PHYSICAL REVIEW D **80,** 104020 (2009)

# Towards a supersymmetric generalization of the Schwarzschild black hole

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(Received 23 June 2009; published 16 November 2009)

The Wheeler-DeWitt (WDW) equation for the Kantowski-Sachs model can also be understood as the WDW equation corresponding to the Schwarzschild black hole due to the well known diffeomorphism between these two metrics. The WDW equation and its solutions are ignorant of the coordinate patch one is using, only by imposing coordinate conditions we can differentiate between cosmological and black hole models. At that point, the foliation parameter $t$ or $r$ will appear in the solution of interest. In this work we supersymmetrize this WDW equation obtaining an extra term in the potential with two possible signs. The WKB method is then applied, giving rise to two classical equations. It is shown that the event horizon can never be reached because very near to it, the extra term in the potential, for each one of the equations, is more relevant than the one that corresponds to Schwarzschild. One can then study the asymptotic cases in which one of the two terms in the Hamiltonian dominates the behavior. One of them corresponds to the usual Schwarzschild black hole. We will study here the other two asymptotic regions; they provide three solutions. All of them have a singularity in $r = 0$ and depending on an integration constant $C$ they can also present a singularity in $r = C^2$. Neither of these solutions have a Newtonian limit. The black hole solution we study is analyzed between the singularity $r = C^2$ and a maximum radius $r_m$. We find an associated mass, considering the related cosmological solution inside $r = C^2$, and based on the holographic principle an entropy can be assigned to this asymptotic solution.

DOI: 10.1103/PhysRevD.80.104020 PACS numbers: 04.70.Bw, 04.60.Ds, 04.65.+e, 12.60.Jv

## I. INTRODUCTION

Black hole physics has been extensively studied in the literature. It is useless to try to address the many interesting aspects, even those related with a single topic. A very rich discussion exists in the literature in one of these topics, namely, that concerning black holes (event) horizons. One can begin by mentioning the fact that in general relativity [1], for stationary vacuum, solutions to the Einstein field equations event horizons arise. Moreover, classical collapse of astrophysical objects results in (future) event horizons [2]. The existence of an event horizon means that one has an inaccessible region, and therefore an external observer must then consider hidden states; pure states become density matrices. So that, seen from outside, the evolution results nonunitary, there is information loss. This is one of those things one has to live with, if one accepts the usual Carter-Penrose diagram. Modifications to this diagram have been proposed based on different theoretical frameworks and models all hinting to a more subtle history for collapse [3]. In classical numerical relativity calculations, event horizons are almost impossible to find with any certainty. Other definitions of horizons like local or quasilocal are used to be able to perform calculations that make sense [4]. It has also been claimed by several authors (see [5] and works cited therein) that there is a variety of physically realistic stellar collapse scenarios in which an event horizon does not in fact forms, so that the singularity remains exposed. Moreover, even though astronomers will recognize that what they have observed so far is compatible with the Schwarzschild or Kerr metrics [6], they will also argue that one cannot unambiguously conclude that the dark objects they observe are black holes in the sense of general relativity. There is an increasing consensus, or at least suspicion, within the general relativity community that event horizons are simply the wrong thing to be looking at. Other possible definitions of horizons have then been proposed; apparent [7], dynamical [8], trapping [9] horizons that make more physical sense. Very powerful and sophisticated methods have been developed since the birth of general relativity searching for solutions to its field equations. For a long time it has been known that changing the structure of spacetime (i.e. interchanging the coordinates $t \leftrightarrow r$), changes a static solution for a cosmological one and vice versa. The best known case is the Schwarzschild metric that under this particular diffeomorphism transforms into the Kantowski-Sachs metric [10]. This interchange of variables has been recently proposed as a method to generate new cosmological models from stationary axisymmetric solutions [11]. In string theory, it has been suggested that by interchanging $r \leftrightarrow it$ we can get time-dependent solutions also from static and stationary solutions. In this way we may relate $Dp$-branes solutions to $S$-brane solutions, i.e. time-dependent backgrounds of the theory [12]. On the other hand, there are proposals to obtain directly $S$-brane solutions [13]; thus, if cosmologi-

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cal solutions (i.e. $S$-branes) can be generated from stationary ones (i.e. $Dp$-branes), this procedure also works the other way around.

In particular, for a Schwarzschild black hole, Kuchař [14] has shown how to reconstruct the curvature coordinates $T$ and $R$ (or the Kruskal coordinate $U$ and $V$) from spherically symmetric initial data. His formalism makes possible a discussion of the action of space-time diffeomorphism on the quantum geometry. A particular interesting example is the interchange of the curvature coordinates $T$ and $R$. This choice of coordinates interchanges the static and dynamical regions for the Kruskal diagram transforming the Kantowski-Sachs cosmological metric into the Schwarzschild metric. This relation was taken into account suggesting a canonical approach based on a foliation in the parameter $r$, by this means a Hamiltonian formalism is developed. This kind of approach was used to find quantum black hole states [15] and a generalization to a noncommutative minisuperspace provides a particular model towards the understanding of noncommutative quantum black holes [16]. The WDW equation solutions are ignorant of the coordinate patch one is using and only when we impose coordinate conditions will there be any difference between cosmological models and black hole solutions. Only at that point will the foliation parameter $t$ and $r$, respectively, appear in the solution of interest.

Several approaches have been suggested to supersymmetrize the WDW equation for cosmological models; the first model proposed [17] was based on the fact that shortly after the invention of supergravity [18] it was shown [19], that this theory provides a natural classical square root of gravity. By this means a method for finding square root equations and their corresponding Hamiltonians in quantum cosmology was proposed in [17], that is the study of supersymmetric quantum cosmology. Later, a superfield formulation was introduced, by means of which it is possible to obtain, in a direct manner, the corresponding fermionic partners and also being able to incorporate matter in a simpler way [20]. A third method allows to define a "square root" of the potential, in the minisuperspace, of the cosmological model of interest and consequently operators which square results in the Hamiltonian [21], other related proposals have been studied [22]. Being the minisuperspace variables in the WDW equation, and consequently the corresponding wave functions, ignorant of the coordinate patch one is using, one imposes coordinate conditions which then produce the difference between cosmological solutions and black hole models.

In this work a WDW equation for a Schwarzschild black hole, is considered [10,15]. It is explicitly shown that, by means of the WKB method, one gets the well-known Schwarzschild solution. Making use of the third method mentioned above [21], a quantum supersymmetric Kantowski-Sachs model, and consequently its corresponding supersymmetric quantum black hole model is found. We get operators which square provides two Hamiltonians that generalize the WDW equation. A simple WKB approach is applied to these Hamiltonians leading to two classical equations, having each one, two asymptotic regions that can be analytically obtained. The Schwarzschild black hole is one of these asymptotic solutions in both cases. However, in general, its horizon can never be reached because when $2m/r$ is very near to one the other two asymptotic regions, of each one of the equations correspondingly are the valid ones. The analytic solutions can be found for both of them, they are singular at $r = 0$; depending on an integration constant $C$, another singularity appears at $r = C^2$. In these asymptotic regions, neither of the solutions have a Newtonian limit. Even though these asymptotic regions are a consequence of supersymmetry, it is interesting to analyze their corresponding classical solutions to understand the behavior of the general classical solution in these asymptotic regions that drastically differ from the Schwarzschild one. It has been shown [23] by solving the Dirac equation, particularly in the Schwarzschild and Kerr backgrounds, that the spinors blow up at the horizon. In our supersymmetric black hole model the fermionic degrees of freedom are intrinsic elements of the theory and they do not allow the presence of the Schwarzschild horizon. Solutions of supergravity theories played a crucial role in important developments in string black hole physics, AdS/CFT and others. It is well known that massive neutral particles cannot be associated with supersymmetric Bogomol'nyi-Prasad-Sommerfield (BPS) states, the most simple spherically symmetric solution that admits Killing spinors to satisfy the constrains that define BPS states is the Reinner-Nordström black hole with $M = |Q|$ [24]. The classical (and quantum) supersymmetric Schwarzschild black hole model we propose is based, as mentioned above, in supersymmetrizing the WDW equation associated with the standard Schwarzschild black hole. This procedure provides a modified (SUSY quantum) Hamiltonian and its corresponding classical equations that, *in this sense* define a supersymmetric generalization of the Schwarzschild black hole. Our proposal seems to provide a starting point to understand and construct a first model of a classical supersymmetric Schwarzschild black hole.

If we would apply whole supergravity ($\mathcal{N} = 1$) to the Kantowski-Sachs-Schwarzschild model, instead of directly supersymmetrizing its WDW equation, it is to be expected to get and equivalent Hamiltonian. As already outlined, in this work we will analyze, in the context of the minisuperspace approximation, the generalized supersymmetric WDW equation for the oldest and most well-known black hole that was discovered by Schwarzschild. We will first review, in Sec. II, the WDW equation for a Schwarzschild black hole, taking advantage of its diffeomorphism with the Kantowski-Sachs model [10,15], and will use the WKB method to obtain the corresponding well-known classical solution. In Sec. III we choose the third and simplest of the three approaches, outlined above, to supersymmetrize the WDW equation and apply it to define the supersymmetric model for the Schwarzschild black hole. The same kind of result would essentially be obtained by using any of the other two approaches (in the case of the first of them, based directly in supergravity $\mathcal{N} = 1$, the fermionic partners are the gravitinos). In this way the corresponding (super) Hamiltonian is obtained. In Sec. IV, the classical analysis, a WKB approach is performed for the only two diagonal components of the (super) Hamiltonian, and the equations corresponding to the asymptotic regions are analytically solved. As already stated, the standard Schwarzschild metric is the solution to one of them and there are other two asymptotic regions corresponding to each one of the two Hamiltonians. One has two solutions and the other, only one, they have a singularity at $r = 0$, and can present also another singularity at $r = C^2$, $(C = \text{const})$. The Schwarzschild horizon can never be reached because when $2m/r$ is very near to one, one must consider the other asymptotic region, in each case, and their corresponding solutions. Even though, in the framework of our proposal, the Schwarzschild solution and the supersymmetric solutions are asymptotic solutions, it is interesting to study the behavior of the last ones; in Sec. V we analyze one of them that has the two singularities at $r = 0$ and at $r = C^2$, find its associated mass in this asymptotic region, and by means of the cosmological model inside $r = C^2$ and making use of the holographic principle, we are able to propose an entropy related to this solution and show its relation with the mass. Section VI is devoted to discussion and conclusions.

## II. WDW EQUATION FOR SCHWARZSCHILD →KANTOWSKI-SACHS METRICS AND THE CLASSICAL LIMIT

Let us begin by reviewing the relationship between the cosmological Kantowski-Sachs metric and the Schwarzschild metric [10,15]. The Schwarzschild solution can be written as

$$ds^2 = -\left(1 - \frac{2m}{r}\right)dt^2 + \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{1}$$

For the case $r < 2m$, the $g_{tt}$ and $g_{rr}$ components of the metric change in sign and $\partial_t$ becomes a spacelike vector, and $\partial_r$ becomes a timelike vector. If we make the coordinate transformation $t \leftrightarrow r$, we find

$$ds^2 = -\left(\frac{2m}{t} - 1\right)^{-1} dt^2 + \left(\frac{2m}{t} - 1\right)dr^2 + t^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{2}$$

On the other hand, the parametrization by Misner [25] appropriate for the Kantowski-Sachs and Schwarzschild metrics is

$$ds^2 = -N^2 dt^2 + e^{2\sqrt{3}\beta} dr^2 + e^{-2\sqrt{3}\beta} e^{-2\sqrt{3}\Omega}(d\theta^2 + \sin^2\theta d\varphi^2). \tag{3}$$

The corresponding WDW equation for the Kantowski-Sachs metric, results in

$$\left[-\frac{\partial^2}{\partial\Omega^2} + \frac{\partial^2}{\partial\beta^2} + 48e^{-2\sqrt{3}\Omega}\right]\psi(\Omega, \beta) = 0. \tag{4}$$

The solution to this equation was given by Misner [25].

Based on the diffeomorphism between the Kanstowski-Sachs and the Schwarzschild metrics, the WDW equation (4) has been applied to find a quantized version of a Schwarzschild black hole [15]. As mentioned, our objective is to find the (super) WDW equation corresponding to (4) that can be traduced in a (super) Hamiltonian for a Schwarzschild black hole and will concentrate our study to its classical solutions, i.e. the supersymmetric generalization of the Schwarzschild black hole. In order to obtain that, first we show how to get the solutions (1) and (2) by applying the WKB method to the WDW equation (4). As is well known, we assume that the wave function has the form

$$\psi = e^{i[S_1(\Omega) + S_2(\beta)]}. \tag{5}$$

The usual procedure results in the Einstein-Hamilton-Jacobi equation

$$-\left(\frac{dS_1(\Omega)}{d\Omega}\right)^2 + \left(\frac{dS_2(\beta)}{d\beta}\right)^2 - 48e^{-2\sqrt{3}\Omega} = 0, \tag{6}$$

one identifies

$$\frac{dS_1(\Omega)}{d\Omega} \to \Pi_\Omega \quad \text{and} \quad \frac{dS_2(\beta)}{d\beta} \to \Pi_\beta, \tag{7}$$

where

$$\Pi_\Omega = -\frac{12}{N} e^{-\sqrt{3}\beta - 2\sqrt{3}\Omega}\dot{\Omega} \quad \text{and} \quad \Pi_\beta = \frac{12}{N} e^{-\sqrt{3}\beta - 2\sqrt{3}\Omega}\dot{\beta}, \tag{8}$$

then the classical equation to be solved is

$$\frac{3}{N^2}(\dot{\Omega}^2 - \dot{\beta}^2) + e^{2\sqrt{3}\Omega + 2\sqrt{3}\beta} = 0. \tag{9}$$

Making use of the Misner parametrization (3), taking $e^{-2\sqrt{3}\Omega - 2\sqrt{3}\beta} = t^2$, and identifying $N^2 = e^{-2\sqrt{3}\beta}$, we get the equation

$$e^{-2\sqrt{3}\Omega}(1 + 2\sqrt{3}t\dot{\Omega}) - t^2 = 0, \tag{10}$$

which solution

$$e^{-2\sqrt{3}\Omega} = 2mt - t^2, \tag{11}$$

with $m = \text{const}$, brings us back to the metric (2), and due to the diffeomorphism between the solutions (1) and (2), choosing the parameter $r$ instead of $t$, we get the Schwarzschild solution (1) as well. So, as it should be, the WKB method applied to the WDW equation (4) gives the classical equation (10), and its solution (11) is the same as that of the classical Einstein equations.

## III. A SUPERSYMMETRIC WDW EQUATION FOR THE MISNER PARAMETRIZATION OF THE KANTOWSKI-SACHS-SCHWARZSCHILD METRICS

In order to generalize the WDW equation (4) to its supersymmetric version the third method outlined in the introduction will be used [21,26,27]. The Hamiltonian for the homogeneous models can in general be written as

$$2H_0 = G^{\mu\nu}\Pi_\mu\Pi_\nu + U(q), \tag{12}$$

where $G^{\mu\nu}$ is the metric in the minisuperspace. It is possible to find a function $\phi$ such that

$$G^{\mu\nu}\frac{\partial\phi}{\partial q^\mu}\frac{\partial\phi}{\partial q^\nu} = U(q). \tag{13}$$

Thus, the minisuperspace Hamiltonian is written in the form

$$H = \frac{1}{2}[Q\bar{Q} + \bar{Q}Q] = H_0 + \frac{1}{2}\frac{\partial^2\phi}{\partial q^\mu \partial q^\nu}[\bar{\theta}^\mu, \theta^\nu], \tag{14}$$

with the non-Hermitian supercharges

$$Q = \theta^\mu\left(\Pi_\mu + i\frac{\partial\phi}{\partial q^\mu}\right), \qquad \bar{Q} = \bar{\theta}^\mu\left(\Pi_\mu - i\frac{\partial\phi}{\partial q^\mu}\right), \tag{15}$$

where $\theta^\nu$ and $\bar{\theta}^\nu$ satisfy the spinor algebra

$$\{\bar{\theta}^\mu, \bar{\theta}^\nu\} = 0, \qquad \{\theta^\mu, \theta^\nu\} = 0, \qquad \{\bar{\theta}^\mu, \theta^\nu\} = G^{\mu\nu}. \tag{16}$$

For our model (4) $U = -48e^{-2\sqrt{3}\Omega}$ and the Hamilton-Jacobi equation is then

$$-\left(\frac{\partial\phi}{\partial\Omega}\right)^2 + \left(\frac{\partial\phi}{\partial\beta}\right)^2 = -48e^{-2\sqrt{3}\Omega}, \tag{17}$$

a solution is

$$\phi = -4e^{\sqrt{3}\Omega}, \tag{18}$$

then according to (15) and (16) the supercharges are given by

$$\begin{aligned} Q &= \theta^\Omega(\Pi_\Omega + i4\sqrt{3}e^{-\sqrt{3}\Omega}) + \theta^\beta\Pi_\beta, \\ \bar{Q} &= \bar{\theta}^\Omega(\Pi_\Omega - i4\sqrt{3}e^{-\sqrt{3}\Omega}) + \bar{\theta}^\beta\Pi_\beta, \end{aligned} \tag{19}$$

where $\Omega$ and $\beta$ are the minisuperspace coordinates.

To obtain the supersymmetric Hamiltonian operator it is necessary to find appropriate representations for the bosonic variables and the fermionic ones $\theta^\Omega$, $\bar{\theta}^\Omega$, $\theta^\beta$ and $\bar{\theta}^\beta$. The momenta will be the usual differential operators $\Pi_\Omega \to -i\frac{\partial}{\partial\Omega}$, $\Pi_\beta \to -i\frac{\partial}{\partial\beta}$ and to realize the fermionic variables algebra (16) we will represent them as matrices, in the following manner

$$\begin{aligned} 2\hat{\theta}^\Omega &= \gamma^1 - i\gamma^2, \qquad 2\hat{\bar{\theta}}^\Omega = \gamma^1 + i\gamma^2, \\ 2\hat{\theta}^\beta &= \gamma^0 + \gamma^3, \qquad 2\hat{\bar{\theta}}^\beta = \gamma^0 - \gamma^3, \end{aligned} \tag{20}$$

for the $\gamma$-matrices we will use the representation

$$\begin{aligned} \gamma^0 &= \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \\ \gamma^1 &= \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix}, \\ \gamma^2 &= \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \\ \gamma^3 &= \begin{pmatrix} 0 & -i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \end{pmatrix}. \end{aligned} \tag{21}$$

Making use of these operators representation, the $\hat{Q}$ and $\hat{\bar{Q}}$ operators can be constructed from (19) and with them, a diagonal Hamiltonian operator $\hat{H}$ is obtained, since the first and third as well as the second and fourth operators are equal, the wave function has only two components. The usual WDW equation (4) is by these means generalized to two quantum equations for the Kantowski-Sachs-Schwarzschild minisuperspace, namely

$$-\frac{\partial^2}{\partial\Omega^2}\psi_\pm + \frac{\partial^2}{\partial\beta^2}\psi_\pm + 12e^{-2\sqrt{3}\Omega}(4 \pm e^{\sqrt{3}\Omega})\psi_\pm = 0, \tag{22}$$

where $\psi_+$ and $\psi_-$ correspond to the wave function associated with the $(+)$ and the $(-)$ signs in the potential.

Note that both quantum equations differ from the usual WDW equation (4) by the same extra term but with different sign. This is similar to what happens in standard supersymmetric quantum mechanics where also extra terms arise in the potential. Even though, the last term in (22), is expected to be relevant only when the supersymmetric contribution is larger or of the order of the usual one, it is of interest to study the behavior of these other asymptotic solutions that considerable differ from the Schwarzschild solution, as will be shown.

It is clear that to certain linear combinations of the fermionic operators $\hat{\theta}^\Omega$, $\hat{\theta}^\beta$, $\hat{\bar{\theta}}^\Omega$, and $\hat{\bar{\theta}}^\beta$, one can associate the eigenvectors

$$\begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \quad \begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \quad \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \quad \text{and} \quad \begin{pmatrix}0\\0\\0\\1\end{pmatrix}. \tag{23}$$

So, for example, the first eigenvector is an eigenstate with eigenvalue $+1$ corresponding to one of these particular combinations, having then that the eigenvectors (23) are linear combinations of the eigenstates of the operators (20) and (21). Through an appropriate rotation one could associate the above four eigenstates directly with the fermionic operators $\hat{\theta}^\mu$ and $\hat{\bar{\theta}}^\mu$ [17,22], and by this means identify their contribution to wave function components.

## IV. THE CLASSICAL LIMIT OF THE SUPERSYMMETRIC REGION

Following the same procedure, to apply the WKB method presented in Sec. II, one obtains two classical equations corresponding to (22), that written together give

$$\frac{12}{N^2}(\dot{\Omega}^2 - \dot{\beta}^2) + e^{2\sqrt{3}\Omega + 2\sqrt{3}\beta}(4 \pm e^{\sqrt{3}\Omega}) = 0. \tag{24}$$

Making use of the Misner parametrization (3), as in Sec. II and using $e^{-2\sqrt{3}\Omega - 2\sqrt{3}\beta} = t^2$ and $N^2 = e^{-2\sqrt{3}\beta}$ one has

$$4e^{-2\sqrt{3}\Omega}(1 + 2t\sqrt{3}\dot{\Omega}) - t^2(4 \pm e^{\sqrt{3}\Omega}) = 0. \tag{25}$$

Both of the classical equations (25) present two asymptotic regions for $4 \gg e^{\sqrt{3}\Omega}$ and another for $4 \ll e^{\sqrt{3}\Omega}$. By interchanging $t$ by $r$, as in Sec. II, the first limit evidently gives the Schwarzschild solution (1), (10), and (11). In the other region, the extra term in the potential (25) dominates and one has two equations. We will see that the corresponding solutions present a singularity at $r = 0$ and, depending on the sign of the integration constant $C$, another singularity could also be present at $r = C^2$ in both cases. In these supersymmetric dominated regions, neither of the solutions can be related with the Newtonian limit. Already, at this stage, Eq. (25) tells us that the Schwarzschild horizon can never be reached.

As we can deduce from (11), the Schwarzschild case, changing $t$ by $r$, gives

$$e^{\sqrt{3}\Omega} = \frac{1}{r(1 - \frac{2m}{r})^{1/2}}. \tag{26}$$

Independently of the value of $r$, for $2m/r$ very near to one, $e^{\sqrt{3}\Omega}$ can be very large, $e^{\sqrt{3}\Omega} \gg 4$ and according to (25) we are then in the other asymptotic regions, not the one corresponding to Schwarzschild and these have different solutions.

The solutions for these asymptotic regions (when $e^{\sqrt{3}\Omega} \gg 4$), are

$$e^{-\sqrt{3}\Omega} = \left(\frac{3}{4}\right)^{1/3} r^{2/3}\left(\pm 1 + \frac{C}{\sqrt{r}}\right)^{1/3}, \tag{27}$$

where $C$ is a constant and taking into account the Misner parametrization (3) and appropriately the coefficients of $dt^2$ and $dr^2$ we get

$$\begin{aligned} ds^2 = &-\left(\frac{3}{4}\right)^{2/3} r^{-2/3}\left(\pm 1 + \frac{C}{\sqrt{r}}\right)^{2/3} dt^2 \\ &+ \left(\frac{4}{3}\right)^{2/3} r^{2/3}\left(\pm 1 + \frac{C}{\sqrt{r}}\right)^{-2/3} dr^2 \\ &+ r^2(d\theta^2 + \sin^2\theta d\phi^2). \end{aligned} \tag{28}$$

As stated, these solutions are valid only in the asymptotic region $e^{\sqrt{3}\Omega} \gg 4$. Taking the solution with the positive sign, in the one of the parenthesis, one should consider two cases $C > 0$ and $C < 0$. In the first of them, solution 1, there is a singularity only at $r = 0$. In the second case there is also a second singularity at $r = C^2$, solution 2. For the solution with the negative sign the constant $C$ can be negative or positive. The first possibility would give a negative value for $e^{-\sqrt{3}\Omega}$, then this is not a valid case. So, we will consider only values $C > 0$. The singularities for this case will be at $r = 0$ and at $r = C^2$, solution 3. In order to exhibit the singularities, the Kretschmann invariant is calculated for the solutions (28), it is given by

$$\begin{aligned} K_\pm = &\frac{1}{864(\pm 1 + \frac{C}{\sqrt{r}})^{8/3} r^{22/3}}\Bigg[38\,88\,6^{1/3}C^4 - 216\,6^{1/3}C^3 r^{1/2}\left[\mp 63 + 8\,6^{1/3}\left(\pm 1 + \frac{C}{\sqrt{r}}\right)^{1/3} r^{2/3}\right] \\ &+ 32r^2\left[71\,6^{1/3} \mp 54\,6^{2/3} r^{2/3}\left(\pm 1 + \frac{C}{\sqrt{r}}\right)^{1/3} + 108 r^{4/3}\left(\pm 1 + \frac{C}{\sqrt{r}}\right)^{2/3}\right] \\ &\pm 24 C r^{3/2}\left[431\,6^{1/3} \mp 216\,6^{2/3} r^{2/3}\left(\pm 1 + \frac{C}{\sqrt{r}}\right)^{1/3} + 288 r^{4/3}\left(\pm 1 + \frac{C}{\sqrt{r}}\right)^{2/3}\right] \\ &+ 27C^2\left[659\,6^{1/3} r \mp 192\,6^{2/3} r^{5/3}\left(\pm 1 + \frac{C}{\sqrt{r}}\right)^{1/3} + 128 r^{7/3}\left(\pm 1 + \frac{C}{\sqrt{r}}\right)^{2/3}\right]\Bigg]. \end{aligned} \tag{29}$$

From (29) we can read the above discussed singularities at the origin $r = 0$ and at the radius $r = C^2$.

The SUSY-WDW equation (22) and their corresponding classical equations (24) have in their own structure the information of certain states related with our fermionic variables ("gravitinos" if we would have used the supergravity $\mathcal{N} = 1$ method), they manifest themselves in the potential. As is well known, by solving the Dirac equation in the Schwarzschild and Kerr backgrounds the spinors blow up at the horizon [23]. It is then reasonable to expect that a model where the fermionic fields are intrinsically incorporated do not allow the existence of a horizon, as we have shown. It is to be noted that our (quantum) supersymmetric approach removes, in the WKB limit (27), in this SUSY asymptotic region, the horizon but not the singularity at $r = 0$ that remains in all the three solutions, in fact, in two of them another singularity arises in $r = C^2$.

## V. ANALYSIS OF SOLUTION 2, AN ASSOCIATED MASS AND A POSSIBLE RELATION WITH THE ENTROPY

Lets us consider

$$ds^2 = -Fdt^2 + F^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{30}$$

with $F = (\frac{3}{4})^{2/3} r^{-2/3}(1 - \frac{C}{\sqrt{r}})^{2/3}$ and $C > 0$. The expression given by Eq. (27) must be positive. This imposes the restriction $\sqrt{r} > C$. Also according to Eq. (25) solution (30) is valid for $e^{\sqrt{3}\Omega} \gg 4$. This implies (27) that there is a maximum radius $r_m > C^2$ (in Planck lengths) where this solution is valid.

Because of the fact that for large $r$ the Minkowski metric is not a limit of (30), one can consider this metric at $r_m$ as the background metric. With this assumption we can follow a well known proposal [28] to associate a mass to our metric. The mass formula can be expressed as

$$M = -\frac{1}{2G_N}\frac{|\tilde{g}_{tt}|^{1/2}}{|\tilde{g}_{rr}|^{3/2}} r(g_{rr} - \tilde{g}_{rr}), \tag{31}$$

where $\tilde{g}_{tt}$ and $\tilde{g}_{rr}$ correspond to the background metric, in our case $\tilde{F}$ and $\tilde{F}^{-1}$, which, for a large enough $r_m$, results in

$$\tilde{F}^{-1} = \left(\frac{4}{3}\right)^{2/3} r_m^{2/3}, \tag{32}$$

for $r = \alpha r_m$, $\alpha \ll 1$, then

$$F^{-1} = \tilde{F}^{-1}\alpha^{2/3}\left(1 + \frac{2}{3}\frac{C}{\alpha^{1/2} r_m^{1/2}}\right), \tag{33}$$

and $M = -\frac{1}{2G_N}\tilde{F}^2 r(F^{-1} - \tilde{F}^{-1})$, which results in

$$M = \left(\frac{3}{4}\right)^{2/3}\frac{1}{2G_N}\left[\alpha(1 - \alpha^{2/3}) r_m^{1/3} - \frac{2}{3}\alpha^{7/6} C r_m^{-1/6}\right]. \tag{34}$$

As $r_m$ is proportional to $C^2$, we have that

$$M \sim C^{2/3}. \tag{35}$$

This relation between the constant of integration $C$ and the mass was to be expected because $F = (\frac{3}{4})^{2/3}(\frac{1}{r} - \frac{C}{r^{3/2}})^{2/3}$, so $C^{2/3}$ would be related with $r$, in the same manner as the mass appears in the usual static metric $\sim \frac{M}{r}$.

The SUSY-WDW equation (22) provides in the asymptotic region $e^{\sqrt{3}\Omega} \gg 4$, the black hole solution (30). This has an associated cosmological model inside $r = C^2$ as it happens in the standard bosonic case (1) and (2). This cosmological solution can be expressed as (30) but now with

$$F = \left(\frac{4}{3}\right)^{2/3} t^{2/3}\left(\frac{C}{\sqrt{t}} - 1\right)^{-2/3}. \tag{36}$$

On the other hand, the holographic principle tells us that for a given volume $V$, the state of maximal entropy is given by the largest black hole that fits inside $V$. 'tHooft and Susskind [29] argued that the microscopic entropy associated with the volume $V$ should not exceed the Bekenstein-Hawking entropy $S \leq \frac{A}{4G}$ of a certain black hole with horizon area $A$ equal to the surface area of the boundary of $V$. A particular model to realize this idea was given by Verlinde [30]. He generalized the Cardy formula [31] to arbitrary spacetime dimensions and proposed that a closed universe has a Casimir contribution to its energy and entropy and that the Casimir energy is bounded from above by the Bekenstein-Hawking energy. He found that $S_{\rm BH} = (n-1)\frac{V}{4\pi GR}$, where $n$ is the number of space dimensions, $V$ is the volume of the universe, and $R$ its radius. This $S_{\rm BH}$ was identified with the holographic Bekenstein-Hawking entropy of a black hole with the size of the universe.

The largest possible radius of the universe (30) and (36) cannot exceed $C^2$. So, the largest standard black hole fitting $V$ would have also $r = C^2$ and accordingly the maximum entropy of this universe should be $S \sim \frac{A}{4G} \sim r^2 \sim C^4$ (note that we cannot assign an entropy to our black hole solution outside $r = C^2$ (30) by following the usual procedure [32], because this solution has no horizon, as already shown). Now taking into account $M \sim C^{2/3}$ (34) and (35), it results that $S \sim C^4 \sim M^6$. As mentioned, we have not found the general solutions to the classical equations (24) and (25), the above resulting mass and entropy were obtained in the extreme supersymmetric (or fermionic) limit. As consequence, this entropy $S \sim M^6$, with $M$ given by (34) and (35), should be interpreted as a correction to the usual one emerging in this (extreme) asymptotic (quantum) supersymmetric regime.

## VI. CONCLUSIONS

Based on previous works [14,15], we have first shown that the WDW equation (4) has a classical limit (10) of which the solution is the Schwarzschild metric (1), [or the

Kantowski-Sachs cosmological model (2)]. A supersymmetrization of (4) results in (22) which classical limit (25) shows already (26) that the event horizon can never be reached because very near to it, the other term in the potential in (25) dominates. It is known, by solving the Dirac equation, that the spinors blow up at the horizon of the Schwarzschild and Kerr metrics [23]. In our supersymmetry black hole proposal the fermionic degrees of freedom (the gravitinos if we would have used the first supergravity $\mathcal{N} = 1$ [17,22] method mentioned in this work) are intrinsic elements of the model and they do not allow the presence of an event horizon. Each Eq. (25) exhibits two asymptotic limits, neither of them correspond to a whole exact solution of this equation. Nevertheless, the Schwarzschild black hole is the solution to one of these asymptotic regions. We have studied the other two possible asymptotic regions, depending on the sign of the extra term in the potential. The three solutions are given by (28). They are singular at $r = 0$ and depending on the sign of $C$ they can also be singular at $r = C^2$ (29). None of these supersymmetric dominated solutions have a Newtonian limit. We then analyzed the second solution (30), the Minkowski space-time is not a background metric of it. However, there is a maximum radius, $r_m > C^2$, the metric (30) calculated at $r_m$ is assumed as the background metric. By this means a mass (34) can be associated to this solution and it results that we can relate it with our constant of integration $C$; $M \sim C^{2/3}$ (35). Inside $r = C^2$ one has the cosmological solution (30) and (36) and according with the holographic principle proposal, the state of maximum entropy should correspond to the entropy associated with the largest standard black hole fitting its volume. The maximum radius of this universe is $r = C^2$, consequently $S \sim C^4$ and by means of $M \sim C^{2/3}$ (35), it results that $S \sim M^6$. This arises in the asymptotic supersymmetric (fermionic) region, so the mass (34) and (35) and its associate entropy correspond to this extreme regime and could be understood as a (quantum) SUSY correction to the standard black hole entropy.

## ACKNOWLEDGMENTS

This work was supported in part by CONACyT México Grant No. 51306 and PROMEP projects.


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