PHYSICAL REVIEW D **80,** 106003 (2009)

Correlation classes on the landscape: To what extent is string theory predictive?

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(Received 8 September 2008; published 16 November 2009)

In light of recent discussions of the string landscape, it is essential to understand the degree to which string theory is predictive. We argue that it is unlikely that the landscape as a whole will exhibit unique correlations amongst low-energy observables, but rather that different regions of the landscape will exhibit different overlapping sets of correlations. We then provide a statistical method for quantifying this degree of predictivity, and for extracting statistical information concerning the relative sizes and overlaps of the regions corresponding to these different correlation classes. Our method is robust and requires no prior knowledge of landscape properties, and can be applied to the landscape as a whole as well as to any relevant subset.

DOI: 10.1103/PhysRevD.80.106003 PACS numbers: 11.25.Wx, 11.25.Mj

Over the past few years, the existence and implications of a vast string theory ''landscape'' have attracted considerable attention [1]. Indeed, research in this area has spanned a considerable range of topics and followed a number of different approaches [2–16]; for recent reviews, see Ref. [17]. However, because the specific low-energy phenomenology that can be expected to emerge from string theory depends critically on the particular choice of vacuum state within the landscape, and because the space of possible string vacua is extremely large (with some estimates putting the number of phenomenologically interesting vacua at 10^{500} or more [2]), the question which naturally arises is a critical one. To what extent can we say that string theory is predictive? In what sense can we say that certain low-energy phenomenological features of the observed universe are predicted by, or derivable from, string theory?

In this paper, we shall begin with a short discussion which outlines some of the recent ideas concerning general notions of predictivity on the string landscape. We will then argue that it is unlikely that the landscape as a whole exhibits unique correlations amongst low-energy observables, but rather that different regions of the landscape will exhibit different overlapping sets of correlations. Finally, we will provide a statistical method for quantifying this degree of predictivity, and for extracting statistical information concerning the relative sizes and overlaps of the regions corresponding to these different correlation classes.

I. INTRODUCTION: PREDICTIVITY AND THE STRING LANDSCAPE

The question of predictivity goes to the heart of what it means to be doing science. As such, there can be no more critical question for string theory than this. Of course, string theory is perhaps unique amongst most theories of physics because of the fact that its characteristic energy scales are essentially unreachable by present-day experimental technology. As a result, many of the direct experimental consequences of string theory lie at presently inaccessible energy scales. However, it is not clear that *all* experimental consequences of string theory will lie at inaccessible energy scales. And even if all of the firm experimental consequences of string theory were somehow proven to lie at scales exceeding those reachable by current accelerator technology, this would not free string theory from its obligations to make predictions which are testable at those higher energy scales—i.e., testable in principle, if not in practice.

On the one hand, even accepting this standard, one might argue that it is too much to ask that string theory be predictive in and of itself. From this perspective, one should rightly compare string theory not with a specific quantum field-theoretic model such as the standard odel, but with quantum field theory itself—indeed, both string theory and quantum field theory can be viewed as languages or frameworks within which the subsequent act of model building takes place. Just as the Lagrangian of the standard model is just one out of many possible self-consistent quantum field-theoretic Lagrangians, the correct string model might be just one out of many possible self-consistent string vacua. Thus, according to this argument, string theory is just as predictive as quantum field theory: neither becomes predictive until a particular model is constructed, and all predictions that ensue can be expected to hold only within that model.

While this argument has some validity, one could just as well argue that it misses a critical point. While quantum field theory tolerates many free parameters, string theory does not: generally, all free parameters in string theory (such as gauge couplings, Yukawa couplings, and so forth) are determined by the vacuum expectation values of scalar

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fields and are thus expected to have dynamical origins within the theory itself. Moreover, while many architectural details of a given model (such as the gauge group, the number of generations, or even the degree of supersymmetry) are uncorrelated within quantum field theory, string theory has deeper underpinnings in terms of the geometric properties and configurations of strings and branes. It therefore becomes meaningful to ask more from string theory than from quantum field theory. Of course, many string models are plagued with flat directions. Such flat directions could imply that not all continuous variables will develop finite, dynamically fixed expectation values. However, the hypothesized existence of the landscape already presupposes that such flat directions are ultimately lifted, and that multiple stable string ground states exist after moduli stabilization. Thus, in the remainder of this paper, we shall assume that all flat directions have been lifted.

Given the existence of the landscape, it is certainly too much to demand that string theory gives rise to predictions for such individual quantities as the number of particle generations. Indeed, we already know that such individual quantities can vary greatly from one string vacuum to the next. However, it is perhaps not too much to ask that string theory manifests its predictive power through the existence of *correlations* between physical observables that would otherwise be uncorrelated in quantum field theory. Such correlations would be the spacetime phenomenological manifestations of the deeper underlying geometric structure that ultimately defines string theory and distinguishes it from a theory whose fundamental degrees of freedom are based on point particles. Of course, it is logically possible that string theory leads to sharp correlations amongst observables at *high* energy scales, but that the mathematical form of the connections between these high-scale observables and experimentally accessible low-scale observables completely washes these correlations away as far as a low-energy physicist might be concerned. However, there is no evidence that nature is so cruel for the low-energy parameters of interest. Thus, our question concerning the predictivity of string theory boils down to a single critical question: to what extent are there correlations between different physical observables on the string-theory landscape?

II. CORRELATION CLASSES ON THE STRING LANDSCAPE

Clearly, the existence of sharp correlations across the string theory landscape would imply that string theory is predictive, while the absence of such correlations would suggest that it is not. Indeed, many recent discussions of this issue have proceeded under the assumption that these are the only two logical options.

However, we believe that neither of these two options is likely to represent the true nature of correlations on the string landscape. Rather, we believe that the true nature of such correlations lies somewhere between these two extremes and is more likely to resemble that shown in Fig. 1. In Fig. 1, some regions of the landscape exhibit certain correlations and other regions of the landscape exhibit other correlations. For example, we can imagine that one region might principally correspond to perturbative heterotic strings (in which world sheet symmetries such as conformal invariance and modular invariance play a decisive role in producing correlations amongst low-energy observables), while another region might principally correspond to intersecting D-brane models (in which decisive roles are instead played by tadpole anomaly constraints). Of course, it is a naïve expectation that different correlation-class regions will correspond neatly to different underlying string-construction methods, and more subtle mappings between construction methodologies and correlation regions will undoubtedly occur. For this reason, it is important that such regions be defined according to their low-energy phenomenological predictions and correlations, not according to their construction methodologies. Thus these regions need not be disjoint, and indeed nontrivial overlaps will occur.

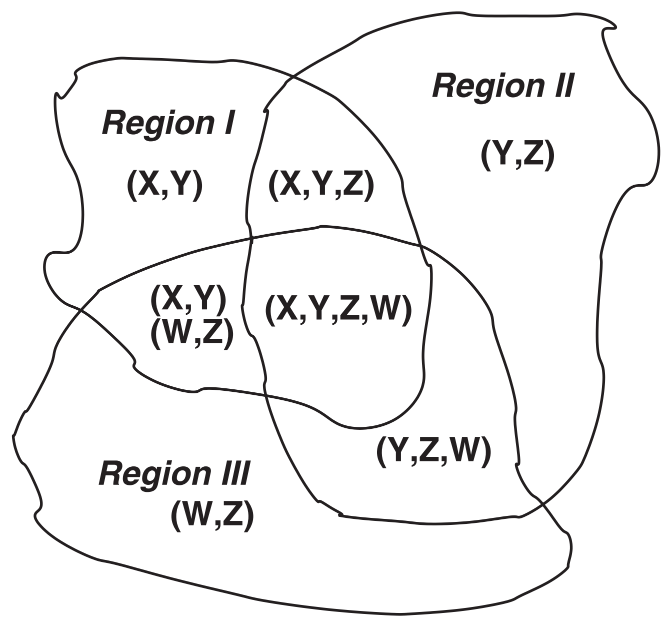

FIG. 1. A sketch of a landscape in which different regions exhibit different correlations between phenomenological observables X, Y, Z, and W. As discussed in the text, the overlaps between these regions can then exhibit correlations amongst larger subsets of observables or multiple independent correlations involving smaller subsets of observables. For example, while each region separately exhibits a correlation amongst two observables, the overlap between regions I and II exhibits a single correlation between three observables, while the overlap between regions I and III exhibits two independent correlations, each involving only two observables. Many other generalizations and geometric configurations are possible.

It is, of course, impossible to speculate at this stage concerning the number of such distinct correlation classes. However, preliminary hints from concrete studies of large sets of actual string models [6–9,11–13] suggest that the number of distinct regions might actually be quite large. Indeed, one feature that emerges from such studies is a relative lack of strong correlations between the various phenomenological quantities which have been examined. It should be noted that the models typically examined in such studies contain unstabilized moduli. However, the relative absence of strong correlations in such models suggests that a number of different correlation classes are present, even over partial regions of the landscape. Given this, there is no reason to believe that such regions are disjoint, and one can expect that many of these regions are also likely to have nontrivial overlaps.

Given such a picture, the precise nature of correlations at a given point on the landscape is likely to depend rather sensitively on the location of that point relative to the boundaries of all possible nearby regions. For example, in Fig. 1, we observe that two phenomenological properties X and Y are correlated in Region I, while Y and Z are correlated in Region II and W and Z are correlated in Region III. Even though each of these regions exhibits only a single correlation involving two phenomenological quantities, we see that the overlapping sections of these regions nevertheless exhibit a number of different correlation patterns:

$$\begin{aligned} \text{overlap I \& II:}&\ \text{single three-quantity correlation}\,(X, Y, Z)\\ \text{overlap II \& III:}&\ \text{single three-quantity correlation}\,(Y, Z, W)\\ \text{overlap I \& III:}&\ \text{two two-quantity correlations}\,(X, Y)\ \text{and}\ (Z, W)\\ \text{overlap I, II, \& III:}&\ \text{single four-quantity correlation}\,(X, Y, Z, W). \end{aligned} \tag{2.1}$$

Strictly speaking, such a situation fails to yield a single correlation which holds across the landscape as a whole. As such, this situation is one in which it might be claimed that string theory as a whole is nonpredictive. However, even in such a situation, we can still claim that string theory is partially predictive if the sizes of these correlation-class regions are relatively large compared with the landscape as a whole. If there exist huge subregions of the landscape across which correlations hold, then we can claim that string theory is entirely predictive within each such domain. Of course, one can also restrict attention to portions of the landscape that can give rise to realistic phenomenologies. The relevant comparison for assessing potential predictivity in this case is then the size of the restricted landscape and the sizes of the subregions with different correlations. At the opposite extreme, however, it may turn out that the fundamental regions across which such correlations hold are relatively small. For example, one could imagine a situation in which each region is so small that it contains no more than a single model. In such a case, we would then claim that string theory is entirely nonpredictive.

III. QUANTIFYING THE STRUCTURE OF THE STRING LANDSCAPE

In the remainder of this paper, we would like to attach a quantitative measure to this notion of predictivity. Specifically, we would like to develop a method for determining the extent to which a situation such as that sketched in Fig. 1 emerges in an actual set of string models. Ideally, such a method should be generally applicable and allow one to determine the number of different correlation classes.

While there are many ways to develop such a mathematical model, we shall proceed in a general fashion as follows. At a practical level, we can imagine that we have sampled a certain number $x \gg 1$ of models, randomly selected across the landscape as a whole.[1] Let us assume that we have analyzed the physical observables predicted from these x models, and we have not observed any correlations that hold across this set of models. Clearly, this means that not all x of our models come from the same correlation-class region; at least one model must originate from a different region.

We can then ask for the probability that there exists a *partitioning* of our data set into two groups of models such that there exist correlations which hold across each group separately. If no such two-way partitions exist, we could then attempt to construct three-way partitions which have the same property, and so forth. In general, we can seek to derive the probability $P_x(n)$ that we can partition our x models into n distinct classes, each of which individually exhibits correlations across the class as a whole. This question is sketched schematically in Fig. 2.

We can immediately make a number of statements concerning $P_x(n)$. First, $P_x(n)$ will clearly grow monotonically as a function of n. This follows from the observation that if a given set of x models can be successfully partitioned into

[1] In stating that these models are selected randomly, we are disregarding the critical issue that arises due to the fact that our sampling techniques will inevitably introduce biases that distort the apparent space of models in nontrivial ways. Methods of overcoming these difficulties were developed in Ref. [10], and we shall assume in the remainder of this paper that such methods have already been utilized and all such distortions have been eliminated as far as possible.

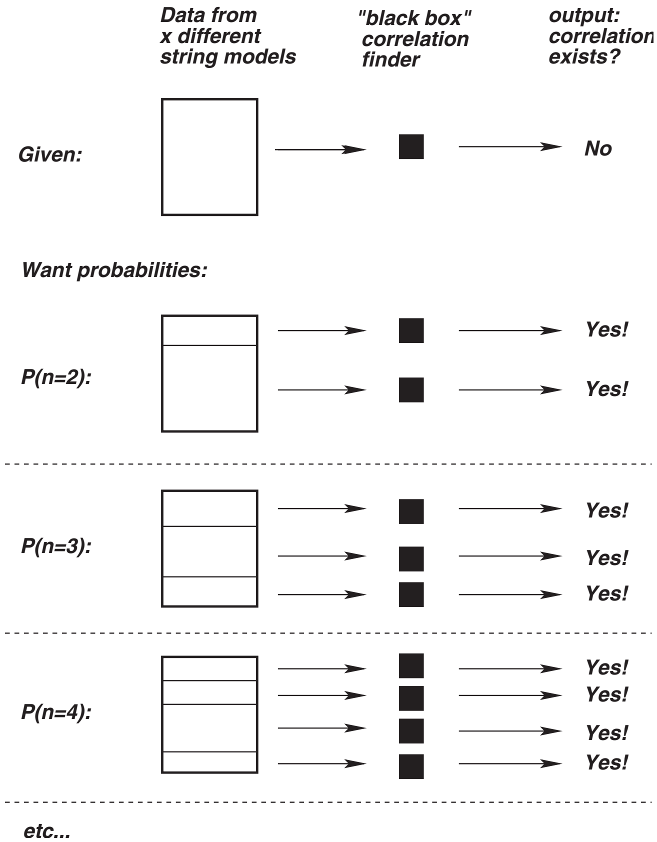

FIG. 2. Schematic illustration of the fundamental problem. Suppose data from x string models does not exhibit any correlations amongst low-energy physical observables which hold across all x models. What is the probability $P_x(n)$ that we can partition our x models into n distinct classes, each of which individually exhibits correlations across the class as a whole? Clearly $P_x(n)$ grows as a function of n, ultimately reaching $P_x(n) = 1$ for $n = x$ (i.e., the case in which each class is no larger than a single model). The behavior of $P_x(n)$ as a function of n for $1 < n < x$ determines the extent to which the landscape sketched in Fig. 1 is predictive, with larger $P_x(n)$ for small n indicating a larger degree of predictivity.

n correlation classes, then it can necessarily be successfully partitioned into any greater number of sets within which correlations hold. Second, we observe that $P_x(n)$ should ultimately reach $P_x(n) = 1$ for $n = x$. This corresponds to the case in which each correlation class is no larger than a single model—although completely nonpredictive, such a partition is indeed guaranteed to be successful. Finally, we are intrinsically assuming that $P_x(1) = 0$. This essentially serves as an initial condition.

What interests us, however, is the *behavior* of $P_x(n)$ as a function of n for $1 < n < x$, as this determines the extent to which the landscape sketched in Fig. 1 is predictive. Indeed, larger values of $P_x(n)$ for small n can be associated with a greater degree of predictivity for the landscape as a whole, in the sense that our correlation classes on the landscape are larger rather than smaller. Ultimately, we are interested in identifying behaviors of $P_x(n)$ which are stable as x increases. Such signatures could then provide a handle through which one could potentially extract the number and distribution of correlation classes on the landscape as a whole.

It is important to reiterate that we are defining our correlation classes of models in terms of their spacetime phenomenological predictions rather than their underlying world sheet or D-brane constructions. Needless to say, it is only in this manner that we can declare two different models to be phenomenologically distinct. But at a deeper level, we observe that this method of defining our correlation classes overcomes whatever theoretical prejudices we might have concerning which phenomenological properties are associated with which model-construction techniques. Indeed, one might argue that the very notion of string theory being predictive rests on the existence of correlation classes which *transcend* the somewhat artificial boundaries associated with different underlying model-construction methods.

We also stress that in this paper, we shall not be concerned with the inner workings of the "correlation finder" sketched in Fig. 2. Likewise, we shall not be concerned with the question of *how* to partition our x models into the n test classes which are then each individually tested for internal correlations. Needless to say, these are very important questions—the former is critical for data analysis in general, and the latter might potentially be addressed through direct enumeration of different partitioning possibilities or on the basis of other external physical information. However, our purpose in this paper is to study the mathematical extent to which we can learn about the properties of the underlying landscape, assuming that such data-analysis tools are at our disposal.

We shall now calculate the probabilities $P_x(n)$. In order to do so, we shall first need to quantify the sizes and overlaps between the correlation regions sketched in Fig. 1. Let us therefore assume that a given randomly selected string model has a probability p_i of being a member of the ith correlation class. In some sense, the p_i quantify the normalized "sizes" of the individual correlation-class regions across the string landscape. We shall also need to quantify the sizes of two-region overlaps, three-region overlaps, and so forth. Towards this end, we shall let p_{ij} denote the probability that a randomly selected string model is simultaneously a member of both the ith and jth correlation classes (where $i \neq j$), p_{ijk} denote the probability that such a string model is simultaneously a member of the ith, jth, and kth correlation classes (where i, j, and k are all unequal), and so forth.

In general, these quantities $p_{ijk...}$ can vary significantly as functions of their indices across the landscape. However, for the purposes of calculating the overall probabilities $P_x(n)$, what really concern us are the "average" values of these quantities. We shall therefore assume a uniform average distribution in which

$$p_{i_1,i_2,\ldots,i_N} = a_N p, \tag{3.1}$$

where p is an overall arbitrary probability, and where the a-coefficients satisfy the constraints

$$0 \le \ldots \le a_4 \le a_3 \le a_2 \le 1, \tag{3.2}$$

with $a_1 \equiv 1$. There is also another constraint on the a-coefficients which will be explained shortly.

In order to understand these assumptions, it will help to consider an abstract geometric picture of the landscape in which each string model occupies a volume of arbitrary dimensionality but fixed, uniform magnitude. For example, we may imagine that each string model is represented by a square of unit area, or by a cube of unit volume. We can also freely reposition these squares or cubes so that models in the same correlation class are adjacent to each other, filling out contiguous regions as in Fig. 1. We shall refer to the entire space of models arranged this way as the "correlation space". Note that the correlation space is *not* the usual geometric picture of the landscape in which the different directions might be parametrized by different low-energy observables, or alternatively by different string-construction parameters (e.g., fluxes). Indeed, in such a picture, models which are in the same correlation classes can be scattered across the landscape and need not occupy contiguous regions. By contrast, in the correlation space, each model occupies an equal volume of arbitrary (irrelevant) dimensionality, and models can be freely repositioned so that models in the same correlation class (according to their low-energy observables) occupy neighboring contiguous regions, as in Fig. 1.

In terms of the correlation space, our probability distributions can be understood geometrically as follows. If we imagine the entire correlation space to occupy a normalized volume $V = 1$, then p_i is nothing but the volume of the ith correlation region, p_{ij} is nothing but the volume of the (i, j) overlap region, and so forth. Likewise, our assumptions in Eqs. (3.1) and (3.2) indicate that p is the average volume of each correlation class, individually, while $a_n p$ is the average volume of each overlap region between n different correlation classes.

Note that the volume of each overlap region must scale linearly with p (the volume of each individual region). This is the case because any two different correlation-class regions of a given dimensionality must have an overlap region which also shares that dimensionality. This in turn follows from the fact that each model occupies the same dimensionality in correlation space, and that all of our regions (whether disjoint or overlapping) are quantized in terms of models. (In other words, any overlap region must still be composed of individual squares or cubes, and thus must have the same dimensionality as the correlation classes themselves.) Indeed, this is the major advantage of working with the correlation space rather than the usual geometric visualization of the landscape in which models are placed along axes parametrized by low-energy observables. In the usual visualization, we would easily expect situations in which our different correlation classes have overlapping regions of reduced dimensionalities. By contrast, all such situations are automatically incorporated within the correlation space without any required changes in dimensionality.

Likewise, the constraint in Eq. (3.2) merely assures that the volume of the average overlap region between n different correlation classes in the correlation space cannot exceed the volume of the average overlap region between $(n-1)$ different correlation classes. This, too, makes intuitive sense since the n-overlap region is by definition more restrictive than the $(n-1)$-overlap region. Note that the limiting case with $a_2 = 0$ corresponds to the situation in which all correlation regions are necessarily disjoint, while the case with $a_2 = 1$ represents a null limit in which all correlation regions overlap completely. This implies that $a_3 = a_4 = \ldots = 1$ as well, which in turn implies that there is really only one correlation region. This implies that $p = 1$.

Given the distributions in Eqs. (3.1) and (3.2), the next step is to calculate the probability ϕ_n that a randomly selected string model is a member of any of n previously selected correlation classes. For example, the probability ϕ_1 that a given model is a member of a single previously specified correlation class i is nothing but

$$\phi_1 = p_i = p, \tag{3.3}$$

while the probability ϕ_2 that a given model is a member of at least one of two previously specified correlation classes (i, j) is given by

$$\phi_2 = p_i + p_j - p_{ij} = 2p - a_2 p = (2 - a_2)p \tag{3.4}$$

and the probability ϕ_3 that a given model is a member of at least one of three previously specified correlation classes (i, j, k) is given by

$$\begin{aligned}\phi_3 &= p_i + p_j + p_k - p_{ij} - p_{jk} - p_{ik} + p_{ijk} \\ &= 3p - 3a_2 p + a_3 p = (3 - 3a_2 + a_3)p. \end{aligned}\tag{3.5}$$

Note that in the correlation space, each of these results has a natural geometric interpretation: ϕ_1 is the volume of a single correlation region; ϕ_2 is the combined volume of two correlation regions [which is the sum of the volume of each region minus the (double-counted) volume of their overlap]; and so forth. In general, for ℓ, previously specified correlation classes $(i, j, k, r, \ldots, s)$, we have

$$\begin{aligned}\phi_\ell &= \sum_i p_i - \sum_{ij} p_{ij} + \sum_{ijk} p_{ijk} - \sum_{ijkr} p_{ijkr} + \ldots + p_{ijkr\ldots s} \\ &= \left[\sum_{m=1}^{\ell} (-1)^{m+1} \frac{\ell!}{m!(\ell - m)!} a_m\right] p, \end{aligned}\tag{3.6}$$

where the summations in the first line of Eq. (3.6) are over all unequal choices from amongst the classes $(i, j, k, r, \ldots, s)$.

Of course, logical consistency requires that $\phi_1 \leq \phi_2 \leq \phi_3 \leq \ldots$. This, in turn, places an additional constraint on the a-coefficients in Eq. (3.1). Thus, while Eq. (3.2) indicates that each a_i cannot exceed a_{i-1}, we now see that each a_i also cannot be too much smaller than a_{i-1}. This new constraint merely reflects the mathematical fact that if all two-region overlaps are large, there is no way to prevent three-region overlaps from also being fairly large, and so forth. For example, while we have $a_3 \leq a_2$, the requirement that $\phi_3 \geq \phi_2$ also requires that $a_3 \geq 2a_2 - 1$. Indeed, we have already noted the limiting case of this phenomenon: if $a_2 = 1$, then $a_n = 1$ for all $n \geq 2$.

Given the result for ϕ_ℓ in Eq. (3.6), we now have all of the ingredients necessary to calculate $P_x(n)$. Let us begin by calculating the *exclusive* probabilities $\hat{P}_x(n)$ and see how they evolve as we examine more and more models in the landscape. Unlike the general probabilities $P_x(n)$ that x models can be successfully partitioned into at most n correlated sets of models and thereby exhibit at most n distinct correlation classes, the exclusive probabilities $\hat{P}_x(n)$ represent the probabilities that x models will exhibit *exactly* n correlation classes and can, thereby, be successfully partitioned into a minimum of n correlated sets.

When $x = 1$, there is only one model and consequently only one correlation class needed. We therefore have $\hat{P}_1(1) = 1$ and $\hat{P}_1(n) = 0$ for all $n > 1$. Next, when we select our second model, there are two possibilities: either it is in the same correlation class as our first model (which happens with probability ϕ_1), or it is not. We thus find that $\hat{P}_2(1) = \phi_1 = p$, while $\hat{P}_2(2) = 1 - \phi_1 = 1 - p$. Proceeding to the third model, we again have the same situation: it may be in the same correlation classes as we have already seen, or it may not. Tallying the possibilities in each case, we then find $\hat{P}_3(1) = \phi_1^2 = p^2$, while $\hat{P}_3(2) = \phi_1(1 - \phi_1) + (1 - \phi_1)\phi_2 = (3 - a_2)(p - p^2)$ and $\hat{P}_3(3) = (1 - \phi_1)(1 - \phi_2) = 1 - (3 - a_2)p + (2 - a_2)p^2$.

This process continues as we select more and more models. Ultimately, all of our exclusive probabilities $\hat{P}_x(n)$ can be generated through the recursion relation

$$\hat{P}_\ell(k) = \hat{P}_{\ell-1}(k)\phi_k + \hat{P}_{\ell-1}(k-1)[1 - \phi_{k-1}], \quad (3.7)$$

with the initial condition $\hat{P}_1(1) = 1$. This recursion relation merely says that there are only two possible ways of finding exactly k correlation classes after ℓ models have been examined: either there were already k classes found from amongst the previous $\ell - 1$ models (and the ℓth model must be in one of these k classes), or there were only $k - 1$ classes found from amongst the previous $\ell - 1$ models (and the ℓth model is not in one of those classes). These possibilities then give rise to the first and second terms on the right side of Eq. (3.7).

Given the recursion relation in Eq. (3.7), we immediately see that $\hat{P}_x(1) = \phi_1^x = p^x$, which is the probability that x models are all in the same correlation class. Likewise, we see that $\hat{P}_x(x) = \prod_{i=1}^{x}(1 - \phi_i)$, which is the probability that each successive model is outside the correlation classes determined by the previous models.

Finally, given the exclusive probabilities $\hat{P}_x(n)$, we can easily calculate the general probabilities $P_x(n)$:

$$P_x(n) = \sum_{m=1}^{n} \hat{P}_x(m). \quad (3.8)$$

It then follows, for example, that while $P_x(1) = \hat{P}_x(1) = p^x$, we have $P_x(x) = 1$, as required.

Using Eqs. (3.6), (3.7), and (3.8), it is straightforward to evaluate $P_x(n)$ as a function of n in the range $1 \leq n \leq x$ for any $\{p, a_2, a_3, \ldots\}$. Note that specific choices for the a_i coefficients are needed only insofar as they enable us to determine closed-form expressions for the ϕ_ℓ. However, the relation between the probabilities $P_x(n)$ and volumes ϕ_ℓ is completely general.

Our results are shown in Fig. 3 for the case with $p = 1/30$ and $a_i = 0$ for all $i \geq 2$, corresponding to a situation in which there are 30 disjoint correlation classes. Already,

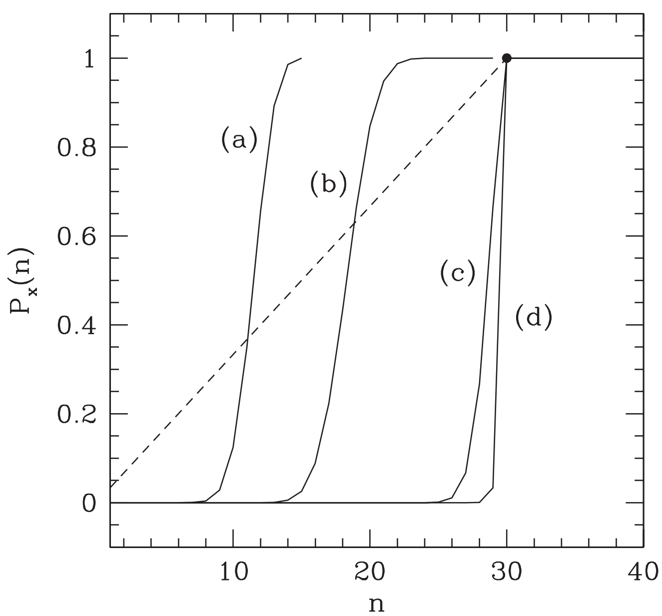

FIG. 3. The probabilities $P_x(n)$, plotted (solid lines) as functions of n in the range $1 \leq n \leq x$ for (a) $x = 15$, (b) $x = 29$, (c) $x = 100$, and (d) $x = 200$. In each case, we have chosen $p = 1/30$ and $a_i = 0$ for all $i \geq 2$, so that our correlation classes are all nonoverlapping (disjoint). The dashed line shows ϕ_n as a function of n. For $x \lesssim 1/p$, we see that $P_x(n)$ reaches 1 when $n = x$; by contrast, for $x \gg 1/p$, we see that $P_x(n)$ reaches 1 near $n \approx 1/p$. As $x \to \infty$, the curve $P_x(n)$ asymptotes to a sharp step function at $n = 1/p$. Thus, as the number of models examined increases beyond $1/p$, measuring $P_x(n)$ can yield an extremely precise measure for the average value of p_i on the string landscape.

we can observe certain general features. For $x \lesssim 1/p$, we see that $P_x(n)$ reaches 1 when $n = x$, as required. However, for $x \gg 1/p$, we see that $P_x(n)$ reaches 1 near $n \approx 1/p$. This, too, makes sense since we expect to achieve a successful partition of our data set when the number of partitions is approximately equal to $1/p$, the number of disjoint correlation classes. Finally, we observe that as $x \rightarrow \infty$, the curve $P_x(n)$ asymptotes to a sharp step function at $n = n_*$, where $n_* \equiv [1/p] + 1$, i.e., where n_* is the smallest integer exceeding $1/p$. This sharpening into a step function also makes intuitive sense since n_* is merely the total number of different correlation classes present on the landscape. As we examine more and more models, it becomes more and more unlikely that we have missed finding at least one representative model from any correlation class. Thus, we are guaranteed to achieve successful partitionings only when the number of partitions equals the number of correlation classes.

This last result provides us with a clear ''experimental'' way of determining the average value of p_i on the string landscape. Indeed, as the number of models increases beyond $1/p$ [which can be determined from the increasing sharpness of the rise of $P_x(n)$], the location of this rise in $P_x(n)$ will be given by n_*, the smallest integer exceeding $1/p$.

These results are valid for the situation in which all correlation classes are disjoint. However, this general situation persists even when the a-coefficients are nonzero and overlaps between regions become significant. Indeed, with nonzero overlap regions, the volumes ϕ_n will no longer grow linearly with n; these volumes will accrue more slowly as a function of n because only part of the volume corresponding to each new correlation class leads to new territory not previously covered. Nevertheless, the previous behavior for $P_x(n)$ persists, provided we more generally identify n_* (the number of distinct correlation classes present on the landscape) as the smallest integer n for which $\phi_n = 1$. Indeed, just as in the disjoint-region case, we find that $P_x(n)$ reaches 1 when $n = x$ for $x \lesssim n_*$, while $P_x(n)$ reaches 1 near $n \approx n_*$ for $x \gg n_*$. Indeed, as $x \rightarrow \infty$, the curve $P_x(n)$ continues to asymptote to a sharp step function at $n = n_*$.

This situation is illustrated in Fig. 4. For this figure, we have taken $a_1 = 1$ and $a_n = r^{n-1}$, where r is a predetermined scale factor; note that such a-coefficients satisfy all of the self-consistency constraints previously discussed. Also, note that even though ϕ_n is growing only very slowly as a function of n, the probabilities $P_x(n)$ still make a relatively sharp transition from 0 to 1, even for $x \leq n_*$. A similar situation emerges for any $r < p$.

Thus, even when there are significant overlaps between correlation regions on the landscape, we see that we can continue to extract sharp experimental data about n_* on the landscape merely by taking a sufficiently large value of $x \gg n_*$ and examining the location of the probability step function which emerges. Indeed, the only difference relative to the disjoint-region case is that the relationship between n_* and $1/p$ has been modified. We are therefore now extracting information about n_* alone, but not necessarily about $1/p$. Of course, the emergence of approximate step-function behavior itself provides experimental verification that x is sufficiently large; as a result, no prior knowledge of n_* is required.

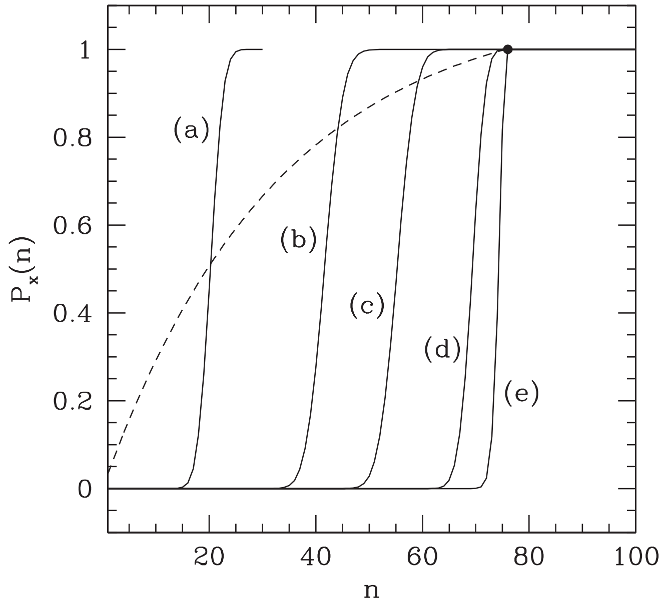

FIG. 4. The probabilities $P_x(n)$, plotted as functions of n in the range $1 \leq n \leq x$ for (a) $x = 30$, (b) $x = 100$, (c) $x = 200$, (d) $x = 500$, and (e) $x = 1000$. In each case, we have chosen $p = 1/30$. However, unlike the plot in Fig. 3, we have taken $a_n = r^{n-1}$ with $r = 3/100$ for all $n \geq 2$, reflecting nonzero overlaps between correlation-class regions. The dashed line shows ϕ_n as a function of n, reaching $\phi_n = 1$ at $n_* = 76$. We see that $P_x(n)$ behaves similarly to the case in Fig. 3, with the primary difference that significantly larger values of x are required in order to ''saturate'' the probability function and trigger the transition to a step function. Despite these differences, however, we see that measuring $P_x(n)$ for $x \gg n_*$ continues to yield an extremely precise measure for n_* on the string landscape.

Moreover, once n_* is determined, the rate at which $P_x(n)$ freezes into a step function gives information about the relative sizes of the *overlaps* between correlation-class regions. Indeed, we see from comparisons between Figs. 3 and 4 that when overlaps are sizable, considerably more models are required (i.e., x must be considerably larger) before $P_x(n)$ develops step-function behavior as a function of n. Thus, through experimental statistical examinations of $P_x(n)$, we see that it is possible to determine not only the number of correlation classes on the landscape but also their relative overlap sizes.

There is only one finely-tuned situation in which this method of measuring $P_x(n)$ fails to yield clear information about the underlying landscape: this occurs if n_* is infinite.

At first glance, it may seem that one cannot ever physically realize a situation in which n_* is infinite. However, it is possible for ϕ_n to approach 1 as an *asymptote* rather than actually hit 1 for finite n. Again considering the case with $a_n = r^{n-1}$ for all $n \geq 2$, it turns out that we can mathematically realize such a situation by taking $r = p$. Such a situation is illustrated in Fig. 5, where we see that our probability function $P_x(n)$ fails to reach a fixed shape no matter how large x becomes.

Physically, taking $r = p$ corresponds to a situation in which each new correlation class adds an incrementally smaller amount of new volume, so that an infinite number of correlation classes are required to saturate the full correlation space. Clearly, such a situation is highly fine-tuned, requiring a landscape exhibiting both an infinite number of models and an infinite number of correlation classes. String theory would have absolutely no predictive power in such a situation. However, there exist general arguments [5] suggesting that the number of string models in the landscape is actually finite. If so, then such a situation cannot arise.

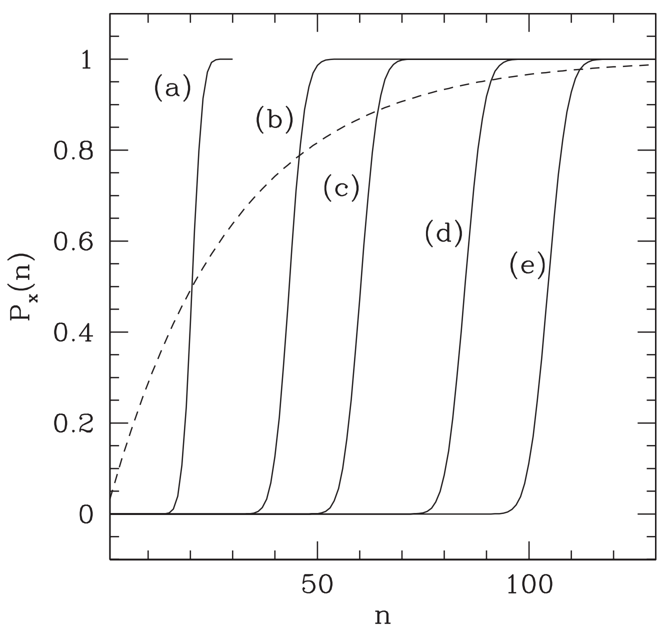

FIG. 5. The probabilities $P_x(n)$, plotted as functions of n in the range $1 \leq n \leq x$ for (a) $x = 30$, (b) $x = 100$, (c) $x = 200$, (d) $x = 500$, and (e) $x = 1000$. This plot is the same as in Fig. 4 except that we have now taken $r = 1/30$ rather than $r = 3/100$. As is evident, this change in the value of r (adjusting its value by a mere few parts in a thousand) has changed the behavior of $P_x(n)$ significantly, shifting $n_* \to \infty$ and entirely eliminating the asymptotic step-function behavior for $P_x(n)$ no matter how large x becomes. As argued in the text, this represents a highly fine-tuned situation in which the landscape consists of an infinite number of models and an infinite number of correlation classes. In such a case, string theory would have no predictive power.

Likewise, for mathematical completeness, we remark that a similar situation with infinite n_* can also arise in our example by taking $r > p$. In such cases, as $n \to \infty$, the function ϕ_n asymptotes to a value *less* than 1, once again implying that n_* is infinite. However, this situation is also clearly unphysical since it corresponds to the self-contradictory claim that there exist nonvanishing regions of the landscape which are not populated by any string models.

IV. THREE EXAMPLES

We now provide three toy-model examples designed to illustrate these techniques.

A. A birthday example

Our first example is a simple one. Let us suppose that we enter a classroom of schoolchildren, randomly select x of them, and ask each selected child to state the date of the month when he/she was born. The responses in principle could range from 1 through 30 (where we shall ignore 31 and other special months). Children with the same number will be defined to be in the same correlation class. We can then use the methods of this paper to determine how many correlation classes exist amongst the children in this classroom. Specifically, by repeatedly sampling different randomly chosen groups of x students, and repeating this procedure for different values of x, we can generate probability distributions $P_x(n)$. According to the above results, we know that for $x \gg 30$, we expect to find that $P_x(n)$ should take the form of a step function located at $n = n_* = 30$. Indeed, because the correlation classes in this example are disjoint, this is exactly the situation which is plotted in Fig. 3, and, indeed, we find that $n_* = 30$.

However, it is possible that the methods discussed above might yield a probability distribution function $P_x(n)$ which freezes onto a step function located at a smaller value of n_*, even though $x \gg 30$. For example, without our knowledge, the children in the classroom might have been preselected in such a manner that the classroom contains only children with even birth dates (while children with odd birth dates were placed elsewhere); in such a case, we would find $n_* = 15$. The emergence of such a smaller value of n_* would then reveal the existence of a hidden preselection or correlation, and would be completely analogous to an extra prediction of string theory that transcends our pre-existing (e.g., field-theoretic) expectations.

Despite its simplicity, this example already illustrates certain unexpected features. For example, we may ask for the probability that we will find n different birth dates if 29 children are polled. Consulting Fig. 3(b), we see that this probability is essentially zero for $n \lesssim 15$, but rises rather dramatically and reaches values very close to 1 for all values $n \gtrsim 22$. Thus, even though this probability will only hit 1 exactly for $n = x = 29$, we see that this probability is already effectively equal to 1 for values as small as

$n \approx 22$. This is somewhat surprising, since we might not have expected to be able to predict that a landscape of 29 randomly selected schoolchildren will share only 22 birth dates with such a high degree of certainty.

Of course, this "birthday example" may seem somewhat trivial. However, this is only because we already know that there are only 30 possible birth dates which are logically possible. We must remember, however, that this information is *a priori* completely unknown to the physicist who has no information regarding the layout of the calendar (i.e., who has no prior knowledge of the structure of the string landscape). Moreover, the method we have developed in this paper—relying on an experimental construction of the function $P_x(n)$—enables us to extract n_* with great precision and confidence, and without any prior knowledge of either the calendar or the collection of children in the classroom. The x-dependence of the rate at which $P_x(n)$ develops into a step function might then also experimentally suggest that these correlation classes are relatively disjoint.

B. The heterotic landscape: shatter

We shall now give an example drawn from the actual landscape associated with perturbative four-dimensional heterotic string models which are realizable using free-field constructions (such as those based on free world sheet bosons or fermions). In several recent papers [9,11], we conducted a random exploration of the landscape associated with such models. These studies resulted in a data set of approximately 10^7 distinct self-consistent four-dimensional heterotic string models, each of which is tachyon-free and hence stable at tree level; in particular, models with all levels of supersymmetry (ranging from $\mathcal{N} = 0$ to $\mathcal{N} = 4$) were included. To the best of our knowledge, this is the largest set of distinct heterotic string models ever constructed. Although these string models do not have fully stabilized moduli, they represent the current state-of-the-art in heterotic string model-building and are as stable as many of the classes of heterotic and Type I string models which have been considered in other random statistical studies [6–8,12,14] of the landscape. Further details concerning this data set of string models can be found in Refs. [9,11].

For the purpose of this example, we shall concentrate on a quantity called "shatter" [9]. For any given string model, we shall define the corresponding shatter f as the number of distinct irreducible factors which comprise the corresponding gauge group. Note that for this purpose, we shall define any gauge-group factor of $SO(4) \sim SU(2) \times SU(2)$ as contributing two units to f. Thus, if a gauge group is broken (or "shattered") from $SO(10)$ to $SU(3) \times SU(2) \times U(1)^2$, we may state that its shatter f has been increased from $f = 1$ to $f = 3$ even though the total rank of the gauge group is unchanged. Roughly speaking, the shatter f can be viewed as a measure of the degree of complexity needed for the construction of the string model, with increasingly smaller individual gauge-group factors tending to require increasingly many nonoverlapping sequences of orbifold twists and nontrivial Wilson lines.

Given this definition of shatter, we may then ask how many different values of shatter might be present in the corresponding string landscape. Equivalently, defining any two models to populate the same correlation class if they share the same shatter value, we may ask how many correlation classes are present in the landscape of such models.

To answer this question following the procedures outlined in this paper, we randomly chose x models from amongst our data set of models and examined how many different shatter values were found. We then repeated this process 5000 times in order to generate probabilities $P_x(n)$, and repeated this process for several different values of x. Our results are shown in Fig. 6. As we see from Fig. 6, step-function behavior emerges rather quickly, yielding the value $n_* = 22$. This value, of course, makes complete sense, since we are dealing with the landscape of perturbative four-dimensional heterotic string models over which the gauge group can range from $SO(44)$ (with a shatter $f = 1$) to $U(1)^{22}$ (with a shatter of $f = 22$). Moreover, we see that this value for n_* emerges for relatively small values of x (compared with the size of our landscape as a whole).

We emphasize that in this landscape, not all values of shatter emerge with equal probability. Indeed, as discussed

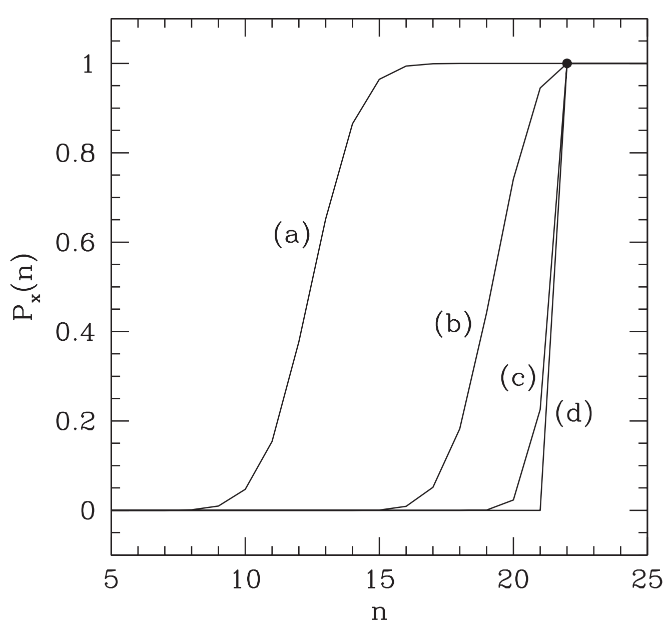

FIG. 6. Probabilities $P_x(n)$, plotted as functions of n, the number of shatter correlation classes found for (a) $x = 20$, (b) $x = 50$, (c) $x = 100$, and (d) $x = 1000$, where x indicates the number of string models selected in each run. It is clear that step-function behavior at $n_* = 22$ emerges rather quickly, and for relatively small values of x compared with the size of our landscape as a whole.

more fully in Refs. [9,11], a random search through the space of such models reveals large hierarchies in the relative probabilities of finding models with different levels of shatter. Nevertheless, our method for determining n_* continues to work even in such cases, with the probability distribution $P_x(n)$ smoothly transitioning to a step function.

Moreover, we emphasize that the sampling techniques that were used for this study introduce a large degree of unavoidable bias which distort the apparent space of models in nontrivial ways. Methods of overcoming these difficulties were developed in Ref. [10]. However, in order to generate the curves in Fig. 6, we employed a simpler method in which we introduced a compensating bias such that each correlation class had equal statistical representation in our random selection of models.

C. The heterotic landscape: gauge-group correlations

Finally, we shall give a third example of our techniques. For this example, we shall *not* adjust for sampling biases; we shall, nevertheless, find that our methods continue to function without difficulty. Moreover, we shall obtain a result which is, *a priori*, unexpected from a field-theory perspective.

Working within the same data set of heterotic string models discussed above, we shall now consider a new quantity g, defined as

$$g \equiv (\#U(1)\text{ factors}) - \sum_{\substack{n=3\\ n\in\mathbb{Z}\geq 5}} (\#SU(n)\text{ factors}). \qquad (4.1)$$

In other words, g counts how many $U(1)$ factors in the gauge group of a given string model cannot be associated with factors $SU(n)$ for $n = 3$ or $n \geq 5$. In general, from a string perspective, we might naïvely expect that g can never be negative; indeed, this result would follow from the expectation that each $SU(n)$ gauge-group factor in a perturbative heterotic string model can only arise from a larger $U(n)$ factor via the decomposition $U(n) \rightarrow SU(n) \times U(1)$. The quantity g can thus indicate the extent to which this expectation is actually realized in the string landscape. [Note that the special case with $n = 4$ is somewhat different since $SU(4)$ is also simultaneously an SO group, namely, $SO(6)$ for which such expectations would not apply.]

In principle, the quantity g can range from a maximum of $g = 22$ [corresponding to the total gauge group $U(1)^{22}$] down to a minimum of $g = -11$ [corresponding to the total gauge group $SU(3)^{11}$]. There are thus 34 different possible values of g. Equivalently, defining models to be in the same correlation class if they have the same values of g, we may expect as many as 34 different correlation classes in the landscape of such models. Note that all 34 values of g are equally likely to emerge on the basis of any low-energy effective field theory description of these models. In particular, gauge groups ranging all the way from $SU(3)^{11}$ to $U(1)^{22}$ satisfy the constraints appropriate for perturbative four-dimensional heterotic string models, with total ranks 22 and total left-moving world sheet central charges $c = 22$ at affine level $k = 1$.

In order to test whether all 34 possible g-values are actually realized in this string mini-landscape, we again followed the procedures developed in this paper and randomly chose x models from amongst our data set of models. We then examined how many different g-values were found in this sample, and repeated this process 5000 times in order to generate probabilities $P_x(n)$. This procedure was then followed for several different values of x, and our results are shown in Fig. 7.

Unlike the case in the previous example, we did not choose to correct for the sampling biases which are inherent in any random probabilistic study of this sort. Moreover, it turns out that there are huge hierarchies in the probabilities with which models with different g-values appear in any random sampling of models; indeed, models with $g \geq 0$ are found [9] to constitute more than 99% of the landscape as a whole (thereby confirming the general expectations discussed above). Nevertheless, we again see that our methods yield probabilities $P_x(n)$ which asymp-

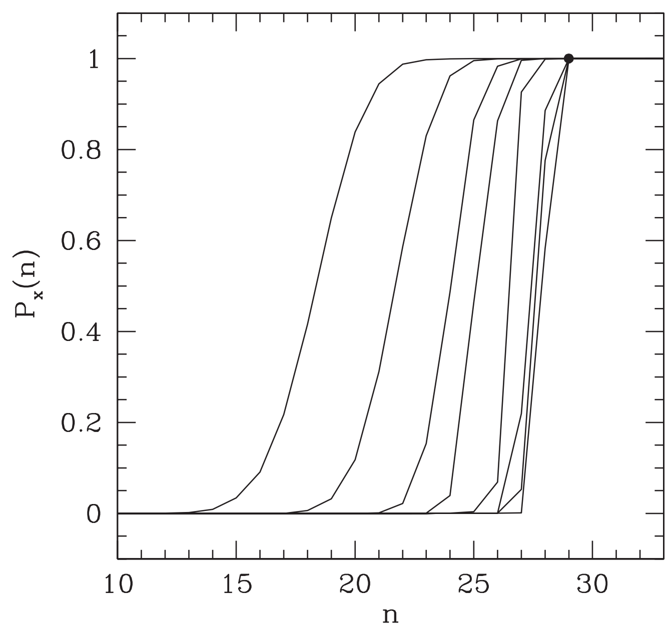

FIG. 7. Probabilities $P_x(n)$, plotted as functions of the number n of different g-value correlation classes found. The curves (left to right) correspond to values $x = 50$, 100, 300, 1000, 3×10^4, 5×10^5, 10^6, and 2×10^6 respectively, where x is the number of string models selected in each run. Despite our naïve field-theoretic expectation that there should be 34 correlation classes on this landscape, we see that these curves asymptote to a step function with $n_* = 29$. Given the fact that the $g \geq 0$ models constitute more than 99% of the landscape, the emergence of such a step function indicates that our method yields meaningful results even when the correlation classes are hierarchically different in size and even when significant sampling biases exist.

tote to a step function for sufficiently large values of x. Moreover, somewhat surprisingly, we find that the corresponding value of n_* is not 34, but is only 29. We thus learn that negative values of g are indeed possible, but not all of them: indeed, the remaining g-values (which in this case turn out to correspond to g-values smaller than -6) belong to the "swampland" [18] rather than to the landscape of these models.

It is clear from Fig. 7 that the values of x which are required in order to observe asymptotic step-function behavior for $P_x(n)$ are significantly greater than they are if sampling biases are corrected at an earlier stage of the analysis. Thus, the utility of the method we have developed in this paper depends on the extent to which we can address the sampling bias issues discussed in Ref. [10]. Ultimately, these issues are best dealt with on a case-by-case basis. Nevertheless, we see that our method is capable, in principle, of yielding sharp results, even in the presence of significant sampling biases. Indeed, we stress that the curves in Fig. 7—just like those in Fig. 6—are not theoretical calculations, but instead represent the results of actual statistical samplings from the data set of heterotic string models generated in Refs. [9,11].

V. CONCLUSIONS

We see, then, that measuring $P_x(n)$ provides a robust practical method of extracting information concerning the average behavior of the different correlation classes across the string landscape. This, in turn, provides a direct and compelling way of quantifying the extent to which string theory is predictive. Perhaps the primary virtue of this method is that it can readily be applied for situations in which only a relatively small number of string models are examined, provided these models are randomly selected from across the entire landscape as a whole. Indeed, all that is required is that x, the number of models examined, exceed n_* by perhaps 1 or 2 orders of magnitude, a proposition which can be verified (without *a priori* knowledge of n_*) by measuring $P_x(n)$ for increasing values of x and observing if and when this function saturates into step-function behavior.

Needless to say, the calculations in this paper may be easily generalized to more complex landscape distributions and correlation-region overlap patterns. Indeed, the only effect of such a generalization would be the modification of the expression for ϕ_ℓ given in Eq. (3.6). However, the central point of this paper is general and remains applicable regardless of such possible generalizations: there will always be a value n_* at which $\phi_n = 1$, and this value, which represents the number of correlation classes on the landscape, can be "experimentally" extracted with great statistical certainty through the methods we have described. Indeed, we have shown this explicitly for landscape distributions at both extremes: distributions in which our correlation-class regions are entirely disjoint, and distributions in which significant overlaps occur.

Note that even our notion of "correlation class" can be generalized without altering the main results of this paper. In this paper, we have implicitly assumed that within a single correlation class, there exists a tight mathematical relation between specific low-energy observables. However, this requirement may also be relaxed: meaningful correlation classes may also exist in which one might be able to say nothing more than that a certain *range* of values for one specific low-energy observable tends to be statistically correlated with a certain range of values for a different low-energy observable. Indeed, evidence that such types of correlation classes exist has recently been presented in Ref. [15]. Nevertheless, the methods we have developed in this paper are applicable to these types of generalized correlation classes as well.

Of course, our method of examining $P_x(n)$ can also be used to examine the properties of any *subset* of the landscape. For example, one might restrict to a class of models which share a common underlying construction methodology. In such cases, the resulting information for n_* then applies to the correlation regions appropriate for that subset. Our method is therefore suitable for examinations of arbitrary subsets of the landscape, without requiring knowledge of the string landscape as a whole.

ACKNOWLEDGMENTS

We wish to thank the CERN Theory Division, where this work was initiated, for hospitality during the July/August 2008 Theory Division Institute on String Phenomenology. The work of K. R. D. was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-04ER-41298 and by the U.S. National Science Foundation under Grant No. PHY/0301998. The work of M. L. was supported in part by ANR Grant No. ANR-05-BLAN-0079-02, RTN Contract Nos. MRTN-CT-2004-005104 and MRTN-CT-2004-503369, CNRS PICS 2530, 3059 and 3747, and by a European Union Excellence Grant No. MEXT-CT-2003-509661. We are happy to thank W. Taylor for discussions.

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