle also have a dependence on q_0 such as $\text{const.} \times q_0^5$, which would result in Eqs. (37) and (41) being multiplied by q_0^{10} . We have checked this case, finding that the results are altered quantitatively, namely, that the mass is shifted upwards while the difference between the different threshold curves increases. Nevertheless, our quantitative discussion above does not have to be changed, as our observation of a large dependence of the results on s_{th} is even more apparent in this case.

The results of this section show that the dependence of $m_{\Theta^+}^2(M, s_{\text{th}})$ on s_{th} provides us with an indicator of how

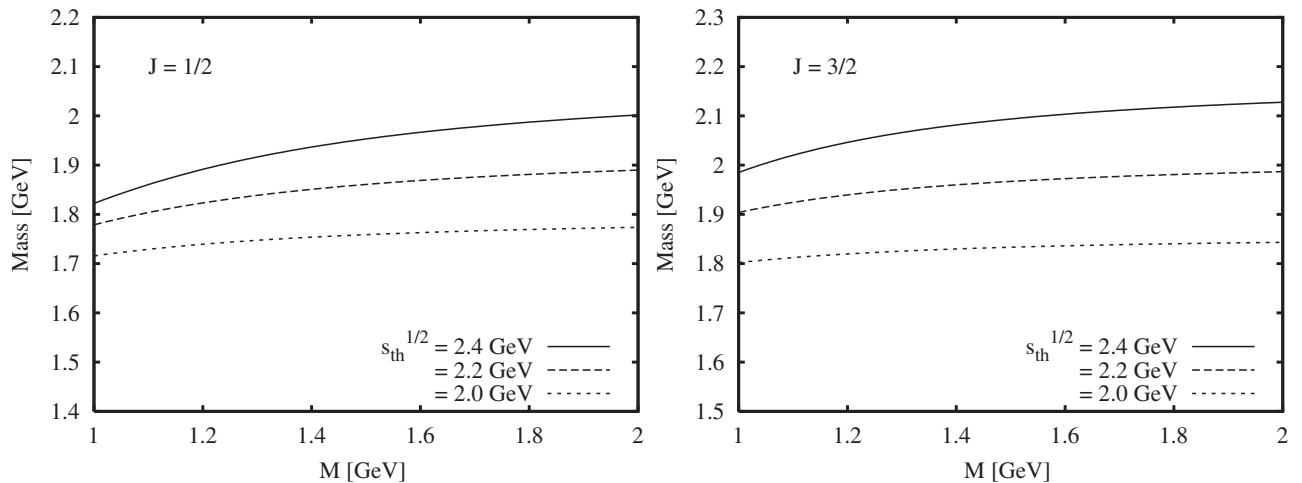


FIG. 1. The “mass of the ground state pole” of spin $\frac{1}{2}$ (left) and $\frac{3}{2}$ (right), obtained when only the KN scattering states contribute to the spectral function. Equation (37) and (41) have been used as the expression of the spectral function of the chiral-even part.

much the KN scattering states contribute to the sum rule: a linear dependence of the same (or larger) extent as in Fig. 1 suggests a strong contribution of the scattering states, while a significantly smaller dependence indicates that a narrow pole exists and is the dominant structure in the spectral function. Similar arguments have already been discussed earlier in [42].

IV. RESULTS

A. Preliminaries

We summarize in this subsection general issues common to the sum rules of all the various quantum numbers, and explain the parameters, conventions and basic approximations used in the calculation.

One important feature of the results for all quantum numbers is, that the perturbative term C_0 vanishes when the difference of the two correlators is taken. This corresponds to the suppression of the contribution of the high-energy states as was discussed in the last section. The detailed results of the OPE of both the chiral-even and chiral-odd part are given in Appendix A.

We will for all quantum numbers first investigate the sum rule for the chiral-even part and after that consider the parity-projected sum rules, where both the results of the chiral-even and chiral-odd parts have to be used. However, as mentioned before, the results of the OPE calculation of the chiral-odd part have turned out to contain some ambiguous terms in the first power of the strange quark mass m_s , related to an infrared divergence. It is important to note here that this kind of divergence is artificially arising because we are expanding our results in m_s and are ignoring higher-order terms. It should thus in principle be possible to remove this divergence by taking the full dependence on m_s into account without resorting to any expansion, although this may be difficult to achieve in practice. In any case, to avoid this kind of artificial ambiguity, the results of the parity-projected sum rules will be given only in the chiral limit ($m_s = 0$, $\langle \bar{s}s \rangle = \langle \bar{q}q \rangle$). Hence, our strategy will be to use the results of the chiral-even part to calculate the mass of the investigated state, while we will employ the parity-projected sum rules to determine only the parity of the state.

The values of the mixing angles and threshold parameters are obtained using the conditions of pole domination of Sec. III B. We will use the same values for both the sum rule of the chiral-even part and the parity-projected sum rules.

The values of the condensates and other used parameters are given in Table II. These are standard values for QCD sum rule calculations [36,43], but they of course all have a certain range and the results will therefore depend on what exact values have been chosen for the condensates and other parameters. In the last part of this result section we will show to what extent the results will be influenced by the uncertainties of these parameters.

TABLE II. Values of all the parameters used throughout this paper, given at a scale of 1 GeV [36,43]. The parameter κ describes the possible breaking of the vacuum saturation approximation and is explained in this subsection.

$\langle \bar{q}q \rangle$	$-(0.23 \pm 0.02 \text{ GeV})^3$
$\frac{\langle \bar{s}s \rangle}{\langle \bar{q}q \rangle}$	0.8 ± 0.2
$\frac{\langle \bar{q}g\sigma \cdot Gq \rangle}{\langle \bar{q}q \rangle}$	$0.8 \pm 0.1 \text{ GeV}^2$
$\frac{\langle \bar{s}g\sigma \cdot Gs \rangle}{\langle \bar{s}s \rangle}$	$0.8 \pm 0.1 \text{ GeV}^2$
$\langle \frac{\alpha_s}{\pi} G^2 \rangle$	$0.012 \pm 0.004 \text{ GeV}^4$
m_s	$0.12 \pm 0.06 \text{ GeV}$
κ	$1 \sim 2$

Finally, κ , the last parameter of Table II, will now be explained. It parametrizes the possible violation of the vacuum saturation approximation and is used as follows:

$$\begin{aligned}
 \langle \bar{q}q\bar{q}q \rangle &= \kappa \langle \bar{q}q \rangle^2, \\
 \langle \bar{q}q\bar{q}q\bar{q}q \rangle &= \kappa^2 \langle \bar{q}q \rangle^3, \\
 \langle \bar{q}q\bar{q}g\sigma \cdot Gq \rangle &= \kappa \langle \bar{q}q \rangle \langle \bar{q}g\sigma \cdot Gq \rangle, \\
 \langle \bar{q}g\sigma \cdot Gq\bar{q}g\sigma \cdot Gq \rangle &= \kappa \langle \bar{q}g\sigma \cdot Gq \rangle^2, \\
 &\dots
 \end{aligned} \tag{42}$$

All the results shown below are obtained with $\kappa = 1$, which means that the vacuum saturation approximation has been assumed. This approximation has been shown to be valid in the leading order of the large N_c expansion, even though the $\frac{1}{N_c}$ corrections may be quite large. We have checked to what extent the results change when this approximation is broken up to values of $\kappa = 2$. These changes will be included in the estimation of the error.

The results of the various sum rules are given in the following. To allow a direct comparison between the different quantum numbers, all the plots corresponding to the same quantity are shown in the same figure.

B. $IJ^\pi = 0\frac{1}{2}^\pm$

We first show our obtained results for the isosinglet, spin $\frac{1}{2}$ case. This quantum number has been already frequently investigated as a possible assignment for $\Theta^+(1540)$ in QCD sum rules [18–23,26,27] and lattice QCD [29–32]. As for QCD sum rules, most of these calculations have problems in the establishment of the Borel window. In [27], this problem was avoided by taking the similar approach with that of the present paper, but we found some mistakes in the computation of the OPE beyond dimension 8. After correcting them, we got the present result which excludes the positive parity state obtained in [27].

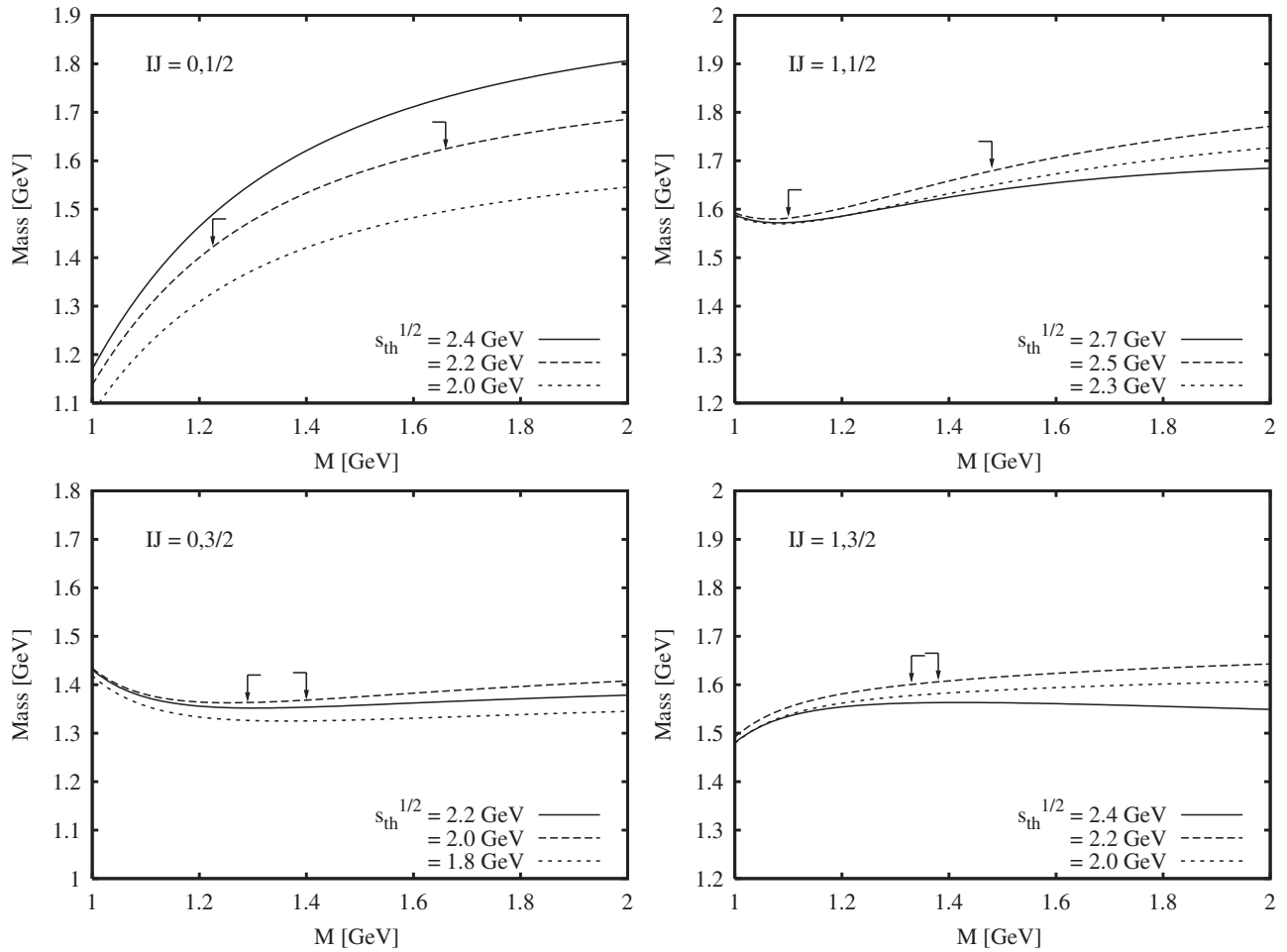


FIG. 2. The mass of the pentaquark with quantum numbers $IJ = 0\frac{1}{2}, 1\frac{1}{2}, 0\frac{3}{2}, 1\frac{3}{2}$ as a function of the Borel mass M , obtained from the sum rule of the chiral-even part. The arrows indicate the boundary of the Borel window for the middle value of the threshold parameter s_{th} .

1. Sum rule for the chiral-even part

Using the operators of Eqs. (16) and (17), we first determine the mixing angle $\phi_{1/2}^0$ and the threshold parameter s_{th} . The values that we have obtained are, $\phi_{1/2}^0 = -0.22$ and $\sqrt{s_{\text{th}}} = 2.2$ GeV. Furthermore, checking the convergence of the OPE and investigating the value of the pole contribution, we have confirmed that a Borel window exists for $1.2 \text{ GeV} \lesssim M \lesssim 1.6 \text{ GeV}$ (for details, consult Figs. of Appendix B).

The calculated value of the ground state mass $m_{\Theta^+}(M, s_{\text{th}})$ of Eq. (10) is given in Fig. 2 (top left) as a function of the Borel mass M . The boundary of the Borel window for the case of $\sqrt{s_{\text{th}}} = 2.2$ GeV are indicated by the two arrows. One can see that the obtained value is about 1.5 GeV within the Borel window. Even though we have found a wide Borel window, the curves shown in Fig. 2 exhibit quite a large dependence on M and s_{th} , which suggests that the spectral function only contains KN scat-

tering states and not a narrow pole. On the other hand, as will be shown later, the result of the parity-projected sum rules are fairly stable against M and s_{th} , which rather points to a narrow pole in the ground state. The interpretation these different results will be discussed below.

The result of the residue, calculated from Eq. (11) is given in Fig. 3. As well as for the mass, the results for the residue depend on M and s_{th} quite strongly.

2. Parity-projected sum rules

As already mentioned before, we will use the parity-projected sum rules in the chiral limit. This is justified, as we have confirmed in the sum rules of the chiral-even part that the qualitative behavior of the results does not change when this limit is taken. To show the strength of the contribution of the positive and negative parity states in the spectral function of the sum rule, the residues of the parity-projected sum rules [Eq. (13)] are given in Fig. 4. It

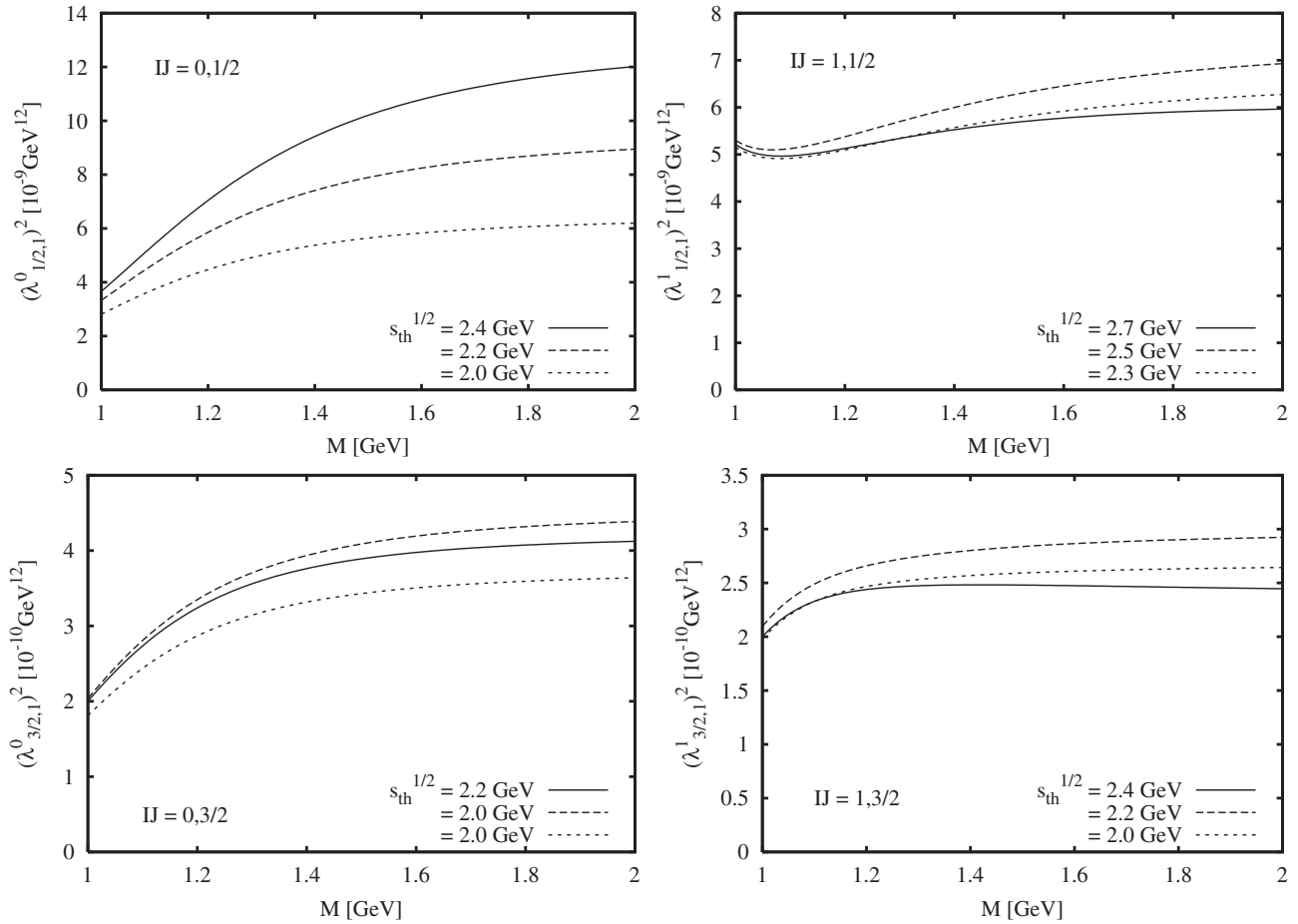


FIG. 3. The residue $(\lambda_{J,1}^I)^2$ for the quantum numbers $IJ = 0\frac{1}{2}, 1\frac{1}{2}, 0\frac{3}{2}, 1\frac{3}{2}$, obtained from Eq. (11) for the sum of the chiral-even part. The value is given for three different threshold parameters.

is clear from this figure that the negative parity states dominate and that therefore negative parity has to be assigned the state investigated in the last section. Furthermore, the mass calculated from the negative parity sum rule of Eq. (13) is shown in Fig. 5, together with the Borel window for the threshold parameter $\sqrt{s_{\text{th}}} = 2.2$ GeV. As is seen in the figure, a valid Borel window is established around $1.2 \text{ GeV} \lesssim M \lesssim 1.3 \text{ GeV}$ and the obtained value is consistent with the one of the chiral-even sum rule. Moreover, the dependencies on both M and s_{th} are very small, which in contrast to the chiral-even case rather points to a narrow ground state pole and not to KN scattering states.

It is puzzling why the behavior of these two sum rules is so different, even though the contribution of the positive parity states is very small, as shown in Fig. 4. Numerically, this can be understood from the fact that the chiral-even part is multiplied by an additional power of q_0 in the parity-projected sum rules [compare Eqs. (2) and (12)], which considerably changes the behavior of the sum rules in this case. Moreover, we have confirmed that even though the residue for positive parity state is small, it numerically has a large influence on Eq. (8) for the low Borel mass region.

To illustrate this point, the contribution of positive and negative parts, calculated from Eq. (13) in the chiral limit, are shown in Fig. 6. The negative parity part clearly shows an unphysical behavior as it is almost constant, while it should be an exponentially increasing function in the case of a narrow ground state pole dominating the sum rules. Nevertheless, around 1.2 GeV, its contribution is comparable to positive parity part and therefore has a strong influence on the result of the chiral-even part. Thus, the most reasonable explanation for these different results seems to be that the positive parity KN scattering states are contaminating the results of the chiral-even part and therefore lead to a large dependence on M and especially on s_{th} . We hence conclude that we have found some real evidence for a narrow ground state pole with $IJ^\pi = 0\frac{1}{2}^-$ —even though the situation is more ambiguous than in the other channels.

C. $IJ^\pi = 1\frac{1}{2}^\pm$

Next, the isotriplet, spin $\frac{1}{2}$ states are studied. As no isospin partners of $\Theta^+(1540)$ have so far been found experimentally, it is currently believed to be an isosinglet

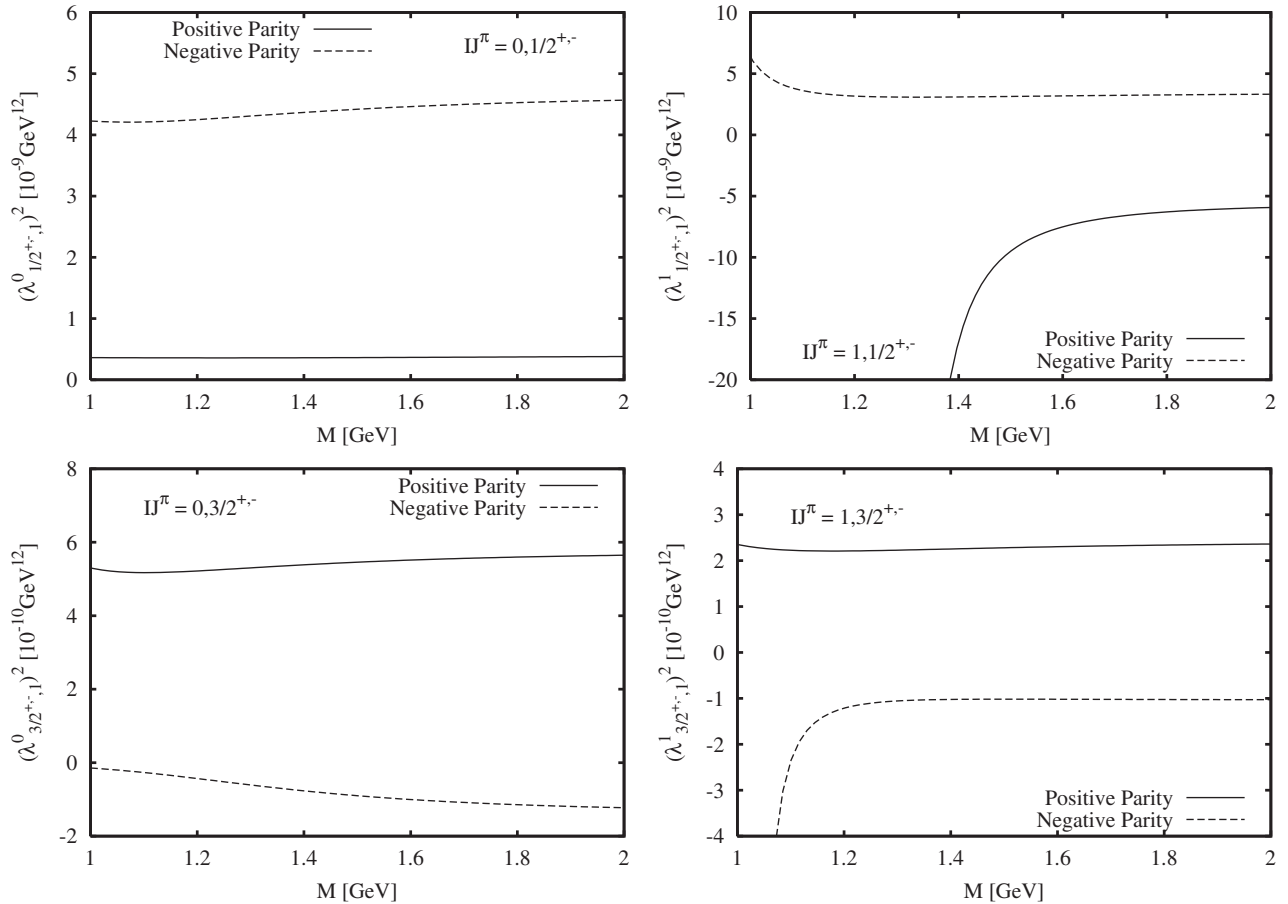


FIG. 4. The residues of the positive and negative parity sum rules for the quantum numbers $IJ = 0\frac{1}{2}, 1\frac{1}{2}, 0\frac{3}{2}, 1\frac{3}{2}$, obtained from Eq. (13). The curves are calculated in the chiral limit ($m_s = 0$, $\langle \bar{s}s \rangle = \langle \bar{q}q \rangle$).

state, but this assignment is not conclusive yet. Furthermore, even if $\Theta^+(1540)$ is an isosinglet state, a different isotriplet pentaquark state could exist at higher energies. We thus consider this state in the following paragraphs.

1. Sum rule for the chiral-even part

The method is essentially parallel to the isosinglet case, the difference being only that we employ the operators of Eqs. (19) and (20) instead of Eqs. (16) and (17). The values of the mixing angle $\phi_{1/2}^1$ and the threshold parameter s_{th} have turned out to be $\phi_{1/2}^1 = -0.079$ and $\sqrt{s_{\text{th}}} = 2.5$ GeV.

The mass calculated from Eq. (10) is shown in Fig. 2 (top right), as before with the Borel window for the middle value of the threshold parameter, indicated by the two arrows. The obtained value is about 1.6 GeV. Compared to the isosinglet spin $\frac{1}{2}$ case of Fig. 2, it is obvious that the dependence on the Borel mass M and especially on the threshold parameter s_{th} is small, which is positive evidence for a narrow ground state pole in the spectral function. The

residues for the three different threshold parameters are given in Fig. 3, where we again get only a similarly mild dependence on M and s_{th} .

2. Parity-projected sum rules

We will follow the same method as in the isosinglet case and calculate the parity-projected sum rules in the chiral limit. As is shown in Table III, the result of the chiral-even sum rule did depend on the strange quark mass m_s quite strongly and one thus may wonder whether the procedure of taking the chiral limit is justified. But, as we will use this sum rule only to determine the parity of the state, we think that it is accurate enough to provide reliable information, because even though the mass value of the state may quantitatively change, it is improbable that the parity of the state will switch when this limit is taken.

The residues of the positive and negative parity sum rules are compared in Fig. 4. In this figure, it is seen that both residues are similar in magnitude. (Note that, as we have taken the difference of two correlators, the residue $(\lambda_{1/2,1}^1)^2$ can become negative. States with negative resi-

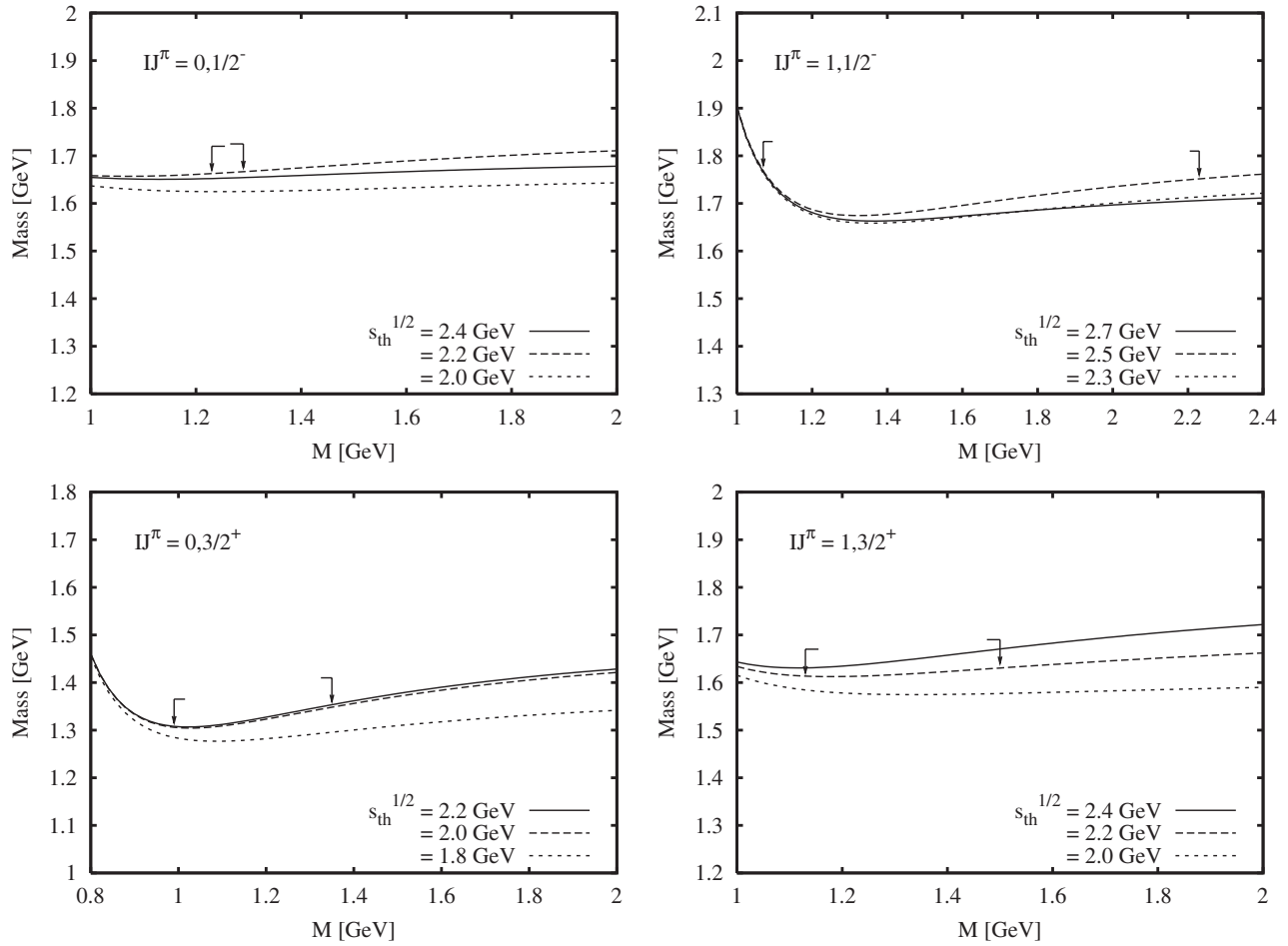


FIG. 5. The mass of the pentaquark for $IJ^\pi = 0\frac{1}{2}^-$, $IJ^\pi = 1\frac{1}{2}^-$, $IJ^\pi = 0\frac{3}{2}^+$, $IJ^\pi = 1\frac{3}{2}^+$ as a function of the Borel mass M . The arrows indicate the boundary of the Borel window for the middle value of the threshold parameter s_{th} . The curves are calculated in the chiral limit.

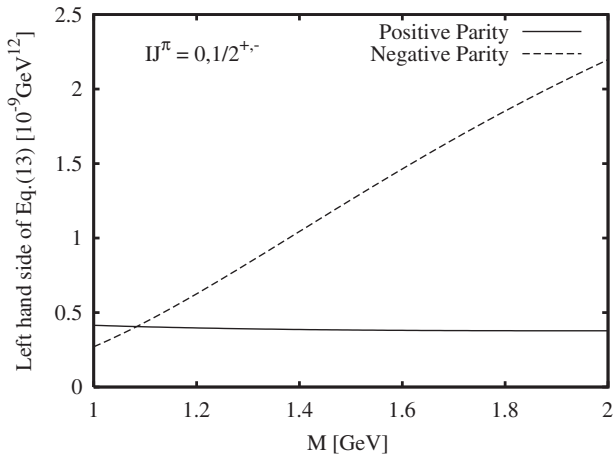


FIG. 6. The contribution of positive and negative parts of Eq. (8) for $IJ = 0\frac{1}{2}^+$, obtained from Eq. (13) in the chiral limit. The value of the threshold parameter is $\sqrt{s_{th}} = 2.2$ GeV.

dues can thus not be ruled out as unphysical like in the ordinary QCD sum rules with just one correlator.) On the other hand, the residue of the positive parity state is very unstable against the variation of the Borel mass, which suggests that it does not correspond to a narrow ground state pole that we are looking for. Meanwhile, the residue with negative parity is fairly stable and thus seems to be consistent with the chiral-even sum rule.

The masses of both parity states are also calculated. As expected from the result of the residues, the calculated mass of positive parity strongly depends on the Borel mass M and no stable region is found. In contrast, the results for negative parity are stable and consistent with the value obtained from the chiral-even sum rule. We therefore conclude that the parity of the state is negative. The mass values for the negative parity case are shown in Fig. 5.

Compared to all other cases studied in this paper, the Borel window here seems to be unnaturally large. The

TABLE III. Contributions of the uncertainties of all parameters appearing in the calculation to the final error. Only values larger than ± 0.05 GeV are explicitly given. These values have been obtained using the sum rule of the chiral-even part.

$IJ^P =$	$0\frac{1}{2}^\pm$	$1\frac{1}{2}^\pm$	$0\frac{3}{2}^\pm$	$1\frac{3}{2}^\pm$
M	± 0.10 GeV	± 0.05 GeV	~ 0	~ 0
s_{th}	± 0.10 GeV	~ 0	± 0.05 GeV	± 0.05 GeV
$\langle \bar{q}q \rangle$	± 0.15 GeV	± 0.20 GeV	~ 0	± 0.10 GeV
$\frac{\langle \bar{s}s \rangle}{\langle \bar{q}q \rangle}$	~ 0	± 0.05 GeV	~ 0	~ 0
$\frac{\langle \bar{q}g\sigma\cdot Gq \rangle}{\langle \bar{q}q \rangle}$	± 0.05 GeV	~ 0	± 0.10 GeV	± 0.05 GeV
$\frac{\langle \bar{s}g\sigma\cdot Gs \rangle}{\langle \bar{s}s \rangle}$	~ 0	~ 0	~ 0	~ 0
$\langle \frac{\alpha_s}{\pi} G^2 \rangle$	± 0.05 GeV	± 0.20 GeV	~ 0	~ 0
m_s	± 0.05 GeV	± 0.20 GeV	~ 0	± 0.05 GeV
κ	$+0.15$ GeV	$+0.20$ GeV	$+0.05$ GeV	$+0.10$ GeV
Combined error	± 0.3 GeV	± 0.4 GeV	± 0.2 GeV	± 0.3 GeV

reason for this is that the same phenomenon as in the upper left part of Fig. 9 has occurred, meaning that due to some cancellation in the integral of the spectral function above s_{th} , a peak has emerged in the function of the pole contribution, which shifts the upper boundary of the Borel window to a high value and therefore leads to this very large Borel window.

D. $IJ^\pi = 0\frac{3}{2}^\pm$

This quantum number has been already investigated in detail by the present authors in a recent paper [28]. We will not repeat the analysis given there and only restate the most important results.

The same strategy as in this paper was followed, meaning that the difference of two correlators was taken, and the values of the mixing angle and threshold parameter were determined from the conditions of pole dominance. The obtained values are $\phi_{3/2}^0 = 0.063$ and $\sqrt{s_{\text{th}}} = 2.0$ GeV. This then leads to the mass values shown in Fig. 2 (bottom left), calculated from the chiral-even sum rule. The obtained value lies at about 1.4 GeV. The result shows both a small dependence on M and s_{th} , which suggests that a narrow ground state pole exists in the spectral function of this quantum number.

The parity of the state is determined with the parity-projected sum rules, leading to Figs. 4 and 5. Figure 4 shows that the pole strength is dominated by the residue of the positive parity state. Figure 5 then confirms that the positive parity sum rules give stable results, which are consistent with the ones obtained from the chiral-even sum rule.

E. $IJ^\pi = 1\frac{3}{2}^\pm$

The existence of states with quantum numbers $IJ^P = 1\frac{3}{2}^\pm$ have been suggested for instance by studies using the quark model [44] and the chiral unitary approach [45]. We

further investigate them here using the QCD sum rule method.

1. Sum rule for the chiral-even part

The operators used are given in Eqs. (26) and (27) and the following values have been obtained for the mixing angle and the threshold parameter: $\phi_{3/2}^1 = 0.024$ and $\sqrt{s_{\text{th}}} = 2.2$ GeV.

The results for the mass are shown in Fig. 2 (bottom right) together with the Borel window for $\sqrt{s_{\text{th}}} = 2.2$ GeV. As can be read off from the figure, a value around 1.6 GeV is obtained for the mass of the state. The dependence of the result on both M and s_{th} is weak, which suggests that a narrow pole is present in the spectrum.

The value of the residue $(\lambda_{3/2,1}^1)^2$ is given in Fig. 3, where again only a small dependence on M and s_{th} is observed.

2. Parity-projected sum rules

We have obtained a consistent result for the positive parity channel, while no state below 2.0 GeV was found with negative parity. The two residues are shown in Fig. 4, where one can see that the magnitude of the positive parity residue is larger than the one of negative parity and that it is an almost completely stable against the variation of M . The calculated mass of the positive parity sum rule, shown in Fig. 5, moreover gives similar values as obtained the chiral-even case, which do not strongly depend on M and s_{th} . We therefore conclude that positive parity has to be assigned to the investigated state.

F. Estimation of the theoretical ambiguity

As the last point, we have to investigate the dependencies of the results on the various parameters of Table II, in order to obtain a quantitative estimate of the error inherent

in our results. We will here use only the results of the chiral-even part for this estimation.

The contributions to the errors for the different quantum numbers are given in Table III. For instance, considering the $IJ^P = 0\frac{1}{2}^{\pm}$ case, we have already seen from Fig. 2, that the dependence of the mass value on M or s_{th} leads to an uncertainty of about ± 0.1 GeV. Among the other parameters, the result depends most strongly on $\langle \bar{q}q \rangle$, which gives an uncertainty of about ± 0.15 GeV. Similarly, raising the breaking parameter of the vacuum saturation approximation to $\kappa = 2$ leads to an increase of the mass of about 0.15 GeV. Similar considerations lead to all the error contributions for the other quantum numbers given in Table III.

Assuming that the various errors are uncorrelated, the final error estimations are then obtained by taking the root of the sum of all squared errors δm_i and rounding up:

$$\text{combined error} \sim \sqrt{\sum_i (\delta m_i)^2}. \quad (43)$$

Note that this is merely a rough estimation, as there are additional errors coming from the truncation of the OPE and possible radiative corrections, that have been neglected in the current calculation.

V. DISCUSSION

The details of the results for the various quantum numbers have been presented in the last section. Putting everything together, these results can be summarized as in Table IV.

A number of comments have to be made here. First of all, the statement ‘‘no state found below 2.0 GeV’’ in Table IV means that either no valid Borel window could be found or that the results of the sum rules did strongly depend on M and s_{th} and that therefore no evidence for a

narrow ground state pole could be found. Concerning this point, in the case of $IJ^\pi = 0\frac{1}{2}^-$, the results of the chiral-even sum rule and the parity-projected sum rule are to a certain extent contradictory and we therefore have to put a question mark behind this conclusion. Furthermore, having found no narrow state in our sum rule calculation does not necessarily mean that such a state does not exist. It could happen that the spectral function is dominated by the KN scattering states and that the narrow states that we are looking for only couple weakly to the interpolating field that we have used. Nevertheless, we have constructed general operators from linear combinations of two independent local operators, and have investigated all possible mixing angles and therefore the nonobservation of states shown in Table IV has been confirmed for quite a wide range of independent operators.

Next, we discuss the physical implications of the obtained results. A question that comes to one’s mind when looking at Table IV is, why we can not observe $J^\pi = \frac{1}{2}^+$ states while we are seeing the ones with $J^\pi = \frac{3}{2}^+$. These states are in some models considered to be spin-orbit partners [10], so if these models are realistic and consistent with QCD, we should be able to observe both of these states. There are at least two possible explanations for our obtained results. One explanation could be that the states with $J^\pi = \frac{1}{2}^+$ in fact exist, but their coupling to the used operators are too small and/or the KN scattering contribution is too large, so that a narrow peak structure cannot be extracted. Another possible interpretation of the missing $J^\pi = \frac{1}{2}^+$ states could be that, the spin-orbit partners of the spin $\frac{3}{2}$ states are not the ones with spin $\frac{1}{2}$ but with spin $\frac{5}{2}$. This would mean that $\Theta^+(1540)$ is indeed a very exotic state, as in this case the $uudd$ quarks have to form a spin 2 state, which would then couple to the remaining \bar{s} . This is of course only a very speculative picture, but it would be interesting to test it by calculating pentaquark states with spin $\frac{5}{2}$.

TABLE IV. Summarized results for all quantum numbers that have been investigated. The allowed KN decay channels of the respective quantum numbers are indicated in brackets. The mass values quoted here are obtained for the sum rule of the chiral-even part.

	Parity		
		+	-
$J = \frac{1}{2}$	$I = 0$	No state found below 2.0 GeV (KN P-wave)	1.5 ± 0.3 GeV (?) (KN S-wave)
	$I = 1$	No state found below 2.0 GeV (KN P-wave)	1.6 ± 0.4 GeV (KN S-wave)
$J = \frac{3}{2}$	$I = 0$	1.4 ± 0.2 GeV (KN P-wave)	No state found below 2.0 GeV (KN D-wave)
	$I = 1$	1.6 ± 0.3 GeV (KN P-wave)	No state found below 2.0 GeV (KN D-wave)

Another important point, that needs to be discussed, is the interpretation of our results on the $J^\pi = \frac{1}{2}^-$ states. Such a state was also found in a lattice study (conducted only for the isosinglet state), where a resonance state was isolated from the KN scattering states [31]. Our results (especially in the isosinglet case) are somewhat ambiguous, and the errors are large, so it is difficult to draw any definite conclusions. In any case, whether such states turn out to be real pentaquark resonances or not, they most possibly do not correspond to the observed $\Theta^+(1540)$ state, because $J^\pi = \frac{1}{2}^-$ states can decay into KN by an S-wave, for which the width is expected to be much larger than the observed value for Θ^+ , which is less than 1 MeV [8]. Of course, in principle there may exist some so far unknown mechanism, which suppresses the width strongly and which would allow to assign the $J^\pi = \frac{1}{2}^-$ quantum numbers to the Θ^+ , but with our present knowledge and experience, this seems to be unlikely.

VI. CONCLUSION

We conclude from our results summarized in Table IV that the most probable quantum number candidate for $\Theta^+(1540)$ is $IJ^\pi = 0\frac{3}{2}^+$. We have also found evidence for an isotriplet state $1\frac{3}{2}^+$ and two states with spin $\frac{1}{2}$ ($0\frac{1}{2}^-$ and $1\frac{1}{2}^-$) at slightly higher energy.

To obtain these results, we have employed the QCD sum rule method, whose reliability is improved by analyzing the difference of two independent correlators, by which the contribution of the high-energy continuum states is suppressed. Furthermore, by calculating the OPE up to dimension 14 it is made sure that the expansion is converging well, and a valid Borel window can be established.

Considering the spin $\frac{1}{2}$ states, although we could observe some evidence for resonance states with $IJ^\pi = 0\frac{1}{2}^-$ and $1\frac{1}{2}^-$ in the region of 1.5 GeV, we have pointed out that the situation concerning the KN scattering states does not seem to be very clear and our predictive power is quantitatively very limited. Furthermore, as discussed in the previous section, we do not believe that these states correspond to $\Theta^+(1540)$, because their width is expected to be too large to be consistent with the experimental value.

Looking at the states with $IJ^\pi = 0\frac{3}{2}^+$ and $1\frac{3}{2}^+$, in both cases the values of the masses and residues show only a weak dependence on the Borel mass M and the threshold parameter s_{th} . For the isosinglet case this was already pointed out in [28]. This suggests that we are really observing narrow resonance states in the spectral functions of these quantum numbers. As no isospin partners of the $\Theta^+(1540)$ have so far been found, it is believed to be an

isosinglet, which leads to our conclusion that the $\Theta^+(1540)$ is likely to be a state with quantum numbers $IJ^\pi = 0\frac{3}{2}^+$. The isotriplet state $1\frac{3}{2}^+$ is predicted to exist somewhere above the isosinglet, so it may be interesting for future experiments to look for this state. One nevertheless has to be cautious when interpreting the current results, as we cannot make any real quantitative prediction about the width of the state with the present method. Therefore it is difficult to say whether the predicted isotriplet state is narrow enough to be unambiguously detected in an experiment.

The width of a state can be obtained from the QCD sum rule technique by calculating three-point functions of appropriate currents, and it would be interesting to see whether it is possible to obtain a value consistent with experiment for the $0\frac{3}{2}^+$ state and whether the $IJ^\pi = 1\frac{3}{2}^+$ state is really narrow enough to be experimentally observed. Furthermore, it is important to check whether our conjecture of the large widths of the $J^\pi = \frac{1}{2}^-$ states is really true or not. These issues are left for further studies.

ACKNOWLEDGMENTS

This work was partially supported by KAKENHI under Contract Nos. 17070002 (Priority area), 19540275, and 20028004. A part of this work was done in the Yukawa International Project for Quark-Hadron Sciences (YIPQS). P. G. acknowledges the support by the Japan Society for the Promotion of Science for Young Scientists and is thankful for the hospitality of the Yukawa Institute for Theoretical Physics at Kyoto University, where part of this work has been completed. T. K. is supported by RIKEN, Brookhaven National Laboratory, and the U.S. Department of Energy under Contract No. DE-AC02-98CH10886.

APPENDIX A: RESULTS OF THE OPERATOR PRODUCT EXPANSION

We obtain the following result for the OPE in terms of the parameters C_i defined in Eq. (6). Note, that we here give the values of C_i after the difference of the two correlators is taken, and that we have used $\theta_J^I - \theta_J^{I'} = \frac{\pi}{2}$ and $\phi_J^I = \theta_J^I + \theta_J^{I'}$. After showing the results of the chiral-even part (up to terms proportional to m_s), the chiral-odd part is given in the chiral limit.

The used abbreviations are $G^2 \equiv G_{\mu\nu}^a G^{a\mu\nu}$ and $\sigma \cdot G \equiv \sigma^{\mu\nu} \frac{\lambda^a}{2} G_{\mu\nu}^a$, the λ^a being the Gell-Mann matrices. g is the coupling constant of QCD, giving $\alpha_s = \frac{g^2}{4\pi}$. The values of the condensates and the strange quark mass are shown in Table II.

1. $IJ^P = 0_{\frac{1}{2}}^{\pm}$ **a. Chiral-even part**

$$\begin{aligned}
C_0 &= 0, & C_4 &= -\frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle}{2^{14} 3 \cdot 5 \pi^6} \cos \phi_{1/2}^0, & C_6 &= -\frac{\langle \bar{q}q \rangle^2}{2^7 3^2 \pi^4} \sin \phi_{1/2}^0 - \frac{m_s \langle \bar{s}g\sigma \cdot Gs \rangle}{2^{13} \pi^6} \cos \phi_{1/2}^0, \\
C_8 &= \frac{\langle \bar{q}q \rangle \langle \bar{q}g\sigma \cdot Gq \rangle}{2^9 3^2 \pi^4} (7 \cos \phi_{1/2}^0 + 34 \sin \phi_{1/2}^0), \\
C_{10} &= -\frac{\langle \bar{q}g\sigma \cdot Gq \rangle^2}{2^{13} 3^2 \pi^4} (22 \cos \phi_{1/2}^0 + 299 \sin \phi_{1/2}^0) - \frac{\langle \bar{q}q \rangle^2 \langle \frac{\alpha_s}{\pi} G^2 \rangle}{2^9 3^3 \pi^2} (6 \cos \phi_{1/2}^0 + 61 \sin \phi_{1/2}^0) \\
&\quad - \frac{13 m_s \langle \frac{\alpha_s}{\pi} G^2 \rangle \langle \bar{s}g\sigma \cdot Gs \rangle}{2^{14} 3 \pi^4} \cos \phi_{1/2}^0 - \frac{m_s \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{2^3 3^2 \pi^2} \sin \phi_{1/2}^0, \\
C_{12} &= \frac{2 \langle \bar{q}q \rangle^4}{3^3} \sin \phi_{1/2}^0 + \frac{\langle \bar{q}q \rangle \langle \bar{q}g\sigma \cdot Gq \rangle \langle \frac{\alpha_s}{\pi} G^2 \rangle}{2^{12} 3^3 \pi^2} (65 \cos \phi_{1/2}^0 + 418 \sin \phi_{1/2}^0) \\
&\quad - \frac{m_s \langle \bar{q}q \rangle^2 \langle \bar{s}g\sigma \cdot Gs \rangle}{2^6 3^3 \pi^2} (3 \cos \phi_{1/2}^0 + 13 \sin \phi_{1/2}^0) + \frac{7 m_s \langle \bar{q}q \rangle \langle \bar{s}s \rangle \langle \bar{q}g\sigma \cdot Gq \rangle}{2^5 3^2 \pi^2} \sin \phi_{1/2}^0, \\
C_{14} &= +\frac{31 \langle \bar{q}q \rangle^3 \langle \bar{q}g\sigma \cdot Gq \rangle}{2^4 3^3} \sin \phi_{1/2}^0 - \frac{m_s \langle \bar{q}q \rangle \langle \bar{q}g\sigma \cdot Gq \rangle \langle \bar{s}g\sigma \cdot Gs \rangle}{2^9 3^3 \pi^2} (17 \cos \phi_{1/2}^0 + 8 \sin \phi_{1/2}^0) \\
&\quad + \frac{19 m_s \langle \bar{s}s \rangle \langle \bar{q}g\sigma \cdot Gq \rangle^2}{2^9 3^2 \pi^2} \sin \phi_{1/2}^0 + \frac{19 m_s \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle \langle \frac{\alpha_s}{\pi} G^2 \rangle}{2^7 3^4} \sin \phi_{1/2}^0. \tag{A1}
\end{aligned}$$

b. Chiral-odd part

$$\begin{aligned}
C_1 &= 0, & C_3 &= \frac{\langle \bar{s}s \rangle}{2^{11} 3^2 5 \pi^6} \sin \phi_{1/2}^0, & C_5 &= -\frac{7 \langle \bar{s}g\sigma \cdot Gs \rangle}{2^{14} 3^2 \pi^6} \sin \phi_{1/2}^0, & C_7 &= -\frac{5 \langle \bar{s}s \rangle \langle \frac{\alpha_s}{\pi} G^2 \rangle}{2^{11} 3^3 \pi^4} \sin \phi_{1/2}^0, \\
C_9 &= \frac{161 \langle \frac{\alpha_s}{\pi} G^2 \rangle \langle \bar{s}g\sigma \cdot Gs \rangle}{2^{15} 3^2 \pi^4} \sin \phi_{1/2}^0 - \frac{\langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{2^2 3^2 \pi^2} \sin \phi_{1/2}^0, \\
C_{11} &= \frac{7 \langle \bar{q}q \rangle^2 \langle \bar{s}g\sigma \cdot Gs \rangle}{2^6 3^2 \pi^2} (\cos \phi_{1/2}^0 + 2 \sin \phi_{1/2}^0) + \frac{7 \langle \bar{q}q \rangle \langle \bar{s}s \rangle \langle \bar{q}g\sigma \cdot Gq \rangle}{2^4 3^2 \pi^2} \sin \phi_{1/2}^0, \\
C_{13} &= -\frac{25 \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle \langle \frac{\alpha_s}{\pi} G^2 \rangle}{2^6 3^3} \sin \phi_{1/2}^0 - \frac{\langle \bar{q}q \rangle \langle \bar{q}g\sigma \cdot Gq \rangle \langle \bar{s}g\sigma \cdot Gs \rangle}{2^{12} 3^3 \pi^2} (424 \cos \phi_{1/2}^0 + 1979 \sin \phi_{1/2}^0) \\
&\quad - \frac{19 \langle \bar{s}s \rangle \langle \bar{q}g\sigma \cdot Gq \rangle^2}{2^8 3^2 \pi^2} \sin \phi_{1/2}^0. \tag{A2}
\end{aligned}$$

2. $IJ^P = 1\frac{1}{2}^{\pm}$ **a. Chiral-even part**

$$\begin{aligned}
C_0 &= 0, & C_4 &= -\frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle}{2^{14} 3 \cdot 5 \pi^6} \cos \phi_{1/2}^1, & C_6 &= -\frac{\langle \bar{q}q \rangle^2}{2^6 3^2 \pi^4} \sin \phi_{1/2}^1 - \frac{m_s \langle \bar{s}g\sigma \cdot Gs \rangle}{2^{13} \pi^6} \cos \phi_{1/2}^1, \\
C_8 &= \frac{\langle \bar{q}q \rangle \langle \bar{q}g\sigma \cdot Gq \rangle}{2^9 3^2 \pi^4} (3 \cos \phi_{1/2}^1 + 44 \sin \phi_{1/2}^1), \\
C_{10} &= -\frac{\langle \bar{q}g\sigma \cdot Gq \rangle^2}{2^{13} 3^2 \pi^4} (15 \cos \phi_{1/2}^1 + 361 \sin \phi_{1/2}^1) + \frac{\langle \bar{q}q \rangle^2 \langle \frac{\alpha_s}{\pi} G^2 \rangle}{2^9 3^3 \pi^2} (18 \cos \phi_{1/2}^1 - 29 \sin \phi_{1/2}^1) - \frac{13 m_s \langle \frac{\alpha_s}{\pi} G^2 \rangle \langle \bar{s}g\sigma \cdot Gs \rangle}{2^{14} 3 \pi^4} \\
&\quad \times \cos \phi_{1/2}^1 - \frac{m_s \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{2^2 3^2 \pi^2} \sin \phi_{1/2}^1, \\
C_{12} &= -\frac{2 \langle \bar{q}q \rangle^4}{3^3} \sin \phi_{1/2}^1 - \frac{\langle \bar{q}q \rangle \langle \bar{q}g\sigma \cdot Gq \rangle \langle \frac{\alpha_s}{\pi} G^2 \rangle}{2^{12} 3^3 \pi^2} (135 \cos \phi_{1/2}^1 - 158 \sin \phi_{1/2}^1) + \frac{m_s \langle \bar{q}q \rangle^2 \langle \bar{s}g\sigma \cdot Gs \rangle}{2^6 3^3 \pi^2} (9 \cos \phi_{1/2}^1 \\
&\quad + 13 \sin \phi_{1/2}^1) + \frac{19 m_s \langle \bar{q}q \rangle \langle \bar{s}s \rangle \langle \bar{q}g\sigma \cdot Gq \rangle}{2^6 3^2 \pi^2} \sin \phi_{1/2}^1, \\
C_{14} &= -\frac{31 \langle \bar{q}q \rangle^3 \langle \bar{q}g\sigma \cdot Gq \rangle}{2^4 3^3} \sin \phi_{1/2}^1 + \frac{m_s \langle \bar{q}q \rangle \langle \bar{q}g\sigma \cdot Gq \rangle \langle \bar{s}g\sigma \cdot Gs \rangle}{2^{10} 3^3 \pi^2} (69 \cos \phi_{1/2}^1 + 122 \sin \phi_{1/2}^1) \\
&\quad + \frac{11 m_s \langle \bar{s}s \rangle \langle \bar{q}g\sigma \cdot Gq \rangle^2}{2^8 3^2 \pi^2} \sin \phi_{1/2}^1 + \frac{23 m_s \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle \langle \frac{\alpha_s}{\pi} G^2 \rangle}{2^7 3^4} \sin \phi_{1/2}^1.
\end{aligned} \tag{A3}$$

b. Chiral-odd part

$$\begin{aligned}
C_1 &= 0, & C_3 &= -\frac{\langle \bar{s}s \rangle}{2^{11} 3^2 5 \pi^6} \sin \phi_{1/2}^1, & C_5 &= +\frac{7 \langle \bar{s}g\sigma \cdot Gs \rangle}{2^{14} 3^2 \pi^6} \sin \phi_{1/2}^1, & C_7 &= +\frac{5 \langle \bar{s}s \rangle \langle \frac{\alpha_s}{\pi} G^2 \rangle}{2^{11} 3^3 \pi^4} \sin \phi_{1/2}^1, \\
C_9 &= -\frac{161 \langle \frac{\alpha_s}{\pi} G^2 \rangle \langle \bar{s}g\sigma \cdot Gs \rangle}{2^{15} 3^2 \pi^4} \sin \phi_{1/2}^1 - \frac{\langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{2 \cdot 3^2 \pi^2} \sin \phi_{1/2}^1, \\
C_{11} &= \frac{\langle \bar{q}q \rangle^2 \langle \bar{s}g\sigma \cdot Gs \rangle}{2^6 3^2 \pi^2} (3 \cos \phi_{1/2}^1 + 19 \sin \phi_{1/2}^1) + \frac{19 \langle \bar{q}q \rangle \langle \bar{s}s \rangle \langle \bar{q}g\sigma \cdot Gq \rangle}{2^5 3^2 \pi^2} \sin \phi_{1/2}^1, \\
C_{13} &= -\frac{29 \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle \langle \frac{\alpha_s}{\pi} G^2 \rangle}{2^6 3^3} \sin \phi_{1/2}^1 - \frac{\langle \bar{q}q \rangle \langle \bar{q}g\sigma \cdot Gq \rangle \langle \bar{s}g\sigma \cdot Gs \rangle}{2^{12} 3^3 \pi^2} (204 \cos \phi_{1/2}^1 + 2503 \sin \phi_{1/2}^1) - \frac{11 \langle \bar{s}s \rangle \langle \bar{q}g\sigma \cdot Gq \rangle^2}{2^7 3^2 \pi^2} \\
&\quad \times \sin \phi_{1/2}^1.
\end{aligned} \tag{A4}$$

3. $IJ^P = 0\frac{3}{2}^{\pm}$

See [28].

4. $IJ^P = 1\frac{3}{2}^+$ **a. Chiral-even part**

$$\begin{aligned}
C_0 &= 0, & C_4 &= \frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle}{2^{16} 3^3 \cdot 5 \pi^6} \cos \phi_{3/2}^1, & C_6 &= \frac{\langle \bar{q}q \rangle^2}{2^6 3^2 5 \pi^4} \sin \phi_{3/2}^1 + \frac{m_s \langle \bar{s}g\sigma \cdot Gs \rangle}{2^{14} 3 \cdot 5 \pi^6} \cos \phi_{3/2}^1, \\
C_8 &= -\frac{\langle \bar{q}q \rangle \langle \bar{q}g\sigma \cdot Gq \rangle}{2^{12} 3^2 \pi^4} (\cos \phi_{3/2}^1 + 54 \sin \phi_{3/2}^1), \\
C_{10} &= \frac{\langle \bar{q}g\sigma \cdot Gq \rangle^2}{2^{14} 3^4 \pi^4} (15 \cos \phi_{3/2}^1 + 677 \sin \phi_{3/2}^1) - \frac{\langle \bar{q}q \rangle^2 \langle \frac{\alpha_s}{\pi} G^2 \rangle}{2^{10} 3^3 \pi^2} (2 \cos \phi_{3/2}^1 + 11 \sin \phi_{3/2}^1) \\
&\quad + \frac{13 m_s \langle \frac{\alpha_s}{\pi} G^2 \rangle \langle \bar{s}g\sigma \cdot Gs \rangle}{2^{15} 3^3 \pi^4} \cos \phi_{3/2}^1 + \frac{m_s \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{2^3 3^3 \pi^2} \sin \phi_{3/2}^1, \\
C_{12} &= \frac{\langle \bar{q}q \rangle^4}{3^3} \sin \phi_{3/2}^1 + \frac{5 \langle \bar{q}q \rangle \langle \bar{q}g\sigma \cdot Gq \rangle \langle \frac{\alpha_s}{\pi} G^2 \rangle}{2^{14} 3^2 \pi^2} (3 \cos \phi_{3/2}^1 + 20 \sin \phi_{3/2}^1) - \frac{m_s \langle \bar{q}q \rangle^2 \langle \bar{s}g\sigma \cdot Gs \rangle}{2^8 3^3 \pi^2} (3 \cos \phi_{3/2}^1 + 10 \sin \phi_{3/2}^1) \\
&\quad - \frac{11 m_s \langle \bar{q}q \rangle \langle \bar{s}s \rangle \langle \bar{q}g\sigma \cdot Gq \rangle}{2^8 3^2 \pi^2} \sin \phi_{3/2}^1, \\
C_{14} &= \frac{97 \langle \bar{q}q \rangle^3 \langle \bar{q}g\sigma \cdot Gq \rangle}{2^5 3^4} \sin \phi_{3/2}^1 - \frac{m_s \langle \bar{q}q \rangle \langle \bar{q}g\sigma \cdot Gq \rangle \langle \bar{s}g\sigma \cdot Gs \rangle}{2^{11} 3^3 \pi^2} (23 \cos \phi_{3/2}^1 + 18 \sin \phi_{3/2}^1) \\
&\quad - \frac{11 m_s \langle \bar{s}s \rangle \langle \bar{q}g\sigma \cdot Gq \rangle^2}{2^9 3^4 \pi^2} \sin \phi_{3/2}^1 + \frac{25 m_s \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle \langle \frac{\alpha_s}{\pi} G^2 \rangle}{2^8 3^4} \sin \phi_{3/2}^1. \tag{A5}
\end{aligned}$$

b. Chiral-odd part

$$\begin{aligned}
C_1 &= 0, & C_3 &= -\frac{\langle \bar{s}s \rangle}{2^{13} 3^3 \pi^6} \sin \phi_{3/2}^1, & C_5 &= +\frac{5 \langle \bar{s}g\sigma \cdot Gs \rangle}{2^{13} 3^3 \pi^6} \sin \phi_{3/2}^1, & C_7 &= -\frac{7 \langle \bar{s}s \rangle \langle \frac{\alpha_s}{\pi} G^2 \rangle}{2^{14} 3^3 \pi^4} \sin \phi_{3/2}^1, \\
C_9 &= +\frac{5 \langle \frac{\alpha_s}{\pi} G^2 \rangle \langle \bar{s}g\sigma \cdot Gs \rangle}{2^{15} 3^4 \pi^4} \sin \phi_{3/2}^1 - \frac{\langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{2^3 3^2 \pi^2} \sin \phi_{3/2}^1, & C_{11} &= -\frac{\langle \bar{q}q \rangle^2 \langle \bar{s}g\sigma \cdot Gs \rangle}{2^8 3^2 \pi^2} (\cos \phi_{3/2}^1 - 15 \sin \phi_{3/2}^1) \\
&\quad + \frac{19 \langle \bar{q}q \rangle \langle \bar{s}s \rangle \langle \bar{q}g\sigma \cdot Gq \rangle}{2^7 3^2 \pi^2} \sin \phi_{3/2}^1, & C_{13} &= -\frac{7 \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle \langle \frac{\alpha_s}{\pi} G^2 \rangle}{2^6 3^3} \sin \phi_{3/2}^1 \\
&\quad + \frac{\langle \bar{q}q \rangle \langle \bar{q}g\sigma \cdot Gq \rangle \langle \bar{s}g\sigma \cdot Gs \rangle}{2^{11} 3^3 \pi^2} (17 \cos \phi_{3/2}^1 - 216 \sin \phi_{3/2}^1) - \frac{11 \langle \bar{s}s \rangle \langle \bar{q}g\sigma \cdot Gq \rangle^2}{2^6 3^4 \pi^2} \sin \phi_{3/2}^1. \tag{A6}
\end{aligned}$$

APPENDIX B: ESTABLISHMENT OF A VALID BOREL WINDOW

In this Appendix, we explicitly show that a Borel window has been obtained for the sum rules of the chiral-even part for the various quantum numbers.

First, the convergence of the OPE is checked. This is done by calculating the left-hand side of Eq. (14). The

results are given in Fig. 7. Additionally, the right-hand side of Eq. (8) added order by order is shown in Fig. 8 to get a better idea of the behavior of the expansion. Subsequently, the pole contribution of Eq. (15) is investigated. The corresponding plots are given in Fig. 9 for the various quantum numbers.

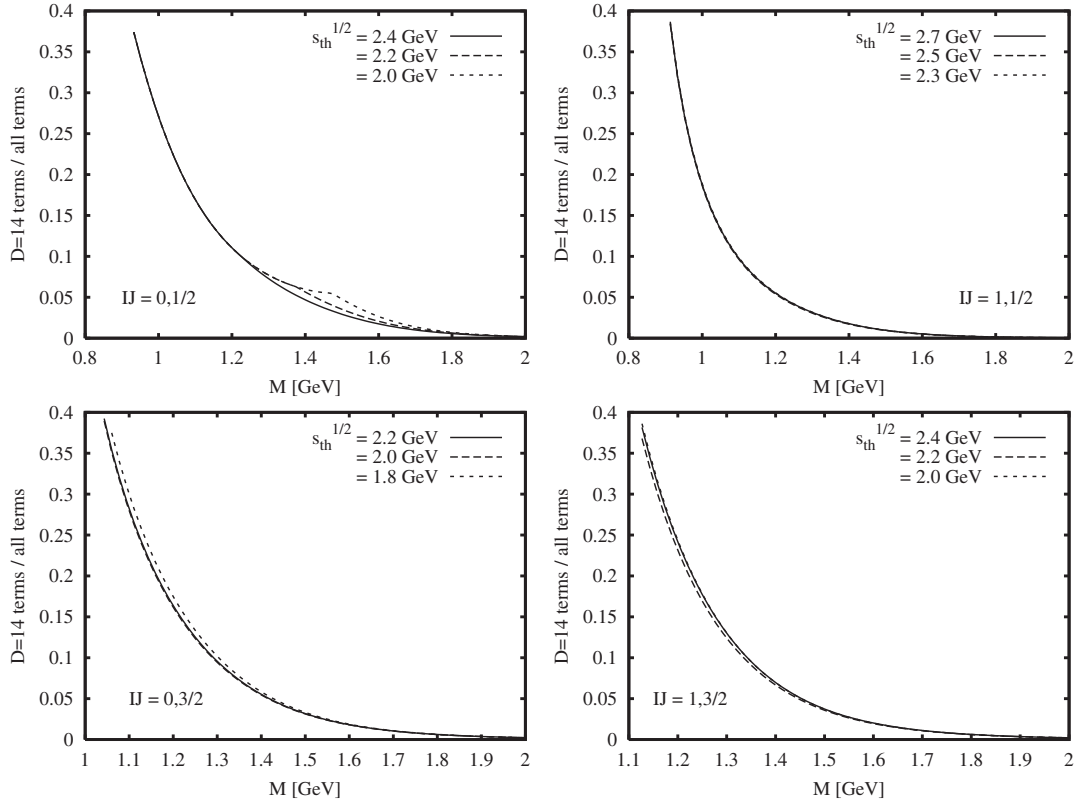


FIG. 7. The highest term in the OPE divided by the whole OPE for $IJ = 0\frac{1}{2}$, $IJ = 1\frac{1}{2}$, $IJ = 0\frac{3}{2}$, $IJ = 1\frac{3}{2}$, as given in the left-hand side of Eq. (14).

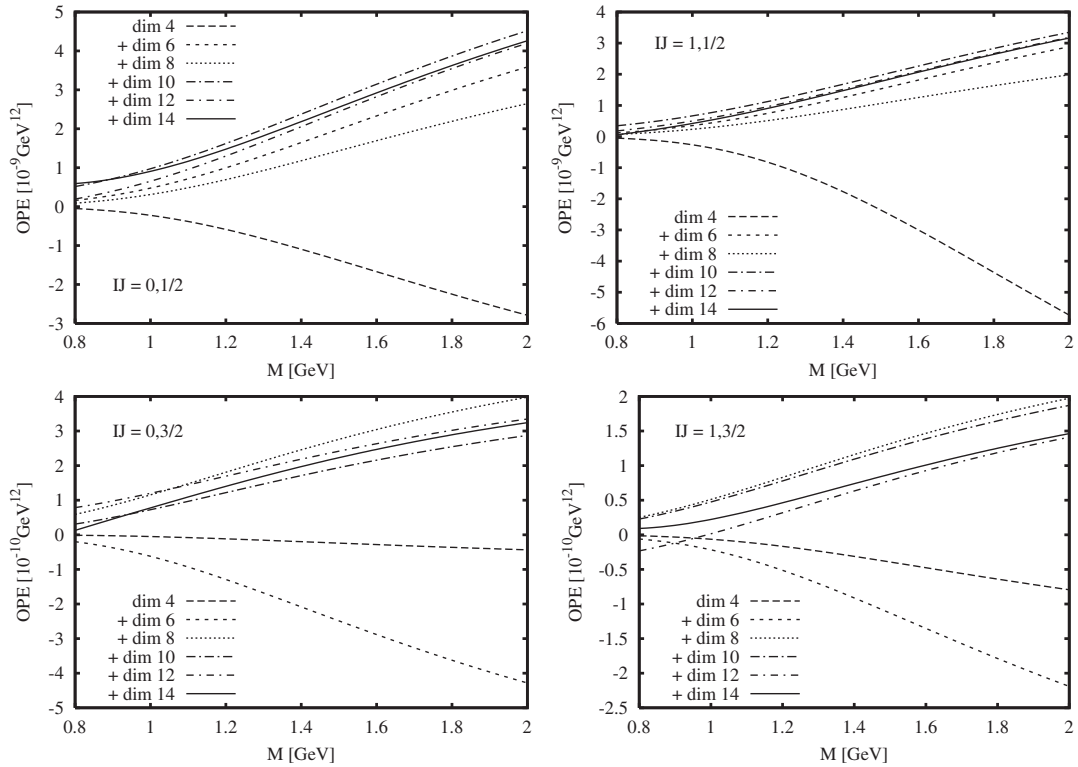


FIG. 8. Contributions of different dimensions to the right-hand side of Eq. (8) for $IJ = 0\frac{1}{2}$, $IJ = 1\frac{1}{2}$, $IJ = 0\frac{3}{2}$, $IJ = 1\frac{3}{2}$, added in succession. The expansion in most cases starts to converge after terms up to dimension 10 are included.

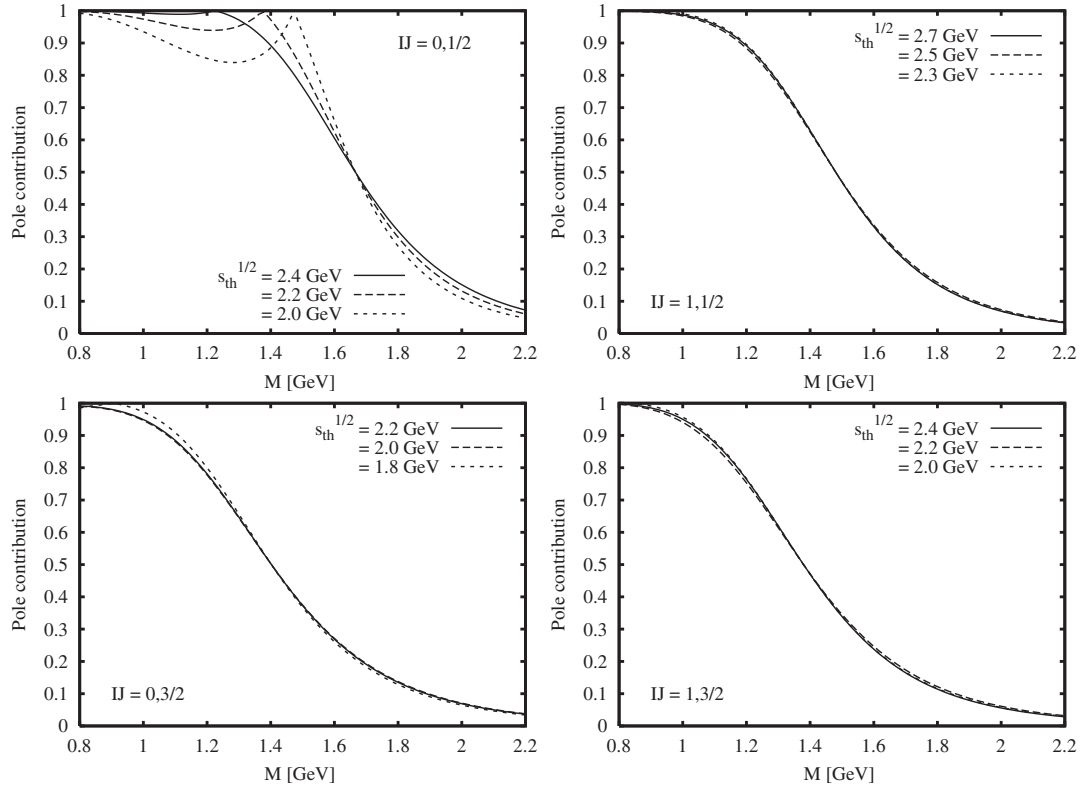


FIG. 9. The pole contribution to the sum rule with $IJ = 0\frac{1}{2}$, $IJ = 1\frac{1}{2}$, $IJ = 0\frac{3}{2}$, $IJ = 1\frac{3}{2}$ in comparison with the contribution of the high-energy continuum states. These plots, together with the ones in Fig. 7 finally lead to the Borel windows referred to in the result section.

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