Revisiting charmless hadronic $B_{u,d}$ decays in QCD factorization

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Within the framework of QCD factorization, we consider two different types of power correction effects in order to resolve the CP puzzles and rate deficit problems with penguin-dominated two-body decays of B mesons and color-suppressed tree-dominated $\pi^0\pi^0$ and $\rho^0\pi^0$ modes: penguin annihilation and soft corrections to the color-suppressed tree amplitude. We emphasize that the electroweak penguin solution to the $B \to K\pi$ CP puzzle via new physics is irrelevant for solving the CP and rate puzzles related to tree-dominated decays. While some channels, e.g. $K^-\pi^+$, $K^-\rho^0$, $\pi^+\pi^-$, $\rho^\pm\pi^\mp$ need penguin annihilation to induce the correct magnitudes and signs for their CP violation, some other decays such as $B^- \to K^-\pi^0$, $\pi^-\eta$, $K^-\eta$ and $\bar{B}^0 \to \bar{K}^{*0}\eta$, $\pi^0\pi^0$ require the presence of both power corrections to account for the measured CP asymmetries. In general, QCD factorization predictions for the branching fractions and direct CP asymmetries of $\bar{B} \to PP$, VP, VV decays are in good agreement with experiment. The predictions of perturbaive QCD and soft-collinear effective theory are included for comparison.

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I. INTRODUCTION

Although the underlying dynamics for the hadronic B decays is extremely complicated, it is greatly simplified in the heavy quark limit. In the $m_b \to \infty$ limit, hadronic matrix elements can be expressed in terms of certain nonperturbative input quantities such as light-cone distribution amplitudes and transition form factors. Consequently, the decay amplitudes of charmless two-body decays of B mesons can be described in terms of decay constants and form factors. However, the leading-order $1/m_b$ predictions encounter three major difficulties: (i) the predicted branching fractions for penguin-dominated $\bar{B} \to PP$, VP, VV decays are systematically below the measurements [1] and the rates for color-suppressed tree-dominated decays $\bar{B}^0 \to \pi^0\pi^0$, $\rho^0\pi^0$ are too small, (ii) direct CP-violating asymmetries for $\bar{B} \to K^-\pi^+$, $\bar{B} \to K^{*-}\pi^+$, $B^- \to K^-\rho^0$, $\bar{B} \to \pi^+\pi^-$, and $\bar{B}_s \to K^+\pi^-$ disagree with experiment in signs, and (iii) the transverse polarization fraction in penguin-dominated charmless $B \to VV$ decays is predicted to be very small, while experimentally it is comparable to the longitudinal polarization one. All these indicate the necessity of going beyond zeroth $1/m_b$ power expansion.

In the QCD factorization (QCDF) approach [2], power corrections often involve end-point divergences. For example, the hard spectator-scattering diagram at twist-3 order is power suppressed and posses soft and collinear divergences arising from the soft spectator quark and the $1/m_b$ annihilation amplitude has endpoint divergences even at the twist-2 level. Since the treatment of endpoint divergences is model dependent, subleading power corrections generally can be studied only in a phenomenological way. Therefore, $1/m_b$ power suppressed effects are generally nonperturbative in nature and hence not calculable by the perturbative method.

As a first step, let us consider power corrections to the QCD penguin amplitude of the $\bar{B} \to PP$ decay, which has the generic expression

$$\begin{aligned} P &= P_{\rm SD} + P_{\rm LD}, \\ &= A_{PP}[\lambda_u(a_4^u + r_\chi^P a_6^u) + \lambda_c(a_4^c + r_\chi^P a_6^c)] \\ &\quad + 1/m_b \text{ corrections}, \end{aligned} \tag{1.1}$$

where $\lambda_p^{(q)} = V_{pb}V_{pq}^*$ with $q = s$, d, $a_{4,6}$ are the effective parameters to be defined below, and r_χ^P is a chiral factor of order unity. Strictly speaking, the penguin contributions associated with the chiral factor r_χ^P are formerly $1/m_b$ suppressed but chirally enhanced. Since they are of order $1/m_b^0$ numerically, their effects are included in the zeroth order calculation. Possible power corrections to penguin amplitudes include long-distance charming penguins, final-state interactions, and penguin annihilation characterized by the parameters $\beta_3^{u,c}$. Because of possible "double counting" problems, one should not take into account all power correction effects simultaneously. As we shall see below in Sec. IV B, CP violation of $K^-\pi^+$ and $\pi^+\pi^-$ arise from the interference between the tree amplitude $\lambda_u{}^{(q)}a_1$ and the penguin amplitude $\lambda_c{}^{(q)}(a_4^c + r_\chi^P a_6^c)$ with $q = s$ for the former and $q = d$ for the latter. The short-distance contribution to $a_4^c + r_\chi^P a_6$ will yield a positive $A_{CP}(K^-\pi^+)$ and a negative $A_{CP}(\pi^+\pi^-)$. Both are wrong in signs when confronted with experiment. In the so-called "S4" scenario of QCDF [1], power corrections to the penguin-annihilation topology characterized by $\lambda_u\beta_3^u + \lambda_c\beta_3^c$ are added to Eq. (1.1). By adjusting the magnitude and phase of β_3 in this scenario, all the above-mentioned discrepancies except for the rate deficit problem with the decays $\bar{B}^0 \to \pi^0\pi^0$, $\rho^0\pi^0$ can be resolved.

However, a scrutiny of the QCDF predictions reveals more puzzles with respect to direct CP violation. While the signs of CP asymmetries in $K^-\pi^+$, $K^-\rho^0$ modes are flipped to the right ones in the presence of power corrections from penguin annihilation, the signs of A_{CP} in $B^- \to K^-\pi^0$, $K^-\eta$, $\pi^-\eta$ and $\bar{B}^0 \to \pi^0\pi^0$, $\bar{K}^{*0}\eta$ will also get reversed in such a way that they disagree with experiment. In other words, in the heavy quark limit the CP asymmetries of these five modes have the right signs when compared with experiment.

The so-called $B \to K\pi$ CP puzzle is related to the difference of CP asymmetries of $B^- \to K^-\pi^0$ and $\bar{B}^0 \to K^-\pi^+$. This can be illustrated by considering the decay amplitudes of $\bar{B} \to \bar{K}\pi$ in terms of topological diagrams

$$
\begin{aligned}
A(\bar{B}^0 \to K^-\pi^+) &= P' + T' + \frac{2}{3}P'^c_{\rm EW} + P'_A,\\
A(\bar{B}^0 \to \bar{K}^0\pi^0) &= \frac{-1}{\sqrt{2}}\left(P' - C' - P'_{\rm EW} - \frac{1}{3}P'^c_{\rm EW} + P'_A\right),\\
A(B^- \to \bar{K}^0\pi^-) &= P' - \frac{1}{3}P'^c_{\rm EW} + A' + P'_A,\\
A(B^- \to K^-\pi^0) &= \frac{1}{\sqrt{2}}\Big(P' + T' + C' + P'_{\rm EW}\\
&\quad + \frac{2}{3}P'^c_{\rm EW} + A' + P'_A\Big),
\end{aligned} \tag{1.2}
$$

where T, C, E, A, $P_{\rm EW}$, and $P^c_{\rm EW}$ are color-allowed tree, color-suppressed tree, W-exchange, W-annihilation, color-allowed, and color-suppressed electroweak penguin amplitudes, respectively, and P_A is the penguin-induced weak annihilation amplitude. We use unprimed and primed symbols to denote $\Delta S = 0$ and $|\Delta S| = 1$ transitions, respectively. We notice that if C', $P'_{\rm EW}$ and A' are negligible compared with T', it is clear from Eq. (1.2) that the decay amplitudes of $K^-\pi^0$ and $K^-\pi^+$ will be the same apart from a trivial factor of $1/\sqrt{2}$. Hence, one will expect that $A_{CP}(K^-\pi^0) \approx A_{CP}(K^-\pi^+)$, while they differ by 5.3σ experimentally, $\Delta A_{K\pi} \equiv A_{CP}(K^-\pi^0) - A_{CP}(K^-\pi^+) = 0.148 \pm 0.028$ [3].

The aforementioned direct CP puzzles indicate that it is necessary to consider subleading power corrections other than penguin annihilation. For example, the large power corrections due to P' cannot explain the $\Delta A_{K\pi}$ puzzle as they contribute equally to both $B^- \to K^-\pi^0$ and $\bar{B}^0 \to K^-\pi^+$. The additional power correction should have little effect on the decay rates of penguin-dominated decays but will manifest in the measurement of direct CP asymmetries. Note that all the "problematic" modes receive a contribution from $c^{(\prime)} = C^{(\prime)} + P^{(\prime)}_{\rm EW}$. Since $A(B^- \to K^-\pi^0) \propto t' + c' + p'$ and $A(\bar{B}^0 \to K^-\pi^+) \propto t' + p'$ with $t' = T' + P'^c_{\rm EW}$ and $p' = P' - \frac{1}{3}P'^c_{\rm EW} + P'_A$, we can consider this puzzle resolved, provided that c'/t' is of order $1.3 \sim 1.4$ with a large negative phase ($|c'/t'| \sim 0.9$ in the standard short-distance effective Hamiltonian approach). There are several possibilities for a large complex c': either a large complex C' or a large complex electroweak penguin $P'_{\rm EW}$ or a combination of them. Various scenarios for accommodating large C' [4–11] or $P'_{\rm EW}$ [12,13] have been proposed. To get a large complex C', one can resort to spectator scattering or final-state interactions (see discussions in Sec. III E). However, the general consensus for a large complex $P'_{\rm EW}$ is that one needs new physics beyond the standard model (SM) because it is well known that $P'_{\rm EW}$ is essentially real in the SM as it does not carry a nontrivial strong phase [14]. In principle, one cannot discriminate between these two possibilities in penguin-dominated decays as it is always the combination $c' = C' + P'_{\rm EW}$ that enters into the decay amplitude except for the decays involving η and/or η' in the final state where both c' and $P'_{\rm EW}$ present in the amplitudes [15]. Nevertheless, these two scenarios will lead to very distinct predictions for tree-dominated decays where $P_{\rm EW} \ll C$. (In penguin-dominated decays, $P'_{\rm EW}$ is comparable to C' due to the fact that $\lambda_c^{(s)} \gg \lambda_u^{(s)}$.) The decay rates of $\bar{B}^0 \to \pi^0\pi^0$, $\rho^0\pi^0$ will be substantially enhanced for a large C but remain intact for a large $P_{\rm EW}$. Since $P_{\rm EW} \ll C$ in tree-dominated channels, CP puzzles with $\pi^-\eta$ and $\pi^0\pi^0$ cannot be resolved with a large $P_{\rm EW}$. Therefore, it is most likely that the color-suppressed tree amplitude is large and complex. In other words, the $B \to K\pi$ CP puzzle can be resolved without invoking New Physics.

In this work we shall consider the possibility of having a large color-suppressed tree amplitude with a sizable strong phase relative to the color-allowed tree amplitude [16]

$$
C = [\lambda_u a_2^u]_{\rm SD} + [\lambda_u a_2^u]_{\rm LD} + {\rm FSIs} + \cdots. \tag{1.3}
$$

As will be discussed below, the long-distance contribution to a_2 can come from the twist-3 effects in spectator rescattering, while an example of final-state rescattering contribution to C will be illustrated below.

Note that our phenomenological study of power corrections to penguin annihilation and to color-suppressed tree topology is in the same spirit of S4 and S2 scenarios, respectively, considered by Beneke and Neubert [1]. In the "large α_2" S2 scenario, the ratio a_2/a_1 is enhanced basically by having a smaller λ_B and a smaller strange quark mass. It turns out that the CP asymmetries of $K^-\pi^+$, $K^{*-}\pi^+$, $K^-\eta$, $K^-\rho^0$, $\pi^+\pi^-$ have correct signs in S4 but not so in S2, whereas the signs of $A_{CP}(K^-\pi^0)$, $A_{CP}(K^-\eta)$,

$A_{CP}(\pi^0\pi^0)$ in S2 (or in the heavy quark limit) agree with experiment but not in S4. In a sense, our study is a combination of S4 and S2. However, there is a crucial difference between our work and [1], namely, our a_2 is not only large in the magnitude but also has a large strong phase. As we shall see, a large and *complex* a_2 is needed to account for all the remaining *CP* puzzles.

It should be remarked that the aforementioned *B-CP* puzzles with the $K^-\pi^0$, $K^-\eta$, $\pi^-\eta$, $\bar{K}^{*0}\eta$, $\pi^0\pi^0$ modes also occur in the approach of soft-collinear effective theory (SCET) [17] where the penguin-annihilation effect in QCDF is replaced by the long-distance charming penguins. Owing to a different treatment of endpoint divergence in penguin-annihilation diagrams, some of the *CP* puzzles do not occur in the approach of perturbative quantum chromodynamics (pQCD) [18]. For example, pQCD predicts the right sign for *CP* asymmetries of $\bar{B}^0\to\pi^0\pi^0$ and $B^-\to\pi^-\eta$ as we shall see below. In this work, we shall show that soft power correction to the color-suppressed tree amplitude will bring the signs of A_{CP} back to the right track. As a bonus, the rates of $\bar{B}^0\to\pi^0\pi^0$, $\rho^0\pi^0$ can be accommodated.

In the past decade, nearly 100 charmless decays of $B_{u,d}$ mesons have been observed at B factories with a statistical significance of at least 4 standard deviations (for a review, see [19]). Before moving to the era of LHCb and Super B factories in the next few years, it is timing to have an overview on charmless hadronic B decays to see what we have learned from the fruitful experimental results obtained by *BABAR* and Belle. In this work, we will update QCDF calculations and compare with experiment and other theoretical predictions.

This work is organized as follows: We outline the QCDF framework in Sec. II and specify various input parameters, such as form factors, light-cone distribution amplitudes (LCDAs), and the parameters for power corrections in Sec. III. Then $B_{u,d}\to PP$, VP, VV decays are analyzed in detail in Secs. IV, V, and VI, respectively. Conclusions are given in Sec. VII.

II. *B* DECAYS IN QCD FACTORIZATION

Within the framework of QCD factorization [2], the effective Hamiltonian matrix elements are written in the form

$$\langle M_1M_2|\mathcal{H}_{\rm eff}|\bar{B}\rangle=\frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(q)}\times\langle M_1M_2|\mathcal{T}_{\mathcal{A}}^{h,p}+\mathcal{T}_{\mathcal{B}}^{h,p}|\bar{B}\rangle, \quad (2.1)$$

where the superscript h denotes the helicity of the final-state meson. For PP and VP final states, $h=0$. $\mathcal{T}_{\mathcal{A}}^{h,p}$ describes contributions from naive factorization, vertex corrections, penguin contractions, and spectator scattering expressed in terms of the flavor operators $a_i^{p,h}$, while $\mathcal{T}_{\mathcal{B}}$ contains annihilation topology amplitudes characterized by the annihilation operators $b_i^{p,h}$. Specifically, [2],

$$\begin{aligned}\mathcal{T}_{\mathcal{A}}^{h}=&\,a_1^p(M_1M_2)\delta_{pu}(\bar{u}b)_{V-A}\otimes(\bar{q}u)_{V-A}+a_2^p(M_1M_2)\delta_{pu}(\bar{q}b)_{V-A}\otimes(\bar{u}u)_{V-A}+a_3^p(M_1M_2)\sum(\bar{q}b)_{V-A}\otimes(\bar{q}'q')_{V-A}\\&+a_4^p(M_1M_2)\sum(\bar{q}'b)_{V-A}\otimes(\bar{q}q')_{V-A}+a_5^p(M_1M_2)\sum(\bar{q}b)_{V-A}\otimes(\bar{q}'q')_{V+A}\\&+a_6^p(M_1M_2)\sum(-2)(\bar{q}'b)_{S-P}\otimes(\bar{q}q')_{S+P}+a_7^p(M_1M_2)\sum(\bar{q}b)_{V-A}\otimes\frac{3}{2}e_q(\bar{q}'q')_{V+A}\\&+a_8^p(M_1M_2)\sum(-2)(\bar{q}'b)_{S-P}\otimes\frac{3}{2}(\bar{q}q')_{S+P}+a_9^p(M_1M_2)\sum(\bar{q}b)_{V-A}\otimes\frac{3}{2}e_q(\bar{q}'q')_{V-A}\\&+a_{10}^p(M_1M_2)\sum(\bar{q}'b)_{V-A}\otimes\frac{3}{2}e_q(\bar{q}q')_{V-A},\end{aligned} \quad (2.2)$$

where $(\bar{q}_1q_2)_{V\pm A}\equiv\bar{q}_1\gamma_\mu(1\pm\gamma_5)q_2$ and $(\bar{q}_1q_2)_{S\pm P}\equiv\bar{q}_1(1\pm\gamma_5)q_2$ and the summation is over $q'=u,d,s$. The symbol $\otimes$ indicates that the matrix elements of the operators in $\mathcal{T}_{\mathcal{A}}$ are to be evaluated in the factorized form. For the decays $\bar{B}\to PP$, VP, VV, the relevant factorizable matrix elements are

$$X^{(\bar{B}P_1,P_2)} \equiv \langle P_2|J^\mu|0\rangle\langle P_1|J'_\mu|\bar{B}\rangle = if_{P_2}(m_B^2 - m_{P_1}^2)F_0^{BP_1}(m_{P_2}^2),$$
$$X^{(\bar{B}P,V)} \equiv \langle V|J^\mu|0\rangle\langle P|J'_\mu|\bar{B}\rangle = 2f_V m_B p_c F_1^{BP}(m_V^2),$$
$$X^{(\bar{B}V,P)} \equiv \langle P|J^\mu|0\rangle\langle V|J'_\mu|\bar{B}\rangle = 2f_P m_B p_c A_0^{BV}(m_P^2),$$
$$X_h^{(\bar{B}V_1,V_2)} \equiv \langle V_2|J^\mu|0\rangle\langle V_1|J'_\mu|\bar{B}\rangle = -if_{V_2}m_2\bigg[(\varepsilon_1^*\cdot\varepsilon_2^*)(m_B+m_{V_1})A_1^{BV_1}(m_{V_2}^2) - (\varepsilon_1^*\cdot p_B)(\varepsilon_2^*\cdot p_B)\frac{2A_2^{BV_1}(m_{V_2}^2)}{(m_B+m_{V_1})} + i\varepsilon_{\mu\nu\alpha\beta}\varepsilon_2^{*\mu}\varepsilon_1^{*\nu}p_B^\alpha p_1^\beta\frac{2V^{BV_1}(m_{V_2}^2)}{(m_B+m_{V_1})}\bigg], \tag{2.3}$$

where we have followed the conventional definition for form factors [20]. For $B\to VP$, PV amplitudes, we have applied the replacement $m_V(\varepsilon^*\cdot p_B)\to m_B p_c$ with p_c being the c.m. momentum. The longitudinal ($h=0$) and transverse ($h=\pm$) components of $X_h^{(\bar{B}V_1,V_2)}$ are given by

$$X_0^{(\bar{B}V_1,V_2)} = \frac{if_{V_2}}{2m_{V_1}}\bigg[(m_B^2 - m_{V_1}^2 - m_{V_2}^2)(m_B+m_{V_1})A_1^{BV_1}(q^2) - \frac{4m_B^2p_c^2}{m_B+m_{V_1}}A_2^{BV_1}(q^2)\bigg],$$
$$X_\pm^{(\bar{B}V_1,V_2)} = -if_{V_2}m_Bm_{V_2}\bigg[\bigg(1+\frac{m_{V_1}}{m_B}\bigg)A_1^{BV_1}(q^2) \mp \frac{2p_c}{m_B+m_{V_1}}V^{BV_1}(q^2)\bigg]. \tag{2.4}$$

The flavor operators $a_i^{p,h}$ are basically the Wilson coefficients in conjunction with short-distance nonfactorizable corrections such as vertex corrections and hard spectator interactions. In general, they have the expressions [1,2]

$$a_i^{p,h}(M_1M_2) = \bigg(c_i + \frac{c_{i\pm1}}{N_c}\bigg)N_i^h(M_2) + \frac{c_{i\pm1}}{N_c}\frac{C_F\alpha_s}{4\pi}\times\bigg[V_i^h(M_2) + \frac{4\pi^2}{N_c}H_i^h(M_1M_2)\bigg] + P_i^{h,p}(M_2), \tag{2.5}$$

where $i=1,\cdots,10$, the upper (lower) signs apply when i is odd (even), c_i are the Wilson coefficients, $C_F=(N_c^2-1)/(2N_c)$ with $N_c=3$, M_2 is the emitted meson, and M_1 shares the same spectator quark with the B meson. The quantities $V_i^h(M_2)$ account for vertex corrections, $H_i^h(M_1M_2)$ for hard spectator interactions with a hard gluon exchange between the emitted meson and the spectator quark of the B meson and $P_i(M_2)$ for penguin contractions. The expression of the quantities $N_i^h(M_2)$ reads

$$N_i^h(M_2) = \begin{cases}0, & i=6,8,\\ 1, & \text{else}.\end{cases} \tag{2.6}$$

The weak annihilation contributions to the decay $\bar{B}\to M_1M_2$ can be described in terms of the building blocks $b_i^{p,h}$ and $b_{i,\rm EW}^{p,h}$

$$\frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p\langle M_1M_2|\mathcal{T}_{\mathcal{B}}{}^{h,p}|\bar{B}^0\rangle = i\frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p f_B f_{M_1} f_{M_2}\sum_i(d_ib_i^{p,h} + d'_ib_{i,\rm EW}^{p,h}). \tag{2.7}$$

The building blocks have the expressions [1]

$$\begin{aligned} b_1 &= \frac{C_F}{N_c^2}c_1A_1^i,\\ b_3 &= \frac{C_F}{N_c^2}[c_3A_1^i + c_5(A_3^i + A_3^f) + N_cc_6A_3^f],\\ b_2 &= \frac{C_F}{N_c^2}c_2A_1^i,\\ b_4 &= \frac{C_F}{N_c^2}[c_4A_1^i + c_6A_2^f],\\ b_{3,\rm EW} &= \frac{C_F}{N_c^2}[c_9A_1^i + c_7(A_3^i + A_3^f) + N_cc_8A_3^i],\\ b_{4,\rm EW} &= \frac{C_F}{N_c^2}[c_{10}A_1^i + c_8A_2^i]. \end{aligned} \tag{2.8}$$

Here, for simplicity we have omitted the superscripts p and h in above expressions. The subscripts 1, 2, 3 of $A_n^{i,f}$ denote the annihilation amplitudes induced from $(V-A)(V-A)$, $(V-A)(V+A)$, and $(S-P)(S+P)$ operators, respectively, and the superscripts i and f refer to gluon emission from the initial and final-state quarks, respectively. Following [1] we choose the convention that M_1 contains an antiquark from the weak vertex, and M_2 contains a quark from the weak vertex.

For the explicit expressions of vertex, hard spectator corrections and annihilation contributions, the reader is referred to [1,2,21] for details. The decay amplitudes of $B\to PP$, VP are given in Appendix A of [1] and can be easily generalized to $B\to VV$ (see [22] for explicit expressions of $B\to VV$ amplitudes). In practice, it is more convenient to express the decay amplitudes in terms of the flavor operators $\alpha_i^{h,p}$ and the annihilation operators β_i^p, which are related to the coefficients $a_i^{h,p}$ and b_i^p by

$$\alpha_1^h(M_1M_2) = a_1^h(M_1M_2), \qquad \alpha_2^h(M_1M_2) = a_2^h(M_1M_2),$$
$$\alpha_3^{h,p}(M_1M_2) = \begin{cases} a_3^{h,p}(M_1M_2) - a_5^{h,p}(M_1M_2) & \text{for } M_1M_2 = PP, VP, \\ a_3^{h,p}(M_1M_2) + a_5^{h,p}(M_1M_2) & \text{for } M_1M_2 = VV, PV, \end{cases}$$
$$\alpha_4^{h,p}(M_1M_2) = \begin{cases} a_4^{h,p}(M_1M_2) + r_\chi^{M_2} a_6^{h,p}(M_1M_2) & \text{for } M_1M_2 = PP, PV, \\ a_4^{h,p}(M_1M_2) - r_\chi^{M_2} a_6^{h,p}(M_1M_2) & \text{for } M_1M_2 = VP, VV, \end{cases} \tag{2.9}$$
$$\alpha_{3,\rm EW}^{h,p}(M_1M_2) = \begin{cases} a_9^{h,p}(M_1M_2) - a_7^{h,p}(M_1M_2) & \text{for } M_1M_2 = PP, VP, \\ a_9^{h,p}(M_1M_2) + a_7^{h,p}(M_1M_2) & \text{for } M_1M_2 = VV, PV, \end{cases}$$
$$\alpha_{4,\rm EW}^{h,p}(M_1M_2) = \begin{cases} a_{10}^{h,p}(M_1M_2) + r_\chi^{M_2} a_8^{h,p}(M_1M_2) & \text{for } M_1M_2 = PP, PV, \\ a_{10}^{h,p}(M_1M_2) - r_\chi^{M_2} a_8^{h,p}(M_1M_2) & \text{for } M_1M_2 = VP, VV, \end{cases}$$

and

$$\beta_i^p(M_1M_2) = \frac{if_B f_{M_1} f_{M_2}}{X^{(\bar{B}M_1,M_2)}} b_i^p. \tag{2.10}$$

The order of the arguments of $\alpha_i^p(M_1M_2)$ and $\beta_i^p(M_1M_2)$ is consistent with the order of the arguments of $X^{(\bar{B}M_1,M_2)} \equiv A_{M_1M_2}$. The chiral factor r_χ is given by

$$r_\chi^P(\mu) = \frac{2m_P^2}{m_b(\mu)(m_2 + m_1)(\mu)}, \qquad r_\chi^V(\mu) = \frac{2m_V}{m_b(\mu)} \frac{f_V^\perp(\mu)}{f_V}. \tag{2.11}$$

The Wilson coefficients $c_i(\mu)$ at various scales, $\mu = 4.4$ GeV, 2.1 GeV, 1.45 GeV, and 1 GeV are taken from [23]. For the renormalization scale of the decay amplitude, we choose $\mu = m_b(m_b)$.[1] However, as stressed in [2], the hard spectator and annihilation contributions should be evaluated at the hard-collinear scale $\mu_h = \sqrt{\mu \Lambda_h}$ with $\Lambda_h \approx 500$ MeV.

[1] In principle, physics should be independent of the choice of μ, but in practice there exists some residual μ dependence in the truncated calculations. We have checked explicitly that the decay rates without annihilation are indeed essentially stable against μ. However, when penguin annihilation is turned on, it is sensitive to the choice of the renormalization scale because the penguin-annihilation contribution characterized by the parameter b_3 is dominantly proportional to $\alpha_s(\mu_h)c_6(\mu_h)$ at the hard-collinear scale $\mu_h = \sqrt{\mu\Lambda_h}$. In our study of $B \to VV$ decays [24], we found that if the renormalization scale is chosen to be $\mu = m_b(m_b)/2 = 2.1$ GeV, we cannot fit the branching ratios and polarization fractions simultaneously for both $B \to K^*\phi$ and $B \to K^*\rho$ decays. In order to ensure the validity of the penguin-annihilation mechanism for describing $B \to VV$ decays, we will confine ourselves to the renormalization scale $\mu = m_b(m_b)$ in the ensuing study.

III. INPUT PARAMETERS

A. Form factors

There exist many model calculations of form factors for $B \to P, V$ transitions. For $B \to P$ transitions, recent light-cone sum rule results for form factors at $q^2 = 0$ are collected in Table I. A small $F_0^{B\pi}$ of order 0.25 is also preferred by the measurement of $B^- \to \pi^-\pi^0$. It is more convenient to express the form factors for $B \to \eta^{(\prime)}$ transitions in terms of the flavor states $q\bar{q} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$, $s\bar{s}$, and $c\bar{c}$ labeled by the η_q, η_s, and η_c^0, respectively. Neglecting the small mixing with η_c^0, we have

$$F^{B\eta} = F^{B\eta_q}\cos\theta, \qquad F^{B\eta'} = F^{B\eta_q}\sin\theta, \tag{3.1}$$

where θ is the η_q-η_s mixing angle defined by

$$|\eta\rangle = \cos\theta|\eta_q\rangle - \sin\theta|\eta_s\rangle, \qquad |\eta'\rangle = \sin\theta|\eta_q\rangle + \cos\theta|\eta_s\rangle, \tag{3.2}$$

with $\theta = (39.3 \pm 1.0)^\circ$ in the Feldmann-Kroll-Stech mixing scheme [29]. From the sum rule results shown in Table I we obtain $F_0^{B\eta_q}(0) = 0.296$. The flavor-singlet contribution to the $B \to \eta^{(\prime)}$ form factors is characterized by the parameter B_2^g, a gluonic Gegenbauer moment. It appears that the singlet contribution to the form factor is small unless B_2^g assumes extreme values ~ 40 [28].

The $B \to \pi, K, \eta_q$ transition form factors to be used in this work are displayed in Table II. We shall use the form factors determined from QCD sum rules for $B \to V$ transitions [32].

B. Decay constants

Decay constants of various vector mesons defined by

$$\langle V(p,\varepsilon)|\bar{q}_2\gamma_\mu q_1|0\rangle = -if_V m_V \varepsilon_\mu^*, \qquad \langle V(p,\varepsilon)|\bar{q}_2\sigma_{\mu\nu} q_1|0\rangle = -f_V^\perp(\varepsilon_\mu^* p^\nu - \varepsilon_\nu^* p^\mu) \tag{3.3}$$

TABLE I. Form factors for $B \to P$ transitions obtained in the QCD sum rules with B_2^g being the gluonic Gegenbauer moment.

$F_0^{B\pi}(0)$	0.258 ± 0.031 [25]	$0.26^{+0.04}_{-0.03}$ [26]
$F_0^{BK}(0)$	0.331 ± 0.041 [25]	$0.36^{+0.05}_{-0.04}$ [27]
$F_0^{B\eta}(0)$	$0.229 \pm 0.024 \pm 0.011$ [28]	
$F_0^{B\eta'}(0)$	$0.188 \pm 0.002 B_2^g \pm 0.019 \pm 0.009$ [28]	

are listed in Table II. They are taken from [33]. For pseudoscalar mesons, we use $f_\pi = 132$ MeV and $f_K = 160$ MeV. Decay constants $f^q_{\eta^{(\prime)}}$, $f^s_{\eta^{(\prime)}}$ and $f^c_{\eta^{(\prime)}}$ defined by

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 q|\eta^{(\prime)}\rangle = i\frac{1}{\sqrt{2}} f^q_{\eta^{(\prime)}} q_\mu, \quad \langle 0|\bar{s}\gamma_\mu\gamma_5 s|\eta^{(\prime)}\rangle = i f^s_{\eta^{(\prime)}} q_\mu, \quad \langle 0|\bar{c}\gamma_\mu\gamma_5 c|\eta^{(\prime)}\rangle = i f^c_{\eta^{(\prime)}} q_\mu \tag{3.4}$$

are also needed in calculations. For the decay constants $f^q_{\eta^{(\prime)}}$ and $f^s_{\eta^{(\prime)}}$, we shall use the values

$$f^q_\eta = 107 \text{ MeV}, \qquad f^s_\eta = -112 \text{ MeV}, \qquad f^q_{\eta'} = 89 \text{ MeV}, \qquad f^s_{\eta'} = 137 \text{ MeV} \tag{3.5}$$

obtained in [29]. As for $f^c_{\eta^{(\prime)}}$, a straightforward perturbative calculation gives [34]

$$f^c_{\eta^{(\prime)}} = -\frac{m^2_{\eta^{(\prime)}}}{12 m_c^2} \frac{f^q_{\eta^{(\prime)}}}{\sqrt{2}}. \tag{3.6}$$

C. LCDAs

We next specify the LCDAs for pseudoscalar and vector mesons. The general expressions of twist-2 LCDAs are

TABLE II. Input parameters. The values of the scale dependent quantities $f_V^\perp(\mu)$ and $a_{1,2}^{\perp,V}(\mu)$ are given for $\mu = 1$ GeV. The values of Gegenbauer moments are taken from [30] and Wolfenstein parameters from [31].

			Light vector mesons			
V	f_V (MeV)	$f_V^\perp$ (MeV)	a_1^V	a_2^V	$a_1^{\perp,V}$	$a_2^{\perp,V}$
ρ	216 ± 3	165 ± 9	0	0.15 ± 0.07	0	0.14 ± 0.06
ω	187 ± 5	151 ± 9	0	0.15 ± 0.07	0	0.14 ± 0.06
ϕ	215 ± 5	186 ± 9	0	0.18 ± 0.08	0	0.14 ± 0.07
K^*	220 ± 5	185 ± 10	0.03 ± 0.02	0.11 ± 0.09	0.04 ± 0.03	0.10 ± 0.08
			Light pseudoscalar mesons			
a_1^π	a_2^π		a_1^K		a_2^K	
0	0.25 ± 0.15		0.06 ± 0.03		0.25 ± 0.15	
			B mesons			
B	m_B (GeV)		τ_B (ps)	f_B (MeV)		λ_B (MeV)
B_u	5.279		1.638	210 ± 20		300 ± 100
B_d	5.279		1.525	210 ± 20		300 ± 100
B_s	5.366		1.472	230 ± 20		300 ± 100
			Form factors at $q^2 = 0$			
$F_0^{BK}(0)$	$A_0^{BK^*}(0)$	$A_1^{BK^*}(0)$	$A_2^{BK^*}(0)$		$V_0^{BK^*}(0)$	
0.35 ± 0.04	0.374 ± 0.033	0.292 ± 0.028	0.259 ± 0.027		0.411 ± 0.033	
$F_0^{B\pi}(0)$	$A_0^{B\rho}(0)$	$A_1^{B\rho}(0)$	$A_2^{B\rho}(0)$		$V_0^{B\rho}(0)$	
0.25 ± 0.03	0.303 ± 0.029	0.242 ± 0.023	0.221 ± 0.023		0.323 ± 0.030	
$F_0^{B\eta_q}(0)$	$A_0^{B\omega}(0)$	$A_1^{B\omega}(0)$	$A_2^{B\omega}(0)$		$V_0^{B\omega}(0)$	
0.296 ± 0.028	0.281 ± 0.030	0.219 ± 0.024	0.198 ± 0.023		0.293 ± 0.029	
			Quark masses			
$m_b(m_b)$/GeV	$m_c(m_b)$/GeV		$m_c^{\rm pole}/m_b^{\rm pole}$		$m_s(2.1\ \text{GeV})$/GeV	
4.2	0.91		0.3		0.095 ± 0.020	
			Wolfenstein parameters			
A	λ	$\bar{\rho}$	$\bar{\eta}$		γ	
0.8116	0.2252	0.139	0.341		$(67.8^{+4.2}_{-3.9})^\circ$	

$$\Phi_P(x,\mu) = 6x(1-x)\left[1 + \sum_{n=1}^{\infty} a_n^P(\mu) C_n^{3/2}(2x-1)\right],$$
$$\Phi_\parallel^V(x,\mu) = 6x(1-x)\left[1 + \sum_{n=1}^{\infty} a_n^V(\mu) C_n^{3/2}(2x-1)\right],$$
$$\Phi_\perp^V(x,\mu) = 6x(1-x)\left[1 + \sum_{n=1}^{\infty} a_n^{\perp,V}(\mu) C_n^{3/2}(2x-1)\right], \tag{3.7}$$

and twist-3 ones

$$\Phi_p(x) = 1, \qquad \Phi_\sigma(x) = 6x(1-x),$$
$$\Phi_v(x,\mu) = 3\left[2x - 1 + \sum_{n=1}^{\infty} a_n^{\perp,V}(\mu) P_{n+1}(2x-1)\right], \tag{3.8}$$

where $C_n(x)$ and $P_n(x)$ are the Gegenbauer and Legendre polynomials, respectively. When three-particle amplitudes are neglected, the twist-3 $\Phi_v(x)$ can be expressed in terms of $\Phi_\perp$

$$\Phi_v(x) = \int_0^x \frac{\Phi_\perp(u)}{\bar{u}} du - \int_x^1 \frac{\Phi_\perp(u)}{u} du. \tag{3.9}$$

The normalization of LCDAs is

$$\int_0^1 dx \Phi_V(x) = 1, \qquad \int_0^1 dx \Phi_v(x) = 0. \tag{3.10}$$

Note that the Gegenbauer moments $a_i^{(\perp),K^*}$ displayed in Table II taken from [30] are for the mesons containing a strange quark.

The integral of the B meson wave function is parameterized as [2]

$$\int_0^1 \frac{d\rho}{1-\rho} \Phi_1^B(\rho) \equiv \frac{m_B}{\lambda_B}, \tag{3.11}$$

where $1-\rho$ is the momentum fraction carried by the light spectator quark in the B meson. The study of hadronic B decays favors a smaller first inverse moment λ_B: a value of 350 ± 150 MeV was employed in [2] and 200^{+250}_{-0} MeV in [21], though QCD sum rule and other studies prefer a larger $\lambda_B \sim 460$ MeV [35]. We shall use $\lambda_B = 300 \pm 100$ MeV.

For the running quark masses we shall use [36,37]

$$m_b(m_b) = 4.2 \text{ GeV}, \qquad m_b(2.1 \text{ GeV}) = 4.94 \text{ GeV},$$
$$m_b(1 \text{ GeV}) = 6.34 \text{ GeV}, \qquad m_c(m_b) = 0.91 \text{ GeV},$$
$$m_c(2.1 \text{ GeV}) = 1.06 \text{ GeV}, \qquad m_c(1 \text{ GeV}) = 1.32 \text{ GeV},$$
$$m_s(2.1 \text{ GeV}) = 95 \text{ MeV}, \qquad m_s(1 \text{ GeV}) = 118 \text{ MeV},$$
$$m_d(2.1 \text{ GeV}) = 5.0 \text{ MeV}, \qquad m_u(2.1 \text{ GeV}) = 2.2 \text{ MeV}. \tag{3.12}$$

Note that the charm quark masses here are smaller than the one $m_c(m_b) = 1.3 \pm 0.2$ GeV adopted in [1,22] and consistent with the high precision mass determination from lattice QCD [38]: $m_c(3 \text{ GeV}) = 0.986 \pm 0.010$ GeV and $m_c(m_c) = 1.267 \pm 0.009$ GeV (see also [39]). Among the quarks, the strange quark gives the major theoretical uncertainty to the decay amplitude. Hence, we will only consider the uncertainty in the strange quark mass given by $m_s(2.1 \text{ GeV}) = 95 \pm 20$ MeV. Notice that for the one-loop penguin contribution, the relevant quark mass is the pole mass rather than the current one [40]. Since the penguin loop correction is governed by the ratio of the pole masses squared $s_i \equiv (m_i^{\rm pole}/m_b^{\rm pole})^2$ and since the pole mass is meaningful only for heavy quarks, we only need to consider the ratio of c and b quark pole masses given by $s_c \approx (0.3)^2$.

D. Penguin annihilation

In the QCDF approach, the hadronic B decay amplitude receives contributions from tree, penguin, electroweak penguin, and weak annihilation topologies. In the absence of $1/m_b$ power corrections except for the chiral enhanced penguin contributions, the leading QCDF predictions encounter three major difficulties as discussed in the Introduction. This implies the necessity of introducing $1/m_b$ power corrections. Soft corrections due to penguin annihilation have been proposed to resolve the rate deficit problem for penguin-dominated decays and the CP puzzle for $\bar{B}^0 \to K^-\pi^+$.[2] However, the penguin-annihilation amplitude involve troublesome endpoint divergences. Hence, subleading power corrections generally can be studied only in a phenomenological way. We shall follow [2] to model the endpoint divergence $X \equiv \int_0^1 dx/\bar{x}$ in the annihilation and hard spectator-scattering diagrams as

$$X_A = \ln\left(\frac{m_B}{\Lambda_h}\right)(1 + \rho_A e^{i\phi_A}), \tag{3.13}$$

with Λ_h being a typical scale of order 500 MeV, and ρ_A, ϕ_A being the unknown real parameters.

A fit to the data of $B_{u,d} \to PP$, VP, PV, and VV decays yields the values of ρ_A and ϕ_A shown in Table III. Basically, it is very similar to the so-called "S4 scenario" presented in [1]. Within the framework of QCDF, one cannot account for all charmless two-body B decay data by a universal set of ρ_A and ϕ_A parameters. Since the penguin-annihilation effects are different for $B \to VP$ and $B \to PV$ decays,

[2]Besides the mechanisms of penguin annihilation, charming penguins and final-state rescattering, another possibility of solving the rate and CP puzzle for $\bar{B}^0 \to K^-\pi^+$ was advocated recently in [41] by adding to the $B \to K\pi$ QCDF amplitude a real and an absorptive part with a strength 10% and 30% of the penguin amplitude, respectively.

TABLE III. The parameters ρ_A and ϕ_A for penguin annihilation. The fitted ρ_A and ϕ_A for $B \to VV$ decays are taken from [24].

Mode	ρ_A	ϕ_A	Mode	ρ_A	ϕ_A
$B \to PP$	1.10	$-50°$	$B \to VP$	1.07	$-70°$
$B \to PV$	0.87	$-30°$	$B \to K\phi$	0.70	$-40°$
$B \to K^*\rho$	0.78	$-43°$	$B \to K^*\phi$	0.65	$-53°$

$$\begin{aligned} A_1^i &\approx -A_2^i \\ &\approx 6\pi\alpha_s\left[3\left(X_A^{VP} - 4 + \frac{\pi^2}{3}\right) + r_\chi^V r_\chi^P((X_A^{VP})^2 - 2X_A^{VP})\right], \\ A_3^i &\approx 6\pi\alpha_s\left[-3r_\chi^V\left((X_A^{VP})^2 - 2X_A^{VP} + 4 - \frac{\pi^2}{3}\right) \right.\\ &\quad \left. + r_\chi^P\left((X_A^{VP})^2 - 2X_A^{VP} + \frac{\pi^2}{3}\right)\right], \\ A_3^f &\approx 6\pi\alpha_s[3r_\chi^V(2X_A^{VP} - 1)(2 - X_A^{VP}) \\ &\quad - r_\chi^P(2(X_A^{VP})^2 - X_A^{VP})] \end{aligned} \tag{3.14}$$

for $M_1M_2 = VP$ and

$$\begin{aligned} A_1^i &\approx -A_2^i \\ &\approx 6\pi\alpha_s\left[3\left(X_A^{PV} - 4 + \frac{\pi^2}{3}\right) + r_\chi^V r_\chi^P((X_A^{PV})^2 - 2X_A^{PV})\right], \\ A_3^i &\approx 6\pi\alpha_s\left[-3r_\chi^P\left((X_A^{PV})^2 - 2X_A^{PV} + 4 - \frac{\pi^2}{3}\right) \right.\\ &\quad \left. + r_\chi^V\left((X_A^{PV})^2 - 2X_A^{PV} + \frac{\pi^2}{3}\right)\right], \\ A_3^f &\approx 6\pi\alpha_s[-3r_\chi^P(2X_A^{PV} - 1)(2 - X_A^{PV}) \\ &\quad + r_\chi^V(2(X_A^{PV})^2 - X_A^{PV})] \end{aligned} \tag{3.15}$$

for $M_1M_2 = PV$, the parameters X_A^{VP} and X_A^{PV} are not necessarily the same. Indeed, a fit to the $B \to VP$, PV decays yields $\rho_A^{VP} \approx 1.07$, $\phi_A^{VP} \approx -70°$ and $\rho_A^{PV} \approx 0.87$, $\phi_A^{PV} \approx -30°$ (see Table III). For the estimate of theoretical uncertainties, we shall assign an error of ± 0.1 to ρ_A and $\pm 20°$ to ϕ_A. Note that penguin-annihilation contributions to $K\phi$ ($K^*\phi$) are smaller than other PV (VV) modes. In general, penguin annihilation is dominated by b_3 or β_3 through $(S-P)(S+P)$ interactions.

E. Power corrections to a_2

As pointed out in [16], while the discrepancies between theory and experiment for the rates of penguin-dominated two-body decays of B mesons and direct CP asymmetries of $\bar{B}_d \to K^-\pi^+$, $B^- \to K^-\rho^0$, and $\bar{B}_d \to \pi^+\pi^-$ are resolved by the power corrections due to penguin annihilation, the signs of direct CP-violating effects in $B^- \to K^-\pi^0$, $B^- \to K^-\eta$, and $\bar{B}^0 \to \pi^0\pi^0$ are flipped to the wrong ones when confronted with experiment. These new B-CP puzzles in QCDF can be explained by the subleading power corrections to the color-suppressed tree amplitudes due to hard spectator interactions and/or final-state interactions that yield not only correct signs for aforementioned CP asymmetries but also accommodate the observed $\bar{B}_d \to \pi^0\pi^0$ and $\rho^0\pi^0$ rates simultaneously.

Following [16], power corrections to the color-suppressed topology are parametrized as

$$a_2 \to a_2(1 + \rho_C e^{i\phi_C}), \tag{3.16}$$

with the unknown parameters ρ_C and ϕ_C to be inferred from experiment. We shall use [16]

$$\rho_C \approx 1.3, 0.8, 0, \qquad \phi_C \approx -70°, -80°, 0 \tag{3.17}$$

for $\bar{B} \to PP$, VP, VV decays, respectively. This pattern that soft power corrections to a_2 are large for PP modes, moderate for VP ones and very small for VV cases is consistent with the observation made in [9] that soft power correction dominance is much larger for PP than VP and VV final states. It has been argued that this has to do with the special nature of the pion, which is a $q\bar{q}$ bound state on the one hand and a nearly massless Nambu-Goldstone boson on the other hand [9].

What is the origin of power corrections to a_2? There are two possible sources: hard spectator interactions and final-state interactions. From Eq. (3.18) we have the expression

$$a_2(M_1M_2) = c_2 + \frac{c_1}{N_c} + \frac{c_1}{N_c}\frac{C_F\alpha_s}{4\pi}\left[V_2(M_2) + \frac{4\pi^2}{N_c}H_2(M_1M_2)\right] + a_2(M_1M_2)_{\rm LD} \tag{3.18}$$

for a_2. The hard spectator term $H_2(M_1M_2)$ reads

$$H_2(M_1M_2) = \frac{if_Bf_{M_1}f_{M_2}}{X^{(\bar{B}M_1,M_2)}}\frac{m_B}{\lambda_B}\int_0^1 dxdy\left(\frac{\Phi_{M_1}(x)\Phi_{M_2}(y)}{\bar{x}\,\bar{y}} + r_\chi^{M_1}\frac{\Phi_{m_1}(x)\Phi_{M_2}(y)}{\bar{x}y}\right), \tag{3.19}$$

where $X^{(\bar{B}M_1,M_2)}$ is the factorizable amplitude for $\bar{B} \to M_1M_2$, $\bar{x} = 1 - x$. Power corrections from the twist-3 amplitude Φ_m are divergent and can be parameterized as

$$X_H \equiv \int_0^1 \frac{dy}{y} = \ln\frac{m_B}{\Lambda_h}(1 + \rho_H e^{i\phi_H}). \tag{3.20}$$

Since $c_1 \sim \mathcal{O}(1)$ and $c_9 \sim \mathcal{O}(-1.3)$ in units of α_{em}, it is clear that hard spectator contributions to a_i are usually very small except for a_2 and a_{10}. Indeed, there is a huge cancellation between the vertex and naive factorizable terms so that the real part of a_2 is governed by spectator interactions, while its imaginary part comes mainly from the vertex corrections [42]. The value of $a_2(K\pi) \approx 0.51e^{-i58°}$ needed to solve the $B \to K\pi$ CP puzzle [see Eq. (4.4)] corresponds to $\rho_H \approx 4.9$ and $\phi_H \approx -77°$. Therefore, there is no reason to restrict ρ_H to the range $0 \le \rho_H \le 1$. A sizable color-suppressed tree amplitude also can be induced via color-allowed decay $B^- \to K^-\eta'$ followed by

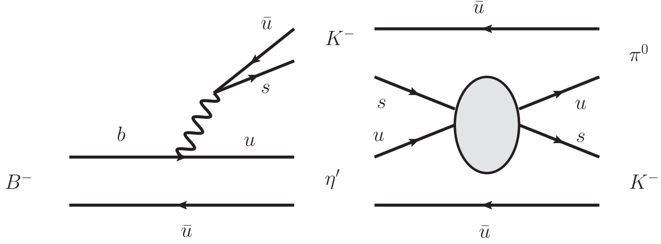

FIG. 1. Contribution to the color-suppressed tree amplitude of $B^- \to K^-\pi^0$ from the weak decay $B^- \to K^-\eta'$ followed by the final-state rescattering of $K^-\eta'$ into $K^-\pi^0$. This has the same topology as the color-suppressed tree diagram.

the rescattering of $K^-\eta'$ into $K^-\pi^0$ as depicted in Fig. 1. Recall that among the 2-body B decays, $B \to K\eta'$ has the largest branching fraction, of order 70×10^{-6}. This final-state rescattering has the same topology as the color-suppressed tree diagram [43]. One of us (C. K. C.) has studied the final-state interaction (FSI) effects through residual rescattering among PP states and resolved the B-CP puzzles [7]. As stressed by Neubert sometime ago, in the presence of soft final-state interactions, there is no color suppression of C with respect to T [44].

Since the chiral factor r_χ^V for the vector meson is substantially smaller than r_χ^P for the pseudoscalar meson (typically, $r_\chi^P = \mathcal{O}(0.8)$ and $r_\chi^V = \mathcal{O}(0.2)$ at the hard-collinear scale $\mu_h = \sqrt{\Lambda_h m_b}$), one may argue that Eq. (3.19) provides a natural explanation as to why the power corrections to a_2 are smaller when M_1 is a vector meson, provided that soft corrections arise from spectator rescattering. Unfortunately, this is not the case. Numerically, we found that, for example, $H(K^*\pi)$ is comparable to $H(K\pi)$. This is due to the fact that $\int_0^1 dx r_\chi^M \Phi_m(x)/(1-x)$ is equal to $X_H r_\chi^P$ for $M = P$ and approximated to $3(X_H - 2) r_\chi^V$ for $M = V$.

We use next leading order (NLO) results for a_2 in Eq. (3.16) as a benchmark to define the parameters ρ_C and ϕ_C. The next to next leading order calculations of spectator-scattering tree amplitudes and vertex corrections at order α_s^2 have been carried out in [45,46], respectively. As pointed out in [47,48], a smaller value of λ_B can enhance the hard spectator interaction and hence a_2 substantially. For example, $a_2(\pi\pi) \sim 0.375 - 0.076i$ for $\lambda_B = 200$ MeV was found in [48]. However, the recent *BABAR* data on $B \to \gamma\ell\bar\nu$ [49] seems to imply a larger λ_B (> 300 MeV at the 90% C.L.). While next to next leading order corrections can in principle push the magnitude of $a_2(\pi\pi)$ up to the order of 0.40 by lowering the value of the B meson parameter λ_B, the strong phase of a_2 relative to a_1 cannot be larger than 15° [47]. In this work we reply on ρ_C and ϕ_C to get a large magnitude and strong phase for a_2.

TABLE IV. CP-averaged branching fractions (in units of 10^{-6}) and direct CP asymmetries (in %) of some selective $B \to PP$ decays obtained in QCD factorization for three distinct cases: (i) without any power corrections, (ii) with power corrections to penguin annihilation, and (iii) with power corrections to both penguin annihilation and color-suppressed tree amplitudes. The parameters ρ_A and ϕ_A are taken from Table III, $\rho_C = 1.3$ and $\phi_C = -70°$. The theoretical errors correspond to the uncertainties due to the variation of (i) Gegenbauer moments, decay constants, quark masses, form factors, the λ_B parameter for the B meson wave function, and (ii) $\rho_{A,H}$, $\phi_{A,H}$, respectively.

Mode	W/o $\rho_{A,C}$, $\phi_{A,C}$	With ρ_A, ϕ_A	With $\rho_{A,C}$, $\phi_{A,C}$	Expt. [3]
$\mathcal{B}(\bar B^0 \to K^-\pi^+)$	$13.1^{+5.8+0.7}_{-3.5-0.7}$	$19.3^{+7.9+8.2}_{-4.8-6.2}$	$19.3^{+7.9+8.2}_{-4.8-6.2}$	19.4 ± 0.6
$\mathcal{B}(\bar B^0 \to \bar K^0\pi^0)$	$5.5^{+2.8+0.3}_{-1.7-0.3}$	$8.4^{+3.8+3.8}_{-2.3-2.9}$	$8.6^{+3.8+3.8}_{-2.2-2.9}$	9.5 ± 0.5
$\mathcal{B}(B^- \to \bar K^0\pi^-)$	$14.9^{+6.9+0.9}_{-4.5-1.0}$	$21.7^{+9.2+9.0}_{-6.0-6.9}$	$21.7^{+9.2+9.0}_{-6.0-6.9}$	23.1 ± 1.0
$\mathcal{B}(B^- \to K^-\pi^0)$	$9.1^{+3.6+0.5}_{-2.3+0.5}$	$12.6^{+4.7+4.8}_{-3.0-3.7}$	$12.5^{+4.7+4.9}_{-3.0-3.8}$	12.9 ± 0.6
$\mathcal{B}(B^- \to K^-\eta)$	$1.6^{+1.1+0.3}_{-0.7-0.4}$	$2.4^{+1.8+1.3}_{-1.1-1.0}$	$2.4^{+1.8+1.3}_{-1.1-1.0}$	2.36 ± 0.27
$\mathcal{B}(\bar B^0 \to \pi^+\pi^-)$	$6.2^{+0.4+0.2}_{-0.6-0.4}$	$7.0^{+0.4+0.7}_{-0.7-0.7}$	$7.0^{+0.4+0.7}_{-0.7-0.7}$	5.16 ± 0.22
$\mathcal{B}(\bar B^0 \to \pi^0\pi^0)$	$0.42^{+0.29+0.18}_{-0.11-0.08}$	$0.52^{+0.26+0.21}_{-0.10-0.10}$	$1.1^{+1.0+0.7}_{-0.4-0.3}$	1.55 ± 0.19 [a]
$\mathcal{B}(B^- \to \pi^-\pi^0)$	$4.9^{+0.9+0.6}_{-0.5-0.3}$	$4.9^{+0.9+0.6}_{-0.5-0.3}$	$5.9^{+2.2+1.4}_{-1.1-1.1}$	$5.59^{+0.41}_{-0.40}$
$\mathcal{B}(B^- \to \pi^-\eta)$	$4.4^{+0.6+0.4}_{-0.3-0.2}$	$4.5^{+0.6+0.5}_{-0.3-0.3}$	$5.0^{+1.2+0.9}_{-0.6-0.7}$	4.1 ± 0.3
$A_{CP}(\bar B^0 \to K^-\pi^+)$	$4.0^{+0.6+1.1}_{-0.7-1.1}$	$-7.4^{+1.7+4.3}_{-1.5-4.8}$	$-7.4^{+1.7+4.3}_{-1.5-4.8}$	$-9.8^{+1.2}_{-1.1}$
$A_{CP}(\bar B^0 \to \bar K^0\pi^0)$	$-4.0^{+1.2+3.5}_{-1.8-3.0}$	$0.75^{+1.88+2.56}_{-0.94-3.32}$	$-10.6^{+2.7+5.6}_{-3.8-4.3}$	-1 ± 10
$A_{CP}(B^- \to \bar K^0\pi^-)$	$0.72^{+0.06+0.05}_{-0.05-0.05}$	$0.28^{+0.03+0.09}_{-0.03-0.10}$	$0.28^{+0.03+0.09}_{-0.03-0.10}$	0.9 ± 2.5
$A_{CP}(B^- \to K^-\pi^0)$	$7.3^{+1.6+2.3}_{-1.2-2.7}$	$-5.5^{+1.3+4.9}_{-1.8-4.6}$	$4.9^{+3.9+4.4}_{-2.1-5.4}$	5.0 ± 2.5
$A_{CP}(B^- \to K^-\eta)$	$-22.1^{+7.7+14.0}_{-16.7-7.3}$	$12.7^{+7.7+13.4}_{-5.0-15.0}$	$-11.0^{+8.4+14.9}_{-21.6-10.1}$	-37 ± 9
$A_{CP}(\bar B^0 \to \pi^+\pi^-)$	$-6.2^{+0.4+2.0}_{-0.5-1.8}$	$17.0^{+1.3+4.3}_{-1.2-8.7}$	$17.0^{+1.3+4.3}_{-1.2-8.7}$	38 ± 6
$A_{CP}(\bar B^0 \to \pi^0\pi^0)$	$33.4^{+6.8+34.8}_{-10.6-37.7}$	$-26.9^{+8.4+48.5}_{-6.0-37.5}$	$57.2^{+14.8+30.3}_{-20.8-34.6}$	43^{+25}_{-24}
$A_{CP}(B^- \to \pi^-\pi^0)$	$-0.06^{+0.01+0.01}_{-0.01-0.02}$	$-0.06^{+0.01+0.01}_{-0.01-0.02}$	$-0.11^{+0.01+0.06}_{-0.01-0.03}$	6 ± 5
$A_{CP}(B^- \to \pi^-\eta)$	$-11.4^{+1.1+2.3}_{-1.0-2.7}$	$11.4^{+0.9+4.5}_{-0.9-9.1}$	$-5.0^{+2.4+8.4}_{-3.4-10.3}$	-13 ± 7

[a] If an S factor is included, the average will become 1.55 ± 0.35.

TABLE V. CP-averaged branching fractions (in units of 10^{-6}) of $B \to PP$ decays obtained in various approaches. The pQCD results are taken from [50–54]. Note that there exist several pQCD calculations for $B \to K\eta^{(\prime)}$ [52,55–57] and here we cite the pQCD results with partial NLO corrections [52]. There are two solution sets with SCET predictions for decays involving η and/or η' [58].

Mode	QCDF (this work)	pQCD	SCET	Expt. [3]
$B^- \to \bar{K}^0\pi^-$	$21.7^{+9.2+9.0}_{-6.0-6.9}$	$23.6^{+14.5}_{-8.4}$	$20.8 \pm 7.9 \pm 0.6 \pm 0.7$	23.1 ± 1.0
$B^- \to K^-\pi^0$	$12.5^{+4.7+4.9}_{-3.0-3.8}$	$13.6^{+10.3}_{-5.7}$	$11.3 \pm 4.1 \pm 1.0 \pm 0.3$	12.9 ± 0.6
$\bar{B}^0 \to K^-\pi^+$	$19.3^{+7.9+8.2}_{-4.8-6.2}$	$20.4^{+16.1}_{-8.4}$	$20.1 \pm 7.4 \pm 1.3 \pm 0.6$	19.4 ± 0.6
$\bar{B}^0 \to \bar{K}^0\pi^0$	$8.6^{+3.8+3.8}_{-2.2-2.9}$	$8.7^{+6.0}_{-3.4}$	$9.4 \pm 3.6 \pm 0.2 \pm 0.3$	9.5 ± 0.5
$B^- \to \pi^-\pi^0$	$5.9^{+2.2+1.4}_{-1.1-1.1}$	$4.0^{+3.4}_{-1.9}$	$5.2 \pm 1.6 \pm 2.1 \pm 0.6$	$5.59^{+0.41}_{-0.40}$
$\bar{B}^0 \to \pi^+\pi^-$	$7.0^{+0.4+0.7}_{-0.7-0.7}$	$6.5^{+6.7}_{-3.8}$	$5.4 \pm 1.3 \pm 1.4 \pm 0.4$	5.16 ± 0.22
$\bar{B}^0 \to \pi^0\pi^0$	$1.1^{+1.0+0.7}_{-0.4-0.3}$	$0.29^{+0.50}_{-0.20}$	$0.84 \pm 0.29 \pm 0.30 \pm 0.19$	1.55 ± 0.19
$B^- \to K^-K^0$	$1.8^{+0.9+0.7}_{-0.5-0.5}$	1.66	$1.1 \pm 0.4 \pm 1.4 \pm 0.03$	$1.36^{+0.29}_{-0.27}$
$\bar{B}^0 \to K^+K^-$	$0.10^{+0.03+0.03}_{-0.02-0.03}$	0.046	-	$0.15^{+0.11}_{-0.10}$
$\bar{B}^0 \to K^0\bar{K}^0$	$2.1^{+1.0+0.8}_{-0.6-0.6}$	1.75	$1.0 \pm 0.4 \pm 1.4 \pm 0.03$	$0.96^{+0.21}_{-0.19}$
$B^- \to K^-\eta$	$2.3^{+1.8+1.3}_{-1.1-1.0}$	$3.2^{+1.2+2.7+1.1}_{-0.9-1.2-1.0}$	$2.7 \pm 4.8 \pm 0.4 \pm 0.3$	2.36 ± 0.27
			$2.3 \pm 4.5 \pm 0.4 \pm 0.3$	
$B^- \to K^-\eta'$	$78.4^{+61.2+26.4}_{-26.8-19.5}$	$51.0^{+13.5+11.2+4.2}_{-8.2-6.2-3.5}$	$69.5 \pm 27.0 \pm 4.4 \pm 7.7$	71.1 ± 2.6
			$69.3 \pm 26.0 \pm 7.1 \pm 6.3$	
$\bar{B}^0 \to \bar{K}^0\eta$	$1.6^{+1.5+1.1}_{-0.9-0.8}$	$2.1^{+0.8+2.3+1.0}_{-0.6-1.0-0.9}$	$2.4 \pm 4.4 \pm 0.2 \pm 0.3$	$1.12^{+0.30}_{-0.28}$
			$2.3 \pm 4.4 \pm 0.2 \pm 0.5$	
$\bar{B}^0 \to \bar{K}^0\eta'$	$74.2^{+56.5+24.7}_{-24.9-18.4}$	$50.3^{+11.8+11.1+4.5}_{-8.2-6.2-2.7}$	$63.2 \pm 24.7 \pm 4.2 \pm 8.1$	66.1 ± 3.1
			$62.2 \pm 23.7 \pm 5.5 \pm 7.2$	
$B^- \to \pi^-\eta$	$5.0^{+1.2+0.9}_{-0.6-0.7}$	$4.1^{+1.3+0.4+0.6}_{-0.9-0.3-0.5}$	$4.9 \pm 1.7 \pm 1.0 \pm 0.5$	4.07 ± 0.32
			$5.0 \pm 1.7 \pm 1.2 \pm 0.4$	
$B^- \to \pi^-\eta'$	$3.8^{+1.3+0.9}_{-0.6-0.6}$	$2.4^{+0.8}_{-0.5} \pm 0.2 \pm 0.3$	$2.4 \pm 1.2 \pm 0.2 \pm 0.4$	$2.7^{+0.5}_{-0.4}$
			$2.8 \pm 1.2 \pm 0.3 \pm 0.3$	
$\bar{B}^0 \to \pi^0\eta$	$0.36^{+0.03+0.13}_{-0.02-0.10}$	$0.23^{+0.04+0.04}_{-0.03-0.03} \pm 0.05$	$0.88 \pm 0.54 \pm 0.06 \pm 0.42$	<1.5
			$0.68 \pm 0.46 \pm 0.03 \pm 0.41$	
$\bar{B}^0 \to \pi^0\eta'$	$0.42^{+0.21+0.18}_{-0.09-0.12}$	$0.19 \pm 0.02 \pm 0.03^{+0.04}_{-0.05}$	$2.3 \pm 0.8 \pm 0.3 \pm 2.7$	1.2 ± 0.4
			$1.3 \pm 0.5 \pm 0.1 \pm 0.3$	
$\bar{B}^0 \to \eta\eta$	$0.32^{+0.13+0.07}_{-0.05-0.06}$	$0.67^{+0.32}_{-0.25}$	$0.69 \pm 0.38 \pm 0.13 \pm 0.58$	<1.0
			$1.0 \pm 0.4 \pm 0.3 \pm 1.4$	
$\bar{B}^0 \to \eta\eta'$	$0.36^{+0.24+0.12}_{-0.10-0.08}$	0.18 ± 0.11	$1.0 \pm 0.5 \pm 0.1 \pm 1.5$	<1.2
			$2.2 \pm 0.7 \pm 0.6 \pm 5.4$	
$\bar{B}^0 \to \eta'\eta'$	$0.22^{+0.14+0.08}_{-0.06-0.06}$	$0.11^{+0.12}_{-0.09}$	$0.57 \pm 0.23 \pm 0.03 \pm 0.69$	<1.7
			$1.2 \pm 0.4 \pm 0.3 \pm 3.7$	

IV. $B \to PP$ DECAYS

Effects of power corrections on penguin annihilation and the color-suppressed tree amplitude for some selective $B \to PP$ decays are shown in Table IV. The implications will be discussed below. Branching fractions and CP asymmetries for all $B \to PP$ decays are shown in Tables V and VII, respectively. The theoretical errors correspond to the uncertainties due to the variation of (i) the Gegenbauer moments, the decay constants, (ii) the heavy-to-light form factors and the strange quark mass, and (iii) the wave function of the B meson characterized by the parameter λ_B, the power corrections due to weak annihilation and hard spectator interactions described by the parameters $\rho_{A,H}$, $\phi_{A,H}$, respectively. To obtain the errors shown in these tables, we first scan randomly the points in the allowed ranges of the above nine parameters (specifically, the ranges $\rho_A^0 - 0.1 \le \rho_A \le \rho_A^0 + 0.1$, $\phi_A^0 - 20° \le \phi_A \le \phi_A^0 + 20°$, $0 \le \rho_H \le 1$, and $0 \le \phi_H \le 2\pi$ are used in this work, where the values of ρ_A^0 and ϕ_A^0 are displayed in Table III) and then add errors in quadrature. More specifically, the second error in the table is referred to the uncertainties caused by the variation of $\rho_{A,H}$ and $\phi_{A,H}$, where all other uncertainties are lumped into the first error. Power corrections beyond the heavy quark limit generally give the major theoretical uncertainties.

A. Branching fractions

1. $B \to K\pi$

The $B \to K\pi$ decays are dominated by penguin contributions because of $|V_{us}V_{ub}^*| \ll |V_{cs}V_{cb}^*| \approx |V_{ts}V_{tb}^*|$ and the large top quark mass. For the ratios defined by

$$R_c \equiv \frac{2\Gamma(B^- \to K^-\pi^0)}{\Gamma(B^- \to \bar{K}^0\pi^-)}, \qquad R_n \equiv \frac{\Gamma(\bar{B}^0 \to K^-\pi^+)}{2\Gamma(\bar{B}^0 \to \bar{K}^0\pi^0)}, \quad (4.1)$$

we have $R_c = R_n \approx 1$ if the other quark-diagram amplitudes are negligible compared with P'. The current experimental measurements give $R_c = 1.12 \pm 0.07$ and $R_n = 0.99 \pm 0.07$. In QCDF we have $R_c = 1.15 \pm 0.03$ and $R_n = 1.12 \pm 0.03$, which are consistent with experiment.

From Table IV, we see that the predicted rates for penguin-dominated $B \to PP$ decays to the zeroth order of $1/m_b$ expansion are usually $(30 \sim 45)\%$ below measurements (see the second column of Table IV). Also the direct CP asymmetry $A_{CP}(K^-\pi^+)$ is wrong in sign. We use penguin annihilation dictated by $\rho_A = 1.10$ and $\phi_A = -50°$ to fix both problems.

2. $B \to K\eta^{(\prime)}$

Among the 2-body B decays, $B \to K\eta'$ has the largest branching fraction, of order 70×10^{-6}, while $\mathcal{B}(B \to \eta K)$ is only $(1–3) \times 10^{-6}$. This can be qualitatively understood as follows: Since the η-η' mixing angle in the quark-flavor basis $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$

$$\eta = \cos\phi\,\eta_q - \sin\phi\,\eta_s, \qquad \eta' = \sin\phi\,\eta_q + \cos\phi\,\eta_s \quad (4.2)$$

is extracted from the data to be $\phi = 39.3°$ [29], it is clear that the interference between the $B \to K\eta_q$ amplitude induced by the $b \to sq\bar{q}$ penguin and the $B \to K\eta_s$ amplitude induced by $b \to ss\bar{s}$ is constructive for $B \to K\eta'$ and destructive for $B \to \eta K$. This explains the large rate of the former and the suppression of the latter. However, most of the model calculations still fall short of the data for $\mathcal{B}(B \to K\eta')$.

Many possible solutions to the puzzle for the abnormally large $K\eta'$ rate have been proposed in the past: (i) a significant flavor-singlet contribution [15,59], (ii) a large $B \to \eta'$ form factor [60], (iii) a contribution from the charm content of the η', (iv) an enhanced hadronic matrix element $\langle 0|\bar{s}\gamma_5 s|\eta'\rangle$ due to the axial U(1) anomaly [61], (v) a large chiral scale m_0^q associated with the η_q [55,56], (vi) a long-distance charming penguin in SCET [58], and (vii) a large contribution from the two-gluon fusion mechanism [62].

Numerically, Beneke and Neubert already obtained $\mathcal{B}(B^- \to K^-\eta') \sim \mathcal{O}(50 \times 10^{-6})$ in QCDF using the default values $\rho_A = \rho_H = 0$ [1]. Here, we found similar results 57×10^{-6} (53×10^{-6}) with (without) the contributions from the "charm content" of the η'. In the presence of penguin annihilation, we obtain $\mathcal{B}(B^- \to K^-\eta') \sim 78 \times 10^{-6}$ (71×10^{-6}) with (without) the "charm content" contributions. Therefore, the observed large $B \to K\eta'$ rates are naturally explained in QCDF without invoking, for example, flavor-singlet contributions. Data on $B \to K\eta$ modes are also well accounted for by QCDF.

3. $B \to \pi\pi$

From Table IV we see that power corrections to the color-suppressed tree amplitude have almost no effect on the decay rates of penguin-dominated decays, but will enhance the color-suppressed tree-dominated decay $B \to \pi^0\pi^0$ substantially owing to the enhancement of $|a_2| \sim \mathcal{O}(0.6)$ [see Eq. (4.4) below]. Since $|P_{EW}/C|$ is of order 0.06 before any power corrections, it is very unlikely that an enhancement of P_{EW} through new physics effects can render $c = C + P_{EW}$ large and complex. Notice that the central values of the branching fractions of $B^0 \to \pi^0\pi^0$ measured by *BABAR* [63] and Belle [64], $(1.83 \pm 0.21 \pm 0.13) \times 10^{-6}$ and $(1.1 \pm 0.3 \pm 0.1) \times 10^{-6}$, respectively, are somewhat different in their central values. The charged mode $B^- \to \pi^-\pi^0$ also gets an enhancement as its amplitude is proportional to $a_1 + a_2$. The prediction of QCDF or pQCD (see Table V) for $\mathcal{B}(B^0 \to \pi^+\pi^-)$ is slightly too large compared to the data. This is a long-standing issue. One possibility for the remedy is that there exists $\pi\pi \to \pi\pi$ meson annihilation contributions in which two initial quark pairs in the zero isospin configuration are destroyed and then created. Indeed, in the topological quark-diagram approach, this corresponds to the vertical W-loop diagram [65]. As shown in [7,43], this additional long-distance contribution may lower the $\pi^+\pi^-$ rate. In the final-state rescattering model considered by Hou and Yang [66] and elaborated more by one of us (C. K. C.) [7], $\bar{B}^0 \to \pi^+\pi^-$ and $\pi^0\pi^0$ rates are reduced and enhanced roughly by a factor of 2, respectively, through FSIs. It should be remarked that in the pQCD approach, it has been shown recently that the color-suppressed tree amplitude will be enhanced by a soft factor arising from the uncanceled soft divergences in the k_T factorization for nonfactorizable hadronic B decays [10]. As a consequence, the $B^0 \to \pi^0\pi^0$ rate can be enhanced to the right magnitude.

4. $B \to K\bar{K}$

The decays $B^- \to K^-K^0$ and $\bar{B}^0 \to \bar{K}^0K^0$ receive $b \to d$ penguin contributions and $\bar{B}^0 \to K^+K^-$ proceeds only through weak annihilation. Hence, the first two modes have branching fractions of order 10^{-6}, while the last one is suppressed to the order of 10^{-8}.

5. $B \to \pi\eta^{(\prime)}$

The decay amplitudes of $B \to \pi\eta$ are

$$\begin{aligned}\sqrt{2}A(B^- \to \pi^-\eta) &\approx A_{\pi\eta_q}[\delta_{pu}(\alpha_2 + \beta_2) + 2\alpha_3^p + \hat{\alpha}_4^p] \\ &\quad + A_{\eta_q\pi}[\delta_{pu}(\alpha_1 + \beta_2) + \hat{\alpha}_4^p], \\ -2A(\bar{B}^0 \to \pi^0\eta) &\approx A_{\pi\eta_q}[\delta_{pu}(\alpha_2 - \beta_1) + 2\alpha_3^p + \hat{\alpha}_4^p] \\ &\quad + A_{\eta_q\pi}[\delta_{pu}(-\alpha_2 - \beta_1) + \hat{\alpha}_4^p],\end{aligned} \quad (4.3)$$

with $\hat{\alpha}_4 = \alpha_4 + \beta_3$ and similar expressions for $B \to \pi\eta'$.

TABLE VI. *CP*-averaged branching fractions (in units of 10^{-6}) of $B \to \pi\eta^{(\prime)}$ decays.

	$\pi^-\eta'$	$\pi^0\eta'$	$\pi^-\eta$	$\pi^0\eta$
BABAR	$3.5 \pm 0.6 \pm 0.2$ [67]	$0.9 \pm 0.4 \pm 0.1$ [68]	$4.00 \pm 0.40 \pm 0.24$ [67]	<1.5 [68]
Belle	$1.8^{+0.7}_{-0.6} \pm 0.1$ [69]	$2.8 \pm 1.0 \pm 0.3$ [69]	$4.2 \pm 0.4 \pm 0.2$ [70]	<2.5 [71]
Average	$2.7^{+0.5}_{-0.4}$	1.2 ± 0.4	4.1 ± 0.3	<1.5

It is clear that the decays $\bar{B}^0 \to \eta^{(\prime)}\pi^0$ have very small rates because of near cancellation of the color-suppressed tree amplitudes, while the charged modes $\eta^{(\prime)}\pi^-$ receive color-allowed tree contributions. From the experimental data shown in Table VI, it is clear that *BABAR*'s measurement of $\mathcal{B}(B^- \to \pi^-\eta') \gg \mathcal{B}(\bar{B}^0 \to \pi^0\eta')$ is in accordance with the theoretical expectation, whereas Belle's results indicate the other way around. Nevertheless, *BABAR* and Bell agree with each other on $\mathcal{B}(B \to \pi\eta)$. QCDF predictions for $B \to \pi\eta^{(\prime)}$ agree well with the *BABAR* data. As for the pQCD approach, it appears that its prediction for $\mathcal{B}(\bar{B}^0 \to \pi^0\eta')$ is too small. At any rate, it is important to have more accurate measurements of $B \to \pi\eta^{(\prime)}$.

B. Direct *CP* asymmetries

For $\rho_C \approx 1.3$ and $\phi_C \approx -70°$, we find that all the *CP* puzzles in $B \to PP$ decays are resolved as shown in fourth column of Table IV. The corresponding a_2's are

$$a_2(\pi\pi) \approx 0.60e^{-i55°}, \qquad a_2(K\pi) \approx 0.51e^{-i58°}. \quad (4.4)$$

They are consistent with the phenomenological determination of $C^{(\prime)}/T^{(\prime)} \sim a_2/a_1$ from a global fit to the available data [15]. Because of the interference between the penguin and the large complex color-suppressed tree amplitudes, it is clear from Table IV that theoretical predictions for direct *CP* asymmetries now agree with experiment in signs even for those modes with the significance of A_{CP} less than 3σ. We shall discuss each case one by one.

1. $A_{CP}(K^-\pi^+)$

Neglecting electroweak penguin contributions, the decay amplitude of $\bar{B}^0 \to K^-\pi^+$ reads

$$A(\bar{B}^0 \to K^-\pi^+) = A_{\pi\bar{K}}(\delta_u\alpha_1 + \alpha_4^p + \beta_3^p). \quad (4.5)$$

Following [1], the *CP* asymmetry of $\bar{B}^0 \to K^-\pi^+$ can be expressed as

$$A_{CP}(\bar{B}^0 \to K^-\pi^+)R_{\rm FM} = -2\sin\gamma\, {\rm Im} r_{\rm FM}, \quad (4.6)$$

with

$$R_{\rm FM} \equiv \frac{\Gamma(\bar{B}^0 \to K^-\pi^+)}{\Gamma(B^- \to \bar{K}^0\pi^-)} = 1 - 2\cos\gamma\,{\rm Re}r_{\rm FM} + |r_{\rm FM}|^2,$$

$$r_{\rm FM} = \left|\frac{\lambda_u^{(s)}}{\lambda_c^{(s)}}\right| \frac{\alpha_1(\pi\bar{K})}{-\alpha_4^c(\pi\bar{K}) - \beta_3^c(\pi\bar{K})}, \quad (4.7)$$

where the small contribution from $\hat{\alpha}_4^u$ has been neglected and the decay amplitude of $B^- \to \bar{K}^0\pi^-$ is given in Eq. (4.11). Theoretically, we obtain $r_{\rm FM} = 0.14$ for $\gamma = 67.8°$ with a small imaginary part and $R_{\rm FM} = 0.91$, to be compared with the experimental value $R_{\rm FM} = 0.84 \pm 0.04$. In the absence of penguin annihilation, direct *CP* violation of $\bar{B}^0 \to K^-\pi^+$ is positive as ${\rm Im}\alpha_4^c \approx 0.013$. When the power correction to penguin annihilation is turned on, we have ${\rm Im}(\alpha_4^c + \beta_3^c) \approx -0.039$ and hence a negative $A_{CP}(K^-\pi^+)$. This also explains why *CP* asymmetries of penguin-dominated decays in the QCDF framework will often reverse their signs in the presence of penguin annihilation.

2. $A_{CP}(K^-\pi^0)$

The decay amplitude is

$$\sqrt{2}A(B^- \to K^-\pi^0) = A_{\pi\bar{K}}(\delta_u\alpha_1 + \alpha_4^p + \beta_3^p) + A_{\bar{K}\pi}(\delta_{pu}\alpha_2 + \tfrac{3}{2}\alpha_{3,{\rm EW}}^p). \quad (4.8)$$

If the color-suppressed tree and electroweak penguin amplitudes are negligible, it is obvious that the amplitude of $K^-\pi^0$ will be the same as that of $K^-\pi^+$ except for a trivial factor of $1/\sqrt{2}$. The *CP* asymmetry difference $\Delta A_{K\pi} \equiv A_{CP}(K^-\pi^0) - A_{CP}(K^-\pi^+)$ arising from the interference between P' and C' and between $P'_{\rm EW}$ and T' is expected to be small, while it is 0.148 ± 0.028 experimentally [3]. To identify the effect due to the color-suppressed tree amplitude, we write

$$\Delta A_{K\pi} = 0.015^{+0.006+0.008}_{-0.006-0.013} - 2\sin\gamma\,{\rm Im}r_C + \cdots, \quad (4.9)$$

where the first term on the right-hand side is due to the interference of the electroweak penguin with color-allowed tree and QCD penguin amplitudes and

$$r_C = \left|\frac{\lambda_u^{(s)}}{\lambda_c^{(s)}}\right| \frac{f_\pi F_0^{BK}(0)}{f_K F_0^{B\pi}(0)} \frac{\alpha_2(\pi\bar{K})}{-\alpha_4^c(\pi\bar{K}) - \beta_3^c(\pi\bar{K})}. \quad (4.10)$$

The imaginary part of r_C is rather small because of the cancellation of the phases between α_2 and $\alpha_4^c + \beta_3^c$. When soft corrections to a_2 are included, we have $r_C \approx 0.078 - 0.063i$. It follows from Eq. (4.9) that $\Delta A_{K\pi}$ will become of order 0.13.

As first emphasized by Lunghi and Soni [72], in the QCDF analysis of the quantity $\Delta A_{K\pi}$, although the theoretical uncertainties due to power corrections from penguin annihilation are large for individual asymmetries $A_{CP}(K^-\pi^0)$ and $A_{CP}(K^-\pi^+)$, they essentially cancel out in their difference, rendering the theoretical prediction more reliable. We find $\Delta A_{K\pi} = (12.3^{+2.2+2.1}_{-0.9-4.7})\%$, while it is only $(1.9^{+0.5+1.6}_{-0.4-1.0})\%$ in the absence of power corrections to a_2 or to the topological amplitude C'.

3. $A_{CP}(\bar{K}^0\pi^0)$ and $A_{CP}(\bar{K}^0\pi^-)$

The decay amplitudes are

$$\begin{aligned}\sqrt{2}A(\bar{B}^0 \to \bar{K}^0\pi^0) &= A_{\pi\bar{K}}(-\alpha_4^p - \beta_3^p) \\ &\quad + A_{\bar{K}\pi}\left(\delta_{pu}\alpha_2 + \frac{3}{2}\alpha^p_{3,\mathrm{EW}}\right) \\ &= -p' + c', \\ A(B^- \to \bar{K}^0\pi^-) &= A_{\pi\bar{K}}(\alpha_4^p + \beta_3^p) = p',\end{aligned} \tag{4.11}$$

where the amplitudes $p' = P' - \frac{1}{3}P'^c_{\mathrm{EW}} + P'_A$, and $c' = C' + P'_{\mathrm{EW}}$ have been introduced in Sec. I. CP violation of $B^- \to \bar{K}^0\pi^-$ is expected to be very small as it is a pure penguin process. Indeed, QCDF predicts $A_{CP}(\bar{K}^0\pi^-) \approx 0.003$. If c' is negligible compared to p', $A_{CP}(\bar{K}^0\pi^0)$ will be very small. Just as the previous case, the CP asymmetry difference of the $\bar{K}^0\pi^0$ and $\bar{K}^0\pi^-$ modes reads

$$\begin{aligned}\Delta A'_{K\pi} &\equiv A_{CP}(\bar{K}^0\pi^0) - A_{CP}(\bar{K}^0\pi^-) \\ &= (0.57^{+0.04+0.14}_{-0.04-0.06})\% + 2\sin\gamma\,\mathrm{Im}r_C + \cdots,\end{aligned} \tag{4.12}$$

where the first term on the right-hand side is due to the interference between the electroweak and QCD penguin amplitudes. To a good approximation, we have $\Delta A'_{K\pi} \sim -\Delta A_{K\pi}$. This together with the measured value of $\Delta A_{K\pi}$ and the smallness of $A_{CP}(\bar{K}^0\pi^-)$ indicates that $A_{CP}(\bar{K}^0\pi^0)$ should be roughly of order -0.15. Using $\mathrm{Im}r_C \approx -0.063$ as discussed before, it follows from the above equation that $A_{CP}(\bar{K}^0\pi^0)$ is of order -11%. More precisely, we predict $A_{CP}(\bar{K}^0\pi^0) = (-10.6^{+2.7+5.6}_{-3.8-4.3})\%$ and $\Delta A'_{K\pi} = (-11.0^{+2.7+5.8}_{-3.8-4.3})\%$, while they are of order 0.0075 and 0.0057, respectively, in the absence of ρ_C and ϕ_C. Therefore, an observation of $A_{CP}(\bar{K}^0\pi^0)$ at the level of $-(10 \sim 15)\%$ will be strong support for the presence of power corrections to c'. This is essentially a model-independent statement.

Experimentally, the current world average -0.01 ± 0.10 is consistent with no CP violation because the *BABAR* and Belle measurements $-0.13 \pm 0.13 \pm 0.03$ [73] and $0.14 \pm 0.13 \pm 0.06$ [74], respectively, are of opposite sign. Nevertheless, there exist several model-independent determinations of this asymmetry: one is the SU(3) relation $\Delta\Gamma(\pi^0\pi^0) = -\Delta\Gamma(\bar{K}^0\pi^0)$ [75], and the other is the approximate sum rule for CP rate asymmetries [76]

$$\Delta\Gamma(K^-\pi^+) + \Delta\Gamma(\bar{K}^0\pi^-) \approx 2[\Delta\Gamma(K^-\pi^0) + \Delta\Gamma(\bar{K}^0\pi^0)], \tag{4.13}$$

based on isospin symmetry, where $\Delta\Gamma(K\pi) \equiv \Gamma(\bar{B} \to \bar{K}\bar{\pi}) - \Gamma(B \to K\pi)$. This sum rule allows us to extract $A_{CP}(\bar{K}^0\pi^0)$ in terms of the other three asymmetries of $K^-\pi^+$, $K^-\pi^0$, $\bar{K}^0\pi^-$ modes that have been measured. From the current data of branching fractions and CP asymmetries, the above SU(3) relation and CP-asymmetry sum rule lead to $A_{CP}(\bar{K}^0\pi^0) = -0.073^{+0.042}_{-0.041}$ and $A_{CP}(\bar{K}^0\pi^0) = -0.15 \pm 0.04$, respectively. An analysis based on the topological quark diagrams also yields a similar result $-0.08 \sim -0.12$ [77]. All these indicate that the direct CP violation $A_{CP}(\bar{K}^0\pi^0)$ should be negative and has a magnitude of order 0.10.

4. $A_{CP}(K\eta^{(\prime)})$

The world average of $A_{CP}(B^- \to K^-\eta) = -0.37 \pm 0.09$ due to the measurements $-0.36 \pm 0.11 \pm 0.03$ from *BABAR* [67] and $-0.39 \pm 0.16 \pm 0.03$ from Belle [70] differs from zero by 4.1σ deviations. The decay amplitude of $B^- \to K^-\eta$ is given by [1]

$$\begin{aligned}\sqrt{2}A(B^- \to K^-\eta) &= A_{\bar{K}\eta_q}[\delta_{pu}\alpha_2 + 2\alpha_3^p] \\ &\quad + \sqrt{2}A_{\bar{K}\eta_s}[\delta_{pu}\beta_2 + \alpha_3^p + \alpha_4^p + \beta_3^p] \\ &\quad + \sqrt{2}A_{\bar{K}\eta_c}[\delta_{pc}\alpha_2 + \alpha_3^p] \\ &\quad + A_{\eta_q\bar{K}}[\delta_{pu}(\alpha_1 + \beta_2) + \alpha_4^p + \beta_3^p],\end{aligned} \tag{4.14}$$

where the flavor states of the η meson, $q\bar{q} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$, $s\bar{s}$ and $c\bar{c}$ are labeled by η_q, η_s, and η_c^0, respectively. Since the two penguin processes $b \to ss\bar{s}$ and $b \to sq\bar{q}$ contribute destructively to $B \to K\eta$ (i.e. $A_{\bar{K}\eta_s} = X^{(\bar{B}\bar{K},\eta_s)}$ has an opposite sign to $A_{\bar{K}\eta_q}$ and $A_{\eta_q\bar{K}}$), the penguin amplitude is comparable in magnitude to the tree amplitude induced from $b \to us\bar{u}$, contrary to the decay $B \to K\eta'$ which is dominated by large penguin amplitudes. Consequently, a sizable direct CP asymmetry is expected in $B^- \to K^-\eta$ but not in $K^-\eta'$ [78].

The decay constants f_η^q, f_η^s and f_η^c are given before in Eqs. (3.5) and (3.6). Although $f_\eta^c \approx -2$ MeV is much smaller than $f_\eta^{q,s}$, its effect is Cabibbo-Kobayashi-Maskawa (CKM) enhanced by $V_{cb}V_{cs}^*/(V_{ub}V_{us}^*)$. In the presence of penguin annihilation, $A_{CP}(K^-\eta)$ is found to be of order 0.127 (see Table IV). When ρ_C and ϕ_C are turned on, $A_{CP}(K^-\eta)$ will be reduced to 0.004 if there is no intrinsic charm content of the η. When the effect of f_η^c is taken into account, $A_{CP}(K^-\eta)$ finally reaches the level of -11% and has a sign in agreement with experiment. Hence, CP violation in $B^- \to K^-\eta$ is the place where the charm content of the η plays a role.

Two remarks are in order. First, the pQCD prediction for $A_{CP}(K^-\eta)$ is very sensitive to m_{qq}, the mass of the η_q, which is generally taken to be of order m_π. It was found in [55] that for $m_{qq} = 0.14$, 0.18 and 0.22 GeV, $A_{CP}(K^-\eta)$ becomes 0.0562, 0.0588, and -0.3064, respectively. There are two issues here: (i) Is it reasonable to have a large value of m_{qq}? and (ii) The fact that $A_{CP}(K^-\eta)$ is so sensitive to m_{qq} implies that the pQCD prediction is not stable. Within the framework of pQCD, the authors of [79] rely on the NLO corrections to get a negative CP asymmetry and avoid the aforementioned issues. At the lowest order, pQCD predicts $A_{CP}(K^-\eta) \approx 9.3\%$. Then NLO corrections will flip the sign and give rise to $A_{CP}(K^-\eta) = (-11.7^{+8.4}_{-11.4})\%$. In view of the sign change of A_{CP} by NLO effects here, this indicates that pQCD calculations should be carried out systematically to NLO in order to have a reliable estimate of CP asymmetries. Second, while both QCDF and pQCD can manage to lead to a correct sign for $A_{CP}(K^-\eta)$, the predicted magnitude still falls short of the measurement -0.37 ± 0.09. At first sight, it appears that the QCDF prediction $A_{CP}(K^-\eta) = -0.221^{+0.160}_{-0.182}$ (see Table IV) obtained in the leading $1/m_b$ expansion already agrees well with the data. However, the agreement is just an accident. Recall that in the absence of power corrections, the calculated CP asymmetries for $K^-\pi^+$ and $\pi^+\pi^-$ modes are wrong in signs. That is why it is important to consider the major power corrections step by step. The QCDF results in the heavy quark limit should not be considered as the final QCDF predictions to be compared with experiment.

5. $A_{CP}(\pi^-\eta)$

As for the decay $B^- \to \pi^-\eta$, it is interesting to see that penguin annihilation will flip the sign of $A_{CP}(\pi^-\eta)$ into a wrong one without affecting its magnitude (see Table IV). Again, soft corrections to a_2 will bring the CP asymmetry back to the right track. Contrary to the previous case of $B^- \to K^-\eta$, the charm content of the η here does not play a role as it does not get a CKM enhancement.

6. $A_{CP}(\pi^+\pi^-)$

It is well known that based on SU(3) flavor symmetry, direct CP asymmetries in $K\pi$ and $\pi\pi$ systems are related as [75]:

$$\Delta\Gamma(K^-\pi^+) = -\Delta\Gamma(\pi^+\pi^-),\qquad \Delta\Gamma(\bar{K}^0\pi^0) = -\Delta\Gamma(\pi^0\pi^0). \tag{4.15}$$

The first relation leads to $A_{CP}(\pi^+\pi^-) = [\mathcal{B}(K^-\pi^+)/\mathcal{B}(\pi^+\pi^-)]A_{CP}(K^-\pi^+) \approx 0.37$, which is in good agreement with the current world average of 0.38 ± 0.06 [3].

The decay amplitude is

$$A(\bar{B}^0 \to \pi^+\pi^-) = A_{\pi\pi}[\delta_{pu}(\alpha_1 + \beta_1) + \alpha_4^p + \beta_3^p + \cdots], \tag{4.16}$$

which is very similar to the amplitude of the $K^-\pi^+$ mode [see Eq. (4.5)] except for the CKM matrix elements. Since the penguin contribution is small compared to the tree one, its CP asymmetry is approximately given by

$$A_{CP}(\pi^+\pi^-) \approx 2\sin\gamma\,\mathrm{Im}\,r_{\pi\pi}, \tag{4.17}$$

with

$$r_{\pi\pi} = \left|\frac{\lambda_c^{(d)}}{\lambda_u^{(d)}}\right| \frac{\alpha_4^c(\pi\pi) + \beta_3^c(\pi\pi)}{\alpha_1(\pi\pi)}. \tag{4.18}$$

Numerically, we obtain $\mathrm{Im}\,r_{\pi\pi} = 0.107\ (-0.033)$ with (without) the annihilation term β_3^c. Hence, one needs penguin annihilation in order to have a correct sign for $A_{CP}(\pi^+\pi^-)$. However, the dynamical calculation of both QCDF and pQCD yields $A_{CP}(\pi^+\pi^-) \approx 0.17 \sim 0.20$. It is hard to push the CP asymmetry to the level of 0.38. Note that the central values of current B factory measurements of CP asymmetry: $-0.25 \pm 0.08 \pm 0.02$ by *BABAR* [80] and $-0.55 \pm 0.08 \pm 0.05$ by Belle [81], differ by a factor of 2.

7. $A_{CP}(\pi^0\pi^0)$

Just like the $\pi^0\eta$ mode, penguin annihilation will flip the sign of $A_{CP}(\pi^0\pi^0)$ into a wrong one (see Table IV). If the amplitude $c = C + P_{\rm EW}$ is large and complex, its interference with the QCD penguin will bring the sign of CP asymmetry into the right one. As mentioned before, $|P_{\rm EW}/C|$ is of order 0.06 before any power corrections. It is thus very unlikely that an enhancement of $P_{\rm EW}$ through new physics can render c large and complex. For the $a_2(\pi\pi)$ given by Eq. (4.4), we find that $A_{CP}(\pi^0\pi^0)$ is of order 0.55, to be compared with the current average, $0.43^{+0.25}_{-0.24}$ [3].

8. $A_{CP}(\pi^-\pi^0)$

It is generally believed that direct CP violation of $B^- \to \pi^-\pi^0$ is very small. This is because the isospin of the $\pi^-\pi^0$ state is $I = 2$ and hence it does not receive QCD penguin contributions and receives only the loop contributions from electroweak penguins. Since this decay is tree dominated, SM predicts an almost null CP asymmetry, of order $10^{-3} \sim 10^{-4}$. What will happen if a_2 has a large magnitude and strong phase? We find that power corrections to the color-suppressed tree amplitude will enhance $A_{CP}(\pi^-\pi^0)$ substantially to the level of 2%. Similar conclusions were also obtained by the analysis based on the diagrammatic approach [15]. However, one must be very cautious about this. The point is that power corrections will affect not only a_2, but also other parameters a_i with $i \neq 2$. Since the isospin of $\pi^-\pi^0$ is $I = 2$, the soft corrections to a_2 and a_i must be conspired in such a way that $\pi^-\pi^0$ is always an $I = 2$ state. As explained below, there are two possible sources of power corrections to a_2: spectator scattering and final-state interactions. For final-state rescattering, it is found in [43] that effects of FSIs on $A_{CP}(\pi^-\pi^0)$ are small, consistent with the requirement followed from the CPT theorem. In the specific residual scattering model considered by one of us (C. K. C.) [7], $\pi^-\pi^0$ can only rescatter into itself, and as a consequence, direct CP violation will not receive any contribution from residual final-state interactions. Likewise, if large ρ_H and ϕ_H are turned on to mimic Eq. (4.4), we find $A_{CP}(\pi^-\pi^0)$ is at most of

TABLE VII. Same as Table V except for direct CP asymmetries (in %) of $B \to PP$ decays obtained in various approaches.

Mode	QCDF (this work)	pQCD	SCET	Expt. [3]
$B^- \to \bar{K}^0\pi^-$	$0.28^{+0.03+0.09}_{-0.03-0.10}$	0	<5	0.9 ± 2.5
$B^- \to K^-\pi^0$	$4.9^{+3.9+4.4}_{-2.1-5.4}$	-1^{+3}_{-6}	$-11 \pm 9 \pm 11 \pm 2$	5.0 ± 2.5
$\bar{B}^0 \to K^-\pi^+$	$-7.4^{+1.7+4.3}_{-1.5-4.8}$	-10^{+7}_{-8}	$-6 \pm 5 \pm 6 \pm 2$	$-9.8^{+1.2}_{-1.1}$
$\bar{B}^0 \to \bar{K}^0\pi^0$	$-10.6^{+2.7+5.6}_{-3.8-4.3}$	-7^{+3}_{-4}	$5 \pm 4 \pm 4 \pm 1$	-1 ± 10
$B^- \to \pi^-\pi^0$	$-0.11^{+0.01+0.06}_{-0.01-0.03}$	0	<4	6 ± 5
$\bar{B}^0 \to \pi^+\pi^-$	$17.0^{+1.3+4.3}_{-1.2-8.7}$	18^{+20}_{-12}	$20 \pm 17 \pm 19 \pm 5$	38 ± 6
$\bar{B}^0 \to \pi^0\pi^0$	$57.2^{+14.8+30.3}_{-20.8-34.6}$	63^{+35}_{-34}	$-58 \pm 39 \pm 39 \pm 13$	43^{+25}_{-24}
$B^- \to K^-K^0$	$-6.4^{+0.8+1.8}_{-0.6-1.8}$	11	$1.1 \pm 0.4 \pm 1.4 \pm 0.03$	12^{+17}_{-18}
$\bar{B}^0 \to K^+K^-$	0	29		
$\bar{B}^0 \to K^0\bar{K}^0$	$-10.0^{+0.7+1.0}_{-0.7-1.9}$	0	$1.0 \pm 0.4 \pm 1.4 \pm 0.03$	
$B^- \to K^-\eta$	$-11.2^{+8.5+15.2}_{-22.0-10.3}$	$-11.7^{+6.8+3.9+2.9}_{-9.6-4.2-5.6}$	$33 \pm 30 \pm 7 \pm 3$ $-33 \pm 39 \pm 10 \pm 4$	-37 ± 9
$B^- \to K^-\eta'$	$0.52^{+0.66+1.14}_{-0.53-0.90}$	$-6.2^{+1.2+1.3+1.3}_{-1.1-1.0-1.0}$	$-10 \pm 6 \pm 7 \pm 5$ $0.7 \pm 0.5 \pm 0.2 \pm 0.9$	$1.3^{+1.6}_{-1.7}$
$\bar{B}^0 \to \bar{K}^0\eta$	$-21.4^{+8.6+11.8}_{-22.9-11.3}$	$-12.7^{+4.1+3.2+3.2}_{-4.1-1.5-6.7}$	$21 \pm 20 \pm 4 \pm 3$ $-18 \pm 22 \pm 6 \pm 4$	
$\bar{B}^0 \to \bar{K}^0\eta'$	$3.0^{+0.6+0.7}_{-0.5-0.8}$	$2.3^{+0.5+0.3+0.2}_{-0.4-0.6-0.1}$	$11 \pm 6 \pm 12 \pm 2$ $-27 \pm 7 \pm 8 \pm 5$	5 ± 5
$B^- \to \pi^-\eta$	$-5.0^{+2.4+8.4}_{-3.4-10.3}$	-37^{+8+4+0}_{-6-4-1}	$5 \pm 19 \pm 21 \pm 5$ $37 \pm 19 \pm 21 \pm 5$	-13 ± 7
$B^- \to \pi^-\eta'$	$1.6^{+5.0+9.4}_{-8.2-11.1}$	-33^{+6+4+0}_{-4-6-2}	$21 \pm 12 \pm 10 \pm 14$ $2 \pm 10 \pm 4 \pm 15$	6 ± 15
$\bar{B}^0 \to \pi^0\eta$	$-5.2^{+2.8+24.6}_{-5.0-15.6}$	$-42^{+9+3+1}_{-12-2-3}$	$3 \pm 10 \pm 12 \pm 5$ $-7 \pm 16 \pm 4 \pm 90$	
$\bar{B}^0 \to \pi^0\eta'$	$-7.3^{+1.0+17.6}_{-1.8-14.0}$	$-36^{+10+2+2}_{-9-1-3}$	$-24 \pm 10 \pm 19 \pm 24$ -	
$\bar{B}^0 \to \eta\eta$	$-63.5^{+10.4+9.8}_{-6.4-12.4}$	$-33^{+2.6+4.1+3.5}_{-2.8-3.8-0.0}$	$-9 \pm 24 \pm 21 \pm 4$ $48 \pm 22 \pm 20 \pm 13$	
$\bar{B}^0 \to \eta\eta'$	$-59.2^{+7.2+3.8}_{-6.8-4.8}$	$77.4^{+0.0+6.9+8.0}_{-5.6-11.2-9.0}$	- $70 \pm 13 \pm 20 \pm 4$	
$\bar{B}^0 \to \eta'\eta'$	$-44.9^{+3.1+8.5}_{-3.1-9.2}$	$23.7^{+10.0+18.5+6.0}_{-6.9-16.9-8.5}$	- $60 \pm 11 \pm 22 \pm 29$	

order 10^{-3}. (The result of $A_{CP}(\pi^-\pi^0)$ in QCDF listed in Tables IV and VII is obtained in this manner.) This is because spectator scattering will contribute to not only a_2 but also a_1 and the electroweak penguin parameters a_{7-10}. Therefore, a measurement of direct CP violation in $B^- \to \pi^-\pi^0$ provides a nice test of the standard model and new physics.

9. CP asymmetries in pQCD and SCET

For most of the $B \to PP$ decays, pQCD predictions of CP asymmetries are similar to the QCDF ones at least in signs except for the $K\bar{K}$, $K^-\pi^0$, $K^-\eta'$, $\pi\eta$, $\eta\eta'$, $\eta'\eta'$ modes. Experimental measurements of A_{CP} in $\pi^-\eta$, $\pi^-\eta'$ modes are in better agreement with QCDF than pQCD. It is known that power corrections such as penguin annihilation in QCDF are often plagued by the endpoint divergence that in turn breaks the factorization theorem. In the pQCD approach, the endpoint singularity is cured by including the parton's transverse momentum. Because of a different treatment of endpoint divergences in penguin-annihilation diagrams, some of the CP puzzles do not occur in the approach of pQCD. For example, pQCD predicts the right sign of CP asymmetries for $\bar{B}^0 \to \pi^0\pi^0$ and $B^- \to \pi^-\eta$ without invoking soft corrections to a_2.

For decays involving η and η', there are two sets of SCET solutions as there exist two different sets of SCET parameters that minimize χ^2. It is clear from Table VII that the predicted signs of CP asymmetries for $K^-\pi^0$, $\pi^0\pi^0$, $\pi^-\eta$ disagree with the data and hence the $\Delta A_{K\pi}$ puzzle is not resolved. Also the predicted CP violation for $\bar{K}^0\pi^0$ and $\bar{K}^{*0}\pi^0$ is of opposite sign to QCDF and pQCD. This is not a surprise because the long-distance charming penguins in SCET mimic the penguin-annihilation effects in QCDF. All the B-CP puzzles occurred in QCDF will also manifest in SCET. (The reader can compare the SCET results of A_{CP} in Tables VII (for $B \to PP$) and XIII and XIV (for $B \to VP$) with the QCDF predictions in the third column of Tables IV and X.) This means that one needs other power

TABLE VIII. Mixing-induced CP violation S_f in $B \to PP$ decays predicted in various approaches. The pQCD results are taken from [50,52–54]. For final states involving η and/or η', there are two solutions with SCET predictions [58]. The parameter $\eta_f = 1$ except for $K_S(\pi^0, \eta, \eta')$ modes where $\eta_f = -1$. Experimental results from *BABAR* (first entry) and Belle (second entry) are listed whenever available. The input values of sin2β used at the time of theoretical calculations are displayed.

Mode sin2β	QCDF (this work) 0.670	pQCD 0.685	SCET 0.725	Expt. [94–98]	Average
$\eta' K_S$	$0.67^{+0.01+0.01}_{-0.01-0.01}$	$0.63^{+0.50}_{-0.91}$	0.706 ± 0.008 0.715 ± 0.010	$0.57 \pm 0.08 \pm 0.02$ $0.64 \pm 0.10 \pm 0.04$	0.59 ± 0.07
ηK_S	$0.79^{+0.04+0.08}_{-0.06-0.06}$	$0.62^{+0.50}_{-0.92}$	0.69 ± 0.16 0.79 ± 0.15		
$\pi^0 K_S$	$0.79^{+0.06+0.04}_{-0.04-0.04}$	$0.74^{+0.02}_{-0.03}$	0.80 ± 0.03	$0.55 \pm 0.20 \pm 0.03$ $0.67 \pm 0.31 \pm 0.08$	0.57 ± 0.17
$\pi^+\pi^-$	$-0.69^{+0.08+0.19}_{-0.10-0.09}$	$-0.42^{+1.00}_{-0.56}$	-0.86 ± 0.10	$-0.68 \pm 0.10 \pm 0.03$ $-0.61 \pm 0.10 \pm 0.04$	-0.65 ± 0.07
$\pi^0\eta$	$0.08^{+0.06+0.19}_{-0.12-0.23}$	$0.067^{+0.005}_{-0.010}$	-0.90 ± 0.24 -0.67 ± 0.82		
$\pi^0\eta'$	$0.16^{+0.05+0.11}_{-0.07-0.14}$	$0.067^{+0.004}_{-0.011}$	-0.96 ± 0.12 -0.60 ± 1.31		
$\eta\eta$	$-0.77^{+0.07+0.12}_{-0.05-0.06}$	$0.535^{+0.004}_{-0.004}$	-0.98 ± 0.11 -0.78 ± 0.31		
$\eta\eta'$	$-0.76^{+0.07+0.06}_{-0.05-0.03}$	$-0.131^{+0.056}_{-0.050}$	-0.82 ± 0.77 -0.71 ± 0.37		
$\eta'\eta'$	$-0.85^{+0.03+0.07}_{-0.02-0.06}$	$0.93^{+0.08}_{-0.12}$	-0.59 ± 1.10 -0.78 ± 0.31		

corrections to resolve the CP puzzles induced by charming penguins. In the current phenomenological analysis of SCET [93], the ratio of $C^{(\prime)}/T^{(\prime)}$ is small and real to the leading order. This constraint should be released.

C. Mixing-induced CP asymmetry

Possible new physics beyond the standard model is being intensively searched via the measurements of time-dependent CP asymmetries in neutral B meson decays into final CP eigenstates defined by

$$\frac{\Gamma(\bar{B}(t) \to f) - \Gamma(B(t) \to f)}{\Gamma(\bar{B}(t) \to f) + \Gamma(B(t) \to f)} = S_f \sin(\Delta m t) - C_f \cos(\Delta m t), \tag{4.19}$$

where Δm is the mass difference of the two neutral B eigenstates, S_f monitors mixing-induced CP asymmetry and A_f measures direct CP violation (note that $C_f = -A_{CP}$). The CP-violating parameters C_f and S_f can be expressed as

$$C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \qquad S_f = \frac{2\,\mathrm{Im}\lambda_f}{1 + |\lambda_f|^2}, \tag{4.20}$$

where

$$\lambda_f = \frac{q_B}{p_B}\frac{A(\bar{B}^0 \to f)}{A(B^0 \to f)}. \tag{4.21}$$

In the standard model $\lambda_f \approx \eta_f e^{-2i\beta}$ for $b \to s$ penguin-dominated or pure penguin modes with $\eta_f = 1\,(-1)$ for final CP-even (odd) states. Therefore, it is anticipated in the standard model that $-\eta_f S_f \approx \sin 2\beta$ and $A_f \approx 0$.

The predictions of S_f of $B \to PP$ decays in various approaches and the experimental measurements from *BABAR* and Belle are summarized in Table VIII. It is clear that $\eta' K_S$ appears theoretically very clean in QCDF and SCET and is close to $\sin 2\beta = 0.672 \pm 0.023$ determined from $b \to c\bar{c}s$ transitions [3]. Note also that the experimental errors on $S_{\eta' K_S}$ are the smallest, and its branching fraction is the largest, making it especially suitable for faster experimental progress in the near future.

Time-dependent CP violation in $\bar{B}^0 \to \pi^0 K_S$ has received a great deal of attention. A correlation between $S_{\pi^0 K_S}$ and $A_{CP}(\pi^0 K_S)$ has been investigated in [99]. Recently, it has been argued that soft corrections to the color-suppressed tree amplitude will reduce the mixing-induced asymmetry $S_{\pi^0 K_S}$ to the level of 0.63 [10]. However, we find that it is the other way around in our case. The asymmetry $S_{\pi^0 K_S}$ is enhanced from 0.76 to $0.79^{+0.06+0.04}_{-0.04-0.04}$ in the presence of power correction effects on a_2. Our result of $S_{\pi^0 K_S}$ is consistent with [7–9] where power corrections were studied.[3] Although this deviates

[3]Since power corrections will affect not only a_2, but also other parameters a_i with $i \neq 2$, we have examined such effects by using $\rho_H \approx 4.9$ and $\phi_H \approx -77°$ [see discussions after Eq. (3.20)] and obtained the same result as before.

TABLE IX. Same as Table VIII except for ΔS_f for penguin-dominated modes. The QCDF results obtained by Beneke [101] are displayed for comparison.

Mode	QCDF (this work) With ρ_C, ϕ_C	W/o ρ_C	QCDF (Beneke)	pQCD	SCET	Expt.	Average
$\eta' K_S$	$0.00^{+0.01}_{-0.01}$	$0.01^{+0.01}_{-0.01}$	$0.01^{+0.01}_{-0.01}$	$-0.06^{+0.50}_{-0.91}$	-0.02 ± 0.01	-0.10 ± 0.08	-0.08 ± 0.07
					-0.01 ± 0.01	-0.03 ± 0.11	
ηK_S	$0.12^{+0.09}_{-0.08}$	$0.12^{+0.04}_{-0.03}$	$0.10^{+0.11}_{-0.07}$	$-0.07^{+0.50}_{-0.92}$	-0.04 ± 0.16		
					0.07 ± 0.15		
$\pi^0 K_S$	$0.12^{+0.07}_{-0.06}$	$0.09^{+0.07}_{-0.06}$	$0.07^{+0.05}_{-0.04}$	$0.06^{+0.02}_{-0.03}$	0.08 ± 0.03	-0.12 ± 0.20	-0.10 ± 0.17
						0.00 ± 0.32	

somewhat from the world average value of 0.57 ± 0.17 [3], it does agree with the Belle measurement of $0.67 \pm 0.31 \pm 0.08$ [96].

In sharp contrast to QCDF and SCET where the theoretical predictions for $S_{\eta' K_S}$ are very clean, the theoretical errors in pQCD predictions for both $S_{\eta' K_S}$ and $S_{\eta K_S}$ arising from uncertainties in the CKM angles α and γ are very large [52]. This issue should be resolved.

For the mixing-induced asymmetry in $B \to \pi^+\pi^-$, we obtain $S_{\pi^+\pi^-} = -0.69^{+0.08+0.19}_{-0.10-0.09}$, in accordance with the world average of -0.65 ± 0.07 [3]. For comparison, the SCET prediction -0.86 ± 0.10 [58] is too large and the theoretical uncertainty of the pQCD result $-0.42^{+1.00}_{-0.56}$ [50] is too large. For $\pi^0\eta^{(\prime)}$ modes, SCET predictions are opposite to QCDF and pQCD in signs. For $\eta\eta'$, the pQCD result is very small compared to QCDF and SCET.

The reader may wonder why the QCDF result $S_{\eta' K_S} \approx 0.67$ presented in this work is smaller than the previous result ≈ 0.74 obtained in [100–102]. This is because the theoretical calculation of S_f depends on the input of the angle β or $\sin 2\beta$. For example, $\sin 2\beta \approx 0.725$ was used in the earlier estimate of S_f around 2005, while a smaller value of 0.670 is used in the present work.[4] Therefore, it is more sensible to consider the difference

$$\Delta S_f \equiv -\eta_f S_f - \sin 2\beta \tag{4.22}$$

for penguin-dominated decays. In the SM, S_f for these decays should be nearly the same as the value measured from the $b \to c\bar{c}s$ decays such as $\bar{B}^0 \to J/\psi K^0$; there is a small deviation *at most* $\mathcal{O}(0.1)$ [104]. In Table VIII we have listed the values of $\sin 2\beta$ used in the theoretical calculations. Writing the decay amplitude in the form

$$M(\bar{B}^0 \to f) = V_{ub}V_{us}^* A_f^u + V_{cb}V_{cs}^* A_f^c \tag{4.23}$$

it is known that to the first order in $r_f \equiv (\lambda_u A_f^u)/(\lambda_c A_f^c)$ [105,106]

$$\Delta S_f = 2|r_f| \cos 2\beta \sin\gamma \cos\delta_f, \tag{4.24}$$

with $\delta_f = \arg(A_f^u/A_f^c)$. Hence, the magnitude of the CP asymmetry difference ΔS_f is governed by the size of A_f^u/A_f^c. In QCDF the dominant contributions to A_f^u/A_f^c are given by [101]

$$\begin{aligned}
\frac{A^u}{A^c}\bigg|_{\eta' K_S} &\sim \frac{[-P^u]-[C]}{[-P^c]} \sim \frac{[-(a_4^u + r_\chi a_6^u)] - [a_2^u R_{\eta' K_S}]}{[-(a_4^c + r_\chi a_6^c)]},\\
\frac{A^u}{A^c}\bigg|_{\eta K_S} &\sim \frac{[P^u]+[C]}{[P^c]} \sim \frac{[-(a_4^u + r_\chi a_6^u)] + [a_2^u R_{\eta K_S}]}{[-(a_4^c + r_\chi a_6^c)]},\\
\frac{A^u}{A^c}\bigg|_{\pi^0 K_S} &\sim \frac{[-P^u]+[C]}{[-P^c]} \sim \frac{[-(a_4^u + r_\chi a_6^u)] + [a_2^u R_{\pi K_S}]}{[-(a_4^c + r_\chi a_6^c)]},
\end{aligned} \tag{4.25}$$

where R's are real and positive ratios of form factors and decay constants and we have followed [101] to denote the complex quantities by square brackets if they have real positive parts. For $\eta' K_S$, $[-P]$ is enhanced because of the constructive interference of various penguin amplitudes. This together with the destructive interference between penguin and color-suppressed tree amplitudes implies the smallness of $\Delta S_{\eta' K_S}$. As explained before, the penguin amplitude of $\bar{B}^0 \to \eta K_S$ is small because of the destructive interference of two penguin amplitudes [see Eq. (4.14)]. This together with the fact that the color-suppressed tree amplitude contributes constructively to A^u/A^c explains why $\Delta S_{\eta K_S}$ is positive and sizable.

Mixing-induced CP asymmetries in various approaches are listed in Table IX where the soft effects due to ρ_C and

[4]The experimental value of $\sin 2\beta$ determined from all B-factory charmonium data is 0.672 ± 0.023 [3]. However, as pointed out by Lunghi and Soni [103], one can use some observables to deduce the value of $\sin 2\beta$: CP-violating parameter ε_K, $\Delta M_s/\Delta M_d$ and V_{cb} from experiment along with the lattice hadronic matrix elements, namely, the kaon B-parameter B_K and the SU(3) breaking ratio ξ_s. A prediction $\sin 2\beta = 0.87 \pm 0.09$ is yielded in the SM. If the ratio $|V_{ub}/V_{cb}|$ is also included as an input, one gets a smaller value 0.75 ± 0.04. The deduced value of $\sin 2\beta$ thus differs from the directly measured value at the 2σ level. If the SM description of CP violation through the CKM paradigm with a single CP-odd phase is correct, then the deduced value of $\sin 2\beta$ should agree with the directly measured value of $\sin 2\beta$ in B-factory experiments.

TABLE X. Same as Table IV except for some selective $B \to VP$ decays with $\rho_C = 0.8$ and $\phi_C = -80°$.

Mode	W/o $\rho_{A,C}$, $\phi_{A,C}$	With ρ_A, ϕ_A	With $\rho_{A,C}$, $\phi_{A,C}$	Expt. [3]
$\mathcal{B}(\bar{B}^0 \to K^- \rho^+)$	$6.5^{+5.4+0.4}_{-2.6-0.4}$	$8.6^{+5.7+7.4}_{-2.8-4.5}$	$8.6^{+5.7+7.4}_{-2.8-4.5}$	$8.6^{+0.9}_{-1.1}$
$\mathcal{B}(\bar{B}^0 \to \bar{K}^0 \rho^0)$	$4.7^{+3.3+0.3}_{-1.7-0.3}$	$5.5^{+3.5+4.3}_{-1.8-2.8}$	$5.4^{+3.3+4.3}_{-1.7-2.8}$	4.7 ± 0.7
$\mathcal{B}(B^- \to \bar{K}^0 \rho^-)$	$5.5^{+6.1+0.7}_{-2.8-0.5}$	$7.8^{+6.3+7.3}_{-2.9-4.4}$	$7.8^{+6.3+7.3}_{-2.9-4.4}$	$8.0^{+1.5}_{-1.4}$
$\mathcal{B}(B^- \to K^- \rho^0)$	$1.9^{+2.5+0.3}_{-1.0-0.2}$	$3.3^{+2.6+2.9}_{-1.1-1.7}$	$3.5^{+2.9+2.9}_{-1.2-1.8}$	$3.81^{+0.48}_{-0.46}$
$\mathcal{B}(\bar{B}^0 \to K^{*-} \pi^+)$	$3.7^{+0.5+0.4}_{-0.5-0.4}$	$9.2^{+1.0+3.7}_{-1.0-3.3}$	$9.2^{+1.0+3.7}_{-1.0-3.3}$	$8.6^{+0.9}_{-1.0}$
$\mathcal{B}(\bar{B}^0 \to \bar{K}^{*0} \pi^0)$	$1.1^{+0.2+0.2}_{-0.2-0.2}$	$3.5^{+0.4+1.7}_{-0.5-1.5}$	$3.5^{+0.4+1.6}_{-0.4-1.4}$	2.4 ± 0.7
$\mathcal{B}(B^- \to \bar{K}^{*0} \pi^-)$	$4.0^{+0.7+0.6}_{-0.9-0.6}$	$10.4^{+1.3+4.3}_{-1.5-3.9}$	$10.4^{+1.3+4.3}_{-1.5-3.9}$	$9.9^{+0.8}_{-0.9}$
$\mathcal{B}(B^- \to K^{*-} \pi^0)$	$3.2^{+0.4+0.3}_{-0.4-0.3}$	$6.8^{+0.7+2.3}_{-0.7-2.2}$	$6.7^{+0.7+2.4}_{-0.7-2.2}$	6.9 ± 2.3
$\mathcal{B}(\bar{B}^0 \to \bar{K}^{*0} \eta)$	$11.0^{+6.9+1.7}_{-3.5-1.0}$	$15.4^{+7.7+9.4}_{-4.0-7.1}$	$15.6^{+7.9+9.4}_{-4.1-7.1}$	15.9 ± 1.0
$\mathcal{B}(\bar{B}^0 \to \omega \bar{K}^0)$	$2.9^{+4.0+0.9}_{-1.6-0.4}$	$3.9^{+4.0+3.3}_{-1.6-2.2}$	$4.1^{+4.2+3.3}_{-1.7-2.2}$	5.0 ± 0.6
$\mathcal{B}(\bar{B}^0 \to \rho^0 \pi^0)$	$0.76^{+0.96+0.66}_{-0.37-0.31}$	$0.58^{+0.88+0.60}_{-0.32-0.22}$	$1.3^{+1.7+1.2}_{-0.6-0.6}$	2.0 ± 0.5 [a]
$\mathcal{B}(B^- \to \rho^- \pi^0)$	$11.6^{+1.2+0.9}_{-0.9-0.5}$	$11.8^{+1.3+1.0}_{-0.9-0.6}$	$11.8^{+1.8+1.4}_{-1.1-1.4}$	$10.9^{+1.4}_{-1.5}$
$\mathcal{B}(B^- \to \rho^0 \pi^-)$	$8.2^{+1.8+1.2}_{-0.9-0.6}$	$8.5^{+1.8+1.2}_{-0.9-0.6}$	$8.7^{+2.7+1.7}_{-1.3-1.4}$	$8.3^{+1.2}_{-1.3}$
$\mathcal{B}(\bar{B}^0 \to \rho^- \pi^+)$	$15.3^{+1.0+0.5}_{-1.5-0.9}$	$15.9^{+1.1+0.9}_{-1.5-1.1}$	$15.9^{+1.1+0.9}_{-1.5-1.1}$	15.7 ± 1.8
$\mathcal{B}(\bar{B}^0 \to \rho^+ \pi^-)$	$8.4^{+0.4+0.3}_{-0.7-0.5}$	$9.2^{+0.4+0.5}_{-0.7-0.7}$	$9.2^{+0.4+0.5}_{-0.7-0.7}$	7.3 ± 1.2
$A_{CP}(\bar{B}^0 \to K^- \rho^+)$	$-1.3^{+0.7+3.8}_{-0.3-3.8}$	$31.9^{+11.5+19.6}_{-11.0-12.7}$	$31.9^{+11.5+19.6}_{-11.0-12.7}$	15 ± 6
$A_{CP}(\bar{B}^0 \to \bar{K}^0 \rho^0)$	$6.8^{+1.1+4.9}_{-1.2-4.9}$	$-5.0^{+3.2+6.0}_{-6.4-4.5}$	$8.7^{+1.2+8.7}_{-1.2-6.8}$	6 ± 20
$A_{CP}(B^- \to \bar{K}^0 \rho^-)$	$0.24^{+0.12+0.08}_{-0.15-0.07}$	$0.27^{+0.19+0.46}_{-0.27-0.17}$	$0.27^{+0.19+0.46}_{-0.27-0.17}$	-12 ± 17
$A_{CP}(B^- \to K^- \rho^0)$	$-8.3^{+3.5+7.0}_{-0.9-7.0}$	$56.5^{+16.1+30.0}_{-18.2-22.8}$	$45.4^{+17.8+31.4}_{-19.4-23.2}$	37 ± 11
$A_{CP}(\bar{B}^0 \to K^{*-} \pi^+)$	$15.6^{+0.9+4.5}_{-0.7-4.7}$	$-12.1^{+0.5+12.6}_{-0.5-16.0}$	$-12.1^{+0.5+12.6}_{-0.5-16.0}$	-18 ± 8
$A_{CP}(\bar{B}^0 \to \bar{K}^{*0} \pi^0)$	$-12.0^{+2.4+11.3}_{-4.6-7.6}$	$-0.87^{+1.71+6.04}_{-0.89-6.79}$	$-10.7^{+1.8+9.1}_{-2.8-6.3}$	-15 ± 12
$A_{CP}(B^- \to \bar{K}^{*0} \pi^-)$	$0.97^{+0.11+0.12}_{-0.07-0.11}$	$0.39^{+0.04+0.10}_{-0.03-0.12}$	$0.39^{+0.04+0.10}_{-0.03-0.12}$	-3.8 ± 4.2
$A_{CP}(B^- \to K^{*-} \pi^0)$	$17.5^{+2.0+6.3}_{-1.3-8.0}$	$-6.7^{+0.7+11.8}_{-1.1-14.0}$	$1.6^{+3.1+11.1}_{-1.7-14.3}$	4 ± 29
$A_{CP}(\bar{B}^0 \to \bar{K}^{*0} \eta)$	$3.0^{+0.4+1.9}_{-0.4-1.8}$	$0.20^{+0.51+2.00}_{-1.00-1.21}$	$3.5^{+0.4+2.7}_{-0.5-2.4}$	19 ± 5
$A_{CP}(\bar{B}^0 \to \omega \bar{K}^0)$	$-5.9^{+1.9+3.4}_{-2.3-4.1}$	$6.6^{+4.7+6.0}_{-3.4-5.3}$	$-4.7^{+1.8+5.5}_{-1.6-5.8}$	32 ± 17
$A_{CP}(\bar{B}^0 \to \rho^0 \pi^0)$	$-2.3^{+2.4+9.9}_{-3.7-9.2}$	$31.5^{+13.3+21.5}_{-12.5-30.9}$	$11.0^{+5.0+23.5}_{-5.7-28.8}$	-30 ± 38
$A_{CP}(B^- \to \rho^- \pi^0)$	$-5.4^{+0.4+2.0}_{-0.3-2.1}$	$16.3^{+1.1+7.1}_{-1.2-10.5}$	$9.7^{+2.1+8.0}_{-3.1-10.3}$	2 ± 11
$A_{CP}(B^- \to \rho^0 \pi^-)$	$6.7^{+0.5+3.5}_{-0.8-3.1}$	$-19.8^{+1.7+12.6}_{-1.2-8.8}$	$-9.8^{+3.4+11.4}_{-2.6-10.2}$	18^{+9}_{-17}
$A_{CP}(\bar{B}^0 \to \rho^- \pi^+)$	$-3.5^{+0.2+1.0}_{-0.2-0.9}$	$4.4^{+0.3+5.8}_{-0.3-6.8}$	$4.4^{+0.3+5.8}_{-0.3-6.8}$	11 ± 6
$A_{CP}(\bar{B}^0 \to \rho^+ \pi^-)$	$0.6^{+0.1+2.2}_{-0.1-2.2}$	$-22.7^{+0.9+8.2}_{-1.1-4.4}$	$-22.7^{+0.9+8.2}_{-1.1-4.4}$	-18 ± 12

[a]If an S factor is included, the average will become 2.0 ± 0.8.

ϕ_C are also displayed. In the QCDF approach, soft corrections to the color-suppressed tree amplitude will enhance $\Delta S_{\pi^0 K_S}$ slightly from $\mathcal{O}(0.09)$ to $\mathcal{O}(0.12)$. It is clear that the QCDF results in the absence of power corrections are consistent with that obtained by Beneke [101], by us [100], and by Buchalla *et al.* [102]. For example, we obtained $S_{\eta' K_S} \approx 0.737$ in 2005 and $S_{\eta' K_S} \approx 0.674$ this time. But the value of $\Delta S_{\eta' K_S}$ remains the same as the value of $\sin 2\beta$ has been changed since 2005.

V. $B \to VP$ DECAYS

Power corrections to a_2 for $B \to VP$ and $B \to VV$ are not the same as that for $B \to PP$ as described by Eq. (4.4). From Table X we see that an enhancement of a_2 is needed to improve the rates of $B \to \rho^0 \pi^0$ and the direct CP asymmetry of $\bar{B}^0 \to \bar{K}^{*0} \eta$. However, it is constrained by the measured rates of $\rho^0 \pi^-$ and $\rho^- \pi^0$ modes. The central values of their branching fractions are already saturated even for vanishing $\rho_C(VP)$. This means that $\rho_C(VP)$ is preferred to be smaller than $\rho_C(PP) = 1.3$. In Table X we show the dependence of the branching fractions and CP asymmetries in $B \to VP$ decays with respect to $\rho_{A,C}$ and $\phi_{A,C}$. The corresponding values of a_2 for $\rho_C = 0.8$ and $\phi_C = -80°$ are

$$a_2(\pi\rho) \approx 0.40 e^{-i51°}, \qquad a_2(\rho\pi) \approx 0.38 e^{-i52°},$$
$$a_2(\rho\bar{K}) \approx 0.36 e^{-i52°}, \qquad a_2(\pi\bar{K}^*) \approx 0.39 e^{-i51°}. \tag{5.1}$$

It is clear from Table X that in the heavy quark limit, the predicted rates for $\bar{B} \to \bar{K}^* \pi$ are too small by a factor of

TABLE XI. Branching fractions (in units of 10^{-6}) of $B \to VP$ decays induced by the $b \to d$ ($\Delta S = 0$) transition. We also cite the experimental data [3,36] and theoretical results given in pQCD [82–85] and in SCET [86].

Mode	QCDF (this work)	pQCD	SCET 1	SCET 2	Expt.
$B^- \to \rho^- \pi^0$	$11.8^{+1.8+1.4}_{-1.1-1.4}$	$6 \sim 9$	$8.9^{+0.3+1.0}_{-0.1-1.0}$	$11.4^{+0.6+1.1}_{-0.6-0.9}$	$10.9^{+1.4}_{-1.5}$
$B^- \to \rho^0 \pi^-$	$8.7^{+2.7+1.7}_{-1.3-1.4}$	$5 \sim 6$	$10.7^{+0.7+1.0}_{-0.7-0.9}$	$7.9^{+0.2+0.8}_{-0.1-0.8}$	$8.3^{+1.2}_{-1.3}$
$\bar{B}^0 \to \rho^\pm \pi^\mp$	$25.1^{+1.5+1.4}_{-2.2-1.8}$	$18 \sim 45$	$13.4^{+0.6+1.2}_{-0.5-1.2}$	$16.8^{+0.5+1.6}_{-0.5-1.5}$	23.0 ± 2.3
$\bar{B}^0 \to \rho^+ \pi^-$	$9.2^{+0.4+0.5}_{-0.7-0.7}$		$5.9^{+0.5+0.5}_{-0.5-0.5}$	$6.6^{+0.2+0.7}_{-0.1-0.7}$	7.3 ± 1.2
$\bar{B}^0 \to \rho^- \pi^+$	$15.9^{+1.1+0.9}_{-1.5-1.1}$		$7.5^{+0.3+0.8}_{-0.1-0.8}$	$10.2^{+0.4+0.9}_{-0.5-0.9}$	15.7 ± 1.8
$\bar{B}^0 \to \rho^0 \pi^0$	$1.3^{+1.7+1.2}_{-0.6-0.6}$	$0.07 \sim 0.11$	$2.5^{+0.2+0.2}_{-0.1-0.2}$	$1.5^{+0.1+0.1}_{-0.1-0.1}$	2.0 ± 0.5
$B^- \to \omega \pi^-$	$6.7^{+2.1+1.3}_{-1.0-1.1}$	$4 \sim 8$	$6.7^{+0.4+0.7}_{-0.3-0.6}$	$8.5^{+0.3+0.8}_{-0.3-0.8}$	6.9 ± 0.5
$\bar{B}^0 \to \omega \pi^0$	$0.01^{+0.02+0.04}_{-0.00-0.01}$	$0.10 \sim 0.28$	$0.0003^{+0.0299+0.0000}_{-0.0000-0.0000}$	$0.015^{+0.024+0.002}_{-0.000-0.002}$	<0.5
$B^- \to K^{*0} K^-$	$0.80^{+0.20+0.31}_{-0.17-0.28}$	$0.32^{+0.12}_{-0.07}$	$0.49^{+0.26+0.09}_{-0.20-0.08}$	$0.51^{+0.18+0.07}_{-0.16-0.06}$	0.68 ± 0.19[a]
$B^- \to K^{*-} K^0$	$0.46^{+0.37+0.42}_{-0.17-0.26}$	$0.21^{+0.14}_{-0.13}$	$0.54^{+0.26+0.10}_{-0.21-0.08}$	$0.51^{+0.21+0.08}_{-0.17-0.07}$	
$\bar{B}^0 \to K^{*+} K^-$	$0.08^{+0.01+0.02}_{-0.01-0.02}$	$0.083^{+0.072}_{-0.067}$			
$\bar{B}^0 \to K^{*-} K^+$	$0.07^{+0.01+0.04}_{-0.01-0.03}$	$0.017^{+0.027}_{-0.011}$			
$\bar{B}^0 \to K^{*0} \bar{K}^0$	$0.70^{+0.18+0.28}_{-0.15-0.25}$	$0.24^{+0.07}_{-0.06}$	$0.45^{+0.24+0.09}_{-0.19-0.07}$	$0.47^{+0.17+0.06}_{-0.14-0.05}$	
$\bar{B}^0 \to \bar{K}^{*0} K^0$	$0.47^{+0.36+0.43}_{-0.17-0.27}$	$0.49^{+0.15}_{-0.09}$	$0.51^{+0.24+0.09}_{-0.20-0.08}$	$0.48^{+0.20+0.07}_{-0.16-0.06}$	<1.9
$B^- \to \phi \pi^-$	≈ 0.043[b]	$0.032^{+0.012}_{-0.014}$	≈ 0.003	≈ 0.003	<0.24
$\bar{B}^0 \to \phi \pi^0$	$0.01^{+0.03+0.02}_{-0.01-0.01}$	$0.0068^{+0.0010}_{-0.0008}$	≈ 0.001	≈ 0.001	<0.28
$B^- \to \rho^- \eta$	$8.3^{+1.0+0.9}_{-0.6-0.9}$	$6.7^{+2.6}_{-1.9}$	$3.9^{+2.0+0.4}_{-1.7-0.4}$	$3.3^{+1.9+0.3}_{-1.6-0.3}$	6.9 ± 1.0
$B^- \to \rho^- \eta'$	$5.6^{+0.9+0.8}_{-0.5-0.7}$	$4.6^{+1.6}_{-1.4}$	$0.37^{+2.46+0.08}_{-0.22-0.07}$	$0.44^{+3.18+0.06}_{-0.20-0.05}$	$9.1^{+3.7}_{-2.8}$
$\bar{B}^0 \to \rho^0 \eta$	$0.10^{+0.02+0.04}_{-0.01-0.03}$	$0.13^{+0.13}_{-0.06}$	$0.04^{+0.20+0.00}_{-0.01-0.00}$	$0.14^{+0.33+0.01}_{-0.13-0.01}$	<1.5
$\bar{B}^0 \to \rho^0 \eta'$	$0.09^{+0.10+0.07}_{-0.04-0.03}$	$0.10^{+0.05}_{-0.05}$	$0.43^{+2.51+0.05}_{-0.12-0.05}$	$1.0^{+3.5+0.1}_{-0.9-0.1}$	<1.3
$\bar{B}^0 \to \omega \eta$	$0.85^{+0.65+0.40}_{-0.26-0.24}$	$0.71^{+0.37}_{-0.28}$	$0.91^{+0.66+0.09}_{-0.49-0.09}$	$1.4^{+0.8+0.1}_{-0.6-0.1}$	$0.94^{+0.36}_{-0.31}$
$\bar{B}^0 \to \omega \eta'$	$0.59^{+0.50+0.33}_{-0.20-0.18}$	$0.55^{+0.31}_{-0.26}$	$0.18^{+1.31+0.04}_{-0.10-0.03}$	$3.1^{+4.9+0.3}_{-2.6-0.3}$	$1.01^{+0.47}_{-0.39}$
$\bar{B}^0 \to \phi \eta$	≈ 0.005[b]	$0.011^{+0.062}_{-0.009}$	≈ 0.0004	≈ 0.0008	<0.5
$\bar{B}^0 \to \phi \eta'$	≈ 0.004	$0.017^{+0.161}_{-0.010}$	≈ 0.0001	≈ 0.0007	<0.5

[a]from the preliminary Belle measurement [87]. [b]due to the ω-ϕ mixing effect.

$2 \sim 3$, while $\mathcal{B}(\bar{B} \to \bar{K}\rho)$ are too small by $(15 \sim 100)\%$ compared with experiment. The rate deficit for penguin-dominated decays can be accounted by the subleading power corrections from penguin annihilation. Soft corrections to a_2 will enhance $\mathcal{B}(B \to \rho^0 \pi^0)$ to the order of 1.3×10^{-6}, while the *BABAR* and Belle results, $(1.4 \pm 0.6 \pm 0.3) \times 10^{-6}$ [107] and $(3.0 \pm 0.5 \pm 0.7) \times 10^{-6}$ [108], respectively, differ in their central values by a factor of 2. Improved measurements are certainly needed for this decay mode.

A. Branching fractions

1. $B \to \rho\pi$, $\omega\pi$

From Table XI it is evident that the calculated $B \to \rho\pi$, $\omega\pi$ rates in QCDF are in good agreement with experiment. The previous QCDF predictions [1] for $B \to \rho\pi$ (except $B^0 \to \pi^0\rho^0$) are too large because of the large form factor $A_0^{B\rho}(0) = 0.37 \pm 0.06$ adopted in [1]. In this work we use the updated sum rule result $A_0^{B\rho}(0) = 0.303 \pm 0.029$ [25]. It appears that there is no updated pQCD calculation for $B \to \rho\pi$ and $B \to \omega\pi$.

2. $B \to (\rho, \omega, \phi)\eta^{(\prime)}$

The relevant decay amplitudes are

$$\begin{aligned}
\sqrt{2}A(B^- \to \rho^- \eta) &\approx A_{\rho\eta_q}[\delta_{pu}(\alpha_2 + \beta_2) + 2\alpha_3^p + \hat{\alpha}_4^p] \\
&\quad + A_{\eta_q\rho}[\delta_{pu}(\alpha_1 + \beta_2) + \hat{\alpha}_4^p], \\
-2A(\bar{B}^0 \to \rho^0 \eta) &\approx A_{\rho\eta_q}[\delta_{pu}(\alpha_2 - \beta_1) + 2\alpha_3^p + \hat{\alpha}_4^p] \\
&\quad + A_{\eta_q\rho}[\delta_{pu}(-\alpha_2 - \beta_1) + \hat{\alpha}_4^p], \\
2A(\bar{B}^0 \to \omega\eta) &\approx A_{\omega\eta_q}[\delta_{pu}(\alpha_2 + \beta_1) + 2\alpha_3^p + \hat{\alpha}_4^p] \\
&\quad + A_{\eta_q\omega}[\delta_{pu}(\alpha_2 + \beta_1) + 2\alpha_3^p + \hat{\alpha}_4^p], \\
\sqrt{2}A(\bar{B}^0 \to \phi\eta) &\approx A_{\eta_q\phi}\alpha_3^p + \sqrt{2}B_{\eta_s\phi}b_4^p + \sqrt{2}B_{\phi\eta_s}b_4^p,
\end{aligned} \tag{5.2}$$

and similar expressions for η'. It is clear that the decays $B^- \to \rho^-\eta^{(\prime)}$ have rates much larger than $\bar{B}^0 \to \rho^0\eta^{(\prime)}$ as the former receive color-allowed tree contributions, while the color-suppressed tree amplitudes in the latter cancel each other. Both QCDF and pQCD lead to the pattern $\Gamma(B^- \to \rho^-\eta) > \Gamma(B^- \to \rho^-\eta')$. This should be tested by more accurate measurements. The SCET prediction of $\mathcal{B}(B^- \to \rho\eta') \sim 0.4 \times 10^{-6}$ is far too small and clearly ruled out by experiment. Since the color-suppressed tree amplitudes in the decay $\bar{B} \to \omega\eta^{(\prime)}$ are added together, one should have $\Gamma(\bar{B}^0 \to \omega\eta^{(\prime)}) > \Gamma(\bar{B}^0 \to \rho^0\eta^{(\prime)})$. It appears that SCET predictions for $(\rho^-, \rho^0, \omega)\eta'$ [86] are at odds with experiment. For example, solution I yields $\Gamma(\bar{B}^0 \to \omega\eta') < \Gamma(\bar{B}^0 \to \rho^0\eta')$ in contradiction to the theoretical expectation and solution II gives $\Gamma(\bar{B}^0 \to \rho^0\eta') > \Gamma(B^- \to \rho^-\eta')$ in disagreement with the data.

The decays $\bar{B}^0 \to \phi\eta^{(\prime)}$ are very suppressed as their amplitudes are governed by $V_{ub}V_{ud}^*(a_3^u - a_5^u)$. For example, we obtain $\mathcal{B}(\bar{B}^0 \to \phi\eta) \approx 10^{-9}$ in the QCDF approach. Since the branching fraction of the $\omega\eta$ mode is of order 10^{-6}, it appears that the ϕ meson can be produced from the decay $\bar{B}^0 \to \omega\eta$ followed by ω-ϕ mixing. This will be possible if ϕ is not a pure $s\bar{s}$ state and contains a tiny $q\bar{q}$ component. Neglecting isospin violation and the admixture with the ρ^0 meson, one can parametrize the ω-ϕ mixing in terms of an angle δ such that the physical ω and ϕ are related to the ideally mixed states $\omega^I \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\phi^I \equiv s\bar{s}$ by

$$\begin{pmatrix} \omega \\ \phi \end{pmatrix} = \begin{pmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} \omega^I \\ \phi^I \end{pmatrix}, \tag{5.3}$$

and the mixing angle is about $|\delta| \sim 3.3°$ [109] (see [110] for the latest determination of δ). Therefore, the production of $\phi\eta$ through ω-ϕ mixing is expected to be

$$\begin{aligned} \mathcal{B}(\bar{B}^0 \to \phi\eta)_{\omega\text{-}\phi\ \text{mixing}} &= \mathcal{B}(\bar{B}^0 \to \omega\eta)\sin^2\delta \\ &\approx 0.85 \times 10^{-6} \times (0.08)^2 \\ &\sim 5.4 \times 10^{-9}. \end{aligned} \tag{5.4}$$

It turns out that the ω-ϕ mixing effect dominates over the short-distance contribution. By the same token, the ω-ϕ mixing effect should also manifest in the decay $B^- \to \phi\pi^-$:

$$\begin{aligned} \mathcal{B}(B^- \to \phi\pi^-)_{\omega\text{-}\phi\ \text{mixing}} &= \mathcal{B}(B^- \to \omega\pi^-)\sin^2\delta \\ &\approx 6.7 \times 10^{-6} \times (0.08)^2 \\ &\sim 4.3 \times 10^{-8}. \end{aligned} \tag{5.5}$$

For this decay, the short-distance contribution is only of order 2×10^{-9}.

3. $B \to K^*\bar{K}$, $K\bar{K}^*$

The decays $B^- \to K^{*-}K^0$, $K^{*0}K^-$ and $\bar{B}^0 \to \bar{K}^{*0}K^0$, $K^{*0}\bar{K}^0$ are governed by $b \to d$ penguin contributions and $\bar{B}^0 \to K^{*+}K^-$, $K^{*-}K^+$ proceed only through weak annihilation. Hence, the last two modes are suppressed relative to the first four decays by 1 order of magnitude. The recent preliminary measurement by Belle [87], $\mathcal{B}(B^- \to K^{*0}K^-) = (0.68 \pm 0.16 \pm 0.10) \times 10^{-6}$, is in agreement with the QCDF prediction (see Table XI).

4. $B \to K^*\pi$, ρK

The relevant decay amplitudes are

$$\begin{aligned} A(\bar{B} \to \rho\bar{K}) &= A_{\rho K}(a_4^c - r_\chi^K a_6^c + \beta_3^c + \cdots), \\ A(\bar{B} \to \pi\bar{K}^*) &= A_{\pi K^*}(a_4^c + r_\chi^{K^*} a_6^c + \beta_3^c + \cdots). \end{aligned} \tag{5.6}$$

Since the chiral factor r_χ^K is of order unity and $r_\chi^{K^*}$ is small, it turns out numerically $\alpha_4^c(\rho K) \sim -\alpha_4^c(\pi K^*)$. Fortunately, $\beta_3^c(\rho K)$ and $\beta_3^c(\pi K^*)$ are also of opposite sign so that penguin annihilation will contribute constructively. As noted before, in order to accommodate the data, penguin annihilation should enhance the rates by $(15 \sim 100)\%$ for ρK modes and by a factor of $2 \sim 3$ for $K^*\pi$ ones. A fit to the $K^*\pi$ and $K\rho$ data including *CP* asymmetries yields $\rho_A(VP) \approx 1.07$, $\phi_A(VP) \approx -70°$, $\rho_A(PV) \approx 0.87$ and $\phi_A(PV) \approx -30°$ as shown in Table III.

The pQCD predictions are too small for the branching fractions of $\bar{K}^{*0}\pi^-$ and $K^{*-}\pi^+$, and too large for ωK^- and $\omega\bar{K}^0$.

5. $B \to \phi K$

A direct use of the parameter set $\rho_A(PV) \approx 0.87$ and $\phi_A(PV) \approx -30°$ gives $\mathcal{B}(B^- \to K^-\phi) \approx 13 \times 10^{-6}$, which is too large compared to the measured value $(8.30 \pm 0.65) \times 10^{-6}$ [3]. This means that penguin-annihilation effects should be smaller for the ϕK case. The values of $\rho_A(K\phi)$ and $\phi_A(K\phi)$ are shown in Table III. It is interesting to notice that a smaller ρ_A for the ϕ meson production also occurs again in *VV* decays.

6. $B \to K^*\eta^{(\prime)}$

In the *PP* sector we learn that $\Gamma(B \to K\eta') \gg \Gamma(B \to K\eta)$. It is the other way around in the *VP* sector, namely, $\Gamma(B \to K^*\eta) \gg \Gamma(B \to K^*\eta')$. This is due to an additional sign difference between $\alpha_4(\eta_q K^*)$ and $\alpha_4(K^*\eta_s)$ as discussed before.

The QCDF prediction for the branching fraction of $B \to K^*\eta'$, of order 1.5×10^{-6}, is smaller compared to pQCD and SCET. The experimental averages quoted in Table XII are dominated by the *BABAR* data [89]. Belle obtained only the upper bounds [90]: $\mathcal{B}(B^- \to K^{*-}\eta') < 2.9 \times 10^{-6}$ and $\mathcal{B}(B^- \to \bar{K}^{*0}\eta') < 2.6 \times 10^{-6}$. Therefore, although our predictions are smaller compared to *BABAR*, they are consistent with Belle. It will be of importance to measure them to discriminate between various model predictions.

B. Direct *CP* asymmetries

1. $A_{CP}(K^*\pi)$ and $A_{CP}(K\rho)$

First of all, *CP* violation for $\bar{K}^{*0}\pi^-$ and $\rho^-\bar{K}^0$ is expected to be very small as they are pure penguin processes

TABLE XII. Branching fractions (in units of 10^{-6}) of $B \to VP$ decays induced by the $b \to s$ ($\Delta S = 1$) transition. We also cite the average of the experimental data [3,36] and theoretical results given in pQCD [55,88] and in SCET [86].

Mode	QCDF (this work)	pQCD	SCET 1	SCET 2	Expt.
$B^- \to K^{*-}\pi^0$	$6.7^{+0.7+2.4}_{-0.7-2.2}$	$4.3^{+5.0}_{-2.2}$	$4.2^{+2.2+0.8}_{-1.7-0.7}$	$6.5^{+1.9+0.7}_{-1.7-0.7}$	6.9 ± 2.3
$B^- \to \bar{K}^{*0}\pi^-$	$10.4^{+1.3+4.3}_{-1.5-3.9}$	$6.0^{+2.8}_{-1.5}$	$8.5^{+4.7+1.7}_{-3.6-1.4}$	$9.9^{+3.5+1.3}_{-3.0-1.1}$	$9.9^{+0.8}_{-0.9}$
$\bar{B}^0 \to \bar{K}^{*0}\pi^0$	$3.5^{+0.4+1.6}_{-0.4-1.4}$	$2.0^{+1.2}_{-0.6}$	$4.6^{+2.3+0.9}_{-1.8-0.7}$	$3.7^{+1.4+0.5}_{-1.2-0.5}$	2.4 ± 0.7
$\bar{B}^0 \to K^{*-}\pi^+$	$9.2^{+1.0+3.7}_{-1.0-3.3}$	$6.0^{+6.8}_{-2.6}$	$8.4^{+4.4+1.6}_{-3.4-1.3}$	$9.5^{+3.2+1.2}_{-2.8-1.1}$	$8.6^{+0.9}_{-1.0}$
$B^- \to \rho^0 K^-$	$3.5^{+2.9+2.9}_{-1.2-1.8}$	$5.1^{+4.1}_{-2.8}$	$6.7^{+2.7+1.0}_{-2.2-0.9}$	$4.6^{+1.8+0.7}_{-1.5-0.6}$	$3.81^{+0.48}_{-0.46}$
$B^- \to \rho^- \bar{K}^0$	$7.8^{+6.3+7.3}_{-2.9-4.4}$	$8.7^{+6.8}_{-4.4}$	$9.3^{+4.7+1.7}_{-3.7-1.4}$	$10.1^{+4.0+1.5}_{-3.3-1.3}$	$8.0^{+1.5}_{-1.4}$
$\bar{B}^0 \to \rho^0 \bar{K}^0$	$5.4^{+3.4+4.3}_{-1.7-2.8}$	$4.8^{+4.3}_{-2.3}$	$3.5^{+2.0+0.7}_{-1.5-0.6}$	$5.8^{+2.1+0.8}_{-1.8-0.7}$	4.7 ± 0.7
$\bar{B}^0 \to \rho^+ K^-$	$8.6^{+5.7+7.4}_{-2.8-4.5}$	$8.8^{+6.8}_{-4.5}$	$9.8^{+4.6+1.7}_{-3.7-1.4}$	$10.2^{+3.8+1.5}_{-3.2-1.2}$	$8.6^{+0.9}_{-1.1}$
$B^- \to \omega K^-$	$4.8^{+4.4+3.5}_{-1.9-2.3}$	$10.6^{+10.4}_{-5.8}$	$5.1^{+2.4+0.9}_{-1.9-0.8}$	$5.9^{+2.1+0.8}_{-1.7-0.7}$	6.7 ± 0.5
$\bar{B}^0 \to \omega \bar{K}^0$	$4.1^{+4.2+3.3}_{-1.7-2.2}$	$9.8^{+8.6}_{-4.9}$	$4.1^{+2.1+0.8}_{-1.7-0.7}$	$4.9^{+1.9+0.7}_{-1.6-0.6}$	5.0 ± 0.6
$B^- \to \phi K^-$	$8.8^{+2.8+4.7}_{-2.7-3.6}$	$7.8^{+5.9}_{-1.8}$	$9.7^{+4.9+1.8}_{-3.9-1.5}$	$8.6^{+3.2+1.2}_{-2.7-1.0}$	8.30 ± 0.65
$\bar{B}^0 \to \phi \bar{K}^0$	$8.1^{+2.6+4.4}_{-2.5-3.3}$	$7.3^{+5.4}_{-1.6}$	$9.1^{+4.6+1.7}_{-3.6-1.4}$	$8.0^{+3.0+1.1}_{-2.5-1.0}$	$8.3^{+1.2}_{-1.0}$
$B^- \to K^{*-}\eta$	$15.7^{+8.5+9.4}_{-4.3-7.1}$	$22.13^{+0.26}_{-0.27}$	$17.9^{+5.5+3.5}_{-5.4-2.9}$	$18.6^{+4.5+2.5}_{-4.8-2.2}$	19.3 ± 1.6
$B^- \to K^{*-}\eta'$	$1.7^{+2.7+4.1}_{-0.4-1.6}$	6.38 ± 0.26	$4.5^{+6.6+0.9}_{-3.9-0.8}$	$4.8^{+5.3+0.8}_{-3.7-0.6}$	$4.9^{+2.1}_{-1.9}$ [a]
$\bar{B}^0 \to \bar{K}^{*0}\eta$	$15.6^{+7.9+9.4}_{-4.1-7.1}$	$22.31^{+0.28}_{-0.29}$	$16.6^{+5.1+3.2}_{-5.0-2.7}$	$16.5^{+4.1+2.3}_{-4.3-2.0}$	15.9 ± 1.0
$\bar{B}^0 \to \bar{K}^{*0}\eta'$	$1.5^{+2.4+3.9}_{-0.4-1.7}$	$3.35^{+0.29}_{-0.27}$	$4.1^{+6.2+0.9}_{-3.6-0.7}$	$4.0^{+4.7+0.7}_{-3.4-0.6}$	3.8 ± 1.2 [b]

[a] This is from the *BABAR* data [89]. Belle obtained an upper limit 2.9×10^{-6} [90].
[b] This is from the *BABAR* data [89]. Belle obtained an upper limit 2.6×10^{-6} [90].

(apart from a W-annihilation contribution). From Table X we see that CP asymmetries for $\rho^0 K^-$, $\rho^+ K^-$, and $K^{*-}\pi^+$ predicted in the heavy quark limit are all wrong in signs when confronted with experiment. For the last two modes, CP asymmetries are governed by the quantity $r_{\rm FM}$ defined in Eq. (4.7) except that PP is replaced by VP or PV. Since $\hat{\alpha}_4^c(\rho K)$ and $\hat{\alpha}_4^c(\pi K^*)$ are of opposite sign, this means that $A_{CP}(\rho^+ K^-)$ and $A_{CP}(K^{*-}\pi^+)$ should have different signs. This is indeed borne out by experiment (see Table XIV). Numerically, we have $\alpha_4^c(\rho K) = 0.041 + 0.001i$, $\hat{\alpha}_4^c(\rho K) = \alpha_4^c(\rho K) + \beta_3^c(\rho K) = 0.045–0.046i$, $\alpha_4^c(\pi K^*) = -0.034 + 0.009i$ and $\hat{\alpha}_4^c(\pi K^*) = -0.066 + 0.013i$. Therefore, one needs the β_3^c terms (i.e. penguin annihilation) to get correct signs for CP violation of above-mentioned three modes. One can check from Eqs. (4.6) and (4.7) that $A_{CP}(\rho^+ K^-)$ is positive, while $A_{CP}(K^{*-}\pi^+)$ is negative.

In order to see the effects of soft corrections to a_2, we consider the following quantities

$$\begin{aligned} \Delta A_{K^*\pi} &\equiv A_{CP}(K^{*-}\pi^0) - A_{CP}(K^{*-}\pi^+) \\ &= 0.036^{+0.002+0.035}_{-0.003-0.045} - 2\sin\gamma\, {\rm Im} r_C(K^*\pi) + \cdots, \\ \Delta A'_{K^*\pi} &\equiv A_{CP}(\bar{K}^{*0}\pi^0) - A_{CP}(\bar{K}^{*0}\pi^-) \\ &= (-0.23^{+0.01+0.01}_{-0.01-0.04})\% + 2\sin\gamma\, {\rm Im} r_C(K^*\pi) + \cdots, \end{aligned} \tag{5.7}$$

defined in analogy to $\Delta A_{K\pi}$ and $\Delta A'_{K\pi}$ with

$$r_C(K^*\pi) = \left|\frac{\lambda_u^{(s)}}{\lambda_c^{(s)}}\right| \frac{f_\pi A_0^{BK^*}(0)}{f_{K^*} F_1^{B\pi}(0)} \frac{\alpha_2(K^*\pi)}{-\alpha_4^c(\pi\bar{K}^*) - \beta_3^c(\pi\bar{K}^*)}. \tag{5.8}$$

The first terms on the right-hand side of Eq. (5.7) come from the interference between QCD and electroweak penguins. We will not consider similar quantities for $K\rho$ modes as the first term there will become large. In other words, as far as CP violation is concerned, $K^*\pi$ mimics $K\pi$ more than $K\rho$. We obtain ${\rm Im} r_C(K^*\pi) = -0.057$ and ${\rm Im} r_C(K\rho) = 0.023$ and predict that $\Delta A_{K^*\pi} = (13.7^{+2.9+3.6}_{-1.4-6.9})\%$ and $\Delta A'_{K^*\pi} = (-11.1^{+1.7+9.1}_{-2.8-6.3})\%$, while it is naively expected that $K^{*-}\pi^0$ and $K^{*-}\pi^+$ have similar CP-violating effects. It will be very important to measure CP asymmetries of these two modes to test our prediction. It is clear from Eqs. (4.6) and (5.7) [see also Tables IV and X] that CP asymmetries of $\bar{K}^{*0}\pi^0$ and $\bar{K}^0\pi^0$ are of order -0.10 and arise dominantly from soft corrections to a_2. As for $A_{CP}(\bar{K}^0\rho^0)$, it is predicted to be ≈ 0.09 (≈ -0.05) with (without) soft corrections to a_2 (cf. Table X).

Power corrections to the color-suppressed tree amplitude is needed to improve the prediction for $A_{CP}(\bar{K}^{*0}\eta)$. The current experimental measurement $A_{CP}(\bar{K}^{*0}\eta) = 0.19 \pm 0.05$ is in better agreement with QCDF than pQCD and SCET.

In the pQCD approach, the predictions for some of the VP modes, e.g. $A_{CP}(K^{*-}\pi^+)$, $A_{CP}(\rho^0 K^-)$ and $A_{CP}(\rho^+ K^-)$ are very large, above 50%. This is because

QCD penguin contributions in these modes are small, and direct CP violation arises from the interference between tree and annihilation diagrams. The strong phase comes mainly from the annihilation diagram in this approach. On the other hand, the predicted $A_{CP}(\bar{K}^{*0}\eta)$ is too small. So far the pQCD results for $A_{CP}(K^*\eta^{(\prime)})$ are quoted from [55] where $m_{qq} = 0.22$ GeV is used. Since the pQCD study of $B \to K\eta^{(\prime)}$ has been carried to the (partial) NLO and a drastic different prediction for $A_{CP}(K^-\eta)$ has been found, it will be crucial to generalize the NLO calculation to the $K^*\eta^{(\prime)}$ sector.

We would like to point out the CP violation of $\bar{B}^0 \to \omega\bar{K}^0$. It is clear from Table X that power correction on a_2 will flip the sign of $A_{CP}(\omega\bar{K}^0)$ to a negative one. The pQCD estimate is similar to the QCDF one. At first sight, it seems that QCDF and pQCD predictions are ruled out by the data $A_{CP}(\omega\bar{K}^0) = 0.032 \pm 0.017$. However, the *BABAR* and Belle measurements $0.52^{+0.22}_{-0.20} \pm 0.03$ [91] and $-0.09 \pm 0.29 \pm 0.06$ [92], respectively, are opposite in sign. Hence, we need to await more accurate experimental studies to test theory predictions.

As for the approach of SCET, the predicted CP asymmetries for the neutral modes $\bar{K}^{*0}\pi^0$, $\rho^0\bar{K}^0$, $\omega\bar{K}^0$, and $\bar{K}^{*0}\eta$ have signs opposite to QCDF and pQCD. Especially, the predicted $A_{CP}(\bar{K}^{*0}\eta)$ is already ruled out by experiment.

2. $A_{CP}(\rho\pi)$

The decay amplitudes of $\bar{B}^0 \to \rho^\pm\pi^\mp$ are given by

$$A(\bar{B}^0 \to \rho^-\pi^+) = A_{\pi\rho}[\delta_{pu}\alpha_1 + \alpha_4^p + \beta_3^p + \cdots],$$
$$A(\bar{B}^0 \to \rho^+\pi^-) = A_{\rho\pi}[\delta_{pu}\alpha_1 + \alpha_4^p + \beta_3^p + \cdots]. \tag{5.9}$$

Since the penguin contribution is small compared to the tree one, its CP asymmetry is approximately given by

$$A_{CP}(\rho^-\pi^+) \approx 2\sin\gamma\,\mathrm{Im}r_{\pi\rho},$$
$$A_{CP}(\rho^+\pi^-) \approx 2\sin\gamma\,\mathrm{Im}r_{\rho\pi}, \tag{5.10}$$

with

$$r_{\pi\rho} = \left|\frac{\lambda_c^{(d)}}{\lambda_u^{(d)}}\right| \frac{\alpha_4^c(\pi\rho) + \beta_3^c(\pi\rho)}{\alpha_1(\pi\rho)},$$
$$r_{\rho\pi} = \left|\frac{\lambda_c^{(d)}}{\lambda_u^{(d)}}\right| \frac{\alpha_4^c(\rho\pi) + \beta_3^c(\rho\pi)}{\alpha_1(\rho\pi)}. \tag{5.11}$$

We obtain the values $\mathrm{Im}r_{\pi\rho} = 0.037$ and $\mathrm{Im}r_{\rho\pi} = -0.134$. Therefore, CP asymmetries for $\rho^+\pi^-$ and $\rho^-\pi^+$ are opposite in signs, and the former is much bigger than the latter. We see from Table XIII that the predicted signs for CP violation of $\rho^+\pi^-$ and $\rho^-\pi^+$ agree with experiment. The $B^- \to \rho^0\pi^-$ decay amplitude reads

TABLE XIII. Same as Table XI except for direct CP asymmetries involving $b \to d$ ($\Delta S = 0$) transitions.

Mode	QCDF (this work)	pQCD	SCET 1	SCET 2	Expt.
$B^- \to \rho^-\pi^0$	$9.7^{+2.1+8.0}_{-3.1-10.3}$	$0 \sim 20$	$15.5^{+16.9+1.6}_{-18.9-1.4}$	$12.3^{+9.4+0.9}_{-10.0-1.1}$	2 ± 11
$B^- \to \rho^0\pi^-$	$-9.8^{+3.4+11.4}_{-2.6-10.2}$	$-20 \sim 0$	$-10.8^{+13.1+0.9}_{-12.7-0.7}$	$-19.2^{+15.5+1.7}_{-13.4-1.9}$	18^{+9}_{-17}
$\bar{B}^0 \to \rho^+\pi^-$	$-22.7^{+0.9+8.2}_{-1.1-4.4}$		$-9.9^{+17.2+0.9}_{-16.7-0.7}$	$-12.4^{+17.6+1.1}_{-15.3-1.2}$	-18 ± 12
$\bar{B}^0 \to \rho^-\pi^+$	$4.4^{+0.3+5.8}_{-0.3-6.8}$		$11.8^{+17.5+1.2}_{-20.0-1.1}$	$10.8^{+9.4+0.9}_{-10.2-1.0}$	11 ± 6
$\bar{B}^0 \to \rho^0\pi^0$	$11.0^{+5.0+23.5}_{-5.7-28.8}$	$-75 \sim 0$	$-0.6^{+21.4+0.1}_{-21.9-0.1}$	$-3.5^{+21.4+0.3}_{-20.3-0.3}$	-30 ± 38
$B^- \to \omega\pi^-$	$-13.2^{+3.2+12.0}_{-2.1-10.7}$	~ 0	$0.5^{+19.1+0.1}_{-19.6-0.0}$	$2.3^{+13.4+0.2}_{-13.2-0.2}$	-4 ± 6
$\bar{B}^0 \to \omega\pi^0$	$-17.0^{+55.4+98.6}_{-22.8-82.3}$	$-20 \sim 75$	$-9.4^{+24.0+1.1}_{-0.0-0.9}$	$39.5^{+79.1+3.4}_{-185.5-3.1}$	
$B^- \to K^{*0}K^-$	$-8.9^{+1.1+2.8}_{-1.1-2.4}$	$-6.9^{+5.6+1.0+9.2+4.0}_{-5.3-0.3-6.5-6.0}$	$-3.6^{+6.1+0.4}_{-5.3-0.4}$	$-4.4^{+4.1+0.2}_{-4.1-0.2}$	
$B^- \to K^{*-}K^0$	$-7.8^{+5.9+4.1}_{-4.1-10.0}$	$6.5^{+7.9+1.1+9.1+2.1}_{-7.3-1.4-7.7-3.9}$	$-1.5^{+2.6+0.1}_{-2.3-0.1}$	$-1.2^{+1.7+0.1}_{-1.7-0.1}$	
$\bar{B}^0 \to K^{*+}K^-$	$-4.7^{+0.1+4.7}_{-0.2-2.7}$				
$\bar{B}^0 \to K^{*-}K^+$	$5.5^{+0.2+7.0}_{-0.2-5.5}$				
$\bar{B}^0 \to K^{*0}\bar{K}^0$	$-13.5^{+1.6+1.4}_{-1.7-2.3}$		$-3.6^{+6.1+0.4}_{-5.3-0.4}$	$-4.4^{+4.1+0.2}_{-4.1-0.2}$	
$\bar{B}^0 \to \bar{K}^{*0}K^0$	$-3.5^{+1.3+0.7}_{-1.7+2.0}$		$-1.5^{+2.6+0.1}_{-2.3-0.1}$	$-1.2^{+1.7+0.1}_{-1.7-0.1}$	
$B^- \to \phi\pi^-$	0	$-8.0^{+0.9+1.5}_{-1.0-0.1}$			
$\bar{B}^0 \to \phi\pi^0$	0	$-6.3^{+0.7+2.5}_{-0.5-2.5}$			
$B^- \to \rho^-\eta$	$-8.5^{+0.4+6.5}_{-0.4-5.3}$	$1.9^{+0.1+0.2+0.1+0.6}_{-0.0-0.3-0.0-0.5}$	$-6.6^{+21.5+0.6}_{-21.3-0.7}$	$-9.1^{+16.7+0.9}_{-15.8-0.8}$	11 ± 11
$B^- \to \rho^-\eta'$	$1.4^{+0.8+14.0}_{-2.2-11.7}$	$-25.0^{+0.4+4.1+0.8+2.1}_{-0.3-1.6-0.7-1.8}$	$-19.8^{+66.5+2.8}_{-37.5-3.1}$	$-21.7^{+135.9+2.1}_{-24.3-1.7}$	4 ± 28
$\bar{B}^0 \to \rho^0\eta$	$86.2^{+3.7+10.4}_{-5.8-21.4}$	$-89.6^{+1.9+13.7+0.7+4.6}_{-0.9-3.9-0.1-9.0}$	$-46.7^{+170.4+2.9}_{-74.3-3.7}$	$33.3^{+66.9+3.1}_{-62.4-2.8}$	
$\bar{B}^0 \to \rho^0\eta'$	$53.5^{+4.5+39.5}_{-7.9-57.6}$	$-75.7^{+5.6+13.1+6.3+12.9}_{-4.8-7.0-4.0-9.9}$	$-51.7^{+103.3+3.4}_{-42.9-3.9}$	$52.2^{+19.9+4.4}_{-80.6-4.1}$	
$\bar{B}^0 \to \omega\eta$	$-44.7^{+13.1+17.7}_{-9.9-11.6}$	$33.5^{+1.0+0.8+5.9+3.9}_{-1.4-4.6-6.8-4.4}$	$-9.4^{+30.7+0.9}_{-30.2-1.0}$	$-9.6^{+17.8+0.9}_{-16.8-0.9}$	
$\bar{B}^0 \to \omega\eta'$	$-41.4^{+2.5+19.5}_{-2.4-14.4}$	$16.0^{+0.1+3.3+2.2+1.7}_{-0.9-3.9-3.2-2.0}$	$-43.0^{+87.5+4.8}_{-38.8-5.1}$	$-27.2^{+18.1+2.4}_{-29.7-2.2}$	
$\bar{B}^0 \to \phi\eta$	0	0			
$\bar{B}^0 \to \phi\eta'$	0	0			

TABLE XIV. Same as Table XII except for direct CP asymmetries (in %) involving $\Delta S = 1$ processes.

Mode	QCDF (this work)	pQCD	SCET 1	SCET 2	Expt.
$B^- \to K^{*-}\pi^0$	$1.6^{+3.1+11.1}_{-1.7-14.4}$	-32^{+21}_{-28}	$-17.8^{+30.3+2.2}_{-24.6-2.0}$	$-12.9^{+12.0+0.8}_{-12.2-0.8}$	4 ± 29
$B^- \to \bar{K}^{*0}\pi^-$	$0.4^{+1.3+4.3}_{-1.6-3.9}$	-1^{+1}_{-0}	0	0	-3.8 ± 4.2
$\bar{B}^0 \to \bar{K}^{*0}\pi^0$	$-10.8^{+1.8+9.1}_{-2.8-6.3}$	-11^{+7}_{-5}	$5.0^{+7.5+0.5}_{-8.4-0.5}$	$5.4^{+4.8+0.4}_{-5.1-0.5}$	-15 ± 12
$\bar{B}^0 \to K^{*-}\pi^+$	$-12.1^{+0.5+12.6}_{-0.5-16.0}$	-60^{+32}_{-19}	$-11.2^{+19.0+1.3}_{-16.2-1.3}$	$-12.2^{+11.4+0.8}_{-11.3-0.8}$	-18 ± 8
$B^- \to \rho^0 K^-$	$45.4^{+17.8+31.4}_{-19.4-23.2}$	71^{+25}_{-35}	$9.2^{+15.2+0.7}_{-16.1-0.7}$	$16.0^{+20.5+1.3}_{-22.4-1.6}$	37 ± 11
$B^- \to \rho^- \bar{K}^0$	$0.3^{+0.2+0.5}_{-0.3-0.2}$	1 ± 1	0	0	-12 ± 17
$\bar{B}^0 \to \rho^0 \bar{K}^0$	$8.7^{+1.2+8.7}_{-1.2-6.8}$	7^{+8}_{-5}	$-6.6^{+11.6+0.8}_{-9.7-0.9}$	$-3.5^{+4.8+0.3}_{-4.8-0.2}$	6 ± 20
$\bar{B}^0 \to \rho^+ K^-$	$31.9^{+11.5+19.6}_{-11.0-12.7}$	64^{+24}_{-30}	$7.1^{+11.2+0.7}_{-12.4-0.7}$	$9.6^{+13.0+0.7}_{-13.5-0.9}$	15 ± 6
$B^- \to \omega K^-$	$22.1^{+13.7+14.0}_{-12.8-13.0}$	32^{+15}_{-17}	$11.6^{+18.2+1.1}_{-20.4-1.1}$	$12.3^{+16.6+0.8}_{-17.3-1.1}$	2 ± 5
$\bar{B}^0 \to \omega \bar{K}^0$	$-4.7^{+1.8+5.5}_{-1.6-5.8}$	-3^{+2}_{-4}	$5.2^{+8.0+0.6}_{-9.2-0.6}$	$3.8^{+5.2+0.3}_{-5.4-0.3}$	32 ± 17[a]
$B^- \to \phi K^-$	$0.6^{+0.1+0.1}_{-0.1-0.1}$	1^{+0}_{-1}	0	0	-1 ± 6
$\bar{B}^0 \to \phi \bar{K}^0$	$0.9^{+0.2+0.2}_{-0.1-0.1}$	3^{+1}_{-2}	0	0	23 ± 15
$B^- \to K^{*-}\eta$	$-9.7^{+3.9+6.2}_{-3.7-7.1}$	$-24.57^{+0.72}_{-0.27}$	$-2.6^{+5.4+0.3}_{-5.5-0.3}$	$-1.9^{+3.4+0.1}_{-3.6-0.1}$	2 ± 6
$B^- \to K^{*-}\eta'$	$65.5^{+10.1+34.2}_{-39.5-50.2}$	$4.60^{+1.16}_{-1.32}$	$2.7^{+27.4+0.4}_{-19.5-0.3}$	$2.6^{+26.7+0.2}_{-32.9-0.2}$	-30^{+37}_{-33}
$\bar{B}^0 \to \bar{K}^{*0}\eta$	$3.5^{+0.4+2.7}_{-0.5-2.4}$	0.57 ± 0.011	$-1.1^{+2.3+0.1}_{-2.4-0.1}$	$-0.7^{+1.2+0.1}_{-1.3-0.0}$	19 ± 5
$\bar{B}^0 \to \bar{K}^{*0}\eta'$	$6.8^{+10.7+33.2}_{-9.2-50.2}$	-1.30 ± 0.08	$9.6^{+8.9+1.3}_{-11.0-1.2}$	$9.9^{+6.2+0.9}_{-4.3-0.9}$	8 ± 25

[a]Note that the measurements of $52^{+22}_{-20} \pm 3$ by *BABAR* [91] and $-9 \pm 29 \pm 6$ by Belle [92] are of opposite sign.

$$A(B^- \to \rho^0\pi^-) = A_{\rho\pi}[\delta_{pu}\alpha_1 + \alpha_4^p + \beta_3^p] + A_{\pi\rho}[\delta_{pu}\alpha_2 - \alpha_4^p - \beta_3^p]. \quad (5.12)$$

As far as the sign is concerned, it suffices to keep terms in the first square bracket on the right-hand side and obtain a negative $A_{CP}(\rho^0\pi^-)$. By the same token, $A_{CP}(\rho^-\pi^0)$ is predicted to be positive.

CP violation of $\bar{B}^0 \to \rho^0\pi^0$ is predicted to be of order 0.11 by QCDF and negative by pQCD and SCET. The

TABLE XV. Mixing-induced CP violation S_f in $\bar{B} \to VP$ decays predicted in various approaches. The pQCD results are taken from [83,88]. There are two solutions with SCET predictions [86]. The parameter $\eta_f = 1$ except for $(\phi, \rho, \omega)K_S$ modes where $\eta_f = -1$. Experimental results from *BABAR* (first entry) and Belle (second entry) are listed whenever available. The input values of sin2β used at the time of theoretical calculations which are needed for the calculation of ΔS_f are displayed.

Decay sin2β	QCDF (this work) 0.670	pQCD 0.687	SCET 0.687	Expt. [94,95,113–118]	Average
ϕK_S	$0.692^{+0.003+0.002}_{-0.000-0.002}$	0.71 ± 0.01	0.69 0.69	$0.26 \pm 0.26 \pm 0.03$ $0.67^{+0.22}_{-0.32}$	$0.44^{+0.17}_{-0.18}$
ωK_S	$0.84^{+0.05+0.04}_{-0.05-0.06}$	$0.84^{+0.03}_{-0.07}$	$0.51^{+0.05+0.02}_{-0.06-0.02}$ $0.80^{+0.02+0.01}_{-0.02-0.01}$	$0.55^{+0.26}_{-0.29} \pm 0.02$ $0.11 \pm 0.46 \pm 0.07$	0.45 ± 0.24
$\rho^0 K_S$	$0.50^{+0.07+0.06}_{-0.14-0.12}$	$0.50^{+0.10}_{-0.06}$	$0.85^{+0.04+0.01}_{-0.05-0.01}$ $0.56^{+0.02+0.01}_{-0.03-0.01}$	$0.35^{+0.26}_{-0.31} \pm 0.06 \pm 0.03$ $0.64^{+0.19}_{-0.25} \pm 0.09 \pm 0.10$	$0.54^{+0.18}_{-0.21}$
$\rho^0\pi^0$	$-0.24^{+0.15+0.20}_{-0.14-0.22}$		$-0.11^{+0.14+0.10}_{-0.14-0.15}$ $-0.19^{+0.14+0.10}_{-0.14-0.15}$	$0.04 \pm 0.44 \pm 0.18$ $0.17 \pm 0.57 \pm 0.35$	0.12 ± 0.38
$\omega\pi^0$	$0.78^{+0.14+0.20}_{-0.20-1.39}$		$-0.87^{+0.44+0.02}_{-0.00-0.01}$ $0.72^{+0.36+0.07}_{-1.54-0.11}$		
$\rho^0\eta$	$0.51^{+0.08+0.19}_{-0.07-0.32}$	$0.23^{+0.30}_{-0.37}$	$0.86^{+0.15+0.03}_{-2.03-0.07}$ $0.29^{+0.36+0.09}_{-0.44-0.15}$		
$\rho^0\eta'$	$0.80^{+0.04+0.24}_{-0.09-0.43}$	$-0.49^{+0.25}_{-0.20}$	$0.79^{+0.20+0.05}_{-1.73-0.09}$ $0.38^{+0.22+0.09}_{-1.24-0.14}$		
$\omega\eta$	$-0.16^{+0.13+0.17}_{-0.13-0.16}$	$0.39^{+0.51}_{-0.66}$	$0.12^{+0.19+0.10}_{-0.20-0.17}$ $-0.16^{+0.14+0.10}_{-0.15-0.15}$		
$\omega\eta'$	$-0.28^{+0.14+0.16}_{-0.13-0.13}$	$0.77^{+0.22}_{-0.53}$	$0.23^{+0.59+0.10}_{-1.10-0.10}$ $-0.27^{+0.17+0.09}_{-0.33-0.14}$		

TABLE XVI. Same as Table XV except for ΔS_f for penguin-dominated modes. The QCDF results obtained by Beneke [101] are included for comparison.

Decay	QCDF (this work) With ρ_C, ϕ_C	W/o ρ_C	QCDF (Beneke)	pQCD	SCET	Expt.	Average
ϕK_S	$0.022^{+0.004}_{-0.002}$	$0.022^{+0.004}_{-0.002}$	$0.02^{+0.01}_{-0.01}$	0.02 ± 0.01	~ 0	-0.43 ± 0.26	$-0.25^{+0.17}_{-0.18}$
					~ 0	$0.02^{+0.22}_{-0.32}$	
ωK_S	$0.17^{+0.06}_{-0.08}$	$0.13^{+0.06}_{-0.04}$	$0.13^{+0.08}_{-0.08}$	$0.15^{+0.03}_{-0.07}$	$-0.18^{+0.05}_{-0.06}$	-0.43 ± 0.26	$-0.14^{+0.26}_{-0.29}$
					$0.11^{+0.02}_{-0.02}$	$0.02^{+0.22}_{-0.32}$	
$\rho^0 K_S$	$-0.17^{+0.09}_{-0.18}$	$-0.11^{+0.07}_{-0.11}$	$-0.08^{+0.08}_{-0.12}$	$-0.19^{+0.10}_{-0.06}$	$0.16^{+0.04}_{-0.05}$	$-0.34^{+0.27}_{-0.31}$	-0.10 ± 0.17
					$-0.13^{+0.02}_{-0.03}$	$-0.05^{+0.23}_{-0.28}$	

current data are $0.10 \pm 0.40 \pm 0.53$ by *BABAR* [111] and $-0.49 \pm 0.36 \pm 0.28$ by Belle [112]. It is of interest to notice that QCDF and pQCD predictions for *CP* asymmetries of $B \to (\rho, \omega)\eta^{(\prime)}$ are opposite in signs.

C. Mixing-induced *CP* asymmetries

Mixing-induced *CP* asymmetries S_f and ΔS_f of $B \to VP$ in various approaches are listed in Tables XV and XVI, respectively. Just as the $\eta' K_S$ mode, ϕK_S is also theoretically very clean as it is a pure penguin process. Although the prediction of $S_{\phi K_S} \sim 0.69$ has some deviation from the world average of $0.44^{+0.17}_{-0.18}$, it does agree with one of the B-factory measurements, namely, $0.67^{+0.22}_{-0.32}$ by Belle [95]. In short, it appears that the theoretical predictions of S_f for several penguin-dominated $B \to PP$, VP decays deviate from the world averages and hence may indicate some new physics effects. However, if we look at the individual measurement made by *BABAR* or Belle, the theory prediction actually agrees with one of the measurements. Hence, in order to uncover new physics effects through the time evolution of *CP* violation, we certainly need more accurate measurements of time-dependent *CP* violation and better theoretical estimates of S_f. This poses a great challenge to both theorists and experimentalists.

The ratio of A^u/A^c for the penguin-dominated decays $(\phi, \omega, \rho^0)K_S$ has the expressions [101]

$$\frac{A^u}{A^c}\bigg|_{\phi K_S} \sim \frac{[-P^u]}{[-P^c]} \sim \frac{[-(a_4^u + r_\chi^\phi a_6^u)]}{[-(a_4^c + r_\chi^\phi a_6^c)]},$$
$$\frac{A^u}{A^c}\bigg|_{\omega K_S} \sim \frac{[P^u]+[C]}{[P^c]} \sim \frac{[(a_4^u - r_\chi^K a_6^u)] + [a_2^u R_{\omega K_S}]}{[(a_4^c - r_\chi^K a_6^c)]},$$
$$\frac{A^u}{A^c}\bigg|_{\rho^0 K_S} \sim \frac{[P^u]-[C]}{[P^c]} \sim \frac{[(a_4^u - r_\chi^K a_6^u)] - [a_2^u R_{\rho K_S}]}{[(a_4^c - r_\chi^K a_6^c)]}. \quad (5.13)$$

As discussed before, the quantity $(a_4^c - r_\chi^K a_6^c)$ in above equations is positive and has a magnitude similar to $|a_4^c|$. Since a_2 is larger than $-a_4^c$, ΔS_f is positive for ωK_S but negative for $\rho^0 K_S$ and both have large magnitude due to the small denominator of ΔS_f. From Table XVI we see that $\Delta S_{\omega K_S} = \mathcal{O}(0.17)$, while $\Delta S_{\rho^0 K_S} = \mathcal{O}(-0.17)$. Effects of soft corrections on them are sizable. For example, $\Delta S_{\rho^0 K_S}$ is shifted from ≈ -0.11 to ≈ -0.17 in the presence of power corrections. This explains why our prediction of $\Delta S_{\rho^0 K_S}$ is substantially different from the Beneke's estimate [101] and our previous calculation [100].

For tree-dominated decays, so far there is only one measurement, namely, $S_{\rho^0\pi^0}$ with a sign opposite to the theoretical predictions of QCDF and SCET.

1. Time-dependent *CP* violation of the $\rho^\pm\pi^\mp$ systems

The study of *CP* violation for $\bar{B}^0 \to \rho^+\pi^-$ and $\rho^-\pi^+$ becomes more complicated as $\rho^\pm\pi^\mp$ are not *CP* eigenstates. The time-dependent *CP* asymmetries are given by

$$\mathcal{A}(t) \equiv \frac{\Gamma(\bar{B}^0(t) \to \rho^\pm\pi^\mp) - \Gamma(B^0(t) \to \rho^\pm\pi^\mp)}{\Gamma(\bar{B}^0(t) \to \rho^\pm\pi^\mp) + \Gamma(B^0(t) \to \rho^\pm\pi^\mp)} = (S \pm \Delta S)\sin(\Delta m t) - (C \pm \Delta C)\cos(\Delta m t), \quad (5.14)$$

where Δm is the mass difference of the two neutral B^0 eigenstates, S is referred to as mixing-induced *CP* asymmetry, and C is the direct *CP* asymmetry, while ΔS and ΔC are *CP*-conserving quantities. Defining

$$A_{+-} \equiv A(B^0 \to \rho^+\pi^-), \qquad A_{-+} \equiv A(B^0 \to \rho^-\pi^+),$$
$$\bar{A}_{-+} \equiv A(\bar{B}^0 \to \rho^-\pi^+), \qquad \bar{A}_{+-} \equiv A(\bar{B}^0 \to \rho^+\pi^-), \quad (5.15)$$

and

$$\lambda_{+-} = \frac{q_B}{p_B}\frac{\bar{A}_{+-}}{A_{+-}}, \qquad \lambda_{-+} = \frac{q_B}{p_B}\frac{\bar{A}_{-+}}{A_{-+}}, \quad (5.16)$$

with $q_B/p_B \approx e^{-2i\beta}$, we have

$$C + \Delta C = \frac{1 - |\lambda_{+-}|^2}{1 + |\lambda_{+-}|^2} = \frac{|A_{+-}|^2 - |\bar{A}_{+-}|^2}{|A_{+-}|^2 + |\bar{A}_{+-}|^2},$$
$$C - \Delta C = \frac{1 - |\lambda_{-+}|^2}{1 + |\lambda_{-+}|^2} = \frac{|A_{-+}|^2 - |\bar{A}_{-+}|^2}{|A_{-+}|^2 + |\bar{A}_{-+}|^2}, \quad (5.17)$$

and

TABLE XVII. Various CP-violating parameters in the decays $\bar{B}^0 \to \rho^\pm\pi^\mp$. SCET results are quoted from [86]. Experimental results are taken from [111,112] and the world average from [3].

Parameter	QCDF (this work)	SCET 1	SCET 2	Expt.
$\mathcal{A}_{\rho\pi}$	$-0.11^{+0.00+0.07}_{-0.00-0.05}$	$-0.12^{+0.04+0.04}_{-0.05-0.03}$	$-0.21^{+0.03+0.02}_{-0.02-0.03}$	-0.13 ± 0.04
C	$0.09^{+0.00+0.05}_{-0.00-0.07}$	$-0.01^{+0.13+0.00}_{-0.12-0.00}$	$0.01^{+0.09+0.00}_{-0.10-0.00}$	0.01 ± 0.07
S	$-0.04^{+0.01+0.10}_{-0.01-0.09}$	$-0.11^{+0.07+0.08}_{-0.08-0.13}$	$-0.01^{+0.06+0.08}_{-0.07-0.14}$	0.01 ± 0.09
ΔC	$0.26^{+0.02+0.02}_{-0.02-0.02}$	$0.11^{+0.12+0.01}_{-0.13-0.01}$	$0.12^{+0.09+0.01}_{-0.10-0.01}$	0.37 ± 0.08
ΔS	$-0.02^{+0.00+0.03}_{-0.00-0.02}$	$-0.47^{+0.08+0.05}_{-0.06-0.04}$	$0.43^{+0.05+0.03}_{-0.07-0.03}$	-0.04 ± 0.10

$$S + \Delta S \equiv \frac{2\,\mathrm{Im}\lambda_{+-}}{1+|\lambda_{+-}|^2} = \frac{2\,\mathrm{Im}(e^{2i\beta}\bar{A}_{+-}A^*_{+-})}{|A_{+-}|^2+|\bar{A}_{+-}|^2},$$
$$S - \Delta S \equiv \frac{2\,\mathrm{Im}\lambda_{-+}}{1+|\lambda_{-+}|^2} = \frac{2\,\mathrm{Im}(e^{2i\beta}\bar{A}_{-+}A^*_{-+})}{|A_{-+}|^2+|\bar{A}_{-+}|^2}. \tag{5.18}$$

Hence, we see that ΔS describes the strong phase difference between the amplitudes contributing to $B^0 \to \rho^\pm\pi^\mp$, and ΔC measures the asymmetry between $\Gamma(B^0 \to \rho^+\pi^-) + \Gamma(\bar{B}^0 \to \rho^-\pi^+)$ and $\Gamma(B^0 \to \rho^-\pi^+) + \Gamma(\bar{B}^0 \to \rho^+\pi^-)$.

Next consider the time- and flavor-integrated charge asymmetry

$$\mathcal{A}_{\rho\pi} \equiv \frac{|A_{+-}|^2+|\bar{A}_{+-}|^2-|A_{-+}|^2-|\bar{A}_{-+}|^2}{|A_{+-}|^2+|\bar{A}_{+-}|^2+|A_{-+}|^2+|\bar{A}_{-+}|^2}. \tag{5.19}$$

Then, following [31] one can transform the experimentally motivated CP parameters $\mathcal{A}_{\rho\pi}$ and $C_{\rho\pi}$ into the physically motivated choices

$$A_{CP}(\rho^+\pi^-) \equiv \frac{|\kappa^{-+}|^2-1}{|\kappa^{-+}|^2+1},$$
$$A_{CP}(\rho^-\pi^+) \equiv \frac{|\kappa^{+-}|^2-1}{|\kappa^{+-}|^2+1}, \tag{5.20}$$

with

$$\kappa^{+-} = \frac{q_B}{p_B}\frac{\bar{A}_{-+}}{A_{+-}}, \qquad \kappa^{-+} = \frac{q_B}{p_B}\frac{\bar{A}_{+-}}{A_{-+}}. \tag{5.21}$$

Hence,

$$A_{CP}(\rho^+\pi^-) = \frac{\Gamma(\bar{B}^0\to\rho^+\pi^-)-\Gamma(B^0\to\rho^-\pi^+)}{\Gamma(\bar{B}^0\to\rho^+\pi^-)+\Gamma(B^0\to\rho^-\pi^+)} = \frac{\mathcal{A}_{\rho\pi}-C_{\rho\pi}-\mathcal{A}_{\rho\pi}\Delta C_{\rho\pi}}{1-\Delta C_{\rho\pi}-\mathcal{A}_{\rho\pi}C_{\rho\pi}},$$
$$A_{CP}(\rho^-\pi^+) = \frac{\Gamma(\bar{B}^0\to\rho^-\pi^+)-\Gamma(B^0\to\rho^+\pi^-)}{\Gamma(\bar{B}^0\to\rho^-\pi^+)+\Gamma(B^0\to\rho^+\pi^-)} = -\frac{\mathcal{A}_{\rho\pi}+C_{\rho\pi}+\mathcal{A}_{\rho\pi}\Delta C_{\rho\pi}}{1+\Delta C_{\rho\pi}+\mathcal{A}_{\rho\pi}C_{\rho\pi}}. \tag{5.22}$$

Therefore, direct CP asymmetries $A_{CP}(\rho^+\pi^-)$ and $A_{CP}(\rho^-\pi^+)$ are determined from the above two equations and shown in Tables X and XIII. Results for various CP-violating parameters in the decays $\bar{B}^0 \to \rho^\pm\pi^\mp$ are displayed in Table XVII. The CP-violating quantity $\mathcal{A}_{\rho\pi}$ with the experimental value -0.13 ± 0.04 is different from zero by 3.3σ deviations. The QCDF prediction is in good agreement with experiment.

VI. $B \to VV$ DECAYS

A. Branching fractions

In two-body decays $B_{u,d} \to PP$, VP, VV, we have the pattern $VV > PV > VP > PP$ for the branching fractions of tree-dominated modes and $PP > PV \sim VV > VP$ for penguin-dominated ones, where $B \to VP(PV)$ here means that the factorizable amplitude is given by $\langle V(P)|J_\mu|B\rangle \times \langle P(V)|J^\mu|0\rangle$. For example,

$$\mathcal{B}(B^-\to\rho^-\rho^0) > \mathcal{B}(B^-\to\rho^-\pi^0) > \mathcal{B}(B^-\to\rho^0\pi^-) > \mathcal{B}(B^-\to\pi^-\pi^0),$$
$$\mathcal{B}(B^-\to\bar{K}^0\pi^-) > \mathcal{B}(B^-\to\bar{K}^{*0}\pi^-) \sim \mathcal{B}(B^-\to\bar{K}^{*0}\rho^-) > \mathcal{B}(B^-\to\bar{K}^0\rho^-), \tag{6.1}$$

for tree- and penguin-dominated B^- decays, respectively. The first hierarchy is due to the pattern of decay constants $f_V > f_P$ and the second hierarchy stems from the fact that the penguin amplitudes are proportional to $a_4 + r_\chi^P a_6$, $a_4 + r_\chi^V a_6$, $a_4 - r_\chi^P a_6$ $a_4 + r_\chi^V a_6$, respectively, for $B \to PP$, PV, VP, VV. Recall that $r_\chi^P \sim \mathcal{O}(1) \gg r_\chi^V$. There are a few exceptions to the above hierarchy patterns. For example, $\mathcal{B}(B^0\to\rho^0\rho^0) \lesssim \mathcal{B}(B^0\to\pi^0\pi^0)$ is observed. This is ascribed to the fact that the latter receives a large soft correction to a_2, while the former does not.

There exist three QCDF calculations of $B \to VV$ [21,22,24]. However, only the longitudinal polarization states of $B \to VV$ were considered in [22]. The analyses in [21,24] differ mainly in (i) the values of the parameters ρ_A and ϕ_A and (ii) the treatment of penguin-annihilation contributions characterized by the parameters β_i [see Eq. (2.10)] for penguin-dominated VV modes. Beneke, Rohrer, and Yang (BRY) applied the values $\rho_A(K^*\phi) = 0.6$ and $\phi_A(K^*\phi) = -40°$ obtained from a fit to the data of $B \to K^*\phi$ to study other $\bar{B} \to VV$ decays. However, as pointed out in [24], the parameters $\rho_A(K^*\rho) \approx 0.78$ and $\phi_A(K^*\rho) \approx -43°$ fit to the data of $B \to K^*\rho$ decays are slightly different from the ones $\rho_A(K^*\phi)$ and $\phi_A(K^*\phi)$.

TABLE XVIII. CP-averaged branching fractions (in units of 10^{-6}) and polarization fractions for $\bar{B} \to VV$ decays. For QCDF, the annihilation parameters are specified to be $\rho_A = 0.78$ and $\phi_A = -43^\circ$ for $K^*\rho$, $K^*\bar{K}^*$ and $\rho_A = 0.65$ and $\phi_A = -53^\circ$ for $K^*\phi$ and $K^*\omega$ by default. The world averages of experimental results are taken from [3]. The pQCD results are taken from [119–122]. There are two distinct pQCD predictions for the branching fractions and longitudinal polarization fractions of $B \to K^*(\rho, \phi, \omega)$ decays, depending on the type of wave functions. Numbers in parentheses are for asymptotic wave functions. Estimates of uncertainties are not available in many of pQCD predictions.

	$\mathcal{B}$			f_L		
Decay	QCDF	pQCD	Expt.	QCDF	pQCD	Expt.
$B^- \to \rho^-\rho^0$	$20.0^{+4.0+2.0}_{-1.9-0.9}$	$16.0^{+15.0\,\mathrm{a}}_{-8.1}$	$24.0^{+1.9}_{-2.0}$	$0.96^{+0.01+0.02}_{-0.01-0.02}$		0.950 ± 0.016
$\bar{B}^0 \to \rho^+\rho^-$	$25.5^{+1.5+2.4}_{-2.6-1.5}$	$25.3^{+25.3\,\mathrm{a}}_{-13.8}$	$24.2^{+3.1}_{-3.2}$	$0.92^{+0.01+0.01}_{-0.02-0.02}$		$0.978^{+0.025}_{-0.022}$
$\bar{B}^0 \to \rho^0\rho^0$	$0.9^{+1.5+1.1}_{-0.4-0.2}$	$0.92^{+1.10\,\mathrm{a}}_{-0.56}$	$0.73^{+0.27}_{-0.28}$	$0.92^{+0.03+0.06}_{-0.04-0.37}$	0.78	$0.75^{+0.12}_{-0.15}$
$B^- \to \rho^-\omega$	$16.9^{+3.2+1.7}_{-1.6-0.9}$	$19 \pm 2 \pm 1$	15.9 ± 2.1	$0.96^{+0.01+0.02}_{-0.01-0.03}$	0.97	0.90 ± 0.06
$\bar{B}^0 \to \rho^0\omega$	$0.08^{+0.02+0.36}_{-0.02-0.00}$	$1.9 \pm 0.2 \pm 0.2$	<1.5	$0.52^{+0.11+0.50}_{-0.25-0.36}$	0.87	
$\bar{B}^0 \to \omega\omega$	$0.7^{+0.9+0.7}_{-0.3-0.2}$	$1.2 \pm 0.2 \pm 0.2$	<4.0	$0.94^{+0.01+0.04}_{-0.01-0.20}$	0.82	
$B^- \to K^{*0}K^{*-}$	$0.6^{+0.1+0.3}_{-0.1-0.3}$	$0.48^{+0.12}_{-0.08}$	1.2 ± 0.5	$0.45^{+0.02+0.55}_{-0.04-0.38}$	0.82	$0.75^{+0.16}_{-0.26}$
$\bar{B}^0 \to K^{*-}K^{*+}$	$0.1^{+0.0+0.1}_{-0.0-0.1}$	$0.064^{+0.005}_{-0.010}$	<2.0	≈ 1	0.99	
$\bar{B}^0 \to K^{*0}\bar{K}^{*0}$	$0.6^{+0.1+0.2}_{-0.1-0.3}$	$0.35^{+0.13}_{-0.07}$	$1.28^{+0.37\,\mathrm{b}}_{-0.32}$	$0.52^{+0.04+0.48}_{-0.07-0.48}$	0.78	$0.80^{+0.12}_{-0.13}$
$B^- \to \bar{K}^{*0}\rho^{-\,\mathrm{c}}$	$9.2^{+1.2+3.6}_{-1.1-5.4}$	17(13)	9.2 ± 1.5	$0.48^{+0.03+0.52}_{-0.04-0.40}$	0.82(0.76)	0.48 ± 0.08
$B^- \to K^{*-}\rho^0$	$5.5^{+0.6+1.3}_{-0.5-2.5}$	9.0 (6.4)	<6.1	$0.67^{+0.02+0.31}_{-0.03-0.48}$	0.85 (0.78)	$0.96^{+0.06\,\mathrm{d}}_{-0.16}$
$\bar{B}^0 \to K^{*-}\rho^+$	$8.9^{+1.1+4.8}_{-1.0-5.5}$	13 (9.8)	<12	$0.53^{+0.02+0.45}_{-0.03-0.32}$	0.78 (0.71)	
$\bar{B}^0 \to \bar{K}^{*0}\rho^0$	$4.6^{+0.6+3.5}_{-0.5-3.5}$	5.9 (4.7)	3.4 ± 1.0	$0.39^{+0.00+0.60}_{-0.00-0.31}$	0.74 (0.68)	0.57 ± 0.12
$B^- \to K^{*-}\phi^{\mathrm{e}}$	$10.0^{+1.4+12.3}_{-1.3-6.1}$	f	10.0 ± 1.1	$0.49^{+0.04+0.51}_{-0.07-0.42}$	f	0.50 ± 0.05
$\bar{B}^0 \to \bar{K}^{*0}\phi$	$9.5^{+1.3+11.9}_{-1.2-5.9}$	f	9.8 ± 0.7	$0.50^{+0.04+0.51}_{-0.06-0.43}$	f	0.480 ± 0.030
$B^- \to K^{*-}\omega$	$3.0^{+0.4+2.5}_{-0.3-1.5}$	7.9 (5.5)	<7.4	$0.67^{+0.03+0.32}_{-0.04-0.39}$	0.81(0.73)	0.41 ± 0.19
$\bar{B}^0 \to \bar{K}^{*0}\omega$	$2.5^{+0.4+2.5}_{-0.4-1.5}$	9.6 (6.6)	2.0 ± 0.5	$0.58^{+0.07+0.43}_{-0.10-0.14}$	0.82(0.74)	0.70 ± 0.13

[a]There exist several pQCD calculations for $\rho\rho$ modes [120,122,123]. Here, we cite the NLO results from [122].
[b]This is from the *BABAR* data [124]. The Belle's new measurement yields $(0.3 \pm 0.3 \pm 0.1) \times 10^{-6}$ [87].
[c]This mode is employed as an input for extracting the parameters ρ_A and ϕ_A for $B \to K^*\rho$ decays.
[d]A recent *BABAR* measurement gives $f_L(K^{*-}\rho^0) = 0.9 \pm 0.2$ [125], but it has only 2.5σ significance.
[e]This mode is employed as an input for extracting the parameters ρ_A and ϕ_A for $B \to K^*\phi$ decays.
[f]See footnote 6 in Sec. VI B.

Indeed, we have noticed before that phenomenologically penguin annihilation should contribute less to ϕK than ρK and πK^*. This explains why the $K^*\rho$ branching fractions obtained by BRY are systematically below the measurements. Second, as noticed in [24], there are sign errors in the expressions of the annihilation terms $A_3^{f,0}$ and $A_3^{i,0}$ obtained by BRY. As a consequence, BRY claimed (wrongly) that the longitudinal penguin-annihilation amplitude β_3^0 is strongly suppressed, while the β_3^- term receives a sizable penguin-annihilation contribution. This will affect the decay rates and longitudinal polarization fractions in some of $B \to K^*\rho$ modes, as discussed in details in [24].

In Table XVIII, QCDF results are taken from [24] except that (i) a new channel $\bar{B}^0 \to \omega\omega$ is added, and (ii) branching fractions and f_L for $B \to (\rho, K^*)\omega$ decays are updated.[5] We see that the overall agreement between QCDF and experiment is excellent. In QCDF, the decay $\bar{B}^0 \to \omega\rho^0$ has a very small rate

$$-2A(\bar{B}^0 \to \omega\rho^0) \approx A_{\rho\omega}[\delta_{pu}(\alpha_2 - \beta_1) + 2\hat{\alpha}_3^p + \hat{\alpha}_4^p] + A_{\omega\rho}[\delta_{pu}(-\alpha_2 - \beta_1) + \hat{\alpha}_4^p], \quad (6.2)$$

due to a near cancellation of the color-suppressed tree amplitudes. In view of this, it seems rather peculiar that the rate of $\bar{B}^0 \to \rho^0\omega$ predicted by pQCD [120] is larger than QCDF by a factor of 20 and exceeds the current experimental upper bound. Likewise, $\mathcal{B}(B^- \to \bar{K}^{*0}\rho^-)$ obtained by pQCD is slightly too large.

We notice that the calculated $B^0 \to \rho^0\rho^0$ rate in QCDF is $\mathcal{B}(B^0 \to \rho^0\rho^0) = (0.88^{+1.46+1.06}_{-0.41-0.20}) \times 10^{-6}$ for $\rho_C = 0$ [24], while *BABAR* and Belle obtained $(0.92 \pm 0.32 \pm 0.14) \times 10^{-6}$ [126] and $(0.4 \pm 0.4^{+0.2}_{-0.3}) \times 10^{-6}$ [127], respectively. Therefore, soft corrections to a_2, i.e. $\rho_C(VV)$ should be very small for $B^0 \to \rho^0\rho^0$. Consequently, a pattern follows: Power corrections to a_2 are large for PP modes, moderate for VP ones, and very small for VV cases. This is consistent with the observation made in [9] that soft power correction dominance is much larger for PP

[5]The $B \to \omega$ transition form factors were mistakenly treated to be the same as that of $B \to \rho$ ones in the computer code of [24]. Here, we use the light-cone sum rule results from [32] for $B \to \omega$ form factors.

than VP and VV final states. It has been argued that this has to do with the special nature of the pion, which is a $q\bar{q}$ bound state on the one hand and a nearly massless Nambu-Goldstone boson on the other hand [9]. The two seemingly distinct pictures of the pion can be reconciled by considering a soft cloud of higher Fock states surrounding the bound valence quarks. From the FSI point of view, since $B \to \rho^+\rho^-$ has a rate much larger than $B \to \pi^+\pi^-$, it is natural to expect that $B \to \pi^0\pi^0$ receives a large enhancement from the weak decay $B \to \rho^+\rho^-$ followed by the rescattering of $\rho^+\rho^-$ to $\pi^0\pi^0$ through the exchange of the ρ particle. Likewise, it is anticipated that $B \to \rho^0\rho^0$ will receive a large enhancement via isospin final-state interactions from $B \to \rho^+\rho^-$. The fact that the branching fraction of this mode is rather small and is consistent with the theory prediction implies that the isospin phase difference of δ_0^ρ and δ_2^ρ and the final-state interaction must be negligible [128].

Both $\bar{B}^0 \to \bar{K}^{*0}K^{*0}$ and $B^- \to K^{*0}K^{*-}$ are $b \to d$ penguin-dominated decays, while $\bar{B}^0 \to K^{*-}K^{*+}$ proceeds only through weak annihilation. Hence, their branching ratios are expected to be small, of order $\lesssim 10^{-6}$. However, the predicted rates for $\bar{K}^{*0}K^{*0}$ and $K^{*0}K^{*-}$ modes are slightly smaller than the data. Note that a new Belle measurement of $\mathcal{B}(\bar{B}^0 \to K^{*0}\bar{K}^{*0}) = (0.3 \pm 0.3 \pm 0.1) \times 10^{-6} < 0.8 \times 10^{-6}$ [87] is smaller than the *BABAR* result $\mathcal{B}(\bar{B}^0 \to K^{*0}\bar{K}^{*0}) = (1.28^{+0.37}_{-0.32}) \times 10^{-6}$ [124]. Hence, the experimental issue with $B \to K^*\bar{K}^*$ decays needs to be resolved.

B. Polarization fractions

For charmless $\bar{B} \to VV$ decays, it is naively expected that the helicity amplitudes $\bar{\mathcal{A}}_h$ (helicities $h = 0, -, +$) for both tree- and penguin-dominated $\bar{B} \to VV$ decays respect the hierarchy pattern

$$\bar{\mathcal{A}}_0 : \bar{\mathcal{A}}_- : \bar{\mathcal{A}}_+ = 1 : \left(\frac{\Lambda_{\rm QCD}}{m_b}\right) : \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2. \qquad (6.3)$$

Hence, they are dominated by the longitudinal polarization states and satisfy the scaling law, namely, [129],

$$f_T \equiv 1 - f_L = \mathcal{O}\left(\frac{m_V^2}{m_B^2}\right), \qquad \frac{f_\perp}{f_\parallel} = 1 + \mathcal{O}\left(\frac{m_V}{m_B}\right), \qquad (6.4)$$

with f_L, $f_\perp$, $f_\parallel$, and f_T being the longitudinal, perpendicular, parallel and transverse polarization fractions, respectively, defined as

$$f_\alpha \equiv \frac{\Gamma_\alpha}{\Gamma} = \frac{|\bar{\mathcal{A}}_\alpha|^2}{|\bar{\mathcal{A}}_0|^2 + |\bar{\mathcal{A}}_\parallel|^2 + |\bar{\mathcal{A}}_\perp|^2}, \qquad (6.5)$$

with $\alpha = L, \parallel, \perp$. In sharp contrast to the $\rho\rho$ case, the large fraction of transverse polarization of order 0.5 observed in $\bar{B} \to \bar{K}^*\phi$ and $\bar{B} \to \bar{K}^*\rho$ decays at B factories is thus a surprise and poses an interesting challenge for any theoretical interpretation. Therefore, in order to obtain a large transverse polarization in $\bar{B} \to \bar{K}^*\phi$, $\bar{K}^*\rho$, this scaling law must be circumvented in one way or another. Various mechanisms such as sizable penguin-induced annihilation contributions [129], final-state interactions [43,130], form-factor tuning [131] and new physics [132–135] have been proposed for solving the $\bar{B} \to VV$ polarization puzzle.

As pointed out by Yang and one of us (H. Y. C.) [24], in the presence of NLO nonfactorizable corrections, e.g. vertex, penguin and hard spectator-scattering contributions, effective Wilson coefficients a_i^h are helicity dependent. Although the factorizable helicity amplitudes X^0, X^-, and X^+ defined by Eq. (2.4) respect the scaling law (6.3) with $\Lambda_{\rm QCD}/m_b$ replaced by $2m_V/m_B$ for the light vector meson production, one needs to consider the effects of helicity-dependent Wilson coefficients: $\mathcal{A}^-/\mathcal{A}^0 = f(a_i^-)X^-/[f(a_i^0)X^0]$. For some penguin-dominated modes, the constructive (destructive) interference in the negative-helicity (longitudinal-helicity) amplitude of the $\bar{B} \to VV$ decay will render $f(a_i^-) \gg f(a_i^0)$ so that $\mathcal{A}^-$ is comparable to $\mathcal{A}^0$ and the transverse polarization is enhanced. For example, $f_L(\bar{K}^{*0}\rho^0) \sim 0.91$ is predicted in the absence of NLO corrections. When NLO effects are turned on, their corrections on a_i^- will render the negative-helicity amplitude $\mathcal{A}^-(\bar{B}^0 \to \bar{K}^{*0}\rho^0)$ comparable to the longitudinal one $\mathcal{A}^0(\bar{B}^0 \to \bar{K}^{*0}\rho^0)$ so that even at the short-distance level, f_L for $\bar{B}^0 \to \bar{K}^{*0}\rho^0$ can be as low as 50%. However, this does not mean that the polarization anomaly is resolved. This is because the calculations based on naive factorization often predict too small rates for penguin-dominated $\bar{B} \to VV$ decays, e.g. $\bar{B} \to \bar{K}^*\phi$ and $\bar{B} \to \bar{K}^*\rho$, by a factor of $2 \sim 3$. Obviously, it does not make sense to compare theory with experiment for $f_{L,T}$ as the definition of polarization fractions depends on the partial rate and hence the prediction can be easily off by a factor of $2 \sim 3$. Thus, the first important task is to have some mechanism to bring up the rates. While the QCD factorization and pQCD [136] approaches rely on penguin annihilation, soft-collinear effective theory invokes charming penguin [17] and the final-state interaction model considers final-state rescattering of intermediate charm states [43,130,137]. A nice feature of the $(S-P)(S+P)$ penguin annihilation is that it contributes to $\mathcal{A}^0$ and $\mathcal{A}^-$ with similar amount. This together with the NLO corrections will lead to $f_L \sim 0.5$ for penguin-dominated VV modes. Hence, within the framework of QCDF we shall assume weak annihilation to account for the discrepancy between theory and experiment, and fit the existing data of branching fractions and f_L simultaneously by adjusting the parameters ρ_A and ϕ_A.

For the longitudinal fractions in $\bar{B} \to \bar{K}^*\rho$ decays, we have the pattern (see also [21])

$$f_L(K^{*-}\rho^0) > f_L(K^{*-}\rho^+) > f_L(\bar{K}^{*0}\rho^-) > f_L(\bar{K}^{*0}\rho^0). \tag{6.6}$$

Note that the quoted experimental value $f_L(K^{*-}\rho^0) = 0.96^{+0.06}_{-0.16}$ in Table XVIII was obtained by *BABAR* in a previous measurement where $K^{*-}\rho^0$ and $K^{*-}f_0(980)$ were not separated [138]. This has been overcome in a recent *BABAR* measurement, but the resultant value $f_L(K^{*-}\rho^0) = 0.9 \pm 0.2$ has only 2.5σ significance [125]. At any rate, it would be important to have a refined measurement of the longitudinal polarization fraction for $K^{*-}\rho^0$ and $\bar{K}^{*0}\rho^0$ and a new measurement of $f_L(K^{*-}\rho^+)$ to test the hierarchy pattern (6.6).

In the QCDF approach, we expect that the $b \to d$ penguin-dominated modes $K^{*0}K^{*-}$ and $K^{*0}\bar{K}^{*0}$ have $f_L \sim 1/2$ similar to the $\Delta S = 1$ penguin-dominated channels. However, the data seem to prefer to $f_L \sim \mathcal{O}(0.75\text{–}0.80)$. Because of the near cancellation of the color-suppressed tree amplitudes, the decay $\bar{B}^0 \to \omega\rho^0$ is actually dominated by $b \to d$ penguin transitions. Hence, it is expected that $f_L(\rho^0\omega) \sim 0.52$. It will be interesting to measure f_L for this mode.

For $\Delta S = 1$ penguin-dominated modes, the pQCD approach predicts $f_L \sim 0.70\text{–}0.80$.[6]

C. Direct *CP* asymmetries

Direct *CP* asymmetries of $B \to VV$ decays are displayed in Table XIX. They are small for color-allowed tree-dominated processes and large for penguin-dominated decays. Direct *CP* violation is very small for the pure penguin processes $\bar{K}^{*0}\rho^-$ and $K^*\phi$.

D. Time-dependent *CP* violation

In principle, one can study time-dependent *CP* asymmetries for each helicity component,

$$\mathcal{A}_h(t) \equiv \frac{\Gamma(\bar{B}^0(t) \to V_hV_h) - \Gamma(B^0(t) \to V_hV_h)}{\Gamma(\bar{B}^0(t) \to V_hV_h) + \Gamma(B^0(t) \to V_hV_h)} = S_h \sin(\Delta m t) - C_h \cos(\Delta m t). \tag{6.7}$$

Time-dependent *CP* violation has been measured for the longitudinally polarized components of $\bar{B}^0 \to \rho^+\rho^-$ and $\rho^0\rho^0$ with the results [140,141]:

TABLE XIX. Direct *CP* asymmetries (in %) of $\bar{B} \to VV$ decays. The pQCD results are taken from [120] for $\rho\rho$, $\rho\omega$, $K^*\bar{K}^*$. *CP* asymmetries of $K^*\rho$ and $K^*\omega$ in the pQCD approach are shown in Fig. 4 of [119] as a function of γ and only the signs of $A_{CP}(K^*\rho)$ and $A_{CP}(K^*\omega)$ are displayed here. Note that the definition of A_{CP} in [119] has a sign opposite to the usual convention.

Decay	QCDF (this work)	pQCD	Expt. [3]
$B^- \to \rho^-\rho^0$	0.06	0	-5.1 ± 5.4
$\bar{B}^0 \to \rho^+\rho^-$	-4^{+0+3}_{-0-3}	-7	6 ± 13
$\bar{B}^0 \to \rho^0\rho^0$	30^{+17+14}_{-16-26}	80	
$B^- \to \rho^-\omega$	-8^{+1+3}_{-1-4}	-23 ± 7	-20 ± 9
$\bar{B}^0 \to \rho^0\omega$	3^{+2+51}_{-6-76}		
$\bar{B}^0 \to \omega\omega$	-30^{+15+16}_{-14-18}		
$B^- \to K^{*0}K^{*-}$	16^{+1+17}_{-3-34}	-15	
$\bar{B}^0 \to K^{*-}K^{*+}$	0	-65	
$\bar{B}^0 \to K^{*0}\bar{K}^{*0}$	-14^{+1+6}_{-1-2}	0	
$B^- \to \bar{K}^{*0}\rho^-$	-0.3^{+0+2}_{-0-0}	+	-1 ± 16
$B^- \to K^{*-}\rho^0$	43^{+6+12}_{-3-28}	+	20^{+32}_{-29}
$\bar{B}^0 \to K^{*-}\rho^+$	32^{+1+2}_{-3-14}	+	
$\bar{B}^0 \to \bar{K}^{*0}\rho^0$	-15^{+4+16}_{-8-14}	$-$	9 ± 19
$B^- \to K^{*-}\phi$	0.05		-1 ± 8
$\bar{B}^0 \to K^{*0}\phi$	$0.8^{+0+0.4}_{-0-0.5}$		1 ± 5
$B^- \to K^{*-}\omega$	56^{+3+4}_{-4-43}	+	29 ± 35
$\bar{B}^0 \to \bar{K}^{*0}\omega$	23^{+9+5}_{-5-18}	+	45 ± 25

$$S_L^{\rho^+\rho^-} = -0.05 \pm 0.17, \qquad C_L^{\rho^+\rho^-} = -0.06 \pm 0.13,$$
$$S_L^{\rho^0\rho^0} = -0.3 \pm 0.7 \pm 0.2, \qquad C_L^{\rho^0\rho^0} = 0.2 \pm 0.8 \pm 0.3. \tag{6.8}$$

In the QCDF approach we obtain

$$\begin{aligned}
\mathcal{B}(\rho^+\rho^-)_L &= (24.7^{+1.6+1.3}_{-2.8-2.8}) \times 10^{-6},\\
S_L^{\rho^+\rho^-} &= -0.19^{+0.01+0.09}_{-0.00-0.10},\\
C_L^{\rho^+\rho^-} &= 0.11^{+0.01+0.11}_{-0.01-0.04},\\
\mathcal{B}(\rho^0\rho^0)_L &= (0.6^{+1.3+0.8}_{-0.3-0.3}) \times 10^{-6},\\
S_L^{\rho^0\rho^0} &= 0.16^{+0.05+0.50}_{-0.11-0.48},\\
C_L^{\rho^0\rho^0} &= -0.53^{+0.23+0.12}_{-0.25-0.48}.
\end{aligned} \tag{6.9}$$

As pointed out in [1], since [see Eq. (33) of [21] and Eq. (106) of [1]]

$$S_L^{\rho^+\rho^-} = \sin 2\alpha + 2r_P \cos\delta_P \sin\gamma \cos 2\alpha + \mathcal{O}(r_P^2), \tag{6.10}$$

with $P = |T|r_P \cos\delta_P$ and $\alpha = \pi - \beta - \gamma$, the measurement of $S_L^{\rho^+\rho^-}$ can be used to fix the angle γ with good accuracy. For the QCDF predictions in Eq. (6.9) we have used $\beta = (21.6^{+0.9}_{-0.8})^\circ$ and $\gamma = (67.8^{+4.2}_{-3.9})^\circ$ [31].

[6]Early pQCD calculations of $B \to K^*\phi$ tend to give a large branching fraction of order 15×10^{-6} and the polarization fraction $f_L \sim 0.75$ [136,139]. Two possible remedies have been considered: a small form factor $A_0^{BK^*}(0) = 0.32$ [131] and a proper choice of the hard scale $\bar{\Lambda}$ in B decays [123]. As shown in [123], the branching fraction of $\bar{B}^0 \to \bar{K}^{*0}\phi$ becomes 8.9×10^{-6} and $f_L \sim 0.63$ for $\bar{\Lambda} = 1.3$ GeV.

VII. CONCLUSION AND DISCUSSION

We have reexamined the branching fractions and CP-violating asymmetries of charmless $\bar{B} \to PP$, VP, VV decays in the framework of QCD factorization. We have included subleading $1/m_b$ power corrections to the penguin-annihilation topology and to color-suppressed tree amplitudes that are crucial for explaining the decay rates of penguin-dominated decays, color-suppressed tree-dominated $\pi^0\pi^0$, $\rho^0\pi^0$ modes and the measured CP asymmetries in the $B_{u,d}$ sectors. A solution to the $\Delta A_{K\pi}$ puzzle requires a large complex color-suppressed tree amplitude and/or a large complex electroweak penguin. These two possibilities can be discriminated in tree-dominated B decays. The CP puzzles with $\pi^-\eta$, $\pi^0\pi^0$ and the rate deficit problems with $\pi^0\pi^0$, $\rho^0\pi^0$ can only be resolved by having a large complex color-suppressed tree topology C. While the new physics solution to the $B \to K\pi$ CP puzzle is interesting, it is irrelevant for tree-dominated decays.

The main results of the present paper are

(i) *Branching fractions*: The observed abnormally large rates of $B \to K\eta'$ decays are naturally explained in QCDF without invoking additional contributions, such as flavor-singlet terms. It is important to have more accurate measurements of $B \to \pi\eta^{(\prime)}$ to confirm the pattern $\mathcal{B}(B^- \to \pi^-\eta') \gg \mathcal{B}(\bar{B}^0 \to \pi^0\eta')$.

(ii) The observed large rates of the color-suppressed tree-dominated decays $\bar{B}^0 \to \pi^0\pi^0$, $\rho^0\pi^0$ can be accommodated due to the enhancement of $|a_2(\pi\pi)| \sim \mathcal{O}(0.6)$ and $|a_2(\pi\rho)| \sim \mathcal{O}(0.4)$.

(iii) The decays $\bar{B}^0 \to \phi\eta$ and $B^- \to \phi\pi^-$ are dominated by the ω-ϕ mixing effect. They proceed through the weak decays $\bar{B}^0 \to \omega\eta$ and $B^- \to \omega\pi^-$, respectively, followed by ω-ϕ mixing.

(iv) QCDF predictions for charmless $B \to VV$ rates are in excellent agreement with experiment.

Direct CP asymmetries

(1) In the heavy quark limit, the predicted CP asymmetries for the penguin-dominated modes $K^-\pi^+$, $K^{*-}\pi^+$, $K^-\rho^+$, $K^-\rho^0$, and tree-dominated modes $\pi^+\pi^-$, $\rho^\pm\pi^\mp$ [with A_{CP} defined in Eq. (5.19)] and $\rho^-\pi^+$ are wrong in signs when confronted with experiment. Their signs can be flipped into the right direction by the power corrections from penguin annihilation.

(2) On the contrary, the decays $K^-\pi^0$, $K^-\eta$, $\bar{K}^{*0}\eta$, $\pi^0\pi^0$, and $\pi^-\eta$ get wrong signs for their direct CP violation when penguin annihilation is turned on. These CP puzzles can be resolved by having soft corrections to the color-suppressed tree coefficient a_2 so that a_2 is large and complex.

(3) The smallness of the CP asymmetry in $B^- \to \pi^-\pi^0$ is not affected by the soft corrections under consideration. This is different from the topological quark-diagram approach where the color-suppressed tree topology is also large and complex, but $A_{CP}(\pi^-\pi^0)$ is predicted to be of order a few percent.

(4) If the color-suppressed tree and electroweak penguin amplitudes are negligible compared to QCD penguins, CP asymmetry differences of $K^-\pi^0$ and $K^-\pi^+$, $\bar{K}^0\pi^0$ and $\bar{K}^0\pi^-$, $K^{*-}\pi^0$ and $K^{*-}\pi^+$, $\bar{K}^{*0}\pi^0$ and $\bar{K}^{*0}\pi^-$ will be expected to be small. Defining $\Delta A_{K^{(*)}\pi} \equiv A_{CP}(K^{(*)-}\pi^0) - A_{CP}(K^{(*)-}\pi^+)$ and $\Delta A'_{K^{(*)}\pi} \equiv A_{CP}(\bar{K}^{(*)0}\pi^0) - A_{CP}(\bar{K}^{(*)0}\pi^-)$, we found $\Delta A_{K\pi} = (12.3^{+3.0}_{-4.8})\%$, $\Delta A'_{K\pi} = (-11.0^{+6.4}_{-5.7})\%$, $\Delta A_{K^*\pi} = (13.7^{+4.6}_{-7.0})\%$ and $\Delta A'_{K^*\pi} = (-11.1^{+9.3}_{-6.9})\%$, while they are very small (less than 2%) in the absence of power corrections to the topological amplitude c'. Experimentally, it will be important to measure the last three CP asymmetry differences.

(5) For both $\bar{B}^0 \to \bar{K}^0\pi^0$ and $\bar{B}^0 \to \bar{K}^{*0}\pi^0$ decays, their CP asymmetries are predicted to be of order -0.10 (less than 1%) in the presence (absence) of power corrections to a_2. The relation $\Delta A'_{K\pi} \approx -\Delta A_{K\pi}$ and the smallness of $A_{CP}(\bar{K}^0\pi^-)$ give a model-independent statement that $A_{CP}(\bar{K}^0\pi^0)$ is roughly of order -0.15. Hence, an observation of $A_{CP}(\bar{K}^0\pi^0)$ at the level of $-(0.10 \sim 0.15)$ will give a strong support for the presence of soft corrections to c'. It is also in agreement with the value inferred from the CP-asymmetry sum rule, or SU(3) relation or the diagrammatical approach. For $\bar{B}^0 \to \bar{K}^0\rho^0$, we obtained $A_{CP}(\bar{K}^0\rho^0) = 0.087^{+0.088}_{-0.069}$.

(6) Power corrections to the color-suppressed tree amplitude is needed to improve the prediction for $A_{CP}(\bar{K}^{*0}\eta)$. The current measurement $A_{CP}(\bar{K}^{*0}\eta) = 0.19 \pm 0.05$ is in better agreement with QCDF than pQCD and SCET.

(7) There are 6 modes in which direct CP asymmetries have been measured with significance above 3σ: $K^-\pi^+$, $\pi^+\pi^-$, $K^-\eta$, $\bar{K}^{*0}\eta$, $K^-\rho^0$ and $\rho^\pm\pi^\mp$. There are also 7 channels with significance between 3.0σ and 1.8σ for CP violation: ρ^+K^-, $K^{*-}\pi^+$, $K^-\pi^0$, $\pi^-\eta$, $\omega\bar{K}^0$, $\pi^0\pi^0$, and $\rho^-\pi^+$. We have shown in this work that the QCDF predictions of A_{CP} for aforementioned 13 decays are in agreement with experiment except the decay $\bar{B}^0 \to \omega\bar{K}^0$. The QCDF prediction $A_{CP}(\omega\bar{K}^0) = -0.047^{+0.058}_{-0.060}$ is not consistent with the experimental average, 0.32 ± 0.17. However, we notice that *BABAR* and Belle measurements of $A_{CP}(\omega\bar{K}^0)$ are of opposite sign.

Mixing-induced CP asymmetries

(a) The decay modes $\eta' K_S$ and ϕK_S appear theoretically very clean in QCDF; for these modes the central value of ΔS_f as well as the uncertainties are rather small.

(b) The QCDF approach predicts $\Delta S_{\pi^0 K_S} \approx 0.12$, $\Delta S_{\omega K_S} \approx 0.17$, and $\Delta S_{\rho^0 K_S} \approx -0.17$. Soft corrections to a_2 have significant effects on these three observables, especially the last one.

(c) For tree-dominated modes, the predicted $S_{\pi^+\pi^-} \approx -0.69$ agrees well with experiment, while $S_{\rho^0\pi^0} \approx -0.24$ disagrees with the data in sign.

Puzzles to be resolved

(i) Both QCDF and pQCD can manage to lead to a correct sign for $A_{CP}(K^-\eta)$, but the predicted magnitude still falls short of the measurement -0.37 ± 0.09. The same is also true for $A_{CP}(\pi^+\pi^-)$.

(ii) The QCDF prediction for the branching fraction of $B \to K^*\eta'$, of order 1.5×10^{-6}, is smaller compared to pQCD and SCET. Moreover, although the QCDF results are smaller than the *BABAR* measurements, they are consistent with Belle's upper limits. It will be crucial to measure them to discriminate between various predictions.

(iii) CP asymmetry of $\bar{B}^0 \to \omega\bar{K}^0$ is estimated to be of order -0.047. The current data $0.52^{+0.22}_{-0.20} \pm 0.03$ by *BABAR* and $-0.09 \pm 0.29 \pm 0.06$ by Belle seem to favor a positive $A_{CP}(\omega\bar{K}^0)$. This should be clarified by more accurate measurements.

(iv) CP violation of $\bar{B}^0 \to \rho^0\pi^0$ is predicted to be of order 0.11 by QCDF and negative by pQCD and SCET. The current data are $0.10 \pm 0.40 \pm 0.53$ by *BABAR* and $-0.49 \pm 0.36 \pm 0.28$ by Belle. This issue needs to be resolved.

In this work we have collected all the pQCD and SCET predictions whenever available and made a detailed comparison with the QCDF results. In general, QCDF predictions for the branching fractions and direct CP asymmetries of $\bar{B} \to PP$, VP, VV decays are in good agreement with experiment except for a few remaining puzzles mentioned above. For the pQCD approach, predictions on the penguin-dominated VV modes and tree-dominated VP channels should be updated. Since the sign of $A_{CP}(K^-\eta)$ gets modified by the NLO effects, it appears that all pQCD calculations should be carried out systematically to the complete NLO (not just partial NLO) in order to have reliable estimates of CP violation.

As for the approach of SCET, its phenomenological analysis so far is not quite successful in several places. For example, the predicted branching fraction $\mathcal{B}(B^- \to \rho^-\eta') \sim 0.4 \times 10^{-6}$ is far below the experimental value of $\sim 9 \times 10^{-6}$. The most serious ones are the CP asymmetries for $K^-\pi^0$, $\pi^0\pi^0$, $\pi^-\eta$, and $\bar{K}^{*0}\eta$. The predicted signs of them disagree with the data (so the $\Delta A_{K\pi}$ puzzle is not resolved). Also the predicted CP violation for $\bar{K}^0\pi^0$ and $\bar{K}^{*0}\pi^0$ is of opposite sign to QCDF and pQCD. As stressed before, all the B-CP puzzles occurred in QCDF will also manifest in SCET because the long-distance charming penguins in the latter mimic the penguin-annihilation effects in the former. This means that one needs other large and complex power corrections to resolve the CP puzzles induced by charming penguins. For example, in the current phenomenological analysis of SCET, the ratio of $C^{(\prime)}/T^{(\prime)}$ is small and real to the leading order. This constraint should be released somehow.

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