

A₄ model for lepton masses and mixings

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We study an extension of the standard model based on the flavor symmetry A_4 only. Neutrino Majorana mass terms arise from a dimension five operator and charged lepton masses from renormalizable Yukawa couplings. We introduce three Higgs doublets that belong to one triplet irreducible representation of A_4 . We study the most general A_4 -invariant scalar potential and the phenomenological consequences of the model. We find that the reactor angle could be as large as $\sin^2\theta_{13_{\max}} \sim 0.03$, while the atmospheric mixing angle θ_{23} is close to maximal, $\sin^2\theta_{23} = 1/2$.

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I. INTRODUCTION

Experiments on neutrino oscillations [1] confirmed that neutrinos are massive and mix among themselves like the quarks do. In contrast to the quark sector, neutrino oscillations have two large mixing angles. In the literature, continuous as well as discrete (Abelian and non-Abelian) flavor symmetries have been extensively studied. However, so far we do not have a unique top-down hint for the choice of the flavor symmetry. In a bottom-up approach simplicity and predictivity are possible criteria that we can use. In the quark sector the heaviness of the top quark suggests that first and second families could belong to a doublet irreducible representation and the third family belongs to a singlet representation of the flavor symmetry group. Discrete groups with such a property are, for instance, D_4 [2] and S_3 [3]. Differently, the large solar and atmospheric neutrino mixing angles suggest that the three neutrino families belong to one triplet irreducible representation of a flavor group. The group of even permutation of four objects, A_4 , is the smallest non-Abelian discrete group with triplet irreducible representation. Most of the models based on A_4 [4] need to introduce extra auxiliary Abelian symmetries and supersymmetry (or extra dimensions) in order to reproduce tri-bimaximal mixing [5].

In this paper we use the A_4 symmetry group as the flavor group but we renounce to predict the tri-bimaximal mixing. The Dirac mass matrices arise from a renormalizable operator coupled with the Higgs, that is the Yukawa interactions. Neutrino Majorana mass terms are generated from a dimension five Weinberg operator [6]. It is a known fact that A_4 can be broken spontaneously into its Z_3 or Z_2 subgroups assuming the vacuum expectation values (vevs) to be real.¹ Recently in [7], a model for quarks mixing has been suggested where the most generic

A_4 -invariant potential has complex solutions. In this case A_4 is completely broken. This solution opens new possibilities for the description of the leptonic sector that deserve further investigation.

In the next section we present the model, in Secs. III and IV we discuss the charged lepton and neutrino mass matrices, respectively. In Sec. V we discuss the implication of the model and finally, in Sec. VI we summarize the results of the model.

II. THE MODEL

In our model the Higgs sector is extended from one $SU(2)_L$ doublet to three $SU(2)_L$ doublets belonging to a triplet irreducible representation of A_4 . The left-handed as well as the right-handed charged leptons belong to the triplet irreducible representation of A_4 . The irreducible representation assignment for the particles is given in Table I.

A_4 is the group of even permutations of 4 objects and is isomorphic to the symmetries of the tetrahedron. A_4 can be generated by two generators S and T with the properties

$$S^2 = T^2 = (ST)^3 = 1. \quad (1)$$

A_4 contains one 3-dimensional representation, 3, and three one-dimensional, 1, 1', and 1'' representations. The product of two 3's gives $3 \otimes 3 = 1 \oplus 1' \oplus 1'' \oplus 3 \oplus 3$ and $1' \otimes 1' = 1''$, $1' \otimes 1'' = 1$, $1'' \otimes 1'' = 1'$, etc. For two triplets $3_a \sim (a_1, a_2, a_3)$, $3_b \sim (b_1, b_2, b_3)$ the irreducible representations obtained from their product are

$$1 = a_1 b_1 + a_2 b_2 + a_3 b_3, \quad (2)$$

TABLE I. Lepton multiplet structure of the model.

Fields	L_i	l_i^c	H_i
$SU(2)_L$	2	1	2
A_4	3	3	3

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¹When A_4 is broken into Z_3 in the charged lepton sector and into Z_2 in the neutrino sector the lepton mixing is tri-bimaximal.

$$l' = a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3, \quad (3)$$

$$l'' = a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3, \quad (4)$$

$$3 \sim (a_2 b_3, a_3 b_1, a_1 b_2), \quad (5)$$

$$3 \sim (a_3 b_2, a_1 b_3, a_2 b_1), \quad (6)$$

this in the basis of S diagonal and where $\omega = e^{i2\pi/3}$; see, for instance, [8].

The most general renormalizable Yukawa Lagrangian for the charged leptons in the model is

$$L_{\text{Yukawa}} = y_1(\bar{L}_1 \phi_3 l_2^c + \bar{L}_2 \phi_1 l_3^c + \bar{L}_3 \phi_2 l_1^c) + y_2(\bar{L}_1 \phi_2 l_3^c + \bar{L}_2 \phi_3 l_1^c + \bar{L}_3 \phi_1 l_2^c). \quad (7)$$

Once the electroweak symmetry (EW) is broken, the charged lepton mass matrix is obtained from this Yukawa

Lagrangian:

$$M_l = \begin{pmatrix} 0 & y_1 \langle \phi_3 \rangle & y_2 \langle \phi_2 \rangle \\ y_2 \langle \phi_3 \rangle & 0 & y_1 \langle \phi_1 \rangle \\ y_1 \langle \phi_2 \rangle & y_2 \langle \phi_1 \rangle & 0 \end{pmatrix}. \quad (8)$$

The most general neutrino dimension five operator invariant under A_4 (see, for instance, [9]), is given by

$$\begin{aligned} \mathcal{L}_{5d} = & \beta(LL)_3(HH)_3 + k(LL)_1(HH)_1 + \alpha'(LL)_{1'}(HH)_{1''} \\ & + \alpha''(LL)_{1''}(HH)_{1'} + [a(LH)_{3^a}(LH)_{3^a} \\ & + b(LH)_{3^a}(LH)_{3^b} + c(LH)_{3^b}(LH)_{3^a} \\ & + d(LH)_{3^b}(LH)_{3^b}] + l(LH)_1(LH)_1 \\ & + l'[(LH)_{1'}(LH)_{1''} + (LH)_{1''}(LH)_{1'}], \end{aligned} \quad (9)$$

where $\beta, k, \alpha, \alpha', \alpha'', a, b, c,$ and d are arbitrary complex couplings. Once the EW symmetry is broken, the Majorana neutrino mass matrix is given by

$$M_\nu = \begin{pmatrix} x \langle \phi_1 \rangle^2 + y \langle \phi_2 \rangle^2 + z \langle \phi_3 \rangle^2 & \kappa \langle \phi_1 \rangle \langle \phi_2 \rangle & \kappa \langle \phi_1 \rangle \langle \phi_3 \rangle \\ \kappa \langle \phi_1 \rangle \langle \phi_2 \rangle & z \langle \phi_1 \rangle^2 + x \langle \phi_2 \rangle^2 + y \langle \phi_3 \rangle^2 & \kappa \langle \phi_2 \rangle \langle \phi_3 \rangle \\ \kappa \langle \phi_1 \rangle \langle \phi_3 \rangle & \kappa \langle \phi_2 \rangle \langle \phi_3 \rangle & y \langle \phi_1 \rangle^2 + z \langle \phi_2 \rangle^2 + x \langle \phi_3 \rangle^2 \end{pmatrix}, \quad (10)$$

with

$$x = k + \alpha' + \alpha'' + l + l', \quad (11)$$

$$y = k + a + \alpha' \omega^2 + \alpha'' \omega, \quad (12)$$

$$z = k + d + \alpha' \omega + \alpha'' \omega^2, \quad (13)$$

$$\kappa = \beta + (b + c) + 2l - l', \quad (14)$$

where $x, y, z,$ and κ are complex parameters.

The most general renormalizable scalar potential invariant under the symmetry A_4 [7] is

$$\begin{aligned} V = & \mu(\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 + \phi_3^\dagger \phi_3) + \lambda_1(\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 \\ & + \phi_3^\dagger \phi_3)^2 + \lambda_2[(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + (\phi_2^\dagger \phi_2)(\phi_3^\dagger \phi_3) \\ & + (\phi_3^\dagger \phi_3)(\phi_1^\dagger \phi_1)] + (\lambda_3 - \lambda_2)[(\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) \\ & + (\phi_2^\dagger \phi_3)(\phi_3^\dagger \phi_2) + (\phi_3^\dagger \phi_1)(\phi_1^\dagger \phi_3)] + \frac{\lambda_4}{2} \\ & \times \{e^{i\epsilon}[(\phi_1^\dagger \phi_2)^2 + (\phi_2^\dagger \phi_3)^2 + (\phi_3^\dagger \phi_1)^2] + \text{H.c.}\}, \end{aligned} \quad (15)$$

where μ and λ_{1-4} are real, and the phase ϵ is arbitrary. The scalar doublets are

$$\phi_j = \begin{pmatrix} \phi_j^+ \\ \phi_j^0 \end{pmatrix}, \quad \tilde{\phi}_j = \begin{pmatrix} \phi_j^{0*} \\ -\phi_j^- \end{pmatrix}. \quad (16)$$

The three nontrivial and interesting minimums found in [7] are

$$v_1 = v_2, \quad v_2 = v_3, \quad v_1 = v_3.$$

We choose the solution $v_2 = v_3$, and therefore

$$\langle \phi_1 \rangle = v_1, \quad \langle \phi_2 \rangle = v e^{i\alpha/2}, \quad \langle \phi_3 \rangle = v e^{-i\alpha/2}. \quad (17)$$

Notice that this solution is different from that used in [7] because we are interested in the breakdown of the $\mu - \tau$ symmetry.

III. CHARGED LEPTONS

With the vevs in Eq. (17), the charged lepton mass matrix in Eq. (8) takes the form

$$M_l = \begin{pmatrix} 0 & a e^{i\alpha} & b e^{-i\alpha} \\ b e^{i\alpha} & 0 & a r \\ a e^{-i\alpha} & b r & 0 \end{pmatrix}, \quad (18)$$

with the parameters in the matrix (18) defined by $a = y_1 v$, $b = y_2 v$, and $r = v_1/v$. Notice that in a matrix with this form, all the phases in the entries can be absorbed in the fields; hence we can write the charged lepton matrix as

$$M_l = \begin{pmatrix} 0 & a & b \\ b & 0 & a r \\ a & b r & 0 \end{pmatrix}, \quad (19)$$

with $a, b,$ and r real parameters. We write the symmetric matrix, $M_l M_l^T$,

$$M_l M_l^T = \begin{pmatrix} a^2 + b^2 & abr & abr \\ abr & b^2 + a^2 r^2 & ab \\ abr & ab & a^2 + b^2 r^2 \end{pmatrix}, \quad (20)$$

which is diagonalized by an orthogonal matrix O_l . It is straightforward to obtain the analytical expressions for a , b , and r as a function of the charged lepton masses; it can be written as

$$\begin{aligned} r &\approx \frac{m_\tau}{\sqrt{m_e m_\mu}} \sqrt{1 - \frac{m_e^2 m_\mu^2}{m_\tau^4}}, \\ a &\approx \frac{m_\mu}{m_\tau} \sqrt{m_e m_\mu} \left[1 + \frac{1}{2} \frac{m_\mu^2}{m_\tau^2} \right], \\ b &\approx \sqrt{m_e m_\mu} \left[1 - \frac{1}{2} \frac{m_\mu^2}{m_\tau^2} \right]. \end{aligned} \quad (21)$$

With

$$\begin{aligned} m_e &= 0.511\,006 \text{ MeV}, & m_\mu &= 105.656 \text{ MeV}, \\ m_\tau &= 1776.96 \text{ MeV}, \end{aligned}$$

we have $a = 0.43474$, $b = 7.3471$, and $r = 241.8582$.

Note that $a < b \ll r$; thus the orthogonal matrix O_l diagonalizing $M_l M_l^T$ is approximately

$$O_{l_{12}} \approx \frac{b}{a} r^{-1}, \quad O_{l_{13}} \approx \frac{a}{b} r^{-1}, \quad O_{l_{23}} \approx \frac{a}{b} r^{-2}. \quad (22)$$

The element, $O_{l_{12}}$, gives a contribution to the reactor mixing angle, θ_{13} ; see Sec. VA. The analytical expression for this element is given as

$$O_{l_{12}} \approx \sqrt{\frac{m_e}{m_\mu}} \left[1 - \left(\frac{m_\mu}{m_\tau} \right)^2 \right]. \quad (23)$$

The numerical expression for the matrix O_l is

$$O_l = \begin{pmatrix} 0.997 & 0.069 & 2.44 \times 10^{-4} \\ -0.069 & 0.997 & 1.075 \times 10^{-6} \\ -2.439 \times 10^{-4} & -1.800 \times 10^{-5} & 0.999 \end{pmatrix}. \quad (24)$$

IV. NEUTRINOS

The mass matrix for the neutrinos in Eq. (10) with the vevs in Eq. (17) takes the form

$$M_\nu = \begin{pmatrix} xr^2 + ye^{-2i\alpha} + ze^{2i\alpha} & \kappa r e^{-i\alpha} & \kappa r e^{i\alpha} \\ \kappa r e^{-i\alpha} & zr^2 + xe^{-2i\alpha} + ye^{2i\alpha} & \kappa \\ \kappa r e^{i\alpha} & \kappa & yr^2 + ze^{-2i\alpha} + xe^{2i\alpha} \end{pmatrix}. \quad (25)$$

From the charged lepton sector we know that r is fixed as shown in Eq. (21), and $r \gg 1$; then we can neglect in the diagonal the terms not proportional to r^2 . With this the mass matrix in Eq. (25) can be written as

$$M_\nu = \begin{pmatrix} xr^2 & \kappa r e^{-i\alpha} & \kappa r e^{i\alpha} \\ \kappa r e^{-i\alpha} & zr^2 & \kappa \\ \kappa r e^{i\alpha} & \kappa & yr^2 \end{pmatrix}. \quad (26)$$

Note that there are 4 complex free parameters, x , y , z , and κ , and one extra phase α coming from the Higgs sector. We can absorb two phases in the fields, and then 7 free parameters remain in the neutrino mass matrix. Neutrino oscillation experiments determine two mass square differences $\Delta m_{12}^2 \equiv m_2^2 - m_1^2$ and $\Delta m_{13}^2 \equiv |m_3^2 - m_1^2|$ with the corresponding three mixing angle [10]. If θ_{13} is different from zero, the Dirac phases could be probed in future experiments [11]. The absolute neutrino mass scale can be probed in future tritium beta decay [12] and neutrinoless double beta decay [13] experiments, while it will be hard to measure the two Majorana phases. There are seven measurable physical observables plus two Majorana physical phases.

Since the charged lepton mass matrix is close to diagonal, see Eq. (24), we already know that in order to be consistent with current experimental data, the neutrino mass matrix should be approximately $\mu \leftrightarrow \tau$ invariant [14–16] in order to give a nearly maximal atmospheric angle and small reactor angle θ_{13} . Thus we expect α to be small and $y \approx z$ (a moderate fine-tuning is required between y and z) and we set $z \equiv y(1 + \delta)$.

In the limit $\alpha, \delta \rightarrow 0$, the matrix in Eq. (26) is reduced to

$$M_\nu = \begin{pmatrix} xr^2 & \kappa r & \kappa r \\ \kappa r & yr^2 & \kappa \\ \kappa r & \kappa & yr^2 \end{pmatrix}, \quad (27)$$

which is $\mu - \tau$ invariant. Consider the orthogonal matrix

$$V = \begin{pmatrix} -\cos\theta & \sin\theta & 0 \\ \sin\theta/\sqrt{2} & \cos\theta/\sqrt{2} & -1/\sqrt{2} \\ \sin\theta/\sqrt{2} & \cos\theta/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \quad (28)$$

and by multiplying the matrix in Eq. (27) we obtain

$$V^T M_\nu V = \begin{pmatrix} \kappa s_\theta(-2\sqrt{2}c_\theta r + s_\theta) + r^2(c_\theta^2 x + s_\theta^2 y) & \sqrt{2}\kappa r(-c_\theta^2 + s_\theta^2) + s_\theta c_\theta(r^2(y-x) + \kappa) & 0 \\ \sqrt{2}\kappa r(-c_\theta^2 + s_\theta^2) + s_\theta c_\theta(r^2(y-x) + \kappa) & \kappa c_\theta(c_\theta + 2\sqrt{2}s_\theta r) + r^2(s_\theta^2 x + c_\theta^2 y) & 0 \\ 0 & 0 & yr^2 - \kappa \end{pmatrix}, \quad (29)$$

where $s_\theta = \sin\theta$ and $c_\theta = \cos\theta$. If we require that the elements 12 and 21 in Eq. (29) are zero, we obtain

$$\tan 2\theta = -\frac{2\sqrt{2}\kappa r}{(r^2(x-y) - \kappa)}. \quad (30)$$

Defining the function K as

$$\kappa = -K(x-y), \quad (31)$$

from Eq. (30) we have

$$K = r^2 \frac{\sin 2\theta}{2\sqrt{2}r \cos 2\theta - \sin 2\theta}. \quad (32)$$

The masses of the neutrinos can be written as

$$\begin{aligned} m_1 &= \frac{1}{2}(x+y)r^2 + (x-y) \\ &\quad \times [\sqrt{2r^2K^2 + 1/4(r^2 + K)^2} - 1/2K], \\ m_2 &= \frac{1}{2}(x+y)r^2 - (x-y) \\ &\quad \times [\sqrt{2r^2K^2 + 1/4(r^2 + K)^2} + 1/2K], \\ m_3 &= \frac{1}{2}(x+y)r^2 + (x-y)[K - \frac{1}{2}r^2], \end{aligned} \quad (33)$$

and the squared masses $|m_i|^2 = m_i m_i^*$ are

$$\begin{aligned} |m_{\nu 1}|^2 &= \frac{1}{4}|x+y|^2 + |x-y|^2(F + \frac{1}{4}K^2 - FK) \\ &\quad + (x^2 - y^2)r^2(F - \frac{1}{2}K), \\ |m_{\nu 2}|^2 &= \frac{1}{4}|x+y|^2 + |x-y|^2(F + \frac{1}{4}K^2 + FK) \\ &\quad - (x^2 - y^2)r^2(F + \frac{1}{2}K), \\ |m_{\nu 3}|^2 &= \frac{1}{4}|x+y|^2 + |x-y|^2(\frac{1}{4}r^4 - r^2K + K^2) \\ &\quad - (x^2 - y^2)r^2(\frac{1}{2}r^2 - K), \end{aligned} \quad (34)$$

where

$$F = \sqrt{2r^2K^2 + 1/4(r^2 + K)^2}. \quad (35)$$

V. PHENOMENOLOGY OF THE MODEL

In our model we have six free parameters, $x, y, \theta, \phi_{xy}, \delta$, and α in the neutrino sector for nine physical parameters, $\Delta m_{12}^2, \Delta m_{13}^2$, and m_{ee} , three mixing angles and two Majorana phases and the Jarlskog invariant J , for Dirac CP violation in the neutrino sector. From Eqs. (34) we can construct the expressions for Δm_{12}^2 and Δm_{13}^2 and find x and y as functions of the observables, $\Delta m_{12}^2, \Delta m_{13}^2$, the mixing parameter θ and its relative phase ϕ_{xy} . It is possible to show from Eqs. (34) that the model is only compatible with an inverted hierarchy neutrino mass spectrum.

In the next sections we present the predictions for the allowed region for mixing angles, the Jarlskog invariant as well as the neutrinoless double beta decay.

A. Neutrino mixing angles

Recently there has been an indication that the reactor neutrino angle θ_{13} could be different from zero [17]

$$\sin^2 \theta_{13} = 0.016 \pm 0.010(1\sigma). \quad (36)$$

The θ_{13}^ν angle coming from the diagonalization of the neutrino mass matrix is exactly zero in the limit $\delta, \alpha = 0$. However, the reactor angle resulting from the product of the unitary matrix that diagonalizes the charged lepton matrix, Eqs. (23) and (24) and the neutrino mass matrix, Eq. (28), is different from zero and we have

$$\sin^2 \theta_{13} \approx \frac{m_e}{2m_\mu} \approx 0.0024. \quad (37)$$

We observe from Eq. (28) that the solar mixing angle is given by the θ parameter up to corrections coming from the charged lepton sector of the order $\mathcal{O}(\sqrt{m_e/m_\mu})$. The parameters α and δ are related to the deviations of θ_{13} and θ_{23} from the zero and maximal values, respectively.

On the left side of Fig. 1, we show the allowed region for the atmospheric mixing angle. The deviation from its maximal value is small. The reactor mixing angle, θ_{13} , can be large. On the right side of Fig. 1 we show the magnitude of the Jarlskog invariant [18], J , of CP violation in neutrino oscillation defined as

$$J = \text{Im}(V_{11}V_{22}V_{12}^*V_{21}^*). \quad (38)$$

J is correlated to the $\sin^2 \theta_{13}$ that can be measured in the next experiments such as Double Chooz [11].

In general, all three mixing angles receive corrections of the same order of magnitude [19].² Since the experimentally allowed departures of the solar mixing angle from the best fit value are at most $\mathcal{O}(\lambda_C^2)$, the correction for the reactor angle is of order $\mathcal{O}(\lambda_C^2)$. In our model we can have reactor angle deviation of order λ_C as shown in Fig. 1.

B. Neutrinoless double beta decay

Neutrinos are guaranteed to have a nonzero Majorana mass if the neutrinoless double beta decay $0\nu\beta\beta$ is observed [21]. The $0\nu\beta\beta$ decay rate m_{ee} is proportional to the $\nu_e - \nu_e$ entry of the Majorana neutrino mass matrix

²A different model with corrections of different magnitude for the mixing angles has been studied, for instance, in [20].

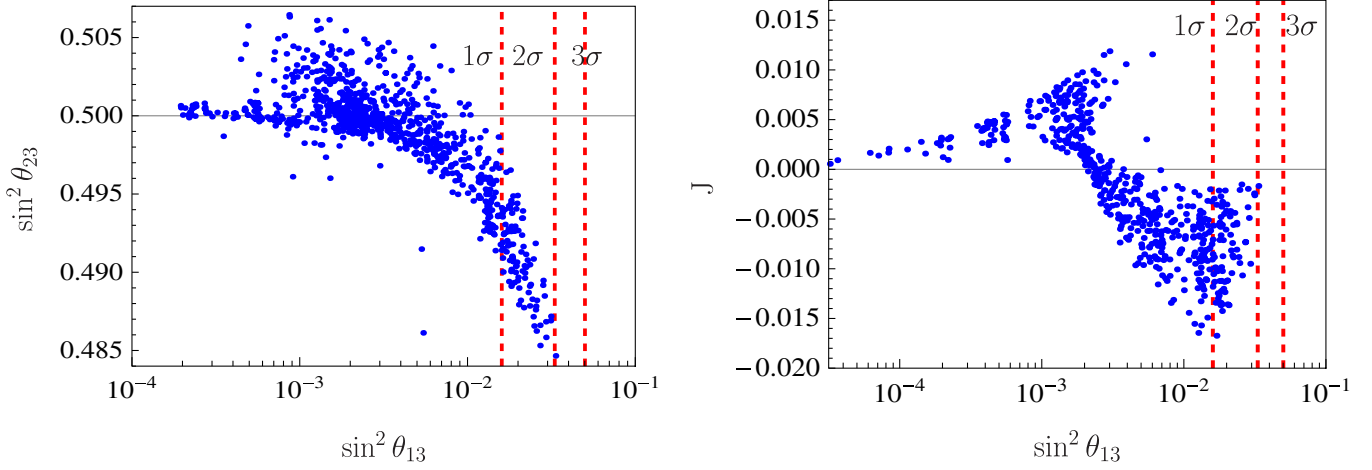


FIG. 1 (color online). The figure on the left shows the allowed region for θ_{23} vs θ_{13} . The figure on the right shows the allowed region for J vs θ_{13} .

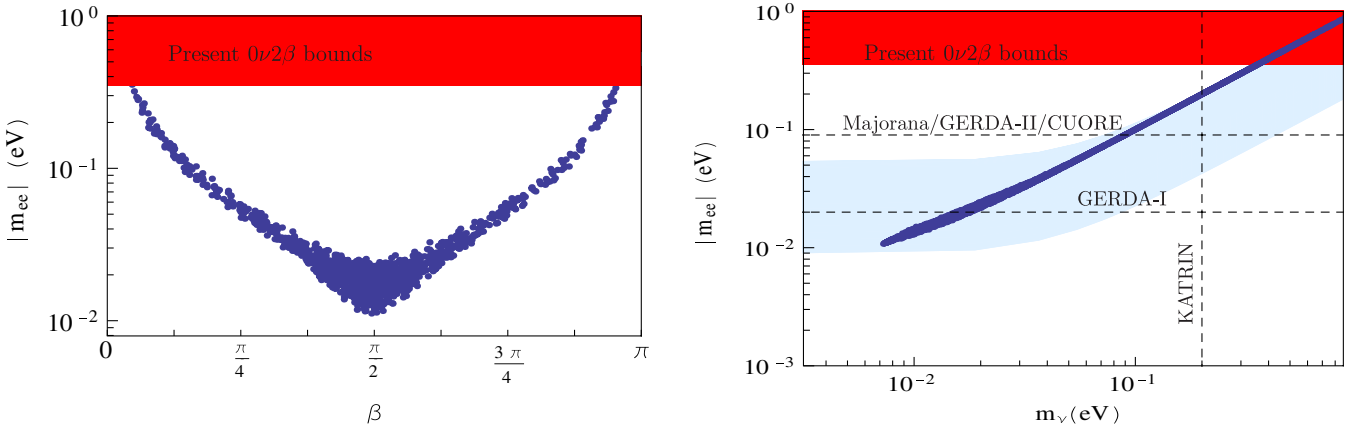


FIG. 2 (color online). The figure on the left shows the allowed range for the $0\nu\beta\beta$ as a function of the physical Majorana phase β . The figure on the right shows $|m_{ee}|$ as a function of the lightest neutrino mass m_{ν_3} . We also present here the future experimental sensitivity [13].

M_ν . For an introduction to the phenomenology of $0\nu\beta\beta$ see, for instance, [22].

We observed that the parameter ϕ_{xy} is related to the Majorana phase β and the m_{ee} is predicted in our model. The allowed region for the neutrino double beta decay is shown in Fig. 2. We show the dependence of the m_{ee} as a function of the Majorana physical phase β . As can be seen from the figure, the values for $\beta = 0$ and $\beta = \pi$ of m_{ee} are forbidden by the upper limits obtained by the HM Collaboration. Other models in the literature excluding $\beta = 0$ but not $\beta = \pi$ have been studied [23]. We note that $m_{\text{light}} > 0.008$.

VI. CONCLUSION

We have studied a model for lepton mixing based on a A_4 flavor symmetry. This constrains the model, reducing the number of free parameters with respect to the case of the standard model. In the scalar sector we introduce three

Higgs doublets that belong to a triplet representation of A_4 . If the vevs of the Higgs fields are assumed to be real, there are only two possible solutions: (i) the three vevs are all equal, or (ii) two vevs are equal to zero. In this paper we consider the most general case with complex vevs. This solution is different from that of real vevs; it is found that one of the vevs is real and the other two are the complex conjugate one of each other, that is, $\langle H \rangle = (v_1, v, v^*)$, where v_1 is different from v as noted also in [7]. This fact opens an interesting scenario in the model building due to the extra CP phase in the Higgs sector [24]. We studied the phenomenological implications of the neutrino masses and mixings. The charged lepton mass matrix arises only from renormalizable Yukawa interactions, while the Majorana neutrino mass matrix arises from a dimension five operator. We do not enter into detail how this dimension five operator is generated. In order to fit the data we assumed a moderate fine-tuning between the free parameters in the neutrino mass matrix. We found that the model is

compatible with inverse hierarchy only. The atmospheric angle is very close to the maximal value $\sin^2\theta_{23} \sim 0.5$ and the maximum allowed value for the reactor angle is close to the current 2σ upper bound, that is $\sin^2\theta_{13} \sim 0.03$. The solar mixing angle can be fitted in the allowed experimental range at 3σ . The maximal value for the CP Jarlskog invariant is $|J| \approx 0.015$. We also found that the current $0\nu\beta\beta$ upper bound restricts the physical Majorana phase β , to be slightly different from zero and π .

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APPENDIX: CORRECTIONS TO θ_{13}

When $\alpha, \delta \neq 0$, then $\theta_{13} \neq 0$. The case with $\alpha = 0$ and $\delta \neq 0$ has been already considered in [25,26].³ An upper bound $\sin^2\theta_{13} < 0.01$ for the inverse hierarchy case was found. Here we study analytically the case $\delta = 0$ and α small.

The squared neutrino mass matrix can be written as

$$M_\nu M_\nu^\dagger = [M_\nu M_\nu^\dagger]^0 + [M_\nu M_\nu^\dagger]^1, \quad (\text{A1})$$

where $[M_\nu M_\nu^\dagger]^0$ is the mass matrix when $\alpha = 0$ and $[M_\nu M_\nu^\dagger]^1$ is the correction to the squared mass matrix when $\alpha \neq 0$. Taking $\sin\alpha = \alpha$ and $\cos\alpha = 1$ the correction matrix is given by

$$|3\rangle^1 = \alpha K r \begin{pmatrix} \frac{i|x-y|^2[-\sqrt{2}(K-r^2)+R(r\sin 2\theta-\sqrt{2}\cos^2\theta(1-r^2))]+2\sqrt{2}r^2(1+R\cos^2\theta)\sin\phi_{xy}}{(1+R)\Delta m_{13}^2} \\ -\frac{i|x-y|^2[\sqrt{2}Kr+KR(\sqrt{2}r\sin^2\theta-\frac{1}{2}r^2\sin 2\theta(1-r^2))]+\frac{1}{2}Rr^2\sin 2\theta\sin\phi_{xy}}{(1+R)\Delta m_{13}^2} \\ -\frac{i|x-y|^2[\sqrt{2}Kr+KR(\sqrt{2}r\sin^2\theta-\frac{1}{2}r^2\sin 2\theta(1-r^2))]+\frac{1}{2}Rr^2\sin 2\theta\sin\phi_{xy}}{(1+R)\Delta m_{13}^2} \end{pmatrix}, \quad (\text{A5})$$

where $R = \frac{\Delta m_{12}^2}{\Delta m_{13}^2}$. The third neutrino eigenvector is given by

$$|V_{i3}\rangle \approx \begin{pmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} + \alpha K r \begin{pmatrix} \frac{i|x-y|^2[-\sqrt{2}(K-r^2)+R(r\sin 2\theta-\sqrt{2}\cos^2\theta(1-r^2))]+2\sqrt{2}r^2(1+R\cos^2\theta)\sin\phi_{xy}}{(1+R)\Delta m_{13}^2} \\ -\frac{i|x-y|^2[\sqrt{2}Kr+KR(\sqrt{2}r\sin^2\theta-\frac{1}{2}r^2\sin 2\theta(1-r^2))]+\frac{1}{2}Rr^2\sin 2\theta\sin\phi_{xy}}{(1+R)\Delta m_{13}^2} \\ -\frac{i|x-y|^2[\sqrt{2}Kr+KR(\sqrt{2}r\sin^2\theta-\frac{1}{2}r^2\sin 2\theta(1-r^2))]+\frac{1}{2}Rr^2\sin 2\theta\sin\phi_{xy}}{(1+R)\Delta m_{13}^2} \end{pmatrix}, \quad (\text{A6})$$

where ϕ_{xy} is the relative phase between x and y . The correction to θ_{13} is given by

$$\sin\theta_{13} \approx \langle O_{1i} | V_{i3} \rangle, \quad (\text{A7})$$

where $|O_{1i}\rangle$ in the first row in Eq. (24),

³A similar case to $\alpha \neq 0$ and $\delta = 0$ was studied by Ma in [14].

$$[M_\nu M_\nu^\dagger]^1 = Kr\alpha(x^2 + y^2 - 2xy\cos\phi_{xy}) \times \begin{pmatrix} 0 & i(K-r^2) & -i(K-r^2) \\ -i(K-r^2) & 0 & -2iKr \\ i(K-r^2) & 2iKr & 0 \end{pmatrix} + 2\alpha Kr^3 xy \sin\phi_{xy} \begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \quad (\text{A2})$$

We can compute the correction to the $\sin\theta_{13}$ mixing angle due to the α parameter as a perturbation. We have the eigenvectors for $\alpha = 0$

$$|1\rangle = \left(-\cos\theta, \frac{\sin\theta}{\sqrt{2}}, \frac{\sin\theta}{\sqrt{2}}\right)^T, \quad |2\rangle = \left(\sin\theta, \frac{\cos\theta}{\sqrt{2}}, \frac{\cos\theta}{\sqrt{2}}\right)^T, \\ |3\rangle = \left(0, -\frac{\sin\theta}{\sqrt{2}}, \frac{\sin\theta}{\sqrt{2}}\right)^T. \quad (\text{A3})$$

There are two corrections to the matrix elements V_{ij} , one coming from the charged leptons and the other from the phase α .

The correction to the third eigenvector is given by

$$|3\rangle^1 = \sum_{i \neq 3} \left(\frac{\langle i | [M_\nu M_\nu^\dagger]^1 | 3 \rangle}{m_i^2 - m_3^2} \right) |i\rangle. \quad (\text{A4})$$

From this and Eqs. (A2) and (A3) we obtain the third vector V_{i3} ,

$$|O_{1i}\rangle \approx \left(1 - \frac{1}{2} \frac{m_e}{m_\mu}, -\sqrt{\frac{m_e}{m_\mu}}, 0\right)^T, \quad (\text{A8})$$

with this the analytical expression for $\sin\theta_{13}$ is given by

$$\sin\theta_{13} \approx \frac{1}{\sqrt{2}} \sqrt{\frac{m_e}{m_\mu}} + \alpha K r (r^2 \sin\phi_{xy} A_1 + i|x-y|^2 A_2), \quad (\text{A9})$$

where

$$\begin{aligned}
A_1 &= \left(1 - \frac{1}{2} \frac{m_e}{m_\mu}\right) \frac{2\sqrt{2}(1 + R\cos^2\theta)}{(1 + R)\Delta m_{13}^2} + \frac{1}{2} \sqrt{\frac{m_e}{m_\mu}} \frac{R \sin 2\theta}{(1 + R)\Delta m_{13}^2}, \\
A_2 &= \frac{[-\sqrt{2}(K - r^2) + R(r \sin 2\theta - \sqrt{2}\cos^2\theta(1 - r^2))](1 - \frac{1}{2} \frac{m_e}{m_\mu})}{(1 + R)\Delta m_{13}^2} + \frac{[\sqrt{2}r + R(\sqrt{2}r \sin^2\theta - \frac{1}{2} r^2 \sin 2\theta(1 - r^2))]K\sqrt{\frac{m_e}{m_\mu}}}{(1 + R)\Delta m_{13}^2}.
\end{aligned}
\tag{A10}$$

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