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Catalysis of Schwinger vacuum pair production

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We propose a new catalysis mechanism for nonperturbative vacuum electron-positron pair production, by superimposing a plane-wave x-ray probe beam with a strongly focused optical laser pulse, such as is planned at the Extreme Light Infrastructure (ELI) facility. We compute the absorption coefficient arising from vacuum polarization effects for photons below threshold in a strong electric field. This setup should facilitate the (first) observation of this nonperturbative QED effect with planned light sources such as ELI yielding an envisioned intensity of order 10^{26} W/cm^2.

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Soon after Dirac's seminal discovery that a consistent relativistic quantum description of electrons entails the existence of positrons, pictured as holes in the Dirac sea [1], it was realized that a strong enough electric field can create electron-positron pairs out of the vacuum [2–6]. For a constant electric field E, the electron-positron pair creation rate (i.e., the number of pairs per unit time and volume) is given by [3,4]

$$\Re_{e^+e^-} = \frac{e^2E^2}{4\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left\{-n\pi\frac{m^2}{eE}\right\}, \quad (1)$$

where m is the electron rest mass, and e is the elementary charge (we use $\hbar = c = \varepsilon_0 = 1$). Apart from the striking experimental possibility to create matter out of the vacuum just by applying a strong electric field, this quantum field theoretical prediction is of fundamental importance due to its purely nonperturbative nature: the rate (1) has no Taylor expansion in (positive) powers of the coupling strength e, so no Feynman diagram $\propto e^{2n}$ of arbitrarily large (finite) order n can describe this phenomenon. While nonperturbative tunneling processes are well studied in quantum chromodynamics (QCD) at both the fundamental and phenomenological level [7], quantum electrodynamics (QED) offers the possibility of a well-controllable direct experimental observation of such nonperturbative vacuum effects [8,9].

The relevant critical field strength scale, $E_{\rm cr} = m^2/e \simeq 1.3\times 10^{18}$ V/m, is set by the exponent in (1), and corresponds to a field intensity $I_{\rm cr} = E_{\rm cr}^2 \simeq 4.3\times 10^{29}$ W/cm^2. For $E \ll E_{\rm cr}$, the pair creation rate (1) is strongly (exponentially) suppressed, even for the dominant $n = 1$ term. For instance, an electric field E corresponding to an intensity of $I = 10^{26}$ W/cm^2 yields a Schwinger exponential factor $\exp\{-\pi m^2/(eE)\} \sim 10^{-90}$, and for $I = 10^{27}$ W/cm^2 this factor is $\sim 5\times 10^{-29}$. It has been argued [10] that for certain focused laser beam configurations, a space-time volume enhancement factor might render observation feasible for $I = 10^{27}$ W/cm^2. Unfortunately, 10^{27} W/cm^2 is extremely difficult to reach, and already with 10^{26} W/cm^2 it is hard to see how the exponential suppression of order 10^{-90} could be compensated by such a four-volume enhancement factor. Thus, while e^+e^- pair creation has already been observed in the perturbative multiphoton regime [11] or due to the Bethe-Heitler process [12], truly nonperturbative QED vacuum effects have so far eluded experimental observation.

Partly motivated by these difficulties, we recently proposed a *dynamically assisted Schwinger mechanism* [13], showing that it is possible to enhance significantly the pair creation rate (1) by superimposing a strong but slow electric field with a weak but fast electric field, resulting in a decrease of the effective spectral gap between the electron states and the Dirac sea. Here, we extend this idea into a potentially realistic experimental scenario, proposing a new catalysis mechanism for nonperturbative vacuum e^+e^- pair production. The setup we propose is a superposition of a plane-wave x-ray probe beam with a strongly focused optical laser pulse, $\sim 10^{26}$ W/cm^2, as may soon be available in the Extreme Light Infrastructure (ELI) project [14]. This superposition leads to a dramatic enhancement of the expected yield of e^+e^- pairs, and brings the vacuum pair-production effect significantly closer to the observable regime.

A different implementation of the dynamically assisted Schwinger mechanism idea has recently been proposed [15], wherein a strong low-frequency laser and a weak high-frequency laser collide with a relativistic nucleus. This configuration effectively lowers the tunneling barrier, thereby increasing the pair-production rate. Strong Coulomb fields from accelerated ions in combination with lasers can also lead to noticeably higher production rates in the multiphoton regime [16].

By contrast, our proposal of a "catalyzed Schwinger mechanism" can be realized with all-optics components, explicitly avoids any above-threshold scales, and fully preserves the nonperturbative character of the Schwinger mechanism. For this, we assume that the optical laser pulse is focused to yield a maximum electric field E such that the magnetic field near the focal point can be neglected (e.g., by colliding two counter-propagating pulses, such that $E = 2E_{\text{laser}}$). Furthermore, since the Schwinger mechanism is dominated by the region with the highest field strength E, and the x-ray wavelength is much smaller than the (optical) focus size, we can approximate the optical laser pulse by a constant [17] electric field E, in which the x-ray photons propagate. And with a large number of coherent photons in the intense pulse, we can treat the strong field as being classical. To compute the expected number of pairs produced, we need the absorption coefficient κ of an x-ray photon propagating in an electric field, which is encoded in the imaginary part of the polarization tensor $\Pi_{\mu\nu}$:

$$\kappa_{\|,\perp} = -\frac{1}{\omega}\Im(\Pi_{\|,\perp}). \tag{2}$$

Here ω is the photon frequency and $\|$, $\perp$ denote the polarizations of the x-ray photons with respect to the electric field direction. The quantities $\Pi_{\|,\perp}$ are eigenvalues of the polarization tensor $\Pi_{\mu\nu}$ corresponding to the eigendirections which reduce to the two physical transversal modes in the free-field case. For weak fields $eE/m^2 \ll 1$, photons in a field propagate essentially on the light cone with small corrections, i.e., the phase velocity is $v \equiv \omega/|\mathbf{k}| = 1 + \mathcal{O}(\alpha(eE)^2/m^4)$. Even at stronger fields $eE \sim m^2$, the phase velocity deviates from 1 only by $\mathcal{O}(\alpha/\pi)$ corrections [18]. Since $\Pi_{\mu\nu}$ itself is of order α, we can safely set $k^2 = \mathbf{k}^2 - \omega^2 \to 0$ inside $\Pi_{\mu\nu}$ for photons propagating in a laboratory electric field.

Given an initial photon amplitude A_{in}, the outgoing amplitude after traversing an electric field E of length L is $A_{\text{out}} = \mathrm{e}^{-\kappa L/2}A_{\text{in}}$. The survival probability of the photon is $P = |A_{\text{out}}|^2/|A_{\text{in}}|^2 = \mathrm{e}^{-\kappa L}$. Thus, given the rate n_{in} of incoming photons, the pair creation rate (i.e., number of pairs per unit time) is (for $\kappa L \ll 1$)

$$n_{e^+e^-} = n_{\text{in}}(1-P) = n_{\text{in}}(1-\mathrm{e}^{-\kappa L}) \simeq \kappa L n_{\text{in}}, \tag{3}$$

where we have ignored multiple-pair production which can occur at higher order. Thus, the technical part of the derivation involves computing the absorption coefficient κ. The imaginary part of the polarization tensor is [18]

$$\Im(\Pi_{\|,\perp}) = \frac{\alpha}{8\pi i}\int_{-1}^{+1} d\nu \int_{\mathcal{C}} \frac{ds}{\sinh s}\mathrm{e}^{-i\varphi(s,\nu)}N_{\|,\perp}(s,\nu), \tag{4}$$

where the contour $\mathcal{C}$ is just below the real s axis: $\mathcal{C} = \mathbb{R} - i\varepsilon$. In the relevant physical limit, the functions $N_{\|,\perp}(s,\nu)$ and $\varphi(s,\nu)$ are analytic apart from simple poles on the imaginary axis at $s \in i\pi\mathbb{N}$:

$$\varphi = \frac{m^2 s}{eE}\left[1 - \frac{\tilde{\omega}^2}{m^2}\left(\frac{\cosh s - \cosh\nu s}{2s\sinh s} - \frac{(1-\nu^2)}{4}\right)\right],$$
$$N_\| = \tilde{\omega}^2\left(\cosh\nu s - \frac{\nu\sinh\nu s}{\tanh s} - 2\frac{(\cosh s - \cosh\nu s)}{\sinh^2 s}\right),$$
$$N_\perp = \tilde{\omega}^2((1-\nu^2)\cosh s - \cosh\nu s + \nu\sinh\nu s\coth s),$$

where $\tilde{\omega}^2 \equiv \omega^2\sin^2\theta$, with θ being the angle between the E field and the propagation direction, $k_z = |\mathbf{k}|\cos\theta$. Notice that the dependence on $\tilde{\omega}^2 = \omega^2\sin^2\theta$ arises from the relativistic invariants $F_{\mu\nu}k^\nu \to \tilde{\omega}^2$, and also $\tilde{F}_{\mu\nu}k^\nu \to \tilde{\omega}^2$ (note that $k_\mu k^\mu \approx 0$).

The presence of these poles at $s \in i\pi\mathbb{N}$ is an important ingredient of Schwinger pair production. The poles are closely related to instantons of the Euclidean theory which describe the tunneling of Dirac-sea electrons to the real continuum [19]. For $\omega \to 0$, the sum over the poles at $s = in\pi$ precisely corresponds to the terms in Eq. (1). If we replace the electric field E by a magnetic field B, the poles would lie on the real axis instead and there would be no pair creation for all frequencies below threshold $\tilde{\omega} < 2m$. Above threshold $\tilde{\omega} > 2m$, however, pair creation mechanisms different from the Schwinger effect set in—that is why the x-ray photons should be below threshold in order to avoid these competing effects, as the x ray may also pass through a region (beside the focus) where the B field dominates.

In the static limit, $eE \ll m^2$ and $\tilde{\omega} \to 0$, the first pole dominates, i.e. the $n = 1$ term in Eq. (1). However, for larger frequencies $\tilde{\omega} = \mathcal{O}(m)$, implying also $eE \ll \tilde{\omega}^2$, this is no longer correct and we have to sum over all poles in the lower complex half-plane. In addition to a full numerical evaluation (see below), this resummation can approximately be accomplished via the saddle point (stationary phase) technique for evaluating the integrals in (4), with eE/m^2 being the small expansion parameter. Starting with the ν integration, we get three saddle points (where $\partial\varphi/\partial\nu = 0$) at $\nu_* = \pm 1$ and $\nu_* = 0$. However, the critical points $\nu_* = \pm 1$ produce exponentially suppressed contributions, such that the dominant saddle point is given by $\nu_* = 0$. Since ν governs the asymmetry between the created electron and positron, this is consistent with phase space arguments. As a result, the ν integration produces the phase $\varphi(s, \nu_* = 0) = s(1 + \tilde{\omega}^2/(4m^2)) - \tanh(s/2)\tilde{\omega}^2/(2m^2)$, and the prefactor $\sqrt{2\pi/|\partial^2\varphi/\partial\nu^2(s,\nu_*=0)|}$, with $|\partial^2\varphi/\partial\nu^2(s,\nu_*=0)| = (\tilde{\omega}^2/2eE)|s(s/\sinh s - 1)|$. The remaining s integral can also be done by the saddle point expansion, deforming the contour $\mathcal{C}$ into the lower half-plane $\Im(s) < 0$, with dominant contribution from the saddle point $\partial\varphi(s,\nu=0)/\partial s = 0 \Rightarrow s_* = -2i\arctan(2m/\tilde{\omega})$. For small $\tilde{\omega}$, the saddle point s_* approaches $-i\pi$, while near threshold where $\tilde{\omega} \to 2m$, we have $s_* \to -i\pi/2$. The exponential factor in $\Im(\Pi)$ is

$$\exp\{-i\varphi(s_*, \nu_*)\} = \exp\left\{-\frac{m^2}{eE}\left[2\left(1+\frac{\tilde{\omega}^2}{4m^2}\right)\arctan\left(\frac{2m}{\tilde{\omega}}\right)-\frac{\tilde{\omega}}{m}\right]\right\} \sim \begin{cases}\exp\{-\frac{m^2}{eE}\pi\}, & \tilde{\omega}\to 0\\ \exp\{-\frac{m^2}{eE}(\pi-2)\}, & \tilde{\omega}\to 2m\end{cases}. \tag{5}$$

At small $\tilde{\omega}$, we recognize the familiar constant-field exponential suppression factor, $\exp\{-m^2\pi/(eE)\}$ from (1). But as $\tilde{\omega}$ approaches the threshold value $\tilde{\omega}\to 2m$, there is a dramatic enhancement, by a factor $\exp\{2m^2/(eE)\}$, which is very large in the $eE \ll m^2$ regime. This exponential enhancement is the key feature of our catalysis proposal. In spite of the exponential enhancement, our result is still purely nonperturbative and thus qualitatively different from all perturbative multiphoton effects $\propto (eE)^n$. The threshold value of the enhancement factor can also be obtained from an interaction picture analysis of the appropriate matrix element [20].

To obtain precise estimates, we need also the prefactors, not just the exponential factor. For general $\tilde{\omega}$ below threshold, the s integration yields a prefactor $\sqrt{2\pi/|\partial^2\varphi/\partial s^2(s_*, \nu_*)|}$, with $|\partial^2\varphi/\partial s^2(s_*, \nu_*)| = (m/2eE)(1+4m^2/\tilde{\omega}^2)$. Combining both prefactors, the exponential factor $e^{-i\varphi(s_*,\nu_*)}$, and $N(s_*, \nu_*)/\sinh s_*$, and the appropriate phase, we obtain

$$\Im(\Pi_{\parallel,\perp}) \approx -\frac{\alpha}{8}\frac{eE}{m^2}|N_{\parallel,\perp}|\frac{\sqrt{m/\tilde{\omega}}\sqrt{1+4m^2/\tilde{\omega}^2}}{\sqrt{|s_*(s_*/\sinh s_*-1)|}} \times \exp\left\{-\frac{m^2}{eE}\left[2\left(1+\frac{\tilde{\omega}^2}{4m^2}\right)\arctan\left(\frac{2m}{\tilde{\omega}}\right)-\frac{\tilde{\omega}}{m}\right]\right\}, \tag{6}$$

where

$$N_{\parallel}(s_*, \nu_*) = \tilde{\omega}^2\left(1-2\frac{\cosh s_*-1}{\sinh^2 s_*}\right) = -4m^2, \tag{7}$$

$$N_{\perp}(s_*, \nu_*) = \tilde{\omega}^2(\cosh s_*-1) = -\frac{8\tilde{\omega}^2m^2}{\tilde{\omega}^2+4m^2}. \tag{8}$$

Expression (6) is our main technical result. Notice that at threshold, $\tilde{\omega}=2m$, the difference between parallel and perpendicular x-ray polarization disappears and (6) simplifies to

$$\Im(\Pi_{\parallel,\perp}) \approx -\frac{\alpha}{\sqrt{\pi(\pi-2)}}eE\exp\left\{-\frac{m^2}{eE}(\pi-2)\right\}. \tag{9}$$

We have also compared the saddle point expression (6) with a direct numerical integration of the double integrals in Eq. (4). A crucial ingredient for the numerical integration consists in a suitable choice of the s contour. A parabolic shape $s=-ic_1+t-ic_2t^2$ with $0<c_1\le\pi/2$ and $c_2>0$ turns out to be convenient for standard integrators; we have typically used $c_1=\pi/2$ and $c_2=3$. In the relevant physical regime where $eE\ll m^2$, the agreement is excellent, as shown in Fig. 1.

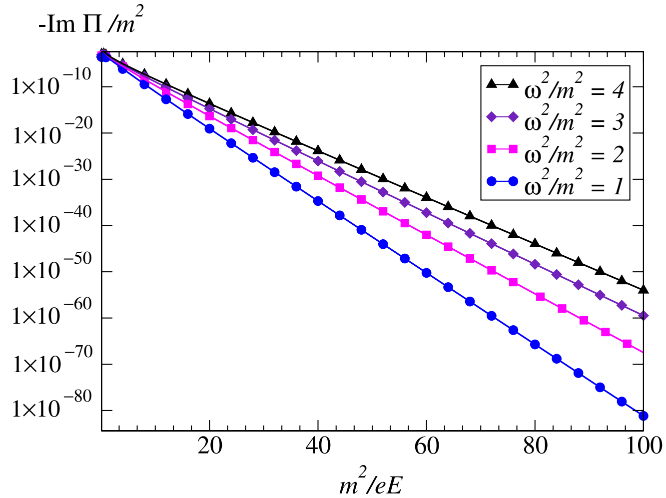

FIG. 1 (color online). Comparison of numerical evaluation of $\Im(\Pi_\parallel)$ [symbols] with the saddle point expression (6) [solid lines].

We now discuss the physical implications of the result (6) for the expected yield of electron-positron pairs, using parameters relevant to the planned ELI optical laser configuration [14]. The yield is most sensitive to the parameter eE/m^2, the ratio of the peak electric field E of the optical laser to the Schwinger critical field E_{cr}. The envisaged peak intensity at ELI is around 10^{25}–10^{26} W/cm^2, so we consider eE/m^2 of the order of 1/100 to 1/10. As the largest possible intensity is reached by maximum focussing in time and space, we assume that the temporal and spatial extent of the pulse is near $L=1$ μm (diffraction limit). Pulse synchronization can be achieved by generating the catalyzing x-ray beam also from ELI by incoherent Thomson backscattering of optical photons off an laser-accelerated electron bunch. We expect a rate of $n_{in}=10^{10}$ x-ray photons per pulse to be realistic. In Fig. 2, the solid lines show the number of pairs produced within 86 400 pulses, corresponding to one day of operation at a repetition rate of 1 Hz (the latter may be realizable, if ELI is built with diode-pumped amplifiers). If the x-ray pulse is at perpendicular incidence $\theta=\pi/2$ and close to threshold, $\omega\simeq 2m$, our x-ray catalysis pair-production sets in at $eE/m^2>0.029$. With $E=2E_{laser}$, this corresponds to a laser intensity of $I\simeq 9\times10^{25}$ W/cm^2. By contrast, standard Schwinger pair production (1) for the same parameters but without an x-ray catalysis is more than 20 orders of magnitude smaller. This is expressed by the dashed curves in Fig. 2, exhibiting the ratio of the catalyzed pair production to the standard Schwinger production. For $eE/m^2>0.101$, i.e., $I>1.1\times10^{27}$, the standard Schwinger mechanism starts to dominate, since it scales with L^4 as compared to the scaling (3) of the x-ray catalysis pair production. In fact, beyond this field strength further corrections arise,

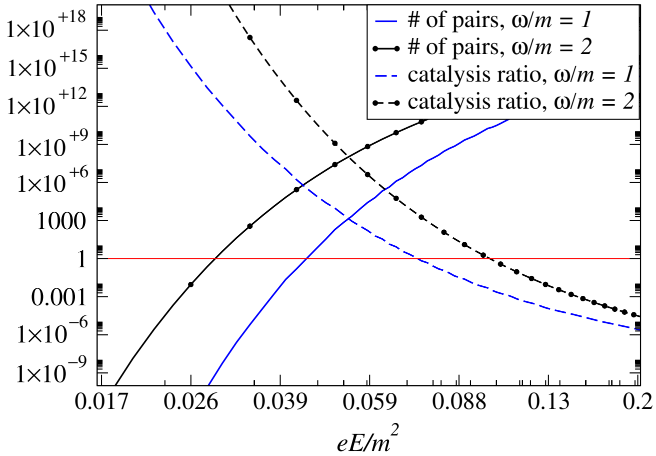

FIG. 2 (color online). Discovery potential for pair production: number of calatyzed pairs (solid lines) and the ratio of catalyzed pairs to pairs produced by the standard Schwinger mechanism (dashed lines) both as a function of the field strength. Further parameters: $n_{\rm in} = 10^{10}$/pulse, $L = 1\ \mu$m, $\theta = \pi/2$, one day of operation at a 1 Hz repetition rate.

e.g., from backreaction and multiple-pair production, but these can be neglected for the present discussion since this is beyond the envisaged next-generation experimental regime. Thus, we conclude that our catalyzed pair-production mechanism can reduce by more than an order of magnitude the required laser intensity, from $I > 10^{27}$ to $I \simeq 9 \times 10^{25}$ W/cm^2, and also yields a drastically enhanced signal in the window up to $I < 1.1 \times 10^{27}$ W/cm^2. We stress that this *exponential* enhancement is a very different mechanism from the *linear* volume factor enhancement proposed in [10] based on the pure Schwinger mechanism with a modified effective volume due to spatial focussing, and with very different laser pulse parameters from those at ELI.

Apart from potentially aiding the first observation of the Schwinger effect, which has posed a long-standing challenge, these theoretical and experimental investigations deepen our understanding of nonperturbative aspects of quantum field theory in general. For example, there is an interesting analogy with the seminal work of Voloshin and Selivanov [21] on the phenomenon of induced decay of a metastable vacuum (see also [22]). There, they considered processes whereby the decay of a metastable vacuum can be induced by the presence of another (massive) particle, which serves as a center for the nucleation process, leading to an exponential enhancement of the decay probability. In our scenario, the massless x-ray photon catalyzes, or induces, the vacuum decay process from the strong optical laser field. The prefactor contributions in [21] and (6), which are crucial for precise estimates of the pair-production yield, are different, but the exponential factors are universal. In the limit $eE \ll m^2$, and away from threshold, $\tilde{\omega} \ll m$, we can evaluate $\Im(\Pi)$ in (4) from the poles of the $1/\sinh(s)$ function, and we find the dominant contribution

$$\Im(\Pi_{\|}) \sim -2\alpha m^2 I_1^2(m\tilde{\omega}/eE)e^{-m^2\pi/eE}, \qquad (10)$$

where I_1 is the modified Bessel function. This result has recently been found also in the metastable decay picture [23].

To conclude, our proposal of a catalyzed Schwinger mechanism on the one hand introduces a strong amplification mechanism for pair production by a tunnel barrier suppression, but on the other hand fully preserves the nonperturbative character of the Schwinger mechanism. For the latter, we use purely electric field components instead of on-shell laser fields and explicitly avoid the presence of any above-threshold scales which would open up phase space for perturbative production schemes.

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