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Hyperquarks and bosonic preon bound states

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In a model in which leptons, quarks, and the recently introduced hyperquarks are built up from two fundamental spin-$\frac{1}{2}$ preons, the standard model weak gauge bosons emerge as preon bound states. In addition, the model predicts a host of new composite gauge bosons, in particular, those responsible for hyperquark and proton decay. Their presence entails a left-right symmetric extension of the standard model weak interactions and a scheme for a partial and grand unification of nongravitational interactions based on, respectively, the effective gauge groups $SU(6)_P$ and $SU(9)_G$. This leads to a prediction of the Weinberg angle at low energies in good agreement with experiment. Furthermore, using evolution equations for the effective coupling strengths, we calculate the partial and grand unification scales, the hyperquark mass scale, as well as the mass and decay rate of the lightest hyperhadron.

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I. INTRODUCTION

In a composite model in which leptons and quarks are bound states of two fundamental, massless spin-$\frac{1}{2}$ preons, called T and V, interacting via color and hypercolor forces [1–3], a new class of fermionic bound states, called hyperquarks, has recently been introduced in order to satisfy a special case of the 't Hooft anomaly matching conditions [4]. In contrast to quarks, which carry zero hypercolor and open color, hyperquarks have zero color and open hypercolor. The matching of the anomalies on the preon and bound state levels that has been achieved by introducing hyperquarks gives an answer to the question why there are exactly three fermionic generations. At the same time it raises new questions, foremost whether there is any experimental evidence for hyperquarks.

Because hyperquarks are subject to confining forces one would expect hyperquark bound states, such as hypermesons and hyperbaryons to exist in nature. The nonobservation of these hyperhadrons with present accelerators indicates that their masses are considerably larger than those of ordinary hadrons, and that hyperquarks are much heavier than quarks. Hyperhadrons might have been produced in the Early Universe. However, because not even the lightest of these is observed today, the lightest hyperquark itself cannot be stable. Consequently, a new class of massive bosons which generate hyperquark decays must exist.

In this paper, we discuss the spectrum of composite spin 1 bosons that can be constructed in the preon model as well as their role in various weak decay processes. We assume that all massive composite bosons including the electroweak gauge bosons W and Z of the standard model, as well as those responsible for hyperquark decay, remain tightly bound at least up to $\cong 10^{16}$ GeV (grand unification scale). Explicit preon degrees of freedom do not appear below this scale so that the corresponding interactions between preon bound states can be described by approximate *effective* gauge theories. In particular, for low energies $<10^3$ GeV (Fermi scale) one recovers the standard model Lagrangian with left-right asymmetric weak interactions, while for $\cong 10^9$ GeV (partial unification scale), it is suggested that an effective SU(6) gauge theory unifies left-right symmetric extended weak interactions including the new gauge bosons generating hyperquark decay with the hypercolor interaction.

We also address the issue of the mass scale, where hyperquarks appear. The momentum dependence of the different gauge couplings is used to predict the energy scale where they converge and a unified gauge theory with a single coupling occurs. This in turn enables us to put limits on the nonperturbative regime of the hypercolor force and the mass of the lightest hypermeson. In short, the purpose of this work is to address the following questions:

(i) How are massive weak gauge bosons described in the preon model?
(ii) Which processes and gauge bosons are responsible for hyperquark instability?
(iii) What is the effect of hyperquarks in various weak interaction processes?
(iv) At what energy scale do hyperquarks and their composites appear?

Important questions such as gauge boson and fermion mass generation are not discussed here. It may suffice to say that in the present model there are no fundamental Higgs fields although there could be composite scalars such as hyperquark bound states. This is reminiscent of technicolor models and appears to be promising. However, closer inspection shows that hyperquarks are singlets under weak isospin. Therefore, they and their scalar bound states do not have the same $SU(2)_{W_L} \times U(1)_Y$ group structure as the standard model fermions and Higgs fields. Thus, hyperquark bound state scalars cannot give mass to any of the

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standard model gauge bosons and fermions. In this paper, we concentrate on the spectrum of vector bosons. We hope to come back to the issue of scalar particles and mass generation in the preon model elsewhere.

The paper is organized as follows. Section II gives a short review of fermionic bound states in the preon model. In Sec. III, taking as straightforward a position as possible, we discuss an extension of the standard electroweak theory, making explicit the preon content of the massive electroweak gauge bosons including those responsible for hyperquark-quark transitions. A further generalization of the extended weak and the hypercolor interactions is discussed in Sec. IV. The resulting theory, which we call partial unification, represents a necessary step towards grand unification, as shown in Sec. V. Section VI provides numerical results for the running couplings and unification constraints from where mass ranges of the heavy bosons, the lightest neutrino, and the hyperquarks are obtained. The predicted hyperhadron masses are within reach of the Large Hadron Collider at CERN. Section VII contains a summary and outlook.

II. FERMIONIC BOUND STATES

In the Harari-Shupe model [1–3] all quarks and leptons are built from just two spin-$\frac{1}{2}$ fermions (preons). According to Harari and Seiberg [2] the two types of preons belong to the following representations of the underlying exact gauge group $SU(3)_H \times SU(3)_C \times U(1)_Q$: T: $(3,3)_{1/3}$ and V: $(3,\bar{3})_0$, where the first (second) argument is the dimension of the representation in hypercolor (color) space and the subscript denotes the electric charge Q. The fundamental Lagrangian of the preon model [3] reads

$$\mathcal{L} = \bar{T}(\not\partial + g_H \not A_H + g_C \not A_C + \tfrac{1}{3} e \not A_Q)T + \bar{V}(\not\partial + g_H \not A_H + g_C \not A_C)V - \tfrac{1}{4}F_H F_H - \tfrac{1}{4}F_C F_C - \tfrac{1}{4}F_Q F_Q, \qquad (2.1)$$

with $\not A = \gamma_\mu A_a^\mu \lambda^a$ representing the three fundamental gauge fields of the theory with their respective coupling strengths g_H, g_C, and e. The last three terms in Eq. (2.1) represent the kinetic energies of the gauge fields, where the field strength tensors F are as usual given in terms of the A^μ. The γ_μ are the Dirac matrices and λ^a are generators of the corresponding gauge groups.

Although there are two degenerate types of preons (T and V) there is no global SU(2) isospin symmetry on the preon level because the charged and neutral preon belong to different representations in color space. To be consistent with the parity assignment for the standard model fermions, the intrinsic parity Π of the T and V preons must be different [4]. The different parities of T and V preons provide a possible explanation for parity violation at low energies as will be discussed in Sec. IV.

Preons and their bound states are characterized by new quantum numbers [5]. These are the preon number $\mathcal{P}$ and Υ number, which are linear combinations of the numbers of T-preons $n(T)$ and V-preons $n(V)$ in a given state

$$\mathcal{P} = \tfrac{1}{3}(n(T) + n(V)) \qquad \Upsilon = \tfrac{1}{3}(n(T) - n(V)). \qquad (2.2)$$

The factor $\frac{1}{3}$ in Eq. (2.2) is a convention. The Υ number is also related to the baryon (B) and lepton (L) numbers of the standard model as $\Upsilon = B - L$. The antipreon numbers $n(\bar{T})$ and $n(\bar{V})$ are defined as $n(\bar{T}) = -n(T)$ and $n(\bar{V}) = -n(V)$. The preon quantum numbers of individual preons are summarized in Table I.

There is a connection between the $\mathcal{P}$ and Υ numbers, and the electric charge Q of the preons

$$Q = \tfrac{1}{2}(\mathcal{P} + \Upsilon), \qquad (2.3)$$

which can be readily verified from Table I. This generalized Gell-Mann-Nishijima relation does not only hold for the preons but for all bound states, such as leptons, quarks, hyperquarks and their bound states, as well as the effective weak gauge bosons.

As shown in Ref. [4] the 't Hooft anomaly condition, which demands that the anomalies on the preon level match those on the bound state level, can be satisfied if in addition to leptons and quarks a third fermionic bound state type, called hyperquarks, is introduced. Hyperquarks have the same electric charge as the corresponding quarks. However, instead of being color triplets and hypercolor singlets as ordinary quarks, they are color singlets and hypercolor triplets. Moreover, because of the different parities of $\tilde{u}$ and $\tilde{d}$ one cannot define a weak SU(2) isospin symmetry for hyperquarks. Therefore, they do not participate in the usual left-right asymmetric weak interaction but must couple left-right symmetrically calling for a left-right symmetric extension of weak interactions.

The same conclusion is also obtained from the anomaly freedom constraint of this extended electroweak theory. In the standard model the anomaly contributions of quarks and leptons cancel. There are no additional fermionic bound states that could cancel an anomaly contribution coming from hyperquarks. Therefore, hyperquarks must not contribute to electroweak anomalies. The only way this can be achieved is that their weak interactions be left-right symmetric. Formally, hyperquarks are obtained from quarks by replacing their neutral preons with their antiparticles. We refer to this process as hyperquark transformation.

TABLE I. The color C, hypercolor H, electric charge Q, preon number $\mathcal{P}$, Υ number, and intrinsic parity Π of preons and antipreons (see also [4]).

preon	H	C	Q	$\mathcal{P}$	Υ	Π
T	3	3	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	-1
V	3	$\bar{3}$	0	$+\frac{1}{3}$	$-\frac{1}{3}$	$+1$
$\bar{V}$	$\bar{3}$	3	0	$-\frac{1}{3}$	$+\frac{1}{3}$	-1
$\bar{T}$	$\bar{3}$	$\bar{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$+1$

TABLE II. Allowed three-preon bound states representing leptons, quarks, and hyperquarks and their quantum numbers (see also [4]). Formally, the hyperquarks are obtained from the corresponding quarks by interchanging: $V \leftrightarrow \bar{V}$ (hyperquark transformation).

state	preon content	bound state	$\mathcal{P}$	Υ	B	L	Q	Π	T_3
leptons	(VVV)	$(\nu_e, \nu_\mu, \nu_\tau)$	$+1$	-1	0	$+1$	0	$+1$	$+\frac{1}{2}$
	$(\bar{T}\bar{T}\bar{T})$	(e^-, μ^-, τ^-)	-1	-1	0	$+1$	-1	$+1$	$-\frac{1}{2}$
quarks	(TTV)	(u, c, t)	$+1$	$+\frac{1}{3}$	$+\frac{1}{3}$	0	$+\frac{2}{3}$	$+1$	$+\frac{1}{2}$
	$(\bar{T}\bar{V}\bar{V})$	(d, s, b)	-1	$+\frac{1}{3}$	$+\frac{1}{3}$	0	$-\frac{1}{3}$	$+1$	$-\frac{1}{2}$
hyperquarks	$(TT\bar{V})$	$(\tilde{u}, \tilde{c}, \tilde{t})$	$+\frac{1}{3}$	$+1$	$+1$	0	$+\frac{2}{3}$	-1	0
	$(\bar{T}VV)$	$(\tilde{d}, \tilde{s}, \tilde{b})$	$+\frac{1}{3}$	-1	-1	0	$-\frac{1}{3}$	$+1$	0

In summary, we can construct two groups of preon bound states (leptons and quarks) with the same intrinsic parities and integer preon number for which a new quantum number ‘‘weak isospin’’ corresponding to an effective SU(2) chiral isospin symmetry can be defined in terms of the preon number $\mathcal{P}$ as

$$T_3 = \tfrac{1}{2}\mathcal{P}. \tag{2.4}$$

The members of the third group (hyperquarks) necessarily have opposite intrinsic parities and fractional preon number and thus do not possess weak isospin. The three different types of fermionic bound states and their quantum numbers are shown in Table II.

III. BOSONIC BOUND STATES AND EXTENDED WEAK INTERACTIONS

Hyperquarks are hypercolored objects and thus cannot exist as free particles but must be confined into hypercolorless bound states such as hypermesons and hyperbaryons. This is in complete analogy to quarks being confined into colorless mesons and baryons. Although hyperhadrons might have been created at sufficiently high energies available in the Early Universe they are no longer observed today and hence cannot be stable. This leads to the question which gauge bosons are responsible for their decay.

We begin our discussion with charged hypermeson decay, which is generated by a new gauge boson $\tilde{W}$ followed by a short exposition of the charged and neutral weak transitions between fermionic bound states. In section III C we discuss the issue of hyperquark decay and show that it is mediated by a six-preon bosonic bound state, called χ, which can be thought of as two-neutrino bound state. The scenario of lepton number violating neutrino-antineutrino oscillations characteristic of Majorana neutrinos is described in section III D. There, also the consequences for the formulation of an extended left-right symmetric weak interaction theory are expounded.

A. Weak meson and hypermeson decays into leptons

As stated in the introduction and shown in Fig. 1, in the preon model the weak decays of hadrons and hyperhadrons are mediated by composite gauge bosons. For example, the weak decay of the positively charged π-meson into a muon and a neutrino can be schematically written as

$$\pi^+\begin{pmatrix} u(TTV) \\ \bar{d}(TVV) \end{pmatrix} \to W^+\begin{pmatrix} TTT \\ VVV \end{pmatrix} \to \mu^+(TTT) + \nu(VVV), \tag{3.1}$$

As is obvious from the notation, the compositeness of quarks and leptons implies that the weak gauge bosons are composed of six preons [3], and furthermore that initial and final states correspond merely to different arrangements of these preons.[1]

Similarly, the corresponding hyper-π-meson decay reads

$$\tilde{\pi}^+\begin{pmatrix} \tilde{u}(TT\bar{V}) \\ \bar{\tilde{d}}(T\bar{V}\bar{V}) \end{pmatrix} \to \tilde{W}^+\begin{pmatrix} TTT \\ \bar{V}\bar{V}\bar{V} \end{pmatrix} \to \mu^+(TTT) + \bar{\nu}(\bar{V}\bar{V}\bar{V}), \tag{3.2}$$

A comparison of the preon content of the bound states in Eq. (3.1) and (3.2) shows that the neutral preons V are replaced by their antiparticles $\bar{V}$ when going from quarks to hyperquarks [4] or from mesons to hypermesons (hyperquark transformation). In addition, in both cases the intermediate gauge bosons are formed by a mere rearrangement of the initial state preons. Hence, we propose that the existence of left-right symmetrically coupling hyperquarks entails the existence of a new class of composite weak gauge bosons, called $\tilde{W}$, which couple left-right symmetrically to fermions.

Although there is a certain analogy between weak meson and hypermeson decays into leptons there is an important difference between them. According to the quantum number assignments in Table II, the latter process simultaneously violates lepton and baryon number ($\Delta B = \Delta L = -2$), which indicates that it occurs only at a higher

[1]For convenience the preon content of the intermediate W boson is denoted by two rows of three preons.

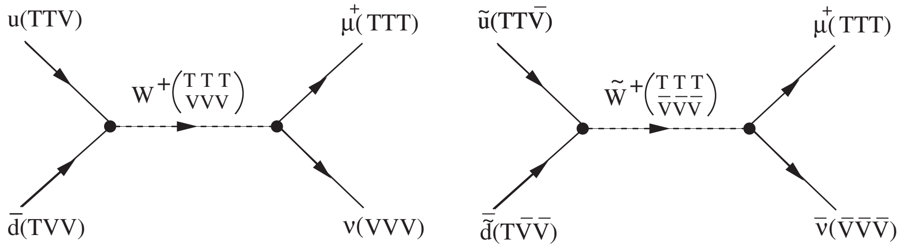

FIG. 1. Weak pion decay into leptons (left) and lepton number violating weak hyperpion decay (right) mediated by bosonic preon bound states.

energy scale where left-right symmetry is restored. In Sec. VI this scale is calculated as $M_P \cong 10^9$ GeV. Note that in both processes the preon $\mathcal{P}$ and $\Upsilon = B - L$ numbers remain conserved.

B. Charged and neutral weak transitions between fermionic preon bound states

In the preon model, the weak transitions among the members of quark and lepton weak isospin doublets caused by the charged weak currents of the standard model are written as

$$\begin{aligned} u(TTV) &\rightarrow d(\bar{T}\,\bar{V}\,\bar{V}) + W^{+}\begin{pmatrix} TTT \\ VVV \end{pmatrix} \\ e^{-}(\bar{T}\,\bar{T}\,\bar{T}) &\rightarrow \nu(VVV) + W^{-}\begin{pmatrix} \bar{T}\,\bar{T}\,\bar{T} \\ \bar{V}\,\bar{V}\,\bar{V} \end{pmatrix}, \end{aligned} \tag{3.3}$$

In these transitions all three preons of the final fermionic bound states are created from the vacuum, while the initial state preons and the antiparticle counterparts of the three vacuum pairs merge to form the weak gauge bosons.

Analogously, the corresponding weak transitions among hyperquarks imply the existence of new gauge bosons, $\tilde{W}$, which may then also generate transitions within lepton doublets

$$\begin{aligned} \tilde{u}(TT\bar{V}) &\rightarrow \tilde{d}(\bar{T}VV) + \tilde{W}^{+}\begin{pmatrix} TTT \\ \bar{V}\,\bar{V}\,\bar{V} \end{pmatrix} \\ e^{-}(\bar{T}\,\bar{T}\,\bar{T}) &\rightarrow \bar{\nu}(\bar{V}\,\bar{V}\,\bar{V}) + \tilde{W}^{-}\begin{pmatrix} \bar{T}\,\bar{T}\,\bar{T} \\ VVV \end{pmatrix}. \end{aligned} \tag{3.4}$$

In contrast to the usual electron-neutrino transition in Eq. (3.3), the $\tilde{W}$ induced process in Eq. (3.4) leads to an antineutrino in the final state and thus violates lepton number conservation ($\Delta L = -2$). On the other hand, $\Upsilon = B - L$ and $\mathcal{P}$ are conserved because $\Upsilon = -2$ and $\mathcal{P} = 0$ for the $\tilde{W}^-$ boson according to Table III. Similarly, in the hyperquark sector the charged weak transition violates baryon number ($\Delta B = -2$) but again $\Upsilon = B - L$ and $\mathcal{P}$ are conserved. One also notices that these charged weak transitions leave the type of fermionic bound state invariant, i.e., a quark remains a quark, a hyperquark remains a hyperquark, and a lepton remains a lepton.

Because neutral currents do not change the internal quantum numbers of the fermionic bound states involved in the transition, the W_0 boson must be a linear combination of the two neutral six-preon states. For the neutral standard model gauge bosons we define

$$\begin{aligned} W_0 &= \frac{1}{\sqrt{2}}\left[\begin{pmatrix} \bar{T}\,\bar{T}\,\bar{T} \\ TTT \end{pmatrix} - \begin{pmatrix} \bar{V}\,\bar{V}\,\bar{V} \\ VVV \end{pmatrix}\right] \\ B_0 &= \frac{1}{\sqrt{2}}\left[\begin{pmatrix} \bar{T}\,\bar{T}\,\bar{T} \\ TTT \end{pmatrix} + \begin{pmatrix} \bar{V}\,\bar{V}\,\bar{V} \\ VVV \end{pmatrix}\right]. \end{aligned} \tag{3.5}$$

Note that applying the hyperquark transformation to Eq. (3.5) leaves these states invariant so that we need not introduce additional neutral bosons $\tilde{W}_0$ and $\tilde{B}_0$. As a linear combination of the two states in Eq. (3.5) the Z-boson is a pure $(3V, 3\bar{V})$ state, while the orthogonal combination $(3T, 3\bar{T})$ state can in some sense be interpreted as an *effective* photon similar to the vector meson dominance model. In this way the standard model weak forces are seen to be mediated by *effective* gauge bosons composed of six preons.

As far as we can see, $6T$ and $6\bar{T}$ states do not occur in any weak process and are not considered here. The same applies to six-preon bound states consisting of one charged preon and five neutral ones or one neutral preon and five charged ones. In Table III we list the composite gauge bosons introduced so far as well as their preon content and quantum numbers.

In summary, for the weak gauge bosons W, Z, and $\tilde{W}$, the following rules apply: (i) leptons and quarks transform via W exchange; (ii) leptons and hyperquarks transform via $\tilde{W}$ exchange; (iii) leptons, quarks, and hyperquarks transform via Z exchange. But neither the standard model gauge

TABLE III. Preon content and quantum numbers of the standard model weak gauge bosons W and B_0 and the newly introduced gauge bosons $\tilde{W}$.

state	content	$\mathcal{P}$	Υ	Q	Π	T_3
W^-	$(3\bar{T}, 3\bar{V})$	-2	0	-1	-1	-1
W^+	$(3T, 3V)$	$+2$	0	$+1$	-1	$+1$
W_0/B_0	$(3(T\bar{T}), 3(V\bar{V}))$	0	0	0	-1	0
$\tilde{W}^-$	$(3\bar{T}, 3V)$	0	-2	-1	$+1$	0
$\tilde{W}^+$	$(3T, 3\bar{V})$	0	$+2$	$+1$	$+1$	0

bosons nor the newly introduced $\tilde{W}$ can transform hyperquarks into quarks.

C. Weak hyperquark decays into quarks

Similar to the case of hypermesons discussed in Sec. III A, the nonobservation of hyperbaryons implies that they are not stable. Consequently, there must be transitions from hyperquarks to known fermionic preon bound states. In principle, hyperquarks can decay into quarks and into leptons. However, below the grand unification (GUT) scale the former process dominates for the following reason. The decay of quarks into leptons, as required for proton decay is suppressed due to the heavy mass of the GUT gauge bosons of the order of 10^{16} GeV, which corresponds to lifetimes of the order of 10^{35} y. Because the preonic substructure of hyperquarks and quarks are very similar, transitions from hyperquarks to leptons are also suppressed at lower energies. The decay of quarks and hyperquarks into leptons mediated by dipreonic bound states U and $\tilde{U}$ occurring at the GUT scale will be considered in Sec. V. Clearly, the fact that hyperquarks are not observed today requires that their lifetime be much shorter than the lifetime of the Universe (10^{10} y), and that the gauge bosons N responsible for hyperquark decay into quarks be lighter than the GUT bosons. The transitions between the three types of fermionic bound states mediated by dipreonic bound states N, U, and $\tilde{U}$ are shown in Fig. 2.

In our previous paper [4], we have seen that the transition from hyperquarks to quarks is formally obtained by interchanging the neutral preon by its antiparticle ($V \leftrightarrow \bar{V}$). Hyperquark decays into quarks are generated via a new class of electrically neutral bosons, called N-bosons or neutralons $N(VV)$ and $\bar{N}(\bar{V}\bar{V})$

$$\begin{aligned} \tilde{u}(TT\bar{V}) &\rightarrow u(TTV) + \bar{N}(\bar{V}\bar{V}) \\ \tilde{d}(\bar{T}VV) &\rightarrow d(\bar{T}\bar{V}\bar{V}) + 2N(VV). \end{aligned} \tag{3.6}$$

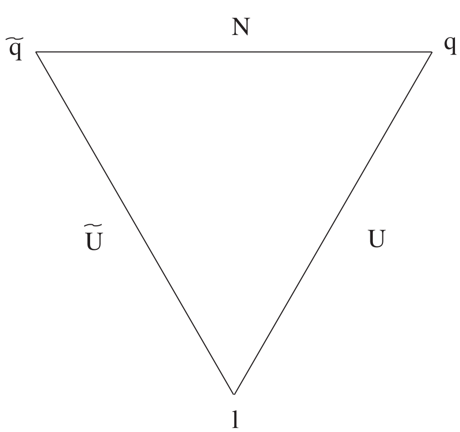

FIG. 2. Fermion triangle. The three fermionic bound states, hyperquarks ($\tilde{q}$), quarks (q), and leptons (l) and their antiparticles are placed at the corners of a triangle. The dipreonic bound states ($N, U, \tilde{U}$) and their antiparticles describing transitions between these fermions are placed along the edges.

As color and hypercolor triplets, neutralons are confined particles. In order to carry away the energy and momentum made available in the $\tilde{q} \rightarrow q$ transition, an unconfined neutral particle must be emitted. Such a hypercolor and color singlet can be formed by an $N\bar{N}$ pair or three neutralons. The emission of two $N\bar{N}$ pairs, as occurring in hyperproton decay according to Eq. (3.6) seems at first sight possible. But an $N\bar{N}$ state has neither weak isospin nor electric charge and thus it cannot couple to any of the known low-energy bosons such as Z or γ into which it could annihilate. Therefore, direct hyperproton decay via the emission of two $N\bar{N}$ pairs does not work. However, annihilation into the vacuum is possible for *three* $N\bar{N}$ pairs. In fact, $3N$ or $3\bar{N}$(and thus $3N\bar{N}$) form colorless and hypercolorless bound states which can decay into neutrinos and antineutrinos. In the following the $3N$ and $3\bar{N}$ states are called χ and $\bar{\chi}$.

More generally, we postulate that in all weak processes occurring below the grand unification scale of 10^{16} GeV, such as hyperquark-quark transitions, as well as in processes involving W, $\tilde{W}$ and Z exchange, the generation or annihilation of preon-antipreon pairs is only possible for integer multiples of three preon-antipreon pairs. This is symbolically written as

$$n_{(\bar{T}T)} + n_{(\bar{V}V)} = 3k \quad (\text{for E} \leq 10^{16} \text{ GeV}), \tag{3.7}$$

where $n_{(\bar{T}T)}$, $n_{(\bar{V}V)}$, and k are natural numbers. Thus, in any weak interaction below the grand unification scale, only six-preon (see previous subsection) or three-dipreon bosons (see next subsection) are involved. We refer to this as ''preon triality rule.''

The simplest hyperbaryon decay process seems to proceed via a $\tilde{\Delta}$ type hyperbaryon in which the six-preon bound states χ and $\bar{\chi}$ are produced

$$\begin{aligned} \tilde{\Delta}^{++}(\tilde{u}\,\tilde{u}\,\tilde{u}) &\rightarrow \Delta^{++}(uuu) + \bar{\chi} \\ \tilde{\Delta}^{-}(\tilde{d}\,\tilde{d}\,\tilde{d}) &\rightarrow \Delta^{-}(ddd) + 2\chi, \end{aligned} \tag{3.8}$$

and where the latter subsequently decay into two neutrinos ($\chi \rightarrow 2\nu$) or antineutrinos ($\bar{\chi} \rightarrow 2\bar{\nu}$). Thus, the transition from hyperquarks to quarks is only possible within bound states of three hyperquarks of the same charge state ($\tilde{u}$, $\tilde{c}$, $\tilde{t}$ or $\tilde{d}$, $\tilde{s}$, $\tilde{b}$). Other baryonic hyperquark bound states, $\tilde{\Delta}^{+}(\tilde{u}\,\tilde{u}\,\tilde{d})$ and $\tilde{\Delta}^{0}(\tilde{u}\,\tilde{d}\,\tilde{d})$, decay via a two-step process, i.e., they first change into a $\tilde{\Delta}^{++}(\tilde{u}\,\tilde{u}\,\tilde{u})$ and $\tilde{\Delta}^{-}(\tilde{d}\,\tilde{d}\,\tilde{d})$ via $\tilde{W}^{\pm}$ emission as in Eq. (3.4) and then decay into ordinary baryons via χ emission as in Eq. (3.8). Because of the vanishing isospin of hyperhadrons the spin-symmetric $\tilde{\Delta}$ baryons are the lightest fermionic hyperhadron states.

D. Left-right symmetric weak interactions and neutrino-antineutrino oscillations

In this section we discuss in more detail how the existence of hyperquarks leads to a left-right symmetric extension of standard model weak interactions. As explained before, hyperquarks do not have weak isospin and therefore they couple left-right symmetrically to the new gauge bosons $\tilde{W}$ and N. At the partial unification scale M_P, where these new left-right symmetric gauge bosons appear, hyperquarks decay into quarks as in Eq. (3.6). Therefore, at this scale quarks must also couple left-right symmetrically to the standard model weak gauge bosons W. This entails an extension of the standard model weak isospin group $SU(2)_{W_L}$ to the gauge group $SU(2)_{W_L} \times SU(2)_{W_R}$.

At the same energy, the effective gauge boson masses M_{W_R}, M_{W_L}, $M_{\tilde{W}}$ are of order M_P, and the left-right symmetric gauge bosons can transform into each other. In particular, the W-bosons can change into the $\tilde{W}$-bosons and vice versa,

$$\begin{aligned} \tilde{W}^+\begin{pmatrix} TTT \\ \bar{V}\bar{V}\bar{V} \end{pmatrix} &\leftrightarrow W^+\begin{pmatrix} TTT \\ VVV \end{pmatrix}_{L/R} \\ \tilde{W}^-\begin{pmatrix} \bar{T}\bar{T}\bar{T} \\ VVV \end{pmatrix} &\leftrightarrow W^-\begin{pmatrix} \bar{T}\bar{T}\bar{T} \\ \bar{V}\bar{V}\bar{V} \end{pmatrix}_{L/R}. \end{aligned} \tag{3.9}$$

In addition, there are transitions between the χ, Z, and $\bar{\chi}$ bosons

$$\chi(\nu\nu) \leftrightarrow Z(\nu\bar{\nu}) \leftrightarrow \bar{\chi}(\bar{\nu}\,\bar{\nu}). \tag{3.10}$$

The transformations in Eq. (3.9) and (3.10) are a reflection of left-right symmetry restoration at the partial unification scale. These left-right symmetric interactions also enable transitions between the left- and right-handed neutrino sectors

$$\nu \leftrightarrow \bar{\nu} \quad \text{or} \quad \nu_L \leftrightarrow \nu_R \tag{3.11}$$

characteristic for two-component Majorana neutrinos [6,7], for which the difference between neutrino and antineutrino is only one of helicity, i.e. $\nu_L \equiv \nu$ and $\nu_R \equiv \bar{\nu}$. Such neutrino-antineutrino transitions can be thought of as proceeding via $\chi \leftrightarrow Z \leftrightarrow \bar{\chi}$ transitions as depicted in Fig. 3. Obviously, at this energy scale the inner parity and weak isospin of preon bound states is no longer a good quantum number.

In the preon model neutrino-antineutrino oscillations may be allowed for the following reasons. As discussed

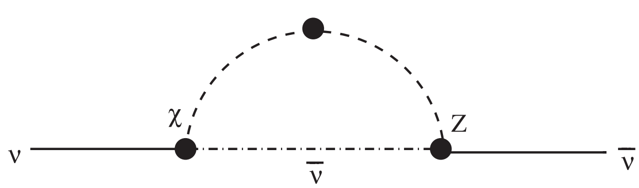

FIG. 3. Neutrino-antineutrino oscillation via χ and Z emission and absorption.

in sect. II, the neutral V and $\bar{V}$ preons have different $SU(3)_C \times SU(3)_H$ and parity assignments (see Table I). Because none of the fundamental gauge interactions of the theory, namely $SU(3)_C$, $SU(3)_H$, and $U(1)_Q$ violates parity conservation, there can be no direct transitions between the neutral V preon and its antiparticle $\bar{V}$. For the same reason, transitions between the neutralons $N(VV)$ and $\bar{N}(\bar{V}\,\bar{V})$ are forbidden. However, for bound states of three V and three $\bar{V}$ preons, which are electrically neutral color and hypercolor singlets, there is from the viewpoint of the fundamental gauge symmetries no distinction between $\nu(VVV)$ and $\bar{\nu}(\bar{V}\,\bar{V}\,\bar{V})$ as pointed out by Harari and Seiberg [5].

As one can readily see from Table I, the additive $\mathcal{P}$ and Υ quantum numbers are simultaneously violated in Eq. (3.11) but in such a way that the total electric charge associated with the fundamental $U(1)_Q$ gauge interaction remains conserved in accordance with the generalized Gell-Mann Nishijima relation Eq. (2.3), i.e., $\Delta Q = 0$ which entails $\Delta\mathcal{P} = -\Delta\Upsilon$. Furthermore, it is observed that this violation can only occur for integer values of these quantum number and processes involving three V or three $\bar{V}$, whereas fractional values of $\mathcal{P}$ and Υ are strictly conserved because of their connection with fundamental gauge interactions (see Table I). Analogous to the $\mathcal{P}$ and Υ number violation associated with the direct neutrino-antineutrino oscillation, the intrinsic parity violation in Eq. (3.11) only occurs at the level of bound states of at least three preons.

Within each chiral sector the $\mathcal{P}$ and Υ quantum numbers are conserved as required by the anomaly equations [4]. For this reason, any violation of these quantum numbers is accompanied by a simultaneous change of the chiral sector, which is only possible for massive particles. The corresponding transition rate is determined by the ratio of the neutrino mass and the partial unification scale, where the right-handed electroweak gauge bosons occur [7,8]. The heavier these bosons, the smaller the helicity changing transition rate and therefore the neutrino rest mass. This corresponds to the so-called seesaw mechanism [8,9]

$$\frac{m_e^2}{M_P} = m_{\nu_e}, \tag{3.12}$$

where m_{ν_e} and m_e are the neutrino and electron mass, and $M_P \cong 10^9$ GeV is the scale of left-right symmetry restoration.

The extended left-right symmetric electroweak interactions, which go hand in hand with $\nu - \bar{\nu}$ and $\chi - Z - \bar{\chi}$ oscillations as depicted in Fig. 3, lead to new $\Upsilon = B - L$ number violating processes, such as, for example, (i) neutrinoless hyperbaryon decay where Υ- and B- numbers are violated and lepton number L is conserved and (ii) neutrinoless double beta decay where Υ- and L- numbers are violated and B is conserved

$$\tilde{\Delta}^{++}(\tilde{u}\,\tilde{u}\,\tilde{u}) \to \Delta^{++}(uuu) + Z; \qquad \Delta Y = \Delta B = -2,$$
$$\tilde{\Delta}^{-}(\tilde{d}\,\tilde{d}\,\tilde{d}) \to \Delta^{-}(ddd) + 2Z; \qquad \Delta Y = \Delta B = 4,$$
$$2n(udd) \to 2p^{+}(uud) + 2e^{-}; \qquad \Delta Y = -\Delta L = -2. \tag{3.13}$$

The discovery of any one of these B or L violating decays would lend some support to the ideas developed here.

IV. PARTIAL UNIFICATION

Having motivated an extension of the standard model weak interaction that accounts for hyperquark decay, left-right symmetric weak gauge bosons, and direct neutrino-antineutrino oscillations, we study in this section further aspects of this extension. In particular, we propose that the generalized weak interactions are part of a larger unification scheme for weak and strong interactions between preon bound states that includes (i) 7 left-right symmetric standard model weak gauge bosons W_L, W_R, B^0, transforming according to a simply extended rank 3 gauge group $\mathrm{SU}(2)_{W_L} \times \mathrm{SU}(2)_{W_R} \times U(1)_Y$, (ii) 20 new left-right symmetric gauge bosons $\tilde{W}$, N, $\bar{N}$, (iii) 8 hypergluons, altogether 35 gauge bosons (see Table V) characteristic of an effective SU(6) gauge group. In addition, we emphasize that in the present model it is possible to connect the left-right symmetry of weak interactions at high energies and its breaking at low energies to the existence of hyperquarks and the new weak gauge bosons. Before this, we discuss the consequences of the new effective gauge interactions and of hyperquarks for the momentum transfer dependence of the electroweak coupling strengths α_W, α_Y, and α_Q and calculate the corresponding Weinberg angle.

A. Extended electroweak couplings

In the standard model the left-handed leptons and quarks form weak isospin doublets transforming according to the gauge group $\mathrm{SU}(2)_{W_L}$ whereas the right-handed leptons and quarks are isospin singlets transforming only according to the gauge group $\mathrm{U}(1)_Y$. The connection between electric charge Q_i, weak isospin T_{3i}, and weak hypercharge Y_{Wi} of particle i is given by the Gell-Mann Nishijima relation

$$Q_i = T_{3i} + Y_{Wi}, \tag{4.1}$$

where the particle index i stands for a member of the lepton or quark doublets. The corresponding quantum numbers are given in Table IV.

It has been shown in Sec. III B that hyperquarks do not participate in the usual charged current interactions mediated by W exchange. Therefore, with respect to the standard electroweak gauge group, the left- and right-handed hyperquarks are isospin singlets ($T_{3i} = 0$) analogous to the right-handed leptons and quarks. With these properties of the composite fermions at hand, we can now proceed and discuss how the coupling constants and their ratios are affected by the presence of the hyperquarks.

TABLE IV. Weak isospin, weak hypercharge, baryon, and lepton quantum number assignments of the left- and right-handed leptons, quarks, and left-right symmetric hyperquarks. The squares of weak isospin, hypercharge, and electric charge are also given.

Fermion	T_3	Y_W	B	L	T_3^2	Y_W^2	Q^2
ν_L	$+\frac{1}{2}$	$-\frac{1}{2}$	0	$+1$	$\frac{1}{4}$	$\frac{1}{4}$	0
e_L^-	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$+1$	$\frac{1}{4}$	$\frac{1}{4}$	1
e_R^-	0	-1	0	$+1$	0	1	1
u_L	$+\frac{1}{2}$	$+\frac{1}{6}$	$+\frac{1}{3}$	0	$\frac{1}{4}$	$\frac{1}{36}$	$\frac{4}{9}$
u_R	0	$+\frac{2}{3}$	$+\frac{1}{3}$	0	0	$\frac{4}{9}$	$\frac{4}{9}$
d_L	$-\frac{1}{2}$	$+\frac{1}{6}$	$+\frac{1}{3}$	0	$\frac{1}{4}$	$\frac{1}{36}$	$\frac{1}{9}$
d_R	0	$-\frac{1}{3}$	$+\frac{1}{3}$	0	0	$\frac{1}{9}$	$\frac{1}{9}$
$\tilde{u}_{L/R}$	0	$+\frac{2}{3}$	$+1$	0	0	$\frac{4}{9}$	$\frac{4}{9}$
$\tilde{d}_{L/R}$	0	$-\frac{1}{3}$	-1	0	0	$\frac{1}{9}$	$\frac{1}{9}$

First, we note the standard definitions of the three electroweak couplings α_Q, α_W, and α' and the relation between them [10,11]

$$\frac{\alpha_Q}{\alpha_W} := \sin^2\Theta_W = \frac{\sum_i T_{3i}^2}{\sum_i Q_i^2} \qquad \frac{\alpha_Q}{\alpha'} := \cos^2\Theta_W = \frac{\sum_i Y_{Wi}^2}{\sum_i Q_i^2}$$
$$\frac{1}{\alpha_Q} = \frac{1}{\alpha_W} + \frac{1}{\alpha'}, \tag{4.2}$$

where the third equation follows from dividing the sum of the first two by α_Q. This implies the relation

$$\sum_i Q_i^2 = \sum_i T_{3i}^2 + \sum_i Y_{Wi}^2, \tag{4.3}$$

which can also be obtained directly from the square of the Gell-Mann Nishijima relation Eq. (4.1) because the sum (over all fermions) of the mixed product terms vanishes.

In must be noted that an equality of α_W and α' cannot be obtained because $\sum T_{3i}^2 \neq \sum_i Y_{Wi}^2$. Therefore, the following coupling α_Y is introduced

$$\frac{1}{\alpha'} = \frac{1}{\alpha_Y} \frac{\sum_i Y_{Wi}^2}{\sum_i T_{3i}^2} \tag{4.4}$$

which, when inserted in Eq. (4.2) gives

$$\frac{1}{\alpha_Q} = \frac{1}{\alpha_W} + \frac{\sum_i Y_{Wi}^2}{\sum_i T_{3i}^2} \frac{1}{\alpha_Y}. \tag{4.5}$$

In the standard electroweak theory (without hyperquarks) we have at the weak scale $M_Z = 91.2$ GeV from Eq. (4.5)

$$\frac{1}{\alpha_Q} = \frac{1}{\alpha_W} + \frac{5}{3}\frac{1}{\alpha_Y}. \tag{4.6}$$

Above this scale, at a certain energy which will be determined in Sec. VI as $m_{\rm hq} \cong 26$ TeV, the additional contributions of the hyperquarks affect the sums in Eq. (4.2) and (4.5) as discussed in the following. First, the sum T_{3i}^2 over (left-handed) quarks and leptons remains unmodified

$$\sum_i T_{3iL}^2 = N_G\Big[\Big(\frac{1}{2}\Big)_\nu^2 + \Big(-\frac{1}{2}\Big)_e^2 + N_C\Big(\Big(\frac{1}{2}\Big)_u^2 + \Big(-\frac{1}{2}\Big)_d^2\Big)\Big], \tag{4.7}$$

because hyperquarks do not have weak isospin. Here, the factor N_C stands for the number of quark colors, and N_G is the number of generations. Second, the sum of the weak hypercharge squares is modified by the presence of hyperquarks as

$$\sum_i Y_{Wi}^2 = N_G\Big[\Big(-\frac{1}{2}\Big)_{\nu_L}^2 + \Big(-\frac{1}{2}\Big)_{e_L}^2 + (-1)_{e_R}^2 + N_C\Big(\Big(\frac{1}{6}\Big)_{u_L}^2 + \Big(\frac{1}{6}\Big)_{d_L}^2 + \Big(\frac{2}{3}\Big)_{u_R}^2 + \Big(-\frac{1}{3}\Big)_{d_R}^2\Big) + N_H\Big(\Big(\frac{2}{3}\Big)_{\tilde{u}_L}^2 + \Big(-\frac{1}{3}\Big)_{\tilde{d}_L}^2 + \Big(\frac{2}{3}\Big)_{\tilde{u}_R}^2 + \Big(-\frac{1}{3}\Big)_{\tilde{d}_R}^2\Big)\Big], \tag{4.8}$$

where N_H is the number of hypercolors. Third, the sum over the squared charges is

$$\sum_i Q_i^2 = 2N_G\Big[(-1)_e^2 + N_C\Big(\Big(\frac{2}{3}\Big)_u^2 + \Big(-\frac{1}{3}\Big)_d^2\Big) + N_H\Big(\Big(\frac{2}{3}\Big)_{\tilde{u}}^2 + \Big(-\frac{1}{3}\Big)_{\tilde{d}}^2\Big)\Big], \tag{4.9}$$

where the factor of 2 is due to equal contributions from left-handed fermion doublets and right-handed fermion singlets, and numerically $N_C = N_H = N_G = 3$ according to Ref. [4].

Because hyperquarks carry electric charge and weak hypercharge but no weak isospin, their inclusion in Eq. (4.2) leads to the following Weinberg angle

$$\sin^2\Theta_W = \frac{\alpha_Q}{\alpha_W} = \frac{\sum_i T_{3i}^2}{\sum_i Q_i^2} = \frac{3}{13}. \tag{4.10}$$

Thus, we have $\sin^2\Theta_W = 3/13 \cong 0.231$, which is close to the experimental Weinberg angle 0.23119(14) at $M_Z = 91.2$ GeV [12]. We take this as an indication for the physical relevance of hyperquarks. In the original preon model of Harari and Seiberg [2] the weak mixing angle was predicted as $\sin^2\Theta_W = 1/4$.

Because of left-right symmetry restoration at the partial unification scale $M_P \cong 10^9$ GeV discussed in sect. III D we have left-right symmetric couplings of the standard weak gauge bosons and the new gauge bosons $\tilde{W}$ and χ to the fermions. Consequently, above this scale, the sum over the squared isospin quantum numbers for leptons and quarks must run over both left- and right- handed fermions

$$\sum_i T_{3i}^2 = \sum_i T_{3iL}^2 + \sum_i T_{3iR}^2, \tag{4.11}$$

and hence is twice as large as in Eq. (4.10). Furthermore, at the energy scale of $M_P \cong 10^9$ GeV, where left-right symmetry is restored, the electroweak coupling constants will necessarily have equal strength (see Fig. 5)

$$\alpha_{W/Y} := \alpha_W = \alpha_Y. \tag{4.12}$$

Using these conditions in Eq. (4.10) one obtains then for the Weinberg angle at M_P

$$\sin^2\Theta_W = \frac{\alpha_Q}{\alpha_{W/Y}} = \frac{6}{13}, \tag{4.13}$$

which is twice the experimental value of $\sin^2\Theta_W$ at $M_Z = 91.2$ GeV. We note that for a simply extended weak gauge group $SU(2)_{W_L} \times SU(2)_{W_R} \times U(1)$ without hyperquarks one gets $\sin^2\Theta_W = 3/4$.

The breaking of left-right symmetry in standard electroweak theory at the M_Z scale leads to a smaller value for the Weinberg angle which is only half of the value at M_P given by Eq. (4.13). This change of the Weinberg angle is accompanied by a mass shift from the left-right symmetrically coupling bosons at M_P down to their low energy counterparts at the Fermi scale M_Z.

At the energy M_P, where left-right symmetry is restored, the existence of an additional $SU(2)_{W_R}$ gauge group leads to an extended electroweak group $SU(2)_{W_L} \times SU(2)_{W_R} \times U(1)_Y$ [5] which itself is embedded in an even larger partial unification group $SU(6)_P$ with a single coupling constant

$$\alpha_P := 2\alpha_{W/Y}. \tag{4.14}$$

Here, α_P is to be evaluated at the scale of left-right symmetry restoration and the factor 2 is due to the ensuing strength doubling according to Eq. (4.11) and depicted by the short dotted line in Fig. 5. Further aspects of the unification of weak with hypergluon interactions will be discussed next.

B. Partial unification group $SU(6)_P$

We have seen that on the level of preon bound states new effective gauge interactions emerge, e.g. those mediated by the $\tilde{W}$ and the N bosons, and that they occur at an energy where a left-right symmetric extension of standard model weak interactions is required. However, we have not yet incorporated the new gauge bosons in an appropriately enlarged effective gauge group. This can be achieved by combining the extended weak and strong hypercolor gauge interactions. Such a combination is also suggested by the fact that the coupling constants of $SU(3)_H$ and $SU(3)_C$ always run in parallel (see Fig. 5) and cannot reach equal strength because both groups have the same structure.

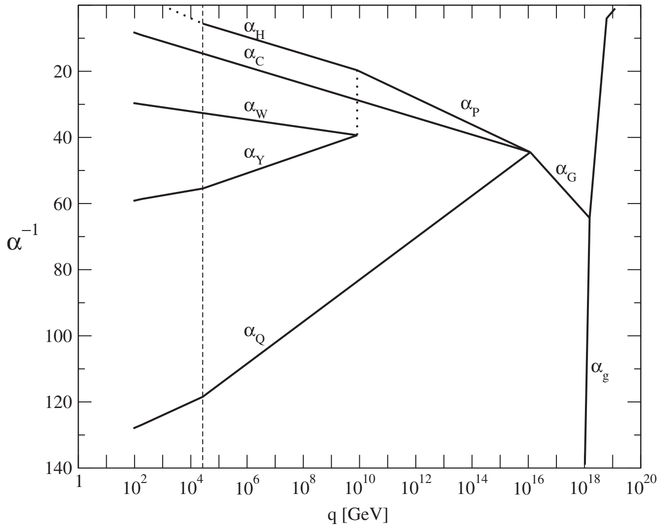

FIG. 5. Running coupling constants. The dashed vertical line indicates the expected hyperquark mass scale. Because hyperquarks are hypercolor triplets and charged weak isosinglets, they only affect the $SU(3)_H$ coupling α_H and the U(1) gauge couplings α_Q and α_Y. The dotted vertical line at 10^9 GeV indicates the partial unification scale where the weak isospin and hypercharge couplings converge and are unified with the hypercolor interaction to a common coupling α_P. The three gauge couplings α_P, α_C, and α_Q meet at the grand unification scale of 10^{16} GeV. Finally, from the grand unification scale to the point where α_G meets the gravitational coupling α_g ($\cong 10^{18}$ GeV), preons behave as quasifree particles.

Consequently, one of these color groups has to be embedded in a larger group if one wishes to unify the strong and weak interactions between preon bound states.

We propose a partial unification by embedding the extended electroweak and hypercolor interactions within a broken gauge group $SU(6)_P \supset SU(3)_H \times SU(2)_{W_L} \times SU(2)_{W_R} \times U(1)_Y$. Note that SU(6) is the simplest rank 5 group having the same rank as its subgroups. At the partial unfication scale M_P, the following equality of coupling constants is required to hold

$$\alpha_P := \alpha_H = 2\alpha_{W/Y}, \tag{4.15}$$

where the last equality follows from Eq. (4.14). The SU(6) group also contains the hypercolored neutralons and the $\tilde{W}^{\pm}$ bosons. The 6×6 matrix representing the effective SU(6) gauge bosons is schematically shown below

$$G_P = \begin{pmatrix} G_H & N \\ \bar{N} & \begin{pmatrix} W_R^0 & W_R^+ & \tilde{W}^+ \\ W_R^- & W_L^0 & W_L^+ \\ \tilde{W}^- & W_L^- & B^0 \end{pmatrix} \end{pmatrix}. \tag{4.16}$$

This scheme includes 3×3 matrices for the hypergluons G_H and the neutralons N and $\bar{N}$ transforming according to $SU(3)_H$. For the latter only the hypercolor degree of freedom is considered when counting their multiplicity. The partial unification group is then a unitary group of dimension 6, comprising 8 hypergluons, 9 left-right symmetric weak gauge bosons, and 18 left-right symmetric dipreonic neutralons, in total 35 generators as discussed above. The corresponding gauge bosons with their respective energy scales are listed in Table V. Note that the diagonal generators G_H, W_R^0, and W_L^0 have a B_0 admixture [11].

The relevant SU(6) representations for the fermionic preon bound states arise from the direct product of three fundamental six-dimensional representations, where the "**6**" is due to the three hypercolors and the two types of preons. We then have for the fermions $\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} = \mathbf{20} \oplus \mathbf{56} \oplus \mathbf{70} \oplus \mathbf{70}$, where only the lowest dimensional (antisymmetric) **20** is needed to represent the first generation of leptons, quarks, and hyperquarks and their antiparticles. The **20** dimensional representation decomposes into 4 hypercolor neutral leptons, 4 hypercolor neutral quarks, and 12 hyperquarks where the multiplicity of hypercolor (H) is included as indicated below

$$\left(\begin{pmatrix} \nu & \bar{\nu} \\ e^- & e^+ \end{pmatrix} \begin{pmatrix} u & \bar{u} \\ d & \bar{d} \end{pmatrix} \begin{pmatrix} \tilde{u} & \bar{\tilde{u}} \\ \tilde{d} & \bar{\tilde{d}} \end{pmatrix}_H \right)_P. \tag{4.17}$$

TABLE V. Gauge groups, gauge bosons, and corresponding energy scales.

gauge group	generators	gauge boson	energy scale
$U(1)_Q$	1	A_Q	$\Lambda_Q = 10^{-3}$ GeV
$SU(3)_C$	8	G_C	$\Lambda_C \cong 0.2$ GeV
$SU(2)_{W_L} \times U(1)_Y$	4	W_L^-; W_L^+; W_L^0; B^0	$M_Z = 91.18$ GeV
$SU(3)_H$	8	G_H	$\Lambda_H \cong 1700$ GeV
$SU(6)_P$	35	G_H	
		W_L^-; W_L^+; W_L^0; W_R^-; W_R^+; W_R^0; B^0	
		$\tilde{W}^-$; $\tilde{W}^+$	
		$(N; \bar{N})_H$	$M_P = 8.3 \times 10^9$ GeV
$SU(9)_G$	80	G_P	
		G_C	
		$(X; Y; \bar{X}; \bar{Y}; U; \bar{U})_C$	
		$(\tilde{X}; \tilde{Y}; \bar{\tilde{X}}; \bar{\tilde{Y}}; \tilde{U}; \bar{\tilde{U}})_H$	
		A_Q	$M_G = 1.2 \times 10^{16}$ GeV

We mention that at energies where $SU(6)_P$ comes into play, anomaly freedom of $SU(6)_P$ is guaranteed because by definition all its generators are left-right symmetric. Furthermore, the charge sum of the above multiplet is zero.

C. Preon intrinsic parity and weak interaction phenomenology

In this subsection we wish to comment on the different intrinsic parity assignment for T and V preons (see Table I) and its consequences for the effective preon bound state interactions that have been discussed so far.

As shown in Table II, leptons and quarks have the same intrinsic parity whereas the two hyperquarks $\tilde{u}$ and $\tilde{d}$ have opposite intrinsic parities. Therefore, one can assign a weak isospin to leptons and quarks but not to hyperquarks. This means that only leptons and quarks can be divided into left- and right-handed sectors carrying different weak isospin T_L and T_R.

At low energies below the partial unification scale M_P, where chiral isospin symmetry is broken, this classification of leptons and quarks in separate chiral sectors having different weak isospin $T_L = 1/2$ and $T_R = 0$ goes hand in hand with left-right asymmetric weak interactions mediated by the standard model gauge bosons W_L and Z. In contrast, hyperquarks do not possess a weak isospin and consequently do not participate in the standard model weak isospin interactions but only in left-right symmetric $\tilde{W}$, N, and Z mediated interactions as discussed before.

For energies above M_P, hyperquark-quark transitions occur, and the left-right symmetric hyperquark interactions entail left-right symmetric weak interactions also for quarks and leptons coupling to the weak isospin T_L and T_R with equal strength because at this scale the effective masses of the W_L, W_R, $\tilde{W}$ are of the same order as M_P. At this point the inner parity of preon bound states ceases to be a good quantum number.

Thus, preon intrinsic parities reveal themselves in different effective interactions of standard model fermions and of hyperquarks. At low energies of 10^2 GeV, we have left-right asymmetric interactions between particles having definite intrinsic parities. At higher energies of 10^9 GeV we have left-right symmetric interactions but intrinsic parity violating transitions involving hyperquarks and neutrino-antineutrino pairs.

These aspects of the extended weak interaction have their origin in the different intrinsic parity assignments of the fundamental preon building blocks. Moreover, the existence of left-right symmetry at high and left-right asymmetry at low energies is seen to be closely connected with the production and decay of hyperquarks and hence to some extent explained in the preon model developed here.

V. GRAND UNIFICATION

As motivated in the previous chapter we are now left with three gauge groups, namely $SU(6)_P$, $SU(3)_C$, and $U(1)_Q$. At the grand unification scale M_G, the corresponding interactions strengths have the same strength and are described by a common coupling constant (see Fig. 5).

$$\alpha_G := \alpha_P = \alpha_H = \alpha_C = \alpha_Q. \tag{5.1}$$

At this scale the direct product, $SU(6)_P \times SU(3)_C \times U(1)_Q$ is embedded in the larger gauge group $SU(9)_G \supset SU(6)_P \times SU(3)_C \times U(1)_Q$. Thus, the unbroken fundamental gauge symmetry $SU(3)_H \times SU(3)_C \times U(1)_Q$ of the preon model is also part of this larger group, which finds its expression in the equality of coupling constants according to Eqs. (5.1).

The grand unification group $SU(9)_G$ contains in addition to the gauge bosons already included in $SU(6)_P$ the following color antitriplet and hypercolor singlet gauge bosons generating transitions between quarks and leptons:

$$X = \begin{pmatrix} TTV \\ TTV \end{pmatrix}; \qquad Y = \begin{pmatrix} \bar{T}\,\bar{V}\,\bar{V} \\ \bar{T}\,\bar{V}\,\bar{V} \end{pmatrix}; \qquad U = \begin{pmatrix} TTV \\ \bar{T}\,\bar{V}\,\bar{V} \end{pmatrix} = (T\bar{V}), \tag{5.2}$$

where the six-preon bound state U reduces to a dipreon bound state with respect to its quantum numbers. These bosons are responsible for proton decay, which has already been predicted by the SU(5) grand unification [13] as well as within the preon model [14].

Furthermore, there are new hypercolor antitriplet and color singlet gauge bosons generating transitions between hyperquarks and leptons

$$\tilde{X} = \begin{pmatrix} TT\bar{V} \\ TT\bar{V} \end{pmatrix}; \qquad \tilde{Y} = \begin{pmatrix} \bar{T}VV \\ \bar{T}VV \end{pmatrix}; \qquad \tilde{U} = \begin{pmatrix} TT\bar{V} \\ \bar{T}VV \end{pmatrix} = (TV). \tag{5.3}$$

Formally, the latter bosons are obtained from those in Eq. (5.2) by the hyperquark transformation. The quantum numbers of the six-preon GUT bosons X and Y generating baryon decay into leptons and of the new GUT bosons $\tilde{X}$ and $\tilde{Y}$ mediating hyperbaryon decay into leptons are listed in Table VI.

The preon content of the dipreonic U and of the neutralons N already introduced in Sec. IV is given in Table VII. The transitions between the preons that are generated by these dipreonic bosons are graphically depicted in Fig. 4.

A. Nucleon decay processes

Nucleon decay into leptons is mediated by six-preon bound states and in an analogous way by dipreon bound states. For example, in proton decay $p(uud) \rightarrow e^+ + \gamma$ the preons in the two u-quarks combine into an intermediate X-boson which then decays by rearrangement into an $\bar{d}$ quark and an e^+

TABLE VI. Quantum numbers of the six-preon GUT gauge bosons X and Y responsible for baryon decay into leptons, and of the new GUT bosons $\tilde{X}$ and $\tilde{Y}$ mediating hyperbaryon decay into leptons.

state	content	$\mathcal{P}$	Υ	Q	Π
X	$(4T, 2V)$	$+2$	$+\frac{2}{3}$	$+\frac{4}{3}$	$+$
$\bar{X}$	$(4\bar{T}, 2\bar{V})$	-2	$-\frac{2}{3}$	$-\frac{4}{3}$	$+$
Y	$(2\bar{T}, 4\bar{V})$	-2	$+\frac{2}{3}$	$-\frac{2}{3}$	$+$
$\bar{Y}$	$(2T, 4V)$	$+2$	$-\frac{2}{3}$	$+\frac{2}{3}$	$+$
$\tilde{X}$	$(4T, 2\bar{V})$	$+\frac{2}{3}$	$+2$	$+\frac{4}{3}$	$+$
$\bar{\tilde{X}}$	$(4\bar{T}, 2V)$	$-\frac{2}{3}$	-2	$-\frac{4}{3}$	$+$
$\tilde{Y}$	$(2\bar{T}, 4V)$	$+\frac{2}{3}$	-2	$-\frac{2}{3}$	$+$
$\bar{\tilde{Y}}$	$(2T, 4\bar{V})$	$-\frac{2}{3}$	$+2$	$+\frac{2}{3}$	$+$

$$u(TTV) + u(TTV) \rightarrow X\begin{pmatrix} TTV \\ TTV \end{pmatrix} \rightarrow \bar{d}(TVV) + e^{+}(TTT). \tag{5.4}$$

The remaining d-quark in the proton annihilates with the final state $\bar{d}$-quark into a photon. An analogous proton decay process of a (ud) quark pair in the proton involves the dipreon U of Eq. (5.2)

$$u(TTV) + d(\bar{T}\,\bar{V}\,\bar{V}) \rightarrow U(T\bar{V}) \rightarrow \bar{u}(\bar{T}\,\bar{T}\,\bar{V}) + e^{+}(TTT), \tag{5.5}$$

where the final state is generated by creating two additional $T\bar{T}$ pairs from the vacuum. Note that the preon triality rule of Eq. (3.7) is broken at the grand unification scale.

Similarly, neutron decay $n \rightarrow \bar{\nu} + \gamma$ proceeds either by rearrangement

$$d(\bar{T}\,\bar{V}\,\bar{V}) + d(\bar{T}\,\bar{V}\,\bar{V}) \rightarrow Y\begin{pmatrix} \bar{T}\,\bar{V}\,\bar{V} \\ \bar{T}\,\bar{V}\,\bar{V} \end{pmatrix} \rightarrow \bar{u}(\bar{T}\,\bar{T}\,\bar{V}) + \bar{\nu}(\bar{V}\,\bar{V}\,\bar{V}), \tag{5.6}$$

or via the dipreon U

$$u(TTV) + d(\bar{T}\,\bar{V}\,\bar{V}) \rightarrow U(T\bar{V}) \rightarrow \bar{d}(TVV) + \bar{\nu}(\bar{V}\,\bar{V}\,\bar{V}) \tag{5.7}$$

requiring the creation of two $V\bar{V}$ pairs from the vacuum. Thus, these nucleon decay processes can be described by

TABLE VII. Dipreon bound states and their quantum numbers.

dipreon	$\mathcal{P}$	Υ	Q	Π
$N(VV)$	$+\frac{2}{3}$	$-\frac{2}{3}$	0	$+$
$\bar{N}(\bar{V}\,\bar{V})$	$-\frac{2}{3}$	$+\frac{2}{3}$	0	$+$
$U(T\bar{V})$	0	$+\frac{2}{3}$	$+\frac{1}{3}$	$+$
$\bar{U}(\bar{T}V)$	0	$-\frac{2}{3}$	$-\frac{1}{3}$	$+$
$\tilde{U}(TV)$	$+\frac{2}{3}$	0	$+\frac{1}{3}$	$-$
$\bar{\tilde{U}}(\bar{T}\,\bar{V})$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	$-$

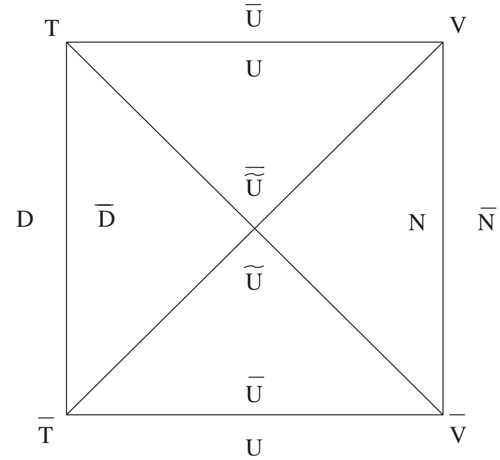

FIG. 4. Preon square. The two fundamental preons (T, V) and their antiparticles are placed at the corners of a square. The dipreon bound states and their antiparticles describing transitions between preons are placed along the edges (N, U) and diagonals $(\tilde{U})$. The dipreon states $D(TT)$ connecting T and $\bar{T}$ can be represented as a linear combination of $\tilde{U}$ and U or U and N. Thus they are not included as independent gauge bosons in Table VII.

six-preon bound states X and Y, as well as by the dipreon bound state $U(T\bar{V})$.

In analogy, hypernucleon decay processes involve the corresponding $\tilde{X}$, $\tilde{Y}$, and $\tilde{U}(TV)$ bosons as discussed before and are obtained from Eq. (5.4), (5.5), (5.6), and (5.7) by hyperquark transformation. Common to these processes is that they cause a simultaneous violation of baryon- and lepton-numbers ($\Delta B = \Delta L = -1$), whereas $\mathcal{P}$ and Υ numbers are conserved. There are also $B - L$ violating proton decays [14] such as $p \rightarrow \nu + \pi^{+}$ which can be thought of as being generated from the $B - L$ conserving $p \rightarrow \bar{\nu} + \pi^{+}$ decay via a $\nu - \bar{\nu}$ oscillation as discussed in sect. III D.

For completeness we mention that between the color triplet six-preon bound states X, Y and the dipreonic U gauge bosons there are the following weak transition processes

$$\begin{aligned} X &\rightarrow U + W^{+}; \qquad Y \rightarrow U + W^{-} \\ \tilde{X} &\rightarrow \tilde{U} + \tilde{W}^{+}; \qquad \tilde{Y} \rightarrow \tilde{U} + \tilde{W}^{-}. \end{aligned} \tag{5.8}$$

The reactions in the second line are generated by applying the hyperquark transformation to the reactions in the first line, emphasizing once again that the hyperquark transformation is applicable to all preon bound states containing neutral V preons.

B. Grand unification group $SU(9)_G$

After having discussed the additional bosons needed to enable transitions from quarks and hyperquarks to leptons, we can now count the number of gauge bosons involved. We find that the unitary group has to be as large as SU(9) to accommodate all gauge bosons that have been introduced. The corresponding 9×9 matrix of gauge bosons is schematically shown below

$$G_G = \begin{pmatrix} G_C & XYU & \tilde{X}\,\tilde{Y}\,\tilde{U} \\ \bar{X}\,\bar{Y}\,\bar{U} & G_H & N \\ \bar{\tilde{X}}\,\bar{\tilde{Y}}\,\bar{\tilde{U}} & \bar{N} & \begin{pmatrix} W_R^0 & W_R^+ & \tilde{W}^+ \\ W_R^- & W_L^0 & W_L^+ \\ \tilde{W}^- & W_L^- & B^0 \end{pmatrix} \end{pmatrix}.$$

This scheme includes the 6×6 matrix of the $\mathrm{SU}(6)_P$ partial unification group G_P of Eq. (4.16), the 3×3 matrix of the QCD gluons G_C transforming according to $\mathrm{SU}(3)_C$, four blocks of 3×3 matrices for the colored GUT bosons X, Y, and U, their antiparticles, as well as their hyperquark transformed states. In addition, we have to include the elementary photon associated with the group $\mathrm{U}(1)_Q$. The generator A_Q appears as an admixture in the diagonal entries and must be included in the counting leading to 80 generators in total.

Representations for the fermionic preon bound states arise from the direct product of three fundamental representations of dimension **9**, where the **9** is due to the three color and three hypercolor degrees of freedom associated with each preon. Note that the preon types T and V need not be included as a separate degree of freedom, i.e. we do not have an SU(18) because specifying the color and hypercolor representations uniquely specifies the preon type (see Table I).

We then have $\mathbf{9} \otimes \mathbf{9} \otimes \mathbf{9} = \mathbf{84} \oplus \mathbf{240} \oplus \mathbf{240} \oplus \mathbf{165}$, where only the lowest dimensional (antisymmetric) **84** is needed to represent the three generations of leptons, quarks, and hyperquarks, as well as their antiparticles:

$$\left(\begin{pmatrix} \nu & \bar{\nu} \\ e^- & e^+ \end{pmatrix}\begin{pmatrix} u & \bar{u} \\ d & \bar{d} \end{pmatrix}_C\begin{pmatrix} \tilde{u} & \bar{\tilde{u}} \\ \tilde{d} & \bar{\tilde{d}} \end{pmatrix}_H\right)_G. \tag{5.9}$$

The **84** dimensional fermion representation decomposes as follows: 12 leptons, 36 quarks, and 36 hyperquarks, where the multiplicities of color (C), hypercolor (H), and generation number (G) for the bound states are included.

VI. RUNNING COUPLINGS AND ENERGY SCALES

On the basis of the model described so far, we discuss in this section the momentum dependence of the couplings $\alpha(q^2)$ described in Sec. IV, where q^2 is the momentum transfer exchanged in the interaction. Our aim is to determine the scale, where the lightest hyperquark bound states appear, that is the mass scale Λ_H of hyperchromodynamics, where α_H is of order $\mathcal{O}(1)$. This is analogous to QCD where the lightest quark bound states (pions) appear at Λ_C, i.e. the scale where α_C is of order $\mathcal{O}(1)$.

In analogy to QCD, $\alpha_H(q^2)$ will decrease with increasing q^2 and the slope b_H of this decrease depends on the number of hyperquark flavors along the way to higher energies in the same manner as the slope b_C of $\alpha_C(q^2)$ depends on the number of quark flavors appearing with increasing q^2. It should be clear that we cannot determine the exact position of each single hyperquark flavor but only an average value of these hyperquark masses denoted as m_{hq}.

To fix the high q^2 end of the running couplings we use the constraints defining partial and grand unification of Eq. (4.15) and (5.1) respectively as detailed in the next section, where we also study how the β-functions of the different effective gauge groups change when Q^2 crosses the hyperquark mass scale.

A. β-functions of the SU(N) and U(1) gauge groups

In non-Abelian gauge theories the slope of the running couplings is determined to first order by the β-function b_i

$$b_i = \frac{11}{3}N_i - \frac{2}{3}n_f. \tag{6.1}$$

where the index i stands for the different gauge groups, N_i is number of degrees of freedom of the $\mathrm{SU}(N_i)$ group, and n_f is the number of fermion flavors. For the running coupling one has in one-loop approximation [15]

$$\frac{1}{\alpha_i(q^2)} = \frac{1}{\alpha_i(\Lambda_i^2)} + \frac{b_i}{4\pi}\ln\left(\frac{q^2}{\Lambda_i^2}\right). \tag{6.2}$$

For the present purposes higher order approximations of α_i can be neglected. Equation (6.2) connects an *a priori* unknown high energy scale $q^2 = M^2$ to a known low-energy scale Λ^2 such as, for example, Λ_C for QCD. Constraints for the high energy scale are obtained from the equality of certain coupling constants at this mass scale.

1. The β-functions of $\mathrm{SU}(3)_C$ *and* $\mathrm{SU}(3)_H$

For the non-Abelian groups $\mathrm{SU}(3)_C$ and $\mathrm{SU}(3)_H$ and where $N_C = N_H = 3$ we have

$$b_C = \tfrac{11}{3}N_C - \tfrac{2}{3}(n_d + n_u) \qquad b_H = \tfrac{11}{3}N_H - \tfrac{2}{3}(n_{\tilde{d}} + n_{\tilde{u}}). \tag{6.3}$$

Because there are three generations for each fermionic bound state type [4]

$$n_d = n_u = n_{\tilde{d}} = n_{\tilde{u}} = 3, \tag{6.4}$$

we obtain for both gauge groups the same b value

$$b_C = b_H = 7. \tag{6.5}$$

2. The β-function of $\mathrm{U}(1)_Q$

The running coupling constant of QED reads

$$\frac{1}{\alpha_Q(q^2)} = \frac{1}{\alpha_Q(\Lambda_Q)} + \frac{b_Q}{4\pi}\ln\left(\frac{q^2}{\Lambda_Q^2}\right). \tag{6.6}$$

Here, b_Q is defined as

$$b_Q = -\frac{2}{3}\sum_i Q_i^2, \tag{6.7}$$

where Q_i^2 is the square of the fermion charge (see Table IV), and the sum extends over all fermions. Thus, the β-function of QED is

$$b_Q = -\frac{2}{3}2\left(n_e + 3\left(\frac{1}{9}n_d + \frac{4}{9}n_u\right) + 3\left(\frac{1}{9}n_{\tilde{d}} + \frac{4}{9}n_{\tilde{u}}\right)\right), \tag{6.8}$$

where $n_e = 3$ is the number of leptonic flavors. The identical contributions of the left- and right-handed fermion charges are taken into account by an overall factor 2, and the color and hypercolor multiplicities are indicated by factors 3 multiplying the quark and hyperquark contributions.

At the hyperquark scale m_{hq} the β-function b_Q changes

$$\begin{aligned} b_Q &= -\frac{32}{3}(\text{without hyperquarks}),\\ b_Q &= -\frac{52}{3}(\text{with hyperquarks}) \end{aligned} \tag{6.9}$$

due to the additional contribution of the hyperquarks.

3. Electroweak β-functions for $\mathrm{U}(1)_Y$ and $\mathrm{SU}(2)_W$

For the $\mathrm{U}(1)_Y$ group we replace in Eq. (6.6) α_Q by α_Y and b_Q by b_Y which is defined as

$$b_Y = -\frac{2}{5}\sum_i Y_i^2, \tag{6.10}$$

where the sum extends over the fermionic preon bound states. In addition, we have to take into account that left- and right-handed fermions give different contributions. As noted before hyperquarks are weak isospin singlets ($T_3 = 0$) and therefore have a left-right symmetric coupling, whereas quarks and leptons couple left-right asymmetrically.

We then obtain for the β-function according to Table IV

$$\begin{aligned} b_Y = -\frac{2}{5}\bigg(&\frac{1}{4}n_\nu + \frac{5}{4}n_e + 3\left(\frac{5}{36}n_d + \frac{17}{36}n_u\right)\\ &+ 3\left(\frac{2}{9}n_{\tilde{d}} + \frac{8}{9}n_{\tilde{u}}\right)\bigg), \end{aligned} \tag{6.11}$$

with $n_\nu = 3$ and where the factor in front of the n_i come from adding left- and right-handed contributions in Eq. (4.8). Note that b_Y changes at the hyperquark mass scale m_{hq} due to the contribution of the hyperquarks as

$$\begin{aligned} b_Y &= -4(\text{without hyperquarks}),\\ b_Y &= -8(\text{with hyperquarks}). \end{aligned} \tag{6.12}$$

For the non-Abelian weak isospin group $\mathrm{SU}(2)_W$ the corresponding β-function reads

$$b_W = \frac{11}{3}N - \frac{1}{3}(n_\nu + n_e + n_d + n_u) \tag{6.13}$$

where $N = 2$. Here, only left-handed leptons and quarks contribute and we obtain the standard model value $b_W = \frac{10}{3}$.

4. The β-function of the partial unification group SU(6)

Here, we deal with the non-Abelian gauge group SU(6) as discussed in Sec. IV. At the M_P scale the Majorana description of the neutrinos implies that the right-handed neutrinos are identical with antineutrinos, i.e., $\nu_R \equiv \bar{\nu}$. Therefore, only the left-handed neutrinos are included in the summation over the left- and right-handed fermions

$$b_P = \tfrac{11}{3}N_P - \tfrac{1}{3}(n_\nu + 2(n_e + n_d + n_u + n_{\tilde{d}} + n_{\tilde{u}})), \tag{6.14}$$

which gives $b_P = 11$ for $N_P = 6$.

We point out that two-state Majorana neutrinos provide the only consistent description in the present framework. A four-state Dirac neutrino would reduce the β-function to $b_P = 10$. As will be seen in Sec. VI B, the latter would lead to inacceptable results for the energy scales M_G, M_P, and m_{hq}. In particular, it would imply a hyperquark mass scale below the masses of W and Z bosons, and a much too short proton lifetime, both of which are in contradiction to experimental facts.

5. The β-function of the grand unification group SU(9)

At this scale the different bosons can be accommodated into a larger gauge group SU(9). By an analogous counting of the fermions as in Eq. (6.14) we obtain

$$b_G = \tfrac{11}{3}N_G - \tfrac{1}{3}(n_\nu + 2(n_e + n_d + n_u + n_{\tilde{d}} + n_{\tilde{u}})) \tag{6.15}$$

which gives $b_G = 22$ for $N_G = 9$.

B. Calculation of energy scales

To obtain numerical values for M_G, M_P, and m_{hq} one starts from the following constraints. First, from Eq. (4.12) we have at M_P

$$\begin{aligned} \frac{1}{\alpha_W(M_P^2)} &= \frac{1}{\alpha_Y(M_P^2)}\\ \frac{1}{\alpha_W(m_t^2)} + \frac{b_W}{4\pi}\ln\left(\frac{M_P^2}{m_t^2}\right) &= \frac{1}{\alpha_Y(m_t^2)} + \frac{b_{Yt}}{4\pi}\ln\left(\frac{m_{\mathrm{hq}}^2}{m_t^2}\right)\\ &\quad + \frac{b_Y}{4\pi}\ln\left(\frac{M_P^2}{m_{\mathrm{hq}}^2}\right), \end{aligned} \tag{6.16}$$

where the evolution starts at the top quark mass m_t. According to Eq. (6.2) the first two terms on the right-hand side can be written as $1/\alpha_Y(m_{\mathrm{hq}})$, and the third term on the right-hand side describes the evolution from m_{hq}, at which point we have to include the hyperquarks to the partial unification scale M_P.

Second, from Eq. (4.14) and (5.1) evaluated at M_G follows

$$\frac{1}{\alpha_C(M_G^2)} = \frac{1}{\alpha_P(M_G^2)}$$
$$\frac{1}{\alpha_C(m_t^2)} + \frac{b_C}{4\pi}\ln\left(\frac{M_G^2}{m_t^2}\right) = \frac{1}{2}\left(\frac{1}{\alpha_W(m_t^2)} + \frac{b_W}{4\pi}\ln\left(\frac{M_P^2}{m_t^2}\right)\right) + \frac{b_P}{4\pi}\ln\left(\frac{M_G^2}{M_P^2}\right), \tag{6.17}$$

where the factor of 2 in the denominator of the first term on the right-hand side comes from the strength doubling of α_W at the partial unification scale according to Eq. (4.15).

Third, from Eq. (5.1) also follows

$$\frac{1}{\alpha_C(M_G^2)} = \frac{1}{\alpha_Q(M_G^2)}$$
$$\frac{1}{\alpha_C(m_t^2)} + \frac{b_C}{4\pi}\ln\left(\frac{M_G^2}{m_t^2}\right) = \frac{1}{\alpha_Q(m_t^2)} + \frac{b_Q}{4\pi}\ln\left(\frac{m_{\rm hq}^2}{m_t^2}\right) + \frac{b_Q}{4\pi}\ln\left(\frac{M_G^2}{m_{\rm hq}^2}\right). \tag{6.18}$$

The energy scales where these conditions hold can now be determined by solving the above three Eqs. (6.16), (6.17), and (6.18) with three unknowns. This gives

$$m_{\rm hq} = 2.6\times 10^4\ \text{GeV} \qquad M_P = 8.3\times 10^9\ \text{GeV} \qquad M_G = 1.2\times 10^{16}\ \text{GeV}. \tag{6.19}$$

In order to calculate the low-energy scale Λ_H we use Eq. (6.2) and start the evolution at the partial unification scale M_P where according to Eq. (4.15) the following equality of coupling constants holds $\alpha_P = \alpha_H = 2\alpha_W$. By backextrapolation we obtain

$$\frac{1}{\alpha_H(\Lambda_H^2)} = \frac{1}{\alpha_H(M_P^2)} - \frac{b_H}{4\pi}\ln\left(\frac{M_P^2}{m_{\rm hq}^2}\right) - \frac{b_{H1}}{4\pi}\ln\left(\frac{m_{\rm hq}^2}{\Lambda_H^2}\right), \tag{6.20}$$

where $b_{H1} = \frac{31}{3}$ is calculated according to Eq. (6.3) assuming $n_{\tilde{d}} = 1$ and $n_{\tilde{u}} = 0$ for the contribution of the lightest hyperquark below the average scale $m_{\rm hq}$. The slope b_{H1} corresponds to the dotted line segment of α_H in the left upper corner of Fig. 5. We then demand $\alpha_H(\Lambda_H^2) \cong \mathcal{O}(1)$ which, using the numerical values in Eq. (6.19), leads to a prediction of the infrared cutoff mass of hypercolor interactions

$$\Lambda_H \cong 1700\ \text{GeV}. \tag{6.21}$$

This is different from the model of Harari-Seiberg which gives a Λ_H of order 10^9 GeV. The numerical values of the β-functions, coupling constants, and energy scales calculated in this section are compiled in Table VIII.

With the constraint $m_{\rm hq1} \geq \Lambda_H$ where $m_{\rm hq1}$ denotes the mass of the lightest hyperquark, we obtain for the weak decay lifetime of hyperquark bound states, e.g., a hyperpion

$$\tau_{\tilde{\pi}} \cong \frac{1}{\alpha_P^2}\frac{M_P^4}{m_{\rm hq1}^5} \leq 100\ \text{s}. \tag{6.22}$$

For the neutrino masses according we obtain according to Eq. (3.12)

$$m_{\nu_e} = 3.3\times 10^{-8}\ \text{eV}, \qquad m_{\nu_\mu} = 1.3\times 10^{-3}\ \text{eV}, \qquad m_{\nu_\tau} = 3.8\times 10^{-1}\ \text{eV} \tag{6.23}$$

compared to $m_{\nu_\mu} = 8.8\times 10^{-3}$ eV and $m_{\nu_\tau} = 5.0\times 10^{-2}$ eV obtained from upper limits of experimental neutrino squared mass differences [16].

Furthermore, the large value of the $SU(9)_G$ boson masses $M_G = 1.2\times 10^{16}$ GeV results in the following value for the proton lifetime

$$\tau_p \cong \frac{1}{\alpha_G^2}\frac{M_G^4}{m_p^5} \cong 10^{35}\ \text{y}, \tag{6.24}$$

where $m_p = 938.3$ MeV is the proton mass. This is in good agreement with the experimental lower limit $\tau_p > 5.5\times 10^{33}$ y [17].

We close this section with a remark on the evolution of α_G for still higher momentum transfers. The grand unification scale M_G calculated here is near the Planck scale of quantum gravity $M_{\rm Pl}$ given by

$$M_{\rm Pl}^2 = \frac{\hbar c}{\gamma_G}, \qquad \alpha_g = \frac{q^2}{M_{\rm Pl}^2}, \tag{6.25}$$

where γ_G is the Newton gravitational constant. The coupling α_g is the only nonrenormalized coupling, having a linear dependence on the momentum transfer q^2.

Explicit preon degrees of freedom will become important at the fundamental preon scale defined by the constraint $\alpha_G(M_{\rm pr}^2) = \alpha_g(M_{\rm pr}^2)$. We can write

$$\frac{1}{\alpha_G(M_G^2)} + \frac{b_G}{4\pi}\ln\left(\frac{M_{\rm pr}^2}{M_G^2}\right) = \frac{M_{\rm Pl}^2}{M_{\rm pr}^2}. \tag{6.26}$$

From this constraint we obtain using $M_{\rm Pl} = 1.22\times 10^{19}$ GeV for the Planck scale

$$M_{\rm pr} = 1.56\times 10^{18}\ \text{GeV} \tag{6.27}$$

for the scale $M_{\rm pr}$ where an asymptotically free preon dynamics is expected.

C. Hyperquark and hyperhadron mass scales

We are now in the position to make statements about the energy scale where the hyperquarks and their bound states presumably occur. As mentioned before, it is reasonable to assume that the new particles do not all appear at the same

TABLE VIII. Numerical values of β functions and coupling constants at the corresponding energy scales. The indicated errors are due to the experimental input values.

$M_Z = 91.18 \pm 0.02$ GeV			
SU(3)$_C$	SU(2)$_W$	U(1)$_Y$	U(1)$_Q$
$b_{Cz} = \frac{23}{3}$	$b_{Wz} = \frac{11}{3}$	$b_{Yz} = -\frac{103}{30}$	$b_{Qz} = -\frac{80}{9}$
$\alpha_{Cz}^{-1} = 8.24(12)$	$\alpha_{Wz}^{-1} = 29.57(05)$	$\alpha_{Yz}^{-1} = 59.00(04)$	$\alpha_{Qz}^{-1} = 127.90(02)$
$m_t = 176.9 \pm 4.0$ GeV			
$b_C = 7$	$b_W = \frac{10}{3}$	$b_{Yt} = -4$	$b_{Qt} = -\frac{32}{3}$
$\alpha_{Ct}^{-1} = 9.05(15)$	$\alpha_{Wt}^{-1} = 29.96(06)$	$\alpha_{Yt}^{-1} = 58.64(05)$	$\alpha_{Qt}^{-1} = 126.96(05)$

$m_{\rm hq} = (26.3 \pm 2.3) \times 10^3$ GeV ($\Lambda_H = (1.66 \pm 0.34) \times 10^3$ GeV)				
SU(3)$_H$	SU(3)$_C$	SU(2)$_W$	U(1)$_Y$	U(1)$_Q$
$b_H = 7$	$b_C = 7$	$b_W = \frac{10}{3}$	$b_Y = -8$	$b_Q = -\frac{52}{3}$
$\alpha_{Hh}^{-1} = 5.58(20)$	$\alpha_{Ch}^{-1} = 14.63(18)$	$\alpha_{Wh}^{-1} = 32.62(08)$	$\alpha_{Yh}^{-1} = 55.45(07)$	$\alpha_{Qh}^{-1} = 118.47(09)$

$M_P = (8.31 \pm 0.88) \times 10^9$ GeV		
SU(6)$_P$	SU(3)$_C$	U(1)$_Q$
$b_P = 11$	$b_C = 7$	$b_Q = -\frac{52}{3}$
$\alpha_P^{-1} = 19.66(08)$	$\alpha_{Cp}^{-1} = 28.73(18)$	$\alpha_{Qp}^{-1} = 83.57(09)$

$M_G = (1.17 \pm 0.12) \times 10^{16}$ GeV
SU(9)$_G$
$b_G = 22$
$\alpha_G^{-1} = 44.48(27)$

$M_{\rm pr} = (1.56 \pm 0.01) \times 10^{18}$ GeV
$\alpha_g^{-1} = 61.20(37)$

threshold level but that their mass values are distributed over a certain range. Thus, the present result $m_{\rm hq} = 26$ TeV provides an average energy level but gives no information concerning the size of the mass range over which the particles are distributed. However, because of the missing isospin degree of freedom the spectrum of hyperhadrons is expected to be much wider than that of ordinary hadrons.

For quark bound states we have the result $m_\pi \sim \Lambda_C$. Analogously, for the bound states of hyperquarks we have $m_{\tilde{\pi}} \sim \Lambda_H$. This leads to the following estimate for the hyperhadron mass

$$m_{\tilde{h}} \cong \frac{\Lambda_H}{\Lambda_C} m_h. \tag{6.28}$$

With the numerical values $m_\pi = 140$ MeV, $\Lambda_C = 100$ MeV, and $\Lambda_H = 1700$ GeV, we arrive at a hyperpion mass $m_{\tilde{\pi}} = 1100$ GeV. Similarly, using the proton mass $m_p = 938$ MeV as input we find hyperbaryon masses to be of order $m_{\tilde{B}} \cong 10^4$ GeV. These results are compiled in Table IX.

We conclude this section with some remarks pertaining to the hyperquark and hyperhadron masses and lifetimes. Because hyperquarks $\tilde{d}$ and $\tilde{u}$ have different intrinsic parities they do not form strong isospin doublets. Thus, unlike the quarks of the first generation their masses need not be close to each other. For the lightest hyperquark ($\tilde{d}$) we find a constituent mass of order $m_{\tilde{d}} \cong \Lambda_H \cong 1700$ GeV. The mass of the hyperquark ($\tilde{u}$) could be substantially higher, and is conjectured to be of order $(m_{\tilde{d}} + \Lambda_H) \cong 3400$ GeV. Consequently, unlike the standard model pions, hyperpions do not form an isospin triplet and we expect that charged hyperpions are heavier than neutral ones. Furthermore, because of the missing isospin degree of freedom, the lowest-lying hyperhadrons have a symmetric spin configuration corresponding to spin $\frac{3}{2}$ for hyperbaryons and spin 1 for hypermesons.

While neutral hyperquark bound states can decay electromagnetically, the charged ones can only decay via $\tilde{W}$ emission as discussed in sect. III A. Therefore, the lifetimes of the charged hyperquark bound states are according to Eq. (6.22) with $\tau_{\rm hq} \lesssim 100$ s substantially longer than the

TABLE IX. The H_0 boson and low lying hyperhadrons.

boson	spin	Π	mass [GeV]	lifetime [s]	decay-mode
H^0	0	+	$\cong$ 350–500	10^{-26}	Z, W^+W^-, $f\bar{f}$
$\tilde{\pi}^0$	1	−	$\cong$ 1100	10^{-27}	H^0,Z, γ
$\tilde{\pi}^\pm$	1	+	$\approx$ 3000	$\lesssim 100$	$f\bar{f}$
$\tilde{\Delta}^-$	$\frac{3}{2}$	+	$\approx 10^4$	$\lesssim 1$	Δ^-
$\tilde{\Delta}^{++}$	$\frac{3}{2}$	−	$\approx 10^4$	$\lesssim 1$	Δ^{++}

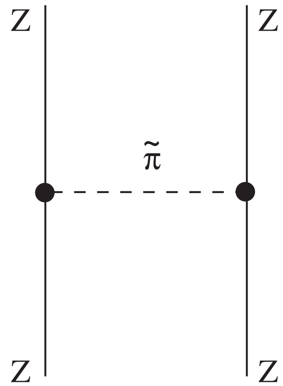

FIG. 6. ZZ bound state H^0 generated via $\tilde{\pi}$ exchange.

lifetimes of the neutral ones. As to the experimental signature, a neutral hyperpion decays electromagnetically into two photons with lifetime $\tau_{\tilde{\pi}} = \hbar/\Gamma_{\tilde{\pi}} \leq 10^{-27}$ s if one assumes a decay width $\Gamma_{\tilde{\pi}} = 800$ GeV derived from $\Gamma_{\tilde{h}} \cong \frac{\Lambda_H}{\Lambda_C}\Gamma_h$ in analogy to Eq. (6.28).

Hypermeson masses of order TeV correspond to a short-range Yukawa coupling interaction with range $\hbar/m_{\tilde{\pi}}c^2 \cong 10^{-19}$ m. At energies of several tens of TeV, hyperquarks couple only to the Z boson and the photon. As a result, there is a coupling of the neutral hyperpion to the Z boson. This short-range interaction could lead to the formation of a scalar ZZ bound state with spin 0, called H^0, as shown in Fig. 6, which could be a viable candidate for solving the unitarity problem in W-W-scattering [18]. For the H^0 boson mass and decay width an estimate according to Ref. [18] using $\lambda = G_f m_{H^0}^2/\sqrt{2}$, where $G_f^{-1/2} \cong 293$ GeV is the Fermi constant and $1 \leq \lambda \leq 2$ gives $m_{H^0} \cong 350$–500 GeV and $\Gamma_{H^0} \cong 40$ GeV corresponding to the lifetime $\tau_{H^0} = \hbar/\Gamma_{H^0} \cong 10^{-26}$ s.

VII. SUMMARY AND OUTLOOK

The standard model leaves many questions unanswered; for example, why leptons and quarks share the same weak interaction, mediated by heavy vector bosons coupling differently to left- and right-handed quarks and leptons. This fact, among others, points to a deeper connection between leptons and quarks. In the preon model, weak interactions are qualitatively understood as a residual force that is associated with the preon number of the bound state. On the other hand, a model which contains only the unbroken gauge interactions $SU(3)_H \times SU(3)_C \times U(1)_Q$ between preons does not readily lend itself to a quantitative description of the left-right asymmetric weak interactions between preon bound states at low energies. Furthermore, it does not provide a dynamical explanation for certain new phenomena predicted by the present theory, for example, the decay of hyperquarks into quarks. Therefore, we have attempted to make some progress by considering approximate effective gauge theories on the level of preon bound states.

In particular, we have investigated the bosonic sector of the preon model in some detail. We have shown that the introduction of hyperquarks in Harari's theory requires new classes of effective gauge bosons, called $\tilde{W}$ and N in order to describe weak transitions among hyperquarks and between hyperquarks and quarks. The presence of these additional gauge bosons leads to an extension of the standard $SU(2)_{W_L} \times U(1)_Y$ electroweak theory to a larger gauge group emerging at an energy $M_P \cong 10^9$ GeV. At this scale, 9 left-right symmetric weak bosons, 8 hypergluons, and 18 neutralons provide the 35 generators of an effective $SU(6)_P$ gauge group, refered to as partial unification group. This scheme predicts a Weinberg angle $\sin^2\Theta_W = 6/13$ at $M_P \cong 10^9$ GeV, which is twice its experimental value at $M_Z = 91.2$ GeV. Furthermore, it shows that the breaking of left-right $SU(2)_{W_L} \times SU(2)_{W_R}$ symmetry into the standard model symmetry $SU(2)_{W_L} \times U(1)_Y$ is closely connected with the production and decay of hyperquarks and thus to some extent explained in the present model.

At the grand unification scale $M_G \cong 10^{16}$ GeV the spectrum of bosonic preon bound states is much larger. In addition to the usual GUT bosons, which provide transitions between quarks and leptons as in proton decay processes, several new gauge bosons appear, the counting of which leads to the grand unification group $SU(9)_G$. Next to the 35 gauge bosons of $SU(6)_P$, there are 8 gluons of $SU(3)_C$, 18 colored bosons (X, Y, and U and their antiparticles), 18 hypercolored bosons ($\tilde{X}$, $\tilde{Y}$, and $\tilde{U}$ and their antiparticles), and the photon A_Q of $U(1)_Q$, altogether 80 generators of $SU(9)_G$.

The hyperquark mass scale has been found from the dimension of these unified gauge groups, the number of fermionic bound states, and the requirement that the coupling constants of the various effective gauge interactions are equal at the two unification scales. These constraints lead to a system of three equations with three unknowns, allowing the determination of both unification scales $M_P \cong 10^9$ GeV and $M_G \cong 10^{16}$ GeV, and the average hyperquark mass scale $m_{\mathrm{hq}} \cong 10^4$ GeV. To obtain a mass constraint for the lightest hyperquark bound state, the hyperpion, we have extrapolated from the point m_{hq} to the point Λ_H where $\alpha_H \cong \mathcal{O}(1)$ and obtained $\Lambda_H \cong 1700$ GeV, as the typical scale for hyperhadron masses, which is within reach of the LHC at CERN.

Finally, from the unification constraints mentioned above, the Majorana description of neutrinos is preferred. For Dirac neutrinos the present theory leads to a proton lifetime which is too short, and furthermore to a hyperquark mass scale below the masses of W and Z bosons, both results contradicting experimental facts.

In summary, based on the introduction of hyperquarks as a new class of fermionic bound states we have depicted a scenario for the unification of forces that partly explains the left-right asymmetry of weak interactions at low energies and fills the large gap between the Fermi scale and partial unification scale with a wide spectrum of hyperquark bound states.

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