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Scalar-tensor theories with pseudoscalar couplings

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We consider scalar-tensor theories of gravity extended by pseudoscalar couplings to matter and gauge fields and derive constraints on the *CP*-odd combinations of scalar and pseudoscalar couplings from laboratory spin precession experiments and from the evolution of photon polarization over cosmological distances. We show the complementary character of local and cosmological constraints, and derive novel bounds on the pseudoscalar couplings to photons from the laboratory experiments. It is also shown that the more accurate treatment of the spin content of nuclei used in the spin precession experiments allows us to tighten bounds on Lorentz-violating backgrounds coupled to the proton spin.

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I. INTRODUCTION

The discovery of the apparent acceleration of the Universe, most naturally attributed to the existence of dark energy [1], instigated many developments in cosmology and particle physics during the last decade. To date, all observational data are consistent with the most economic possibility: the dark energy is just a cosmological constant, and as such does not evolve over the cosmological time scales. On the other hand, it is intriguing to think about the alternative explanations related to a drastic change of the infrared physics. In parallel to the attempts of modifying gravity on large scales [2], there is a renewed interest in the cosmological scalar fields that are nearly massless and that manifest themselves as a "dark energy" component over large cosmological distances [3].

An interesting twist to the well-known story of cosmological scalars comes from the possibility of their interaction with matter and gauge fields. (For purely cosmological signatures of "interacting" quintessence, see e.g. [4].) In fact, such theories exhibit a rich plethora of phenomena that go beyond pure cosmological effects, which we would like to illustrate with the following toy example. Let us consider a Lagrangian for the scalar field ϕ interacting with a standard model (SM) fermion ψ (e.g. electron) and a gauge field A_μ (e.g. photon),

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \bar{\psi}(iD_\mu\gamma^\mu - m)\psi \\ &\quad - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - c_{S\psi}\phi\bar{\psi}\psi - c_{P\psi}\phi\bar{\psi}i\gamma_5\psi \\ &\quad - c_{S\gamma}\phi F_{\mu\nu}F^{\mu\nu} - c_{P\gamma}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}. \qquad (1.1)\end{aligned}$$

Here c_{Si} and c_{Pi} parametrize the strengths of the scalar and pseudoscalar couplings to photons and fermions, while $F_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$ denote the usual and dual field strengths, and D_μ is the covariant derivative. Written in flat space, Lagrangian (1.1) can be trivially generalized to curved backgrounds, and to nonlinear couplings to matter. This is a low energy Lagrangian, and in a bigger particle physics context, it can be embedded in an $SU(2)\times U(1)$-symmetric theory where e.g. the chirality flipping terms $\phi\bar{\psi}\psi$ will be endowed with the Higgs field. Starting from (1.1), one can immediately infer a number of interesting consequences, a partial list of which is given below.

(1) *The existence of a new long-range force distinguishable from spin-two gravity.* The scalar field contributes to the gravitational force, adding $\sim c_s^2$ on top of the familiar Newtonian constant mediated by gravitons. Such a force leaves distinguishable imprints via relativistic corrections and/or composition dependence (effective violation of the equivalence principle).

(2) *The existence of a preferred Lorentz frame associated with $\partial_t\phi$.* If ϕ is a very light quintessencelike field, then there is a preferred frame where, cosmologically, $\partial_\mu\phi = (\dot{\phi}, 0, 0, 0)$. For most of the models this frame coincides with the frame of the CMB, and $|\dot{\phi}|$ is limited by $(\rho_{\rm d.e.}(1+w))^{1/2}$, where w is the dark energy equation-of-state parameter.

(3) *Variation of masses and couplings in time and space*. Effective values of masses and coupling constants vary in space and time, $m_{\rm phys}(t, \mathbf{x}) = m + c_{S\psi}\phi(t, \mathbf{x})$, following the ϕ profile.

(4) *Coupling of polarization to velocity relative to the CMB frame*. A particle moving relative to the CMB frame acquires a helicity-dependent interaction, $H_{\rm int} \sim (\mathbf{Sn})\dot{\phi}$, where $(\mathbf{Sn})$ is the projection of spin on the direction of propagation n. This way, the $c_{P\gamma}$-proportional interaction would result in the rotation of polarization for photons propagating over varying ϕ backgrounds.

(5) *Photon-scalar conversion.* In the presence of an external electromagnetic field, a photon can "oscillate" to a quantum of the scalar field thereby, e.g., reducing the luminosity of distant objects or providing additional channels for star cooling.

(6) *Coupling of spin to the local gravitational force*. The scalar coupling $c_{S\psi}$ will lead to the local field gradient $\nabla\phi$ generated by massive bodies, which is closely parallel to the vector of local free-fall acceleration. The pseudoscalar couplings then create a Zeeman-like splitting for the spin of ψ particles in the direction of the local gravitational acceleration, $H_{\rm int} \sim (\mathbf{Sg})$.

It is remarkable that such a simple Lagrangian leads to a number of quite different phenomena. Unfortunately, at this stage the exciting phenomenology of "interacting dark energy" lives in a pure theoretical realm: there is no confirmed experimental evidence for any of the effects on our list.[1] Consequently, there are only upper limits on the combinations of the couplings in Lagrangian (1.1) that can be quoted. Nevertheless, many of the effects on our list have found an extensive coverage in the theoretical works. Most notably, the changing couplings were discussed, for example, in Refs. [9–12], the photon-scalar conversion was considered in Refs. [13], and the fixed frame effects versus the cosmological evolution of the photon polarization were addressed in a series of papers [7,8,14–16]. For the limits on scalar-induced corrections to gravitational interactions, we refer the reader to recent reviews [17] and references therein. In contrast, the last item on our list, the spin coupling to the local gradient of the scalar field, received far less attention (see e.g. [18,19]). Reference [19] contains very useful generic parametrization of long-range forces induced by any spin carriers and analyzes constraints that can be imposed on the combination of the pseudoscalar and scalar couplings, and as such has strong overlap with some of the analysis done in the present paper.

The purpose of this paper is to show that the pseudoscalar couplings of the Brans-Dicke-type scalar can indeed be subjected to stringent laboratory constraints that are complementary to the cosmological limits. The high-precision spin precession experiments constrain pseudoscalar interactions both in the fermion and photon sectors. In the rest of this paper we present the setup for our model, briefly review the effects created by the cosmological evolution of $\phi(t)$, investigate the local spin effects created by the gradient of ϕ, and set the limits on the admissible size of the pseudoscalar couplings.

Before we delve into studying the physical effects induced by the pseudoscalar couplings, we would like to add a word of caution addressed to all models of "interacting quintessence." The models of light scalar fields represent a formidable challenge at the quantum level, as there are no fundamental reasons for a scalar to remain massless or nearly massless. The scalar interaction of such a field makes the whole problem even more difficult, if not impossible, from the point of view of "technical naturalness": the loops of SM fields tend to generate big corrections to $V(\phi)$ even with a relatively small ultraviolet cutoff parameter, which would be in conflict with requirements, $m_\phi \sim H$ [10,15,20]. There is no clear resolution to this problem, which essentially prevents the fully consistent study of ϕ dynamics. Instead, one has to hope, perhaps too optimistically, that the problem of near masslessness of the scalar field could be cured by the same mechanisms that make the cosmological constant small, and meanwhile keep $V(\phi)$ fixed by hand. To finish this "disclaimer" on an optimistic note, we would like to remark that the pseudoscalar couplings do not make this problem worse. Indeed, in essence the pseudoscalar couplings give only derivative interactions, and therefore do not affect the potential $V(\phi)$ at the perturbative level.

II. ADDING SPIN COUPLINGS TO SCALAR-TENSOR THEORIES

We would like to formulate our reference Lagrangian at the normalization scale just below the QCD scale, so that the effective matter degrees of freedom are electrons, photons, nucleons, and neutrinos. Splitting the ϕ-field Lagrangian into the scalar and pseudoscalar parts,

$$\mathcal{L} = \mathcal{L}_S + \mathcal{L}_P, \tag{2.1}$$

we choose the following parametrization,

$$\mathcal{L}_S = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \sum_{j=e,p,n}\frac{\phi}{M_{Sj}}m_j\bar\psi_j\psi_j - \frac{\phi}{M_{S\gamma}}F_{\mu\nu}F^{\mu\nu} \tag{2.2}$$

and

$$\mathcal{L}_P = \sum_{j=e,p,n,\nu}\frac{\partial_\mu\phi}{M_{Pj}}\bar\psi_j\gamma_\mu\gamma_5\psi_j - \frac{\phi}{M_{P\gamma}}F_{\mu\nu}\tilde F^{\mu\nu}. \tag{2.3}$$

Lagrangian (2.3) includes all possible pseudoscalar interactions at the mass dimension-five level. Notice that the pseudoscalar interactions can be chosen in a slightly different form, $\bar\psi\gamma_5\psi$, as in (1.1). This does not mean, however, that our set of operators should be enlarged. The two type of operators, pseudoscalar and axial vector, are related on the equations of motion. These equations are, in general, anomalous, but since we include the interaction with $F\tilde F$ explicitly, we can assert that Lagrangian (2.3) is indeed complete in a given dimension of the operators.

The scalar part of the Lagrangian (2.2) leads to new contributions to the gravitational force, and to a change of masses and couplings. Since in this paper our main interest is in spin effects, we are going to make simplifying assumptions of approximate universality of the ϕ-mediated attractive force,

[1] A tantalizing hint on the redshift evolution of the fine-structure constant was reported in Ref. [5], which so far has not been corroborated by other searches [6]. Also, an earlier claim of the nonzero pseudoscalar-induced anisotropy in the polarization signal [7] was disputed in the literature [8].

$$M_{Se} = M_{Sp} = M_{Sn} \equiv M_S \quad \text{and} \quad M_{S\gamma} \gg M_S. \tag{2.4}$$

At distances shorter than the Compton wavelength of ϕ quanta, the Newtonian constant receives contributions from both spin-two and spin-zero exchanges,

$$G_N = G_N^0\left(1 + \frac{2M_{\rm Pl}^2}{M_S^2}\right), \tag{2.5}$$

where G_N^0 is the unperturbed Newtonian constant due to graviton exchange, and the Planck mass is defined as $M_{\rm Pl} = (8\pi G_N^0)^{-1/2} = 2.4 \times 10^{18}$ GeV.

If needed, the pseudoscalar couplings could be "lifted" from the nucleon level to the level of individual quarks. Using the experimental results for the spin content of the nucleon combined with $SU(3)$-flavor relations [21], one gets

$$\begin{aligned} M_{Pp}^{-1} &\simeq 0.8M_{Pu}^{-1} - 0.4M_{Pd}^{-1} - 0.1M_{Ps}^{-1}; \\ M_{Pn}^{-1} &\simeq 0.8M_{Pd}^{-1} - 0.4M_{Pu}^{-1} - 0.1M_{Ps}^{-1}, \end{aligned} \tag{2.6}$$

where the light quark couplings are normalized at the scale of 1 GeV.

Using the appropriate field content, one can determine the renormalization group evolution of the pseudoscalar couplings. In general, the equations governing this evolution take the following form,

$$\begin{aligned} \frac{dM_{Pi}^{-1}}{d\log(\Lambda/\mu)} &= a_{ij}M_{Pj}^{-1} + b_{i\alpha}M_{P\alpha}^{-1}, \\ \frac{dM_{P\alpha}^{-1}}{d\log(\Lambda/\mu)} &= c_{\alpha i}M_{Pi}^{-1} + d_{\alpha\beta}M_{P\beta}^{-1}, \end{aligned} \tag{2.7}$$

where the logarithm is taken between the ultraviolet scale Λ and the infrared scale μ, Latin indices indicate fermionic fields, and Greek indices indicate the gauge bosons of the SM group. The renormalization group coefficients a_{ij}, $b_{i\alpha}$, $c_{\alpha i}$, and $d_{\alpha\beta}$ depend on charge assignments and coupling constants of the field running inside the loops. The precise form of these coefficients is not of immediate interest to us, but we would like to emphasize the following important observation: at any loop level the derivative couplings to fermions *do not* generate couplings to $F_{\mu\nu}\tilde{F}^{\mu\nu}$. In other words,

$$c_{\alpha i} \equiv 0. \tag{2.8}$$

Whatever size of the pseudoscalar couplings between photons and ϕ is generated by some (perhaps anomalous) ultraviolet scale physics at energies order Λ, it is preserved by the subsequent evolution to the lower scales. In fact, this refers both to the logarithmic running and to the threshold corrections. This observation delineates two important classes of models: those in which both fermion and photon pseudoscalar couplings are present in the Lagrangian, and those in which only couplings to fermions are present. The models in which ϕ couples only to gauge bosons would necessarily be fine-tuned, as quantum effects in (2.7) would definitely generate induced couplings to fermions.

Existing constraints on the model can be divided into pseudoscalar and scalar constraints. The constraints on the universal scalar coupling M_S can be derived from the constraint imposed by the Cassini satellite data on the post-Newtonian parameter $\bar{\gamma}$ [22],

$$|\bar{\gamma}| < 4 \times 10^{-5} \Rightarrow M_S > 400M_{\rm Pl}. \tag{2.9}$$

The constraints on the nonuniversal part of the scalar coupling are several orders of magnitude stronger. The scalar coupling to photons is constrained via the limits on the time variation of the coupling constant and, less directly, via the composition-dependent contribution to local acceleration. Typically, one has $M_{S\gamma} > 10^3 M_{\rm Pl}$. In contrast, the pseudoscalar couplings are far less constrained. The leading sources of constraints are the energy loss mechanisms in stars [23], and for electrons, photons, and nucleons, all constraints are in the ballpark of

$$|M_P| \gtrsim (10^{10}\text{–}10^{12}) \text{ GeV} \sim (10^{-8}\text{–}10^{-6})M_{\rm Pl}. \tag{2.10}$$

In the next section, we are going to show that if both pseudoscalar and scalar couplings are present, some constraints on M_P can be significantly improved.

III. COSMOLOGICAL CONSTRAINTS ON THE MODEL

To derive cosmological constraints on pseudoscalar couplings, we remind the reader that the presence of a time-evolving scalar field with a pseudoscalar coupling to photons leads to a rotation of polarization for photons. The resulting angular change in the linear polarization for a photon propagating from point 1 to point 2 is simply related to the change of ϕ between the two points,

$$\Delta\theta = \frac{2\Delta\phi}{M_{P\gamma}}. \tag{3.1}$$

Following the work of Carroll [15] and the original analysis of Ref. [24], we use the limit on the extra rotation of polarization from a distant source (3C 9) at redshift $z = 2.012$ as $|\Delta\theta| < 6°$,

$$\frac{|\phi(z=2) - \phi(z=0)|}{M_{P\gamma}} < 0.052. \tag{3.2}$$

Even more distant sources of polarization are available in the studies of the cosmic microwave background. The E-mode polarization map of the sky has been produced [25], and it agrees well with the expectation based on the temperature map. This constrains the amount of extra rotation of polarization introduced by the $\phi F\tilde{F}$ interaction between the surface of the last scattering and $z = 0$. Recent numerical analyses of the CMB data provide a constraint on the amount of extra rotation at the level of $|\Delta\theta| < 6°$ [16] (the same limit of 6° is purely coincidental), which allows us to extend (3.2) to the redshifts of photon decoupling, $z_{\rm dec} \simeq 1100$,

$$\frac{|\phi(z_{\rm dec}) - \phi(z=0)|}{M_{P\gamma}} < 0.052. \qquad (3.3)$$

Finally, we would like to point out that the CMB polarization signal is generated in the narrow window of redshifts that correspond to the "last scattering" surface, and therefore the existing measurements constrain the amount of extra rotation within the thickness of this surface,

$$\frac{2}{M_{P\gamma}}|\Delta\phi(z_{\rm dec} \pm \Delta z_{\rm dec}/2)| < O(1), \qquad (3.4)$$

where $\Delta z_{\rm dec} \simeq 200$ corresponds to the thickness of the last scattering surface. The violation of this bound would *suppress* the strength of the polarization signal, which is well measured.

With these bounds at hand, we are ready to translate them into the constraints on the parameters of our model. However, the cosmological constraints depend very sensitively on what we assume about the scalar couplings of ϕ to dark matter and even more so on the choice of the potential $V(\phi)$. Since the number of options is infinite, we would like to consider in detail two well-motivated cases.

Case 1. The simplest case is when the potential for ϕ is nearly flat and the evolution of ϕ is slow. In this case one can linearize $V(\phi)$,

$$V(\phi) \simeq \rho_\Lambda\left(1 + \frac{\phi}{M_\Lambda}\right), \qquad (3.5)$$

where ρ_Λ is approximately equal to the measured value of dark energy density, and M_Λ is a new parameter on the order of the Planck scale and/or M_S. In the limit when the backreaction of ρ_ϕ on Friedmann's equations is neglected, one can find an analytic expression for the evolution of ϕ in the flat Universe [10]. In this approximation the time evolution of the scale factor can be expressed via the scale factor and the Hubble parameter today ($t_{\rm now} \equiv t_0$): $H_0 = H(t = t_0) = \dot{a}/a|_{t=t_0}$ and $a(t = t_0) \equiv a_0$, as well as the current energy densities of matter and the cosmological constant relative to the critical density, $\Omega_m = \rho_m/\rho_c$ and $\Omega_\Lambda = \rho_\Lambda/\rho_c$:

$$a(t)^3 = a_0^3\frac{\Omega_m}{\Omega_\Lambda}\left[\sinh\left(\frac{3}{2}\Omega_\Lambda^{1/2}H_0 t\right)\right]^2. \qquad (3.6)$$

The equation of motion for the scalar field receives forcing terms directly related to dark energy and matter densities:

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\rho_m}{M_S} - \frac{\rho_\Lambda}{M_\Lambda} = -\rho_c\left[\frac{\Omega_m}{M_S}\left(\frac{a_0}{a}\right)^3 + \frac{\Omega_\Lambda}{M_\Lambda}\right], \qquad (3.7)$$

where we made an assumption of the universal strength of ϕ coupling to matter, including dark matter. This equation can be integrated out explicitly [10] to give

$$\phi(t) = \frac{4}{3}M_{\rm Pl}^2\left[\left(\frac{1}{2M_\Lambda} - \frac{1}{M_S}\right)(bt_0\coth(bt_0) - bt\coth(bt)) - \frac{1}{M_S}\ln\frac{\sinh(bt)}{\sinh(bt_0)}\right], \qquad (3.8)$$

where the following notation has been introduced:

$$b = \tfrac{3}{2}\Omega_\Lambda^{1/2}H_0. \qquad (3.9)$$

This solution implies boundary conditions $\dot{\phi}|_{t\to 0}\sim$ not too large and $\phi(t_0) = 0$. The first condition is automatically satisfied as ϕ does not evolve rapidly during the radiation domination, and the second condition is simply a choice, possible since ϕ enters linearly in the Lagrangian. It is easy to see that in the limit of $t \ll t_0$ the dependence of ϕ on the redshift is logarithmic,

$$\phi(a) \simeq \text{const} - \frac{2M_{\rm Pl}^2}{M_S}\ln(a/a_0) \quad \text{at} \quad t_{\rm eq} \ll t \ll t_0. \qquad (3.10)$$

Now we can use the evolution (3.8) to impose limits on the combination of M_P and $M_{S(\Lambda)}$ parameters using the observational constraint (3.2). We do this for three separate representative cases: for equal couplings to matter and dark energy densities, and for couplings to dark energy and matter only:

$$M_\Lambda = M_S, \qquad |M_{P\gamma}M_S| > 36M_{\rm Pl}^2; \qquad (3.11)$$

$$|M_\Lambda| \to \infty, \qquad |M_{P\gamma}M_S| > 30M_{\rm Pl}^2; \qquad (3.12)$$

$$|M_S| \to \infty, \qquad |M_{P\gamma}M_\Lambda| > 6.1M_{\rm Pl}^2. \qquad (3.13)$$

These limits generalize the analysis of Ref. [15] where only the $V(\phi)$-induced optical rotation was considered. We also notice that both (3.11) and (3.12) are about 1 order of magnitude stronger than (3.13), which is a consequence of $(a_0/a)^3 \sim 8$ enhancement of matter density over the cosmological constant at redshifts ~ 2.

Because of the logarithmic dependence on redshifts at $t \ll t_0$ (3.10), there is about 1 order of magnitude gain in the strength of the constraint when using the CMB limit (3.3) for the case of finite M_S,

$$M_\Lambda = M_S \quad \text{or} \quad |M_\Lambda| \to \infty \Rightarrow |M_{P\gamma}M_S| > 255M_{\rm Pl}^2. \qquad (3.14)$$

In order to see the maximal sensitivity to M_P, we can saturate the constraint on M_S (2.9), which results in $M_{P\gamma} \gtrsim O(M_{\rm Pl})$ at maximally allowed M_S.

Case 2. Going away from the linearized case, we consider the cosmological evolution of the ϕ field approaching some local minimum of $V(\phi)$,

$$V(\phi) = \rho_\Lambda + \frac{m_\phi^2}{2}(\phi - \phi_0)^2. \tag{3.15}$$

If the mass of the field is well above the current Hubble parameter, $m_\phi \gg H_0$, then the evolution of ϕ starts long before the present epoch. Well-known solutions for ϕ in this case are the oscillations around the minimum with the amplitude that redshifts as $a^{-3/2}$. If the initial deviation of ϕ from equilibrium was $\phi_{\rm in}$ at the time $t_{\rm in}$ when oscillations began, $H(t_{\rm in}) \sim m_\phi$, then the subsequent evolution in the radiation domination will be given by

$$\phi(t) \simeq \phi_0 + \phi_{\rm in}\left(\frac{a_{\rm in}}{a(t)}\right)^{3/2} \cos[m_\phi(t - t_{\rm in}) + \alpha], \tag{3.16}$$

where α is some phase factor. Because of the redshifted amplitude in (3.16), the constraints provided by the CMB are clearly more advantageous than the low z constraints. However, the oscillations of ϕ (3.16) make it difficult to define $\phi(z_{\rm dec})$; consequently, the analyses of [16] with the limits (3.3) are not directly applicable, and instead one should resort to the limits (3.4). Still, if the initial deviation of the ϕ field from its minimum is on the order or less than the pseudoscalar coupling $M_{P\gamma}$, and oscillations begin earlier than the decoupling, then the cosmological evolution of polarization provides *no constraints* on the size of the pseudoscalar coupling,

$$|\phi_{\rm in}| < |M_{P\gamma}|; \qquad t_{\rm in} \ll t_{\rm dec} \Rightarrow \text{no constraints on } M_{P\gamma}. \tag{3.17}$$

This is an important observation, since the first condition $|\phi_{\rm in}| < |M_{P\gamma}|$ is quite natural if the ϕ field has a phaselike origin similar to e.g. the QCD axion, and $t_{\rm in} \ll t_{\rm dec}$ is satisfied for all masses of ϕ in excess of 10^{-28} eV.

IV. LOCAL SPIN PRECESSION CONSTRAINTS

As we have shown in the two previous sections, the cosmological constraints on pseudoscalar couplings apply only to $M_{P\gamma}$, and not to fermionic couplings. Moreover, all cosmological constraints will be eliminated if the field starts oscillating much earlier than the decoupling of the CMB photons (3.17). This leaves a large domain of parameter space where only the local experiments are going to be sensitive to the pseudoscalar couplings. We wish to consider them in this section. Before we do that, we would like to note that the couplings of spins to the local gravitational (spin-two) field have been extensively studied in the literature [26–28]. Our main interest is the conclusion reached in these works that the $\mathbf{g}\cdot\mathbf{S}$ coupling does not arise in general relativity. Therefore, if detected, it can be thought of as a distinct signature of the scalar exchange.

Since most of the experiments deal with nonrelativistic atoms and nuclei, it is convenient to use the nonrelativistic Hamiltonian,

$$H_{\rm int} = -\sum_{j=n,p,e} \frac{(\sigma_j \cdot \nabla\phi)}{M_{Pj}} + \int d^3x \frac{4(\mathbf{E}\cdot\mathbf{B})\phi}{M_{P\gamma}}, \tag{4.1}$$

where $\vec{\sigma} = \mathbf{S}/|S| = 2\mathbf{S}$. The local gradient of ϕ is one-to-one related to the gravitational acceleration,

$$\nabla\phi = \mathbf{g}\frac{2M_{\rm Pl}^2}{M_S}, \tag{4.2}$$

so that the strength of the interaction of each spin to the gravitational field is given by $g \times 2M_{\rm Pl}^2/(M_S M_{Pj})$. Gravitational acceleration has dimension of energy in particle physics units of $c = \hbar = 1$, and corresponds to the frequency splitting of spin-up and spin-down states $\nu_g = 2 \times 9.8\,10^2~\text{cm/s}^2/(2\pi \times 3\,10^{10}~\text{cm/s}) = 10.4$ nHz. Unlike most problems in quantum mechanics where "up" and "down" are usually a matter of convention, in this theory these words should be used literally. Only a handful of spin precession experiments ever reached a sensitivity lower than 10 nHz; among them are experiments searching for the permanent electric dipole moments of diamagnetic atoms [29], where the statistical sensitivity is comparable to or better than 10 nHz. Unfortunately, this sensitivity is related to the energy difference of spins in parallel and antiparallel electric and magnetic fields and does not translate into the limits on spin interaction with the vertical direction.

A dedicated search for the $\mathbf{g}\cdot\mathbf{S}$ interaction was pursued in [30] (and earlier in [31]), where a $\sim\mu$Hz accuracy was achieved. In particular, Ref. [30] compared the precession frequencies of two mercury isotope spins, ^{199}Hg and ^{201}Hg, for different orientations of the magnetic field, and set a limit of 2.2×10^{-30} GeV for the spin-dependent component of gravitational energy. Other measurements that can be used to limit the pseudoscalar couplings are the spin precession experiments that searched for the effects of Lorentz violation [32,33] and the experiment with a spin-polarized pendulum [34]. The absence of sidereal modulation of spin precession, confirmed by these experiments, sets the limit on the coupling of spins to any direction in space that *does not* change as the Earth rotates around its axis. Besides useful limits on Lorentz-violating theories [35], such effects will constrain the pseudoscalar couplings, in combination with $\nabla\phi$, created by astronomical bodies other than the Earth. The solar contribution to $\nabla\phi$ is smaller than $\nabla\phi_{\rm Earth}$ by a factor of $\sim 6 \times 10^{-4}$, thereby reducing the strength of the constraints extracted from sidereal variations by the same amount. Putting different results together, and assuming that the range of the force is comparable to or larger than the solar system, we arrive at the following set of constraints,

$$|M_{Pn}M_S| > 1.5 \times 10^{-2} M_{\rm Pl}^2, \quad \text{Ref. [30]}, \tag{4.3}$$

$$|M_{Pn}M_S| > 1 \times 10^{-4} M_{\rm Pl}^2, \quad \text{Refs. [32, 33]}, \tag{4.4}$$

$$|M_{Pe}M_S| > 2 \times 10^{-6} M_{\rm Pl}^2, \quad \text{Ref. [34]}. \tag{4.5}$$

Bounds (4.3) and (4.4) are derived from the assumption of Ref. [36] that the spin of the nucleus is given by the angular momentum of the outside nucleon, which happens to be a neutron for all nuclei used in the most sensitive searches (^{3}He, ^{129}Xe, ^{199}Hg, ^{201}Hg). Consequently, the limits are formulated on the pseudoscalar coupling to neutrons, as it is also the case for the limits on the external Lorentz-violating axial-vector backgrounds [36]. These limits are in broad agreement with a similar analysis done in Ref. [19].

In fact, one can refine these bounds and impose separate constraints on the strength of the pseudoscalar coupling for protons and neutrons. Although most of the nuclei in atoms used in experiments [30–33] have a valence neutron outside of closed shells, one can use the information on the magnetic moments of these nuclei, together with a simple theoretical model of nuclear structure, to deduce the proton contribution to the total nuclear spin. To be specific, we shall assume that the magnetic moment of the nucleus is composed entirely from the spin magnetic moment of the valence neutron and the spin magnetism of the polarized nuclear core,

$$\mu = \mu_n \langle \sigma_z^{(n)} \rangle + \mu_p \langle \sigma_z^{(p)} \rangle, \qquad \langle \sigma_z^{(n)} \rangle + \langle \sigma_z^{(p)} \rangle = \langle \sigma_z^{(0)} \rangle. \tag{4.6}$$

In these equations, μ, μ_p, μ_n are the magnetic moments of the nucleus, proton, and neutron. Numerical estimates show that the orbital contribution to the magnetic moment μ in the nuclei of interest is less important than the spin contribution since the neutron orbital contribution is zero and the proton orbital contribution is small in comparison with the proton spin contribution. The latter is enhanced by the large value of the proton magnetic moment $\mu_p = 2.8$, which justifies the neglection of the proton orbital magnetism for low l orbitals. Neglection of the spin-orbit interaction makes the total spin conserved and its total value equal to the average spin of the neutron above the unpolarized core, $\langle \sigma_z^{(0)} \rangle$. The latter is equal to 1 for $j = l + 1/2$ and $-j/(j+1)$ for $j = l - 1/2$, where j is the value of the nuclear angular momentum, and l is the orbital quantum number of the valence neutron. Using these simple formulas (4.6), we determine $\langle \sigma_z^{(n)} \rangle$ and $\langle \sigma_z^{(p)} \rangle$ for observationally relevant cases of ^{199}Hg, ^{201}Hg, ^{129}Xe, and ^{3}He as shown in Table I.

One can see that the contribution of the proton spin to the total spin of these nuclei, especially ^{129}Xe and ^{201}Hg, can be as high as 30%, and therefore the proton pseudoscalar coupling is also limited in these experiments. For example, Ref. [30] limits the following combination of the proton and neutron couplings:

TABLE I. Composition of the nuclear spin.

Nucleus	μ	j, l	$\langle \sigma_z^{(0)} \rangle$	$\langle \sigma_z^{(n)} \rangle$	$\langle \sigma_z^{(p)} \rangle$
^{3}He	−2.13	1/2, 0	1	1.04	−0.04
^{129}Xe	−0.78	1/2, 0	1	0.76	0.24
^{199}Hg	0.50	1/2, 1	−1/3	−0.31	−0.03
^{201}Hg	−0.56	3/2, 1	1	0.71	0.29

$$|M_{P\rm eff} M_S| > 1.5 \times 10^{-2} M_{\rm Pl}^2, \quad \text{where } M_{P\rm eff}^{-1} = 0.6 M_{Pn}^{-1} - 0.4 M_{Pp}^{-1}. \tag{4.7}$$

The relative enhancement of the proton contribution is due to a rather close cancellation of the neutron contribution to the differential frequency of spin precession for ^{199}Hg and ^{201}Hg.

As a by-product of our analysis, we can improve the bounds on the Lorentz-violating axial-vector couplings in the Colladay-Kostelecky parametrization [35]. Indeed, the spatial components of the axial-vector background to protons, b_μ, is constrained in the same experiments, Refs. [32,33], in particular, because of the substantial contribution of the proton spin to the spin of ^{129}Xe. For example, the interpretation of the null result of the most sensitive experiment [33] with the use of the analysis [36] that assumes $\langle \sigma_z^{(n)} \rangle = \langle \sigma_z^{(0)} \rangle$, $\langle \sigma_z^{(p)} \rangle$,

$$2\pi \nu_{\rm LV} = 2 b_i^{(n)} \left(1 - \frac{\mu_{\rm He}}{\mu_{\rm Xe}}\right) = -3.5 b_i^{(n)}, \quad \text{Ref.[36]}, \tag{4.8}$$

leads to the conclusion that there is no experimental sensitivity to the proton spin content. In contrast, we find that

$$\begin{aligned} 2\pi\nu_{\rm LV} &= 2\bigg(0.76 b_i^{(n)} + 0.24 b_i^{(p)} \\ &\qquad - \frac{\mu_{\rm He}}{\mu_{\rm Xe}} (1.04 b_i^{(n)} - 0.04 b_i^{(p)})\bigg) \\ &= -4.2 b_i^{(n)} + 0.7 b_i^{(p)}, \end{aligned} \tag{4.9}$$

where $\nu_{\rm LV} = 53 \pm 45$ nHz is the experimentally measured (consistent with zero) Lorentz-violating frequency shift [33]. Obviously, the contribution of proton $b_i^{(p)}$ to $\nu_{\rm LV}$ is non-negligible, and implies that $|b_i^{(p)}| <$ few $\times\, O(10^{-31})$ GeV, which is far better than the results of the dedicated searches of Lorentz violation in the proton sector with e.g. a hydrogen maser [37].

Besides the constraints on nucleon and electron couplings, the same clock comparison experiments allow us to set limits on $M_{P\gamma}$. For example, for an atom (or nucleus) with the total angular momentum J, the matrix element of the $\phi F\tilde{F}$ interaction is not zero,

$$\langle J| \int d^3x \frac{4(\mathbf{E}\cdot\mathbf{B})\phi}{M_{P\gamma}} |J\rangle = \frac{\kappa}{M_{P\gamma}} \left(\frac{\mathbf{J}}{|J|} \cdot \nabla\phi\right), \tag{4.10}$$

where κ is a dimensionless matrix element that can be calculated explicitly. For the ground state of the hydrogen atom, κ is given by

$$\kappa = \frac{8e\mu_B}{3a_0} = \frac{4\alpha^2}{3}, \tag{4.11}$$

where a_0 and μ_B are the Bohr radius and magneton, and α is the fine-structure constant. This calculation takes into account the magnetic field generated by the electron magnetic moment, and the electric field of the proton. If we consider both **E** and **B** created by the electron, we discover that the result has a logarithmic divergence in the ultraviolet regime that has the interpretation of $1/M_{Pe}$ being generated by $1/M_{P\gamma}$. Even with a modestly low choice of the cutoff, the coefficient is going to be on the order of $\alpha/\pi \sim O(10^{-3})$ and thus parametrically larger than (4.11).

What happens if instead of an atomic electron we consider a nucleus where the electric field is considerably stronger? To understand the scaling of the effect with Z, we consider a simplified case of a single s-wave neutron above the closed nuclear shells with a "uniform" distribution of its wave function inside the nucleus, which also has a uniform charge distribution within a sphere of radius $R_N \simeq 1.2\ \mathrm{fm}(A)^{1/3}$. The resulting κ can be expressed in terms of the neutron magnetic moment,

$$\kappa = \frac{8}{5}\frac{2\mu_n Ze}{R_N} = \frac{4}{5}\frac{g_n Z\alpha}{m_p R_N} = 0.05\text{–}0.07 \quad \text{for } Z \sim 80, \tag{4.12}$$

where the overall numerical coefficient follows from the approximation of the radial matrix element, $\langle r^2/(2R_N^2) - 3/2\rangle_{r<R_N} = -6/5$. Although an overall numerical coefficient in estimate (4.12) cannot be taken very seriously, the parametric dependence on Z, μ_n, and R_N is certainly expected to hold for large nuclei. For mercury this effect is larger than the loop-induced admixture of the photon coupling into the nucleon coupling. Thus we can deduce the sensitivity of spin precession experiments to the pseudoscalar couplings to photons at the 5% level from the coupling to neutrons:

$$|M_{P\gamma}M_S| \gtrsim O(10^{-4})M_{\rm Pl}^2, \quad \text{Ref. [29]}. \tag{4.13}$$

One can see that the combined bounds from the clock comparison experiments are comparable to or better than the product of separate bounds (2.9) and (2.10). Unfortunately, these bounds do not allow us to probe the pseudoscalar coupling to fermions all the way to the "natural" scale $M_P \sim M_{\rm Pl}$.

V. CONCLUSIONS

Our paper considers the constraints on the combination of scalar and pseudoscalar couplings in the scalar-tensor theories of gravity. The strongest constraints come from the considerations of the cosmological evolution of polarized light, and in the best case scenario of the maximal scalar coupling, consistent with constraints on Brans-Dicke theories, the sensitivity to the pseudoscalar coupling to photons can be as large as the Planck scale. However, the cosmological constraints are not sensitive to the derivative pseudoscalar couplings to fermions, as they do not induce corresponding photon couplings even at the loop level. We also point out that for a wide range of pseudoscalar masses, one can avoid cosmological constraints due to the redshifting of ϕ oscillations. Therefore, the laboratory constraints on spin precession from locally generated gradients of ϕ are complementary to cosmological bounds. We revisited lab bounds to find that the most sensitive experiments are still a few orders of magnitude below the sensitivity to Planck-scale-suppressed couplings. We also note that the local spin precession experiments provide sensitivity to the pseudoscalar coupling to photons, through the relatively large matrix element of the $\phi\mathbf{B}\cdot\mathbf{E}$ interaction inside atomic nuclei. As a separate remark, we have shown that the nuclei of atoms used in the high-precision clock comparison experiments have significant proton contribution to their spins. This allows us to set separate constraints on pseudoscalar couplings to neutrons and protons, and improve the limit on Lorentz-violating axial-vector backgrounds in the proton sector. Further progress in experiments searching for a preferred Lorentz frame would also provide better sensitivity to the scalar-tensor theories extended by pseudoscalar couplings.

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