PHYSICAL REVIEW D **80,** 104009 (2009)

Ultrahigh precision cosmology from gravitational waves

Curt Cutler[1] and Daniel E. Holz[2]

[1]*Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91109, USA*

[2]*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*

(Received 23 June 2009; published 9 November 2009)

We show that the Big Bang Observer (BBO), a proposed space-based gravitational-wave (GW) detector, would provide ultraprecise measurements of cosmological parameters. By detecting $\sim 3 \times 10^5$ compact-star binaries, and utilizing them as standard sirens, BBO would determine the Hubble constant to $\sim 0.1\%$, and the dark-energy parameters w_0 and w_a to ~ 0.01 and ~ 0.1, respectively. BBO's dark-energy figure-of-merit would be approximately an order of magnitude better than all other proposed, dedicated dark-energy missions. To date, BBO has been designed with the primary goal of searching for gravitational waves from inflation, down to the level $\Omega_{\rm GW} \sim 10^{-17}$; this requirement determines BBO's frequency band (deci-Hz) and its sensitivity requirement (strain measured to $\sim 10^{-24}$). To observe an inflationary GW background, BBO would first have to detect and subtract out $\sim 3 \times 10^5$ merging compact-star binaries, out to a redshift $z \sim 5$. It is precisely this carefully measured foreground which would enable high-precision cosmology. BBO would determine the luminosity distance to each binary to $\sim$ percent accuracy. In addition, BBO's angular resolution would be sufficient to uniquely identify the host galaxy for the majority of binaries; a coordinated optical/infrared observing campaign could obtain the redshifts. Combining the GW-derived distances and the electromagnetically-derived redshifts for such a large sample of objects, out to such high redshift, naturally leads to extraordinarily tight constraints on cosmological parameters. We emphasize that such ''standard siren'' measurements of cosmology avoid many of the systematic errors associated with other techniques: GWs offer a *physics-based*, absolute measurement of distance. In addition, we show that BBO would also serve as an exceptionally powerful gravitational-lensing mission, and we briefly discuss other astronomical uses of BBO, including providing an early warning system for all short/hard gamma-ray bursts.

DOI: 10.1103/PhysRevD.80.104009 PACS numbers: 04.30.−w, 04.80.Nn, 95.35.+d, 95.36.+x

I. INTRODUCTION

Improving our understanding of the dark energy responsible for the observed accelerating expansion of the Universe is one of the foremost challenges in physics. Our current theoretical models are exceedingly inadequate. In addition, given its extremely low energy density, there are no plausible scenarios for the direct detection of dark energy. Thus progress on this critical issue must be made through indirect observations. In this paper we show that the Big Bang Observer (BBO), a proposed space-based gravitational-wave (GW) mission designed primarily to search for inflation-generated stochastic GWs in the band 0.03 Hz–3 Hz [1], would *also* be an ultraprecise cosmology mission, measuring the Hubble constant H_0 and the dark-energy parameters w_0 and w_a far more accurately than other proposed dark-energy missions.

BBO has been proposed as a follow-on mission to the Laser Interferometer Space Antenna (LISA), which will be most sensitive to GWs in the band $\sim 10^{-4.5} - 10^{-1.5}$ Hz. In the LISA band, any primordial GWs from standard inflation are likely to be buried under the GW foreground from the short-period white dwarf-white dwarf (WD-WD) binaries in the universe. The WD-WD foreground is practically absent in the BBO band, since the WD-WD contribution falls very rapidly for $f > 0.01$ Hz, and disappears entirely for $f > 0.25$ Hz, where the most massive (and hence smallest) WDs merge [1–3]. Instead, the dominant astrophysical foreground in the BBO band is mergers of compact binaries composed of neutron stars (NSs) or black holes (BHs); i.e., NS-NS, NS-BH, and BH-BH binaries. BBO would be sufficiently sensitive to individually detect and subtract out essentially every merging compact binaries out to high redshift, thereby uncovering any primordial GW background in its band that has energy density $\Omega_{\rm GW} \gtrsim 10^{-17}$ [1,4,5].

As far as the search for primordial GWs is concerned, merging compact binaries represent a foreground that must first be removed. In this paper we show that this foreground is a cosmological gold mine, allowing astronomers to measure the expansion history of the Universe (out to at least $z \sim 5$) far more accurately than the most ambitious currently-proposed cosmology missions. The basic argument is this: GW detectors in general, and LISA and BBO, in particular, will provide high-accuracy measurements of the luminosity distances, D_L, to detected merging binaries. However, the GW signals contain (almost) no information regarding the source redshifts. Thus the situation in GW astronomy is the exact reverse of optical/electromagnetic astronomy: accurate distances will be relatively easy to come by, while determining redshifts will be much more challenging. For both the ground-based LIGO/VIRGO network and the space-based LISA, the angular resolution for detected binaries will typically be several degrees [6,7], so the error box on the sky would contain $\sim 10^{5-6}$ possible host galaxies per event. Therefore plans for doing cosmology with GW observations have usually hinged on finding some electromagnetic outburst associated with the GW events, in order to identify the host galaxy and obtain its redshift [8] (though see Finn and Chernoff [9], MacLeod and Hogan [10], and Seto *et al.* [11] for suggestions on how to evade this requirement). GW-detected merging binaries for which one can *also* determine a redshift have been dubbed ''gold-plated'' binaries; Holz and Hughes have shown that, with LISA, even a handful of such ''gold-plated'' detections of massive black hole binaries could make significant contributions to cosmology [12].

BBO should detect $\sim 10^5$ merging NS-NS binaries per year [4], out to $z \sim 5$. BBO's angular resolution for NS-NS mergers will typically be a few arcsec: a small enough error box to uniquely identify the host galaxy in most cases. This is especially true since one can also use the source's luminosity distance, D_L (measured to several percent), and even a crude $D_L - z$ relation, to rule out galaxies at the right position on the sky but with significantly different redshifts (e.g., differing by $>10\%$). While a unique host galaxy may not be identifiable in very dense galaxy clusters, in such cases one can substitute the average redshift of the cluster; this will lead to a typical redshift error of order $\Delta z \sim 2 \times 10^{-3}$, which is negligible for our purposes. Thus—in stark contrast with LISA and ground-based GW detectors—BBO will detect $\sim 10^5$ ''gold-plated'' binaries per year! BBO will measure the luminosity distance to each NS-NS binary with a relative accuracy of several percent. For example, for a NS-NS binary at $z = 1.5$, the median distance error due to detector noise will be $\sim 2\%$, while the distance error due to weak lensing (WL) will be $\sim 7\%$. For a large sample of sources, both of these errors ''average out'', and the extent to which they do not average out can be readily modeled and accounted for. By contrast, the ambitious, space-based dark-energy mission SNAP would be expected to observe roughly 2000 SNe over the lifetime of the mission, out to a maximum redshift of $z \sim 1.7$ [13]. BBO provides an overwhelmingly larger and deeper data set, with each individual distance measured significantly more precisely. It is thus to be expected that BBO will vastly outperform other proposed dark-energy missions; in fact, BBO might well provide better constraints than all other proposed dark-energy missions *combined*. As we show below, with the inclusion of an (expected) Planck prior, BBO data should measure H_0 to $\sim 0.1\%$, w_0 to 0.01 and w_a to 0.1.

Perhaps even more importantly, we argue that the systematic errors associated with GW detections are generally much smaller, and much easier to characterize, than with any other proposed methods (e.g., weak lensing, supernovae, or baryon acoustic oscillations). Type Ia supernovae (SNe) are arguably the state of the art in cosmological distance measurement. The intrinsic luminosity of a type Ia SN can be *empirically* calibrated to roughly 10%. The physics underlying this calibration is only poorly understood, and possible evolutionary systematic effects are a grave cause for concern (see, e.g., [14–16]). By contrast, the compact binaries detectable by BBO are exceptionally simple sources. At the orbital separations at which BBO observes them, the compact objects can be treated as point masses (with spin), whose dynamics are very accurately described by the post-Newtonian approximation. Systematic distance errors arising from the detector itself will also be negligible, since BBO (like LISA) is fundamentally self-calibrating [17]. The optical scheme is somewhat complicated in practice, but in essence one is measuring the spacecraft arm lengths (more precisely, small time-varying changes in the differences between spacecraft arm lengths) in units of the laser wavelength. For LISA, the laser wavelength (or equivalently its frequency) will be known to an accuracy of $\sim 10^{-6} - 10^{-5}$; this small uncertainty is expected to dominate LISA's calibration error, which is therefore also at the $\sim 10^{-6} - 10^{-5}$ level. (For the same reason, LISA Pathfinder's calibration is expected to be accurate to better than 10^{-5} [18].) One could reduce this small inaccuracy by, say, stabilizing the laser to a transition line in an iodine gas cell [19], if there were sufficient motivation. However there is one caveat concerning BBO's calibration accuracy, which we discuss in Sec. III C.

In addition to determining the equation of state of dark energy, BBO should also be an excellent probe of the growth of structure. BBO's measurements of gravitational lensing will be as sensitive as dedicated weak-lensing missions. The basic idea is that once a $D_L - z$ relation has been derived from the entire NS-NS sample, the dispersion of the $\sim 3 \times 10^5$ NS-NS data points about this curve is dominated by magnification by gravitational lensing. That is, one has obtained $\sim 3 \times 10^5$ independent measurements of the lensing magnification along different lines-of-sight, one for each binary. We show below that the typical SNR for each measurement is ~ 3.4, and therefore the SNR for the whole NS-NS population is $\sim 3.4 \times \sqrt{3 \times 10^5} \sim 2 \times 10^3$. Although the NS-NS merger rate most likely exceeds the BH-BH rate, the lensing SNR from the BH-BH population may exceed that of NS-NS mergers. The BH-BH merger rate is poorly constrained by observation; a reasonable estimate from population synthesis models is that the BH-BH rate is a factor ~ 20 smaller than the NS-NS rate [20]. However, the typical SNR for each BH-BH merger is larger by a factor ~ 5.3 (for our fiducial NS and BH masses). We thus estimate a lensing SNR for the whole BH-BH population of $\sim 2.2 \times 10^3$, and a total SNR for both populations combined of $\sim 3 \times 10^3$. This total lensing SNR could be significantly higher, if the BH-BH merger rate is near the high end of the estimated range. (Of course, advanced ground-based GW detectors will *measure* these rates, for $z \lesssim 0.4$, many years before BBO flies.) In Sec. III B we investigate BBO's sensitivity as a gravitational lensing mission, and compare it with other lensing missions.

While obtaining redshifts for 3×10^5 host galaxies would be a highly ambitious goal at present, we are optimistic that it will be far less daunting by the time BBO flies. By then LSST may already have determined photometric redshifts (accurate to $\sim$2–3%) for a large fraction of the host galaxies in $\sim 1/3$ of the sky. In addition, there are many proposed wide-field spectroscopic surveys; for example, BigBOSS would measure $\sim$5 million spectroscopic redshifts/yr (4000 at a time) for galaxies in the range $0.2 < z < 3.5$, over an area of 14, 000 deg^2 [21]. The success of BBO as a cosmological probe is dependent upon the determination of host redshifts. It will thus be critical to secure the necessary optical resources and develop an efficient strategy for obtaining redshifts for a large sample of binary host galaxies.

After this work was mostly completed we found some brief remarks in the literature that partly anticipate our results. Crowder & Cornish [22] point out (in one sentence) that BBO should be able to localize the host galaxy for most observed compact-binary mergers, but the profound implications of this fact for physical cosmology are not developed. One slide in a workshop talk on Decigo by Seto [23] lists in brief bullets that Decigo measurements of NS-NS mergers could be used to probe dark energy though the $D_L - z$ relation, and that some of these detections will likely be GW-counterparts to gamma-ray bursts, allowing one to get a redshift. But these slides give no estimates of the resulting cosmological constraints, nor any sense of the potentially revolutionary implications of this observation (perhaps because the current version of Decigo would likely have relatively poor calibration accuracy, and so the cosmological measurements would suffer from large systematics; see Sec. III C). Seto *et al.* [11] also proposed using deci-Hz GW detections of inspiralling NS-NS binaries to measure the universe's expansion, using a fundamentally different approach: observing the small fractional change in source redshift over the course of the inspiral (detected as a small time-varying phase shift). Because this phase shift is such a small effect, the cosmological constraints that these authors estimated as obtainable are orders of magnitude less sensitive than the constraints we find (for comparable mission sensitivities).

In this paper we focus on BBO, although it is to be noted that the Japanese GW community is proposing a similar mission called Decigo. Our basic conclusions apply to any deci-Hz GW mission of roughly BBO-level sensitivity, including a Decigo-like mission, so long as the design ensures excellent calibration accuracy. We discuss this further in Sec. II A.

The rest of this paper is organized as follows. In Sec. II we give a brief overview of the BBO mission, its design sensitivity, the foreground produced by NS-NS binaries, and how accurately BBO can measure the distance and sky location of inspiralling compact binaries. In Sec. III we derive the cosmological measurement accuracy obtainable by BBO from $\sim 3 \times 10^5$ NS-NS binaries. We also investigate BBO's performance as a weak-lensing mission, and discuss some caveats accompanying our conclusions. In Sec. IV we briefly discuss several other astrophysical uses of the BBO compact-binary data: as an early warning system for *all* short/hard gamma-ray bursts, in searches for the earliest intermediate-mass black hole mergers, and in studies of hundreds of strongly lensed GW sources. Our conclusions and plans for future work are outlined in Sec. V.

We use units in which $G = c = 1$; everything can be measured in the fundamental unit of seconds. However, for the sake of familiarity, we also sometimes express quantities in terms of yr, Mpc, or $M_\odot$, which are related to our fundamental unit by $1\ \mathrm{yr} = 3.1556 \times 10^7\ \mathrm{s}$, $1\ \mathrm{Mpc} = 1.029 \times 10^{14}\ \mathrm{s}$, and $1 M_\odot = 4.926 \times 10^{-6}\ \mathrm{s}$. For concreteness, in our simulations we adopt the following fiducial values for the cosmological parameters: $H_0 = 70\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\Omega_\mathrm{m} = 0.3$, and $\Omega_\Lambda = 0.7$.

II. OVERVIEW OF BBO, DECIGO, AND THE NS-NS MERGER FOREGROUND

A.BBO

BBO will be most sensitive in the band $\sim 0.03 - 3$ Hz, which is dictated by BBO's main design goal: to detect primordial GWs generated by inflation. BBO has been proposed as a follow-on mission to the Laser Interferometer Space Antenna, which will operate at $\sim 10^{-4.5} - 10^{-1.5}$ Hz. In the BBO band, the dominant astrophysical foreground will be mergers of NS-NS, NS-BH, and BH-BH binaries. The BBO mission is designed so that essentially each and every merging compact binaries in the observable universe (i.e., on BBO's past light cone) can be detected and subtracted out. Cutler & Harms [4] have presented a detection/subtraction algorithm and showed by an analytical calculation that it should work; Harms *et al.* [5] implemented the algorithm and demonstrated that it worked well on simulated BBO data, albeit their demonstration was on a much reduced data set because of computational limitations. A very different algorithm for detecting and subtracting the binary signals from BBO data is also being developed by N. Kanda and collaborators (unpublished).

The current BBO design calls for four constellations of three satellites each, all following heliocentric orbits at a distance of 1 AU from the Sun (see Fig. 1). Each 3-satellite constellation can be thought of as a highly sensitive "mini-LISA", since the arm lengths for each constellation are 2 orders of magnitude smaller than LISA's, and BBO's sensitivity band is correspondingly 2 orders of magnitude higher in frequency. Two of the constellations overlap to

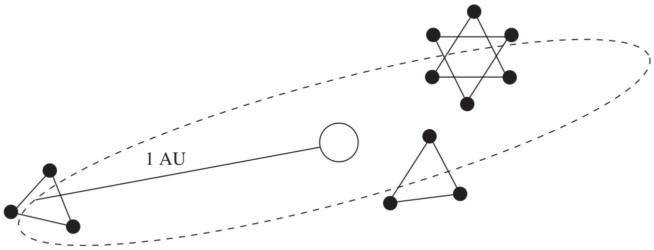

FIG. 1. Big-Bang Observer (BBO) consists of four LISA-like triangular constellations orbiting the Sun at 1 AU. The GW background is measured by cross-correlating the outputs of the two overlapping constellations, while time-of-flight across the Solar System gives BBO its angular resolution. A schematic of Decigo (a Japanese proposal similar to BBO) would be almost identical, except that the constellations are 50 times smaller than BBO's, and their arms form Fabry-Perot cavities.

form a "star of David"; the other two are ahead and behind by $2\pi/3$ radians, respectively. Briefly, the idea behind this orbital geometry is that the energy density of the primordial GW background, $\Omega_{\rm GW}(f)$, will be measured by cross-correlating the outputs of the two overlapping constellations in the star of David (much as LIGO attempts to measure $\Omega_{\rm GW}(f)$ by cross-correlating the outputs of the Livingston and Hanford interferometers [24]). The other two constellations give BBO its angular resolution, which is useful for characterizing and removing the merging compact-binary foreground. The source's angular position on the sky is mostly determined by triangulation, using the differences in arrival times of the GWs at the different constellations.

As explained in Cutler & Harms [4], for compact-binary mergers the science output of each mini-LISA is in practice equivalent to the output of two synthetic Michelson detectors, represented by the time-delay interferometry (TDI) variables X and $(Y-Z)/\sqrt{3}$. We can therefore regard BBO, which is made up of 4 mini-LISAs, as formally equivalent to 8 synthetic Michelson interferometers. To construct the instrumental noise curve, $S_{\rm h}(f)$, of *each* of these synthetic Michelsons, we use Larson's on-line "Sensitivity curve generator" [25] and plug in BBO's instrumental parameters, which are taken from the BBO Concept Study [1] and also listed in Table I. These parameters will be subject to change as the mission evolves, but for now they provide a convenient baseline. The BBO Concept Study [1] also lists parameters for less and more ambitious versions of the BBO mission, referred to as "BBO-lite" and "BBO-grand", respectively, but in this paper we restrict attention to the intermediate version, or "standard BBO". In using the on-line generator, we have specified that the high-frequency part of $S_{\rm h}$ is 4 times larger than the contribution from photon shot noise alone; this factor 4 accounts for high-frequency noise components *other* than shot noise, such as beam pointing jitter and stray light effects. This is the same choice made in Fig. 1 of the BBO proposal [1], and is consistent with the standard assumptions made in drawing the LISA noise curve. This BBO instrumental noise curve is shown in Fig. 2.

TABLE I. BBO parameters.

	Symbol	Value
Laser power	P	300 W
Mirror diameter	D	3.5 m
Optical efficiency	ϵ	0.3
Arm length	L	$5\cdot 10^7$ m
Wavelength of laser light	λ	0.5 μm
Acceleration noise	$\sqrt{S_{\rm acc}}$	$3\cdot 10^{-17}$ m/(s$^2\sqrt{\rm Hz}$)

B. Decigo

BBO is seen as a follow-on mission to LISA in the U.S. and Europe, but in the Japanese GW community there is a strong push to launch a deci-Hz GW mission first. The current plan is for Decigo to be a factor ~2–3 less sensitive than BBO, but for it to launch earlier, in ~2024. A small Decigo precursor mission, Decigo Pathfinder (DPF), was recently among the final two missions competing for the second launch slot (in ~2012) in the JAXA (the Japanese space agency) small satellite science series, but lost that competition. The DPF will now compete for the third launch slot.

Our research to date has concentrated on BBO, but it would be straightforward to generalize our BBO analysis to a mission with Decigo-level sensitivity. Indeed, we expect that pursuing a BBO-style mission, but with a somewhat less ambitious sensitivity goal, might be advisable from a cost/benefit standpoint; Decigo may have a comparable cosmological reach to BBO, if designed to ensure excellent calibration accuracy. In follow-up work,

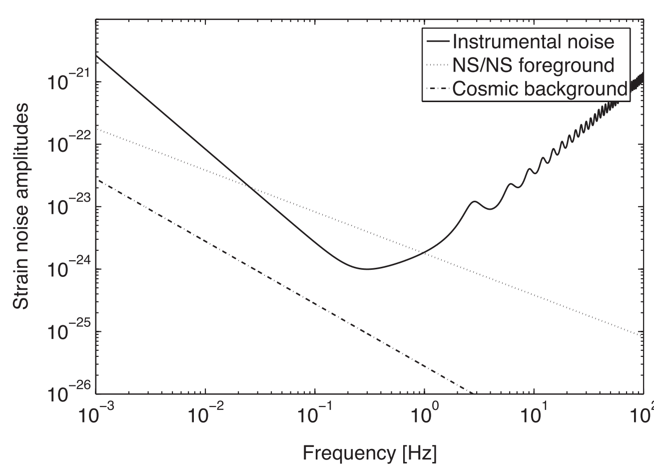

FIG. 2. Amplitude of BBO's instrumental noise, $\sqrt{fS_{\rm h}(f)}$, compared to the amplitude of the (presubtraction) NS binary foreground (plotted for $\dot{n}_0 = 10^{-7}$ Mpc^{-3} yr^{-1}) and the sought-for cosmic GW background (plotted for $\Omega_{\rm GW}(f) = 10^{-16}$). To reveal this cosmic background, the NS foreground must be subtracted off, with fractional residual of $\lesssim 10^{-2.5}$.

we plan to investigate how the science payoff from a deci-Hz GW mission varies with its sensitivity.

C. The NS-NS merger rate over time

BBO would be able to observe BH-BH and BH-NS mergers to significantly higher redshifts than NS-NS mergers, but the rates for BH-BH and BH-NS mergers are more uncertain (and probably a factor ~20 lower) than NS-NS mergers, so our discussion in this section will focus mostly on the NS-NS case. The extension of our work to NS-BH and BH-BH binaries is straightforward.

We denote the NS-NS merger rate (per unit proper time, per unit comoving volume) at redshift z by $\dot{n}(z)$. It is convenient to regard $\dot{n}(z)$ as the product of two factors:

$$\dot{n}(z) = \dot{n}_0 \cdot r(z), \tag{1}$$

where $\dot{n}_0$ is the merger rate today and $r(z)$ encapsulates the rate's time-evolution. For $r(z)$, we adopt the following piecewise linear fit to the rate evolution estimated in [26]:

$$r(z) = \begin{cases} 1 + 2z & z \leq 1 \\ \frac{3}{4}(5 - z) & 1 \leq z \leq 5 \\ 0 & z \geq 5 \end{cases}. \tag{2}$$

The current NS-NS merger rate, $\dot{n}_0$, is also usefully regarded as the product of two factors: the current merger rate in the Milky Way, and a factor that extrapolates from the Milky Way rate to the average rate in the universe. The NS-NS merger rate in the Milky Way has been estimated by several authors; it is still highly uncertain, but most estimates are in the range 10^{-6}–10^{-4} yr^{-1} [27–29]. To extrapolate to the rest of the universe, Kalogera *et al.* [28] estimate that one should multiply the Milky Way rate by 1.1–$1.6 \times 10^{-2} \cdot h_{70}$ Mpc^{-3}. This factor is obtained by extrapolating from the B-band luminosity density of the universe, and it is only a little larger than the extrapolation factor derived by Phinney in [30]. For the estimates in this paper we adopt a rough geometric mean of these rates: $\dot{n}_0 = 10^{-7}$ Mpc^{-3} yr^{-1}.

How many NS-NS merger events, $\Delta N_{\rm m}$, enter the BBO band during some observation time, $\Delta\tau_0$? Integrating the contributions from all redshifts, the rate is given by [4]:

$$\Delta N_{\rm m} = 3.0 \cdot 10^5 \left(\frac{\Delta\tau_0}{3\ \rm yr}\right)\left(\frac{\dot{n}_0}{10^{-7}\ {\rm Mpc}^{-3}\ {\rm yr}^{-1}}\right). \tag{3}$$

Based on our fiducial merger rate $\dot{n}(z)$, Fig. 3 plots the number of observable mergers during three years of observation that occur closer than (any given) redshift z. We see that roughly 15% are at $z < 1$, the median redshift is $z \sim 1.6$, and roughly two-thirds are between $z = 1$ and 3. This distribution is well suited for probing the evolution of dark energy, since it fully samples the evolution history during the transition from the matter dominated to the dark-energy dominated era [31].

The time required for a NS-NS inspiral signal to sweep through the BBO band will typically be comparable to

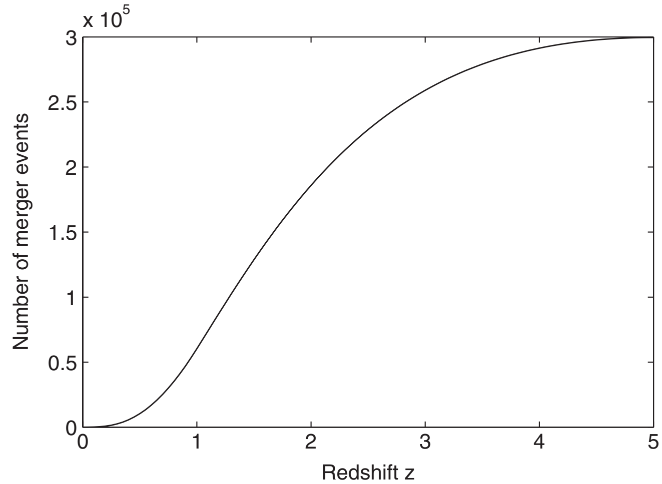

FIG. 3. The total number of NS-NS mergers closer than redshift z. The results are normalized to a 3-yr observation period and $\dot{n}_0 = 10^{-7}$ Mpc^{-3} yr^{-1}.

BBO's lifetime. More specifically, the time remaining until merger, from the moment the GW frequency sweeps through a given frequency, f, is (to lowest post-Newtonian order)

$$t(f) = 4.64 \times 10^5\ {\rm s}\left(\frac{\mathcal{M}(1+z)}{1.22 M_\odot}\right)^{-5/3}\left(\frac{f}{1\ \rm Hz}\right)^{-8/3}, \tag{4}$$

where $\mathcal{M} \equiv \mu^{3/5} M^{2/5}$ is the so-called "chirp mass" of the binary (with M the binary's total mass and μ its reduced mass). For two $1.4 M_\odot$ NSs, $f \approx 0.205$ Hz, 0.136 Hz and 0.112 H2 at one year, three years, and five years before merger, respectively.

D. BBO's accuracy of parameter estimation

Although it is straightforward to calculate BBO's angular resolution and distance measurement accuracy using the Fisher matrix approximation, for pedagogical reasons we begin with some simple, back-of-the-envelope estimates. If the source's luminosity distance, D_L, were the only unknown parameter, it could be measured to a relative accuracy of 1/SNR, where SNR is the (amplitude) signal-to-noise ratio of the detection. For a merging NS-NS binary at $z = 1$, the median SNR will be roughly 180, suggesting $\Delta D_L/D_L \sim 0.6\%$. This is a lower limit on the error, since correlations with other unknown parameters increase the uncertainty. Experience from similar problems in GW data analysis suggests an increase by a factor of ~2, or ~1% distance uncertainty at $z = 1$.

The source's angular position on the sky will be inferred mostly by triangulation, based on the GW time of flight between the different mini-LISA constellations. This idealization suggests the estimate $\Delta\theta \sim (1/{\rm SNR}) \times (1/2\pi 500 f_0) \sim 1$ arcsec, where $f_0 \sim 0.3$ Hz is BBO's most sensitive frequency and we have used 1 AU $\approx$ 500 s in geometric units. This estimate neglects both the information about the source's sky location that is encoded in the waveform by the time-varying antenna patterns of the constellations, as well as the increased uncertainty that results from correlations with the other unknown parameters.

These back-of-the-envelope values turn out to be reasonably good estimates of the BBO's distance and angular position accuracies. We now turn to a more careful, Fisher matrix calculation of the BBO's parameter estimation accuracy. The application of Fisher matrix techniques to LISA measurements of inspiralling binaries was demonstrated in detail in Cutler [7]. The generalization to BBO, which is just four mini-LISAs with higher sensitivity, is straightforward; our exposition will therefore be brief, and we refer the reader to [7] for more detail. As mentioned previously, we can regard the BBO's output as formally equivalent to that of 8 independent, synthetic Michelson interferometers; we represent BBO's output as $s_\alpha(t)$, for $\alpha = 1 \cdots 8$. We use $\mathbf{s}$ to abstractly represent these 8 time-series. For simplicity we assume that the detector noise is stationary, Gaussian, and the same for all four mini-LISA's (in practice, we expect that the noise levels will be somewhat different and slowly time-varying, in a manner that we can fit for). Under these assumptions, we obtain the following natural inner product on the vector space of signals. Given two signals $\mathbf{g}$ and $\mathbf{k}$ we define $\langle \mathbf{g}|\mathbf{k}\rangle$ by

$$\langle \mathbf{g}|\mathbf{k}\rangle = 2\sum_{\alpha=1}^{8}\int_{-\infty}^{\infty} df \frac{(3/20)\tilde{g}_\alpha^*(f)\tilde{k}_\alpha(f)}{S_{\rm h}(f)}, \tag{5}$$

where $\tilde{g}_\alpha(f)$ and $\tilde{k}_\alpha(f)$ are the Fourier transforms of $g_\alpha(t)$ and $k_\alpha(t)$, respectively. We follow the usual convention of taking $S_h(f)$ to be the *single-sided, sky-averaged* noise spectrum for each synthetic Michelson interferometer. The factor $3/20$ in Eq. (5) is the product of a factor $1/5$ due to the sky-average convention and a factor $3/4 = \sin^2(\pi/3)$ arising from the $\pi/3$ angle between the arms in each constellation; see Sec. V.A of Barack & Cutler [32] for a fuller explanation.

In this notation, the rms SNR for any waveform $\mathbf{h}$ is

$$\mathrm{SNR}[\mathbf{h}] = \langle \mathbf{h}|\mathbf{h}\rangle^{1/2}. \tag{6}$$

For a given incident gravitational wave, different realizations of the noise will give rise to somewhat different best-fit parameters. However, for large SNR, the best-fit parameters will have a Gaussian distribution centered on the correct values. Specifically, let $\tilde{\lambda}^\mu$ be the "true" values of the physical parameters, and let $\tilde{\lambda}^\mu + \Delta\lambda^\mu$ be the best-fit parameters in the presence of some realization of the noise. Then for large SNR, the parameter-estimation errors $\Delta\lambda^\mu$ have a nearly Gaussian probability distribution whose covariance matrix is given by

$$\overline{\Delta\lambda^\mu \Delta\lambda^\nu} = (\Gamma^{-1})^{\mu\nu}(1 + \mathcal{O}(\mathrm{SNR})^{-1}), \tag{7}$$

where the overline "" means "expectation value", and where $\Gamma_{\mu\nu}$ is the Fisher matrix, defined by

$$\Gamma_{\mu\nu} \equiv \left\langle \frac{\partial \mathbf{h}}{\partial \lambda^\mu} \,\middle|\, \frac{\partial \mathbf{h}}{\partial \lambda^\nu} \right\rangle. \tag{8}$$

Cutler and Harms [4] have shown that the effects of orbital eccentricity on the NS-NS GW signal will typically amount to less than 1 rad of phase over the entire $\sim 10^8$ radians of observed inspiral, while the precession of the orbital plane due to the Lense-Thirring effect will typically be $\lesssim 10^{-3}$ radians in the BBO band. We can therefore model the binaries as quasicircular, and neglect spin-precession effects (but we do include spin-orbit effects on the waveform phase). Our signal waveform, $h_\alpha(t)$, thus depends on 10 physical parameters describing the binary: $\hat{M}_1$, $\hat{M}_2$, β, θ_S, ϕ_S, θ_L, ϕ_L, ϕ_c, t_c, and D_L. Here $\hat{M}_1$ and $\hat{M}_2$ are the "redshifted mass" ($\hat{M}_i = (1+z)M_i$), β is a spin-orbit coupling parameter defined in Cutler & Flanagan [6], (θ_S, ϕ_S) give the direction to the source, (θ_L, ϕ_L) describe the orientation of the orbital plane, D_L is the luminosity distance to the source, t_c is the time of merger, and ϕ_c is a constant of integration in the evolution of the binary's orbital phase. Our signal model is given by:

$$\tilde{h}_\alpha(f) = \frac{\sqrt{3}}{2}\mathcal{A} f^{-7/6}\Lambda_\alpha(t) e^{i(\Psi(f) - \varphi_\alpha^p(t) - \varphi_\alpha^D(t))}(f > 0), \tag{9}$$

where $M \equiv M_1 + M_2$, $\mu \equiv M_1 M_2/M$, $\mathcal{M} \equiv \mu^{3/5}M^{2/5}$, and

$$\mathcal{A} \equiv (5/96)^{1/2}\pi^{-2/3}D_L^{-1}[\mathcal{M}(1+z)]^{5/6}. \tag{10}$$

The relation between time and frequency, $t = t(f)$, is given through $\mathcal{O}([v/c]^3)$ by [6]

$$t(f) = t_c - 5(8\pi f)^{-8/3}[\mathcal{M}(1+z)]^{-5/3} \times \left[1 + \frac{4}{3}\left(\frac{743}{336} + \frac{11\mu}{4M}\right)x - \frac{32\pi}{5}x^{3/2} + O(x^2)\right], \tag{11}$$

and the waveform phase is given by

$$\Psi(f) = 2\pi f t_c - \phi_c - \pi/4 + \frac{3}{4}(8\pi\mathcal{M}(1+z)f)^{-5/3} \times \left[1 + \frac{20}{9}\left(\frac{743}{336} + \frac{11\mu}{4M}\right)x + (4\beta - 16\pi)x^{3/2}\right], \tag{12}$$

where the PN expansion parameter $x(f)$ is defined by

$$x(f) \equiv (\pi M(1+z)f)^{2/3}. \tag{13}$$

Equations for the modulation factor $\Lambda_\alpha(t(f))$ and "polarization phase" $\varphi_\alpha^p(t(f))$ (both arising from the rotation of each mini-LISA constellation as it orbits the Sun) as well as for the "Doppler phase" Φ_α^D (due to a time delay between the passage of a particular wave front over the Solar System barycenter and its passage over each constellation) are given explicitly in Secs. III and V of Cutler [7]. The exact expressions depend on the two angles (β_0, α_0) that describe each constellation's position around the Sun, and the orientation of each detector-triangle within its plane at some fiducial time, t_0. For definiteness, for the four mini-LISA's we choose: $(\beta_0, \alpha_0) = (0, 0)$, $(0, \pi)$, $(2\pi/3, 2\pi/3)$, and $(4\pi/3, 4\pi/3)$.

The uncertainty in the source's angular position, $\Delta\Omega_S$ (in solid angle), is given by [32]

$$\Delta\Omega_S = 2\pi\sqrt{(\Delta\mu_S)^2(\Delta\phi_S)^2 - (\overline{\Delta\mu_S\Delta\phi_S})^2}. \qquad (14)$$

The 2π factor on the right-hand side of Eq. (14) is conventional; with this definition, the probability that the source lies *outside* an (appropriately shaped) error ellipse enclosing solid angle $\Delta\Omega$ is $e^{-\Delta\Omega/\Delta\Omega_S}$. That is, as defined above, $\Delta\Omega_S$ is very good approximation to the size of the 1σ error ellipse.

BBO's median SNR angular resolution and distance accuracy (both 1σ) for NS and BH mergers and a range of z are shown in Figs. 4 and 5, respectively. These figures were produced as follows. Each NS was taken to have mass $1.4M_\odot$, and each BH to have mass $10M_\odot$. For BBO's (sky-averaged) noise spectral density, $S_h(f)$, we adopted the fitting function

$$S_h(f) = 6.15\times10^{-51}f^{-4} + 1.95\times10^{-48} + 1.2\times10^{-48}f^2, \qquad (15)$$

where f is in units of Hz. For each z, we chose 250 random angle sets $(\theta_S, \phi_S, \theta_L, \phi_L)$, and computed the SNR, the Fisher matrix, and its inverse. Since the Fisher matrices are nearly degenerate, we tested robustness by using both Matlab's standard matrix inversion function and Matlab's Cholesky-factorization inversion routine; these were found to give essentially identical results. While we argued above that the spin-spin coupling will have a negligible impact on NS-NS waveforms for typical cases, as a further test of robustness we added an additional spin-spin parameter (usually denoted "σ" in the literature), and recomputed parameter-estimation accuracies. The results shown in Figs. 4 and 5 turn out to be essentially independent of the presence/absence of a spin-spin term in the waveform model. Note that the results in Figs. 4 and 5 are in reasonably good agreement with the $z = 3$ results in Table 5 of Cornish & Crowder [22] (considering that we model the high-frequency part of BBO's noise curve somewhat differently).

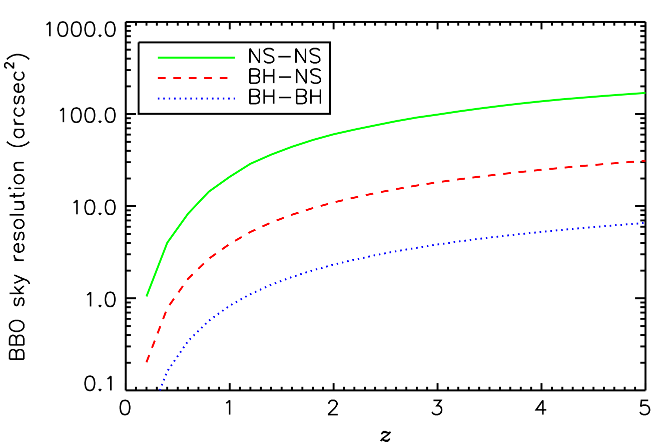

FIG. 4 (color online). BBO's angular resolution as a function of redshift, z. The three curves show BBO's median 1σ angular resolution for three fiducial types of merging compact binaries: BH-BH, BH-NS, and NS-NS.

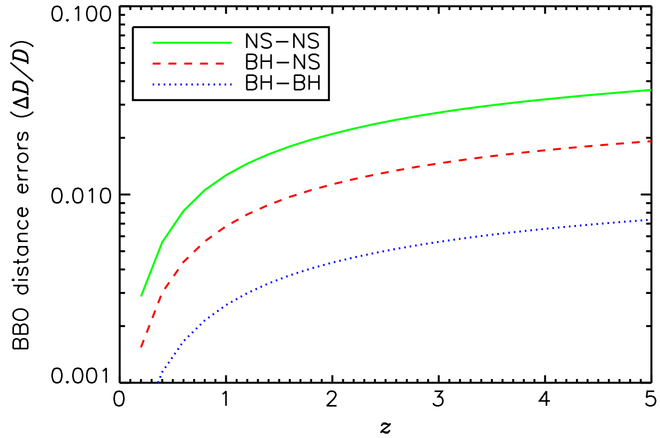

FIG. 5 (color online). BBO's median 1σ distance accuracy as a function of redshift, z, for merging BH-BH, BH-NS, and NS-NS binaries.

Let us suppose that BBO identifies a binary system somewhere in the universe. We now determine the number of potential host galaxies for the binary to be found in the BBO error volume. We closely follow the approach of Holz & Hughes [12], updating their value for the projected number density to the Hubble Ultra Deep Field number: $dN/d\Omega = 1,000$ galaxies/arcmin^2 [33]. Since BBO measures distances at the percent level, the depth of the BBO error box is dominated by the distance uncertainty due to gravitational lensing. Following Eqs. (6)–(8) and Fig. 8 of [12], we calculate the total number of galaxies in the BBO error box, per arcmin^2. We then multiply this by the size of the BBO error box (shown in Fig. 4), to arrive at the total number of galaxies in the BBO error box, as a function of redshift. This result is shown in Fig. 6. The largest number of galaxies in a BBO error box is in the case of NS-NS binaries at $z \sim 1.5$, but even in this case there is less than "half" of a galaxy present. Thus even at the "worst" redshift, the median occupation fraction is less than one—it will be possible to identify the *unique* host for the majority of BBO sources, and hence associate the appropriate redshift for the majority of distance measurements. This is in contrast to the case of LIGO or LISA, where the error boxes are large enough that associated electromagnetic activity (such as a gamma-ray burst, or activity associated with a supermassive binary black hole

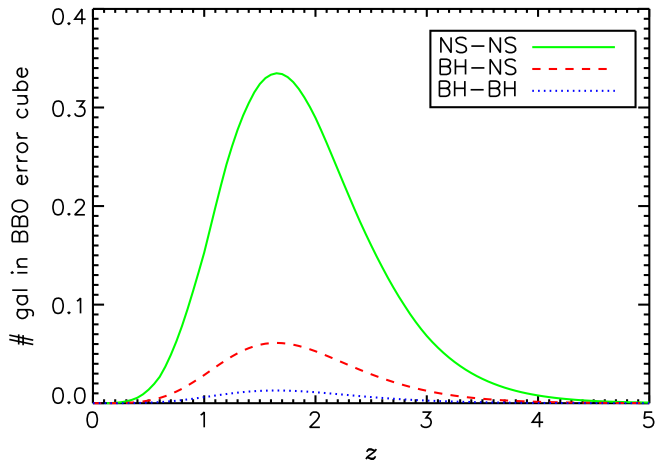

FIG. 6 (color online). Number of galaxies in the BBO error cube, as a function of redshift. Even in the worst case, there is less than one galaxy within 1σ of a given binary on the sky, and therefore it should be possible to robustly identify the unique host galaxy.

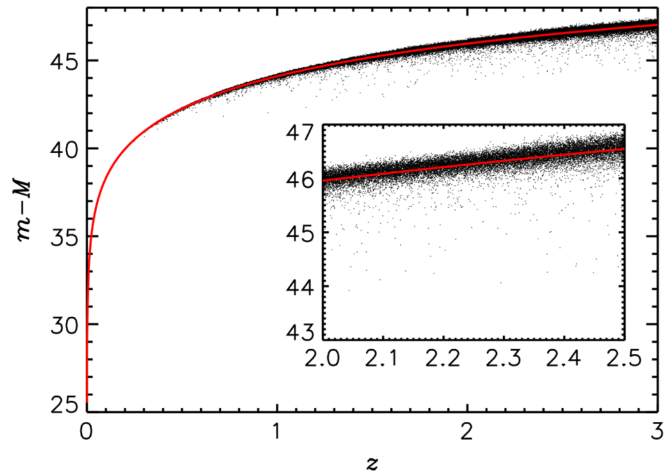

FIG. 7 (color online). Distance versus redshift for a sample BBO binary population. Distance is shown as distance modulus, and includes both BBO errors and gravitational lensing. The red curve is the true luminosity distance-redshift relation. Notice that lensing causes a small number of binaries to become tremendously magnified (to lower distance modulus), but there is a lower limit to the amount of demagnification.

merger) is required to uniquely identify the counterpart [12,34,35]. Obviating the need for an independent identification of the counterpart sharply increases the expected number of usable standard sirens, and hence significantly improves the accuracy of the cosmological measurements. Galaxy misidentifications will generally be seen as large outliers, and thus their influence can be mitigated by the use of robust statistics, such as the Hough Transform (see, e.g., Storkey *et al.* [36])

III. ULTRAHIGH-PRECISION COSMOLOGICAL PARAMETERS FROM BBO

A. The $D_L - z$ relation

We begin by considering BBO's measurements of the luminosity distance-redshift relation (see Fig. 7). This relation is a direct measure of the evolution history of the Universe: redshift provides the size of the Universe at emission, and luminosity distance provides the time since emission. Thus a precise measurement of this relation is sensitive to dark energy; indeed, it is this method that enabled the initial discovery of the accelerating expansion of the Universe now associated with dark energy.

We consider a fiducial population of 2.5×10^5 NS/NS binaries distributed according to Eq. (2), out to $z = 3$. We assume that the distance measurement errors due to detector noise for each individual binary are those shown in Fig. 4. Because BBO does such an exquisite measurement of distance, the errors on the true distance to a given binary will be dominated by the effects of gravitational-lensing magnification [37,38]. We incorporate the lensing errors following the approach of [39], which is entirely appropriate given the very high-number statistics we are considering. For each individual binary we take the dispersion in flux due to lensing to be given by $\sigma_{\rm lensing} = 0.088z$ (see Eq. 9 of [39]). We have explicitly checked that this approach is equivalent to drawing magnification values from the full, non-Gaussian lensing probability distribution functions derived in [38]. We assume that the sky localization is sufficient for the identification of a unique host galaxy (and hence redshift) for each binary (as in Fig. 4). The redshift determination will need to be done independently of BBO, in the electromagnetic band. While in practice there will be some host galaxy misidentifications, for simplicity in this study we assume that perfect redshifts have been obtained for all of our sources. (This simplification is partly based on our belief that a robust cosmological parameter-estimation method will substantially mitigate the effects of a fractionally small set of misidentifications—enough so that in estimating BBO's performance, to a first approximation it is reasonable to neglect them.) We Monte Carlo generate populations of observed binaries, and then for each population we determine the best-fit cosmological parameters (varying the number of free parameters of interest). We repeat this procedure for a large ($> 10^5$) number of runs, and plot the resulting error contours. In what follows, the 1σ contours contain 68.3% of the best-fit values, and the 2σ contours contain 95.5% of the models.

We follow the common convention of parametrizing the dark-energy equation of state in the two-parameter form [40]

$$w(z) = w_0 + w_a \frac{z}{(1+z)}. \tag{16}$$

We fit each data set to five cosmological parameters: the Hubble constant $H_0 = h \times 100$ km/s/Mpc, the dark-matter density Ω_m, the dark-energy density Ω_x, and the dark-energy phenomenological parameters w_0 and w_a. As is standard in assessing the power of proposed cosmology missions, we include a forecasted Planck CMB prior, which constrains the angular diameter distance at $z = 1080$ to 0.01%, and constrains $\Omega_m h^2$ to 1% [41,42].

Figure 8 shows the resulting constraints on h and Ω_m, assuming our fiducial population of binaries, and a 5-parameter fit to the data. We find that BBO will measure the Hubble constant to $\sim$0.1%, even when marginalizing over two dark-energy parameters. For comparison, the best current estimate of H_0 is 74.2 ± 3.6 km/sec/Mpc (so $\sim$5% uncertainty) [43,44]. It is to be noted that, if we fit the data to a ΛCDM model (e.g., setting $w_0 = -1$ and $w_a = 0$), we determine the Hubble constant to $\sim$0.025%. As emphasized in [44], precision measurements of the Hubble constant can be a key component of dark-energy studies; BBO would provide the most precise measurement of H_0 that has ever been contemplated.

In addition to the Hubble constant, BBO will directly constrain the dark-energy equation of state. Figure 8 shows the BBO constraint on w_0 and w_a, for our fiducial binary sample, with the inclusion of Planck CMB priors. We find a $\sim$0.01 constraint on w_0 and a $\sim$0.1 constraint on w_a. We note that we have not assumed a flat Universe in these fits, nor do we incorporate any other cosmological measurements (beyond Planck). For comparison, we consider the stage IV dark-energy missions (supernovae, baryon acoustic oscillations, and weak lensing), as listed by the dark-energy task force [45], representing the state of the art in future dark-energy missions. The combination of all stage IV missions improves the task-force figure-of-merit by a factor 8 to 15 with respect to stage II missions (see pp. 18–20 and pp. 77–78 of [45]). For comparison, BBO finds an equivalent figure-of-merit enhancement of $\sim$100, roughly an order of magnitude better than all of the stage IV missions, *combined*. It is also to be emphasized that there are still fundamental concerns regarding possible systematic errors in all of the stage IV missions, and thus their combined figure-of-merit is undoubtedly optimistic. As discussed above, we expect the systematic errors associated with BBO measurements to be negligible, as it should be possible to build BBO such that calibration errors are much smaller than $\sim 10^{-4}$.

B. Weak gravitational lensing and growth of structure

In addition to providing precision measurements of the fundamental cosmological parameters (H_0, Ω_m, Ω_k, w_0, and w_a), BBO will also directly measure the effects of gravitational lensing, and thus place strong constraints on the primordial dark matter power spectrum, $P(k)$, and the growth of structure. The growth of inhomogeneities is particularly sensitive to gravity, and thus is a powerful way to constrain theories that modify gravity as an alternative to assuming a dark-energy component [46–48].

One of the most powerful ways to measure the growth of density perturbations is through gravitational-lensing shear maps. This is done by observing the shapes of large numbers ($\sim 10^9$) of background galaxies, and measuring the subtle correlations in the shapes of these galaxies due to the shear from gravitational lensing. The shear power spectrum at any redshift is sensitive not only to the distances between observer, lens, and source (and thus, to the dark-energy component), but also to the distribution of lenses. This lens distribution is a direct measure of the dark-matter power spectrum as a function of redshift, which is in turn sensitive to the growth function of perturbations, and thus the gravitational force [49,50].

BBO would provide definitive measurements of the gravitational-lensing convergence power spectrum, comparable to state-of-the-art proposed measurements of the lensing shear power spectrum. BBO measures an absolute luminosity distance to each of the $\sim 10^5$ binaries. The error on this measurement is almost entirely dominated by the effects of gravitational-lensing magnification. Once the

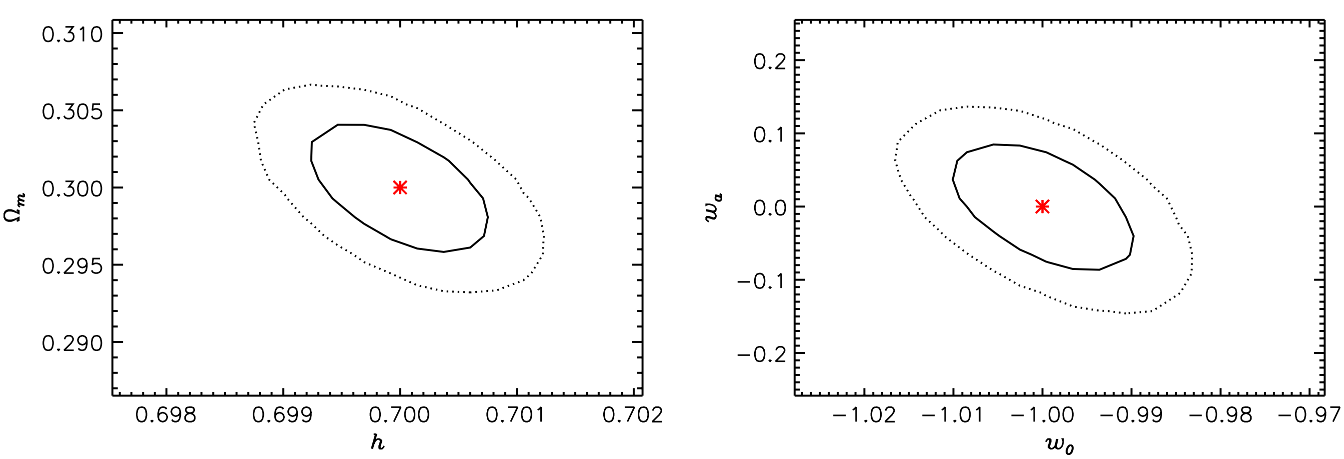

FIG. 8 (color online). Top: Measurement accuracy of the Hubble constant, h, and the dark-matter density, Ω_m. The solid and dashed curves map the 1σ and 2σ contours, respectively. The red star denotes the true underlying model. Bottom: Measurement accuracy of the dark-energy equation-of-state parameters w_0 and w_a.

average luminosity distance–redshift relation is determined (as discussed in the previous section), it is possible to measure the deviations from the background relation. Because the intrinsic uncertainty in the distance measured by BBO is negligible when compared with lensing (see Fig. 5), each individual binary thus becomes a direct measure of the gravitational-lensing magnification along the given line of sight. The population of binaries thus provides a few times 10^5 individual measurements of the magnification out to $z \sim 3$. By evaluating the two-point correlation function of these magnification measurements, it is possible to directly measure the convergence power spectrum (which is equivalent to the shear power spectrum; convergence, κ, is related to magnification, μ, by $\mu = 1 + 2\kappa$ in the weak-lensing limit [e.g. [51]]. This approach has been discussed, for the case of Type Ia supernova distance measurements, in [52]. Here we follow an identical approach, using binary standard sirens instead of supernova standard candles. In our case each individual distance measurement is at least an order of magnitude better, and we have an order of magnitude more sources, even compared to the very ambitious supernova sample considered in [52]. We note that in what follows we focus on the weak-lensing power spectrum, and for simplicity neglect strong lensing. The latter will be discussed in more detail in Sec. IV C.

In the Introduction we provided a rough estimate that BBO could measure weak lensing (WL) with SNR of $\sim 2 \times 10^3$ for its NS-NS data set and also $\sim 2 \times 10^3$ for its BH-BH data set, for a total SNR of $\sim 3 \times 10^3$. The JDEM design has not yet been determined, and the WL capability of the mission varies quite significantly over the range of possibilities. The designs that are best-suited for WL measurements contain $\sim$5–6×10^8 pixels in the focal plane and would have a goal of measuring galaxy ellipticities to $\sim 0.1\%$, and thus would require ellipticity correlation measurements on ~ 100 galaxies to measure the WL effect to SNR of order 1. (This is because galaxies typically have *intrinsic* ellipticities $\epsilon \sim 0.3$, while the correlated ellipticity due to WL is a factor ~ 10 smaller, and SNR builds up as the square root of the number of galaxies observed.) Ideally, JDEM would measure shear for $\sim 10^9$ galaxies, covering $\sim 10^4$ deg^2 on the sky, leading to a total SNR of $\rho_{\rm SNe} \sim 3000$. We note that LSST is expected to measure weak lensing for $\sim 2 \times 10^9$ galaxies, out to $z = 3$, over $\sim 2 \times 10^4$ deg^2, and is thus comparable to the most optimistic space-based lensing missions. These estimates of the power of weak-lensing shear measurements assume that systematic errors (including telescope distortion, shear calibration, point-spread-function correction, and redshift calibration) can be beaten down to the $\sim 0.1\%$, which is quite optimistic (and far better than is currently possible) [53].

The two methods of measuring WL are rather different—individual magnification measurements versus correlated ellipticity measurements—and a proper Fisher-matrix calculation is required to accurately compare the science yield from either method. Such a calculation for BBO is now underway and will be published in a follow-up paper. But, crudely, we expect the ratio of cosmological parameter-estimation errors to be comparable to the ratio of SNRs for the two methods, which is of order one, when BBO is compared to JDEM missions designed to maximize the WL science.

We next calculate how accurately BBO could measure the convergence power spectrum. We following closely the approach of [52]. The weak-lensing angular power spectrum for magnification can be written as

$$C_l^{\mu-\mu} = \int dr \frac{W^2(r)}{d_A^2} P_{\rm dm}\left(k = \frac{l}{d_A}, r\right), \qquad (17)$$

where

$$W(r) = 3 \int dr' n(r') \Omega_m \frac{H_0^2}{c^2 a(r)} \frac{d_A(r) d_A(r' - r)}{d_a(r')}. \qquad (18)$$

Here r is the comoving distance, d_A is the angular diameter distance, $n(r)$ is the number density of binary systems (normalized so that $\int dr n(r) = 1$), and $P_{\rm dm}$ is the three-dimension dark-matter power spectrum (calculated following the approach of [54]). The error on the measurement of the magnification power spectrum is given by

$$\Delta C_l^{\mu-\mu} = \sqrt{\frac{2}{(2l+1) f_{\rm sky} \Delta l}} \left(C_l^{\mu-\mu} + \frac{\sigma_\mu^2}{N_{\rm binaries}} \right), \qquad (19)$$

where $f_{\rm sky}$ is the fraction of the sky covered by the survey, Δl is the binning width in multipole space, σ_μ is the RMS uncertainty of the magnification measurement from each binary, and $N_{\rm binaries}$ is the surface density of the binaries. The first term represents the error from cosmic variance, while the second term represents the error from shot-noise.

Since the BH-BH merger rate is poorly known, to be conservative in the remainder of this section we shall consider WL measurements of the NS-NS population only. (Similar calculations for the BH-BH case, for a range of rates, will be published in later work.) In Fig. 9 we show the BBO's projected measurement of the weak-lensing magnification map for NS-NS mergers. We note that the error bars are for each individual l mode; no binning has been performed in this figure. Defining the signal-to-noise ratio for this measurement as

$$\frac{S}{N} = \sqrt{\sum_l \left(\frac{C_l^i}{\Delta C_l^i} \right)^2} \qquad (20)$$

we find $S/N \approx 120$, over an order of magnitude improvement over the equivalent measurement using 10 000 SNe over 10 deg^2 [52].

We emphasize that our technique for measuring WL through the magnification of GW sources is entirely inde-

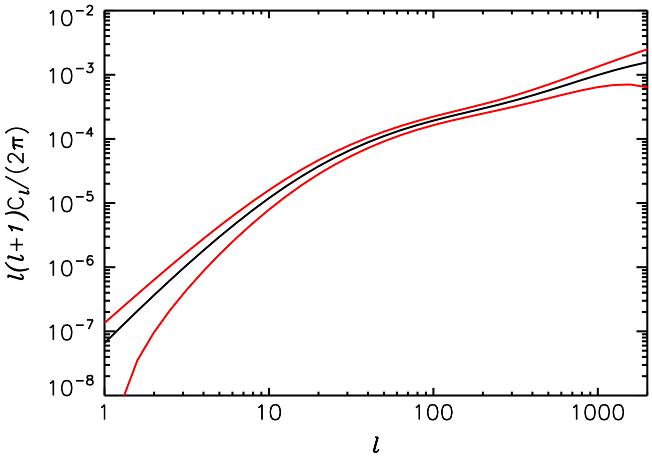

FIG. 9 (color online). BBO's measurement of the gravitational-lensing convergence power spectrum, based just on NS-NS mergers. The red curves show the error bars, for each individual l mode. Each binary is a measure of the gravitational-lensing magnification (convolved with the intrinsic error) along that given line of sight. By observing many binaries, BBO produces a "magnification map" of the sky. This plot shows the power spectrum of this magnification map, which is sensitive to the growth of inhomogeneities in the Universe, as well as potential modifications to gravity.

pendent from the more traditional WL shear measurements, which observe the correlations in galaxy shapes. Since the systematics are a major source of concern for shear measurements, an equally powerful but fundamentally independent technique for measuring WL would be highly desirable. It will also be of interest to cross-correlate the two independent measures of WL (amplification and shear). This will enable interesting consistency tests, and directly test for the presence of systematics [55]. In addition, fundamental tests may be possible, since the relation between shear and amplification is a robust, and thus far untested, prediction of general relativity.

C. Calibration issues and galaxy misidentifications

We have argued that BBO has revolutionary potential as both a dark energy and a gravitational-lensing mission. There are a number of potentially important systematics which must be addressed, foremost of which are ensuring calibration accuracy and the potential misidentification of host galaxies (and hence redshift of the binaries). We comment on these in turn.

The analysis in this paper is based on the basic BBO design put forth in the BBO Concept Study [1]. That design was extremely "LISA-like" in that the test masses are freely floating, with no forces applied along the arm directions. In a later paper, Harry *et al.* [56] pointed out a flaw in the Concept Study design—the laser power arriving at the photodiodes would saturate them—and proposed a shift to a more "LIGO-like" design, with forces applied to the test masses parallel to the arm axes, to keep the photodiode operating near a dark fringe. Because it is difficult to measure this applied force accurately, this redesign would compromise the self-calibrating quality that is one of LISA's strong points. LIGO's strain calibration is accurate to within ~8% [57], which is well below the desired level for the standard siren measurements considered here. An author of [56] has indicated that calibration issues were not a major consideration during the redesign; until the work described in this paper, the motivation for a highly accurate BBO calibration has not been recognized [58]. We have consulted with interferometry experts, and they suggested several plausible solutions to the saturation problem that would preserve the LISA-like calibration accuracy [59]. One possibility is to use optics to widen the beam, spreading the interfered light over an array of photodiodes. Another possibility is to keep the latest LIGO-like design, but introduce an additional small cavity behind the test mass to calibrate the force applied [59]. Our confidence that a good technical solution can be found is enhanced by the fact that LISA-level calibration accuracy is actually overkill for BBO; relative errors of 10^{-4}, and perhaps larger, are acceptable for the cosmological applications described here.

We also note that the current Decigo design is decidedly "LIGO-like" (or "TAMA-like"), with far greater laser power in the arms than for BBO, and so would have to be modified to achieve the calibration accuracy required for doing ultra-high-precision cosmology. We hope that the prospect of precision cosmology will spur instrumentalists to publish improved designs for BBO and Decigo that have (or approach) LISA-level calibration accuracy.

Another possible source of error arises from the misidentifications of the galaxies hosting the compact-binary mergers. Misidentifications could arise for several reasons: the field is particularly crowded; the position error is several σ (for 3×10^5 sources, one expects position errors to range up to $\sim 4.5\sigma$); or the host galaxy is very dim, and so the binary is incorrectly ascribed to a brighter galaxy within the error box [e.g. [60]]. There are strategies for mitigating some of these effects. For instance, misidentifications would naively seem to be most troublesome when they lead to large outliers, but, for estimating cosmological parameters like H_0, w_0, and w_a, one could clearly employ robust statistics that diminish the effect of such outliers. Also, since for each detection one will *know* the size and shape of the error box and the density of galaxies within it, one should be able to reweight the data based upon the confidence of the identification. This reweighting would enhance the importance of BH-BH mergers and gamma-ray bursts, since in those cases the error volumes will be extremely small. Of course, misidentifications will lead to some degradation of BBO's WL results as well, but the impact here is probably less, since a good fraction of the WL SNR will likely come from BH-BH mergers, and BBO's angular resolution for these is ~25 times better (in solid angle) than for the NS-NS case.

The "dim-galaxy" problem is potentially more troublesome, since one is more likely to miss a galaxy on the "far side" rather than the near side of the error volume, which could lead to bias. One could attempt to quantify this effect by comparing results for varying exposure times (since in the limit of infinite exposure time, the dim-galaxy problem goes away). We suspect that this bias is quite small, and it is to be emphasized that this is a bias on the host identification (and hence redshift determination), not on the primary observation of the GW binary. Addressing this potential bias requires the development of a fairly detailed plan for searching for optical counterparts, plus a detailed, robust data analysis algorithm that mitigates misidentifications, which are beyond the scope of this paper.

IV. FURTHER ASTRONOMY FROM BBO

Although BBO has been primarily conceived as a detector of inflation-generated GWs, in this paper we argue that BBO would also be an unrivaled dark-energy mission. In this section we briefly discuss some other unique astronomical opportunities that would be afforded by BBO.

A. Gamma-ray bursts

Short/hard gamma-ray bursts are widely believed to result from (some subset of) NS-NS or BH-NS mergers [61,62]. If this is the case, then BBO will serve as an "early warning system" for short/hard bursts, predicting the precise time and sky location of *every* burst, months in advance. This advance warning allows the bursts to be monitored with a full panoply of telescopes *before* they burst, and will permit searches for any "preburst" electromagnetic activity. BBO will also tell us very precisely the masses of the two bodies and the geometry of the system, which one will be able to correlate with the electromagnetic signals. This should fully resolve the question of short/hard gamma-ray burst progenitors, as well as elucidate other important issues (such as the nature and amount of beaming). In fact, BBO may potentially *overpredict* the short/hard bursts, since not all mergers will necessarily lead to observable bursts. Our knowledge of the orbital geometry of the inspiralling binary should help predict which mergers will be observable (e.g., if the gamma-ray beaming is perpendicular to the orbital plane). BBO would allow us to completely characterize the properties of orphan afterglows, resulting from GRB events beamed such that they are not visible to us. In addition, BBO would tell us which compact-binary mergers do *not* lead to observable bursts and/or afterglows.

It is to be noted that gamma-ray bursts will also serve as "verification sources" that confirm that BBO is working as expected: short/hard bursts should all "go off" with merger times and sky-locations that are consistent with BBO's very precise predictions.

B. IMBHs to high redshift

Another interesting (but more speculative) BBO source is the merger of intermediate-mass black holes (IMBHs) at high redshift. It is predicted that the death of Population III stars could result in BHs weighing a few hundred solar masses [63,64]. Some of these IMBHs may have grown (presumably mostly by gas accretion) into the very massive BHs that now exist in the nuclei of nearly all large galaxies. In the early buildup of galaxies from smaller subhalos, when the subhalos merge the IMBHs they contain may be expected to merge as well. BBO would detect these BH mergers with very high SNR, out to $z = 20$ and beyond. In Table II we list BBO's SNR for IMBH mergers at $z = 20$, for several mass combinations.

Note that for binaries with GW frequency at the last stable orbit satisfying $f_{\rm LSO} \gtrsim 0.5$ mHz (i.e., lying rightwards of the minimum in the BBO noise curve), one expects the SNR of the inspiral waveform to scale like $\mathcal{M}^{5/6}$. However, requiring $f_{\rm LSO} \gtrsim 0.5$ mHz requires $M(1+z) \lesssim 2.2 \times 10^3 M_\odot$, which is satisfied only by the first column in the table. That is, the tabulated SNRs are the results of two competing effects: as the masses increase, so does the signal amplitude, but simultaneously less and less of the inspiral waveform lies in the sensitive portion of the BBO band.

M. Volonteri has supplied us with the results of her merger-tree model, based on two assumptions: 1) today's massive and supermassive BHs began as initial seed BHs of a few hundred $M_\odot$, and 2) these seeds formed at $z \gtrsim 20$, and the seed BHs grew efficiently from accretion. Using Volonteri's model, we estimate that the BBO's detection rate for IMBH mergers in the mass range $M_1 + M_2 < 1,000 M_\odot$ would be $\sim 30/\mathrm{yr}$, of which $\sim 25/\mathrm{yr}$ would be at $z > 10$. By comparison, Sesana *et al* [65] calculate that a network of three third-generation ground-based interferometers would detect IMBH mergers at a rate of $\sim 2/\mathrm{yr}$ (based on a very similar merger-tree model of Volonteri), and almost all of these detected IMBHs would be at $z < 8$. Thus BBO offers a unique opportunity to directly observer the very first seed black holes in the universe.

C. Strong lensing

In addition to a direct measurement of the convergence power spectrum, BBO will measure the full magnification probability distribution function as a function of redshift,

TABLE II. Median matched-filtering SNRs for inspiralling intermediate-mass black hole binaries (IMBHs) at redshift $z = 20$. The masses are the locally measured ones (i.e., *not* redshifted masses), given in units of $M_\odot$.

M_1	1e2	3e2	3e2	1e3	1e3	1e3
M_2	1e2	1e2	3e2	1e2	3e2	1e3
med SNR	1.4e3	1.9e3	2.7e3	1.6e3	2.2e3	2.2e3

including resolving the high-magnification tail. These high-magnification effects can be very sensitive to the dark-matter (and baryon) profiles in galaxies (see, e.g., [66] for an optical version of this). For example, galaxies with more concentrated, cuspy profiles may be more likely to engender strong lensing than those with more flattened cores.

The fraction of multiply-imaged quasars is $\sim 2 \times 10^{-3}$ [67]. One expects a comparable fraction of multiply-lensed NS-NS binaries: i.e., ~ 600 multiply-imaged binaries. In the BBO data set, these will appear as pairs of binaries having nearly identical sky locations and essentially identical masses, spins, and orientations (i.e., identical to within the error bars), but with different apparent distances (according to the magnification of each image), and with arrival times differences of order months [68]. The observed rate of multiple-imaging is an important probe of dark-matter density profiles, as well as the overall dark-matter distribution (e.g., σ_8), and BBO will provide an extremely clean measurement. BBO's determination of the time-delays between the multiple images should be accurate to $\lesssim 0.1$ sec, a fractional accuracy of better than one part in $\sim 10^8$. Thus independent estimates of the Hubble constant from time delays will be possible for ~ 300 multiply-imaged mergers, modulo the standard difficulty of accurately modeling the density profiles of the lensing galaxies. In addition, the relative magnifications should help constrain the dark-matter density profiles, as well as directly break the mass-sheet degeneracy [69].

V. SUMMARY, CONCLUSIONS AND FUTURE WORK

Studies of the dark energy generally take one of two approaches: The first approach is to measure the luminosity distance-redshift relation (or angular diameter distance–redshift) to high accuracy. Type Ia supernovae and baryon-acoustic-oscillation measurements fall into this category. The second approach to exploring the dark energy is to measure the weak-lensing shear power spectrum, and infer values for the growth of structure and cosmological distance ratios. In this paper we have shown that BBO will provide unprecedented measurements of both the luminosity distance-redshift curve as well as the weak-lensing power spectrum.

As discussed in Sec. I, the success of BBO as a cosmological probe is dependent upon obtaining redshifts for a large number of the host galaxies to the binary sources. It is to be emphasized that any subsample of galaxies is sufficient (e.g., LRGs), since there are no standard siren systematics expected to be associated with the nature of the host galaxy. Although obtaining redshifts for 3×10^5 host galaxies is certainly a demanding requirement, as mentioned in Sec. I, we expect that surveys such as LSST or BigBoss may provide the required data set in due course. Future work will be required to refine the estimates of BBO's performance given here, and to consider ways that the mission design might be modified to improve its price/performance ratio from a cosmological perspective. We hope that this work will encourage GW instrumentalists to develop workable designs with calibration accuracy comparable to LISA's. We plan on improving the analyses presented here in several ways. First, in this paper we made simplifying approximations in calculating BBO's parameter-estimation accuracy. Although we expect that these simplifications will have little impact on the results, we will explicitly check this by 1) including the effects of nonzero eccentricity and Lense-Thirring precession in our waveform model, 2) going beyond the Fisher matrix approximation in calculating the expected sizes of the errors [70], and 3) calculating how cross-correlations between different binaries affects parameter estimation for each. Regarding the cosmological constraints derivable from BBO's data set, we intend to 1) explore BBO's performance as measured by other dark-energy figures of merit, 2) improve the sophistication of our method for incorporating priors (and, in particular, Planck CMB priors) into the BBO analysis, 3) incorporate robust parameter-estimation methods that mitigate the effects of host galaxy misidentifications, and 4) generalize our approach to a model-independent, multiparameter description of the dark energy [71–73], instead of the arbitrary two-parameter form (w_0 and w_a) considered here. In addition, we will investigate whether there are special synergies in cross-correlating BBO's gravitational-lensing magnification measurements with the weak-lensing shear measurements made by a JDEM-like mission. We also plan to further explore BBO's potential impact upon other areas of astronomy (some of which were sketched out in Sec. IV). Lastly, we will investigate how various figures of merit vary with BBO's sensitivity and other mission design parameters. There are simple variants to the current design which may retain many of the high-precision cosmology applications while considerably reducing the price or risk of the mission. For instance, having four constellations of mini-LISAs is overkill for high-precision cosmology; three constellations are certainly sufficient, and two may be sufficient, if combined with measurements from planned, third-generation ground-based GW detectors (such as the Einstein Telescope [74]). In the latter case, the mission would require six satellites instead of twelve. And while a noise curve at the level shown in Fig. 2 is probably necessary for detecting a stochastic GW background, it is somewhat overqualified for the task of doing ultra-high-precision cosmology: as currently conceived BBO could detect all NS-NS mergers out to $z = 5$, while for doing high-precision cosmology it is probably sufficient to detect and localize a reasonable fraction of mergers out to $z \sim 2$ [75]. Since BBO would surely be a multi-billion dollar mission, it is important to look for ways of significantly reducing mission cost while retaining a large fraction of the science.

In this paper we have shown that BBO is a particularly powerful mission, as it will provide revolutionary measurements of both the luminosity distance-redshift relation and the growth of structure (through gravitational-lensing measurements). Unfortunately, a deci-Hz GW mission like BBO is, optimistically, at least 15 years from being built. Nevertheless, the power of a BBO-like mission for precision cosmology—stemming from very precise, unbiased distance measurements of $\sim 3 \times 10^5$ NS-NS binaries to $z \sim 5$—is so revolutionary that BBO could represent the future of high-precision cosmology.

ACKNOWLEDGMENTS

CC's work was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract to the National Aeronautics and Space Administration. He acknowledges support from a JPL Research and Technology Development grant, as well as support from NASA grant 06-BEFS06-31. We thank Peter Bender, Peter Fritschel, Salman Habib, Jan Harms, Noboyuki Kanda, Seiji Kawamura, Lexi Moustakis, Hans-Reiner Schulte, Tom Prince, Jason Rhodes, Naoki Seto, Daniel Shaddock, and Michele Vallisneri for helpful discussions. We particularly wish to thank Marta Volonteri for sharing her merger-tree results (which we used to estimate BBO's detection rate for high-z IMBH inspirals), Dragan Huterer and Licia Verde for help with the inclusion of Planck priors, and Lee Lindblom for his observation that BBO would be an all-sky monitor for short gamma-ray bursts.

[1] E. S. Phinney *et al.* Big Bang Observer Mission Concept Study (NASA), 2003.
[2] C. Ungarelli and A. Vecchio, Phys. Rev. D **63**, 064030 (2001).
[3] A. Farmer and E. S. Phinney, Mon. Not. R. Astron. Soc. **346**, 1197 (2003).
[4] C. Cutler and J. Harms, Phys. Rev. D **73**, 042001 (2006).
[5] J. Harms, C. Mahrdt, M. Otto, and M. Priess, Phys. Rev. D **77**, 123010 (2008).
[6] C. Cutler and E. E. Flanagan, Phys. Rev. D **49**, 2658 (1994).
[7] C. Cutler, Phys. Rev. D **57**, 7089 (1998).
[8] B. F. Schutz, Nature (London) **323**, 310 (1986).
[9] L. S. Finn and D. Chernoff, Bull. Am. Astron. Soc. **25**, 1395 (2003).
[10] C. L. MacLeod and C. J. Hogan, Phys. Rev. D **77**, 043512 (2008).
[11] N. Seto, S. Kawamura, and T. Nakamura, Phys. Rev. Lett. **87**, 221103 (2001).
[12] D. E. Holz and S. A. Hughes, Astrophys. J. **629**, 15 (2005).
[13] SNAP Collaboration, arXiv:astro-ph0507458.
[14] M. Sullivan *et al.*, Astrophys. J. **648**, 868 (2006).
[15] B. Dilday *et al.*, Astrophys. J. **682**, 262 (2008).
[16] D. Sarkar, A. Amblard, A. Cooray, and D. E. Holz, Astrophys. J. Lett. **684**, L13 (2008).
[17] J. Baker *et al.*, LISA: Probing the Universe with Gravitational Waves (LISA Science Case Document), 2007, www.srl.caltech.edu/lisa/documents/lisa_science_case.pdf.
[18] S. Vitale *et al.*, technical note S2-UTN-TN-3025, 2005.
[19] R. Spero (private communication).
[20] A. Sadowski, K. Belczynski, T. Bulik, N. Ivanova, F. A. Rasio, and R. O'Shaughnessy, Astrophys. J. **676**, 1162 (2008).
[21] D. J. Schlegel *et al.*, arXiv:0904.0468.
[22] N. J. Cornish and J. Crowder, Phys. Rev. D **72**, 083005 (2005).
[23] N. Seto, www.tamago.mtk.nao.ac.jpl/decigo/LISA-DECIGO/viewgraph08/Seto.ppt (previously publicly accessible, but now requiring a TAMA password for access 2008).
[24] B. Allen and J. D Romano, Phys. Rev. D **59**, 102001 (1999).
[25] S. L. Larson (2005), www.srl.caltech.edu/ shane/sensitivity/index.html.
[26] R. Schneider, V. Ferrari, S. Matarrese, and S. F. Portegies Zwart, Mon. Not. R. Astron. Soc. **324**, 797 (2001).
[27] K. Belczynski, V. Kalogera, and T. Bulik, Astrophys. J. **572**, 407 (2002).
[28] V. Kalogera, R. Narayan, D. N. Spergel, and J. H. Taylor, Astrophys. J. **556**, 340 (2001).
[29] R. Voss and T. M. Tauris, Mon. Not. R. Astron. Soc. **342**, 1169 (2003).
[30] E. S. Phinney, Astrophys. J. **380**, L17 (1991).
[31] G. Aldering *et al.*, Astropart. Phys. **27**, 213 (2007).
[32] L. Barack and C. Cutler, Phys. Rev. D **69**, 082005 (2004).
[33] S. V. W. Beckwith *et al.*, Astron. J. **132**, 1729 (2006).
[34] N. Dalal, D. E. Holz, S. A. Hughes, and B. Jain, Phys. Rev. D **74**, 063006 (2006).
[35] S. Nissanke *et al.*, arXiv:0904.1017.
[36] A. J. Storkey *et al.*, Mon. Not. R. Astron. Soc. **347**, 36 (2004).
[37] J. A. Frieman, Comments Astrophys. **18**, 323 (1997).
[38] D. E. Holz and R. M. Wald, Phys. Rev. D **58**, 063501 (1998).
[39] D. E. Holz and E. V. Linder, Astrophys. J. **631**, 678 (2005).
[40] E. Linder, Phys. Rev. Lett. **90**, 091301 (2003).
[41] A. F. Heavens, T. D. Kitching, and L. Verde, Mon. Not. R. Astron. Soc. **380**, 1029 (2007).
[42] P. Mukherjee *et al.*, Phys. Rev. D **78**, 083529 (2008).
[43] W. Freedman *et al.*, Astrophys. J. **553**, 47 (2001).
[44] A. Riess *et al.*, Astrophys. J. **699**, 539 (2009).
[45] A. Albrecht *et al.*, arXiv:astro-ph/0609591.
[46] E. Linder, Phys. Rev. D **72**, 043529 (2005).
[47] M. Ishak, A. Upadhye, and D. N. Spergel, Phys. Rev. D **74**, 043513 (2006).

[48] D. Huterer, Phys. Rev. D **65**, 063001 (2002).
[49] N. Kaiser, Astrophys. J. **498**, 26 (1998).
[50] P. Schneider *et al.*, Mon. Not. R. Astron. Soc. **296**, 873 (1998).
[51] Y. Mellier, Annu. Rev. Astron. Astrophys. **37** (1998).
[52] A. Cooray, D. E. Holz, and D. Huterer, Astrophys. J. Lett. **637**, L77 (2006).
[53] D. Huterer *et al.*, Mon. Not. R. Astron. Soc. **366**, 101 (2006).
[54] J. A. Peacock and S. J. Dodds, Mon. Not. R. Astron. Soc. **280**, L19 (1996).
[55] L. Van, Waerbeke arXiv:0906.1583.
[56] G. M. Harry, P. Fritschel, D. A Shaddock, W. Folkner, and E. S. Phinney, Classical Quantum Gravity **23**, 4887 (2006).
[57] B. Abbott *et al.* (The LIGO Scientific Collaboration), Phys. Rev. D **79**, 022001 (2009).
[58] P. Fritschel (private communication).
[59] D. Shaddock and R. Spero (private communications).
[60] A. J. Drake *et al.*, arXiv:0908.1990.
[61] K. Belczynski *et al.*, Astrophys. J. **648**, 1110 (2006).
[62] E. Nakar, Phys. Rep. **442**, 166 (2007).
[63] A. Heger, S. E. Woosley, C. L. Fryer, and N. Langer, in *From Twilight to Highlight: The Physics of Supernovae*, edited by W. Hillebrandt and B. Leibundgut (2003); arXiv: astro-ph/0211062.
[64] V. Bromm and R. B. Larson, Annu. Rev. Astron. Astrophys. **42**, 79 (2004).
[65] A. Sesana, J. Gair, I. Mandel, and A. Vecchio, Astrophys. J. **698**, L129 (2009).
[66] M. Oguri *et al.*, Astrophys. J. **699**, 1038 (2009).
[67] M. Treyer and J. Wambsganss, Astron. Astrophys. **416**, 19 (2004).
[68] P. Saha, J. Coles, A. V. Maccio, and L. L. R. Williams, Astrophys. J. **650**, L17 (2006).
[69] D. E. Holz, Astrophys. J. Lett. **556**, L71 (2001).
[70] M. Vallisneri, Phys. Rev. D **77**, 042001 (2008).
[71] D. Huterer and A. Cooray, Phys. Rev. D **71**, 023506 (2005).
[72] C. Li, D. Holz, and A. Cooray, Phys. Rev. D **75**, 103503 (2007).
[73] D. Sarkar *et al.*, Phys. Rev. Lett. **100**, 241302 (2008).
[74] Einstein Telescope website, www.et-gw.eu.
[75] E. V. Linder and D. Huterer, Phys. Rev. D **67**, 081303 (2003).