

Exact null tachyons from renormalization group flowsAllan Adams,¹ Albion Lawrence,² and Ian Swanson¹¹*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*²*Theory Group, Martin Fisher School of Physics, Brandeis University, MS057, 415 South Street, Waltham, Massachusetts 02454, USA*

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We construct exact two-dimensional conformal field theories, corresponding to closed string tachyon and metric profiles invariant under shifts in a null coordinate, which can be constructed from any two-dimensional renormalization group flow. These solutions satisfy first order equations of motion in the conjugate null coordinate. The direction along which the tachyon varies is identified precisely with the world sheet scale, and the tachyon equations of motion are the renormalization group flow equations.

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I. INTRODUCTION

It is an old idea that renormalization group (RG) flows in two-dimensional quantum field theories can be lifted to time-dependent solutions of string theory. When the RG flow describes the evolution of couplings to relevant operators, the string theory background corresponds to a nontrivial tachyon profile. This identification is precisely true in the limit of large dilaton slope [1–4]. Away from this limit, the map between RG flows and real tachyon profiles is modified: in particular, spacetime equations of motion are second order in time and space, while the RG flow equations are first order in scale [4,5].¹ One may construct the full spacetime profile in a derivative expansion for slowly varying tachyons or in conformal perturbation theory for small tachyon expectation values [2,4].

In this work we pursue a modification of these arguments for closed string tachyons in a background with a null shift symmetry. It was pointed out in [6–8] that for sigma models with such a symmetry, the equations of motion would be first order in the null directions and look more like renormalization group flows. A wide class of exact conformal field theories (CFTs) of this kind, with a timelike linear dilaton, has been worked out in [9–15] (see also [16–19] for related studies). Because of the null shift symmetry and the relatively simple tachyon profile, the beta functions receive no corrections beyond one-loop in α' (much as the beta functions for the plane wave backgrounds of [20] are one-loop exact). We generalize this work to describe null tachyon profiles with a null isometry and a timelike dilaton given any renormalization group flow. In these flows, the direction along which the tachyon varies is mapped *precisely* to the world sheet scale by a Lagrange multiplier constraint, and the profile satisfies first order equations equivalent to the RG equations.

Note that these backgrounds will be exact in α' , but not necessarily in g_s . In particular, we expect the dilaton to run to strong coupling in the past or future of these solutions. As usual, however, we can adjust the constant mode of the

dilaton to push this strong coupling region as far into the past or future as we wish.

II. WORLD SHEET DESCRIPTION OF NULL TACHYONS

In constructing our string theory background, we begin with the tensor product of a two-dimensional target space and a conformal field theory \mathcal{C} . Assume this conformal theory has a set of local primary operators \mathcal{O}_a . Let us write the two-dimensional target space with the metric

$$ds^2 = -2dX^+ dX^-. \quad (2.1)$$

We now couple these theories by:

- (i) Deforming \mathcal{C} by couplings $\int d^2\sigma u^a(X^+) \mathcal{O}_a$ depending only on X^+ .
- (ii) Turning on a dilaton of the form $\Phi(X^+, X^-) = \gamma X^- + \tilde{\Phi}(X^+)$. The coefficient γ is arbitrary, and can be shifted by rescaling $X^- \rightarrow \lambda X^-$, $X^+ \rightarrow X^+/\lambda$; γ has mass dimension 1 if X^\pm has length dimension 1.

As in [10–13,16–18], the dilaton is linear in X^- . The X^+ dependence of the tachyon will be nonlinear, as the slope must shift between $X^+ = \pm\infty$ to make up for the change in the central charge associated with the sector \mathcal{C} [4,16].

The full world sheet action is²

$$S_{\text{pert}} = S_{\text{CFT}} + \frac{1}{4\pi\alpha'} \int d^2z \sqrt{g} [-2g^{\alpha\beta} \partial_\alpha X^+ \partial_\beta X^- + \alpha'(\gamma X^- + \tilde{\Phi}(X^+))R^{(2)} + u^a(X^+) \mathcal{O}_a]. \quad (2.2)$$

Note that \mathcal{O}_a will generally include the identity operator. Here S_{CFT} is the action for \mathcal{C} . However, our discussion will

²In Refs. [10–13,16–18], there is also a term $(\partial X^+)^2$ in the action corresponding to a metric G_{++} which is induced at large X^+ . This does not appear in our calculation. We believe this amounts in part to a choice of scheme. A term $G_{++}(X^+)(\partial X^+)^2$ can be removed by a field redefinition $X^- \rightarrow X^- + f(X^+)$, where $\partial_+ f = G_{++}$, without otherwise changing the form of the action. We would like to thank A. Frey for asking about this point.

¹That is, $[R, G] \neq 0$.

work just as well if \mathcal{C} has no Lagrangian description. In this latter case, the presence of S_{CFT} merely denotes that for fixed X^+ , X^- , the theory is the conformal field theory \mathcal{C} perturbed by $\int d^2\sigma u^a(X^+) \mathcal{O}_a$.

The tachyon and metric couplings in (2.2) have a symmetry under shifts of X^- . This appears to be broken by the term in the dilaton linear in X^- . However, note that a shift in X^- simply adds a total derivative (the Euler character of the world sheet) to the action. Thus, this action respects an overall shift symmetry in X^- , which will be reflected in the world sheet beta functions. Note, however, that the string genus expansion will not respect this shift symmetry, so one must treat the strong coupling region with caution.

We will assume that we know, in advance, complete information about S_{CFT} perturbed by arbitrary couplings which might depend on the world sheet coordinates. In particular, this means that we assume knowledge of the full set of beta functions β^q for \mathcal{C} perturbed by the terms $\int d^2\sigma \bar{u}^a \mathcal{O}_a$, where \bar{u} are constant (c -number) couplings. This includes the beta functions for the identity and for the dilaton. The dilaton beta function is proportional to the Zamolodchikov c -function for the perturbed CFT, as we will discuss below.

The beta functions are also dependent on the contribution of the degrees of freedom of the perturbed theory $S_{\text{CFT}} + \frac{1}{4\pi\alpha'} \int d^2z u^a(X^+) \mathcal{O}_a$ to the beta function for the operator $(\partial X^+)^2$ (e.g., for the spacetime metric G_{++}). Even so, as we will argue below, this beta function is determined completely by the above information.

Finally, we will consider (2.2) fixed to conformal gauge: namely, the world sheet metric is

$$g_{\alpha\beta} = e^{2\phi} \hat{g}_{\alpha\beta}. \quad (2.3)$$

The partition function is given by

$$Z = \int d\phi \bar{Z}, \quad (2.4)$$

$$\bar{Z} = \int DX^+ DX^- DYDbDc e^{-S_{\text{pert}}(X^+, X^-, Y) - S_{\text{FP}}}.$$

Here b , c are the conformal ghosts, and S_{FP} is their action. Y stands for the degrees of freedom on \mathcal{C} . (Again, this is for ease of exposition: the central arguments of this work do not require that \mathcal{C} have a Lagrangian description.) For a good string background, \bar{Z} must be independent of ϕ .

III. INTEGRATING OUT X^-

In the action (2.2), X^- appears as a Lagrange multiplier. By analytically continuing the theory to Lorentzian signature to do this integral, we find that

$$\bar{Z} = \int DX^+ + DbDcDY$$

$$\times \delta(\sqrt{g} \partial^2 X^+ - \frac{\gamma\alpha'}{2} \sqrt{g} R) e^{-\bar{S} - S_{\text{FP}}}, \quad (3.1)$$

where

$$\bar{S} = \int d^2\sigma \sqrt{g} \left[\frac{1}{4\pi} \tilde{\Phi}(X^+) R^{(2)} + u^a(X^+) \mathcal{O}_a \right]. \quad (3.2)$$

In conformal gauge, we have

$$\sqrt{g} R = \sqrt{\hat{g}} (\hat{R} - 2\hat{\Delta}^2 \phi), \quad (3.3)$$

where \hat{R} is the two-dimensional curvature for the fiducial metric \hat{g} , and $\hat{\Delta}^2$ is the associated Laplacian. In two dimensions, $\sqrt{g} \partial^2 = \sqrt{\hat{g}} \hat{\Delta}^2$. We can therefore write the delta function in (3.1) as

$$\frac{1}{\det(\sqrt{g} \partial^2)} \delta(X^+ + \gamma\alpha' \phi + Q), \quad (3.4)$$

where Q depends only on the fiducial metric. Note that while $\sqrt{g} \partial^2$ is Weyl invariant, the determinant requires a regulator and thus contributes to the Weyl anomaly.

The main lesson of this section is that for the action at hand, X^+ is identified precisely with world sheet scale. This is the physical basis of the observation below that, for good string backgrounds, $u^a(X^+)$ will satisfy the first order RG equations, with X^+ functioning as the renormalization group scale.

IV. BETA FUNCTIONS

We wish to find the conditions under which $\frac{\delta}{\delta\phi(\sigma)} \bar{Z} = 0$:

$$\frac{\delta}{\delta\phi(x)} \bar{Z} = \int DX^+ DYDbDc ([\delta_{\phi(x)} - (\delta_{\phi(x)} \tilde{S})]$$

$$\times \delta(X^+ + \gamma\alpha' \phi - Q)) \frac{1}{\det \partial^2} e^{-\bar{S} - S_{\text{FP}}}$$

$$+ \delta_{\phi(x)}^{(q)} \bar{Z} = 0. \quad (4.1)$$

Here $\delta_{\phi(x)}^{(q)}$ denotes the part of the variation induced by quantum effects³:

$$\delta_{\phi(x)}^{(q)} \bar{Z} = \frac{1}{8\pi} \beta^{q,\Phi} [u, \tilde{\Phi}, \gamma] \sqrt{g} R^{(2)} + \frac{1}{8\pi\alpha'}$$

$$\times \beta_{++}^q [u, \tilde{\Phi}, \gamma] \sqrt{g} (\partial X^+)^2 + \frac{1}{2} \beta^{q,a} [u] \sqrt{g} \mathcal{O}_a. \quad (4.2)$$

The quantum contribution $\beta^{q,\Phi}$ to the dilaton beta function comes from the determinant in (3.4), and from the perturbations to \mathcal{C} . It can thus be written as

$$\beta^{q,\Phi} = \frac{c[u] - 24}{6}, \quad (4.3)$$

where $c[u]$ is the Zamolodchikov c -function for the theory \mathcal{C} ; perturbed by the operators $u^a \mathcal{O}_a$; the factor of 24 comes from the Fadeev-Popov ghosts and from the determinant

³We are using the conventions found in [4,21].

factor in (3.4) and (4.1), that arose from integrating out X^- . The beta function β_{++}^q comes from the contribution of this perturbed CFT to the beta function for the operator $(\partial X^+)^2$. Finally, $\beta^{q,a}$ are the contributions to the beta functions for the couplings u^a , coming from this perturbed CFT. These

are believed to be gradients of c , but to date there is no proof of that beyond leading order in conformal perturbation theory [22].⁴

We can exchange the ϕ derivative of the delta function in (4.1) for an X^+ derivative and integrate by parts:

$$\begin{aligned} & \int DX^+ DYDbDc \delta_{\phi(x)} \delta(X^+ + \gamma\alpha'\phi - Q) \frac{1}{\det\partial^2} e^{-\tilde{S}-S_{\text{FP}}} \\ &= \gamma\alpha' \int DX^+ DYDbDc \delta_{X^+} \delta(X^+ + \gamma\alpha'\phi - Q) \frac{1}{\det\partial^2} e^{-\tilde{S}-S_{\text{FP}}} \\ &= \int DX^+ DYDbDc \delta(X^+ + \gamma\alpha'\phi - Q) \frac{1}{\det\partial^2} e^{-\tilde{S}-S_{\text{FP}}} \gamma\alpha' \left(\frac{1}{4\pi} \dot{\Phi}(X^+(\sigma)) \sqrt{g}R + \dot{u}^a(X^+) \mathcal{O}_a(\sigma) \right), \end{aligned} \quad (4.4)$$

where the dots denote derivatives with respect to X^+ . Next, the classical variation of \tilde{S} with respect to ϕ is

$$\begin{aligned} \delta_{\phi(x)} \tilde{S} &= -\frac{1}{2\pi} \sqrt{g} \partial^2 \tilde{\Phi} \\ &= -\frac{1}{2\pi} \sqrt{g} (\ddot{\Phi}(X^+) (\partial X^+)^2 + \dot{\Phi} \partial^2 X^+), \end{aligned} \quad (4.5)$$

where we have used (3.3) in the first line. This expression will multiply the delta function in (4.1). As applied to Eq. (4.1), the second term in (4.5) can therefore be replaced by

$$\frac{\gamma\alpha'}{4\pi} \dot{\Phi} \sqrt{g} R, \quad (4.6)$$

i.e., by a contribution to the dilaton beta function.

Collecting all of the terms in (4.1), we find that the variation of \tilde{Z} vanishes if

$$\begin{aligned} 2\gamma\alpha' \dot{u}^a(X^+) + \beta^{q,a} &= 0, \\ 4\gamma\alpha' \dot{\Phi} + \beta^{q,\Phi} &= 0, \\ -4\alpha' \ddot{\Phi}(X^+) + \beta_{++} &= 0. \end{aligned} \quad (4.7)$$

The left-hand sides are the beta functions for the full theory (2.2). The first line is the full beta function for \mathcal{O} , the second line is the dilaton beta function, and the final line is the beta function for $(\partial X^+)^2$.

We claim that the β^q arise entirely from the divergences in the perturbed CFT due to singular operator products of the \mathcal{O}^a , so that as functions of u , β^q are the same as for constant couplings. This is true if there are no additional divergences arising from contractions of X^+ . (If X^+ was instead timelike, for example, such divergences would appear [4].) If the beta functions are computed perturbatively, this should be the result of the diagrammatic arguments given in [9–13]. We have two additional arguments.

One is to note that there are no X^+X^+ correlators by the following argument. The delta function in (3.1) equates X^+ to the scale factor, ϕ , plus a constant. But when the beta functions vanish, the integrand \tilde{Z} is completely independent of ϕ , and so the two-point function $\langle X^+(\sigma)X^+(\sigma') \rangle$ is independent of σ, σ' (the factor Q can be absorbed into ϕ). While the beta functions are computed away from the conformal point, this means that any divergences from contractions of X^+ must vanish at the conformal point, and do not yield independent terms in the beta function equations.

Another argument is as follows. It would appear that the path integral (3.1), together with (3.2), allows for non-vanishing operator product expansions (OPEs) of X^+ , since the delta function makes the dilaton coupling equivalent (after integrating by parts) to a standard kinetic term for X^+ ,

$$S_{\text{kin},\Phi} \propto \int d^2\sigma \dot{\Phi}(\partial X^+)^2. \quad (4.8)$$

Near the RG fixed points at $X^+ = \pm\infty$, such a term is also induced from integrating out the degrees of freedom in C (see for example [12]), which cancels the dilaton contribution. As one flows in X^+ , the second line of (4.7) indicates that the additional contributions to G_{++} from C are canceled by the dilaton contribution. Again, one should be careful since the beta functions are computed away from the conformal point. As above, however, the additional divergences from X^+X^+ correlators should not give any additional contributions to the beta function equations describing the conformal point itself.

We should also make sure that the arguments in this section and in Sec. III hold for correlation functions—that is, that the beta functions which we compute hold for the Callan-Symanzik equation for correlation functions of local operators.⁵ The essential point is that for local correlators [string scattering amplitudes are finite-dimensional integrals of local correlators; recall also that three such

⁴Note that this proof breaks down at higher orders [4]. Furthermore, in at least one case it is likely that the beta functions are not gradients of any function of the relevant and marginal couplings alone [23].

⁵We would like to thank Joe Polchinski for reminding us of this issue.

operators are used to fix the $SL(2, \mathbb{C})$ invariance of the world sheet CFT], the delta function constraint (3.4) should only be modified at the point of the operator insertions. For example, if the world sheet was regulated with a lattice cutoff, one would integrate over all values of X^- point by point. Therefore, the delta function (3.4) is only modified at the operator insertion points. In the Callan-Symanzik equations, modifications of the scale transformations which occur at the points of operator insertions give contributions to the anomalous dimensions of operators, rather than to the beta functions.⁶

Finally, Eqs. (4.7) appear to have more equations than unknowns. This is typical of the beta function equations in string theory. In fact, the left-hand side of the second equation in (4.7) is conserved as a consequence of the first and third equations: the vanishing of the beta functions on a flat two-dimensional world sheet is sufficient to ensure that the theory is a CFT. One may then set it to zero by adjusting the linear term in $\tilde{\Phi}$. This fact has been checked at one-loop for sigma models in [22], and for tachyons in [4]. Furthermore, one can compute β_{++}^q to leading order in X^+ derivatives following the discussion in [4]. Combined with the leading order result [24]

$$\partial_a c[u] = 24\pi^2 g_{ab} \beta^b[u], \quad (4.9)$$

one can show that the third line in (4.7) follows from the first two lines, specifically by taking the derivative of the second line. More generally, one can argue that the vanishing of the first two equations in (4.7) implies the vanishing of the last equation, as follows. The Wess-Zumino condition implies that the derivative of the second term with respect to X^+ is proportional to a linear combination of the beta functions of the theory. Indeed, it should be a linear combination of *all* of the beta functions of the theory; if any one relevant or marginally relevant coupling is turned on, the theory will flow, and $\beta^{q,\Phi}$ will cease to be constant. Thus, if one sets the beta function for all G_{++} to zero, the derivative of the dilaton beta function will be proportional to G_{++} .

It is worth noting that the beta function β_{++}^q is determined by consistency of (4.7): taking the derivative of the second equation, and using the first equation together with (4.3), we find that

$$\beta_{++}^q = \frac{1}{12\gamma^2 \alpha'} \beta^{q,a}, \partial_a c. \quad (4.10)$$

It would be interesting to prove this directly. The γ dependence arises from the relationship between X^+ and the scale factor ϕ , induced by integrating out X^- .

⁶Similarly, if the beta functions are constrained due to a global symmetry, the constraints *on the beta functions* remain even when one computes correlation functions of local operators that transform nontrivially under this global symmetry.

The final result is that the spacetime evolution generated by a null tachyon profile is completely determined by the *first* order equations of the associated RG flow, with the dependence of the tachyon on X^+ identical to that of the renormalized coupling on the RG scale $\phi = X^+ / (\gamma \alpha')$.

V. CONCLUSIONS

We have shown that any RG flow arising from the perturbation of a CFT by a relevant operator also defines an *exact* CFT describing the spacetime evolution of a null tachyon condensate, and that the resulting tachyon and dilaton profiles satisfy *first* order equations determining their evolution along the null direction. This generalizes the work of [9–19]. As pointed out in [12], these are stringy “bubbles of nothing” (when the coupling to the identity operator in \mathcal{C} flows)⁷—analogs of [25]—in which dimensions of spacetime are destroyed by an expanding domain of tachyon condensate satisfying first order equations of motion. It would be interesting to study further examples, such as the null tachyon generated from the RG flow of the $\mathbb{C}\mathbb{P}^n$ model.⁸ In this example, the Kähler class is known to flow precisely logarithmically with RG scale [26,27], and it will therefore evolve linearly in X^+ . This flow goes from a $c = 2n$ sigma model in the ultraviolet to a trivial $c = 0$ Landau-Ginzburg theory in the infrared [27].

We should note that there often exists a scheme in which the tachyon beta functions can be linearized, unless the tachyons mix nontrivially under the OPEs, and are marginally relevant or satisfy some kind of resonance condition (see [4] for a review and discussion). When the beta functions can be linearized, the X^+ dependence of the tachyon will be a simple exponential. Furthermore, unlike the timelike examples in [4], X^+ is identified precisely with the RG scale so that the infrared fixed point will be reached only at null infinity. In this scheme there is less to be lost. When there are universal higher-order terms in the beta function, the X^+ dependence will of course be more complicated.

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⁷We would like to thank Simeon Hellerman for reminding us of this.

⁸We would like to thank Eva Silverstein for suggesting this example.

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