

QCD factorization for charmless hadronic B_s decays revisitedHai-Yang Cheng^{1,2} and Chun-Kiang Chua³¹*Institute of Physics, Academia Sinica, Taipei, Taiwan 115, Republic of China*²*Physics Department, Brookhaven National Laboratory, Upton, New York 11973*³*Department of Physics, Chung Yuan Christian University, Chung-Li, Taiwan 320, Republic of China*

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Branching fractions and CP -violating asymmetries of charmless $\bar{B}_s \rightarrow PP, VP, VV$ decays (P and V denoting pseudoscalar and vector mesons, respectively) are reexamined in the framework of QCD factorization (QCDF). We take into account subleading power corrections to the penguin annihilation topology and to color-suppressed tree amplitudes that are crucial for resolving the CP puzzles and rate deficit problems with penguin-dominated two-body decays and color-suppressed tree-dominated $\pi^0\pi^0$ and $\rho^0\pi^0$ modes in the $B_{u,d}$ sector. Many of the $B_s \rightarrow h_1h_2$ decays can be related to $B_d \rightarrow h_1h_2$ ones via U -spin or $SU(3)$ symmetry. Some useful model-independent relations can be derived and tested. Mixing-induced CP asymmetries for many of the penguin-dominated decays are predicted to be very small in the standard model. They are sensitive to new physics and offer rich possibilities of new discoveries. Measurements of direct CP -violating asymmetries can be used to discriminate QCDF from other competing approaches such as pQCD and soft-collinear effective theory.

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I. INTRODUCTION

The phenomenology of nonleptonic two-body decays of B mesons offers rich opportunities for our understanding of the underlying mechanism for hadronic weak decays and CP violation. In the past decade, nearly 100 charmless decays of $B_{u,d}$ mesons have been observed at B factories, *BABAR* and *Belle*, with a statistical significance of at least 4 standard deviations (for a review, see [1]). The CDF Collaboration has made unique contributions to the measurements of charmless hadronic B_s decays. Recently, *Belle* has also started to study the weak decays of the B_s meson.

Many of the $B_s \rightarrow h_1h_2$ decays can be related to $B_d \rightarrow h'_1h'_2$ ones via U -spin or $SU(3)$ symmetry. Some useful model-independent relations can be derived and tested. For example, direct CP asymmetries of $\bar{B}_s \rightarrow K^+\pi^-$ and $\bar{B}_d \rightarrow K^-\pi^+$ are related to each other by U -spin symmetry. Therefore, the use of flavor symmetry will be helpful to control the hadronic uncertainties in $\bar{B}_s \rightarrow h_1h_2$ decay amplitudes.

Analogous to the neutral B_d system, CP violation in B_s decays also occurs through the interference of decay amplitudes with and without $B_s - \bar{B}_s$ mixing. It is known that the mixing-induced CP violation of $B_d \rightarrow J/\psi K$ is governed by $\sin 2\beta$. Likewise, the decay $B_s \rightarrow J/\psi \phi$ is the benchmark in the B_s system with mixing-induced CP asymmetry characterized by $\sin 2\beta_s$. In the standard model (SM), the phase β_s is very small, of order 1° . Consequently, $B_s \rightarrow J/\psi \phi$ and several charmless penguin-dominated B_s decays e.g. $\bar{B}_s \rightarrow K^{(*)0}\bar{K}^{(*)0}$, $\eta^{(\prime)}\eta^{(\prime)}$, $\phi\phi$ are the ideal places to search for new physics as CP violation from physics beyond the SM can compete

or even dominate over the small SM CP phase. Recently, both CDF [2] and D0 [3] have observed 1–2 σ deviations from the SM prediction for β_s . Because of the possibilities of new discoveries, the search for new physics in the B_s system will be the main focus of the forthcoming experiments at Fermilab, LHCb and Super B factories.

Theoretically, two-body B_s decays have been studied in the framework of generalized factorization [4], QCD factorization (QCDF) [5–9], perturbative QCD (pQCD) [10–12] and soft-collinear effective theory (SCET) [13,14]. In this work we will reexamine and update the QCDF predictions. Especially, we shall pay attention to the issue of power corrections. From the study of charmless hadronic $B_{u,d}$ decays, we learned that two subleading $1/m_b$ power corrections are needed in QCDF in order to account for the observed rates and CP asymmetries. Power corrections to the penguin annihilation topology, corresponding to the so-called “scenario S4” in [6], are crucial for accommodating the branching fractions of penguin-dominated $B_{u,d} \rightarrow PP, VP, VV$ decays on the one hand and direct CP asymmetries of $\bar{B}_d \rightarrow K^-\pi^+$, $\bar{B}_d \rightarrow K^{*-}\pi^+$, $B^- \rightarrow K^-\rho^0$ and $\bar{B}_d \rightarrow \pi^+\pi^-$ on the other hand. Otherwise, the predicted rates will be too small and CP -violating asymmetries of above-mentioned modes will be wrong in signs when confronted with experiment. However, power corrections due to penguin annihilation will bring new CP puzzles for the decays $B^- \rightarrow K^-\pi^0$, $K^-\eta$, $\pi^-\eta$, $\bar{B}_d \rightarrow \bar{K}^{*0}\eta$ and $\bar{B}_d \rightarrow \pi^0\pi^0$: Signs of their A_{CP} 's are flipped into the wrong ones when compared with experiment. It has been shown in [15] that soft corrections to the color-suppressed tree amplitude due to spectator scattering and/or final-state interactions will bring the aforementioned CP asymmetries to the right track and accommodate the observed $\pi^0\pi^0$ and

$\rho^0 \pi^0$ rates simultaneously.¹ Recently we have given a detailed study of charmless hadronic $B_{u,d} \rightarrow PP, VP, VV$ decays within the framework of QCDF incorporating aforementioned power corrections [17]. In this work we shall generalize the study to B_s decays. So far $\bar{B}_s \rightarrow K^+ \pi^-$ is the only hadronic decay mode in the B_s sector that its direct CP violation has been measured [18]. The resulting CP asymmetry $A_{CP}(\bar{B}_s \rightarrow K^+ \pi^-) = 0.39 \pm 0.17$ differs from zero by 2.2σ deviations. Just as the decay $\bar{B}_d \rightarrow K^- \pi^+$, the predicted CP asymmetry for $\bar{B}_s \rightarrow K^+ \pi^-$ in the heavy quark limit is wrong in sign and too small in magnitude. As we shall see below, we need penguin annihilation to get the right sign and magnitude for $A_{CP}(\bar{B}_s \rightarrow K^+ \pi^-)$.

This work is organized as follows. We outline the QCDF framework in Sec. II and specify various input parameters, such as form factors, light-cone distribution amplitudes and the parameters for power corrections in Sec. III. Then $B_s \rightarrow PP, VP, VV$ decays are analyzed in details in Secs. IV, V, and VI, respectively. Conclusions are given in Sec. VII.

II. B DECAYS IN QCD FACTORIZATION

Within the framework of QCD factorization [19], the effective Hamiltonian matrix elements are written in the form

$$\langle M_1 M_2 | \mathcal{H}_{\text{eff}} | \bar{B} \rangle = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(q)} \times \langle M_1 M_2 | (\mathcal{T}_{\mathcal{A}}^{h,p} + \mathcal{T}_{\mathcal{B}}^{h,p}) | \bar{B} \rangle, \quad (2.1)$$

where $\lambda_p^{(q)} \equiv V_{pb} V_{pq}^*$ with $q = d, s$, and the superscript h denotes the helicity of the final-state meson. For PP and VP final states, $h = 0$. $\mathcal{T}_{\mathcal{A}}^{h,p}$ describes contributions from naive factorization, vertex corrections, penguin contractions and spectator scattering expressed in terms of the flavor operators $a_i^{p,h}$, while $\mathcal{T}_{\mathcal{B}}$ contains annihilation topology amplitudes characterized by the annihilation operators $b_i^{p,h}$. Specifically [19]

$$\begin{aligned} \mathcal{T}_{\mathcal{A}}^h &= a_1^p(M_1 M_2) \delta_{pu} (\bar{u}b)_{V-A} \otimes (\bar{q}u)_{V-A} + a_2^p(M_1 M_2) \delta_{pu} (\bar{q}b)_{V-A} \otimes (\bar{u}u)_{V-A} + a_3^p(M_1 M_2) \sum (\bar{q}b)_{V-A} \otimes (\bar{q}'q')_{V-A} \\ &+ a_4^p(M_1 M_2) \sum (\bar{q}'b)_{V-A} \otimes (\bar{q}q')_{V-A} + a_5^p(M_1 M_2) \sum (\bar{q}b)_{V-A} \otimes (\bar{q}'q')_{V+A} \\ &+ a_6^p(M_1 M_2) \sum (-2)(\bar{q}'b)_{S-P} \otimes (\bar{q}q')_{S+P} + a_7^p(M_1 M_2) \sum (\bar{q}b)_{V-A} \otimes \frac{3}{2} e_q (\bar{q}'q')_{V+A} \\ &+ a_8^p(M_1 M_2) \sum (-2)(\bar{q}'b)_{S-P} \otimes \frac{3}{2} (\bar{q}q')_{S+P} + a_9^p(M_1 M_2) \sum (\bar{q}b)_{V-A} \otimes \frac{3}{2} e_q (\bar{q}'q')_{V-A} \\ &+ a_{10}^p(M_1 M_2) \sum (\bar{q}'b)_{V-A} \otimes \frac{3}{2} e_q (\bar{q}q')_{V-A}, \end{aligned} \quad (2.2)$$

where $(\bar{q}_1 q_2)_{V\pm A} \equiv \bar{q}_1 \gamma_\mu (1 \pm \gamma_5) q_2$ and $(\bar{q}_1 q_2)_{S\pm P} \equiv \bar{q}_1 (1 \pm \gamma_5) q_2$ and the summation is over $q' = u, d, s$. The symbol \otimes indicates that the matrix elements of the operators in $\mathcal{T}_{\mathcal{A}}$ are to be evaluated in the factorized form. For the decays $\bar{B} \rightarrow PP, VP, VV$, the relevant factorizable matrix elements are

¹It is well known that a large complex electroweak penguin can also solve the $B \rightarrow K\pi$ CP puzzle with the difference of $A_{CP}(B^- \rightarrow \bar{K}^0 \pi^-)$ and $A_{CP}(\bar{B}^0 \rightarrow K^- \pi^+)$ (see e.g. [16]). Since the electroweak penguin amplitude P_{EW} is essentially real in the standard model, one needs new physics to produce new *strong* and *weak* phases for P_{EW} . In principle, it will be difficult to discriminate between large complex color-suppressed tree C and large P_{EW} scenarios in the penguin-dominated decays. However, as pointed out in [17], the two schemes can lead to very distinct predictions for tree-dominated decays where $P_{EW} \ll C$. The observed decay rates of $\bar{B}^0 \rightarrow \pi^0 \pi^0$, $\rho^0 \pi^0$ and the CP puzzles with $\pi^- \eta$ and $\pi^0 \pi^0$ can be resolved by a large complex C but not P_{EW} . In the $B_{u,d}$ sector, there are 13 modes in which CP asymmetries have been measured with significance above 1.8σ : $K^- \pi^+$, $\pi^+ \pi^-$, $K^- \eta$, $\bar{K}^{*0} \eta$, $K^- \rho^0$, $\rho^\pm \pi^\mp$ and $\rho^+ K^-$, $K^{*-} \pi^+$, $K^- \pi^0$, $\pi^- \eta$, $\omega \bar{K}^0$, $\pi^0 \pi^0$, $\rho^- \pi^+$. We have shown in [17] that the QCDF predictions of A_{CP} for aforementioned 13 decays are in agreement with experiment except the decay $\bar{B}^0 \rightarrow \omega \bar{K}^0$. However, we notice that *BABAR* and *Belle* measurements of $A_{CP}(\omega \bar{K}^0)$ are opposite in sign.

$$\begin{aligned}
 X^{(\bar{B}P_1, P_2)} &\equiv \langle P_2 | J_\mu | 0 \rangle \langle P_1 | J'^\mu | \bar{B} \rangle = if_{P_2} (m_B^2 - m_{P_1}^2) F_0^{BP_1}(m_{P_2}^2), \\
 X^{(\bar{B}P, V)} &\equiv \langle V | J_\mu | 0 \rangle \langle P | J'^\mu | \bar{B} \rangle = 2f_V m_B p_c F_1^{BP}(m_V^2), \\
 X^{(\bar{B}V, P)} &\equiv \langle P | J_\mu | 0 \rangle \langle V | J'^\mu | \bar{B} \rangle = 2f_P m_B p_c A_0^{BV}(m_P^2), \\
 X_h^{(\bar{B}V_1, V_2)} &\equiv \langle V_2 | J_\mu | 0 \rangle \langle V_1 | J'^\mu | \bar{B} \rangle = -if_{V_2} m_2 \left[(\varepsilon_1^* \cdot \varepsilon_2^*) (m_B + m_{V_1}) A_1^{BV_1}(m_{V_2}^2) - (\varepsilon_1^* \cdot p_B) (\varepsilon_2^* \cdot p_B) \frac{2A_2^{BV_1}(m_{V_2}^2)}{(m_B + m_{V_1})} \right. \\
 &\quad \left. + i \varepsilon_{\mu\nu\alpha\beta} \varepsilon_2^{*\mu} \varepsilon_1^{*\nu} p_B^\alpha p_1^\beta \frac{2V^{BV_1}(m_{V_2}^2)}{(m_B + m_{V_1})} \right], \tag{2.3}
 \end{aligned}$$

where we have followed the conventional definition for form factors [20]. For $B \rightarrow VP$, PV amplitudes, we have applied the replacement $m_V \varepsilon^* \cdot p_B \rightarrow m_B p_c$ with p_c being the c.m. momentum. The longitudinal ($h = 0$) and transverse ($h = \pm$) components of $X_h^{(\bar{B}V_1, V_2)}$ are given by

$$\begin{aligned}
 X_0^{(\bar{B}V_1, V_2)} &= \frac{if_{V_2}}{2m_{V_1}} \left[(m_B^2 - m_{V_1}^2 - m_{V_2}^2) (m_B + m_{V_1}) A_1^{BV_1}(q^2) \right. \\
 &\quad \left. - \frac{4m_B^2 p_c^2}{m_B + m_{V_1}} A_2^{BV_1}(q^2) \right], \\
 X_\pm^{(\bar{B}V_1, V_2)} &= -if_{V_2} m_B m_{V_2} \left[\left(1 + \frac{m_{V_1}}{m_B} \right) A_1^{BV_1}(q^2) \right. \\
 &\quad \left. \mp \frac{2p_c}{m_B + m_{V_1}} V^{BV_1}(q^2) \right]. \tag{2.4}
 \end{aligned}$$

The flavor operators $a_i^{p,h}$ are basically the Wilson coefficients in conjunction with short-distance nonfactorizable corrections such as vertex corrections and hard spectator interactions. In general, they have the expressions [6,19]

$$\begin{aligned}
 a_i^{p,h}(M_1 M_2) &= \left(c_i + \frac{c_{i\pm 1}}{N_c} \right) N_i^h(M_2) + \frac{c_{i\pm 1}}{N_c} \frac{C_F \alpha_s}{4\pi} \\
 &\quad \times \left[V_i^h(M_2) + \frac{4\pi^2}{N_c} H_i^h(M_1 M_2) \right] \\
 &\quad + P_i^{h,p}(M_2), \tag{2.5}
 \end{aligned}$$

where $i = 1, \dots, 10$, the upper (lower) signs apply when i is odd (even), c_i are the Wilson coefficients, $C_F = (N_c^2 - 1)/(2N_c)$ with $N_c = 3$, M_2 is the emitted meson and M_1 shares the same spectator quark with the B meson. The quantities $V_i^h(M_2)$ account for vertex corrections, $H_i^h(M_1 M_2)$ for hard spectator interactions with a hard gluon exchange between the emitted meson and the spectator quark of the B meson and $P_i(M_2)$ for penguin contractions. The expression of the quantities $N_i^h(M_2)$ reads

$$N_i^h(M_2) = \begin{cases} 0, & i = 6, 8, \\ 1, & \text{else.} \end{cases} \tag{2.6}$$

The weak annihilation contributions to the decay $\bar{B} \rightarrow M_1 M_2$ can be described in terms of the building blocks $b_i^{p,h}$ and $b_{i,\text{EW}}^{p,h}$

$$\begin{aligned}
 &\frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(q)} \langle M_1 M_2 | \mathcal{T}_B^{h,p} | \bar{B}^0 \rangle \\
 &= i \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(q)} f_B f_{M_1} f_{M_2} \sum_i (d_i b_i^{p,h} + d'_{i,\text{EW}} b_{i,\text{EW}}^{p,h}). \tag{2.7}
 \end{aligned}$$

The building blocks have the expressions [6]

$$\begin{aligned}
 b_1 &= \frac{C_F}{N_c^2} c_1 A_1^i, \\
 b_3 &= \frac{C_F}{N_c^2} [c_3 A_1^i + c_5 (A_3^i + A_3^f) + N_c c_6 A_3^f], \\
 b_2 &= \frac{C_F}{N_c^2} c_2 A_1^i, \\
 b_4 &= \frac{C_F}{N_c^2} [c_4 A_1^i + c_6 A_2^f], \\
 b_{3,\text{EW}} &= \frac{C_F}{N_c^2} [c_9 A_1^i + c_7 (A_3^i + A_3^f) + N_c c_8 A_3^f], \\
 b_{4,\text{EW}} &= \frac{C_F}{N_c^2} [c_{10} A_1^i + c_8 A_2^f]. \tag{2.8}
 \end{aligned}$$

Here for simplicity we have omitted the superscripts p and h in above expressions. The subscripts 1, 2, 3 of $A_n^{i,f}$ denote the annihilation amplitudes induced from $(V - A)(V - A)$, $(V - A)(V + A)$ and $(S - P)(S + P)$ operators, respectively, and the superscripts i and f refer to gluon emission from the initial and final-state quarks, respectively. Following [6] we choose the convention that M_1 contains an antiquark from the weak vertex and M_2 contains a quark from the weak vertex.

For the explicit expressions of vertex, hard spectator corrections and annihilation contributions, the reader is referred to [6,8,19] for details. The decay amplitudes of $\bar{B}_s \rightarrow PP$, VP are given in Appendix A of [6] and can be easily generalized to $\bar{B}_s \rightarrow VV$ (see [9] for explicit expressions of $\bar{B}_s \rightarrow VV$ amplitudes). In practice, it is more convenient to express the decay amplitudes in terms of the flavor operators $\alpha_i^{h,p}$ and the annihilation operators β_i^p which are related to the coefficients $a_i^{h,p}$ and b_i^p by

$$\begin{aligned}
\alpha_1^h(M_1 M_2) &= a_1^h(M_1 M_2), & \alpha_2^h(M_1 M_2) &= a_2^h(M_1 M_2), \\
\alpha_3^{h,p}(M_1 M_2) &= \begin{cases} a_3^{h,p}(M_1 M_2) - a_5^{h,p}(M_1 M_2) & \text{for } M_1 M_2 = PP, VP, \\ a_3^{h,p}(M_1 M_2) + a_5^{h,p}(M_1 M_2) & \text{for } M_1 M_2 = VV, PV, \end{cases} \\
\alpha_4^{h,p}(M_1 M_2) &= \begin{cases} a_4^{h,p}(M_1 M_2) + r_\chi^{M_2} a_6^{h,p}(M_1 M_2) & \text{for } M_1 M_2 = PP, PV, \\ a_4^{h,p}(M_1 M_2) - r_\chi^{M_2} a_6^{h,p}(M_1 M_2) & \text{for } M_1 M_2 = VP, VV, \end{cases} \\
\alpha_{3,EW}^{h,p}(M_1 M_2) &= \begin{cases} a_9^{h,p}(M_1 M_2) - a_7^{h,p}(M_1 M_2) & \text{for } M_1 M_2 = PP, VP, \\ a_9^{h,p}(M_1 M_2) + a_7^{h,p}(M_1 M_2) & \text{for } M_1 M_2 = VV, PV, \end{cases} \\
\alpha_{4,EW}^{h,p}(M_1 M_2) &= \begin{cases} a_{10}^{h,p}(M_1 M_2) + r_\chi^{M_2} a_8^{h,p}(M_1 M_2) & \text{for } M_1 M_2 = PP, PV, \\ a_{10}^{h,p}(M_1 M_2) - r_\chi^{M_2} a_8^{h,p}(M_1 M_2) & \text{for } M_1 M_2 = VP, VV, \end{cases}
\end{aligned} \tag{2.9}$$

and

$$\beta_i^p(M_1 M_2) = \frac{if_{Bf_{M_1}f_{M_2}}}{X^{(\bar{B}M_1, M_2)}} b_i^p. \tag{2.10}$$

The order of the arguments of $\alpha_i^p(M_1 M_2)$ and $\beta_i^p(M_1 M_2)$ is consistent with the order of the arguments of $X^{(\bar{B}M_1, M_2)} \equiv A_{M_1 M_2}$. The chiral factor r_χ is given by

$$\begin{aligned}
r_\chi^p(\mu) &= \frac{2m_p^2}{m_b(\mu)(m_2 + m_1)(\mu)}, \\
r_\chi^V(\mu) &= \frac{2m_V}{m_b(\mu)} \frac{f_V^\perp(\mu)}{f_V}.
\end{aligned} \tag{2.11}$$

III. INPUT PARAMETERS

It is clear from Eq. (2.3) that we need the information on decay constants and form factors in order to evaluate the factorizable matrix elements of 4-quark operators. Moreover, we also need to know the light-cone distribution amplitudes of light hadrons in order to evaluate the non-factorizable contributions.

A. Form factors

There exist one lattice and several model calculations of form factors for $B_s \rightarrow P, V$ transitions:

- (1) In the pQCD approach, the relevant form factors obtained at $q^2 = 0$ are [11] (for simplicity, form factors hereafter are always referred to the ones at $q^2 = 0$, unless specified otherwise)

$$\begin{aligned}
F_0^{B_s K} &= 0.24_{-0.04-0.01}^{+0.05+0.00}, & F_0^{B_s \eta_s} &= 0.30_{-0.05-0.01}^{+0.06+0.01}, \\
V^{B_s K^*} &= 0.21_{-0.03-0.01}^{+0.04+0.00}, & A_0^{B_s K^*} &= 0.25_{-0.05-0.01}^{+0.05+0.00}, \\
A_1^{B_s K^*} &= 0.16_{-0.03-0.01}^{+0.03+0.00}, & V^{B_s \phi} &= 0.25_{-0.04-0.01}^{+0.05+0.00}, \\
A_0^{B_s \phi} &= 0.30_{-0.05-0.01}^{+0.05+0.00}, & A_1^{B_s \phi} &= 0.19_{-0.03-0.01}^{+0.03+0.00}.
\end{aligned} \tag{3.1}$$

- (2) Form factors obtained by QCD sum rules are

$$F_0^{B_s K} = 0.30_{-0.03}^{+0.04}, \tag{3.2}$$

for the $B_s \rightarrow K$ transition [21] and

$$\begin{aligned}
V^{B_s K^*} &= 0.311 \pm 0.026, \\
A_0^{B_s K^*} &= 0.360 \pm 0.034, \\
A_1^{B_s K^*} &= 0.233 \pm 0.022, \\
V^{B_s \phi} &= 0.434 \pm 0.035, \\
A_0^{B_s \phi} &= 0.474 \pm 0.033, \\
A_1^{B_s \phi} &= 0.311 \pm 0.030,
\end{aligned} \tag{3.3}$$

for $B_s \rightarrow V$ transitions [22].

- (3) Another light-cone sum-rule calculation based on heavy quark effective theory gives [23]

$$F_0^{B_s K} = 0.296 \pm 0.018, \quad F_0^{B_s \eta} = 0.281_{-0.016}^{+0.015}, \tag{3.4}$$

and

$$\begin{aligned}
V^{B_s K^*} &= 0.285_{-0.013}^{+0.013}, & A_0^{B_s K^*} &= 0.222_{-0.010}^{+0.011}, \\
A_1^{B_s K^*} &= 0.227_{-0.012}^{+0.010}, & V^{B_s \phi} &= 0.339_{-0.017}^{+0.016}, \\
A_0^{B_s \phi} &= 0.269_{-0.014}^{+0.014}, & A_1^{B_s \phi} &= 0.271_{-0.014}^{+0.014}.
\end{aligned} \tag{3.5}$$

It is clear that form factors obtained by sum rules are larger than the pQCD ones.

- (4) A light-cone quark model in conjunction with soft-collinear effective theory was constructed in [24]. The predictions are

$$\begin{aligned}
 F_0^{B_s K} &= 0.290, & F_0^{B_s \eta_s} &= 0.288, \\
 V^{B_s K^*} &= 0.323, & A_0^{B_s K^*} &= 0.279, \\
 A_1^{B_s K^*} &= 0.228, & V^{B_s \phi} &= 0.329, \\
 A_0^{B_s \phi} &= 0.279, & A_1^{B_s \phi} &= 0.232.
 \end{aligned}
 \tag{3.6}$$

(5) A straightforward application of the covariant light-front quark model of [25] yields [26]

$$\begin{aligned}
 V^{B_s K^*} &= 0.23, & A_0^{B_s K^*} &= 0.26, \\
 A_1^{B_s K^*} &= 0.19, & V^{B_s \phi} &= 0.30, \\
 A_0^{B_s \phi} &= 0.32, & A_1^{B_s \phi} &= 0.26,
 \end{aligned}
 \tag{3.7}$$

all with errors estimated to be ± 0.01 .

(6) A recent lattice QCD calculation yields $F_0^{B_s K} = 0.23 \pm 0.05 \pm 0.04$ [27].

For comparison, Beneke and Neubert [6] used

$$\begin{aligned}
 F_0^{B_s K} &= 0.31 \pm 0.05, & A_0^{B_s K^*} &= 0.29 \pm 0.05, \\
 A_0^{B_s \phi} &= 0.34 \pm 0.05,
 \end{aligned}
 \tag{3.8}$$

and

$$F_0^{B_s \rightarrow \eta^{(\prime)}} = F_0^{BK} \frac{f_{\eta^{(\prime)}}^q}{f_\pi} + F_2 \frac{\sqrt{2}f_{\eta^{(\prime)}}^q + f_{\eta^{(\prime)}}^s}{\sqrt{3}f_\pi},
 \tag{3.9}$$

while Beneke, Rohrer and Yang [8] employed

$$A_0^{B_s K^*} = 0.33 \pm 0.05, \quad A_0^{B_s \phi} = 0.38_{-0.02}^{+0.10}.
 \tag{3.10}$$

Note that it is most convenient to express the form factors for $B \rightarrow \eta^{(\prime)}$ transitions in terms of the flavor states $q\bar{q} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$, $s\bar{s}$ and $c\bar{c}$ labeled by the η_q , η_s and η_c^0 , respectively. Neglecting the small mixing with η_c^0 , we have

$$F^{B_s \eta} = -F^{B_s \eta_s} \sin\theta, \quad F^{B_s \eta'} = F^{B_s \eta_s} \cos\theta,
 \tag{3.11}$$

where θ is the $\eta_q - \eta_s$ mixing angle defined by

$$\begin{aligned}
 |\eta\rangle &= \cos\theta|\eta_q\rangle - \sin\theta|\eta_s\rangle, \\
 |\eta'\rangle &= \sin\theta|\eta_q\rangle + \cos\theta|\eta_s\rangle,
 \end{aligned}
 \tag{3.12}$$

with $\theta = (39.3 \pm 1.0)^\circ$ in the Feldmann-Kroll-Stech mixing scheme [28].

From the above discussions we see that the form factor $F_0^{B_s K}$ at $q^2 = 0$ ranges from 0.23 to 0.31. In the QCDF approach, if $F_0^{B_s K}(0) = 0.31$ is employed, we find that the predicted branching fractions $\mathcal{B}(\bar{B}_s \rightarrow K^+ \pi^-) \approx 9.1 \times 10^{-6}$ and $\mathcal{B}(\bar{B}_s \rightarrow K^+ K^-) \approx 34 \times 10^{-6}$ will be far above the experimental measurements of $(5.0 \pm 1.1) \times 10^{-6}$ [29] and $(25.7 \pm 3.6) \times 10^{-6}$ [30,31], respectively. Hence we shall use $F_0^{B_s K}(0) = 0.24$ obtained by the lattice calculation. Note that a χ^2 analysis by one of us (C. K. C.)

with the available data of $B_s \rightarrow PP$ also yields $F_0^{B_s K}(0) = 0.240_{-0.007}^{+0.021}$ [32]. For other form factors, we shall use $F_0^{B_s \eta_s}(0) = 0.28$ and $B_s \rightarrow V$ transition form factors given by Eq. (3.7) with some modifications on $B_s \rightarrow K^*$ ones (see Table I).

B. Decay constants

Decay constants of various vector mesons defined by

$$\begin{aligned}
 \langle V(p, \epsilon) | \bar{q}_2 \gamma_\mu q_1 | 0 \rangle &= -if_V m_V \epsilon_\mu^*, \\
 \langle V(p, \epsilon) | \bar{q}_2 \sigma_{\mu\nu} q_1 | 0 \rangle &= -f_V^\perp (\epsilon_\mu^* p_\nu - \epsilon_\nu^* p_\mu),
 \end{aligned}
 \tag{3.13}$$

are listed in Table I. They are taken from [35]. For pseudoscalar mesons, we use $f_\pi = 132$ MeV and $f_K = 160$ MeV. Decay constants $f_{\eta^{(\prime)}}^q$, $f_{\eta^{(\prime)}}^s$ and $f_{\eta^{(\prime)}}^c$ defined by

$$\begin{aligned}
 \langle 0 | \bar{q} \gamma_\mu \gamma_5 q | \eta^{(\prime)} \rangle &= i \frac{1}{\sqrt{2}} f_{\eta^{(\prime)}}^q q_\mu, \\
 \langle 0 | \bar{s} \gamma_\mu \gamma_5 s | \eta^{(\prime)} \rangle &= i f_{\eta^{(\prime)}}^s q_\mu, \\
 \langle 0 | \bar{c} \gamma_\mu \gamma_5 c | \eta^{(\prime)} \rangle &= i f_{\eta^{(\prime)}}^c q_\mu
 \end{aligned}
 \tag{3.14}$$

are also needed in calculations. For the decay constants $f_{\eta^{(\prime)}}^q$ and $f_{\eta^{(\prime)}}^s$, we shall use the values

$$\begin{aligned}
 f_\eta^q &= 107 \text{ MeV}, & f_\eta^s &= -112 \text{ MeV}, \\
 f_{\eta'}^q &= 89 \text{ MeV}, & f_{\eta'}^s &= 137 \text{ MeV}
 \end{aligned}
 \tag{3.15}$$

obtained in [28]. As for $f_{\eta^{(\prime)}}^c$, a straightforward perturbative calculation gives [36]

$$f_{\eta^{(\prime)}}^c = -\frac{m_{\eta^{(\prime)}}^2}{12m_c^2} \frac{f_{\eta^{(\prime)}}^q}{\sqrt{2}}.
 \tag{3.16}$$

C. LCDAs

We next specify the light-cone distribution amplitudes (LCDAs) for pseudoscalar and vector mesons. The general expressions of twist-2 LCDAs are

$$\begin{aligned}
 \Phi_P(x, \mu) &= 6x(1-x) \left[1 + \sum_{n=1}^{\infty} a_n^P(\mu) C_n^{3/2}(2x-1) \right], \\
 \Phi_{\parallel}^V(x, \mu) &= 6x(1-x) \left[1 + \sum_{n=1}^{\infty} a_n^V(\mu) C_n^{3/2}(2x-1) \right], \\
 \Phi_{\perp}^V(x, \mu) &= 6x(1-x) \left[1 + \sum_{n=1}^{\infty} a_n^{\perp, V}(\mu) C_n^{3/2}(2x-1) \right],
 \end{aligned}
 \tag{3.17}$$

and twist-3 ones

TABLE I. Input parameters. The values of the scale dependent quantities $f_V^\perp(\mu)$ and $a_{1,2}^{\perp,V}(\mu)$ are given for $\mu = 1$ GeV. The values of Gegenbauer moments are taken from [33] and Wolfenstein parameters from [34].

| | | | | | | |
|---------------------------|-------------|-----------------------|---------------------|---------------------------------------|-----------------|-----------------------------------|
| Light vector mesons | | | | | | |
| V | f_V (MeV) | f_V^\perp (MeV) | a_1^V | a_2^V | $a_1^{\perp,V}$ | $a_2^{\perp,V}$ |
| ρ | 216 ± 3 | 165 ± 9 | 0 | 0.15 ± 0.07 | 0 | 0.14 ± 0.06 |
| ω | 187 ± 5 | 151 ± 9 | 0 | 0.15 ± 0.07 | 0 | 0.14 ± 0.06 |
| ϕ | 215 ± 5 | 186 ± 9 | 0 | 0.18 ± 0.08 | 0 | 0.14 ± 0.07 |
| K^* | 220 ± 5 | 185 ± 10 | 0.03 ± 0.02 | 0.11 ± 0.09 | 0.04 ± 0.03 | 0.10 ± 0.08 |
| Light pseudoscalar mesons | | | | | | |
| a_1^π | | a_2^π | | a_1^K | | a_2^K |
| 0 | | 0.25 ± 0.15 | | 0.06 ± 0.03 | | 0.25 ± 0.15 |
| B mesons | | | | | | |
| B | m_B (GeV) | | τ_B (ps) | | f_B (MeV) | λ_B (MeV) |
| B_u | 5.279 | | 1.638 | | 210 ± 20 | 300 ± 100 |
| B_d | 5.279 | | 1.525 | | 210 ± 20 | 300 ± 100 |
| B_s | 5.366 | | 1.472 | | 230 ± 20 | 300 ± 100 |
| Form factors at $q^2 = 0$ | | | | | | |
| $F_0^{B_s K}(0)$ | | $A_0^{B_s K^*}(0)$ | $A_1^{B_s K^*}(0)$ | $A_2^{B_s K^*}(0)$ | | $V_0^{B_s K^*}(0)$ |
| 0.24 | | 0.30 ± 0.01 | 0.24 ± 0.01 | 0.22 ± 0.01 | | 0.28 ± 0.01 |
| $F_0^{B_s \eta_s}(0)$ | | $A_0^{B_s \phi}(0)$ | $A_1^{B_s \phi}(0)$ | $A_2^{B_s \phi}(0)$ | | $V_0^{B_s \phi}(0)$ |
| 0.28 | | 0.32 ± 0.01 | 0.26 ± 0.01 | 0.23 ± 0.01 | | 0.30 ± 0.01 |
| Quark masses | | | | | | |
| $m_b(m_b)/\text{GeV}$ | | $m_c(m_b)/\text{GeV}$ | | $m_c^{\text{pole}}/m_b^{\text{pole}}$ | | $m_s(2.1 \text{ GeV})/\text{GeV}$ |
| 4.2 | | 0.91 | | 0.3 | | 0.095 ± 0.020 |
| Wolfenstein parameters | | | | | | |
| A | | λ | $\bar{\rho}$ | $\bar{\eta}$ | | γ |
| 0.8116 | | 0.2252 | 0.139 | 0.341 | | $(67.8_{-3.9}^{+4.2})^\circ$ |

$$\Phi_p(x) = 1, \quad \Phi_\sigma(x) = 6x(1-x),$$

$$\Phi_v(x, \mu) = 3 \left[2x - 1 + \sum_{n=1}^{\infty} a_n^{\perp,V}(\mu) P_{n+1}(2x-1) \right], \quad (3.18)$$

where $C_n(x)$ and $P_n(x)$ are the Gegenbauer and Legendre polynomials, respectively. When three-particle amplitudes are neglected, the twist-3 $\Phi_v(x)$ can be expressed in terms of Φ_\perp

$$\Phi_v(x) = \int_0^x \frac{\Phi_\perp(u)}{\bar{u}} du - \int_x^1 \frac{\Phi_\perp(u)}{u} du. \quad (3.19)$$

The normalization of LCDAs is

$$\int_0^1 dx \Phi_V(x) = 1, \quad \int_0^1 dx \Phi_v(x) = 0. \quad (3.20)$$

Note that the Gegenbauer moments $a_i^{(\perp),K^*}$ displayed in Table I taken from [33] are for the mesons containing a strange quark.

The integral of the B meson wave function is parameterized as [19]

$$\int_0^1 \frac{d\rho}{1-\rho} \Phi_1^B(\rho) \equiv \frac{m_B}{\lambda_B}, \quad (3.21)$$

where $1-\rho$ is the momentum fraction carried by the light spectator quark in the B meson. We shall use $\lambda_B = 300 \pm 100$ MeV.

For the running quark masses we shall use [37,38]

$$\begin{aligned} m_b(m_b) &= 4.2 \text{ GeV}, & m_b(2.1 \text{ GeV}) &= 4.94 \text{ GeV}, \\ m_b(1 \text{ GeV}) &= 6.34 \text{ GeV}, & m_c(m_b) &= 0.91 \text{ GeV}, \\ m_c(2.1 \text{ GeV}) &= 1.06 \text{ GeV}, & m_c(1 \text{ GeV}) &= 1.32 \text{ GeV}, \\ m_s(2.1 \text{ GeV}) &= 95 \text{ MeV}, & m_s(1 \text{ GeV}) &= 118 \text{ MeV}, \\ m_d(2.1 \text{ GeV}) &= 5.0 \text{ MeV}, & m_u(2.1 \text{ GeV}) &= 2.2 \text{ MeV}. \end{aligned} \quad (3.22)$$

Note that the charm quark masses here are smaller than the one $m_c(m_b) = 1.3 \pm 0.2$ GeV adopted in [6,9] and consistent with the high precision mass determination from lattice QCD [39]: $m_c(3 \text{ GeV}) = 0.986 \pm 0.010$ GeV and $m_c(m_c) = 1.267 \pm 0.009$ GeV (see also [40]). Among the quarks, the strange quark gives the major theoretical uncertainty to the decay amplitude. Hence, we will only consider the uncertainty in the strange quark mass given by $m_s(2.1 \text{ GeV}) = 95 \pm 20$ MeV. Notice that for the one-loop penguin contribution, the relevant quark mass is the pole mass rather than the current one [41]. Since the penguin loop correction is governed by the ratio of the

pole masses squared $s_i \equiv (m_i^{\text{pole}}/m_b^{\text{pole}})^2$ and since the pole mass is meaningful only for heavy quarks, we only need to consider the ratio of c and b quark pole masses given by $s_c \approx (0.3)^2$.

D. Penguin annihilation

In the QCDF approach, the hadronic B decay amplitude receives contributions from tree, penguin, electroweak penguin and weak annihilation topologies. In the absence of $1/m_b$ power corrections except for the chiral enhanced penguin contributions, the leading QCDF predictions encounter three major difficulties: (i) the predicted branching fractions for penguin-dominated $B \rightarrow PP, VP, VV$ decays are systematically below the measurements, (ii) direct CP -violating asymmetries for $\bar{B}_d \rightarrow K^- \pi^+$, $\bar{B}_d \rightarrow K^{*-} \pi^+$, $B^- \rightarrow K^- \rho^0$, $\bar{B}_d \rightarrow \pi^+ \pi^-$ and $\bar{B}_s \rightarrow K^+ \pi^-$ have signs in disagreement with experiment, and (iii) the predicted longitudinal polarization fractions in penguin-dominated $B \rightarrow VV$ decays are usually too large and do not agree with the data. This implies the necessity of introducing $1/m_b$ power corrections. Unfortunately, there are many possible $1/m_b$ power suppressed effects and they are generally nonperturbative in nature and hence not calculable by the perturbative method.

Power corrections in QCDF always involve troublesome endpoint divergences. For example, the annihilation amplitude has endpoint divergences even at twist-2 level and the hard spectator scattering diagram at twist-3 order is power suppressed and possesses soft and collinear divergences arising from the soft spectator quark. Since the treatment of endpoint divergences is model dependent, subleading power corrections generally can be studied only in a phenomenological way. We shall follow [19] to model the endpoint divergence $X \equiv \int_0^1 dx/\bar{x}$ in the annihilation and hard spectator scattering diagrams as

$$\begin{aligned} X_A &= \ln\left(\frac{m_B}{\Lambda_h}\right)(1 + \rho_A e^{i\phi_A}), \\ X_H &= \ln\left(\frac{m_B}{\Lambda_h}\right)(1 + \rho_H e^{i\phi_H}), \end{aligned} \quad (3.23)$$

with Λ_h being a typical scale of order 500 MeV, and $\rho_{A,H}, \phi_{A,H}$ being the unknown real parameters.

A fit to the data of $B_{u,d} \rightarrow PP, VP, PV$ and VV decays yields the values of ρ_A and ϕ_A shown in Table II. Basically,

TABLE II. The parameters ρ_A and ϕ_A for penguin annihilation.

| Modes | ρ_A | ϕ_A | Modes | ρ_A | ϕ_A |
|--------------------------|----------|-------------|----------------------|----------|-------------|
| $B \rightarrow PP$ | 1.10 | -50° | $B_s \rightarrow PP$ | 1.00 | -55° |
| $B \rightarrow VP$ | 1.07 | -70° | $B_s \rightarrow VP$ | 0.90 | -65° |
| $B \rightarrow PV$ | 0.87 | -30° | $B_s \rightarrow PV$ | 0.85 | -30° |
| $B \rightarrow K^* \rho$ | 0.78 | -43° | $B_s \rightarrow VV$ | 0.70 | -55° |
| $B \rightarrow K^* \phi$ | 0.65 | -53° | | | |

it is very similar to the so-called ‘‘S4 scenario’’ presented in [6]. The fitted ρ_A and ϕ_A for $B \rightarrow VV$ decays are taken from [42]. Since the penguin annihilation effects are different for $B \rightarrow VP$ and $B \rightarrow PV$ decays,

$$\begin{aligned} A_1^i &\approx -A_2^i \\ &\approx 6\pi\alpha_s \left[3\left(X_A^{VP} - 4 + \frac{\pi^2}{3}\right) + r_\chi^V r_\chi^P \left((X_A^{VP})^2 - 2X_A^{VP} \right) \right], \\ A_3^i &\approx 6\pi\alpha_s \left[-3r_\chi^V \left((X_A^{VP})^2 - 2X_A^{VP} + 4 - \frac{\pi^2}{3} \right) \right. \\ &\quad \left. + r_\chi^P \left((X_A^{VP})^2 - 2X_A^{VP} + \frac{\pi^2}{3} \right) \right], \\ A_3^f &\approx 6\pi\alpha_s [3r_\chi^V(2X_A^{VP} - 1)(2 - X_A^{VP}) \\ &\quad - r_\chi^P(2(X_A^{VP})^2 - X_A^{VP})], \end{aligned} \quad (3.24)$$

for $M_1 M_2 = VP$ [the definition for the parameters r_χ^P and r_χ^V can be found in Eq. (2.11) below] and

$$\begin{aligned} A_1^i &\approx -A_2^i \\ &\approx 6\pi\alpha_s \left[3\left(X_A^{PV} - 4 + \frac{\pi^2}{3}\right) + r_\chi^V r_\chi^P \left((X_A^{PV})^2 - 2X_A^{PV} \right) \right], \\ A_3^i &\approx 6\pi\alpha_s \left[-3r_\chi^P \left((X_A^{PV})^2 - 2X_A^{PV} + 4 - \frac{\pi^2}{3} \right) \right. \\ &\quad \left. + r_\chi^V \left((X_A^{PV})^2 - 2X_A^{PV} + \frac{\pi^2}{3} \right) \right], \\ A_3^f &\approx 6\pi\alpha_s [-3r_\chi^P(2X_A^{PV} - 1)(2 - X_A^{PV}) \\ &\quad + r_\chi^V(2(X_A^{PV})^2 - X_A^{PV})], \end{aligned} \quad (3.25)$$

for $M_1 M_2 = PV$, the parameters X_A^{VP} and X_A^{PV} are not necessarily the same. Indeed, a fit to the $B \rightarrow VP, PV$ decays yields $\rho_A^{VP} \approx 1.07$, $\phi_A^{VP} \approx -70^\circ$ and $\rho_A^{PV} \approx 0.87$, $\phi_A^{PV} \approx -30^\circ$ (see Table II). For $B_s \rightarrow PP, VP, VV$ decays, we shall assume that their default values are similar to that in $B_{u,d}$ decays as shown in Table II. For the estimate of theoretical uncertainties, we shall assign an error of ± 0.1 to ρ_A and $\pm 20^\circ$ to ϕ_A .

E. Power corrections to a_2

As pointed out in [15], while the discrepancies between experiment and theory in the heavy quark limit for the rates of penguin-dominated two-body decays of B mesons and direct CP asymmetries of $\bar{B}_d \rightarrow K^- \pi^+$, $B^- \rightarrow K^- \rho^0$ and $\bar{B}_d \rightarrow \pi^+ \pi^-$ are resolved by introducing power corrections coming from penguin annihilation, the signs of direct CP -violating effects in $B^- \rightarrow K^- \pi^0$, $B^- \rightarrow K^- \eta$ and $\bar{B}^0 \rightarrow \pi^0 \pi^0$ are flipped to the wrong ones when confronted with experiment. These new B - CP puzzles in QCDF can be explained by the subleading power corrections to the color-suppressed tree amplitudes due to spectator interactions and/or final-state interactions that not only reproduce correct signs for aforementioned CP asymmetries but also

accommodate the observed $\bar{B}_d \rightarrow \pi^0 \pi^0$ and $\rho^0 \pi^0$ rates simultaneously.

Following [15], power corrections to the color-suppressed topology are parametrized as

$$a_2 \rightarrow a_2(1 + \rho_C e^{i\phi_C}), \quad (3.26)$$

with the unknown parameters ρ_C and ϕ_C to be inferred from experiment. We shall use $\phi_C \approx -70^\circ$ and $\rho_C \approx 1.3, 0.8, 0$ for $B \rightarrow PP, VP, VV$ decays [15,17], respectively. This pattern that soft power corrections to a_2 are large for PP modes, moderate for VP ones and very small for VV cases is consistent with the observation made in [43] that soft power correction dominance is much larger for PP than VP and VV final states. It has been argued that this has to do with the special nature of the pion which is a $q\bar{q}$ bound state on the one hand and a nearly massless Nambu-Goldstone boson on the other hand [43].

IV. $B_s \rightarrow PP$ DECAYS

Before proceeding to the numerical results of QCDF calculations, we discuss some model-independent flavor symmetry relations in which many of $B_s \rightarrow PP$ decays can be related to $B_d \rightarrow PP$ ones by either U -spin or $SU(3)$ symmetry. Hence these relations can be used to cross-check the dynamical calculations.

A. U -spin symmetry

In the limit of U -spin symmetry, some of B_s decays can be related to B_d ones. For example,

$$\begin{aligned} A(\bar{B}_s \rightarrow K^+ \pi^-) &= V_{ub}^* V_{ud} \langle K^+ \pi^- | O_d^u | \bar{B}_s \rangle \\ &\quad + V_{cb}^* V_{cd} \langle K^+ \pi^- | O_d^c | \bar{B}_s \rangle, \\ A(\bar{B}_d \rightarrow K^- \pi^+) &= V_{ub}^* V_{us} \langle K^- \pi^+ | O_s^u | \bar{B}_d \rangle \\ &\quad + V_{cb}^* V_{cs} \langle K^- \pi^+ | O_s^c | \bar{B}_d \rangle, \end{aligned} \quad (4.1)$$

where the 4-quark operator O_s is for the $b \rightarrow q\bar{q}s$ transi-

tion and O_d for the $b \rightarrow q\bar{q}d$ transition. The assumption of U -spin symmetry implies that under $d \leftrightarrow s$ transitions,

$$\begin{aligned} \langle K^+ \pi^- | O_d^u | \bar{B}_s \rangle &= \langle K^- \pi^+ | O_s^u | \bar{B}_d \rangle, \\ \langle K^+ \pi^- | O_d^c | \bar{B}_s \rangle &= \langle K^- \pi^+ | O_s^c | \bar{B}_d \rangle. \end{aligned} \quad (4.2)$$

Using the relation

$$\text{Im}(V_{ub}^* V_{ud} V_{cb} V_{cd}^*) = -\text{Im}(V_{ub}^* V_{us} V_{cb} V_{cs}^*), \quad (4.3)$$

it is straightforward to show that [44–46]

$$\begin{aligned} |A(\bar{B}_s \rightarrow K^+ \pi^-)|^2 - |A(B_s \rightarrow K^- \pi^+)|^2 \\ = |A(B_d \rightarrow K^+ \pi^-)|^2 - |A(\bar{B}_d \rightarrow K^- \pi^+)|^2, \end{aligned} \quad (4.4)$$

and, consequently,

$$\begin{aligned} A_{CP}(\bar{B}_s \rightarrow K^+ \pi^-) &= -A_{CP}(\bar{B}_d \rightarrow K^- \pi^+) \\ &\quad \times \frac{\mathcal{B}(\bar{B}_d \rightarrow K^- \pi^+) \tau(B_s)}{\mathcal{B}(\bar{B}_s \rightarrow K^+ \pi^-) \tau(B_d)}. \end{aligned} \quad (4.5)$$

From the current world averages, $A_{CP}(\bar{B}_d \rightarrow K^- \pi^+) = -0.098_{-0.011}^{+0.012}$, $\mathcal{B}(\bar{B}_d \rightarrow K^- \pi^+) = (19.4 \pm 0.6) \times 10^{-6}$ [47] and the CDF measurement $\mathcal{B}(\bar{B}_s \rightarrow K^+ \pi^-) = (5.0 \pm 1.1) \times 10^{-6}$ [29], it follows that the prediction $A_{CP}(\bar{B}_s \rightarrow K^+ \pi^-) \approx 0.37$ under U -spin symmetry is in good agreement with the experimental result $0.39 \pm 0.15 \pm 0.08$ obtained by CDF [29]. Besides $A_{CP}(\bar{B}_s \rightarrow K^+ \pi^-)$, CDF has also measured direct CP violation in the decay $\bar{B}_d \rightarrow K^- \pi^+$ and obtained [30]

$$\begin{aligned} \frac{\Gamma(\bar{B}_d \rightarrow K^- \pi^+) - \Gamma(B_d \rightarrow K^+ \pi^-)}{\Gamma(\bar{B}_s \rightarrow K^+ \pi^-) - \Gamma(B_s \rightarrow K^- \pi^+)} \\ = -0.83 \pm 0.41 \pm 0.12, \end{aligned} \quad (4.6)$$

which is equal to -1 under U -spin symmetry. Obviously, the experimental measurement is still limited by statistics.

By the same token, we also have the following U -spin relations

$$\begin{aligned} A_{CP}(\bar{B}_s \rightarrow K^+ K^-) &= -A_{CP}(\bar{B}_d \rightarrow \pi^+ \pi^-) \frac{\mathcal{B}(\bar{B}_d \rightarrow \pi^+ \pi^-) \tau(B_s)}{\mathcal{B}(\bar{B}_s \rightarrow K^+ K^-) \tau(B_d)}, \\ A_{CP}(\bar{B}_s \rightarrow K^0 \bar{K}^0) &= -A_{CP}(\bar{B}_d \rightarrow K^0 \bar{K}^0) \frac{\mathcal{B}(\bar{B}_d \rightarrow K^0 \bar{K}^0) \tau(B_s)}{\mathcal{B}(\bar{B}_s \rightarrow K^0 \bar{K}^0) \tau(B_d)}, \\ A_{CP}(\bar{B}_s \rightarrow K^0 \pi^0) &= -A_{CP}(\bar{B}_d \rightarrow \bar{K}^0 \pi^0) \frac{\mathcal{B}(\bar{B}_d \rightarrow \bar{K}^0 \pi^0) \tau(B_s)}{\mathcal{B}(\bar{B}_s \rightarrow K^0 \pi^0) \tau(B_d)}, \\ A_{CP}(\bar{B}_s \rightarrow \pi^+ \pi^-) &= -A_{CP}(\bar{B}_d \rightarrow K^+ K^-) \frac{\mathcal{B}(\bar{B}_d \rightarrow K^+ K^-) \tau(B_s)}{\mathcal{B}(\bar{B}_s \rightarrow \pi^+ \pi^-) \tau(B_d)}. \end{aligned} \quad (4.7)$$

Unlike the first U -spin symmetry relation (4.5), the above relations cannot be tested by the present available data. Nevertheless, they can be checked by our dynamical calculations as shown in Sec. IV C 5.

B. $SU(3)$ symmetry

There are some cases where two-body decays of B_d and B_s can be related to each other in the limit of $SU(3)$ symmetry provided that some of the annihilation effects can be neglected. Let us consider the decay amplitudes of the following three

pairs in QCDF [6]:

$$\begin{aligned}
 A(\bar{B}_s \rightarrow K^+ \pi^-) &= \sum_{p=u,c} V_{pb}^* V_{pd} A_{K\pi} \left(\delta_{pu} \alpha_1 + \alpha_4^p + \alpha_{4,\text{EW}}^p + \beta_3^p - \frac{1}{2} \beta_{3,\text{EW}}^p \right), \\
 A(\bar{B}_d \rightarrow \pi^+ \pi^-) &= \sum_{p=u,c} V_{pb}^* V_{pd} A_{\pi\pi} \left(\delta_{pu} \alpha_1 + \alpha_4^p + \alpha_{4,\text{EW}}^p + \delta_{pu} \beta_1 + \beta_3^p - \frac{1}{2} \beta_{3,\text{EW}}^p + \frac{1}{2} \beta_{4,\text{EW}}^p \right),
 \end{aligned} \tag{4.8}$$

and

$$\begin{aligned}
 A(\bar{B}_s \rightarrow K^+ K^-) &= \sum_{p=u,c} V_{pb}^* V_{ps} \left[A_{K\bar{K}} \left(\delta_{pu} \alpha_1 + \alpha_4^p + \alpha_{4,\text{EW}}^p + \beta_3^p + \beta_4^p - \frac{1}{2} \beta_{3,\text{EW}}^p - \frac{1}{2} \beta_{4,\text{EW}}^p \right) \right. \\
 &\quad \left. + B_{\bar{K}\bar{K}} (\delta_{pu} b_1^p + b_4^p + b_{4,\text{EW}}^p) \right], \\
 A(\bar{B}_d \rightarrow K^- \pi^+) &= \sum_{p=u,c} V_{pb}^* V_{ps} A_{\pi\bar{K}} \left(\delta_{pu} \alpha_1 + \alpha_4^p + \alpha_{4,\text{EW}}^p + \beta_3^p - \frac{1}{2} \beta_{3,\text{EW}}^p \right),
 \end{aligned} \tag{4.9}$$

and

$$\begin{aligned}
 A(\bar{B}_s \rightarrow K^0 \bar{K}^0) &= \sum_{p=u,c} V_{pb}^* V_{ps} \left[A_{K\bar{K}} \left(\alpha_4^p - \frac{1}{2} \alpha_{4,\text{EW}}^p + \beta_3^p + \beta_4^p - \frac{1}{2} \beta_{3,\text{EW}}^p - \frac{1}{2} \beta_{4,\text{EW}}^p \right) + B_{\bar{K}\bar{K}} \left(b_4^p - \frac{1}{2} b_{4,\text{EW}}^p \right) \right], \\
 A(B^- \rightarrow \bar{K}^0 \pi^-) &= \sum_{p=u,c} V_{pb}^* V_{ps} A_{\pi\bar{K}} \left(\alpha_4^p - \frac{1}{2} \alpha_{4,\text{EW}}^p + \delta_{pu} \beta_2 + \beta_3^p + \beta_{3,\text{EW}}^p \right),
 \end{aligned} \tag{4.10}$$

with $A_{h_1 h_2} \equiv X(\bar{B}_s, h_1, h_2)$, where the expressions of the flavor operators α_i in terms of a_i and the annihilation operators β_i in terms of b_i are shown in Eq. (2.9). Roughly speaking, α_1 is due to the tree topology, α_4 comes from the QCD penguin operators O_4 and O_6 , $\alpha_{4,\text{EW}}^p$ receives contributions from the electroweak operators O_8 and O_{10} . From the study of hadronic $B_{u,d}$ decays we learn that annihilation effects are negligible in tree-dominated modes and dominated by the β_3 term in penguin-dominated decays. Hence, under the approximation of negligible annihilation contributions to tree-dominated decays and keeping only the dominant penguin annihilations in penguin-dominated decays, SU(3) symmetry (or U -spin symmetry acting on the spectator quark of the B meson) implies [45,48]

$$\begin{aligned}
 A(\bar{B}_s \rightarrow K^+ \pi^-) &\approx A(\bar{B}_d \rightarrow \pi^+ \pi^-), \\
 A(\bar{B}_s \rightarrow K^+ K^-) &\approx A(\bar{B}_d \rightarrow K^- \pi^+), \\
 A(\bar{B}_s \rightarrow K^0 \bar{K}^0) &\approx A(B^- \rightarrow \bar{K}^0 \pi^-).
 \end{aligned} \tag{4.11}$$

As will be discussed later, it turns out that among the relations

$$\begin{aligned}
 \mathcal{B}(\bar{B}_s \rightarrow K^+ \pi^-) &\approx \mathcal{B}(\bar{B}_d \rightarrow \pi^+ \pi^-), \\
 A_{CP}(\bar{B}_s \rightarrow K^+ \pi^-) &\approx A_{CP}(\bar{B}_d \rightarrow \pi^+ \pi^-), \\
 \mathcal{B}(\bar{B}_s \rightarrow K^+ K^-) &\approx \mathcal{B}(\bar{B}_d \rightarrow K^- \pi^+), \\
 A_{CP}(\bar{B}_s \rightarrow K^+ K^-) &\approx A_{CP}(\bar{B}_d \rightarrow K^- \pi^+), \\
 \mathcal{B}(\bar{B}_s \rightarrow K^0 \bar{K}^0) &\approx \mathcal{B}(B^- \rightarrow \bar{K}^0 \pi^-), \\
 A_{CP}(\bar{B}_s \rightarrow K^0 \bar{K}^0) &\approx A_{CP}(B^- \rightarrow \bar{K}^0 \pi^-),
 \end{aligned} \tag{4.12}$$

the first three ones are experimentally fairly satisfied.

C. Numerical results and comparison with other approaches

We list in Tables III and IV the branching fractions and CP asymmetries of $\bar{B}_s \rightarrow PP$ decays evaluated in the frameworks of QCD factorization (this work), pQCD to the lowest order (LO) [11] and to the next-to-leading order (NLO) [12] and SCET [13]. For the decays involving an η and/or η' , two different sets of SCET results are quoted from [13], corresponding to two distinct SCET parameters regarding to the strong phases of the gluonic charming penguin. The expression for the decay amplitudes of $\bar{B}_s \rightarrow PP$ and VP decays in the QCDF approach can be found in the appendix of [6].

The theoretical errors in QCDF calculations correspond to the uncertainties due to the variation of (i) the Gegenbauer moments, the decay constants, (ii) the heavy-to-light form factors and the strange quark mass, and (iii) the wave function of the B meson characterized by the parameter λ_B , the power corrections due to weak annihilation and hard spectator interactions described by the parameters $\rho_{A,H}$, $\phi_{A,H}$, respectively. To obtain the errors shown in Tables III, IV, V, VI, VII, VIII, IX, X, XI, XII, XIII, and XIV, we first scan randomly the points in the allowed ranges of the above nine parameters and then add errors in quadrature. As noted in passing, we assign an error ± 0.1 and $\pm 20^\circ$ to the default values of ρ_A and ϕ_A , respectively, while ρ_H and ϕ_H lie in the ranges $0 \leq \rho_H \leq 1$ and $0 \leq \phi_H \leq 2\pi$. Specifically, the second error in the table is referred to the uncertainties caused by the variation of $\rho_{A,H}$ and $\phi_{A,H}$, where all other uncertainties are lumped into the first error. Power corrections beyond the heavy quark limit generally give the major theoretical uncertain-

TABLE III. CP -averaged branching fractions (in units of 10^{-6}) of $\bar{B}_s \rightarrow PP$ decays obtained in various approaches. In the QCDF calculations, the parameters ρ_A and ϕ_A are taken from Table II, $\rho_C = 1.3$ and $\phi_C = -70^\circ$. Sources of theoretical uncertainties are discussed in the text. The pQCD predictions to LO and (partial) NLO are taken from [11,12], respectively. For the decays involving an η and/or η' , two different sets of SCET results are quoted from [13].

| Modes | Class | QCDF (this work) | pQCD (LO) | pQCD (NLO) | SCET | Experiment [18,29] |
|---|--------------|----------------------------------|------------------------|------------------------|--|--------------------|
| $\bar{B}_s^0 \rightarrow K^+ \pi^-$ | T | $5.3^{+0.4+0.4}_{-0.8-0.5}$ | $7.6^{+3.3}_{-2.5}$ | $6.3^{+2.6}_{-1.9}$ | $4.9 \pm 1.2 \pm 1.3 \pm 0.3$ | 5.0 ± 1.1 |
| $\bar{B}_s^0 \rightarrow K^0 \pi^0$ | C | $1.7^{+2.5+1.2}_{-0.8-0.5}$ | $0.16^{+0.12}_{-0.07}$ | $0.25^{+0.10}_{-0.07}$ | $0.76 \pm 0.26 \pm 0.27 \pm 0.17$ | |
| $\bar{B}_s^0 \rightarrow K^0 \eta$ | C | $0.75^{+1.10+0.51}_{-0.35-0.22}$ | $0.11^{+0.08}_{-0.11}$ | $0.08^{+0.03}_{-0.02}$ | $0.80 \pm 0.48 \pm 0.29 \pm 0.18$ $0.59 \pm 0.34 \pm 0.24 \pm 0.15$ | |
| $\bar{B}_s^0 \rightarrow K^0 \eta'$ | C | $2.8^{+2.5+1.1}_{-1.0-0.8}$ | $0.72^{+0.36}_{-0.24}$ | $1.87^{+0.45}_{-0.56}$ | $4.5 \pm 1.5 \pm 0.4 \pm 0.5$ $3.9 \pm 1.3 \pm 0.5 \pm 0.4$ | |
| $\bar{B}_s^0 \rightarrow K^+ K^-$ | P | $25.2^{+12.7+12.5}_{-7.2-9.1}$ | $13.6^{+8.6}_{-5.2}$ | $15.6^{+5.1}_{-3.9}$ | $18.2 \pm 6.7 \pm 1.1 \pm 0.5$ | 25.7 ± 3.6^a |
| $\bar{B}_s^0 \rightarrow K^0 \bar{K}^0$ | P | $26.1^{+13.5+12.9}_{-8.1-9.4}$ | $15.6^{+9.7}_{-6.0}$ | $18.0^{+4.7}_{-5.9}$ | $17.7 \pm 6.6 \pm 0.5 \pm 0.6$ | |
| $\bar{B}_s^0 \rightarrow \eta \eta$ | P | $10.9^{+6.3+5.7}_{-4.0-4.2}$ | $8.0^{+5.4}_{-3.1}$ | $10.0^{+3.4}_{-2.6}$ | $7.1 \pm 6.4 \pm 0.2 \pm 0.8$ $6.4 \pm 6.3 \pm 0.1 \pm 0.7$ | |
| $\bar{B}_s^0 \rightarrow \eta \eta'$ | P | $41.2^{+27.3+17.8}_{-12.9-13.1}$ | $21.0^{+11.7}_{-7.2}$ | $34.9^{+11.6}_{-9.5}$ | $24.0 \pm 13.6 \pm 1.4 \pm 2.7$ $23.8 \pm 13.2 \pm 1.6 \pm 2.9$ | |
| $\bar{B}_s^0 \rightarrow \eta' \eta'$ | P | $47.9^{+41.6+20.9}_{-17.1-15.3}$ | $14.0^{+7.0}_{-4.1}$ | $25.2^{+8.3}_{-6.5}$ | $44.3 \pm 19.7 \pm 2.3 \pm 17.1$ $49.4 \pm 20.6 \pm 8.4 \pm 16.2$ | |
| $\bar{B}_s^0 \rightarrow \pi^0 \eta$ | P_{EW} | $0.05^{+0.03+0.02}_{-0.01-0.01}$ | $0.05^{+0.02}_{-0.02}$ | $0.03^{+0.01}_{-0.01}$ | $0.014 \pm 0.004 \pm 0.005 \pm 0.004$ $0.016 \pm 0.0007 \pm 0.005 \pm 0.006$ | |
| $\bar{B}_s^0 \rightarrow \pi^0 \eta'$ | P_{EW} | $0.04^{+0.01+0.01}_{-0.00-0.00}$ | $0.11^{+0.05}_{-0.03}$ | $0.08^{+0.03}_{-0.02}$ | $0.006 \pm 0.003 \pm 0.002^{+0.064}_{-0.006}$ $0.038 \pm 0.013 \pm 0.016^{+0.260}_{-0.036}$ | |
| $\bar{B}_s^0 \rightarrow \pi^+ \pi^-$ | Annihilation | $0.26^{+0.00+0.10}_{-0.00-0.09}$ | $0.57^{+0.18}_{-0.16}$ | $0.57^{+0.24}_{-0.22}$ | | <1.2 |
| $\bar{B}_s^0 \rightarrow \pi^0 \pi^0$ | Annihilation | $0.13^{+0.0+0.05}_{-0.0-0.05}$ | $0.28^{+0.09}_{-0.08}$ | $0.29^{+0.12}_{-0.12}$ | | |

^aThis is the average of the CDF and Belle measurements, $(24.4 \pm 1.4 \pm 3.5) \times 10^{-6}$ [30] and $(38_{-9}^{+10} \pm 7) \times 10^{-6}$ [31], respectively. The old CDF result on $B_s \rightarrow K^+ K^-$ can be found in [49].

TABLE IV. Same as Table III except for the direct CP asymmetries (in %) in $\bar{B}_s \rightarrow PP$ decays.

| Modes | Class | QCDF (this work) | pQCD (LO) | pQCD (NLO) | SCET | Experiment [18] |
|---|--------------|----------------------------------|-----------------------|-----------------------|--|-------------------|
| $\bar{B}_s^0 \rightarrow K^+ \pi^-$ | T | $20.7^{+5.0+3.9}_{-3.0-8.8}$ | $24.1^{+5.6}_{-4.8}$ | $25.8^{+5.1}_{-6.3}$ | $20 \pm 17 \pm 19 \pm 5$ | $39 \pm 15 \pm 8$ |
| $\bar{B}_s^0 \rightarrow K^0 \pi^0$ | C | $36.3^{+17.4+26.6}_{-18.2-24.3}$ | $59.4^{+7.9}_{-12.5}$ | $88.0^{+4.8}_{-8.2}$ | $-58 \pm 39 \pm 39 \pm 13$ | |
| $\bar{B}_s^0 \rightarrow K^0 \eta$ | C | $33.4^{+22.8+25.7}_{-23.8-21.6}$ | $56.4^{+8.0}_{-9.3}$ | $96.7^{+1.6}_{-2.5}$ | $-56 \pm 46 \pm 14 \pm 6$ $61 \pm 59 \pm 12 \pm 8$ | |
| $\bar{B}_s^0 \rightarrow K^0 \eta'$ | C | $-49.3^{+6.2+16.0}_{-5.0+13.0}$ | $-19.9^{+5.5}_{-5.3}$ | $-35.4^{+3.2}_{-2.5}$ | $-14 \pm 7 \pm 16 \pm 2$ $37 \pm 8 \pm 14 \pm 4$ | |
| $\bar{B}_s^0 \rightarrow K^+ K^-$ | P | $-7.7^{+1.6+4.0}_{-1.2-5.1}$ | $-23.3^{+5.0}_{-4.6}$ | $-15.6^{+1.9}_{-1.6}$ | $-6 \pm 5 \pm 6 \pm 2$ | |
| $\bar{B}_s^0 \rightarrow K^0 \bar{K}^0$ | P | $0.40^{+0.04+0.10}_{-0.04-0.04}$ | 0 | 0.4 ± 0.1 | <10 | |
| $\bar{B}_s^0 \rightarrow \eta \eta$ | P | $-5.0^{+1.5+3.8}_{-2.5-2.8}$ | $-0.6^{+0.6}_{-0.5}$ | $0.6^{+0.2}_{-0.0}$ | $7.9 \pm 4.9 \pm 2.7 \pm 1.5$ $-1.1 \pm 5.0 \pm 3.9 \pm 1.0$ | |
| $\bar{B}_s^0 \rightarrow \eta \eta'$ | P | $-0.6^{+0.3+0.5}_{-0.4-0.3}$ | $-1.3^{+0.1}_{-0.2}$ | $-0.2^{+0.1}_{-0.1}$ | $0.04 \pm 0.14 \pm 0.39 \pm 0.43$ $2.7 \pm 0.9 \pm 0.8 \pm 7.6$ | |
| $\bar{B}_s^0 \rightarrow \eta' \eta'$ | P | $3.2^{+0.8+1.0}_{-0.6-1.2}$ | $1.9^{+0.4}_{-0.5}$ | $1.4^{+0.2}_{-0.2}$ | $0.9 \pm 0.4 \pm 0.6 \pm 1.9$ $-3.7 \pm 1.0 \pm 1.2 \pm 5.6$ | |
| $\bar{B}_s^0 \rightarrow \pi^0 \eta$ | P_{EW} | $96.1^{+1.6+1.8}_{-14.3-37.1}$ | $-0.4^{+0.3}_{-0.3}$ | $40.4^{+4.0}_{-7.4}$ | | |
| $\bar{B}_s^0 \rightarrow \pi^0 \eta'$ | P_{EW} | $42.9^{+2.3+31.0}_{-8.1-40.9}$ | $20.6^{+3.4}_{-2.9}$ | $52.5^{+3.2}_{-2.5}$ | | |
| $\bar{B}_s^0 \rightarrow \pi^+ \pi^-$ | Annihilation | 0 | $-1.2^{+1.2}_{-1.3}$ | $0.2^{+2.0}_{-1.5}$ | | |
| $\bar{B}_s^0 \rightarrow \pi^0 \pi^0$ | Annihilation | 0 | $-1.2^{+1.2}_{-1.2}$ | $0.2^{+0.1}_{-1.5}$ | | |

ties. For theoretical uncertainties in pQCD and SCET approaches, the reader is referred to the references cited in the table captions.

1. $\bar{B}_s \rightarrow K^+ \pi^-, K^0 \pi^0, K^0 \eta^{(\prime)}$

As mentioned before, in this work we shall use the form factor $F_0^{B_s K}(0) = 0.24$ obtained by both lattice and pQCD calculations. If a larger B_s to K transition form factor, say, $F_0^{B_s K}(0) = 0.31$, is employed, the predicted $\mathcal{B}(\bar{B}_s \rightarrow K^+ \pi^-)$ and $\mathcal{B}(\bar{B}_s \rightarrow K^+ K^-)$ will be far above the experimental results.² For $F_0^{B_s K}(0) = 0.24$, the calculated $\mathcal{B}(\bar{B}_s \rightarrow K^+ \pi^-) = (5.3_{-0.8-0.5}^{+0.4+0.4}) \times 10^{-6}$ is in good agreement with the measurement $(5.0 \pm 0.7 \pm 0.8) \times 10^{-6}$ [29]. Notice that although the same value of $F_0^{B_s K}$ was used in the leading order pQCD calculation, a larger branching fraction of order 7.6×10^{-6} was obtained (see Table III).

A recent detailed analysis in [51] indicates that SU(3) and factorization only remain approximately valid if the branching fraction of $\bar{B}_s \rightarrow K^+ \pi^-$ exceeds its current value of $(5.0 \pm 1.1) \times 10^{-6}$ by at least 50% or if the parameter ξ defined by

$$\xi \equiv \frac{f_K F_0^{B\pi}(m_K^2)}{f_\pi F_0^{B,K}(m_\pi^2)} \frac{m_B^2 - m_\pi^2}{m_B^2 - m_K^2} \quad (4.13)$$

is more than about 1.2. The analysis goes as follows. Writing the amplitudes $A(B^- \rightarrow \bar{K}^0 \pi^-) = V_{cs} V_{cb}^* P$ and $A(\bar{B}_d \rightarrow K^- \pi^+) = V_{us} V_{ub}^* T e^{i\delta} + V_{cs} V_{cb}^* P$, the measured $B^- \rightarrow \bar{K}^0 \pi^-$ rate sets a constraint on the penguin topology P . Since $V_{ub} = |V_{ub}| e^{-i\gamma}$, the measurement of $\bar{B}_d \rightarrow K^- \pi^+$ will put a constraint on T as a function of the unitarity angle γ . Under U -spin symmetry, the amplitude $A(\bar{B}_s \rightarrow K^+ \pi^-) = V_{ud} V_{ub}^* T' e^{i\delta'} + V_{cd} V_{cb}^* P'$ can be related to the $\bar{B}_d \rightarrow K^- \pi^+$ one by the relations: $T' = T$, $P' = P$ and $\delta' = \delta$. The data of $\bar{B}_s \rightarrow K^+ \pi^-$ will be helpful for pinning down the ratio of P/T . The analysis of [51] shows that for the value of γ to be consistent with other determinations and for the strong phases δ and δ' not different much from each other, then either $\mathcal{B}(\bar{B}_s \rightarrow K^+ \pi^-)$ is at least 50% larger than the current measured value or the parameter ξ is larger than 1.2. Our results of $\xi = 1.24$ and $\mathcal{B}(\bar{B}_s \rightarrow K^+ \pi^-) \approx 5.3 \times 10^{-6}$ are thus consistent with the analysis of [51].

It is known that the predicted direct CP violation for $\bar{B}_d \rightarrow K^- \pi^+$ and $\bar{B}_s \rightarrow K^+ \pi^-$ modes in naive QCDF is wrong in sign when compared with experiment (see the predictions in [6]). This discrepancy together with the rate deficit problem for penguin-dominated decays can be resolved by introducing power corrections coming from penguin annihilation, corresponding to the ‘‘S4 sce-

nario’’ of [6]. Using the values given in Table II for the parameters ρ_A and ϕ_A , we obtain $A_{CP}(\bar{B}_d \rightarrow K^- \pi^+) = -(7.4_{-1.5-4.8}^{+1.7+4.3})\%$ and $A_{CP}(\bar{B}_s \rightarrow K^+ \pi^-) = (20.7_{-3.0-8.8}^{+5.0+3.9})\%$, to be compared with the data $-0.098_{-0.011}^{+0.012}$ [47] and $0.39 \pm 0.15 \pm 0.08$ [29], respectively.

The inclusion of soft corrections to the color-suppressed tree topology has two effects: First, it will enhance the rates of $\bar{B}_s \rightarrow K^0 \pi^0, K^0 \eta$ by a factor of about 2.5 and $\bar{B}_s \rightarrow K^0 \eta'$ slightly. Second, it will flip the sign of CP -violating asymmetries of the former two modes. For example, $\mathcal{B}(\bar{B}_s \rightarrow K^0 \pi^0)$ is enhanced from 0.7×10^{-6} to 1.7×10^{-6} , while $A_{CP}(\bar{B}_s \rightarrow K^0 \pi^0)$ is changed from -0.214 to the order of 0.363 (see Tables III and IV). Note that pQCD predictions of branching fractions for the color-suppressed tree-dominated decays $\bar{B}_s \rightarrow K^0 \pi^0, K^0 \eta^{(\prime)}$ are much smaller than QCDF and SCET. Nevertheless, pQCD results of A_{CP} 's for the above three modes agree in signs with QCDF.

We see from Table IV that SCET predicts a negative sign for $A_{CP}(\bar{B}_s \rightarrow K^0 \pi^0)$, contrary to QCDF and pQCD. This deserves a special discussion. The negative sign of $A_{CP}(\bar{B}_s \rightarrow K^0 \pi^0)$ has to do with the fact that SCET predicts $A_{CP}(\bar{B}_d \rightarrow \bar{K}^0 \pi^0) = (5 \pm 4 \pm 4 \pm 1)\%$ [13]. From the U -spin symmetry relation (4.7) we learn that CP asymmetries of $\bar{B}_s \rightarrow K^0 \pi^0$ and $\bar{B}_d \rightarrow \bar{K}^0 \pi^0$ are of opposite sign. Although the current world average $A_{CP}(\bar{B}_d \rightarrow \bar{K}^0 \pi^0) = -0.01 \pm 0.10$ from the BABAR and Belle measurements, $-0.13 \pm 0.13 \pm 0.03$ [52] and $0.14 \pm 0.13 \pm 0.06$ [53], respectively, is consistent with no CP violation, there exist several model-independent determinations of this asymmetry: one is the SU(3) relation [54]

$$\Delta\Gamma(\bar{B}_d \rightarrow \pi^0 \pi^0) = -\Delta\Gamma(\bar{B}_d \rightarrow \bar{K}^0 \pi^0), \quad (4.14)$$

and the other is the approximate sum rule for CP rate asymmetries [55]

$$\begin{aligned} &\Delta\Gamma(\bar{B}_d \rightarrow K^- \pi^+) + \Delta\Gamma(B^- \rightarrow \bar{K}^0 \pi^-) \\ &\approx 2[\Delta\Gamma(B^- \rightarrow K^- \pi^0) + \Delta\Gamma(\bar{B}_d \rightarrow \bar{K}^0 \pi^0)], \end{aligned} \quad (4.15)$$

based on isospin symmetry, where $\Delta\Gamma(B \rightarrow K\pi) \equiv \Gamma(\bar{B} \rightarrow \bar{K}\pi) - \Gamma(B \rightarrow K\pi)$. This sum rule allows us to extract $A_{CP}(\bar{B}_d \rightarrow \bar{K}^0 \pi^0)$ in terms of the other three asymmetries in $K^- \pi^+, K^- \pi^0, \bar{K}^0 \pi^-$ modes that have been measured. From the current data of branching fractions and CP asymmetries, the above SU(3) relation and CP -asymmetry sum rule lead to $-0.073_{-0.041}^{+0.042}$ and -0.15 ± 0.04 , respectively, for $A_{CP}(\bar{B}_d \rightarrow \bar{K}^0 \pi^0)$. An analysis based on the topological quark diagrams yields a similar result $-0.08 \sim -0.12$ [56]. All these indicate that direct CP violation should be negative for $\bar{B}_d \rightarrow \bar{K}^0 \pi^0$ and hence positive for $\bar{B}_s \rightarrow K^0 \pi^0$.

2. $\bar{B}_s \rightarrow K^+ K^-, K^0 \bar{K}^0$

The penguin-dominated decays $\bar{B}_s \rightarrow K^+ K^-, K^0 \bar{K}^0$ have sizable branching fractions of order 25×10^{-6} in

²A larger branching fraction $\mathcal{B}(\bar{B}_s \rightarrow K^+ \pi^-) = (10.2_{-5.2}^{+6.0}) \times 10^{-6}$ was obtained in [6] within the framework of QCDF using the form factor $F_0^{B_s K}(0) = 0.31 \pm 0.05$.

QCDF. The corresponding pQCD and SCET predictions are slightly smaller (Table III).³ From Eqs. (4.9) and (4.10) we see that K^+K^- and $K^0\bar{K}^0$ modes differ mainly in the tree contribution α_1 and the annihilation term β_1 induced by the operator O_1 , both existing in the former but not in the latter. Since these contributions are Cabibbo-Kobayashi-Maskawa (CKM) suppressed relative to the penguin terms, the above two modes should have similar rates but rather distinct CP asymmetries. Because of the absence of interference between tree and penguin amplitudes, CP asymmetry is very small in $\bar{B}_s \rightarrow K^0\bar{K}^0$, less than 1%. Using the world average of $A_{CP}(\bar{B}_d \rightarrow \pi^+\pi^-) = 0.38 \pm 0.06$, $\mathcal{B}(\bar{B}_d \rightarrow \pi^+\pi^-) = (5.16 \pm 0.22) \times 10^{-6}$ [47] and $\mathcal{B}(\bar{B}_s \rightarrow K^+K^-) = (25.7 \pm 3.6) \times 10^{-6}$ [47], we find from the first U -spin relation in Eq. (4.7) that $A_{CP}(\bar{B}_s \rightarrow K^+K^-) \approx -0.077$ in the U -spin limit, which is in excellent agreement with the QCDF prediction. It is very important to measure the direct CP asymmetry for this mode.

In the pQCD approach, direct CP violation of $\bar{B}_s \rightarrow K^0\bar{K}^0$ vanishes to the lower order as there is only one type of CKM matrix element in its decay amplitude, say, $V_{tb}V_{ts}^*$ [11]. To the NLO, penguin loop corrections allow other CKM matrix elements enter into the decay amplitude and induce CP asymmetry [12]. It turns out that the predicted $A_{CP}(\bar{B}_s \rightarrow K^0\bar{K}^0)$ is very similar in both QCDF and pQCD (to NLO) approaches. It has been argued that the decay $\bar{B}_s \rightarrow K^0\bar{K}^0$ is a very promising place to look for effects of new physics through the measurement of its direct CP violation [58,59]. For example, it was shown in [58] that $A_{CP}(\bar{B}_s \rightarrow K^0\bar{K}^0)$, which is not more than 1% in the SM, can be 10 times larger in the presence of SUSY while its rate remains unaffected.

3. $\bar{B}_s \rightarrow \eta^{(\prime)}\eta^{(\prime)}$

The penguin-dominated $\eta^{(\prime)}\eta^{(\prime)}$ modes have sizable rates, especially $B_s \rightarrow \eta'\eta'$, the analog of $B \rightarrow K\eta'$ in the B_s sector, has the largest branching fraction of order $\sim 50 \times 10^{-6}$ in two-body hadronic decays of the B_s meson. The QCDF predictions in [6] within the S4 scenario are much bigger, 78×10^{-6} and 66×10^{-6} , respectively, for $\eta\eta'$ and $\eta'\eta'$ modes. This is because Eq. (3.9) rather than (3.11) is employed there for describing the $B_s \rightarrow \eta^{(\prime)}$ transition form factors. One of us (C. K. C.) found that the $B_s \rightarrow \eta'\eta'$ branching fraction can even reach the level of 1.0×10^{-4} in the residual final-state scattering model [32]. It is evident from Table III that the pQCD approach to lowest order predicts much smaller $\eta^{(\prime)}\eta^{(\prime)}$ rates even though the form factor $F_0^{B_s, \eta_s}(0) = 0.30$ is used there. A

³An early theoretical estimate yielded $\mathcal{B}(\bar{B}_s \rightarrow K^+K^-) = (35 \pm 7) \times 10^{-6}$ using the measured $B^0 \rightarrow K^+\pi^-$ branching fraction [57]. Based on QCDF and a combination of U -spin and isospin arguments, a result of $(20 \pm 8 \pm 4 \pm 2) \times 10^{-6}$ was obtained in [7].

recent pQCD calculation involving some NLO corrections from vertex corrections, quark loops and chormo-magnetic penguins exhibits some improvements [12]: the branching fractions of $\eta\eta$, $\eta\eta'$ and $\eta'\eta$ are enhanced from 8.0, 21.0 and 14.0 (in units of 10^{-6}) to 10.0, 34.9 and 25.2, respectively. The gap between pQCD and QCDF is thus improved. However, the NLO corrections calculated so far in pQCD are still not the complete results as some other pieces of NLO corrections such as hard spectator and annihilation have not been considered. It is important for the pQCD community to carry out the complete NLO calculations.

Since the decays $\bar{B}_s \rightarrow \eta^{(\prime)}\eta^{(\prime)}$ are penguin dominated and their tree amplitudes are color suppressed, their direct CP asymmetries are not large.

4. $\bar{B}_s \rightarrow \pi\pi$

The decays $\bar{B}_s \rightarrow \pi\pi$ proceed only through annihilation with the amplitudes [6]

$$A_{\bar{B}_s \rightarrow \pi^+\pi^-} \approx \sqrt{2}A_{\bar{B}_s \rightarrow \pi^0\pi^0} \propto 2B_{\pi\pi}b_4^c. \quad (4.16)$$

The predicted $\mathcal{B}(\bar{B}_s \rightarrow \pi^+\pi^-) = 2.6 \times 10^{-7}$ in QCDF is consistent with the current upper limit of 1.2×10^{-6} [29]. Note that in the absence of power corrections i.e. $\rho_A = 0$, the branching ratio will become too small, of order 5×10^{-8} .

5. $\bar{B}_s \rightarrow \pi^0\eta^{(\prime)}$

Since the isospin of the final state is $I = 1$, the electro-weak penguin is the only loop contribution that can contribute to the decays $\bar{B}_s \rightarrow \pi^0\eta^{(\prime)}$, in analog to the $B^- \rightarrow \pi^-\pi^0$ transition. However, unlike the latter, the electroweak penguin amplitude in the former gains a CKM enhancement $\lambda_c^{(s)}/\lambda_u^{(s)}$. Indeed, P_{EW} dominates over C in $\bar{B}_s \rightarrow \pi^0\eta^{(\prime)}$ decays. It is well known that CP asymmetry of $B^- \rightarrow \pi^-\pi^0$ is very small, of order 10^{-3} . This is ascribed to the fact that the electroweak penguin there is very suppressed with respect to the color-suppressed tree amplitude C . On the contrary, CP violation of $\bar{B}_s \rightarrow \pi^0\eta^{(\prime)}$ is very sizable due to the dominant P_{EW} . From Tables III and IV we see that the approaches of QCDF and pQCD have similar results for the rates of $\bar{B}_s \rightarrow \pi^0\eta^{(\prime)}$ but quite different predictions for $A_{CP}(\bar{B}_s \rightarrow \pi^0\eta^{(\prime)})$.

6. Test of U -spin and $SU(3)$ symmetries

There are five U -spin relations shown in Eqs. (4.5) and (4.7). We have pointed out before that the relation (4.5) is experimentally verified. For other relations, we are still lack of the measurements of CP asymmetries. Nevertheless, since the U -spin and $SU(3)$ symmetry breaking is already included in QCDF calculations, we can test quantitatively how good the symmetry is. In Table V we show some of direct CP asymmetries in B_s decays eval-

TABLE V. Direct CP asymmetries (in %) in $\bar{B}_s \rightarrow PP$ decays via U -spin symmetry. Theoretical results of branching fractions and CP asymmetries for $\bar{B}_d \rightarrow PP$ are taken from [17].

| Modes | $\mathcal{B}(10^{-6})$ | A_{CP} (%) | Modes | A_{CP} (%) (U -spin) | A_{CP} (%) (QCDF) |
|---|----------------------------------|-------------------------------|---|---------------------------|----------------------------------|
| $\bar{B}_d^0 \rightarrow K^- \pi^+$ | $19.3^{+7.9+8.2}_{-4.8-6.2}$ | $-7.4^{+1.7+4.3}_{-1.5-4.8}$ | $\bar{B}_s^0 \rightarrow K^+ \pi^-$ | 25.9 | $20.7^{+5.0+3.9}_{-3.0-8.8}$ |
| $\bar{B}_d^0 \rightarrow \pi^+ \pi^-$ | $7.0^{+0.4+0.7}_{-0.7-0.7}$ | $17.0^{+1.3+4.3}_{-1.7-8.7}$ | $\bar{B}_s^0 \rightarrow K^+ K^-$ | -4.5 | $-7.7^{+1.6+4.0}_{-1.2-5.1}$ |
| $\bar{B}_d^0 \rightarrow \bar{K}^0 \pi^0$ | $8.6^{+3.8+3.8}_{-2.2-2.9}$ | $-10.6^{+2.7+5.6}_{-3.8-4.3}$ | $\bar{B}_s^0 \rightarrow K^0 \pi^0$ | 51.5 | $36.3^{+17.4+26.6}_{-18.2-24.3}$ |
| $\bar{B}_d^0 \rightarrow K^0 \bar{K}^0$ | $2.1^{+1.0+0.8}_{-0.6-0.6}$ | $-10.0^{+0.7+1.0}_{-0.7-1.9}$ | $\bar{B}_s^0 \rightarrow K^0 \bar{K}^0$ | 0.77 | $0.40^{+0.04+0.10}_{-0.04-0.04}$ |
| $\bar{B}_d^0 \rightarrow K^+ K^-$ | $0.10^{+0.03+0.03}_{-0.02-0.03}$ | 0 | $\bar{B}_s^0 \rightarrow \pi^+ \pi^-$ | 0 | 0 |

uated using the U -spin relations Eqs. (4.5) and (4.7) and theoretical inputs for the branching fractions of $B_{d,s} \rightarrow PP$ decays and CP asymmetries of $B_d \rightarrow PP$. We see that in general A_{CP} obtained by U -spin symmetry is consistent with that obtained from direct QCDF calculations. In [11] two parameters,

$$R_3 \equiv \frac{|A(B_s \rightarrow \pi^+ K^-)|^2 - |A(\bar{B}_s \rightarrow \pi^- K^+)|^2}{|A(B_s \rightarrow \pi^+ K^-)|^2 + |A(\bar{B}_s \rightarrow \pi^- K^+)|^2},$$

$$\Delta \equiv \frac{A_{CP}(\bar{B}_d \rightarrow \pi^+ K^-)}{A_{CP}(\bar{B}_s \rightarrow \pi^- K^+)} + \frac{\mathcal{B}(\bar{B}_s \rightarrow \pi^- K^+)}{\mathcal{B}(\bar{B}_d \rightarrow \pi^+ K^-)} \frac{\tau(B_d)}{\tau(B_s)},$$
(4.17)

are defined to quantify the U -spin violation through the deviation of R_3 from -1 and Δ from 0 . However, it is not suitable for the U -spin pair $(\bar{B}_s \rightarrow K^0 \bar{K}^0, \bar{B}_d \rightarrow K^0 \bar{K}^0)$ for which we find $\Delta \approx -12$. In this case, it is better to compare $A_{CP}(\bar{B}_s \rightarrow K^0 \bar{K}^0)$ obtained from the U -spin relation with the QCDF prediction as we have done in Table V.

As for the test of $SU(3)$ symmetry, the first three relations in (4.12) are experimentally satisfied:

$$5.0 \pm 1.1 \doteq 5.16 \pm 0.22,$$

$$0.39 \pm 0.17 \doteq 0.38 \pm 0.06, \quad (4.18)$$

$$24.4 \pm 4.8 \doteq 19.4 \pm 0.6,$$

where the branching fractions are in units of 10^{-6} and the data are taken from [47]. For the last three relations of (4.12) we have

$$-0.077^{+0.043}_{-0.052} \doteq -0.098^{+0.012}_{-0.011},$$

$$26.1^{+18.7}_{-12.4} \doteq 19.4 \pm 0.6, \quad (4.19)$$

$$0.004^{+0.001}_{-0.006} \doteq 0.009 \pm 0.025,$$

where we have used the theoretical inputs for B_s decays and experimental inputs for B_d ones. Again, it appears that $SU(3)$ symmetry relations are satisfactorily respected.

7. Mixing-induced CP asymmetry

Measurements of time-dependent CP asymmetries in neutral B_s meson decays into a final CP eigenstate f that is common to B_s and \bar{B}_s will provide the information on two interesting quantities: mixing-induced CP asymmetry S_f and direct CP violation A_f which can be expressed as

$$A_f = -\frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad S_f = \frac{2 \text{Im} \lambda_f}{1 + |\lambda_f|^2}, \quad (4.20)$$

where

$$\lambda_f = \frac{q_{B_s}}{p_{B_s}} \frac{A(\bar{B}_s \rightarrow f)}{A(B_s \rightarrow f)} = \frac{V_{ib}^* V_{ts}}{V_{tb} V_{ts}^*} \frac{A(\bar{B}_s \rightarrow f)}{A(B_s \rightarrow f)}. \quad (4.21)$$

Now let $q_{B_s}/p_{B_s} = e^{2i\beta_s}$ and

$$\bar{A}(\bar{B}_s \rightarrow f) = A_1 e^{i(\phi_{A1} + \delta_1)} + A_2 e^{i(\phi_{A2} + \delta_2)},$$

$$A(B_s \rightarrow f) = \eta_f (A_1 e^{i(-\phi_{A1} + \delta_1)} + A_2 e^{i(-\phi_{A2} + \delta_2)}),$$
(4.22)

where $CP|f\rangle = \eta_f |f\rangle$ with $\eta_f = 1$ (-1) for final CP -even (odd) states, $\phi_{A1, A2}$ are weak phases and $\delta_{1,2}$ strong phases. It follows that (see e.g. [60])

$$\lambda_f = \eta_f e^{2i\phi_1} \frac{1 + r e^{i(\phi_1 - \phi_2)} e^{i\delta}}{1 + r e^{-i(\phi_1 - \phi_2)} e^{i\delta}}, \quad (4.23)$$

with $\phi_{1,2} = \phi_{A1, A2} + \beta_s$, $\delta = \delta_2 - \delta_1$ and $r = A_2/A_1$.

For B_s decays, the phase β_s due to the $B_s - \bar{B}_s$ mixing is very small in the SM, of order 1° . For the decays $\bar{B}_s \rightarrow K^0 \bar{K}^0, \eta\eta, \eta\eta', \eta'\eta'$ dominated by penguin diagrams (tree contributions to $\eta^{(\prime)}\eta^{(\prime)}$ are color suppressed), $r \simeq 0$ and the phase ϕ_{A1} due to $V_{cb} V_{cs}^*$ or $V_{tb} V_{ts}^*$ is also very small. Consequently, S_f are small for penguin-dominated $\bar{B}_s \rightarrow PP$ decays except for $\bar{B}_s \rightarrow K^+ K^-$ which receives a tree contribution with $\phi_{A2} = \gamma$. We see from Table VI that QCDF, pQCDF and SCET all predict $S_{\bar{B}_s \rightarrow K^+ K^-} \approx 0.20$. Recently, both CDF [2] and D0 [3] have reported fits to angular and time distributions of flavor-tagged $B_s \rightarrow J/\psi\phi$ decays which favor a larger value of β_s deviated from the SM by $1-2\sigma$ effects. If this is the case, then mixing-induced CP violation in $\bar{B}_s \rightarrow K^0 \bar{K}^0, \eta\eta, \eta\eta', \eta'\eta'$ could be sizable. Hence, these modes offer rich possibilities of testing new physics beyond the SM.

Because of the large magnitude and strong phase of a_2 induced from soft power corrections to the color-suppressed tree amplitude, for example, $a_2(K\pi) = 0.77e^{-i52^\circ}$ (or $0.41e^{-i11^\circ}$ before corrections),⁴ we find that such corrections will flip the sign of S_f into the positive one for the color-suppressed decays $\bar{B}_s \rightarrow K_S(\pi^0, \eta, \eta')$, while they are all negative in the pQCD

⁴In the $B_{u,d}$ systems, $a_2(K\pi) = 0.51e^{-i58^\circ}$ (or $0.27e^{-i17^\circ}$ before corrections).

TABLE VI. Same as Table III except for the mixing-induced CP asymmetries S_f in $\bar{B}_s \rightarrow PP$ decays. The parameter $\eta_f = 1$ except for $K_S(\pi^0, \eta, \eta')$ modes where $\eta_f = -1$.

| Modes | Class | QCDF (this work) | pQCD (LO) | pQCD (NLO) | SCET |
|---|--------------|-----------------------------------|-------------------------|-------------------------|--|
| $\bar{B}_s^0 \rightarrow K_S \pi^0$ | C | $0.08^{+0.29+0.23}_{-0.27-0.26}$ | $-0.61^{+0.24}_{-0.20}$ | $-0.41^{+0.09}_{-0.13}$ | $-0.16 \pm 0.41 \pm 0.33 \pm 0.17$ |
| $\bar{B}_s^0 \rightarrow K_S \eta$ | C | $0.26^{+0.33+0.21}_{-0.44-0.30}$ | $-0.43^{+0.23}_{-0.23}$ | $-0.18^{+0.12}_{-0.23}$ | $0.82 \pm 0.32 \pm 0.11 \pm 0.04$ $0.63 \pm 0.61 \pm 0.16 \pm 0.08$ |
| $\bar{B}_s^0 \rightarrow K_S \eta'$ | C | $0.08^{+0.21+0.20}_{-0.17-0.16}$ | $-0.68^{+0.06}_{-0.05}$ | $-0.46^{+0.12}_{-0.23}$ | $0.38 \pm 0.08 \pm 0.10 \pm 0.04$ $0.24 \pm 0.09 \pm 0.15 \pm 0.05$ |
| $\bar{B}_s^0 \rightarrow K^- K^+$ | P | $0.22^{+0.04+0.05}_{-0.05-0.03}$ | $0.28^{+0.05}_{-0.05}$ | $0.22^{+0.04}_{-0.03}$ | $0.19 \pm 0.04 \pm 0.04 \pm 0.01$ |
| $\bar{B}_s^0 \rightarrow K^0 \bar{K}^0$ | P | $0.004^{+0.0+0.002}_{-0.0-0.001}$ | 0.04 | $0.04^{+0.00}_{-0.00}$ | |
| $\bar{B}_s^0 \rightarrow \eta \eta$ | P | $-0.07^{+0.03+0.04}_{-0.06-0.05}$ | $0.03^{+0.01}_{-0.01}$ | $0.02^{+0.00}_{-0.00}$ | $-0.026 \pm 0.040 \pm 0.030 \pm 0.014$ $-0.077 \pm 0.061 \pm 0.022 \pm 0.026$ |
| $\bar{B}_s^0 \rightarrow \eta \eta'$ | P | $-0.01^{+0.00-0.00}_{-0.01-0.00}$ | $0.04^{+0.00}_{-0.00}$ | $0.04^{+0.00}_{-0.00}$ | $0.041 \pm 0.004 \pm 0.002 \pm 0.051$ $0.015 \pm 0.010 \pm 0.008 \pm 0.069$ |
| $\bar{B}_s^0 \rightarrow \eta' \eta'$ | P | $0.04^{+0.01+0.01}_{-0.01-0.01}$ | $0.04^{+0.01}_{-0.01}$ | $0.05^{+0.00}_{-0.01}$ | $0.049 \pm 0.005 \pm 0.005 \pm 0.031$ $0.051 \pm 0.009 \pm 0.017 \pm 0.039$ |
| $\bar{B}_s^0 \rightarrow \pi^0 \eta$ | P_{EW} | $0.26^{+0.06+0.48}_{-0.23-0.47}$ | $0.17^{+0.11}_{-0.13}$ | $0.28^{+0.05}_{-0.05}$ | $0.45 \pm 0.14 \pm 0.42 \pm 0.30$ $0.38 \pm 0.20 \pm 0.42 \pm 0.37$ |
| $\bar{B}_s^0 \rightarrow \pi^0 \eta'$ | P_{EW} | $0.88^{+0.03+0.04}_{-0.15-0.29}$ | $-0.17^{+0.08}_{-0.09}$ | $-0.18^{+0.12}_{-0.23}$ | $0.45 \pm 0.14 \pm 0.42 \pm 0.30$ $0.38 \pm 0.20 \pm 0.42 \pm 0.37$ |
| $\bar{B}_s^0 \rightarrow \pi^+ \pi^-$ | Annihilation | $0.15^{+0.00+0}_{-0.00-0}$ | $0.14^{+0.12}_{-0.06}$ | $0.09^{+0.02}_{-0.00}$ | |
| $\bar{B}_s^0 \rightarrow \pi^0 \pi^0$ | Annihilation | $0.15^{+0.00+0}_{-0.00-0}$ | $0.14^{+0.12}_{-0.06}$ | $0.08^{+0.00}_{-0.00}$ | |

approach. Recently, it has been claimed that in the pQCD approach there exist uncanceled soft divergences in the k_T factorization for the nonfactorizable B meson decay amplitudes [61]. This will enhance the nonfactorizable color-suppressed tree amplitudes. It remains to check if the signs of $S_{\bar{B}_s \rightarrow K_S(\pi^0, \eta, \eta')}$ in pQCD will be flipped again under this “ a_2 ” enhancement.

V. $B_s \rightarrow VP$ DECAYS

A. Branching fractions

The tree-dominated decays $\bar{B}_s \rightarrow K^{*+} \pi^-$ and $\rho^- K^+$ with the amplitudes

$$A(\bar{B}_s \rightarrow K^{*+} \pi^-) \approx A_{K^* \pi}(\delta_{\rho u} a_1 + a_4 - r_\chi^\pi a_6), \quad (5.1)$$

$$A(\bar{B}_s \rightarrow \rho^- K^+) \approx A_{K \rho}(\delta_{\rho u} a_1 + a_4 + r_\chi^\rho a_6),$$

have branching fractions of order 10^{-5} . Since $A_{K^* \pi} \equiv X^{(\bar{B}_s, K^* \pi)} \approx f_\pi A_0^{B_s, K^*}(0) m_{B_s}^2$ and $A_{K \rho} \equiv X^{(\bar{B}_s, \rho K)} \approx f_\rho F_0^{B_s, K}(0) m_{B_s}^2$ [see Eq. (2.3)], it is clear that the $\rho^- K^+$ mode has a rate larger than $K^{*+} \pi^-$ due to the hierarchy of the decay constants $f_\rho \gg f_\pi$. The penguin-dominated $\bar{B}_s \rightarrow VP$ decays such as $K^{*-} K^+$ and $K^{*0} \bar{K}^0$ have rates smaller than the counterparts in the PP sector as the amplitudes are proportional to $a_4 + r_\chi^{K^*} a_6$ or $a_4 - r_\chi^K a_6$ for the former and $a_4 + r_\chi^{K^*} a_6$ for the latter. Since a_4 and a_6 are of the same sign and $r_\chi^K > r_\chi^{K^*}$, it is evident that the interference of the penguin terms is constructive for PP and either destructive or less constructive for VP . The decay $\bar{B}_s \rightarrow \phi K^0$ is dominated by the $b \rightarrow d$ penguin

transition and its rate is thus much smaller compared to $b \rightarrow s$ dominated $\bar{B}_s \rightarrow K^* K$ decays.

We see from Table VII that the pQCD predictions for the color-suppressed tree-dominated decays $\bar{B}_s \rightarrow K^{*0} \pi^0$, $\rho^0 K^0$, ωK^0 , $K^{*0} \eta'$ are 1 order of magnitude smaller than QCDF and SCET in rates. For example, $\mathcal{B}(\bar{B}_s \rightarrow \rho^0 K^0)$ is predicted to be of order 1.9×10^{-6} in the approach of QCDF, but it is only about 0.08×10^{-6} in pQCD. The calculated branching fractions in pQCD for $K^{*0} \eta$ and some of the penguin-dominated decays e.g. $\bar{B}_s \rightarrow K^{*+} K^-$, $K^{*0} \bar{K}^0$, ϕK^0 , $\phi \eta'$ are also much smaller than QCDF. In the following we will comment on the decays $\bar{B}_s \rightarrow \phi \eta^{(\prime)}$. While the QCDF approach leads to $\mathcal{B}(\bar{B}_s \rightarrow \phi \eta^{(\prime)}) > \mathcal{B}(\bar{B}_s \rightarrow \phi \eta)$, pQCD and SCET predict very different patterns: $\mathcal{B}(\bar{B}_s \rightarrow \phi \eta) \gg \mathcal{B}(\bar{B}_s \rightarrow \phi \eta^{(\prime)})$ in the pQCD approach and it is the other way around in SCET (see Table VIII). We should stress that the decay rate of $\bar{B}_s \rightarrow \phi \eta'$ is sensitive to the form factor $A_0^{B_s, \phi}(0)$. The decay amplitudes of $\bar{B}_s \rightarrow \phi \eta^{(\prime)}$ are given by

$$A(\bar{B}_s \rightarrow \phi \eta) = \cos\theta A(\bar{B}_s \rightarrow \phi \eta_q) - \sin\theta A(\bar{B}_s \rightarrow \phi \eta_s),$$

$$A(\bar{B}_s \rightarrow \phi \eta') = \sin\theta A(\bar{B}_s \rightarrow \phi \eta_q) + \cos\theta A(\bar{B}_s \rightarrow \phi \eta_s), \quad (5.2)$$

with

$$A(\bar{B}_s \rightarrow \phi \eta_s) = A_{\phi \eta_s}(\alpha_3^p + \alpha_4^p) + A_{\eta_s \phi}(\alpha_3^p + \alpha_4^p),$$

$$\sqrt{2}A(\bar{B}_s \rightarrow \phi \eta_q) = A_{\phi \eta_q}(\delta_{\rho u} \alpha_2 + 2\alpha_3^p). \quad (5.3)$$

Since $\alpha_4^c(\phi \eta_s) = a_4 - r_\chi^{\eta_s} a_6$ and $\alpha_4^c(\eta_s \phi) = a_4 + r_\chi^\phi a_6$ are of opposite sign [numerically, $\alpha_4^c(\phi \eta_s) \approx 0.038$ and

TABLE VII. CP -averaged branching fractions (in units of 10^{-6}) of $\bar{B}_s \rightarrow PV$ decays calculated in various approaches. The LO pQCD predictions are taken from [11], while two different sets of SCET results are quoted from [14].

| Modes | Class | QCDF (this work) | pQCD | SCET 1 | SCET 2 |
|--|--------------|----------------------------------|--|---------------------------------------|---------------------------------------|
| $\bar{B}_s^0 \rightarrow K^{*+} \pi^-$ | T | $7.8^{+0.4+0.5}_{-0.7-0.7}$ | $7.6^{+2.9+0.4+0.5}_{-2.2-0.5-0.3}$ | $5.9^{+0.5+0.5}_{-0.5-0.5}$ | $6.6^{+0.2+0.7}_{-0.1-0.7}$ |
| $\bar{B}_s^0 \rightarrow \rho^- K^+$ | T | $14.7^{+1.4+0.9}_{-1.9-1.3}$ | $17.8^{+7.7+1.3+1.1}_{-5.6-1.6-0.9}$ | $7.6^{+0.3+0.8}_{-0.1-0.8}$ | $10.2^{+0.4+0.9}_{-0.5-0.9}$ |
| $\bar{B}_s^0 \rightarrow K^{*0} \pi^0$ | C | $0.89^{+0.80+0.84}_{-0.34-0.35}$ | $0.07^{+0.02+0.04+0.01}_{-0.01-0.02-0.01}$ | $0.90^{+0.07+0.10}_{-0.01-0.11}$ | $1.07^{+0.16+0.10}_{-0.15-0.09}$ |
| $\bar{B}_s^0 \rightarrow \rho^0 K^0$ | C | $1.9^{+2.9+1.4}_{-0.9-0.6}$ | $0.08^{+0.02+0.07+0.01}_{-0.02-0.03-0.00}$ | $2.0^{+0.2+0.2}_{-0.2-0.2}$ | $0.81^{+0.05+0.08}_{-0.02-0.09}$ |
| $\bar{B}_s^0 \rightarrow \omega K^0$ | C | $1.6^{+2.2+1.0}_{-0.7-0.5}$ | $0.15^{+0.05+0.07+0.02}_{-0.04-0.03-0.01}$ | $0.90^{+0.08+0.10}_{-0.01-0.11}$ | $1.3^{+0.1+0.1}_{-0.1-0.1}$ |
| $\bar{B}_s^0 \rightarrow K^{*-} K^+$ | P | $10.3^{+3.0+4.8}_{-2.2-4.2}$ | $6.0^{+1.7+1.7+0.7}_{-1.5-1.2-0.3}$ | $8.4^{+4.4+1.6}_{-3.4-1.3}$ | $9.5^{+3.2+1.2}_{-2.8-1.1}$ |
| $\bar{B}_s^0 \rightarrow K^{*+} K^-$ | P | $11.3^{+7.0+8.1}_{-3.5-5.1}$ | $4.7^{+1.1+2.5+0.0}_{-0.8-1.4-0.0}$ | $9.8^{+4.6+1.7}_{-3.7-1.4}$ | $10.2^{+3.8+1.5}_{-3.2-1.2}$ |
| $\bar{B}_s^0 \rightarrow \bar{K}^{*0} K^0$ | P | $10.5^{+3.4+5.1}_{-2.8-4.5}$ | $7.3^{+2.5+2.1+0.0}_{-1.7-1.3-0.0}$ | $7.9^{+4.4+1.6}_{-3.4-1.3}$ | $9.3^{+3.2+1.2}_{-2.8-1.0}$ |
| $\bar{B}_s^0 \rightarrow K^{*0} \bar{K}^0$ | P | $10.1^{+7.5+7.7}_{-3.6-4.8}$ | $4.3^{+0.7+2.2+0.0}_{-0.7-1.4-0.0}$ | $8.7^{+4.4+1.6}_{-3.5-1.4}$ | $9.4^{+3.7+1.4}_{-3.1-1.2}$ |
| $\bar{B}_s^0 \rightarrow \phi K^0$ | P | $0.6^{+0.5+0.4}_{-0.2-0.3}$ | $0.16^{+0.04+0.09+0.02}_{-0.03-0.04-0.01}$ | $0.44^{+0.23+0.08}_{-0.18-0.07}$ | $0.54^{+0.21+0.08}_{-0.17-0.07}$ |
| $\bar{B}_s^0 \rightarrow \phi \pi^0$ | P_{EW} | $0.12^{+0.02+0.04}_{-0.01-0.02}$ | $0.16^{+0.06+0.02+0.00}_{-0.05-0.02-0.00}$ | $0.07^{+0.00+0.01}_{-0.00-0.01}$ | $0.09^{+0.00+0.01}_{-0.00-0.01}$ |
| $\bar{B}_s^0 \rightarrow \rho^+ \pi^-$ | Annihilation | $0.02^{+0.00+0.01}_{-0.00-0.01}$ | $0.22^{+0.05+0.04+0.00}_{-0.05-0.06-0.01}$ | | |
| $\bar{B}_s^0 \rightarrow \rho^- \pi^+$ | Annihilation | $0.02^{+0.00+0.01}_{-0.00-0.01}$ | $0.24^{+0.05+0.05+0.00}_{-0.05-0.06-0.01}$ | | |
| $\bar{B}_s^0 \rightarrow \rho^0 \pi^0$ | Annihilation | $0.02^{+0.00+0.01}_{-0.00-0.01}$ | $0.23^{+0.05+0.05+0.00}_{-0.05-0.06-0.01}$ | | |
| $\bar{B}_s^0 \rightarrow K^{*0} \eta$ | C | $0.56^{+0.33+0.35}_{-0.14-0.17}$ | $0.17^{+0.04+0.10+0.03}_{-0.04-0.06-0.01}$ | $1.7^{+0.3+0.2}_{-0.3-0.1}$ | $0.62^{+0.14+0.07}_{-0.14-0.08}$ |
| $\bar{B}_s^0 \rightarrow K^{*0} \eta'$ | C | $0.90^{+0.69+0.72}_{-0.30-0.41}$ | $0.09^{+0.02+0.03+0.01}_{-0.02-0.02-0.01}$ | $0.64^{+0.33+0.11}_{-0.26-0.11}$ | $0.87^{+0.35+0.10}_{-0.32-0.08}$ |
| $\bar{B}_s^0 \rightarrow \phi \eta$ | P | $1.0^{+1.3+3.0}_{-0.1-1.2}$ | $3.6^{+1.5+0.8+0.0}_{-1.0-0.6-0.0}$ | $0.59^{+2.02+0.12}_{-0.59-0.10}$ | $0.94^{+1.89+0.16}_{-0.97-0.13}$ |
| $\bar{B}_s^0 \rightarrow \phi \eta'$ | P | $2.2^{+4.5+8.3}_{-1.9-2.5}$ | $0.19^{+0.06+0.19+0.00}_{-0.01-0.13-0.00}$ | $7.3^{+7.7+1.6}_{-5.4-1.3}$ | $4.3^{+5.2+0.7}_{-3.6-0.6}$ |
| $\bar{B}_s^0 \rightarrow \omega \eta$ | P, C | $0.03^{+0.12+0.06}_{-0.02-0.01}$ | $0.04^{+0.03+0.05+0.00}_{-0.01-0.02-0.00}$ | $0.04^{+0.04+0.00}_{-0.02-0.00}$ | $0.007^{+0.011+0.001}_{-0.002-0.001}$ |
| $\bar{B}_s^0 \rightarrow \omega \eta'$ | P, C | $0.15^{+0.27+0.15}_{-0.08-0.06}$ | $0.44^{+0.18+0.15+0.00}_{-0.13-0.14-0.01}$ | $0.001^{+0.095+0.000}_{-0.000-0.000}$ | $0.20^{+0.34+0.02}_{-0.17-0.02}$ |
| $\bar{B}_s^0 \rightarrow \rho^0 \eta$ | P_{EW} | $0.10^{+0.02+0.02}_{-0.01-0.01}$ | $0.06^{+0.03+0.01+0.00}_{-0.02-0.01-0.00}$ | $0.08^{+0.04+0.01}_{-0.03-0.01}$ | $0.06^{+0.03+0.00}_{-0.02-0.00}$ |
| $\bar{B}_s^0 \rightarrow \rho^0 \eta'$ | P_{EW} | $0.16^{+0.06+0.03}_{-0.02-0.03}$ | $0.13^{+0.06+0.02+0.00}_{-0.04-0.02-0.01}$ | $0.003^{+0.082+0.000}_{-0.000-0.000}$ | $0.14^{+0.24+0.01}_{-0.11-0.01}$ |

$\alpha_4^c(\eta_s \phi) \approx -0.033$], there is a cancellation between the two penguin amplitudes of $\bar{B}_s \rightarrow \phi \eta_s$. Note that $\alpha_3^c(\phi \eta_s)$ and $\alpha_3^c(\eta_s \phi)$ also are of opposite sign. It turns out that the sign of $A(\bar{B}_s \rightarrow \phi \eta_s)$ depends on the form factor $A_0^{B_s \phi}(0)$. For $A_0^{B_s \phi}(0) = 0.32$ as employed in the present work, $\bar{B}_s \rightarrow \phi \eta_s$ and $\bar{B}_s \rightarrow \phi \eta_q$ will contribute constructively to $\bar{B}_s \rightarrow \phi \eta'$ so that $\mathcal{B}(\bar{B}_s \rightarrow \phi \eta') = 2.2 \times 10^{-6}$. However, if we use the sum-rule prediction $A_0^{B_s \phi}(0) = 0.474$ from Eq. (3.3), then a near cancellation between $\bar{B}_s \rightarrow \phi \eta_s$ and $\bar{B}_s \rightarrow \phi \eta_q$ occurs in the decays $\bar{B}_s \rightarrow \phi \eta'$, so that its branching fraction, of order 10^{-7} , becomes very small. Hence, it is very important to measure the branching fractions of $\bar{B}_s \rightarrow \phi \eta^{(\prime)}$ to gain the information on the form factor $A_0^{B_s \phi}$.

One unique feature of the B_s decays is that there exist several modes dominated by electroweak penguins: $\bar{B}_s \rightarrow \pi^0 \eta^{(\prime)}$, $\phi \pi^0$, $\rho^0 \eta^{(\prime)}$ and $\phi \rho^0$. The isospin for the final states of these decays is $I = 1$ and hence the electroweak penguin is the only loop contribution that one can have. It dominates over the color-suppressed tree contribution due to the large CKM matrix element associated with the electroweak penguin amplitude. Since a large complex electroweak penguin amplitude due to new physics is also a possible solution to the $B \rightarrow K \pi$ CP puzzle, it has been advocated that this hypothesis can be tested in the decays $\bar{B}_s \rightarrow \phi \pi^0$, $\phi \rho^0$ whose rates may get an enhancement by an order of magnitude [62].

B. Direct CP asymmetries

Direct CP asymmetries of $\bar{B}_s \rightarrow VP$ decays estimated in various approaches are summarized in Table VIII. In QCDF calculations, the signs of CP asymmetries for color-suppressed tree-dominated decays $\bar{B}_s \rightarrow K^{*0} \pi^0$, $\rho^0 K^0$, ωK^0 and $K^{*0} \eta$ are governed by the soft corrections to a_2 [see Eq. (3.26)]. We see that QCDF and pQCD results agree with each other in signs, whereas SCET predicts opposite signs for these modes. Since the corresponding rates of these decays are very small in pQCD, as a consequence, the CP -violating asymmetries predicted by pQCD are very large, of order 0.50 or even bigger.

In the pQCD approach, the penguin-dominated decays $\bar{B}_s \rightarrow K^0 \phi$, $\bar{K}^{*0} K^0$, $K^{*0} \bar{K}^0$ have no direct CP asymmetry as their decay amplitudes are governed by one type of CKM matrix elements, e.g. $V_{tb} V_{td}^*$ for the first mode and $V_{tb} V_{ts}^*$ for the last two. As noticed before for the decay $\bar{B}_s \rightarrow K^0 \bar{K}^0$, NLO corrections from penguin loop interactions can bring a weak phase necessary for a nonvanishing CP violation. Therefore, it is important to carry out pQCD calculations to NLO for those three modes. In the approach of SCET, CP asymmetries of the decays $\bar{B}_s \rightarrow \pi^0 \phi$ and $\bar{B}_s \rightarrow \rho^0(\omega)(\eta, \eta')$ also vanish. As explained in [14], there is no charming penguins in these 5 channels and hence no direct CP violation due to the lack of strong phases.

We use this chance to clarify one misconception about CP violation under isospin symmetry. The isospin of the final-state is $I = 1$ for $\bar{B}_s \rightarrow \phi \pi^0$, $\rho^0 \eta^{(\prime)}$ and $I = 0$ for

TABLE VIII. Same as Table VII except for the direct CP asymmetries (in %) in $\bar{B}_s \rightarrow PV$ decays.

| Modes | Class | QCDF (this work) | PQCD | SCET 1 | SCET 2 |
|--|--------------|-----------------------------------|---|---------------------------------|---------------------------------|
| $\bar{B}_s^0 \rightarrow K^{*+} \pi^-$ | T | $-24.0^{+1.2+7.7}_{-1.5-3.9}$ | $-19.0^{+2.5+2.7+0.9}_{-2.6-3.4-1.4}$ | $-9.9^{+17.2+0.9}_{-16.7-0.7}$ | $-12.4^{+17.5+1.1}_{-15.3-1.2}$ |
| $\bar{B}_s^0 \rightarrow \rho^- K^+$ | T | $11.7^{+3.5+10.1}_{-2.1-11.6}$ | $14.2^{+2.4+2.3+1.2}_{-2.2-1.6-0.7}$ | $11.8^{+17.5+1.2}_{-20.0-1.1}$ | $10.8^{+9.4+0.9}_{-10.2-1.0}$ |
| $\bar{B}_s^0 \rightarrow K^{*0} \pi^0$ | C | $-26.3^{+10.8+42.2}_{-10.9-36.7}$ | $-47.1^{+7.4+35.5+2.9}_{-8.7-29.8-7.0}$ | $22.9^{+33.1+2.1}_{-40.2-1.9}$ | $13.4^{+18.6+0.8}_{-18.8-1.2}$ |
| $\bar{B}_s^0 \rightarrow \rho^0 K^0$ | C | $28.9^{+14.6+25.0}_{-14.5-23.7}$ | $73.4^{+6.4+16.2+2.2}_{-11.7-47.8-3.9}$ | $-12.0^{+20.1+1.0}_{-19.6-0.7}$ | $-32.5^{+30.7+2.7}_{-23.4-2.9}$ |
| $\bar{B}_s^0 \rightarrow \omega K^0$ | C | $-32.0^{+18.9+23.6}_{-17.5-26.2}$ | $-52.1^{+3.2+22.7+3.2}_{-0.0-15.1-2.0}$ | $24.4^{+33.7+2.2}_{-41.4-2.0}$ | $18.2^{+16.4+1.2}_{-17.0-1.7}$ |
| $\bar{B}_s^0 \rightarrow K^{*-} K^+$ | P | $-11.0^{+0.5+14.0}_{-0.4-18.8}$ | $-36.6^{+2.3+2.8+1.3}_{-2.3-3.5-1.2}$ | $-11.2^{+19.1+1.3}_{-16.2-1.3}$ | $-12.3^{+11.4+0.8}_{-11.3-0.8}$ |
| $\bar{B}_s^0 \rightarrow K^{*+} K^-$ | P | $25.5^{+9.2+16.3}_{-8.8-11.3}$ | $55.3^{+4.4+8.5+5.1}_{-4.9-9.8-2.5}$ | $7.1^{+11.2+0.7}_{-12.4-0.7}$ | $9.6^{+13.0+0.7}_{-13.5-0.9}$ |
| $\bar{B}_s^0 \rightarrow \bar{K}^{*0} K^0$ | P | $0.49^{+0.08+0.09}_{-0.07-0.12}$ | 0 | 0 | 0 |
| $\bar{B}_s^0 \rightarrow K^{*0} \bar{K}^0$ | P | $0.10^{+0.08+0.05}_{-0.07-0.02}$ | 0 | 0 | 0 |
| $\bar{B}_s^0 \rightarrow \phi K^0$ | P | $-3.2^{+1.2+0.6}_{-1.4-1.3}$ | 0 | $-3.0^{+5.3+0.3}_{-4.7-0.3}$ | $-2.2^{+3.0+0.1}_{-2.9-0.1}$ |
| $\bar{B}_s^0 \rightarrow \phi \pi^0$ | P_{EW} | $82.2^{+10.9+9.0}_{-14.0-55.3}$ | $13.3^{+0.3+2.1+1.5}_{-0.4-1.7-0.7}$ | 0 | 0 |
| $\bar{B}_s^0 \rightarrow \rho^+ \pi^-$ | Annihilation | $10.2^{+0.8+12.7}_{-0.7-12.8}$ | $4.6^{+0.0+2.9+0.6}_{-0.6-3.5-0.3}$ | 0 | 0 |
| $\bar{B}_s^0 \rightarrow \rho^- \pi^+$ | Annihilation | $-11.1^{+0.7+13.9}_{-0.8-15.7}$ | $-1.3^{+0.9+2.8+0.1}_{-0.4-3.5-0.2}$ | 0 | 0 |
| $\bar{B}_s^0 \rightarrow \rho^0 \pi^0$ | Annihilation | 0 | $1.7^{+0.2+2.8+0.2}_{-0.8-3.6-0.1}$ | 0 | 0 |
| $\bar{B}_s^0 \rightarrow K^{*0} \eta$ | C | $40.0^{+11.1+53.1}_{-19.2-64.5}$ | $51.2^{+6.2+14.1+2.0}_{-6.4-12.4-3.3}$ | $-25.7^{+23.4+2.0}_{-22.0-1.3}$ | $-62.7^{+28.1+2.6}_{-22.5-3.9}$ |
| $\bar{B}_s^0 \rightarrow K^{*0} \eta'$ | C | $-62.5^{+6.0+24.7}_{-5.5-20.2}$ | $-51.1^{+4.6+15.0+3.2}_{-6.6-18.2-4.1}$ | $-35.2^{+63.3+3.1}_{-49.4-3.8}$ | $-32.1^{+22.8+2.6}_{-23.2-1.7}$ |
| $\bar{B}_s^0 \rightarrow \phi \eta$ | P | $-12.4^{+14.1+64.9}_{-5.7-39.8}$ | $-1.8^{+0.0+0.6+0.1}_{-0.1-0.6-0.2}$ | $21.3^{+53.5+2.5}_{-83.2-2.6}$ | $16.9^{+13.8+1.6}_{-18.3-1.6}$ |
| $\bar{B}_s^0 \rightarrow \phi \eta'$ | P | $13.9^{+15.4+28.5}_{-4.2-89.7}$ | $7.8^{+1.5+1.2+0.1}_{-0.5-8.6-0.4}$ | $4.4^{+5.3+0.6}_{-7.1-0.6}$ | $7.8^{+5.0+0.8}_{-4.9-0.8}$ |
| $\bar{B}_s^0 \rightarrow \omega \eta$ | P, C | $-64.8^{+24.4+44.0}_{-3.4-31.6}$ | $-16.7^{+5.8+15.4+0.8}_{-3.2-19.1-1.7}$ | 0 | 0 |
| $\bar{B}_s^0 \rightarrow \omega \eta'$ | P, C | $-39.4^{+4.4+10.4}_{-3.0-11.7}$ | $7.7^{+0.4+4.5+9.4}_{-0.1-4.2-0.4}$ | 0 | 0 |
| $\bar{B}_s^0 \rightarrow \rho^0 \eta$ | P_{EW} | $75.7^{+15.3+13.3}_{-17.6-37.5}$ | $-9.2^{+1.0+2.8+0.4}_{-0.4-2.7-0.7}$ | 0 | 0 |
| $\bar{B}_s^0 \rightarrow \rho^0 \eta'$ | P_{EW} | $87.4^{+3.4+5.7}_{-10.6-30.3}$ | $25.8^{+1.3+2.8+3.4}_{-2.0-3.6-1.5}$ | 0 | 0 |

$\bar{B}_s \rightarrow (\phi, \omega) \eta^{(\prime)}$. One may argue that there is no CP violation for these decays as they have only one isospin strong phase (see e.g. [63]).⁵ On the contrary, we found large direct CP -violating effects in some of above decays (see Table VIII). The point is that isospin phases should not be confused with other possible strong phases in each of topological amplitudes. In our study, CP asymmetries of $\bar{B}_s \rightarrow \rho \eta^{(\prime)}$ are large since the electroweak penguins dominate over the color-suppressed tree amplitudes.

C. Test of U -spin and $SU(3)$ symmetries

The pairs related by U -spin symmetry are [64] ($\bar{B}_d \rightarrow K^{*-} \pi^+, \bar{B}_s \rightarrow \rho^- K^+$), ($\bar{B}_d \rightarrow K^- \rho^+, \bar{B}_s \rightarrow K^{*+} \pi^-$), ($\bar{B}_d \rightarrow \rho^- \pi^+, \bar{B}_s \rightarrow K^{*-} K^+$), ($\bar{B}_d \rightarrow \rho^+ \pi^-, \bar{B}_s \rightarrow K^{*+} K^-$), ($\bar{B}_d \rightarrow K^{*-} K^+, \bar{B}_s \rightarrow \rho^- \pi^+$), ($\bar{B}_d \rightarrow K^{*+} K^-, \bar{B}_s \rightarrow \rho^+ \pi^-$), ($\bar{B}_d \rightarrow \bar{K}^{*0} K^0, \bar{B}_s \rightarrow K^{*0} \bar{K}^0$), ($\bar{B}_d \rightarrow K^{*0} \bar{K}^0, \bar{B}_s \rightarrow \bar{K}^{*0} K^0$). Note that unlike PP and VV modes, $\bar{B}_s \rightarrow K^{*0} \pi^0$ and $\bar{B}_s \rightarrow K^0 \rho^0$ are not related to $\bar{B}_d \rightarrow \rho^0 \bar{K}^0$ and $\bar{B}_d \rightarrow \bar{K}^{*0} \pi^0$, respectively. Direct CP asymmetries of the pairs listed above are related by U -spin symmetry in analogue to Eq. (4.5) or Eq. (4.7). The test of U -spin symmetry in $B_s \rightarrow VP$ decays is shown in Table IX. It turns out that U -spin symmetry is in general acceptable.

⁵By the same token, it has been (wrongly) claimed that the direct CP asymmetry is strictly zero in the charged $B^- \rightarrow \pi^- \pi^0$ decay.

Just as $B_s \rightarrow PP$ decays, under the approximation of negligible annihilation contributions to tree-dominated decays and keeping only the dominant penguin annihilation terms in penguin-dominated decays, $SU(3)$ symmetry leads to [45,48]

$$\begin{aligned}
A(\bar{B}_s \rightarrow K^{*+} \pi^-) &\approx A(\bar{B}_d \rightarrow \rho^+ \pi^-), \\
A(\bar{B}_s \rightarrow \rho^- K^+) &\approx A(\bar{B}_d \rightarrow \rho^- \pi^+), \\
A(\bar{B}_s \rightarrow K^{*+} K^-) &\approx A(\bar{B}_d \rightarrow \rho^+ K^-), \\
A(\bar{B}_s \rightarrow K^{*-} K^+) &\approx A(\bar{B}_d \rightarrow K^{*-} \pi^+).
\end{aligned} \tag{5.4}$$

Thus, we have the relations

$$\begin{aligned}
\mathcal{B}(\bar{B}_s \rightarrow K^{*+} \pi^-) &\approx \mathcal{B}(\bar{B}_d \rightarrow \rho^+ \pi^-), \\
\mathcal{B}(\bar{B}_s \rightarrow \rho^- K^+) &\approx \mathcal{B}(\bar{B}_d \rightarrow \rho^- \pi^+), \\
\mathcal{B}(\bar{B}_s \rightarrow K^{*+} K^-) &\approx \mathcal{B}(\bar{B}_d \rightarrow \rho^+ K^-), \\
\mathcal{B}(\bar{B}_s \rightarrow K^{*-} K^+) &\approx \mathcal{B}(\bar{B}_d \rightarrow K^{*-} \pi^+),
\end{aligned} \tag{5.5}$$

and

$$\begin{aligned}
A_{CP}(\bar{B}_s \rightarrow K^{*+} \pi^-) &\approx A_{CP}(\bar{B}_d \rightarrow \rho^+ \pi^-), \\
A_{CP}(\bar{B}_s \rightarrow \rho^- K^+) &\approx A_{CP}(\bar{B}_d \rightarrow \rho^- \pi^+), \\
A_{CP}(\bar{B}_s \rightarrow K^{*+} K^-) &\approx A_{CP}(\bar{B}_d \rightarrow \rho^+ K^-), \\
A_{CP}(\bar{B}_s \rightarrow K^{*-} K^+) &\approx A_{CP}(\bar{B}_d \rightarrow K^{*-} \pi^+).
\end{aligned} \tag{5.6}$$

Numerically,

TABLE IX. Direct CP asymmetries (in %) in $B_s \rightarrow VP$ decays via U -spin symmetry.

| Modes | $\mathcal{B}(10^{-6})$ | A_{CP} (%) | Modes | A_{CP} (%) (U -spin) | A_{CP} (%) (QCDF) |
|--|----------------------------------|----------------------------------|--|---------------------------|----------------------------------|
| $\bar{B}_d^0 \rightarrow K^{*-} \pi^+$ | $9.2^{+1.0+3.7}_{-1.0-3.3}$ | $-12.1^{+0.5+12.6}_{-0.5-16.0}$ | $\bar{B}_s^0 \rightarrow \rho^- K^+$ | 9.1 | $11.7^{+3.5+10.1}_{-2.1-11.6}$ |
| $\bar{B}_d^0 \rightarrow \rho^+ K^-$ | $8.6^{+5.7+7.4}_{-2.8-4.5}$ | $31.9^{+11.5+19.6}_{-11.0-12.7}$ | $\bar{B}_s^0 \rightarrow K^{*+} \pi^-$ | -39.6 | $-24.0^{+1.2+7.7}_{-1.5-3.9}$ |
| $\bar{B}_d^0 \rightarrow K^{*+} K^-$ | $0.08^{+0.01+0.02}_{-0.01-0.02}$ | $-4.7^{+0.1+4.7}_{-0.2-2.7}$ | $\bar{B}_s^0 \rightarrow \rho^+ \pi^-$ | 18.9 | $10.2^{+0.8+12.7}_{-0.7-12.8}$ |
| $\bar{B}_d^0 \rightarrow K^{*-} K^+$ | $0.07^{+0.01+0.04}_{-0.01-0.03}$ | $5.5^{+0.2+7.0}_{-0.2-5.5}$ | $\bar{B}_s^0 \rightarrow \rho^- \pi^+$ | -20.5 | $-11.1^{+0.7+13.9}_{-0.8-15.7}$ |
| $\bar{B}_d^0 \rightarrow K^{*0} \bar{K}^0$ | $0.70^{+0.18+0.28}_{-0.15-0.25}$ | $-13.5^{+1.6+1.4}_{-1.7-2.3}$ | $\bar{B}_s^0 \rightarrow \bar{K}^{*0} K^0$ | 0.86 | $0.49^{+0.08+0.09}_{-0.07-0.12}$ |
| $\bar{B}_d^0 \rightarrow \bar{K}^{*0} K^0$ | $0.47^{+0.36+0.43}_{-0.17-0.27}$ | $-3.5^{+1.3+0.7}_{-1.7-2.0}$ | $\bar{B}_s^0 \rightarrow K^{*0} \bar{K}^0$ | 0.17 | $0.10^{+0.08+0.05}_{-0.07-0.02}$ |
| $\bar{B}_d^0 \rightarrow \rho^+ \pi^-$ | $9.2^{+0.4+0.5}_{-0.7-0.7}$ | $-22.7^{+0.9+8.2}_{-1.1-4.4}$ | $\bar{B}_s^0 \rightarrow K^{*+} K^-$ | 19.0 | $25.5^{+9.2+16.3}_{-8.8-11.3}$ |
| $\bar{B}_d^0 \rightarrow \rho^- \pi^+$ | $15.9^{+1.1+0.9}_{-1.5-1.1}$ | $4.4^{+0.3+5.8}_{-0.3-6.8}$ | $\bar{B}_s^0 \rightarrow K^{*-} K^+$ | -6.6 | $-11.0^{+0.5+14.0}_{-0.4-18.8}$ |

$$\begin{aligned}
 7.8^{+0.6}_{-1.0} &\doteq 9.2^{+0.6}_{-1.0}, & 14.7^{+1.7}_{-2.3} &\doteq 15.9^{+1.4}_{-1.9}, \\
 11.3^{+10.7}_{-6.2} &\doteq 8.6^{+9.3}_{-5.3}, & 10.3^{+5.7}_{-4.7} &\doteq 9.2^{+3.8}_{-3.4},
 \end{aligned}
 \tag{5.7}$$

$$\begin{aligned}
 A(\bar{B}_s \rightarrow K_S \phi) &\propto V_{ub} V_{ud}^* [A_{K\phi} \alpha_3^u + A_{\phi K} (\alpha_4^u + \beta_3^u)] \\
 &+ V_{cb} V_{cd}^* [A_{K\phi} \alpha_3^c + A_{\phi K} (\alpha_4^c + \beta_3^c)].
 \end{aligned}
 \tag{5.9}$$

for branching fractions in units of 10^{-6} and

$$\begin{aligned}
 -24.0^{+7.8}_{-4.2} &\doteq -22.7^{+8.2}_{-4.5}, & 11.7^{+10.7}_{-11.8} &\doteq 4.4^{+5.8}_{-6.8}, \\
 25.5^{+18.7}_{-14.3} &\doteq 31.9^{+22.7}_{-16.8}, & -11.0^{+14.0}_{-18.8} &\doteq -12.1^{+12.6}_{-16.0},
 \end{aligned}
 \tag{5.8}$$

for direct CP asymmetries in %. Hence, the above $SU(3)$ relations are generally respected.

D. Mixing-induced CP asymmetry

As discussed before, due to the tiny phase in the $B_s - \bar{B}_s$ mixing and in the CKM matrix element $V_{cb} V_{cs}^*$ or $V_{tb} V_{ts}^*$, mixing-induced CP violation S_f is expected to be very small in the penguin-dominated $\bar{B}_s \rightarrow \phi \eta'$ decays. This is indeed borne out in all model calculations. The $b \rightarrow dg$ penguin-dominated decay $\bar{B}_s \rightarrow K_S \phi$ has a large mixing-induced CP asymmetry due to the fact that the CKM matrix element $V_{ub} V_{ud}^*$ has a weak phase $-\gamma$. More specifically,

To the approximation that $\alpha_{3,4}^c \approx \alpha_{3,4}^u$ and $\beta_3^c \approx \beta_3^u$, it is clear that $A(\bar{B}_s \rightarrow K_S \phi) \propto V_{tb} V_{td}^* = |V_{tb} V_{td}^*| e^{i\beta}$. Thus, $S_{\bar{B}_s \rightarrow K_S \phi} \approx -\sin 2(\beta_s + \beta) = -0.71$ for $\beta_s \approx 1^\circ$ and $\beta = 21.58^\circ$ [34]. In the pQCD approach, this decay is dominated by the $(S-P)(S+P)$ penguin annihilation process with the CKM matrix element proportional to $V_{tb} V_{td}^*$. Therefore, both QCDF and pQCD predict $S_{\bar{B}_s \rightarrow K_S \phi} \sim \mathcal{O}(0.70)$. (However, no error estimate is done in the pQCD calculation [11].) On the contrary, the SCET result of $S_{\bar{B}_s \rightarrow K_S \phi} \sim 0.09$ or -0.13 is dramatically different from the QCDF and pQCD predictions. As explained in [14], charming penguin contributions to $\bar{B}_s \rightarrow K_S \phi$ dominates over penguin operators and the CKM matrix element associated with charming penguins is $V_{cb} V_{cd}^*$. Hence, $S_{\bar{B}_s \rightarrow K_S \phi} = -\sin 2\beta_s = -0.03$ is predicted by SCET when penguin contributions are neglected. It should be stressed that although both QCDF and pQCD approaches have similar results for $S_{\bar{B}_s \rightarrow K_S \phi}$, they differ in the prediction of $A_{CP}(\bar{B}_s \rightarrow K_S \phi)$: it is of order -0.03 in QCDF and vanishes in pQCD for reasons mentioned above.

TABLE X. Same as Table VII except for mixing-induced CP asymmetries S_f in $\bar{B}_s \rightarrow PV$ decays. The parameter $\eta_f = 1$ except for $K_S(\rho^0, \omega, \phi)$ modes where $\eta_f = -1$. Note that the error estimate of $S_{\bar{B}_s \rightarrow K_S \phi}$ is not available in the pQCD calculation [11].

| Modes | Class | QCDF (this work) | pQCD | SCET 1 | SCET 2 |
|--|--------------|-----------------------------------|---|-----------------------------------|-----------------------------------|
| $\bar{B}_s^0 \rightarrow K_S \rho^0$ | C | $0.29^{+0.23+0.16}_{-0.24-0.21}$ | $-0.57^{+0.22+0.51+0.02}_{-0.17-0.39-0.05}$ | $0.99^{+0.00+0.00}_{-0.05-0.01}$ | $-0.03^{+0.22+0.17}_{-0.17-0.12}$ |
| $\bar{B}_s^0 \rightarrow K_S \omega$ | C | $0.92^{+0.03+0.08}_{-0.07-0.15}$ | $-0.63^{+0.09+0.28+0.01}_{-0.09-0.11-0.02}$ | $-0.11^{+0.28+0.18}_{-0.22-0.14}$ | $0.98^{+0.02+0.00}_{-0.04-0.01}$ |
| $\bar{B}_s^0 \rightarrow K_S \phi$ | P | $-0.69^{+0.01+0.01}_{-0.01-0.01}$ | -0.72 | $0.09^{+0.04+0.01}_{-0.03-0.01}$ | $-0.13^{+0.02+0.01}_{-0.02-0.01}$ |
| $\bar{B}_s^0 \rightarrow \phi \eta$ | P | $0.21^{+0.08+0.61}_{-0.11-0.25}$ | $-0.03^{+0.02+0.07+0.01}_{-0.01-0.20-0.02}$ | $-0.39^{+0.43+0.04}_{-0.15-0.04}$ | $0.23^{+0.35+0.02}_{-0.16-0.02}$ |
| $\bar{B}_s^0 \rightarrow \phi \eta'$ | P | $0.08^{+0.05+0.48}_{-0.06-0.81}$ | $0.00^{+0.00+0.02+0.00}_{-0.00-0.02-0.00}$ | $-0.07^{+0.06+0.01}_{-0.06-0.01}$ | $0.10^{+0.07+0.01}_{-0.05-0.01}$ |
| $\bar{B}_s^0 \rightarrow \omega \eta$ | P, C | $-0.76^{+0.16+0.52}_{-0.03-0.22}$ | $-0.02^{+0.01+0.02+0.00}_{-0.03-0.08-0.00}$ | $-0.62^{+0.41+0.08}_{-0.18-0.12}$ | $0.93^{+0.04+0.03}_{-0.98-0.04}$ |
| $\bar{B}_s^0 \rightarrow \omega \eta'$ | P, C | $-0.84^{+0.06+0.04}_{-0.05-0.03}$ | $-0.11^{+0.01+0.04+0.02}_{-0.00-0.04-0.03}$ | $-0.25^{+0.23+0.10}_{-0.74-0.16}$ | $-1.00^{+0.04+0.01}_{-0.00-0.00}$ |
| $\bar{B}_s^0 \rightarrow \pi^0 \phi$ | P_{EW} | $0.40^{+0.04+0.32}_{-0.10-0.53}$ | $-0.07^{+0.01+0.08+0.02}_{-0.01-0.09-0.03}$ | $0.89^{+0.00+0.04}_{-0.00-0.05}$ | $0.90^{+0.00+0.02}_{-0.00-0.03}$ |
| $\bar{B}_s^0 \rightarrow \rho^0 \eta$ | P_{EW} | $0.35^{+0.09+0.22}_{-0.16-0.40}$ | $0.15^{+0.06+0.14+0.01}_{-0.06-0.16-0.01}$ | $1.00^{+0.00+0.00}_{-0.06-0.01}$ | $0.60^{+0.30+0.03}_{-0.53-0.03}$ |
| $\bar{B}_s^0 \rightarrow \rho^0 \eta'$ | P_{EW} | $0.45^{+0.05+0.30}_{-0.13-0.35}$ | $-0.16^{+0.00+0.10+0.04}_{-0.00-0.12-0.05}$ | $0.95^{+0.00+0.02}_{-1.60-0.02}$ | $-0.41^{+0.75+0.10}_{-0.75-0.15}$ |
| $\bar{B}_s^0 \rightarrow \rho^0 \pi^0$ | Annihilation | $-0.65^{+0.03+0.00}_{-0.03-0.00}$ | $-0.19^{+0.00+0.02+0.01}_{-0.00-0.02-0.02}$ | | |

The study of CP violation for $\bar{B}_s \rightarrow K^{*+}K^-$ and $K^{*-}K^+$ is more complicated as $K^{*\pm}K^\mp$ are not CP eigenstates. The time-dependent CP asymmetries are given by

$$\begin{aligned} \mathcal{A}(t) &\equiv \frac{\Gamma(\bar{B}_s^0(t) \rightarrow K^{*+}K^-) - \Gamma(B_s^0(t) \rightarrow K^{*+}K^-)}{\Gamma(\bar{B}_s^0(t) \rightarrow K^{*+}K^-) + \Gamma(B_s^0(t) \rightarrow K^{*+}K^-)} \\ &= (S \pm \Delta S) \sin(\Delta m_s t) - (C \pm \Delta C) \cos(\Delta m_s t), \end{aligned} \quad (5.10)$$

where Δm_s is the mass difference of the two neutral B_s eigenstates, S is referred to as mixing-induced CP asymmetry and C is the direct CP asymmetry ($C = -A_{CP}$), while ΔS and ΔC are CP -conserving quantities. In writing the above equation we have neglected the effects of the width difference of the B_s mesons. Defining

$$\begin{aligned} A_{+-} &\equiv A(B_s^0 \rightarrow K^{*+}K^-), & A_{-+} &\equiv A(B_s^0 \rightarrow K^{*-}K^+), \\ \bar{A}_{-+} &\equiv A(\bar{B}_s^0 \rightarrow K^{*-}K^+), & \bar{A}_{+-} &\equiv A(\bar{B}_s^0 \rightarrow K^{*+}K^-), \end{aligned} \quad (5.11)$$

and

$$\lambda_{+-} = \frac{q_{B_s} \bar{A}_{+-}}{p_{B_s} A_{+-}}, \quad \lambda_{-+} = \frac{q_{B_s} \bar{A}_{-+}}{p_{B_s} A_{-+}}, \quad (5.12)$$

we have

$$\begin{aligned} C + \Delta C &= \frac{1 - |\lambda_{+-}|^2}{1 + |\lambda_{+-}|^2} = \frac{|A_{+-}|^2 - |\bar{A}_{+-}|^2}{|A_{+-}|^2 + |\bar{A}_{+-}|^2}, \\ C - \Delta C &= \frac{1 - |\lambda_{-+}|^2}{1 + |\lambda_{-+}|^2} = \frac{|A_{-+}|^2 - |\bar{A}_{-+}|^2}{|A_{-+}|^2 + |\bar{A}_{-+}|^2}, \end{aligned} \quad (5.13)$$

and

$$\begin{aligned} S + \Delta S &\equiv \frac{2 \operatorname{Im} \lambda_{+-}}{1 + |\lambda_{+-}|^2} = \frac{2 \operatorname{Im}(e^{2i\beta_s} \bar{A}_{+-} A_{+-}^*)}{|A_{+-}|^2 + |\bar{A}_{+-}|^2}, \\ S - \Delta S &\equiv \frac{2 \operatorname{Im} \lambda_{-+}}{1 + |\lambda_{-+}|^2} = \frac{2 \operatorname{Im}(e^{2i\beta_s} \bar{A}_{-+} A_{-+}^*)}{|A_{-+}|^2 + |\bar{A}_{-+}|^2}. \end{aligned} \quad (5.14)$$

Hence we see that ΔS describes the strong phase difference between the amplitudes contributing to $B_s^0 \rightarrow K^{*\pm}K^\mp$ and ΔC measures the asymmetry between $\Gamma(\bar{B}_s^0 \rightarrow K^{*+}K^-) + \Gamma(\bar{B}_s^0 \rightarrow K^{*-}K^+)$ and $\Gamma(B_s^0 \rightarrow K^{*+}K^-) + \Gamma(B_s^0 \rightarrow K^{*-}K^+)$.

Next consider the time- and flavor-integrated charge asymmetry

$$\mathcal{A}_{K^*K} \equiv \frac{|A_{+-}|^2 + |\bar{A}_{+-}|^2 - |A_{-+}|^2 - |\bar{A}_{-+}|^2}{|A_{+-}|^2 + |\bar{A}_{+-}|^2 + |A_{-+}|^2 + |\bar{A}_{-+}|^2}. \quad (5.15)$$

Then, following [34] one can transform the experimentally motivated CP parameters \mathcal{A}_{K^*K} and C_{K^*K} into the physically motivated choices

$$A_{K^{*+}K^-} \equiv \frac{|\kappa^{-+}|^2 - 1}{|\kappa^{-+}|^2 + 1}, \quad A_{K^{*-}K^+} \equiv \frac{|\kappa^{+-}|^2 - 1}{|\kappa^{+-}|^2 + 1}, \quad (5.16)$$

TABLE XI. Various CP -violating parameters in the decays $\bar{B}_s^0 \rightarrow K^{*\pm}K^\mp$. SCET results are quoted from [14].

| Parameter | QCDF (this work) | SCET 1 | SCET 2 |
|----------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| \mathcal{A}_{K^*K} | $0.19_{-0.04-0.11}^{+0.03+0.14}$ | | |
| C | $-0.08_{-0.04-0.14}^{+0.04+0.15}$ | $0.02_{-0.11-0.00}^{+0.10+0.00}$ | $0.01_{-0.09-0.00}^{+0.09+0.00}$ |
| S | $-0.05_{-0.01-0.09}^{+0.01+0.13}$ | $-0.02_{-0.07-0.01}^{+0.07+0.01}$ | $0.02_{-0.05-0.00}^{+0.05+0.01}$ |
| ΔC | $-0.03_{-0.14-0.49}^{+0.12+0.46}$ | $-0.09_{-0.10-0.01}^{+0.11+0.01}$ | $-0.11_{-0.09-0.01}^{+0.09+0.01}$ |
| ΔS | $0.33_{-0.10-0.48}^{+0.09+0.30}$ | $0.38_{-0.07-0.04}^{+0.07+0.04}$ | $-0.41_{-0.05-0.03}^{+0.05+0.03}$ |

with

$$\kappa^{+-} = \frac{q_{B_s} \bar{A}_{-+}}{p_{B_s} A_{+-}}, \quad \kappa^{-+} = \frac{q_{B_s} \bar{A}_{+-}}{p_{B_s} A_{-+}}. \quad (5.17)$$

Hence,

$$\begin{aligned} A_{K^{*+}K^-} &= \frac{\Gamma(\bar{B}_s^0 \rightarrow K^{*+}K^-) - \Gamma(B_s^0 \rightarrow K^{*-}K^+)}{\Gamma(\bar{B}_s^0 \rightarrow K^{*+}K^-) + \Gamma(B_s^0 \rightarrow K^{*-}K^+)} \\ &= \frac{\mathcal{A}_{K^*K} - C_{K^*K} - \mathcal{A}_{K^*K} \Delta C_{K^*K}}{1 - \Delta C_{K^*K} - \mathcal{A}_{K^*K} C_{K^*K}}, \\ A_{K^{*-}K^+} &= \frac{\Gamma(\bar{B}_s^0 \rightarrow K^{*-}K^+) - \Gamma(B_s^0 \rightarrow K^{*+}K^-)}{\Gamma(\bar{B}_s^0 \rightarrow K^{*-}K^+) + \Gamma(B_s^0 \rightarrow K^{*+}K^-)} \\ &= -\frac{\mathcal{A}_{K^*K} + C_{K^*K} + \mathcal{A}_{K^*K} \Delta C_{K^*K}}{1 + \Delta C_{K^*K} + \mathcal{A}_{K^*K} C_{K^*K}}. \end{aligned} \quad (5.18)$$

Note that the quantities $A_{K^{*\pm}K^\mp}$ here correspond to $A_{K^{*\mp}K^\pm}$ defined in [34]. Therefore, direct CP asymmetries $A_{K^{*+}K^-}$ and $A_{K^{*-}K^+}$ are determined from the above two equations. Results for various CP -violating parameters in the decays $\bar{B}_s^0 \rightarrow K^{*\pm}K^\mp$ are shown in Table XI.

VI. $B_s \rightarrow VV$ DECAYS

A. Branching fractions

In two-body decays $B_{u,d} \rightarrow PP, VP, VV$, we have the pattern $VV > PV > VP > PP$ for the branching fractions of tree-dominated modes and $PP > PV \sim VV > VP$ for penguin-dominated ones, where the factorizable amplitude for $B \rightarrow VP(PV)$ here is given by $\langle V(P) | J_\mu | B \rangle \times \langle P(V) | J^\mu | 0 \rangle$. The first hierarchy is due to the difference of decay constants $f_V > f_P$ and the second hierarchy stems from the fact that the penguin amplitudes are proportional to $a_4 + r_\chi^P a_6$, $a_4 + r_\chi^V a_6$, $a_4 - r_\chi^P a_6$, $a_4 + r_\chi^V a_6$, respectively, for $B \rightarrow PP, PV, VP, VV$ with $r_\chi^P \sim \mathcal{O}(1) \gg r_\chi^V$. The same is also true in the B_s sector. From Tables III, VII, and XII we find

$$\begin{aligned} \mathcal{B}(\bar{B}_s \rightarrow \rho^- K^{*+}) &> \mathcal{B}(\bar{B}_s \rightarrow \rho^- K^+) > \mathcal{B}(\bar{B}_s \rightarrow \pi^- K^{*+}) \\ &> \mathcal{B}(\bar{B}_s \rightarrow \pi^- K^+), \\ \mathcal{B}(\bar{B}_s \rightarrow K^+ K^-) &> \mathcal{B}(\bar{B}_s \rightarrow K^{*-} K^+) \sim \mathcal{B}(\bar{B}_s \rightarrow K^{*+} K^-) \\ &> \mathcal{B}(\bar{B}_s \rightarrow K^{*-} K^+), \end{aligned} \quad (6.1)$$

for tree- and penguin-dominated \bar{B}_s decays, respectively.

TABLE XII. CP -averaged branching ratios in $\bar{B}_s \rightarrow VV$ decays (in units of 10^{-6}) obtained in various approaches. Presented are the pQCD predictions taken from [11] and the QCDF predictions from this work and from [8] denoted by BRY.

| Channel | Class | QCDF (this work) | QCDF (BRY) | pQCD | Experiment [38,50] |
|---|--------------|-----------------------------------|----------------------------------|---|--------------------|
| $\bar{B}_s \rightarrow \rho^- K^{*+}$ | T | $21.6^{+1.3+0.9}_{-2.8-1.5}$ | $25.2^{+1.5+4.7}_{-1.7-3.1}$ | $20.9^{+8.2+1.4+1.2}_{-6.2-1.4-1.1}$ | |
| $\bar{B}_s \rightarrow \rho^0 K^{*0}$ | C | $1.3^{+2.0+1.7}_{-0.6-0.3}$ | $1.5^{+1.0+3.1}_{-0.5-1.5}$ | $0.33^{+0.09+0.14+0.00}_{-0.07-0.09-0.01}$ | <767 |
| $\bar{B}_s \rightarrow \omega K^{*0}$ | C | $1.1^{+1.5+1.3}_{-0.5-0.3}$ | $1.2^{+0.7+2.3}_{-0.3-1.1}$ | $0.31^{+0.10+0.12+0.07}_{-0.07-0.06-0.02}$ | |
| $\bar{B}_s \rightarrow K^{*-} K^{*+}$ | P | $7.6^{+1.0+2.3}_{-1.0-1.8}$ | $9.1^{+2.5+10.2}_{-2.2-5.9}$ | $6.7^{+1.5+3.4+0.5}_{-1.2-1.4-0.2}$ | |
| $\bar{B}_s \rightarrow K^{*0} \bar{K}^{*0}$ | P | $6.6^{+1.1+1.9}_{-1.4-1.7}$ | $9.1^{+0.5+11.3}_{-0.4-6.8}$ | $7.8^{+1.9+3.8+0.0}_{-1.5-2.2-0.0}$ | <1681 |
| $\bar{B}_s \rightarrow \phi K^{*0}$ | P | $0.37^{+0.06+0.24}_{-0.05-0.20}$ | $0.4^{+0.1+0.5}_{-0.1-0.3}$ | $0.65^{+0.16+0.27+0.10}_{-0.13-0.18-0.04}$ | <1013 |
| $\bar{B}_s \rightarrow \phi\phi$ | P | $16.7^{+2.6+11.3}_{-2.1-8.8}$ | $21.8^{+1.1+30.4}_{-1.1-17.0}$ | $35.3^{+8.3+16.7+0.0}_{-6.9-10.2-0.0}$ | 24.0 ± 8.9 |
| $\bar{B}_s \rightarrow \phi\omega$ | P, C | $0.18^{+0.44+0.47}_{-0.12-0.04}$ | $0.10^{+0.05+0.48}_{-0.03-0.12}$ | $0.16^{+0.09+0.10+0.01}_{-0.05-0.04-0.00}$ | |
| $\bar{B}_s \rightarrow \phi\rho^0$ | P_{EW} | $0.18^{+0.01+0.09}_{-0.01-0.04}$ | $0.40^{+0.12+0.25}_{-0.10-0.04}$ | $0.23^{+0.09+0.03+0.00}_{-0.07-0.01-0.01}$ | <617 |
| $\bar{B}_s \rightarrow \rho^+ \rho^-$ | Annihilation | $0.68^{+0.04+0.73}_{-0.04-0.53}$ | $0.34^{+0.03+0.60}_{-0.03-0.38}$ | $1.0^{+0.2+0.3+0.0}_{-0.2-0.2-0.0}$ | |
| $\bar{B}_s \rightarrow \rho^0 \rho^0$ | Annihilation | $0.34^{+0.02+0.36}_{-0.02-0.26}$ | $0.17^{+0.01+0.30}_{-0.01-0.19}$ | $0.51^{+0.12+0.17+0.01}_{-0.11-0.10-0.01}$ | <320 |
| $\bar{B}_s \rightarrow \rho^0 \omega$ | Annihilation | $0.004^{+0.0+0.005}_{-0.0-0.003}$ | <0.01 | $0.007^{+0.002+0.001+0.000}_{-0.001-0.001-0.000}$ | |
| $\bar{B}_s \rightarrow \omega\omega$ | Annihilation | $0.19^{+0.02+0.21}_{-0.02-0.15}$ | $0.11^{+0.01+0.20}_{-0.01-0.12}$ | $0.39^{+0.09+0.13+0.01}_{-0.08-0.07-0.00}$ | |

There exist two QCDF calculations of $\bar{B}_s \rightarrow VV$ [8,9]. However, only the longitudinal polarization states of $\bar{B}_s \rightarrow VV$ were considered in [9]. The analysis in this work differs from Beneke, Rohrer and Yang (BRY) [8] mainly in three places: (i) the choice of form factors, (ii) the values of the parameters ρ_A and ϕ_A , and (iii) the treatment of penguin annihilation contributions characterized by the parameters β_i [see Eq. (2.10)] for penguin-dominated VV modes. First, the form factors for $B_s \rightarrow K^*$ and $B_s \rightarrow \phi$ transitions we employ in Eq. (3.6) are smaller than the ones (3.10) used by BRY. Second, BRY applied the values $\rho_A(K^*\phi) = 0.6$ and $\phi_A(K^*\phi) = -40^\circ$ obtained from a fit to the data of $B \rightarrow K^*\phi$ to study $B \rightarrow \bar{K}^*\rho$ and $\bar{B}_s \rightarrow VV$ decays. However, as pointed out in [42], the parameters $\rho_A(K^*\rho) \approx 0.78$ and $\phi_A(K^*\rho) \approx -43^\circ$ fit to the data of $B \rightarrow K^*\rho$ decays are slightly different from the ones $\rho_A(K^*\phi)$ and $\phi_A(K^*\phi)$. Therefore, within the framework of QCDF, one cannot account for all charmless $B \rightarrow VV$ data by a universal set of ρ_A and ϕ_A parameters. This explains why the $B \rightarrow K^*\rho$ branching fractions obtained by BRY are systematically below the measurements. In this work, we choose $\rho_A = 0.70$ and $\phi_A = -55^\circ$ (cf. Table II) to describe $B_s \rightarrow VV$ decays. Third, as noticed in [42], there are sign errors in the expressions of the annihilation terms $A_3^{f,0}$ and $A_3^{i,0}$ obtained by BRY. As a consequence, BRY claimed (wrongly) that the longitudinal penguin annihilation amplitude β_3^0 is strongly suppressed, while the β_3^- term receives sizable penguin annihilation contribution. This will affect the decay rates and longitudinal polarization fractions in some of $B \rightarrow K^*\rho$ modes, as discussed in details in [42]. In spite of the above-mentioned three major differences in the calculations of this work and BRY, it turns out that the calculated rates and f_L shown in Tables XII and XIV, respectively, are similar for most of the $B_s \rightarrow VV$ modes.

Recently CDF has reported a new measurement of $B_s \rightarrow \phi\phi$ [50]

$$\frac{\mathcal{B}(\bar{B}_s \rightarrow \phi\phi)}{\mathcal{B}(\bar{B}_s \rightarrow J/\psi\phi)} = (1.78 \pm 0.14 \pm 0.20) \times 10^{-2}. \quad (6.2)$$

Using the branching fraction of $\bar{B}_s \rightarrow J/\psi\phi$ from Particle Data Group (PDG) [38], updated to current values of f_s/f_d , this leads to

$$\mathcal{B}(\bar{B}_s \rightarrow \phi\phi) = (24.0 \pm 2.1 \pm 2.7 \pm 8.2) \times 10^{-6}, \quad (6.3)$$

where the error is dominated by the last uncertainty coming from the $J/\psi\phi$ branching fraction error. This new measurement is slightly larger than the previous one of $(14^{+8}) \times 10^{-6}$ [65]. Our prediction $\mathcal{B}(\bar{B}_s \rightarrow \phi\phi) \approx 16.7 \times 10^{-6}$ is consistent with experiment.

A few words on the penguin-dominated decays $\bar{B}_s \rightarrow \phi K^{*0}$ and $\bar{B}_s \rightarrow \omega\phi$. Their branching fractions of order 10^{-7} are much smaller than other penguin-dominated $K^*\bar{K}^*$ and $\phi\phi$ modes. This is because $\bar{B}_s \rightarrow \phi K^{*0}$ is induced by the $b \rightarrow d$ penguin transition. The amplitude of $\bar{B}_s \rightarrow \omega\phi$ reads

$$\sqrt{2}A_{\bar{B}_s \rightarrow \omega\phi} = A_{\phi\omega} \left[\delta_{pu} \alpha_2 + 2\alpha_3^p + \frac{1}{2} \alpha_{3,EW}^p \right]. \quad (6.4)$$

The branching fraction due to the QCD penguin $\alpha_3 = a_3 + a_5$ is small, only at the level of 10^{-7} . Moreover, there is a partial cancellation between QCD and electroweak penguin contributions, making its rate even smaller.⁶

As seen from Table XII, pQCD predictions for the color-suppressed tree-dominated modes $\rho^0 K^{*0}$ and ωK^{*0} are much smaller than the QCDF results, whereas $\mathcal{B}(B_s \rightarrow \phi\phi) = \mathcal{O}(35 \times 10^{-6})$ is much larger than QCDF and the CDF measurement [65].

⁶It was argued in [8] that the color-suppressed tree amplitude α_2 is the largest partial amplitude in the decay $\bar{B}_s \rightarrow \omega\phi$. We found that this decay is still dominated by the QCD penguin, though the contribution from α_2 is not negligible.

In analog to Eq. (4.12), there are three SU(3) relations relating the rates of $B_s \rightarrow VV$ and $B_d \rightarrow VV$:

$$\begin{aligned} \mathcal{B}(\bar{B}_s \rightarrow K^{*+} \rho^-) &\approx \mathcal{B}(\bar{B}_d \rightarrow \rho^+ \rho^-), \\ \mathcal{B}(\bar{B}_s \rightarrow K^{*+} K^{*-}) &\approx \mathcal{B}(\bar{B}_d \rightarrow K^{*-} \rho^+), \\ \mathcal{B}(\bar{B}_s \rightarrow K^{*0} \bar{K}^{*0}) &\approx \mathcal{B}(B^- \rightarrow \bar{K}^{*0} \rho^-). \end{aligned} \quad (6.5)$$

Numerically, we have

$$\begin{aligned} 21.6_{-3.2}^{+1.6} \doteq 24.2_{-3.2}^{+3.1}, \quad 7.4_{-2.1}^{+2.5} \doteq 8.9_{-5.6}^{+4.9}, \\ 6.6 \pm 2.2 \doteq 9.2 \pm 1.5 \end{aligned} \quad (6.6)$$

in units of 10^{-6} , where use of the theoretical calculation of $\mathcal{B}(\bar{B}_d \rightarrow K^{*-} \rho^+)$ from [42] has been made.

B. Direct CP violation

Direct CP asymmetries in QCDF and pQCD approaches are summarized in Table XIII.

C. Polarization fractions

For charmless $\bar{B} \rightarrow VV$ decays, it is naively expected that the helicity amplitudes $\bar{\mathcal{A}}_h$ (helicities $h = 0, -, +$) for both tree- and penguin-dominated $\bar{B} \rightarrow VV$ respect the hierarchy pattern

$$\bar{\mathcal{A}}_0 : \bar{\mathcal{A}}_- : \bar{\mathcal{A}}_+ = 1 : \left(\frac{\Lambda_{\text{QCD}}}{m_b}\right) : \left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^2. \quad (6.7)$$

Hence, they are dominated by the longitudinal polarization states and satisfy the scaling law, namely [66],

$$f_T \equiv 1 - f_L = \mathcal{O}\left(\frac{m_V^2}{m_B^2}\right), \quad \frac{f_\perp}{f_\parallel} = 1 + \mathcal{O}\left(\frac{m_V}{m_B}\right), \quad (6.8)$$

with $f_L, f_\perp, f_\parallel$ and f_T being the longitudinal, perpendicular, parallel and transverse polarization fractions, respectively, defined as

$$f_\alpha \equiv \frac{\Gamma_\alpha}{\Gamma} = \frac{|\bar{\mathcal{A}}_\alpha|^2}{|\bar{\mathcal{A}}_0|^2 + |\bar{\mathcal{A}}_\parallel|^2 + |\bar{\mathcal{A}}_\perp|^2}, \quad (6.9)$$

with $\alpha = L, \parallel, \perp$. In sharp contrast to the $\rho\rho$ case, the large fraction of transverse polarization of order 0.5 observed in $\bar{B} \rightarrow \bar{K}^* \rho$ and $\bar{B} \rightarrow \bar{K}^* \phi$ decays at B factories is thus a surprise and poses an interesting challenge for any theoretical interpretation. Therefore, in order to obtain a large transverse polarization in $\bar{B} \rightarrow \bar{K}^* \rho, \bar{K}^* \phi$, this scaling law must be circumvented in one way or another.

As pointed out by Yang and one of us (H. Y. C.) [42], in the presence of NLO nonfactorizable corrections e.g. vertex, penguin and hard spectator scattering contributions, effective Wilson coefficients a_i^h are helicity dependent. Although the factorizable helicity amplitudes X^0, X^- and X^+ defined by Eq. (2.4) respect the scaling law (6.7) with Λ_{QCD}/m_b replaced by $2m_V/m_B$ for the light vector meson production, one needs to consider the effects of helicity-dependent Wilson coefficients: $\mathcal{A}^-/\mathcal{A}^0 = f(a_i^-)X^-/[f(a_i^0)X^0]$. For some penguin-dominated modes, the constructive (destructive) interference in the negative-helicity (longitudinal-helicity) amplitude of the $\bar{B} \rightarrow VV$ decay will render $f(a_i^-) \gg f(a_i^0)$ so that \mathcal{A}^- is comparable to \mathcal{A}^0 and the transverse polarization is enhanced. For example, $f_L(\bar{K}^{*0} \rho^0) \sim 0.91$ is predicted in the absence of NLO corrections. When NLO effects are turned on, their corrections on a_i^- will render the negative-helicity amplitude $\mathcal{A}^-(\bar{B}^0 \rightarrow \bar{K}^{*0} \rho^0)$ comparable to the longitudinal one $\mathcal{A}^0(\bar{B}^0 \rightarrow \bar{K}^{*0} \rho^0)$ so that even at the short-distance level, f_L for $\bar{B}^0 \rightarrow \bar{K}^{*0} \rho^0$ can be as low as 50%. However, this does not mean that the polarization anomaly is resolved. This is because the calculations based on naive factorization often predict too small rates for penguin-dominated $\bar{B} \rightarrow VV$ decays, e.g. $\bar{B} \rightarrow \bar{K}^* \phi$ and $\bar{B} \rightarrow \bar{K}^* \rho$, by a factor of $2 \sim 3$. Obviously, it does not make sense to compare theory with experiment for $f_{L,T}$ as the definition of polarization fractions depends on the partial

TABLE XIII. Same as Table XII except for direct CP asymmetries (in %) in the $\bar{B}_s \rightarrow VV$ decays.

| Channel | Class | QCDF (this work) | QCDF (BRY) | pQCD |
|---|---------------|-----------------------------|------------------------|---|
| $\bar{B}_s \rightarrow \rho^- K^{*+}$ | <i>T</i> | -11_{-1-1}^{+1+4} | -3_{-1-3}^{+1+2} | $-8.2_{-1.2-1.7-1.1}^{+1.0+1.2+0.4}$ |
| $\bar{B}_s \rightarrow \rho^0 K^{*0}$ | <i>C</i> | 46_{-17-25}^{+15+10} | 27_{-7-27}^{+5+34} | $61.8_{-4.7-22.8-2.3}^{+3.2+17.1+4.4}$ |
| $\bar{B}_s \rightarrow \omega K^{*0}$ | <i>C</i> | -50_{-15-6}^{+20+21} | -34_{-7-43}^{+10+31} | $-62.1_{-3.9-12.6-1.9}^{+4.8+19.7+5.5}$ |
| $\bar{B}_s \rightarrow K^{*-} K^{*+}$ | <i>P</i> | 21_{-2-4}^{+1+2} | 2_{-0-15}^{+0+40} | $9.3_{-0.7-3.6-0.2}^{+0.4+3.3+0.3}$ |
| $\bar{B}_s \rightarrow K^{*0} \bar{K}^{*0}$ | <i>P</i> | $0.4_{-0.5-0.4}^{+0.8+0.6}$ | 1_{-0-0}^{+0+1} | 0 |
| $\bar{B}_s \rightarrow \phi K^{*0}$ | <i>P</i> | -9_{-1-6}^{+3+4} | -17_{-5-9}^{+4+9} | 0 |
| $\bar{B}_s \rightarrow \phi \phi$ | <i>P</i> | $0.2_{-0.3-0.2}^{+0.4+0.5}$ | 1_{-0-0}^{+0+1} | 0 |
| $\bar{B}_s \rightarrow \phi \omega$ | <i>P, C</i> | -8_{-1-15}^{+3+20} | 8_{-3-56}^{+3+102} | $3.6_{-0.6-2.4-0.2}^{+0.6+2.4+0.6}$ |
| $\bar{B}_s \rightarrow \phi \rho^0$ | <i>P_{EW}</i> | 83_{-0-36}^{+1+10} | 19_{-5-67}^{+5+56} | $10.1_{-0.9-1.8-0.5}^{+0.9+1.6+1.3}$ |
| $\bar{B}_s \rightarrow \rho^+ \rho^-$ | Annihilation | 0 | | $-2.1_{-0.1-1.3-0.1}^{+0.2+1.7+0.1}$ |
| $\bar{B}_s \rightarrow \rho^0 \rho^0$ | Annihilation | 0 | | $-2.1_{-0.1-1.3-0.1}^{+0.2+1.7+0.1}$ |
| $\bar{B}_s \rightarrow \rho^0 \omega$ | Annihilation | 0 | | $6.0_{-0.5-3.9-0.4}^{+0.7+2.7+1.0}$ |
| $\bar{B}_s \rightarrow \omega \omega$ | Annihilation | 0 | | $-2.0_{-0.1-1.3-0.1}^{+0.1+1.7+0.1}$ |

rate and hence the prediction can be easily off by a factor of $2 \sim 3$. Thus, the first important task is to have some mechanism to bring up the rates. While the QCD factorization approach relies on penguin annihilation [66], soft-collinear effective theory invokes charming penguin [67] and the final-state interaction model considers final-state rescattering of intermediate charm states [68–70]. A nice feature of the $(S - P)(S + P)$ penguin annihilation is that it contributes to \mathcal{A}^0 and \mathcal{A}^- with similar amount. This together with the NLO corrections will lead to $f_L \sim 0.5$ for penguin-dominated VV modes. Hence, within the framework of QCDF we shall assume weak annihilation to account for the discrepancy between theory and experiment, and fit the existing data of branching fractions and f_L simultaneously by adjusting the parameters ρ_A and ϕ_A . Then using this set of annihilation parameters as a guideline, we can proceed to predict the rates and f_L for other VV decays of the $B_{u,d,s}$ mesons.

The longitudinal polarization fractions in $\bar{B}_s \rightarrow VV$ decays obtained in the QCDF and pQCD approaches are summarized in Table XIV. Transverse polarization effects are sizable in penguin-dominated $\bar{B}_s \rightarrow VV$ as expected. However, the pQCD calculations indicate that $f_L \sim f_T \sim \frac{1}{2}$ even for the color-suppressed tree-dominated decays $\bar{B}_s \rightarrow K^{*0}(\rho^0, \omega)$. This is an astonishing result and should be checked by experiment. Polarization fractions of $\bar{B}_s \rightarrow \phi\phi$ will be studied soon by CDF. It will be very interesting to see if the transverse polarization is also important in the penguin-dominated B_s decays.

D. U -spin symmetry

Analogous to the $\bar{B}_s \rightarrow PP$ sector, U -spin symmetry leads to the following relations:

$$\begin{aligned}
 A_{CP}(\bar{B}_s \rightarrow K^{*+} \rho^-) &= -A_{CP}(\bar{B}_d \rightarrow K^{*-} \rho^+) \frac{\mathcal{B}(\bar{B}_d \rightarrow K^{*-} \rho^+) \tau(B_s)}{\mathcal{B}(\bar{B}_s \rightarrow K^{*+} \rho^-) \tau(B_d)}, \\
 A_{CP}(\bar{B}_s \rightarrow K^{*+} K^{*-}) &= -A_{CP}(\bar{B}_d \rightarrow \rho^+ \rho^-) \frac{\mathcal{B}(\bar{B}_d \rightarrow \rho^+ \rho^-) \tau(B_s)}{\mathcal{B}(\bar{B}_s \rightarrow K^{*+} K^{*-}) \tau(B_d)}, \\
 A_{CP}(\bar{B}_s \rightarrow K^{*0} \bar{K}^{*0}) &= -A_{CP}(\bar{B}_d \rightarrow K^{*0} \bar{K}^{*0}) \frac{\mathcal{B}(\bar{B}_d \rightarrow K^{*0} \bar{K}^{*0}) \tau(B_s)}{\mathcal{B}(\bar{B}_s \rightarrow K^{*0} \bar{K}^{*0}) \tau(B_d)}, \\
 A_{CP}(\bar{B}_s \rightarrow K^{*0} \rho^0) &= -A_{CP}(\bar{B}_d \rightarrow \bar{K}^{*0} \rho^0) \frac{\mathcal{B}(\bar{B}_d \rightarrow \bar{K}^{*0} \rho^0) \tau(B_s)}{\mathcal{B}(\bar{B}_s \rightarrow K^{*0} \rho^0) \tau(B_d)}, \\
 A_{CP}(\bar{B}_s \rightarrow \rho^+ \rho^-) &= -A_{CP}(\bar{B}_d \rightarrow K^{*+} K^{*-}) \frac{\mathcal{B}(\bar{B}_d \rightarrow K^{*+} K^{*-}) \tau(B_s)}{\mathcal{B}(\bar{B}_s \rightarrow \rho^+ \rho^-) \tau(B_d)}.
 \end{aligned}
 \tag{6.10}$$

In Table XV we compare the results of CP asymmetries inferred from U -spin relations with the direct QCDF calculations. It appears that U -spin symmetry works well in the VV sector.

Assuming that the transverse amplitude can be expressed as a single dominant contribution which may arise from new physics, U -spin symmetry implies that the transverse amplitudes of $B_s \rightarrow VV$ can be related to the U -spin

TABLE XIV. Same as Table XII except for the longitudinal polarization fractions in the $\bar{B}_s \rightarrow VV$ decays.

| Channel | Class | QCDF (this work) | QCDF (BRY) | pQCD |
|---|--------------|----------------------------------|----------------------------------|---|
| $\bar{B}_s \rightarrow \rho^- K^{*+}$ | T | $0.92^{+0.01+0.01}_{-0.02-0.03}$ | $0.92^{+0.01+0.05}_{-0.01-0.08}$ | $0.937^{+0.001+0.002+0.000}_{-0.002-0.003-0.002}$ |
| $\bar{B}_s \rightarrow \rho^0 K^{*0}$ | C | $0.90^{+0.04+0.03}_{-0.05-0.23}$ | $0.93^{+0.02+0.05}_{-0.03-0.54}$ | $0.455^{+0.004+0.069+0.006}_{-0.003-0.043-0.009}$ |
| $\bar{B}_s \rightarrow \omega K^{*0}$ | C | $0.90^{+0.03+0.03}_{-0.04-0.23}$ | $0.93^{+0.02+0.05}_{-0.04-0.49}$ | $0.532^{+0.003+0.035+0.023}_{-0.002-0.029-0.013}$ |
| $\bar{B}_s \rightarrow K^{*-} K^{*+}$ | P | $0.52^{+0.03+0.20}_{-0.05-0.21}$ | $0.67^{+0.04+0.31}_{-0.05-0.26}$ | $0.438^{+0.051+0.021+0.037}_{-0.040-0.023-0.015}$ |
| $\bar{B}_s \rightarrow K^{*0} \bar{K}^{*0}$ | P | $0.56^{+0.04+0.22}_{-0.07-0.26}$ | $0.63^{+0.00+0.42}_{-0.00-0.29}$ | $0.497^{+0.057+0.006+0.000}_{-0.048-0.038-0.000}$ |
| $\bar{B}_s \rightarrow \phi K^{*0}$ | P | $0.43^{+0.02+0.21}_{-0.02-0.18}$ | $0.40^{+0.01+0.67}_{-0.01-0.35}$ | $0.712^{+0.032+0.027+0.000}_{-0.030-0.037-0.000}$ |
| $\bar{B}_s \rightarrow \phi\phi$ | P | $0.36^{+0.03+0.23}_{-0.04-0.18}$ | $0.43^{+0.00+0.01}_{-0.00-0.34}$ | $0.619^{+0.036+0.025+0.000}_{-0.032-0.033-0.000}$ |
| $\bar{B}_s \rightarrow \phi\omega$ | P, C | $0.95^{+0.01+0.00}_{-0.02-0.42}$ | | $0.443^{+0.000+0.054+0.009}_{-0.075-0.061-0.004}$ |
| $\bar{B}_s \rightarrow \phi\rho^0$ | P_{EW} | $0.88^{+0.01+0.02}_{-0.00-0.18}$ | $0.81^{+0.03+0.09}_{-0.04-0.12}$ | $0.870^{+0.002+0.009+0.009}_{-0.002-0.003-0.004}$ |
| $\bar{B}_s \rightarrow \rho^+ \rho^-$ | Annihilation | 1 | | ~ 1 |
| $\bar{B}_s \rightarrow \rho^0 \rho^0$ | Annihilation | 1 | | ~ 1 |
| $\bar{B}_s \rightarrow \rho^0 \omega$ | Annihilation | 1 | | ~ 1 |
| $\bar{B}_s \rightarrow \omega\omega$ | Annihilation | 1 | | ~ 1 |

TABLE XV. Direct CP asymmetries (in %) in $\bar{B}_s \rightarrow VV$ decays via U -spin symmetry.

| Modes | $\mathcal{B}(10^{-6})$ | A_{CP} (%) | Modes | A_{CP} (%) (U -spin) | A_{CP} (%) (QCDF) |
|---|----------------------------------|-----------------------|---|---------------------------|---------------------|
| $\bar{B}_d^0 \rightarrow K^{*-} \rho^+$ | $8.9^{+1.1+4.8}_{-1.0-5.5}$ | 32^{+1+5}_{-3-24} | $\bar{B}_s^0 \rightarrow K^{*+} \rho^-$ | -10.2 | -11^{+4}_{-1} |
| $\bar{B}_d^0 \rightarrow \bar{K}^{*0} \rho^0$ | $4.6^{+0.6+3.5}_{-0.5-3.5}$ | -15^{+4+16}_{-8-14} | $\bar{B}_s^0 \rightarrow K^{*0} \rho^0$ | 42.3 | 46^{+18}_{-30} |
| $\bar{B}_d^0 \rightarrow \rho^+ \rho^-$ | $25.5^{+1.5+2.4}_{-2.6-1.5}$ | -4^{+0+3}_{-0-3} | $\bar{B}_s^0 \rightarrow K^{*+} K^{*-}$ | 18.7 | 21^{+2}_{-3} |
| $\bar{B}_d^0 \rightarrow K^{*0} \bar{K}^{*0}$ | $0.6^{+0.1+0.2}_{-0.1-0.3}$ | -14^{+1+6}_{-1-2} | $\bar{B}_s^0 \rightarrow K^{*0} \bar{K}^{*0}$ | 0.5 | $0.4^{+1.0}_{-0.6}$ |
| $\bar{B}_d^0 \rightarrow K^{*+} K^{*-}$ | $0.15^{+0.02+0.11}_{-0.01-0.12}$ | 0 | $\bar{B}_s^0 \rightarrow \rho^+ \rho^-$ | 0 | 0 |

related decays in the B_d sector via [71]

$$\frac{\mathcal{A}_T(\bar{B}_s \rightarrow K^{*0} \bar{K}^{*0})}{\mathcal{A}_T(\bar{B}_d \rightarrow \bar{K}^{*0} K^{*0})} \approx \left| \frac{V_{ts}}{V_{td}} \right| \frac{f_{B_s}}{f_{B_d}}, \quad (6.11)$$

$$\frac{\mathcal{A}_T(\bar{B}_d \rightarrow \phi \bar{K}^{*0})}{\mathcal{A}_T(\bar{B}_d \rightarrow \phi K^{*0})} \approx \left| \frac{V_{ts}}{V_{td}} \right| \frac{f_{B_d}}{f_{B_s}}.$$

Therefore,

$$\frac{f_T(\bar{B}_s \rightarrow K^{*0} \bar{K}^{*0})}{f_T(\bar{B}_d \rightarrow \bar{K}^{*0} K^{*0})} \approx (25.5 \pm 6.5) \frac{\mathcal{B}(\bar{B}_d \rightarrow \bar{K}^{*0} K^{*0})}{\mathcal{B}(\bar{B}_s \rightarrow K^{*0} \bar{K}^{*0})},$$

$$\frac{f_T(\bar{B}_d \rightarrow \phi \bar{K}^{*0})}{f_T(\bar{B}_s \rightarrow \phi K^{*0})} \approx (19.3 \pm 4.9) \frac{\mathcal{B}(\bar{B}_s \rightarrow \phi K^{*0})}{\mathcal{B}(\bar{B}_d \rightarrow \phi \bar{K}^{*0})}. \quad (6.12)$$

The polarization measurement in the B_d decay thus allows one to predict the transverse polarization in the B_s decay.⁷ Using the data [47]

$$\mathcal{B}(\bar{B}_d \rightarrow \bar{K}^{*0} K^{*0}) = (1.28^{+0.37}_{-0.32}) \times 10^{-6},$$

$$f_L(\bar{B}_d \rightarrow \bar{K}^{*0} K^{*0}) = 0.80^{+0.12}_{-0.13}, \quad (6.13)$$

$$\mathcal{B}(\bar{B}_d \rightarrow \phi \bar{K}^{*0}) = (9.8 \pm 0.7) \times 10^{-6},$$

$$f_L(\bar{B}_d \rightarrow \phi \bar{K}^{*0}) = 0.48 \pm 0.03,$$

and QCDF predictions for $\mathcal{B}(\bar{B}_s \rightarrow K^{*0} \bar{K}^{*0})$ and $\mathcal{B}(\bar{B}_s \rightarrow \phi K^{*0})$, we obtain

$$f_T(\bar{B}_s \rightarrow K^{*0} \bar{K}^{*0}) = 1.02 \pm 0.28, \quad (6.14)$$

$$f_T(\bar{B}_s \rightarrow \phi K^{*0}) = 0.73 \pm 0.19.$$

It is obvious that the central value of the predicted $f_T(\bar{B}_s \rightarrow K^{*0} \bar{K}^{*0})$ via U -spin symmetry is too large. Note that there is a discrepancy between the QCDF prediction of $\mathcal{B}(\bar{B}_d \rightarrow \bar{K}^{*0} K^{*0}) = (0.6^{+0.2}_{-0.3}) \times 10^{-6}$ [42] and the *BABAR* measurement $\mathcal{B}(\bar{B}_d \rightarrow \bar{K}^{*0} K^{*0}) = (1.28^{+0.37}_{-0.32}) \times 10^{-6}$ [73]. We need to await a more precise measurement of $\bar{B}_d \rightarrow \bar{K}^{*0} K^{*0}$ in order to have a more accurate prediction of its transverse polarization fraction via U -spin symmetry.

⁷Based on $SU(3)$ flavor symmetry, it has been shown in [72] that the transverse polarizations of $\bar{B}_s \rightarrow \phi \phi$ and $\bar{B}_s \rightarrow \phi K^{*0}$ can be related to $\bar{B}_d \rightarrow \phi \bar{K}^{*0}$ and $\bar{B}_d \rightarrow K^{*0} \bar{K}^{*0}$, respectively.

E. Time-dependent CP violation

In principle, one can study time-dependent CP asymmetries for each helicity component,

$$\mathcal{A}_{h(t)} \equiv \frac{\Gamma(\bar{B}_s^0(t) \rightarrow V_h V_h') - \Gamma(B_s^0(t) \rightarrow V_h V_h')}{\Gamma(\bar{B}_s^0(t) \rightarrow V_h V_h') + \Gamma(B_s^0(t) \rightarrow V_h V_h')}$$

$$= S_h \sin(\Delta m_s t) - C_h \cos(\Delta m_s t), \quad (6.15)$$

where the effects of the width difference of the B_s mesons have been neglected. From Table XII we see that there is only one decay mode of particular interest, namely, $\bar{B}_s \rightarrow \phi \phi$. Indeed, this could be the most promising channel for the forthcoming LHCb experiment. This channel is a pure $b \rightarrow s\bar{s}s$ penguin-induced process and hence provides an ideal place for exploring the signal of new physics via $B_s - \bar{B}_s$ mixing and/or the penguin process. The other decays such as $\bar{B}_s \rightarrow \rho \rho$, $\rho^0 \omega$, $\omega \omega$ proceed through weak annihilation. The modes $\phi \omega$ and $\phi \rho^0$ receive QCD penguin and electroweak penguin contributions, respectively, but their rates are too small. A straightforward calculation gives

$$\mathcal{B}_L = (5.9^{+1.0+5.3}_{-0.8-5.7}) \times 10^{-6}, \quad C_L = (-0.5^{+0.1+1.4}_{-0.2-1.5})\%,$$

$$S_L = (-0.5^{+0.1+1.1}_{-0.1-1.8})\%, \quad (6.16)$$

for the longitudinal component of $\bar{B}_s \rightarrow \phi \phi$. Note that S_L is found to be positive and small ≤ 0.02 in [9], while our result is negative for S_L . An observation of large CP violation in this decay will rule out the scenario of minimal flavor violation. Time-dependent CP violation will be studied at LHC. If LHCb is upgraded to accumulate data sample of 100 fb^{-1} , the sensitivity of $S_{B_s \rightarrow \phi_L \phi_L}$ will reach the level of $0.01 \sim 0.02$.

VII. CONCLUSIONS

We have reexamined the branching fractions and CP -violating asymmetries of charmless $\bar{B}_s \rightarrow PP$, VP , VV decays in the framework of QCD factorization. We have included subleading power corrections to the penguin annihilation topology and to color-suppressed tree amplitudes that are crucial for resolving the CP puzzles and rate deficit problems with penguin-dominated two-body decays and color-suppressed tree-dominated $\pi^0 \pi^0$ and $\rho^0 \pi^0$ modes in the $B_{u,d}$ sector. Our main results are

- (i) Many model-independent relations for CP asymmetries and branching fractions of \bar{B}_d and \bar{B}_s decays can be derived under U -spin and $SU(3)$ symmetries for PP , VP , VV modes. In general, they are either experimentally verified or theoretically satisfied. There are also a few U -spin relations for transverse polarizations in $B_s \rightarrow VV$ decays.
- (ii) For the $B_s \rightarrow K$ transition form factor, we use a smaller one, $F^{B,K} \approx 0.24$ at $q^2 = 0$ obtained by the lattice calculation, to avoid too large rates for $\bar{B}_s \rightarrow K^+ \pi^-, K^+ K^-$ decays.
- (iii) Both QCDF and SCET indicate that the penguin-dominated decay $B_s \rightarrow \eta' \eta'$, the analog of $B \rightarrow K \eta'$ in the B_s sector, has the largest branching fraction of order $\sim 50 \times 10^{-6}$ in two-body hadronic decays of the B_s meson, whereas the pQCD approach claims that $\mathcal{B}(\bar{B}_s \rightarrow \eta \eta') \approx 35 \times 10^{-6}$ is the largest one.
- (iv) Even at the decay rate level, there are some noticeable differences between various approaches. The branching fractions of the color-suppressed tree-dominated decays obtained by pQCD, for example, $\bar{B}_s \rightarrow K^0 \pi^0, K^0 \eta^{(\prime)}, K^{*0} \pi^0, \rho^0 K^0, \omega K^0, K^{*0} \eta'$ are typically smaller by 1 order of magnitude than that of QCDF and SCET. For example, $\mathcal{B}(\bar{B}_s \rightarrow \rho^0 K^0)$ is predicted to be of order 1.9×10^{-6} by QCDF, but it is only about 0.08×10^{-6} in pQCD. In the QCDF approach, many of the above-mentioned decays get a substantial enhancement from the power corrections to the color-suppressed tree topology.
- (v) The decay rate of $\bar{B}_s \rightarrow \phi \eta'$ is sensitive to the $B_s \rightarrow \phi$ transition form factor $A_0^{B_s, \phi}(0)$. For $A_0^{B_s, \phi}(0) = 0.474$ obtained by the light-cone sum-rule method, a near cancellation between $\bar{B}_s \rightarrow \phi \eta_s$ and $\bar{B}_s \rightarrow \phi \eta_q$ occurs in the decays $\bar{B}_s \rightarrow \phi \eta'$, so that its branching fraction, of order 10^{-7} , becomes very small. However, if the value $A_0^{B_s, \phi}(0) = 0.30$ favored by many other model calculations is employed, then $\bar{B}_s \rightarrow \phi \eta_s$ and $\bar{B}_s \rightarrow \phi \eta_q$ will contribute constructively to $\bar{B}_s \rightarrow \phi \eta'$ so that $\mathcal{B}(\bar{B}_s \rightarrow \phi \eta') = 2.2 \times 10^{-6}$ and $\mathcal{B}(\bar{B}_s \rightarrow \phi \eta) = 1.0 \times 10^{-6}$. Hence, it is very important to measure the branching fractions of $\bar{B}_s \rightarrow \phi \eta^{(\prime)}$ to gain the information on the form factor $A_0^{B_s, \phi}$.
- (vi) Measurements of CP -violating asymmetries can be used to discriminate between QCDF, pQCD and SCET approaches:
 - (a) Both QCDF and pQCD predict a positive sign for $A_{CP}(\bar{B}_s \rightarrow K^0 \pi^0)$, whereas SCET leads to a negative one. This can be traced back to fact that $A_{CP}(\bar{B}_d \rightarrow \bar{K}^0 \pi^0)$ is positive in SCET, while it is negative inferred from the CP -asymmetry sum-rule, $SU(3)$ relation and the topological quark diagram analysis.
 - (b) For color-suppressed tree-dominated decays $\bar{B}_s \rightarrow K^{*0} \pi^0, \rho^0 K^0, \omega K^0, K^{*0} \eta'$, QCDF and pQCD results are of the same sign, whereas SCET predicts opposite signs for these modes. In the QCDF approach, the signs of these CP asymmetries are governed by the soft corrections to a_2 . Since the corresponding rates of these decays are very small in pQCD, as a consequence, the CP -violating asymmetries predicted by pQCD are very large, of order 0.50 or even bigger.
 - (c) In the QCDF framework, the penguin-dominated decays $\bar{B}_s \rightarrow K^0 \phi, \bar{K}^{*0} K^0, K^{*0} \bar{K}^0$ have nonvanishing CP asymmetries, though very small for the last two modes, whereas leading order pQCD predicts no CP violation for these three decays.
 - (vii) Mixing-induced CP asymmetries of the penguin-dominated decays $\bar{B}_s \rightarrow K^0 \bar{K}^0, \eta^{(\prime)} \eta^{(\prime)}, \phi \eta', \phi \phi$ are predicted to be very small in the SM. Especially, we found $S_{\bar{B}_s \rightarrow \phi_L \phi_L} \sim -0.5\%$. They are sensitive to new physics and provide possibilities of new discoveries. While both QCDF and pQCD approaches predict $S_{\bar{B}_s \rightarrow K_S \phi} \sim \mathcal{O}(0.70)$, the SCET result of 0.09 or -0.13 is dramatically different.
 - (viii) Because of soft power corrections to the color-suppressed tree amplitude, we find that such effects will convert the sign of mixing-induced CP violation S_f into the positive one for the color-suppressed decays $\bar{B}_s \rightarrow K_S(\pi^0, \eta, \eta')$. Therefore, even the measurements of the sign of $S_{\bar{B}_s \rightarrow K_S(\pi^0, \eta, \eta')}$ will be helpful to test if “ a_2 ” has a large magnitude and strong phase.
 - (ix) Transverse polarization effects are sizable in penguin-dominated $\bar{B}_s \rightarrow VV$ as expected. However, the pQCD approach predicts that $f_L \sim f_T \sim \frac{1}{2}$ even for the color-suppressed tree-dominated decays $\bar{B}_s \rightarrow K^{*0}(\rho^0, \omega)$. This should be tested by experiment.

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