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Pseudoscalar-meson decuplet-baryon coupling constants in light cone QCD

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Taking into account the $SU(3)_f$ breaking effects, the strong coupling constants of the π, K, and η mesons with decuplet baryons are calculated within the light cone QCD sum rules method. It is shown that all coupling constants, even in the case of $SU(3)_f$ breaking, are described in terms of only one universal function. It is shown that for $\Xi^{*0} \to \Xi^{*0}\eta$, transition violation of $SU(3)_f$ symmetry is very large and for other channels when $SU(3)_f$ symmetry is violated, its maximum value constitutes $10\% \div 15\%$.

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I. INTRODUCTION

Exciting experimental results are obtained on pion and kaon photo and electric production of nucleon during the last several years. These experiments are performed at different centers, such as MAMI, MIT, Bates, BNL, and Jefferson laboratories. To study the properties of the resonances from the existing data, the coupling constants of π, K, and η mesons with baryon resonances are needed.

In extracting the properties of baryon resonances, the hadronic reactions also play an important role. Therefore, for a more accurate description of the experimental data, reliable determination of the strong coupling constants of pseudoscalar mesons is needed. Calculation of the strong coupling constants of baryon-baryon-pseudoscalar meson (BBP) using the fundamental theory of strong interactions, QCD, constitutes a very important problem. The strong coupling constants of BBP belong to the nonperturbative sector of QCD and for estimating these couplings, we need some nonperturbative approaches. Among all nonperturbative approaches, the most predictive and powerful one is the QCD sum rules method [1]. In the present work, we calculate the strong coupling constants of the pseudoscalar mesons with the decuplet baryons within the framework of the light cone QCD sum rules (LCSR) method. In this method, the operator product expansion is performed over twist rather than dimension of the operators, which is carried out in the traditional sum rules. In the LCSR, there appears matrix elements of the nonlocal operators between the vacuum and the corresponding one-particle state, which are defined in terms of the, so-called, distribution amplitudes (DAs). These DAs are the main nonperturbative parameters of the LCSR method (more about LCSR can be found in [2,3]). Note that the coupling constants of pseudoscalar and vector mesons with octet baryons is investigated within the framework of the LCSR in [4,5], respectively.

The paper is organized as follows. In Sec. II, the strong coupling constants of the pseudoscalar mesons with the decuplet baryons are calculated within the framework of the LCSR method, and relations between these coupling constants are obtained where $SU(3)_f$ symmetry breaking takes place. In Sec. III, the numerical analysis of the obtained sum rules for the pseudoscalar-meson decuplet-baryon coupling constants is performed.

II. LIGHT CONE QCD SUM RULES FOR THE PSEUDOSCALAR-MESON DECUPLET-BARYON COUPLING CONSTANTS

In this section, we obtain LCSR for the pseudoscalar-meson decuplet-baryon coupling constants. For this aim, we consider the following correlation function:

$$\Pi_{\mu\nu}^{B_1\to B_2\mathcal{P}} = i\int d^4x e^{ipx}\langle \mathcal{P}(q)|\mathcal{T}\{\eta_\mu^{B_2}(x)\bar{\eta}_\nu^{B_1}(0)\}|0\rangle, \quad (1)$$

where $\mathcal{P}(q)$ is the pseudoscalar meson with momentum q and η_μ^B is the interpolating current of the considered decuplet baryon. The sum rules for the above-mentioned correlation function can be obtained, on the one side, by calculating it in terms of the physical states of hadrons (phenomenological part), and on the other side, calculating it at $p^2 \to -\infty$ in the deep Euclidean region in terms of quarks and gluons (theoretical part), and equating both representations through the dispersion relations.

First, let us concentrate on the calculation of the phenomenological side of the correlation function (1). The phenomenological part can be obtained by inserting a complete set of baryon states having the same quantum numbers as the interpolating current η_μ^B. Isolating the ground state of baryons, we obtain

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$$\Pi_{\mu\nu}^{B_1\to B_2\mathcal{P}} = \frac{\langle 0|\eta_\mu^{B_2}|B_2(p_2)\rangle}{p_2^2 - m_2^2}\langle B_2(p_2)\mathcal{P}(q)|B_1(p_1)\rangle \times \frac{\langle B_1(p_1)|\bar{\eta}_\nu^{B_1}|0\rangle}{p_1^2 - m_1^2} + \cdots, \tag{2}$$

where $p_1 = p_2 + q$, m_i is the mass of baryon B_i, and $\cdots$ represents the contributions of the higher states and the continuum.

The matrix elements of the interpolating current between vacuum and the hadron states is determined as

$$\langle 0|\eta_\mu|B(p,s)\rangle = \lambda_B u_\mu(p,s), \tag{3}$$

where λ_B is the overlap amplitude, and $u_\mu(p,s)$ is the Rarita–Schwinger tensor spinor with spin s. The matrix element $\langle B_2(p_2)\mathcal{P}(q)|B_1(p_1)\rangle$ is parametrized as

$$\langle B_2(p_2)\mathcal{P}(q)|B_1(p_1)\rangle = g_{B_1B_2\mathcal{P}}\bar{u}_\alpha(p_2)\gamma_5 u^\alpha(p_1). \tag{4}$$

In order to obtain the expression for the phenomenological part of the correlation function, the summation over the spins of the Rarita–Schwinger fields is performed, i.e.,

$$\sum_s u_\mu(p,s)\bar{u}_\nu(p,s) = (\not{p} + m)\bigg(-g_{\mu\nu} + \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2p_\mu p_\nu}{3m^2} - \frac{p_\mu\gamma_\nu - p_\nu\gamma_\mu}{3m}\bigg). \tag{5}$$

In principle, Eqs. (2)–(5) allow us to write down the phenomenological part of the correlation function. However, here the following two principal problems appear: (1) not all Lorentz structures are independent; (2) not only spin-3/2, but also spin-1/2 states contribute. Indeed, the matrix element of the current η_μ, sandwiched between the vacuum and the spin-1/2 states, is different than zero and determined in the following way:

$$\langle 0|\eta_\mu|B(p, s = 1/2)\rangle = A(4p_\mu - m\gamma_\mu)u(p, s = 1/2), \tag{6}$$

where the condition $\gamma_\mu\eta^\mu = 0$ has been used.

There are two different alternatives to remove the unwanted spin-1/2 contribution and take into account only the independent structures: (1) ordering the Dirac matrices in a specific way and eliminate the ones that receive contributions from spin-1/2 states; (2) introduce projection operators for the spin-3/2, that do not contain spin-1/2 contribution.

In the present work, we have used the first approach and choose the $\gamma_\mu\not{p}\not{q}\gamma_\nu\gamma_5$ ordering of the Dirac matrices. Having chosen this ordering for the Dirac matrices, we obtain

$$\Pi_{\mu\nu} = \frac{\lambda_{B_1}\lambda_{B_2}g_{B_1B_2\mathcal{P}}}{(p_1^2 - m_1^2)(p_2^2 - m_2^2)}(g_{\mu\nu}\not{p}\not{q}\gamma_5 + \text{other structures with } \gamma_\mu \text{ at the beginning and } \gamma_\nu \text{ at the end, or terms that are proportional to } p_{1\nu} \text{ or } p_{2\mu}). \tag{7}$$

The advantage of choosing the structure $g_{\mu\nu}\not{p}\not{q}\gamma_5$ is in the fact that, the spin-1/2 states do not give contribution to this structure. This fact immediately follows from Eq. (6), which tells that spin-1/2 states contribution is proportional to p_μ or γ_μ.

In order to calculate the theoretical part of the correlation function (1) from the QCD side, we need the explicit expressions of the interpolating currents of the decuplet baryons. The interpolating currents have the following forms [6]:

$$\eta_\mu = A\epsilon^{abc}[(q_1^{aT}C\gamma_\mu q_2^b)q_3^c + (q_2^{aT}C\gamma_\mu q_3^b)q_1^c + (q_3^{aT}C\gamma_\mu q_1^b)q_2^c], \tag{8}$$

where a, b, c are the color indices and C is the charge conjugation operator. The values of A and the quark flavors q_1, q_2, and q_3 for each decuplet baryon are presented in Table I.

Before presenting detailed calculation of the correlation function from the QCD side for determination of the coupling constants of pseudoscalar mesons with decuplet baryons, let us establish the relation among the correlation functions, more precisely, relations among the coefficients of the invariant functions for the structure $g_{\mu\nu}\not{p}\not{q}\gamma_5$. For this aim, we will follow the works of [4,5], and we will show that all correlation functions which describe the strong coupling constants of pseudoscalar mesons with decuplet baryons can be written in terms of only one invariant function. It should especially be noted that the approach we present below automatically takes into account the $SU(3)_f$ symmetry breaking effects.

TABLE I. The values of A and the quark flavors q_1, q_2, and q_3.

	A	q_1	q_2	q_3
Σ^{*0}	$\sqrt{2/3}$	u	d	s
Σ^{*+}	$-\sqrt{1/3}$	u	u	s
Σ^{*-}	$-\sqrt{1/3}$	d	d	s
Δ^{+}	$\sqrt{1/3}$	u	u	d
Δ^{++}	1	u	u	u
Δ^{0}	$\sqrt{1/3}$	d	d	u
Δ^{-}	1	d	d	d
Ξ^{*0}	$\sqrt{1/3}$	s	s	u
Ξ^{*-}	$\sqrt{1/3}$	s	s	d
Ω^{-}	1	s	s	s

In obtaining relations among the invariant functions, similar to works [4,5], we start by considering the correlation function describing the $\Sigma^{*0} \to \Sigma^{*0}\pi^0$ transition. This correlation function can formally be written in the following form:

$$\Pi^{\Sigma^{*0}\to\Sigma^{*0}\pi^0} = g_{\pi uu}\Pi_1(u,d,s) + g_{\pi dd}\Pi_1'(u,d,s) + g_{\pi ss}\Pi_2(u,d,s), \tag{9}$$

where the π^0 current can formally be written as

$$J = \sum_{q=u,d,s} g_{\pi qq}\bar{q}\gamma_5 q, \tag{10}$$

where $g_{\pi^0 uu} = -g_{\pi^0 dd} = 1/\sqrt{2}$ and $g_{\pi^0 ss} = 0$ for the π^0 meson. The functions Π_1, Π_1', and Π_2 describe radiation the π^0 meson from u, d, and s quarks of the Σ^{*0} baryon, respectively.

The interpolating current $\eta^{\Sigma^{*0}}$ is symmetric under the change $u \leftrightarrow d$, and therefore $\Pi_1'(u,d,s) = \Pi_1(d,u,s)$. Hence, Eq. (9) can be written as

$$\Pi^{\Sigma^{*0}\to\Sigma^{*0}\pi^0} = \frac{1}{\sqrt{2}}[\Pi_1(u,d,s) - \Pi_1(d,u,s)]. \tag{11}$$

For convenience, let us introduce the notations

$$\begin{aligned} \Pi_1(u,d,s) &= \langle \bar{u}u|\Sigma^{*0}\Sigma^{*0}|0\rangle, \\ \Pi_2(u,d,s) &= \langle \bar{s}s|\Sigma^{*0}\Sigma^{*0}|0\rangle. \end{aligned} \tag{12}$$

Obviously, $\Pi_2 \equiv 0$ for the transition $\Sigma^{*0} \to \Sigma^{*0}\pi^0$.

In the transition with the η meson, the situation is more complicated, since strange quark is in the quark content of the η meson. In the present work, we neglect the mixing between the η and η' mesons and the η meson current is taken to have the following form:

$$J_\eta = \frac{1}{\sqrt{6}}(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d - 2\bar{s}\gamma_5 s). \tag{13}$$

A simple analysis shows that the $\Sigma^{*0} \to \Sigma^{*0}\eta$ transition has the similar form as is given in Eq. (9)

$$\Pi^{\Sigma^{*0}\to\Sigma^{*0}\eta} = g_{\eta uu}\Pi_1(u,d,s) + g_{\eta dd}\Pi_1'(u,d,s) + g_{\eta ss}\Pi_2(u,d,s). \tag{14}$$

Using the definition given in Eq. (12), one can easily show that

$$\Pi_2(u,d,s) = \Pi_1(s,d,u). \tag{15}$$

For this reason, using Eqs. (13) and (15), we get from Eq. (14)

$$\Pi^{\Sigma^{*0}\to\Sigma^{*0}\eta} = \frac{1}{\sqrt{6}}[\Pi_1(u,d,s) + \Pi_1(d,u,s) - 2\Pi_1(s,d,u)]. \tag{16}$$

The invariant function describing the $\Sigma^{*+} \to \Sigma^{*+}\pi^0$ transition can be obtained from Eq. (9) with the help of the replacements $d \to u$ in $\Pi_1(u,d,s)$ and using the fact $\Sigma^{*0} = -\sqrt{2}\Sigma^{*+}$, which results in

$$4\Pi(u,u,s) = 2\langle \bar{u}u|\Sigma^{*+}\Sigma^{*+}|0\rangle. \tag{17}$$

The presence of factor 4 on the left-hand side of Eq. (17) can be explained as follows. Each Σ^{*+} contains two u quarks and therefore there are 4 ways that the π^0 meson can be radiated. Since Σ^{*+} does not contain the d quark, for the $\Sigma^{*0} \to \Sigma^{*0}\pi^0$ transition, it can be written from Eq. (9) that

$$\begin{aligned} \Pi^{\Sigma^{*+}\to\Sigma^{*+}\pi^0} &= g_{\pi^0 uu}\langle \bar{u}u|\Sigma^{*+}\Sigma^{*+}|0\rangle \\ &\quad + g_{\pi^0 ss}\langle \bar{s}s|\Sigma^{*+}\Sigma^{*+}|0\rangle \\ &= \sqrt{2}\Pi_1(u,u,s). \end{aligned} \tag{18}$$

The result for the $\Sigma^{*-} \to \Sigma^{*-}\pi^0$ transition can easily be obtained by making the replacement $u \to d$ in Eq. (9) and using $\Sigma^{*0}(u \to d) = \sqrt{2}\Sigma^{*-}$, from which we obtain

$$\begin{aligned} \Pi^{\Sigma^{*-}\to\Sigma^{*-}\pi^0} &= g_{\pi^0 dd}\langle \bar{d}d|\Sigma^{*-}\Sigma^{*-}|0\rangle \\ &\quad + g_{\pi^0 ss}\langle \bar{s}s|\Sigma^{*-}\Sigma^{*-}|0\rangle \\ &= -\sqrt{2}\Pi_1(d,d,s). \end{aligned} \tag{19}$$

In the case of exact isospin symmetry, it follows from Eqs. (11), (18), and (19) that $\Pi^{\Sigma^{*0}\to\Sigma^{*0}\pi^0}=0$ and $\Pi^{\Sigma^{*+}\to\Sigma^{*+}\pi^0}=-\Pi^{\Sigma^{*-}\to\Sigma^{*-}\pi^0}$.

Let us now calculate the invariant function responsible for the $\Delta^+\to\Delta^+\pi^0$ transition. Since $\Delta^+=\Sigma^{*+}(s\to d)$, we get from Eq. (18)

$$\begin{aligned}\Pi^{\Delta^+\to\Delta^+\pi^0} &= g_{\pi^0 uu}\langle\bar{u}u|\Sigma^{*+}\Sigma^{*+}|0\rangle(s\to d)\\ &\quad+g_{\pi^0 ss}\langle\bar{s}s|\Sigma^{*+}\Sigma^{*+}|0\rangle(s\to d)\\ &=\sqrt{2}\Pi_1(u,u,d)-\frac{1}{\sqrt{2}}\Pi_1(d,u,u).\end{aligned}\tag{20}$$

Similarly, it is not difficult to obtain the relations for the transitions in which Δ^0, Δ^{++}, and Δ^- decuplet baryons and the π^0 meson participate:

$$\begin{aligned}\Pi^{\Delta^0\to\Delta^0\pi^0} &= \Pi^{\Sigma^{*-}\to\Sigma^{*-}\pi^0}(s\to u)\\ &=-\sqrt{2}\Pi_1(d,d,u)+\frac{1}{\sqrt{2}}\Pi_1(u,d,d),\\ \Pi^{\Delta^{++}\to\Delta^{++}\pi^0} &= \Pi^{\Sigma^{*+}\to\Sigma^{*+}\pi^0}(s\to u)=\frac{3}{\sqrt{2}}\Pi_1(u,u,u),\\ \Pi^{\Delta^-\to\Delta^-\pi^0} &= \Pi^{\Sigma^{*-}\to\Sigma^{*-}\pi^0}(s\to d)=-\frac{3}{\sqrt{2}}\Pi_1(d,d,d),\\ \Pi^{\Xi^0\to\Xi^0\pi^0} &= \frac{1}{\sqrt{2}}\Pi_1(u,s,s),\\ \Pi^{\Xi^-\to\Xi^-\pi^0} &= -\frac{1}{\sqrt{2}}\Pi_1(d,s,s).\end{aligned}\tag{21}$$

We can proceed now to obtain similar relations in the presence of charged the π meson. In order to obtain these relations, we consider the matrix element $\langle\bar{d}d|\Sigma^{*0}\Sigma^{*0}|0\rangle$, where d quarks from each Σ^{*0} form the final $\bar{d}d$ state and, u and s quarks are the spectators. In the matrix element $\langle\bar{u}d|\Sigma^{*+}\Sigma^{*0}|0\rangle$, the d quark from the Σ^{*0} and u quark from Σ^{*+} form the $\bar{u}d$ state and the other u and s quarks are the spectators. For these reasons, it is natural to expect that these matrix elements should be proportional to each other. Direct calculations confirm this expectation, i.e.,

$$\begin{aligned}\Pi^{\Sigma^{*0}\to\Sigma^{*+}\pi^-} &= \langle\bar{u}d|\Sigma^{*+}\Sigma^{*0}|0\rangle=\sqrt{2}\langle\bar{d}d|\Sigma^{*0}\Sigma^{*0}|0\rangle\\ &=\sqrt{2}\Pi_1'(u,d,s)=\sqrt{2}\Pi_1(d,u,s).\end{aligned}\tag{22}$$

Making the replacement $u\leftrightarrow d$ in Eq. (22), we get

$$\begin{aligned}\Pi^{\Sigma^{*0}\to\Sigma^{*-}\pi^+} &= \langle\bar{d}u|\Sigma^{*-}\Sigma^{*0}|0\rangle=\sqrt{2}\langle\bar{u}u|\Sigma^{*0}\Sigma^{*0}|0\rangle\\ &=\sqrt{2}\Pi_1(u,d,s).\end{aligned}\tag{23}$$

Along the same lines of reasoning, similar calculations for Δ and Ξ decuplet baryons are summarized below:

$$\begin{aligned}\Pi^{\Xi^{*0}\to\Xi^{*-}\pi^+} &= \langle\bar{d}u|\Xi^{*0}\Xi^{*-}|0\rangle=-\sqrt{2}\langle\bar{u}u|\Xi^{*0}\Xi^{*0}|0\rangle\\ &=\Pi_1(d,s,s),\\ \Pi^{\Xi^{*-}\to\Xi^{*0}\pi^-} &= \langle\bar{u}d|\Xi^{*-}\Xi^{*0}|0\rangle=\Pi_1(u,s,s),\\ \Pi^{\Delta^+\to\Delta^0\pi^+} &= 2\Pi_1(d,d,u),\\ \Pi^{\Delta^{++}\to\Delta^+\pi^+} &= \sqrt{3}\Pi_1(d,u,u),\\ \Pi^{\Delta^0\to\Delta^-\pi^+} &= \sqrt{3}\Pi_1(u,d,d),\\ \Pi^{\Delta^0\to\Delta^+\pi^-} &= 2\Pi_1(u,u,d),\\ \Pi^{\Delta^+\to\Delta^{++}\pi^-} &= \sqrt{3}\Pi_1(u,u,u),\\ \Pi^{\Delta^-\to\Delta^0\pi^-} &= \sqrt{3}\Pi_1(d,d,d).\end{aligned}\tag{24}$$

The correlation function involving the K meson can be obtained from the previous results as follows:

$$\begin{aligned}\Pi^{\Xi^{*0}\to\Sigma^{*+}K^-} &= \Pi^{\Delta^0\to\Delta^+\pi^-}(s\leftrightarrow d)=2\Pi_1(u,u,s)\\ \Pi^{\Xi^{*-}\to\Sigma^{*-}K^0} &= \Pi^{\Xi^{*0}\to\Sigma^+K^-}(u\to d)=2\Pi_1(d,d,s)\\ \Pi^{\Sigma^{*+}\to\Xi^{*0}K^+} &= \Pi^{\Xi^{*0}\to\Sigma^+K^-}(u\leftrightarrow s)=2\Pi_1(s,s,u).\end{aligned}\tag{25}$$

The remaining correlation functions involving π and K mesons are presented in the Appendix. It follows from the results presented above that all coupling constants of pseudoscalar mesons with decuplet baryons can be expressed by only one independent invariant function, which constitutes the main result of the present work.

Having obtained this result, our next task is the calculation of the correlation function from the QCD side. The correlation function in deep Euclidean domain $p_1^2\to-\infty$, $p_2^2\to-\infty$, can be calculated using the operator product expansion. For this purpose the propagators of light quarks, as well as their DAs are needed. The matrix elements $\langle\mathcal{P}(q)|\bar{q}(x_1)\Gamma q'(x_2)|0\rangle$ that parametrized in terms of DAs are given in [7–9]

$$
\begin{aligned}
\langle \mathcal{P}(p)|\bar{q}(x)\gamma_\mu\gamma_5 q(0)|0\rangle &= -if_{\mathcal{P}}q_\mu\int_0^1 du e^{i\bar{u}qx}\Big(\varphi_{\mathcal{P}}(u)+\frac{1}{16}m_{\mathcal{P}}^2x^2\mathbb{A}(u)\Big)-\frac{i}{2}f_{\mathcal{P}}m_{\mathcal{P}}^2\frac{x_\mu}{qx}\int_0^1 du e^{i\bar{u}qx}\mathbb{B}(u),\\
\langle \mathcal{P}(p)|\bar{q}(x)i\gamma_5 q(0)|0\rangle &= \mu_{\mathcal{P}}\int_0^1 du e^{i\bar{u}qx}\varphi_P(u),\\
\langle \mathcal{P}(p)|\bar{q}(x)\sigma_{\alpha\beta}\gamma_5 q(0)|0\rangle &= \frac{i}{6}\mu_{\mathcal{P}}(1-\tilde{\mu}_{\mathcal{P}}^2)(q_\alpha x_\beta-q_\beta x_\alpha)\int_0^1 du e^{i\bar{u}qx}\varphi_\sigma(u),\\
\langle \mathcal{P}(p)|\bar{q}(x)\sigma_{\mu\nu}\gamma_5 g_s G_{\alpha\beta}(vx)q(0)|0\rangle &= i\mu_{\mathcal{P}}\Big[q_\alpha q_\mu\Big(g_{\nu\beta}-\frac{1}{qx}(q_\nu x_\beta+q_\beta x_\nu)\Big)-q_\alpha q_\nu\Big(g_{\mu\beta}-\frac{1}{qx}(q_\mu x_\beta+q_\beta x_\mu)\Big)\\
&\quad -q_\beta q_\mu\Big(g_{\nu\alpha}-\frac{1}{qx}(q_\nu x_\alpha+q_\alpha x_\nu)\Big)+q_\beta q_\nu\Big(g_{\mu\alpha}-\frac{1}{qx}(q_\mu x_\alpha+q_\alpha x_\mu)\Big)\Big]\\
&\quad\times\int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}(\alpha_i),\ \langle \mathcal{P}(p)|\bar{q}(x)\gamma_\mu\gamma_5 g_s G_{\alpha\beta}(vx)q(0)|0\rangle\\
&= q_\mu(q_\alpha x_\beta-q_\beta x_\alpha)\frac{1}{qx}f_{\mathcal{P}}m_{\mathcal{P}}^2\int\mathcal{D}\alpha e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{A}_\parallel(\alpha_i)\\
&\quad+\Big[q_\beta\Big(g_{\mu\alpha}-\frac{1}{qx}(q_\mu x_\alpha+q_\alpha x_\mu)\Big)-q_\alpha\Big(g_{\mu\beta}-\frac{1}{qx}(q_\mu x_\beta+q_\beta x_\mu)\Big)\Big]f_{\mathcal{P}}m_{\mathcal{P}}^2\\
&\quad\times\int\mathcal{D}\alpha e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{A}_\perp(\alpha_i),\\
\langle \mathcal{P}(p)|\bar{q}(x)\gamma_\mu i g_s G_{\alpha\beta}(vx)q(0)|0\rangle &= q_\mu(q_\alpha x_\beta-q_\beta x_\alpha)\frac{1}{qx}f_{\mathcal{P}}m_{\mathcal{P}}^2\int\mathcal{D}\alpha e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{V}_\parallel(\alpha_i)\\
&\quad+\Big[q_\beta\Big(g_{\mu\alpha}-\frac{1}{qx}(q_\mu x_\alpha+q_\alpha x_\mu)\Big)-q_\alpha\Big(g_{\mu\beta}-\frac{1}{qx}(q_\mu x_\beta+q_\beta x_\mu)\Big)\Big]f_{\mathcal{P}}m_{\mathcal{P}}^2\\
&\quad\times\int\mathcal{D}\alpha e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{V}_\perp(\alpha_i),
\end{aligned}
\tag{26}
$$

where

$$
\mu_{\mathcal{P}} = f_{\mathcal{P}}\frac{m_{\mathcal{P}}^2}{m_{q_1}+m_{q_2}}, \qquad \tilde{\mu}_{\mathcal{P}} = \frac{m_{q_1}+m_{q_2}}{m_{\mathcal{P}}},
$$

and q_1 and q_2 are the quarks in the meson $\mathcal{P}$, $\mathcal{D}\alpha = d\alpha_{\bar{q}}d\alpha_q d\alpha_g\delta(1-\alpha_{\bar{q}}-\alpha_q-\alpha_g)$, and the DAs $\varphi_{\mathcal{P}}(u)$, $\mathbb{A}(u)$, $\mathbb{B}(u)$, $\varphi_P(u)$, $\varphi_\sigma(u)$, $\mathcal{T}(\alpha_i)$, $\mathcal{A}_\perp(\alpha_i)$, $\mathcal{A}_\parallel(\alpha_i)$, $\mathcal{V}_\perp(\alpha_i)$, and $\mathcal{V}_\parallel(\alpha_i)$ are functions of definite twist and their expressions are given in the next section.

For the calculation of the correlation function, we use the following expression for the light quark propagator,

$$
\begin{aligned}
S_q(x) &= \frac{i\not{x}}{2\pi^2x^4}-\frac{m_q}{4\pi^2x^2}-\frac{\langle\bar{q}q\rangle}{12}\Big(1-i\frac{m_q}{4}\not{x}\Big)-\frac{x^2}{192}m_0^2\langle\bar{q}q\rangle\Big(1-i\frac{m_q}{6}\not{x}\Big)\\
&\quad -ig_s\int_0^1 du\Big[\frac{\not{x}}{16\pi^2x^2}G_{\mu\nu}(ux)\sigma_{\mu\nu}-ux^\mu G_{\mu\nu}(ux)\gamma^\nu\frac{i}{4\pi^2x^2}-i\frac{m_q}{32\pi^2}G_{\mu\nu}\sigma^{\mu\nu}\Big(\ln\Big(\frac{-x^2\Lambda^2}{4}\Big)+2\gamma_E\Big)\Big],
\end{aligned}
\tag{27}
$$

where $\gamma_E \simeq 0.577$ is the Euler constant. In the numerical calculations, the scale parameter Λ is chosen as factorization scale, i.e., $\Lambda = 0.5 \div 1.0$ GeV. This point is discussed in detail in [10,11].

Using Eqs. (26) and (27) and separating the coefficient of the structure $g_{\mu\nu}\not{p}\not{q}\gamma_5$, the theoretical part of the correlation function can be calculated straightforwardly. Equating the coefficients of the structure $g_{\mu\nu}\not{p}\not{q}\gamma_5$ from physical and theoretical parts, and performing Borel transformation in the variables $p_2^2 = p^2$ and $p_1^2 = (p+q)^2$ in order to suppress the higher states and continuum contributions [12,13], we get the sum rules for the corresponding pseudoscalar-meson decuplet-baryon coupling constants.

As the result of our calculations, we obtain the following expression for the invariant function $\Pi_1(u, d, s)$:

$$\Pi_1(u,d,s) = \frac{1}{54\pi^2}M^4E_1(x)[9f_{\mathcal{P}}m_s\phi_{\mathcal{P}}(u_0) + 2(1-\tilde{\mu}_{\mathcal{P}}^2)\mu_{\mathcal{P}}\phi_\sigma(u_0)] + \frac{1}{36\pi^2}\{f_{\mathcal{P}}M^2E_0(x)[-3m_{\mathcal{P}}^2m_s(A(u_0) - 4i(A_\parallel, 1-2v)) - 16\pi^2\phi_{\mathcal{P}}(u_0)(\langle\bar{d}d\rangle + \langle\bar{s}s\rangle)\} + \frac{1}{216M^6}\langle\bar{d}d\rangle\langle g^2G^2\rangle m_s(-1+\tilde{\mu}_{\mathcal{P}}^2)\phi_\sigma(u_0)(m_0^2+2M^2) + \frac{1}{7776\pi^2M^2}[27f_{\mathcal{P}}m_{\mathcal{P}}^2m_s\langle g^2G^2\rangle(A(u_0) - 4i(A_\parallel, 1-2v) + 4i(V_\parallel, 1)) + 32\pi^2m_0^2m_s\mu_{\mathcal{P}}(1-\tilde{\mu}_{\mathcal{P}}^2)(\langle\bar{d}d\rangle + 2\langle\bar{s}s\rangle)\phi_\sigma(u_0)] + \frac{1}{72\pi^2}\left[f_{\mathcal{P}}m_s\left(\gamma_E + \ln\frac{\Lambda^2}{M^2}\right)(24m_{\mathcal{P}}^2M^2E_0(x)i(V_\parallel, 1) + \langle g^2G^2\rangle\phi_{\mathcal{P}}(u_0))\right] + \frac{1}{9}f_{\mathcal{P}}m_{\mathcal{P}}^2A(u_0)(\langle\bar{d}d\rangle + \langle\bar{s}s\rangle) - 4f_{\mathcal{P}}m_{\mathcal{P}}^2(\langle\bar{d}d\rangle + \langle\bar{s}s\rangle)[i(A_\parallel, 1-2v) - i(V_\parallel, 1)] + \frac{1}{324\pi^2}f_{\mathcal{P}}\left[-3m_s\langle g^2G^2\rangle + 40m_0^2\pi^2(\langle\bar{d}d\rangle + \langle\bar{s}s\rangle)\phi_{\mathcal{P}}(u_0) - \frac{2}{9}m_s\mu_{\mathcal{P}}(1-\tilde{\mu}_{\mathcal{P}}^2)\langle\bar{d}d\rangle\phi_\sigma\right], \tag{28}$$

where

$$\mu_{\mathcal{P}} = \frac{f_{\mathcal{P}}m_{\mathcal{P}}^2}{m_{q_1}+m_{q_2}}, \qquad \tilde{\mu}_{\mathcal{P}} = \frac{m_{q_1}+m_{q_2}}{m_{\mathcal{P}}},$$

and the function $i(\varphi, f(v))$ is defined as follows:

$$i(\varphi, f(v)) = \int \mathcal{D}\alpha_i \int_0^1 dv\,\varphi(\alpha_{\bar{q}}, \alpha_q, \alpha_g)f(v)\delta(k-u_0),$$

where

$$k = \alpha_q + \alpha_g\bar{v}, \qquad u_0 = \frac{M_1^2}{M_1^2+M_2^2},$$

$$M^2 = \frac{M_1^2M_2^2}{M_1^2+M_2^2}.$$

In calculating the coupling constants of pseudoscalar mesons with decuplet baryons, the value of the overlap amplitude λ_B of the hadron is needed. This overlap amplitude is determined from the analysis of the two-point function which is calculated in [12,13]. Our earlier considerations reveal that the interpolating currents of decuplet baryons can all be obtained from the Σ^{*0} current, and for this reason we shall present the result only for the overlap amplitude of Σ^{*0}:

$$M_{\Sigma^{*0}}\lambda_{\Sigma^{*0}}^2e^{-((m_\Sigma^2)/(M^2))} = (\langle\bar{u}u\rangle + \langle\bar{d}d\rangle + \langle\bar{s}s\rangle)\frac{M^4}{9\pi^2}E_1(x) - (m_u+m_d+m_s)\frac{M^6}{32\pi^4}E_2(x) - (\langle\bar{u}u\rangle + \langle\bar{d}d\rangle + \langle\bar{s}s\rangle)m_0^2\frac{M^2}{18\pi^2}E_0(x) - \frac{2}{3}\left(1+\frac{5m_0^2}{72M^2}\right)(m_u\langle\bar{d}d\rangle\langle\bar{s}s\rangle + m_d\langle\bar{s}s\rangle\langle\bar{u}u\rangle + m_s\langle\bar{d}d\rangle\langle\bar{u}u\rangle) + (m_s\langle\bar{d}d\rangle\langle\bar{s}s\rangle + m_u\langle\bar{d}d\rangle\langle\bar{u}u\rangle + m_d\langle\bar{s}s\rangle\langle\bar{u}u\rangle)\frac{m_0^2}{12M^2}, \tag{29}$$

where $x = s_0/M^2$.

The contribution of the higher states and continuum in Π_1 are subtracted by taking into account the following replacements:

$$\begin{aligned}
e^{-m_{\mathcal{P}}^2/4M^2}M^2\left(\ln\frac{M^2}{\Lambda^2} - \gamma_E\right) &\to \int_{m_{\mathcal{P}}^2/4}^{s_0} dse^{-s/M^2}\ln\frac{s-m_{\mathcal{P}}^2/4}{\Lambda^2}\\
e^{-m_{\mathcal{P}}^2/4M^2}\left(\ln\frac{M^2}{\Lambda^2} - \gamma_E\right) &\to \ln\frac{s_0-m_{\mathcal{P}}^2/4}{\Lambda^2}e^{-s_0/M^2} + \frac{1}{M^2}\int_{m_{\mathcal{P}}^2/4}^{s_0} dse^{-s/M^2}\ln\frac{s-m_{\mathcal{P}}^2/4}{\Lambda^2}\\
e^{-m_{\mathcal{P}}^2/4M^2}\frac{1}{M^2}\left(\ln\frac{M^2}{\Lambda^2} - \gamma_E\right) &\to \frac{1}{M^2}\ln\frac{s_0-m_{\mathcal{P}}^2/4}{\Lambda^2}e^{-s_0/M^2} + \frac{1}{s_0-m_{\mathcal{P}}^2/4}e^{-s_0/M^2} + \frac{1}{M^4}\int_{m_{\mathcal{P}}^2/4}^{s_0} dse^{-s/M^2}\ln\frac{s-m_{\mathcal{P}}^2/4}{\Lambda^2}\\
e^{-m_{\mathcal{P}}^2/4M^2}M^{2n} &\to \frac{1}{\Gamma(n)}\int_{m_{\mathcal{P}}^2/4}^{s_0} dse^{-s/M^2}(s-m_{\mathcal{P}}^2/4)^{n-1}.
\end{aligned} \tag{30}$$

III. NUMERICAL ANALYSIS

In this section, we present the numerical calculations for the sum rules for the couplings of the pseudoscalar mesons with decuplet baryons. The main nonperturbative parameters of LCSR are the DAs of the pseudoscalar mesons, whose explicit forms entering Eq. (26) are given in [7–9]:

$$\begin{aligned}
\phi_{\mathcal{P}}(u) &= 6u\bar{u}[1 + a_1^{\mathcal{P}} C_1(2u-1) + a_2^{\mathcal{P}} C_2^{3/2}(2u-1)], \qquad \mathcal{T}(\alpha_i) = 360\eta_3\alpha_{\bar{q}}\alpha_q\alpha_g^2\Big[1 + w_3\frac{1}{2}(7\alpha_g - 3)\Big],\\
\phi_P(u) &= 1 + \Big[30\eta_3 - \frac{5}{2}\frac{1}{\mu_{\mathcal{P}}^2}\Big]C_2^{1/2}(2u-1), + \Big(-3\eta_3 w_3 - \frac{27}{20}\frac{1}{\mu_{\mathcal{P}}^2} - \frac{81}{10}\frac{1}{\mu_{\mathcal{P}}^2}a_2^{\mathcal{P}}\Big)C_4^{1/2}(2u-1),\\
\phi_\sigma(u) &= 6u\bar{u}\Big[1 + \Big(5\eta_3 - \frac{1}{2}\eta_3 w_3 - \frac{7}{20}\mu_{\mathcal{P}}^2 - \frac{3}{5}\mu_{\mathcal{P}}^2 a_2^{\mathcal{P}}\Big)C_2^{3/2}(2u-1)\Big],\\
\mathcal{V}_{\parallel}(\alpha_i) &= 120\alpha_q\alpha_{\bar{q}}\alpha_g(v_{00} + v_{10}(3\alpha_g - 1)),\\
\mathcal{A}_{\parallel}(\alpha_i) &= 120\alpha_q\alpha_{\bar{q}}\alpha_g(0 + a_{10}(\alpha_q - \alpha_{\bar{q}})),\\
\mathcal{V}_{\perp}(\alpha_i) &= -30\alpha_g^2\Big[h_{00}(1-\alpha_g) + h_{01}(\alpha_g(1-\alpha_g) - 6\alpha_q\alpha_{\bar{q}}) + h_{10}\Big(\alpha_g(1-\alpha_g) - \frac{3}{2}(\alpha_{\bar{q}}^2 + \alpha_q^2)\Big)\Big],\qquad (31)\\
\mathcal{A}_{\perp}(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q)\Big[h_{00} + h_{01}\alpha_g + \frac{1}{2}h_{10}(5\alpha_g - 3)\Big],\\
B(u) &= g_{\mathcal{P}}(u) - \phi_{\mathcal{P}}(u), \qquad g_{\mathcal{P}}(u) = g_0 C_0^{1/2}(2u-1) + g_2 C_2^{1/2}(2u-1) + g_4 C_4^{1/2}(2u-1),\\
\mathbb{A}(u) &= 6u\bar{u}\Big[\frac{16}{15} + \frac{24}{35}a_2^{\mathcal{P}} + 20\eta_3 + \frac{20}{9}\eta_4 + \Big(-\frac{1}{15} + \frac{1}{16} - \frac{7}{27}\eta_3 w_3 - \frac{10}{27}\eta_4\Big)C_2^{3/2}(2u-1)\\
&\quad + \Big(-\frac{11}{210}a_2^{\mathcal{P}} - \frac{4}{135}\eta_3 w_3\Big)C_4^{3/2}(2u-1)\Big], + \Big(-\frac{18}{5}a_2^{\mathcal{P}} + 21\eta_4 w_4\Big)[2u^3(10 - 15u + 6u^2)\ln u\\
&\quad + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln\bar{u} + u\bar{u}(2 + 13u\bar{u})],
\end{aligned}$$

where $C_n^k(x)$ are the Gegenbauer polynomials, and

$$\begin{aligned}
&h_{00} = v_{00} = -\tfrac{1}{3}\eta_4, \qquad a_{10} = \tfrac{21}{8}\eta_4 w_4 - \tfrac{9}{20}a_2^{\mathcal{P}}, \qquad v_{10} = \tfrac{21}{8}\eta_4 w_4, \qquad h_{01} = \tfrac{7}{4}\eta_4 w_4 - \tfrac{3}{20}a_2^{\mathcal{P}},\\
&h_{10} = \tfrac{7}{4}\eta_4 w_4 + \tfrac{3}{20}a_2^{\mathcal{P}}, \qquad g_0 = 1, \qquad g_2 = 1 + \tfrac{18}{7}a_2^{\mathcal{P}} + 60\eta_3 + \tfrac{20}{3}\eta_4, \qquad g_4 = -\tfrac{9}{28}a_2^{\mathcal{P}} - 6\eta_3 w_3.
\end{aligned}\qquad (32)$$

The values of the parameters $a_1^{\mathcal{P}}$, $a_2^{\mathcal{P}}$, η_3, η_4, w_3, and w_4 entering Eqs. (32) are given in Table II for the π, K, and η mesons.

In the numerical calculations, we set $M_1^2 = M_2^2 = 2M^2$ due to the fact that the masses of the initial and final baryons are close to each other. With this choice, we have $u_0 = 1/2$. The values of the other input parameters entering the sum rules are $\langle\bar{q}q\rangle = -(0.24 \pm 0.01\text{ GeV})^3$, $m_0^2 = (0.8 \pm 0.2)\text{ GeV}^2$ [12], $f_\pi = 0.131$ GeV, $f_K = 0.16$ GeV, and $f_\eta = 0.13$ GeV [7].

TABLE II. Parameters of the wave function calculated at the renormalization scale $\mu = 1$ GeV.

	π	K	η
$a_1^{\mathcal{P}}$	0	0.050	0
$a_2^{\mathcal{P}}$	0.44	0.16	0.2
η_3	0.015	0.015	0.013
η_4	10	0.6	0.5
w_3	−3	−3	−3
w_4	0.2	0.2	0.2

The sum rules for the coupling constant of pseudoscalar mesons with decuplet baryons contain two auxiliary, namely, Borel parameters M^2 and the continuum threshold s_0. Obviously, we need to find such regions of these parameters where coupling constants are practically independent of them.

The upper limit of M^2 can be found by requiring that the higher states and continuum contributions to the correlation function should be less than 40%–50% of the total value of the correlation function. The lower bound of M^2 can be obtained by demanding that the contribution of the highest term with power $1/M^2$ is less than, say, 20%–25% of the highest power of M^2. Using these two conditions, one can find regions of M^2 where the results for the coupling constants are insensitive to the variation of M^2.

As has already been noted, another auxiliary parameter of the sum rules is the continuum threshold, and in the present work we will follow the standard procedure in

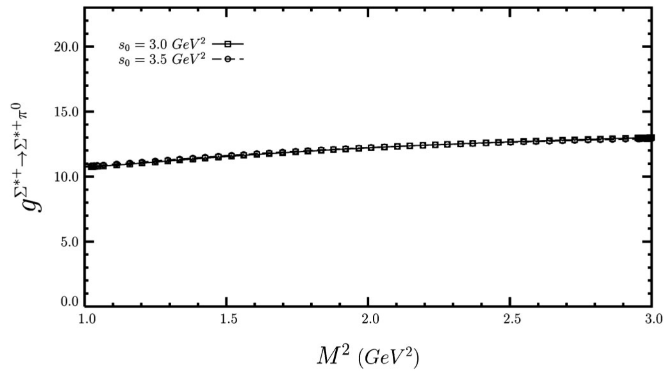

FIG. 1. The dependence of the $g^{\Sigma^{*+}\to\Sigma^{*+}\pi^0}$ coupling constant on M^2 at fixed values of the continuum threshold.

choosing it, i.e., s_0 is taken to be independent of the M^2 and q^2 whose value is varied in the range 2.5 GeV$^2 \leq s_0 \leq$ 4.0 GeV^2. In this connection it is shown in [14] that the continuum threshold is strongly dependent on M^2 and q^2. This modification leads to the standard criteria in the sum rules, namely, stability of the results with respect to the variation in M^2 does not provide realistic errors, and in fact the actual error turns out to be large. Following [14], we consider that these systematic errors are around 15%. Furthermore, in [14] it is also shown that the standard procedure works very well (better that 2%) at low q^2 ($q^2 <$ 2 GeV^2) in determining the q^2 dependence of the form factors. In the present work, since $q^2 = m_{\mathcal{P}}^2 < 2$ GeV^2, the standard procedure explained in [14] should work rather well, and for this reason we prefer this approach in determining the value of s_0.

As an example, in Fig. 1, we depict the dependence of the $g^{\Sigma^{*+}\to\Sigma^{*+}\pi^0}$ coupling constant on M^2 at fixed values of the continuum threshold. From this figure, one can see that the $g^{\Sigma^{*+}\to\Sigma^{*+}\pi^0}$ coupling constant demonstrates good stability to the variation in M^2. The numerical results for the coupling constants of pseudoscalar mesons with decuplet baryons are presented in Table III. Note that in this table, we give only those results which are not obtained from each other by $SU(2)$ and isotopic spin relations. It should be remembered that the sum rules cannot fix the signs of the residues and for this reason the signs of the couplings are not fixed. However, they can be fixed if we use $SU(3)_f$ symmetry (for more about this issue, see [4]).

TABLE III. Coupling constants of pseudoscalar mesons with decuplet baryons.

Channel	Coupling	Coupling in $SU(3)$ limit
$\Sigma^{*+} \to \Sigma^{*+}\pi^0$	11.3 ± 2.5	11.0 ± 2.5
$\Delta^+ \to \Delta^+\pi^0$	5.5 ± 1.6	5.5 ± 1.4
$\Xi^{*0} \to \Xi^{*0}\pi^0$	5.2 ± 1.4	5.5 ± 1.5
$\Sigma^{*0} \to \Delta^+ K^-$	-17.4 ± 4.1	-18.0 ± 4.3
$\Xi^{*0} \to \Sigma^{*+} K^-$	-25.4 ± 6.1	-27.0 ± 6.3
$\Sigma^{*+} \to \Delta^{++} K^-$	-21.2 ± 5.2	-22.0 ± 5.4
$\Omega^- \to \Xi^{*0} K^-$	-20.7 ± 5.1	-22.2 ± 5.4
$\Sigma^{*+} \to \Xi^{*0} K^+$	-22.2 ± 5.3	-27.0 ± 5.6
$\Sigma^{*0} \to \Sigma^{*0}\eta$	0.65 ± 0.15	0.0 ± 0.0
$\Delta^+ \to \Delta^+\eta$	12.5 ± 3.2	12.6 ± 3.2
$\Xi^{*0} \to \Xi^{*0}\eta$	-11.2 ± 2.6	-13.2 ± 3.1

The errors in the results in Table III are coming from the variation of s_0, and Borel parameter M^2, as well as from the systematic uncertainties. From this table, we can deduce the following conclusions:

(i) In all considered couplings except $\Sigma^{*0} \to \Sigma^{*0}\eta$ our predictions consist with the $SU(3)_f$ symmetry. Maximum violation of $SU(3)_f$ symmetry is about 15%.

(ii) In $SU(3)_f$ symmetry the limit coupling constant for $\Sigma^{*0} \to \Sigma^{*0}\eta$ transition is equal to zero, but our prediction on this constant differs from zero considerably when violation of $SU(3)_f$ symmetry is taken into account. Only for this channel, violation of $SU(3)_f$ symmetry is huge. In principle, investigation of this coupling constant can shed light on the structure of the η meson.

(iii) Sign of coupling constant of decuplet baryons to the K meson and also $\Xi^{*0} \to \Xi^{*0}\eta$ is negative, but for all other cases is positive.

In summary, considering the $SU(3)_f$ symmetry breaking effects, the coupling constants of the decuplet baryons with pseudoscalar π, K, and η mesons have been calculated in the framework of light cone QCD sum rules. It was shown that all aforementioned coupling constants is described with the help of one universal function. We obtained that for the $\Xi^{*0} \to \Xi^{*0}\eta$ transition, violation of $SU(3)_f$ is very large.

APPENDIX

In this Appendix, we present the correlation functions involving π, K, and η mesons which is not given in the main text.

(i) Correlation functions for the couplings involving the π^+ meson

$$\Pi^{\Sigma^{*+}\to\Sigma^{*0}\pi^+} = \sqrt{2}\Pi_1(d, u, s).$$

(ii) Correlation functions for the couplings involving the π^- meson

$$\Pi^{\Sigma^{*-}\to\Sigma^{*0}\pi^-} = \sqrt{2}\Pi_1(u, d, s),$$
$$\Pi^{\Delta^0\to\Delta^+\pi^-} = 2\Pi_1(u, u, d).$$

(iii) Correlation functions for the couplings involving the K meson

$$
\begin{aligned}
\Pi^{\Sigma^{*0}\to\Delta^+K^-} &= \sqrt{2}\Pi_1(s,u,d),\\
\Pi^{\Xi^{*-}\to\Sigma^{*0}K^-} &= \sqrt{2}\Pi_1(u,d,s),\\
\Pi^{\Delta^-\to\Sigma^{*0}K^-} &= 0,\\
\Pi^{\Sigma^{*+}\to\Delta^{++}K^-} &= \sqrt{3}\Pi_1(u,u,u),\\
\Pi^{\Sigma^{*-}\to\Delta^0K^-} &= \Pi_1(s,d,d),\\
\Pi^{\Sigma^{*-}\to\Xi^{*0}K^-} &= 0,\\
\Pi^{\Omega^-\to\Xi^{*0}K^-} &= \sqrt{3}\Pi_1(s,s,s),\\
\Pi^{\Delta^0\to\Sigma^{*+}K^-} &= 0,\\
\Pi^{\Delta^+\to\Sigma^{*0}K^+} &= \sqrt{2}\Pi_1(s,u,d),\\
\Pi^{\Delta^0\to\Sigma^{*-}K^+} &= \Pi_1(s,d,d),\\
\Pi^{\Sigma^{*+}\to\Delta^0K^+} &= 0,\\
\Pi^{\Delta^{++}\to\Sigma^{*+}K^+} &= \sqrt{3}\Pi_1(u,u,u),\\
\Pi^{\Xi^{*0}\to\Sigma^{*-}K^+} &= 0,\ \Pi^{\Xi^{*0}\to\Sigma^{*0}\bar{K}^0} = \Pi^{\Sigma^{*0}\to\Xi^{*0}\bar{K}^0}\\
&= \Pi^{\Xi^{*0}\to\Sigma^{*0}K^0} = \Pi^{\Sigma^{*0}\to\Xi^{*0}K^0}\\
&= \sqrt{2}\Pi_1(d,u,s),\\
\Pi^{\Sigma^{*-}\to\Xi^{*-}\bar{K}^0} &= \Pi^{\Xi^{*-}\to\Sigma^{*-}K^0} = 2\Pi_1(s,s,d),\\
\Pi^{\Omega^-\to\Xi^{*-}\bar{K}^0} &= \Pi^{\Xi^{*-}\to\Omega^-\bar{K}^0} = \Pi^{\Xi^{*0}\to\Omega^-K^+}\\
&= \Pi^{\Omega^-\to\Xi^{*-}K^0} = \Pi^{\Xi^{*-}\to\Omega^-K^0}\\
&= \sqrt{3}\Pi_1(s,s,s),\\
\Pi^{\Sigma^{*0}\to\Delta^0\bar{K}^0} &= \Pi^{\Delta^0\to\Sigma^{*0}\bar{K}^0} = \Pi^{\Sigma^{*0}\to\Delta^0K^0}\\
&= \sqrt{2}\Pi_1(s,d,u),\ \Pi^{\Sigma^{*+}\to\Delta^+\bar{K}^0}\\
&= \Pi^{\Delta^+\to\Sigma^{*+}\bar{K}^0} = \Pi_1(s,u,u),\\
\Pi^{\Delta^-\to\Sigma^{*-}\bar{K}^0} &= \Pi^{\Sigma^{*-}\to\Delta^-\bar{K}^0} = \sqrt{3}\Pi_1(s,d,d),\\
\Pi^{\Delta^+\to\Sigma^{*+}K^0} &= \Pi^{\Sigma^{*+}\to\Delta^+K^0} = \Pi_1(s,u,u),\\
\Pi^{\Delta^-\to\Sigma^{*-}K^0} &= \Pi^{\Sigma^{*-}\to\Delta^-K^0} = \sqrt{3}\Pi_1(s,d,d),\\
\Pi^{\Delta^0\to\Sigma^{*0}K^0} &= \sqrt{2}\Pi_1(s,u,d),\\
\Pi^{\Sigma^{*-}\to\Xi^-K^0} &= 2\Pi_1(d,d,s).
\end{aligned}
$$

(iv) Correlation functions for the couplings involving the η meson

$$
\begin{aligned}
\Pi^{\Sigma^{*0}\to\Sigma^{*0}\eta} &= \frac{1}{\sqrt{6}}[\Pi_1(u,d,s) + \Pi_1(d,u,s)\\
&\quad - 2\Pi_1(s,d,u)],\\
\Pi^{\Sigma^{*+}\to\Sigma^{*+}\eta} &= \frac{2}{\sqrt{6}}[\Pi_1(u,u,s) - \Pi_1(s,u,u)],\\
\Pi^{\Sigma^{*-}\to\Sigma^{*-}\eta} &= \frac{2}{\sqrt{6}}[\Pi_1(d,d,s) - \Pi_1(s,d,d)],\\
\Pi^{\Delta^+\to\Delta^+\eta} &= \frac{1}{\sqrt{6}}[2\Pi_1(u,u,d) + \Pi_1(d,u,u)],\\
\Pi^{\Delta^{++}\to\Delta^{++}\eta} &= \frac{\sqrt{6}}{2}\Pi_1(u,u,u),\ \Pi^{\Delta^-\to\Delta^-\eta}\\
&= \frac{\sqrt{6}}{2}\Pi_1(d,d,d),\\
\Pi^{\Delta^0\to\Delta^0\eta} &= \frac{1}{\sqrt{6}}[2\Pi_1(d,d,u) + \Pi_1(u,d,d)],\\
\Pi^{\Xi^{*0}\to\Xi^{*0}\eta} &= \frac{1}{\sqrt{6}}[\Pi_1(u,s,s) - 4\Pi_1(s,s,u)],\\
\Pi^{\Xi^{*-}\to\Xi^{*-}\eta} &= \frac{1}{\sqrt{6}}[\Pi_1(d,s,s) - 4\Pi_1(s,s,d)].
\end{aligned}
$$

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