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Deuteron as a Skyrmion with a generalized mass term

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We calculate the energies of the $B = 1$ and $B = 2$ Skyrmions with a generalized mass term proposed in [V. B. Kopeliovich, B. Piette, and W. J. Zakrzewski, Phys. Rev. D **73**, 014006 (2006).], allowing for (iso-)rotational deformations within the axially-symmetric ansatz. We show that this modification of the chiral symmetry breaking term is not sufficient to accommodate for the experimental value of the binding energy of the deuteron. Also, a computation of the different vibrational modes and energies reveals how the deuteron rigidity is affected as a function of the mass parameter D.

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I. INTRODUCTION

Since its proposal by T. H. R. Skyrme nearly 50 years ago, the Skyrme model [1–4] has acquired the status of one of the most attractive candidate for an effective theory of QCD in the low-energy limit as well as a prototype for a solitonic approach to particle physics. The reasons for such a longevity is probably that the model involves solitonic solutions, or Skyrmions, which have a number of attractive features, such as spatial extension and a conserved topological charge that can be associated to quantum numbers. With the help of semiclassical quantization, the "QCD Skyrmions" considered here are identified to baryons, and their topological charge to the baryon number. The success of the model is also built on the fact that this model possesses the same symmetries as QCD in the large N_c limit, relies on very few parameters and that the nucleon properties can be calculated within a 30% accuracy with respect to experimental data [5,6]. Yet, the Skyrme model remains an effective theory with its limitations and caveats.

Recently the model has been used to describe nuclei in the semiclassical approach. It was noted [7,8] that the rotational term would lead to instabilities unless one takes for the pion mass m_π a value several times larger than the physical one. Furthermore, the energy densities of the classical solutions turn out to have shell-like configurations even for relatively large baryon number. These problems may be avoided by modifying the pion mass term. One approach is to consider the pion mass as a free parameter that must be adjusted to fit experimental data [7,8]. A second more general approach [9] is based on the recognition that the standard mass term in the Skyrme model is only one in a family of such terms that have the correct asymptotic behavior to describe pion fields. Such modification of the original Skyrme model leads to the proposal that with a proper set of parameters, one may solve yet another inconsistency with experimental observations, the difference between the computed binding energy of the deuteron (~ 80 MeV) and the experimental value (2.224 MeV). To resolve this issue, extension to the Skyrme model can be studied and we concentrate in this work on a mass term generalizing the one first proposed by [6] to break chiral symmetry of the Skyrme Model.

The initial work of Kopeliovich, Piette, and Zakrzewski [10] proposed a new mass term regulated by a dimensionless parameter labeled D. Some of these authors [9] also analyzed the binding energy of the deuteron using the rational map ansatz and their results suggested that the it could be significantly lowered and fitted to agree with the experimental value. Indeed the $B = 2$ solution even becomes unstable for larger values of the parameter D. Recently however, a full numerical calculations [11] including some rotational deformations (for the nucleon) revealed that the binding energy was not as sensitive to the parameter D as suggested by the rational map approximation and that the binding energy could not be arbitrarily reduce to zero, which would lead to unstable the $B = 2$ solution. Some questions remain open however: (a) The calculations in [11] relied on fixed values for the parameters of the Skyrme model (except for D) mostly for comparison purposes with previous works. These parameters are usually chosen to reproduce the experimental values of the mass of the nucleon and delta or other physical quantities. So, some the conclusions in [11] may no longer hold for a more physical choice of Skyrme parameters despite the fact that they were mostly based on the ratio of the nucleon and deuteron masses. (b) The second problem involves (iso-)rotational deformations which were neglected for the $B = 2$ deuteron solution, so that one could not reliably conclude in [11] whether or not the binding energy decreases with D. The aim of this work is to address these two problems by adjusting the Skyrme model parameters consistently and allows for (iso-)rotational deformations in both the nucleon and deuteron solutions.

In the next section, we begin by a brief review of the Skyrme model with rotational deformations and a description of the mass term proposed in [10], introducing the parameter D that sets the relative weight of the mass term. In Sec. III, we present the computation of the energies of the nucleon and the deuteron which are performed using a

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simulated-annealing (SA) algorithm assuming axial-symmetric symmetry. In each case, rotational energy will be included in the minimization to allow deformation. Although the axial symmetry turns out to be a property of the nucleon solution, one may question the use of this ansatz for the deuteron as nonaxial deformations could arise in the case. It turns out that the energy difference between the axial-symmetric deuteron solution and the exact solution were found to be at most of the order of 1% [12]. Nonetheless, we shall see that, whether or not nonaxial deformations are taken into account, it will not affect our main conclusion, i.e. the deuteron binding energy increases with the parameter D contrary to what was conjectured in [9]. Next, from the $B = 2$ solutions at hand, we analyze the vibrational modes of the deuteron with respect to variations of the parameter D.

From then on, we shall refer as *rotational* effects, any effects coming from rotational and/or isorotational contributions unless mentioned otherwise.

II. THE SKYRME MODEL AND ITS MASS TERM

Let us first review briefly the Skyrme model with a modified mass term (for a more elaborate description, the reader should refer to [10,11]). The standard Skyrme Lagrangian is given by:

$$L_{\rm Sky} = \int d^3x \left[-\frac{F_\pi^2}{16} \operatorname{Tr}(L_\mu L^\mu) + \frac{1}{32e^2} \operatorname{Tr}([L_\mu, L_\nu]^2) \right] \tag{1}$$

where $L_\mu = U^\dagger \partial_\mu U$, U is an $SU(2)$ matrix related to the pion fields by:

$$U = \sigma + i\tau \cdot \pi. \tag{2}$$

with τ, the three Pauli matrices. The scalar field σ must respect:

$$\sigma^2 + \pi \cdot \pi = 1 \tag{3}$$

to avoid additional unphysical degrees of freedom and allow the possibility of solitonic solutions. The first term in (1) is the so-called nonlinear-σ model. Alone, it would lead to solutions which are unstable under scale transformations so Skyrme proposed a second term to stabilize the solitons solutions and prevent them from shrinking to zero size. Finally, one can add the term

$$L_m = \frac{m_\pi^2 F_\pi^2}{8} \operatorname{Tr}(\mathbf{1} - U) \tag{4}$$

first introduced by Adkins and Nappi [6] to break chiral symmetry. Configurations with boundary conditions:

$$U(\mathbf{r}, t) \rightarrow 1 \quad \text{as} \quad |\mathbf{r}| \rightarrow \infty \tag{5}$$

fall in topological sectors which are identified by their topological charge corresponding to the baryon number in the Skyrme model. Three parameters appear in the full Lagrangian, F_π, e and m_π, which are, respectively, the pion decay constant, a dimensionless parameter fixing the strength of Skyrme interaction term, which is set to $e = 4.84$ here, and the pion mass $m_\pi = 138$ MeV.

The mass term in (4) is certainly not unique. Any such term should have the correct asymptotic behavior to describe pion fields and provide typically non shell-like configurations of the Skyrme field, a desired feature since it is certainly more compatible with observations on nucleons where a roughly even matter density is observed. On the other hand, it was argued in recent works [7,8] that the rotational term is to large and leads to disruption of the soliton unless the mass term can provide a more restraining effect. This requires values of the pion mass m_π in (4) which are several times larger than the physical one. It was also found that the energy densities of the classical solutions exhibit shell-like features even for relatively large baryon number. These problems brought to light the prospects of modifying the pion mass term and perhaps use such a modification to close in on the difference between the computed binding energy of the deuteron (~ 80 MeV) and its experimental value (2.224 MeV).

A first and simple approach is to consider the pion mass m_π as a free parameter, along with the other two parameters of the model F_π and e, that must be chosen to fit experimental data [7,8,12]. A most common fit consist in getting $B = 1$ Skyrmions in the $I = J = \frac{1}{2}$ and $I = J = \frac{3}{2}$ states with the exact nucleon and delta masses, respectively. This led Battye *et al.* [7] to conclude that the pion mass must be set at more than twice its experimental value. Marleau and Fortier [12] repeated the calculations by targeting the nucleon and the more stable deuteron instead and found again that it is not possible to get a common set of parameters F_π and e which would fit both nucleon and deuteron masses simultaneously unless $m_\pi > 500$ MeV. Such large value of the pion mass compared to the nucleon mass challenge somehow our perception of chiral symmetry and how it is broken so it may be instructive to consider other approaches.

One such approach is to modify the form of the mass term. Let us recall here some of the features of a chiral symmetry breaking term. Since pions are usually interpreted as small fluctuations around the chirally invariant vacuum $U_0(x) \equiv 1$, one can approximate U as

$$U = \exp(i\tau \cdot \pi(\mathbf{x})) \simeq 1 + i\tau \cdot \pi(\mathbf{x})$$

in the limit $\pi \ll 1$. Then the sum of Lagrangians (1) and (4) gives rise to the kinematic term for free pions

$$L_\pi = \int d^3x \left(\frac{1}{2} \partial_\mu \pi \cdot \partial^\mu \pi. - \frac{1}{2} m_\pi^2 \pi \cdot \pi + \mathcal{O}(\pi^4) \right). \tag{6}$$

As an indication, the soliton mass for the $B = 1$ hedgehog solution $U = \exp(i\tau \cdot \hat{\mathbf{r}} F(r))$ gets a contribution from the mass term

$$L_m = \frac{m_\pi^2 F_\pi^2}{2} \int d^3x \sin^2\frac{F}{2}. \quad (7)$$

This term will drastically change the behavior of F at infinity, from $F(r \to \infty) \simeq C_1 r^{-2}$ (no mass term) to $F(r \to \infty) \simeq C_2 r^{-1}\exp(-m_\pi r)$ and certainly affects the size of the soliton. This exponential behavior is expected since a force mediated by pion exchange is characterized by a range of $R \sim 1/m_\pi$ but it also prevents disruption against centrifugal effect due to rotation provided m_π is large enough. Clearly the chiral symmetry breaking term achieving the form (6) in the small fluctuations limit is not uniquely defined. An attempt to generalize such term was proposed in [13] with the construction

$$L_m = \int d^3x \sum_{k=1}^{\infty} C_k \operatorname{Tr}(U^k + U^{\dagger k} - 2).$$

Here, C_k are constant parameters which obey the constraint

$$\sum_{k=1}^{\infty} k^2 C_k = \frac{m_\pi^2 f_\pi^2}{4}$$

in order to insure that the pion mass term coincide with (6) in the limit of small pion field fluctuations.

More recently Kopeliovich, Piette, and Zakrzewski [10] have shown that a more general form of the mass term obeying (5) can be written in the form:

$$L_m = \frac{m_\pi^2 F_\pi^2}{8K} \operatorname{Tr}\left(1 - \int_{-\infty}^{+\infty} g(p) U^p dp\right), \quad (8)$$

where $g(p)$ and K are given by

$$\int_{\infty}^{\infty} g(p)dp = 1 \quad \text{and} \quad K = \int_{\infty}^{\infty} g(p)p^2 dp. \quad (9)$$

As in [11], we will focus our analysis on a particular mass term characterized by a new parameter, D, and known to disfavor shell-like configurations [10],

$$L_m = \frac{m_\pi^2 F_\pi^2}{8(1-5D)} \operatorname{Tr}(\mathbf{1} - U - D(U^2 - U^3)). \quad (10)$$

This form follows from the choice of function $g(p)$

$$g(p) = \delta(p-1) + D(\delta(p-2) - \delta(p-3))$$

so that $K = 1 - 5D$ which constrains the parameter D to be in the range [0, 0.2] to keep the mass term from becoming infinite. In the limit $D \to 0$, we recover the original mass term in (4). This yields the static energy

$$E_s^B = E_{\text{Sky}}^B + E_m^B,$$

each piece coming, respectively, from the Skyrme Lagrangian (1) and the chiral symmetry breaking term (10), respectively

$$E_{\text{Sky}}^B = \int d^3x \left[-\frac{F_\pi^2}{16}\operatorname{Tr}(L_i L^i) + \frac{1}{32e^2}\operatorname{Tr}([L_i, L_j]^2)\right] \quad (11)$$

$$E_m^B = \frac{m_\pi^2 F_\pi^2}{8(1-5D)} \operatorname{Tr}(\mathbf{1} - U - D(U^2 - U^3)), \quad (12)$$

where i and j run over spatial components only and B is the baryonic number.

III. BINDING ENERGIES

The lowest static energy configurations for the $B = 1$ and $B = 2$ Skyrmions turn out to have spherical and axial symmetry, respectively. When rotational deformations are taken into account, these symmetries are lost. The $B = 1$ configurations still retains axial symmetry whereas the $B = 2$ solution may show possible nonaxial deformations due to the form of the rotational energy [Eq. (18)]. Yet, it was shown in [12] that the contribution coming from nonaxial deformation is bounded to be at the very most 1% of the deuteron mass and axial symmetry should represent a very good approximation for the $B = 2$ configuration. In that view, we use the axially-symmetric ansatz to perform our calculations reducing considerably computation time with respect to a full 3D computation. We write the σ and π fields accordingly in terms of the unit vector $\psi(\rho, z) = (\psi_1, \psi_2, \psi_3)$ [14]:

$$\sigma = \psi_3, \qquad \pi_1 = \psi_1 \cos n\theta,$$
$$\pi_2 = \psi_1 \sin n\theta, \qquad \pi_3 = \psi_2. \quad (13)$$

Using the mass term in (10) the expressions for the static energy becomes

$$E_{\text{Sky}}^B = 2\pi \int dz d\rho \rho \left\{(\partial_\rho \boldsymbol{\psi} \cdot \partial_\rho \boldsymbol{\psi} + \partial_z \boldsymbol{\psi} \cdot \partial_z \boldsymbol{\psi}) \times \left(1 + n^2 \frac{\psi_1^2}{2\rho^2}\right) + \frac{1}{2}|\partial_z \boldsymbol{\psi} \times \partial_\rho \boldsymbol{\psi}|^2 + n^2 \frac{\psi_1^2}{\rho^2}\right\} \quad (14)$$

$$E_\pi^B = 2\pi \frac{2\beta^2}{(1-5D)} \int dz d\rho \rho (1 - \psi_3) \times (1 + D(1 - 2\psi_3 - 4\psi_3^2)) \quad (15)$$

where we have used $2\sqrt{2}/eF_\pi$ and $F_\pi/(2\sqrt{2}e)$ as units of length and energy, respectively. Here $\beta = \frac{2\sqrt{2}m_\pi}{eF_\pi}$ and the baryon number given by

$$B = \frac{n}{\pi} \int dz d\rho \psi_1 |\partial_\rho \boldsymbol{\psi} \times \partial_z \boldsymbol{\psi}|. \quad (16)$$

In order for the Skyrmions to represent the desired physical states, we must add to the static energy a contribution, E_{rot}^B, coming from the rotational energy due to nucleon and deuteron spin and isospin. The nucleon and the deuteron masses are respectively

$$E_N = E_s^1 + E_{\rm rot}^1 \quad \text{with}$$
$$E_{\rm rot}^1 = \frac{1}{4}\left[\frac{(1-\frac{W_{11}}{U_{11}})^2}{V_{11}-\frac{W_{11}^2}{U_{11}}} + \frac{1}{U_{11}} + \frac{1}{2U_{33}}\right] \tag{17}$$

$$E_D = E_s^2 + E_{\rm rot}^2 \quad \text{with} \quad E_{\rm rot}^2 = \frac{1}{V_{11}}. \tag{18}$$

Here U_{ii}, V_{ii} and W_{ii} are moment of inertia given in the axial-symmetric ansatz by [8,12]:

$$U_{11} = \frac{2\pi}{e^4}\int dz d\rho \rho \bigg\{\psi_1^2 + 2\psi_2^2 + \frac{1}{2}\bigg[\bigg(\partial_\rho \boldsymbol{\psi}\cdot\partial_\rho\boldsymbol{\psi} + \partial_z\boldsymbol{\psi}\cdot\partial_z\boldsymbol{\psi} + n^2\frac{\psi_1^2}{\rho^2}\bigg)\psi_2^2 \tag{19}$$

$$+ (\partial_\rho\psi_3)^2 + (\partial_z\psi_3)^2 + n^2\frac{\psi_1^4}{\rho^2}\bigg]\bigg\}, \tag{20}$$

$$U_{33} = \frac{2\pi}{e^4}\int dz d\rho \rho \psi_1^2(\partial_\rho\boldsymbol{\psi}\cdot\partial_\rho\boldsymbol{\psi} + \partial_z\boldsymbol{\psi}\cdot\partial_z\boldsymbol{\psi} + 2), \tag{21}$$

$$V_{11} = \frac{2\pi}{e^4}\int dz d\rho \rho\bigg\{|\rho\partial_z\boldsymbol{\psi} - z\partial_\rho\boldsymbol{\psi}|^2\bigg(1 + n^2\frac{\psi_1^2}{2\rho^2}\bigg) + z^2 n^2\frac{\psi_1^2}{\rho^2} + \frac{1}{2}(\rho^2+z^2)|\partial_\rho\boldsymbol{\psi}\times\partial_z\boldsymbol{\psi}|^2\bigg\}, \tag{22}$$

$$W_{11} = \frac{2\pi}{e^4}\int dz d\rho \rho\bigg\{[\psi_1(\rho\partial_z\psi_2 - z\partial_\rho\psi_2) - \psi_2(\rho\partial_z\psi_1 - z\partial_\rho\psi_1)]\bigg(1 + \frac{1}{2}\bigg[(\partial_z\psi_3)^2 + (\partial_\rho\psi_3)^2 + \frac{\psi_1^2}{\rho^2}\bigg]\bigg) \tag{23}$$

$$+ \frac{\psi_3}{2}(z\partial_z\psi_3 + \rho\partial_\rho\psi_3)[\partial_\rho\psi_2\partial_z\psi_1 - \partial_\rho\psi_1\partial_z\psi_2] + \frac{z\psi_1\psi_2}{2\rho}(2 + \partial_\rho\boldsymbol{\psi}\cdot\partial_\rho\boldsymbol{\psi} + \partial_z\boldsymbol{\psi}\cdot\partial_z\boldsymbol{\psi})\bigg\}. \tag{24}$$

A solution for the nucleon and deuteron correspond to the configuration of $\boldsymbol{\psi}(\rho, z)$ which minimize E_N and E_D respectively and obeys (16). In this work, this is achieved using a simulated-annealing algorithm, a procedure which allows computing the energies of the nucleon and the deuteron directly without having to solve an integro-differential equation. A grid of 250×500 points is used, corresponding to $0 < \rho < 10$ and $-10 < z < 10$ respectively in units of $2\sqrt{2}/eF_\pi$. As opposed to previous work in [11], both the static and rotational energy are included in the minimization of E_N and E_D, thus allowing for rotational deformations (within the axial approximation) for the $B = 2$ as well as for the $B = 1$ Skyrmions. During computation, configurations that lead to large deviation of the baryonic number, larger that 0.1%, are rejected in order to guarantee that the solution remains in the right topological sector. The simulated annealing process comes to an end when the configuration is cooled down to a temperature of 10^{-8} which corresponds to variations of the total energy of the Skyrmion that are smaller than 0.01%.

Calculations are performed requesting a precision of less than 1000th on the baryonic number to insure that we stay in the right topological sector and the simulations come to an end until the variations on the value of the total energy of the Skyrmion are also less than 1000th of its value.

The solutions are found for a finite set of values of D within the interval [0, 0.2[. Our procedure requires first that the lowest energy configuration for the nucleon possesses a mass of $E_N = 939$ MeV. So, for each value of D, we find a solution for a given value of the parameter F_π and iterate by adjusting F_π until it replicates the nucleon mass at 939 MeV. The value of $e = 4.84$ remains constant throughout all our calculations.[1] The second step consists in evaluating the mass of the deuteron for this set of parameters. The results are presented in the form of the ratio $R = \frac{E_D}{2E_N}$ as in [11]. We recall that in this latter work, fixed values of the model parameters ($F_\pi = 129$ MeV and $e = 5.44$) were used so the information regarding the relative importance of the binding energy was the most meaningful contrary to here where the value of E_D would suffice. The main purpose of this work is to look at that ratio as the value of D increases and most importantly, to verify if the mass term in (10) can accommodate for both the nucleon and deuteron mass i.e. to obtain the experimental value $R = 0.9989$.

The results for the nucleon and deuteron masses are presented in Table I. These are dissected into contributions coming from the static energy E_s^B, which contains the mass term E_m^B, and the rotational energy $E_{\rm rot}^B$. The ratio $R = \frac{E_D}{2E_N}$ is also shown in order to verify if the generalized mass term proposed in [10] can match the experimental value of 0.9989 for an appropriate value of the parameter D. The second of column of data shows the precision of the fit of F_π in favor of the target value of $E_N = 939.0$ MeV. So the uncertainties on absolute value of the nucleon and deuteron masses are close to 0.05%. On the other hand, the uncertainty of the ratio $R = \frac{E_D}{2E_N}$ is less dependent on the scale F_π and should be closer to the precision attainable with the simulated annealing minimization i.e. 0.01% or better. We see that the relative weight of the mass term in both the nucleon and the deuteron masses increases with the parameter D. At first sight, Eq. (12) suggest such a dependence on D however one must keep in mind that the dependence is much more complex since the choice of

[1]We recall that $F_\pi = 108$ MeV, $e = 4.84$, $m_\pi = 138$ MeV and $D = 0$ leads to correct values for the nucleon and delta masses [6].

TABLE I. Nucleon and deuteron masses obtained by minimizing the are presented in Table I. These are dissected into contributions coming from the static energy E_s^B, the mass term E_π^B and the rotational energy $E_{\rm rot}^B$.

D	F_π	E_N	E_s^1	E_π^1	$E_{\rm rot}^1$	E_D	E_s^2	E_π^2	$E_{\rm rot}^2$	$E_D/(2E_N)$
0	109.04	938.82	879.18	53.90	59.65	938.82	1680.44	75.96	13.39	0.9021
0.02	108.65	939.36	879.01	56.27	60.35	939.36	1681.22	80.30	13.35	0.9020
0.04	108.04	939.03	877.99	59.32	61.04	939.03	1680.29	85.57	13.27	0.9018
0.06	107.33	939.01	877.07	62.97	61.94	939.01	1679.86	92.02	13.18	0.9015
0.08	106.46	939.35	876.29	67.59	63.05	939.35	1679.90	99.91	13.08	0.9011
0.1	105.15	938.69	874.16	73.43	64.53	938.69	1677.89	110.28	12.92	0.9006
0.12	103.47	939.07	872.58	81.39	66.49	939.07	1677.33	123.92	12.71	0.8999
0.14	100.80	938.87	869.74	92.84	69.13	938.87	1674.86	143.25	12.38	0.8985
0.16	96.20	938.67	865.68	110.66	72.99	938.67	1670.69	173.18	11.82	0.8962
0.18	86.18	938.96	864.09	144.94	74.87	938.96	1664.68	228.76	10.59	0.8921
0.19	73.68	939.06	862.87	176.54	76.20	939.06	1658.78	283.03	9.05	0.8880
0.195	60.05	938.66	863.33	204.35	75.33	938.66	1656.44	329.49	7.38	0.8863
0.1975	46.65	938.83	862.76	223.73	76.07	938.83	1653.69	362.59	5.73	0.8838

F_π (and hence β) relies on the fit of the nucleon mass for each value of D and this affect non trivially the field configuration. On the contrary, the total static energies $E_s^B = E_{\rm Sky}^B + E_\pi^B$ diminish with D which means that the contribution of the dynamical part of the Lagrangian $L_{\rm Sky}$ is significantly decreasing. As for the rotational energy for the nucleon and the deuteron, we observe a distinct behavior. Whereas $E_{\rm rot}^1$ increases regularly as a function of D, the deuteron rotational energy $E_{\rm rot}^2$ shrinks considerably. So it would seem that a larger mass term reduces the moments of inertia for the nucleon. This is intuitively what one expects since a large mass term compels the nucleon to a smaller size. But the effect is quite the opposite for the deuteron where the rotational energy is suppressed by more than a factor of 2 in the interval from $D = 0$ to $D = 0.1975$ despite the fact that the toroidal configuration shrinks in size as D increases.

Finally, the results for the ratio $R = \frac{E_D}{2E_N}$ ($R = \frac{E_s^2}{2E_s^1}$) as a function of the mass parameter D are illustrated as black squares (black triangles) in Fig. 1 which come from the same family of solutions that were computed by minimizing (17) and (18). Also presented for comparison are data for ratios $R = \frac{E_D}{2E_N}$ as empty circles ($R = \frac{E_s^2}{2E_s^1}$ as empty diamonds) obtained in [11] from the minimization of the solution static energy alone with fixed values of $F_\pi = 129$ MeV and $e = 5.44$. The solid line at $R = 1$ corresponds to the limit of instability of the bound state whereas the experimental value for $\frac{E_D}{2E_N} = 0.99979$, i.e. very close to 1. The set of points $R = \frac{E_D}{2E_N}$ (black squares) clearly show that instead of having a ratio increasing with the value of D to match the experimental value, it is decreasing, meaning that the deuteron is bounded even more strongly as D increases. This leaves no possibility for a model with such a mass term to comply with data on the deuteron mass. A question immediately arises as to whether nonaxial deformations for the $B = 2$ solution would deter this conclusion. The answer is no since such solution would only decrease the computed mass of the deuteron E_D and proportionately increase its binding energy. On the other hand, comparing the sets of data in Fig. 1 indicates that the minimization of the solution static energy with $F_\pi = 129$ MeV (empty diamonds) was rather successful at describing the general behavior of $R = \frac{E_D}{2E_N}$ (black squares).

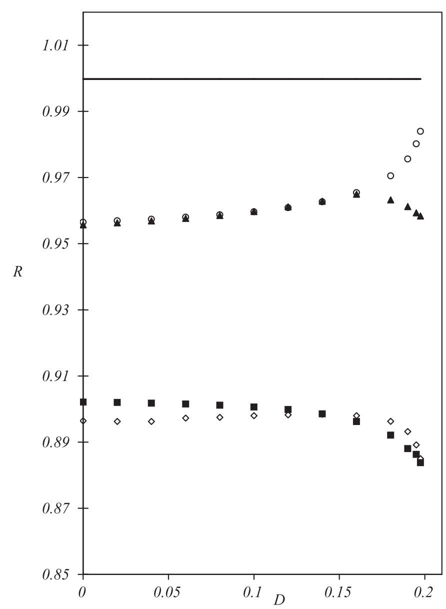

FIG. 1. Ratio $R = \frac{E_D}{2E_N}$ ($R = \frac{E_s^2}{2E_s^1}$) as a function of the mass parameter D are illustrated as black squares (black triangles) in Figure 1 which come from the same family of solutions that were computed by minimizing (17) and (18). Also presented for comparison are data for ratios $R = \frac{E_D}{2E_N}$ as empty circles ($R = \frac{E_s^2}{2E_s^1}$ as empty diamonds) obtained in [11] from the minimization of the solution static energy alone. The dashed line at $R = 1$ corresponds to the limit of instability of the bound state whereas the experimental value for $\frac{E_D}{2E_N} = 0.99979$, i.e. very close to 1.

So the procedure in [11] could serve as a good indication when a full calculation including rotational deformation is prohibitive. Despite the limitations of the model regarding its ability to replicate with precision the nucleon and deuteron masses, it remains a convenient prototype for low-energy QCD and perhaps nuclear physics.

IV. VIBRATIONAL MODES AND THEIR ENERGIES

We consider here briefly the vibrational modes of the $B = 1$ and $B = 2$, where (iso-)rotations have been included in the minimization, in order to see if the $B = 2$ Skyrmion, which correspond to the deuteron in the axial symmetry approximation, tends to be more rigid as the parameter D increases. The vibrational modes are determined by applying a time dependent global scale transformations in the x, y and z directions in the form $x_i \to \beta_i x_i$ as first introduced by Hadjuk and Schwesinger [15] and later used by Marleau and Davies [11]. The fields change according to

$$\sigma(x) \to \sigma(\beta_k x_k) \quad \text{and} \quad \pi_a(x) \to \pi_a(\beta_k x_k), \tag{25}$$

Substitution of (25) and (1) with the mass term (10) results in the following Lagrangian

$$L = \frac{1}{2} M_{ij}(\beta) \frac{\dot{\beta}_i \dot{\beta}_j}{\beta_i \beta_j} - V(\beta), \tag{26}$$

where the matrices $M_{ij}(\beta)$ and $V(\beta)$ are obtained by direct inspection. We then proceed according to the procedure described in [11,15] and consider only small amplitude oscillations η_i around the stable configuration $\beta_i = \beta_i^0$

$$\beta_i = \beta_i^0 e^{\eta_i} = \beta_i^0 \left(1 + \eta_i + \frac{1}{2}\eta_i^2 + \ldots\right)$$

and get the Lagrangian

$$L = \frac{1}{2} M_{ij}(\beta^0) \dot{\eta}_i \dot{\eta}_j - \frac{1}{2} \eta_i \eta_j v_{ij}(\beta^0). \tag{27}$$

Performing the change of coordinates $\eta = A\xi$, with A such as $A^T M(\beta^0) A = 1$ to obtain the Hamiltonian in a standard form

$$H_{\text{vib}} = \frac{1}{2} \sum_i \frac{\partial^2}{\partial \xi_i^2} + \frac{1}{2} (A^T v A)_{ij} \xi_i \xi_j. \tag{28}$$

such that diagonalizing the matrix $(A^T v A)_{ij}$,

$$(B^T A^T v A B)_{ij} = \omega_i^2 \delta_{ij}, \tag{29}$$

leads directly to the eigenvalues and eigenvectors of the vibrational modes. One can then write the energies for the different vibrational modes as

$$E_i^{\text{vib}} = h n_i \omega_i, \tag{30}$$

where the zero point of energy have been set to zero. Each of three eigenvectors obtained from (29) corresponds to one type of vibration. They are orthogonal in the ξ_i basis but it is useful to express them in terms of the Cartesian directions x_i, allowing us to see in which directions and amplitudes the Skyrmions are deformed. For the $B = 2$ Skyrmion, the first type of eigenvector has the form

$$\propto \begin{pmatrix} 1 & 1 & 9.715 \end{pmatrix} \tag{31}$$

in Cartesian coordinates, which could be identified to some sort of axially-symmetric breathing mode since the compression-expansion is in phase in all direction. This configuration is expected given the toroidal shape of the $B = 2$ Skyrmion. As the D increases, the value of the third component goes from 9.715 for $D = 0.02$ to 1.002 for $D = 0.1975$. For $D = 0$, the oscillation amplitude is almost entirely suppressed in the $x - y$ plane while remaining important along the z- direction. The second eigenvector has the same form as the one for the $B = 1$ Skyrmion

$$\propto \begin{pmatrix} 1 & -1 & 0 \end{pmatrix} \tag{32}$$

corresponding to an compression along the x axis while there is expansion along the y axis and vice-versa. Finally, the third eigenvector looks like

$$\propto \begin{pmatrix} 1 & 1 & -0.757 \end{pmatrix} \tag{33}$$

with the third component ranging from -0.757 for $D = 0$ to -1.95 for $D = 0.1975$. All eigenvectors are consistent with the notion of small oscillation around an axial configuration.

We analyzed vibrational energies of the $B = 1$ Skyrmion for solutions obtained both with and without the rotational energy in the minimization in [11]. Vibrational energies, and thus rigidity, were found to increase with D. We concentrate here on the $B = 2$ Skyrmion, including rotational energy minimization in the axially-symmetric ansatz, to verify if the same behavior regarding increases still holds. The results are presented in Table II. We can see from the data that similarly to the nucleon, the energy of the breathing mode increases as the

TABLE II. Vibrational energies for the $B = 2$ Skyrmion including rotational energy minimization.

D	ω_{br}	ω_2	ω_3
0	209.52	296.37	196.67
0.02	211.43	297.01	197.4
0.04	214.08	297.42	197.51
0.06	217.9	297.98	197.22
0.08	223.15	298.74	196.48
0.1	230.11	299.17	194.91
0.12	239.98	299.98	192.75
0.14	254.27	300.42	189.08
0.16	277.36	300.05	182.7
0.18	322.83	295.7	169.09
0.19	369.68	284.53	152.51
0.195	410.31	266.62	134.65
0.1975	435.51	242.9	116.55

value of D gets larger. However, a totally different behavior is observed for the two remaining modes of vibration of the deuteron. The energies from the ω_2 and ω_3 modes get smaller with the increasing D, as can be seen on Fig. 2. Furthermore, the breathing mode is not the lowest energy vibrational mode for the deuteron. For the nucleon, the energies of the ω_2 and ω_3 modes were shown to increase with D [11], supporting the idea that an increasing rigidity would result from the inclusion of a large mass term. For the deuteron, however, the apparent contradictory behavior of ω_i's suggest a more elaborate explanation. To this end, some insight could be provided by the form of the vector representing the deformation in Cartesian coordinates. Let us first we consider the breathing mode. This mode represents simultaneous expansions (or contractions) of the Skyrmion in all directions around some ground state, thus modifying the total volume of the soliton. But as mentioned in the previous section, the increase of D should compel the Skyrmion to occupy a smaller volume. Since this vibrational mode involves expansion of the Skyrmion, the energy required for excitation should increase as is readily observed. The case of the ω_2 mode is somewhat different. It can be visualized as a torus going from a circular cross section in the $x-y$ plane to an ellipsoid shape, the major axis oscillating from one axis to the other. For small vibrations, the total volume occupied by the Skyrmion should not change very much so the constraining effect of the mass term should not be as important. The variation of ω_2 expressed in MeV is somewhat misleading in that respect, as it is obtained from the ω_i's resulting from (30) multiplied by the conversion factor $F_\pi e/(2\sqrt{2})$. It turns out that the decrease of the ω_2 with respect to D is entirely due to the changes in the parameter F_π. Finally, for the third mode, the oscillations involve expansion in x and y-directions while there is compression in the z-direction, with the relative importance of the z displacement increasing as D get larger. Once again, this mode allows the changes in volume of the Skyrmion to remain minimal during the vibration even for large D and as for ω_2, the parameter F_π is responsible for the declining behavior of ω_3. The mass term propose by [10] thus seems to imply lower vibrational states for the deuteron in the ω_2 and ω_3 mode while becoming more rigid against the breathing oscillations.

V. CONCLUSION

We have computed the nucleon and deuteron masses in the framework of the Skyrme model with a chiral symmetry breaking term proposed by Kopeliovich, Piette and Zakrzewski [10]. The calculations allowed axial deformations due to rotational and/or isorotational contributions. For a value of the Skyrme parameter $e = 4.84$, it was not possible to find a set of values for F_π and D which would replicate the small experimental value for the binding energy of the deuteron. Furthermore, these results combined to previous calculations in [11] suggests that the ratio R is not very sensitive to the Skyrme parameter e which would relegate the model only to a gross approximation or a prototype model of nuclear matter. There are of course other alternatives to the simple form of Lagrangian in (1). A Skyrme-like effective Lagrangian, derived from QCD, would most likely include higher or all orders in deriva-

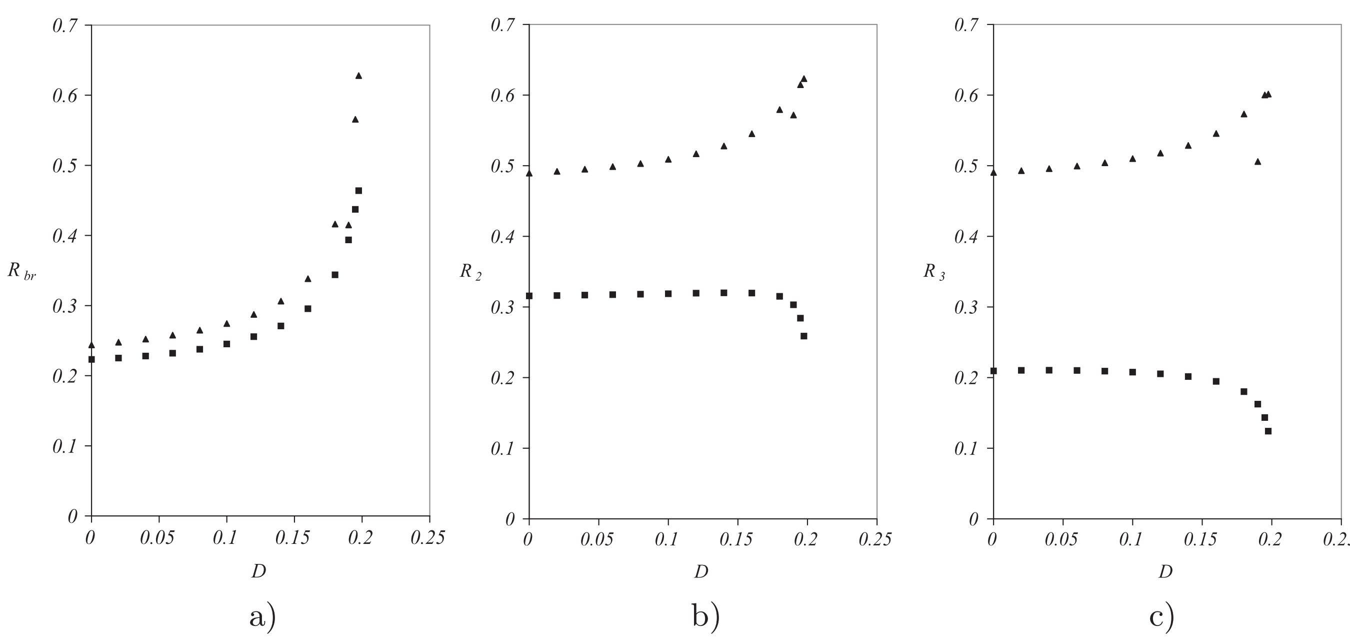

FIG. 2. Ratios of vibrational energies with respect to the total energy of the nucleon $R_i = \frac{\omega_i}{E_N}$ as a function of the mass parameter D where in (a) i labels the vibrational mode $i = br$, in (b) $i = 2$ and in (c) $i = 3$. Triangles (squares) correspond to the nucleon (deuteron) where rotational energy have been included in the minimization for both cases.

tives of the pion field such as those proposed in [16,17]. It remains to be seen if such Lagrangians can accommodate both the nucleon and deuteron masses. On the other hand, all our calculations were obtained assuming null zero-point energy. Going beyond zero-mode quantization could account for the small deuteron binding energy as suggested by the work of Leese *et al* [18]. This work also presented a computation of the vibrational modes and energies for the deuteron which showed that its toroidal configuration results in an increased complexity in the vibrational modes.

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[1] T. H. R. Skyrme, Proc. R. Soc. A **247**, 260 (1958).
[2] T. H. R. Skyrme, Proc. R. Soc. A **260**, 127 (1961).
[3] T. H. R. Skyrme, Proc. R. Soc. A **262**, 237 (1961).
[4] T. H. R. Skyrme, Nucl. Phys. **31**, 556 (1962).
[5] G. S. Adkins, C. R. Nappi, and E. Witten, Nucl. Phys. **B228**, 552 (1983).
[6] G. S. Adkins and C. R. Nappi, Nucl. Phys. **B233**, 109 (1984).
[7] R. A. Battye, S. Krusch, and P. M. Sutcliffe, Phys. Lett. B **626**, 120 (2005).
[8] C. Houghton and S. Magee, Phys. Lett. B **632**, 593 (2006).
[9] B. Piette and W. J. Zakrzewski, Phys. Rev. D **77**, 074009 (2008).
[10] V. B. Kopeliovich, B. Piette, and W. J. Zakrzewski, Phys. Rev. D **73**, 014006 (2006).
[11] M. C. Davies and L. Marleau, Phys. Rev. D **79**, 074003 (2009).
[12] J. Fortier and L. Marleau, Phys. Rev. D **77**, 054017 (2008).
[13] L. Marleau, Phys. Rev. D **43**, 885 (1991).
[14] S. Krusch and P. Sutcliffe, J. Phys. A **37**, 9037 (2004).
[15] C. Hajduk and B. Schwesinger, Nucl. Phys. **B453**, 620 (1986).
[16] L. Marleau, Phys. Lett. B **235**, 141 (1990); L. Marleau and J. F. Rivard, Phys. Rev. D **63**, 036007 (2001).
[17] A. Jackson, A. D. Jackson, A. S. Goldhaber, G. S. Brown, and L. C. Castillo, Phys. Lett. B **154**, 101 (1985).
[18] R. A. Leese, N. S. Manton, and B. J. Schroers, Nucl. Phys. **B442**, 228 (1995).