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Protecting unparticles from the MSSM Higgs sector

Ann E. Nelson,[1,2,*] Maurizio Piai,[1,3,†] and Christopher Spitzer[1,2,‡]

[1]*Department of Physics, University of Washington, Seattle, Washington 98195-1560, USA*
[2]*Instituto de Fisica Teorica, IFT-UAM/CSIC, Facultad de Ciencias, C-XVI, Universidad Autónoma de Madrid, Cantoblanco, Madrid 28049, Spain*
[3]*Swansea University, School of Physical Sciences, Singleton Park, Swansea, Wales, United Kingdom*
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We construct a model of an unparticle sector consisting of a supersymmetric $SU(N)$ gauge theory with the number of flavors in the Seiberg conformal window. We couple this sector to the minimal supersymmetric standard model via heavy messengers. The resulting low energy theory has a Higgs coupling to unparticles. The Higgs vev drives the hidden Seiberg sector to a new conformal fixed point. The coupling to the Higgs mediates supersymmetry breaking to the Seiberg sector, and breaks conformal invariance at a lower scale. The low energy theory contains light stable and metastable mesons. Higgs decay into this sector gives signatures which are similar to those of ''hidden valley'' models. Decays of the lightest superpartner of standard model particles into the hidden sector reveal potentially observable unparticle kinematics.

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I. INTRODUCTION

There is a variety of compelling, albeit indirect, evidence for new physics beyond the minimal standard model. Such physics might conceivably remain undiscovered, even if produced in high energy collisions, as finding new physics generally requires understanding what experimental signatures to look for. There is a standard set of searches which have been developed for the traditional theories of physics beyond the standard model, such as weakly coupled supersymmetry (SUSY). It is however important to consider new physics models with experimental signatures which are as diverse as possible. Strassler and Zurek have emphasized the novel signals of ''hidden valleys''—new light sectors with strong self-interactions which are weakly coupled to standard model particles at low energies [1,2]. Similarly, Georgi has explored the unusual phenomenological signals of ''unparticles''—a hidden strongly coupled scale invariant sector which decouples from the standard model at low energy [3,4]. Although there is no theoretical necessity for these particular sectors, consideration of such models has the virtue of stimulating new analysis techniques and experimental searches, which are relevant for a broad class of theories.

Elucidating the observational features of new strongly coupled sectors is theoretically challenging. One approach, called ''deconstruction'' [5], replaces a strongly coupled scale invariant sector with a closely spaced discrete spectrum of weakly coupled scalars. Another approach is to use a five dimensional dual description [6]. Strassler has criticized these approaches as applying to only an extreme large N limit which may fail to capture all the phenomenologically important effects [7]. Sannino and Zwicky [8] considered an unparticle sector consisting of a gauge theory at a Banks-Zaks fixed point [9]. Another option is to consider strongly coupled supersymmetric theories in the Seiberg [10,11] conformal window, since supersymmetry can then be used to gain some theoretical knowledge of the dynamics in a realistic strongly coupled model [12]. Using this approach, Fox *et al.* argued that couplings between the unparticle sector and the Higgs sector would push the unparticle dynamics away from the infrared scale invariant fixed point, and introduce a mass scale of order the weak scale into the unparticle sector. The argument that a Higgs-unparticle coupling will drive the unparticle sector away from the conformal fixed point is rather generic [13,14]. In this paper, however, we will illustrate a different possibility, also using a supersymmetric unparticle sector with a coupling to the Higgs. In contrast with the approach of Fox *et al.*, we arrange the coupling to the Higgs in such a way that the approximate scale invariance of the unparticle sector survives down to an energy scale Λ which is parametrically lower than the weak scale. Furthermore, we argue that below the scale Λ the theory is driven towards a new superconformal fixed point. A different approximately scale invariant supersymmetric theory describes the dynamics between Λ and a lower scale f_s. At still lower energies, below f_s the theory is a nonsupersymmetric, confining QCD-like theory of light stable and metastable particles. We briefly discuss the possibilities for discovery of unparticles via the decay of the Higgs or the decay of the lightest superpartner of the minimal supersymmetric standard model (MSSM), and the effects of unparticles on Higgs discovery. We also briefly sketch some of the consequences of a dark matter candidate in the unparticle sector. We introduce this realistic, predictive model as a laboratory for illustrating and testing generic claims about unparticle phenomenology.

*anelson@phys.washington.edu
†m.piai@swansea.ac.uk
‡cspitzer@u.washington.edu

II. MODEL WITH A HIDDEN SUPERSYMMETRIC CONFORMAL SECTOR

We wish to use a strongly coupled field theory whose dynamics are well understood for our conformal hidden sector. As a simple, familiar example, we will assume a hidden $SU(N_c)$ gauge theory with N_f flavors of massless quarks. A variety of consistency checks [10] lead to the conclusion that supersymmetric $SU(N_c)$ gauge theory in the "conformal window" with N_f satisfying

$$3/2 < \frac{N_f}{N_c} < 3, \tag{1}$$

must run to a conformal infrared fixed point in the infrared. Furthermore, this fixed point has dual descriptions–the original "electric" description, which is weakly coupled for

$$\frac{3N_c - N_f}{N_c} \ll 1, \tag{2}$$

and a "magnetic" description, with gauge group $SU(N_f - N_c)$ and with the dual quarks coupled to a gauge singlet "meson" field. The magnetic description is weakly coupled for

$$\frac{N_f - \frac{3}{2}N_c}{N_c} \ll 1. \tag{3}$$

We extend the standard model to the MSSM with gauge mediated soft supersymmetry breaking [15–17] terms of order the weak scale. To couple the MSSM to the unparticles we introduce heavy gauge singlet superfields S and T.

The superpotential of the model in the magnetic description is

$$\mathcal{W} = W_{\rm MSSM} + \lambda \sum_{ij} \bar{q}_i A_{ij} q_j + \lambda_S S H_u H_d + \mu' ST + \sum_{ij} g_{ij} \mu'^{1-\gamma} S A_{ij}, \tag{4}$$

where $W_{\rm MSSM}$ is the superpotential of the MSSM, $H_{u,d}$ are the up and down type Higgs superfields, q_j, $\bar{q}_i$ are the unparticle quark and antiquark superfields, A_{ij} is the unparticle meson field, the index i runs from 1 to N_f, and we have suppressed the color indices. We assume the number of unparticle quark flavors is in the conformal window. In the absence of the deformations induced by the visible sector, there is an infrared attractive fixed point in the hidden sector, where the theory is described by the conformal theory based on operators with scaling dimension $d(A_{ij}) = 1 + \gamma$, and $d(\bar{q}_i q_j) = 2 - \gamma$, with γ constant. We assume that μ' is well above the weak scale, of order a few TeV, and that at this high scale the unparticle sector is nearly at the conformal fixed point. In our analysis we will assume that the matrix g_{ij} has nonvanishing entries only for $i, j = N_f - n + 1, \ldots, N_f$, all of the same order of magnitude (but different). Without loss of generality, we may use flavor transformations on the hidden sector fields to diagonalize g_{ij}, and make the diagonal entries real and positive. The factor of $\mu'^{1-\gamma}$ is inserted so that the g_{ij} are effectively dimensionless. Here the couplings g_{ij} are renormalized at the scale μ', and are assumed to be perturbative at this scale. We use field reparametrizations to make the coupling λ_S real and positive.

The approximate global flavor symmetry of the unparticle sector is explicitly broken by the g_{ii} couplings to a global $SU(N_f - n) \times SU(N_f - n) \times U(1)^n \times U(1)_B \times U(1)_R$. [The $U(1)_R$ symmetry is not exact when supersymmetry breaking effects are considered.]

A. Effective field theory analysis

We now discuss the low energy effective theory below the scale μ', at which we integrate out the S and T superfields. As mentioned, we assume that μ' is large compared with the weak scale and the MSSM SUSY breaking scale. We also assume that the mediation of SUSY breaking to the hidden sector occurs indirectly via the S and T couplings, and so the scale of SUSY breaking in the hidden sector is suppressed below the weak scale. We can therefore neglect supersymmetry breaking as we integrate them out. Note that the S and T fields only have weak couplings, and so it is a good approximation to integrate them out at tree level, by solving the classical equations which result from minimizing the potential. The equations of motion are

$$F_S = F_T = 0, \tag{5}$$

$$S = 0, \tag{6}$$

$$T = -\frac{1}{\mu'}\left(\lambda_S H_u H_d + \sum_i g_{ii} \mu'^{1-\gamma} A_{ii}\right). \tag{7}$$

The resulting effective theory has a superpotential

$$\mathcal{W} = W_{\rm MSSM} + \sum_{ij} \lambda \bar{q}_i A_{ij} q_j. \tag{8}$$

Unlike in the model of Fox *et al.*, there is now no direct superpotential coupling between the hidden and visible sectors. Note that this lack of direct coupling results from our inclusion of the T superfield with a nongeneric superpotential. The specific form of our superpotential is technically natural.

The low energy effective theory does couple the hidden and MSSM sectors, but only via terms in the effective Kahler potential. These terms induce an effective supersymmetric superpotential term in the hidden sector Lagrangian, in a manner reminiscent of the Giudice-Masiero [18] mechanism for generating the μ term in the MSSM. The additional terms arising at the leading order in an expansion in powers of $1/\mu'$ in the Kahler potential are obtained by replacing the solutions for T and S:

$$\begin{aligned}\mathcal{K}_{\text{eff}} &\supset S^\dagger S + T^\dagger T \\ &= \frac{1}{\mu'^2}\left|\lambda_S H_u H_d + \sum_i g_{ii}\mu'^{1-\gamma} A_{ii}\right|^2 \\ &= \left|\sum_i \frac{g_{ii}}{\mu'^\gamma} A_{ii}\right|^2 + \frac{\lambda_S^2}{\mu'^2}|H_u H_d|^2 + \sum_i \frac{\lambda_S g_{ii}}{\mu'^{1+\gamma}} \\ &\quad\times (H_u H_d A_{ii}^\dagger + H_u^\dagger H_d^\dagger A_{ii}).\end{aligned} \tag{9}$$

We now run this effective theory down to the weak scale $v \equiv 175$ GeV, which we take to also be of order the scale of SUSY breaking in the MSSM sector (the effective SUSY breaking scale in the hidden sector will be much smaller).

The superpotential terms do not run, since the MSSM terms are weakly coupled, while the hidden sector terms are at a fixed point. We do not know how the first term in Eq. (9) runs, but we do know that it is an irrelevant term with negligible effect on hidden sector dynamics at all scales below μ'. The second term only involves weakly coupled fields and does not run. The third term in Eq. (9), which is linear in the strongly coupled fields A_{ij}, has anomalous dimension γ. At the scale v this term becomes

$$\mathcal{K}_{\text{eff}} \supset \frac{v^\gamma}{\mu'^{1+2\gamma}} \sum_i (\lambda_S g_{ii} H_u H_d A_{ii}^\dagger + \lambda_S g_{ii} H_u^\dagger H_d^\dagger A_{ii}). \tag{10}$$

At the scale v we integrate out the Higgs superfields. These are weakly coupled fields, and we can integrate them out at tree level by setting the scalar and F components to their vacuum values. We define the parameters

$$\epsilon_{ii} \equiv \lambda_S g_{ii} \left(\frac{v}{\mu'}\right)^{1+2\gamma}, \tag{11}$$

which control the strength of quantum corrections in the hidden sector due to the coupling between the two sectors, and choose parameters such that $\epsilon_{ii} \ll 1$. Quantum loop corrections are suppressed by the ϵ coefficient squared, times a loop factor, in units of the scale v. To leading order in the ϵ parameters, the Higgs scalar and auxiliary field F term expectation values are independent of the hidden sector fields. Neglecting tree level and quantum contributions of order ϵ^2, the only terms from the MSSM sector that survive in Eq. (10) are proportional to $\langle F_{H_u H_d}\rangle$, defined below. (Because of the soft weak scale supersymmetry breaking in the MSSM sector, we do not set the F terms for the Higgs fields to zero.) Below the weak scale, we can then use the trick of Giudice and Masiero [18] to rewrite the term (10) as a contribution to the effective superpotential for the hidden sector

$$\mathcal{W}_{\text{eff}} = \lambda \sum_{ij} \bar{q}_i A_{ij} q_j + \sum_i \frac{\langle F_{H_u H_d}\rangle}{v^{1+\gamma}} \epsilon_{ii} A_{ii}, \tag{12}$$

where

$$\langle H_u \rangle = v_W \sin\beta, \tag{13}$$

$$\langle H_d \rangle = v_W \cos\beta, \tag{14}$$

$$\langle F_{H_u} \rangle = \mu_W v_W \cos\beta, \tag{15}$$

$$\langle F_{H_d} \rangle = \mu_W v_W \sin\beta \tag{16}$$

$$\langle F_{H_u H_d} \rangle = \langle H_u \rangle \langle F_{H_d} \rangle + \langle H_d \rangle \langle F_{H_u} \rangle, \tag{17}$$

and μ_W is the usual μ term of the MSSM. Remarkably, to leading order in the coupling between the hidden and visible sectors, the supersymmetry breaking in the MSSM leads to a term in the effective theory for the hidden sector which can be written as supersymmetric, although not scale invariant. Because this term is relevant, it will drive the hidden sector away from the fixed point in the infrared. However, because the theory is still effectively supersymmetric, we still have theoretical control over the dynamics [10]. In a supersymmetric theory with a supersymmetric vacuum, the vacuum may be found by setting the F terms derived from the effective superpotential to zero, along D-flat directions.

$$\langle \bar{q}_i q_i \rangle = \epsilon_{ii} \frac{\langle F_{H_u H_d}\rangle}{\lambda v^{1+\gamma}}. \tag{18}$$

In this vacuum, n flavors of magnetic quarks obtain expectation values, partially Higgsing the magnetic gauge group [10]. In Eq. (18), all fields and couplings are renormalized at the scale v. As long as n is not too large, that is, as long as

$$\frac{3}{2} < \frac{N_f - n}{N_c} < 3, \tag{19}$$

the theory flows to a new fixed point, with different but still nontrivial anomalous dimensions. Note that straightforward computation of the F terms shows that no tadpoles are produced in the unparticle scalar potential by the Higgs vevs, and the theory still has a well-defined supersymmetric vacuum below the scale Λ. To find the physical value of Λ at which the theory is driven to a new fixed point, we note between v and Λ, the A fields have anomalous dimension γ. We take $F_{H_u H_d}$ to be of order v^3, and all the ϵ_{ii} to be of similar size. We can determine the order of magnitude of Λ by using dimensional analysis. The coefficient of A_{ii} in Eq. (12) is a relevant parameter with physical scaling dimension $2 - \gamma$. At a renormalization scale μ where the renormalized coefficient of A_{ii} is small compared with $\mu^{2-\gamma}$, this term may be treated as a perturbation which does not significantly affect the dynamics at the scale μ. At the scale Λ where the perturbation becomes important the renormalized coefficient of A_{ii} in Eq. (12) is of order $\Lambda^{2-\gamma}$. We can therefore use

$$\epsilon_{ii} v^{2-\gamma} \sim \Lambda^{2-\gamma} \tag{20}$$

to find

$$\Lambda \sim \epsilon_{ii}^{1/(2-\gamma)} v. \tag{21}$$

Note that as long as the ϵ_{ii} parameters are less than 1, Λ is below the weak scale. Furthermore, the more strongly coupled the magnetic description is (the closer γ is to 1) the smaller Λ is.

Loop effects and tree effects which are higher order in the weak ϵ_{ii} couplings will induce supersymmetry breaking effects into the hidden sector, at a scale $f_s < \Lambda$. At the weak scale, such nonsupersymmetric tree corrections are proportional to a factor of ϵ_{ii}^2, times powers of v given by dimensional analysis, and loop corrections are roughly proportional to powers of $\epsilon_{ii}^2/(16\pi)^2$. A rough estimate for the upper bound on the scale of SUSY breaking in the hidden sector is (ϵv), where $\epsilon = \max \epsilon_{ii}$, which is lower than Λ by a factor of $\epsilon^{(1-\gamma)/(2-\gamma)}$.

We have several theoretical handles on the effective theory between f_s and Λ. In the limit that $N_f - n$ of the g_{ii} couplings vanish, this theory possesses an $SU(N_f - n) \times SU(N_f - n)$ flavor symmetry, which is also respected by the supersymmetry breaking terms for the fields in this effective theory. In Ref. [19], Nelson and Strassler showed that all soft supersymmetry breaking terms in a superconformal effective theory which respect the flavor symmetry have positive anomalous dimension. Certain linear combinations of quark masses squared, i.e. those proportional to global symmetry generators, do not run [19], however generating such symmetry violating terms is not possible when the supersymmetry breaking is mediated by flavor symmetric couplings. The anomalous dimensions for the supersymmetry breaking terms cannot be nonperturbatively computed, but, since these are known to be positive at an infrared attractive fixed point, renormalization group scaling will lower the supersymmetry breaking scale f_s below (ϵv). Furthermore if the infrared fixed point is weakly coupled in the magnetic description, as is possible at large N_c with the appropriate number of flavors, the scale of SUSY breaking will be suppressed by additional factors of $1/N_c$. (In this limit, however, the anomalous dimension γ is small, making unparticle kinematics less exotic.) We conclude that in the small ϵ limit the physical supersymmetry breaking scale f_s is significantly below Λ. We treat f_s as a free parameter subject to the (ϵv) upper bound, although in principle it is determined by the parameters and the strong dynamics. Below f_s, we assume the unparticle sector is no longer conformal, and, depending on the sign of the soft supersymmetry breaking squark masses, either the magnetic description is in a Higgs phase and the electric description in a QCD-like confining phase, or vice versa.

B. Hidden sector spectrum

As seen in the previous section, electroweak symmetry breaking introduces several scales into the hidden sector, and a spectrum of strongly coupled particles. Most of these are unstable, however some will be stable due to unbroken flavor symmetries. In this section we give an overview of the long-lived and stable spectrum of particle states.

We begin with the lightest particles. Below the hidden sector supersymmetry breaking scale f_s we have a strongly coupled nonsupersymmetric theory, with a global $SU(N_f - n) \times SU(N_f - n) \times U(1)_B$ flavor symmetry, where $U(1)_B$ refers to baryon number. We assume that either the magnetic or the electric description of the low energy nonsupersymmetric effective theory is confining at the scale f_s. Depending on the value of the gluino mass, there may also be an approximate spontaneously broken chiral $U(1)$ symmetry, with the anomaly from the quark sector canceled by the gluino. We will assume QCD-like dynamics, with spontaneous breaking of the chiral $SU(N_f - n) \times SU(N_f - n)$ symmetry to the diagonal subgroup, giving $(N_f - n)^2 - 1$ massless Nambu-Goldstone bosons.

The chiral symmetry is due to our assumption that the coupling matrix g_{ij} is rank n. The discussion of the previous section is still valid when the matrix g is rank N_f, provided there is a hierarchy in the couplings, with n of the eigenvalues of the same order of magnitude and with the other eigenvalues much smaller, so they do not affect the dynamics at the scales Λ or f_s. Such small couplings explicitly break $SU(N_f - n) \times SU(N_f - n) \times U(1)_B$ to the diagonal $U(1)^{N_f - n}$, and give small masses to all the Nambu-Goldstone bosons, of order $\sqrt{g_{jj}}\xi f_s$, where the g_{jj} are the smaller couplings which were neglected in the previous section, and ξ is a factor from scaling the g_{jj} couplings. The lightest particles of the theory are therefore a multiplet of light pseudoscalar bosons, analogous to the pions, kaons and eta of QCD. The unbroken $U(1)$ symmetries will make $(N_f - n - 1)(N_f - n)$ of these stable, while the remaining $N_f - n - 1$ pseudoscalars can mix with the pseudoscalar Higgs and decay to lighter standard model particles. Other stable particles of the theory are the baryons, with masses of order $N_c f_s$.

We also expect heavier particles associated with the scale Λ, the scale associated with the condensate of some of the magnetic quarks, which partially breaks both the global and the magnetic-color symmetries. These particles are analogous to the mesons of QCD containing heavy quarks, with the novel feature that they come in approximately degenerate supermultiplets. There remains an exact global $U(1)^n$ which is not spontaneously broken, yielding stable mesons and baryons with masses of order Λ. There is also a spectrum of unstable resonances.

We can be more definite about the spectrum in the limit where the magnetic description is weakly coupled (see also the Appendix for a discussion of large N_c scaling). In this weakly coupled limit, at the scale Λ the magnetic $SU(N_c - N_f)$ gauge symmetry is Higgsed down to $SU(N_c - N_f - n)$, and the massive particles at the scale Λ come in vector supermultiplets. There are n^2 vector supermultiplets which are uncharged under the low energy gauge group, and n vector supermultiplets transforming in the fundamental and antifundamental representations, which at long distances will form bound states with each other and with lighter particles. At the scale f_s, small splittings are introduced into the spectrum of heavy supermultiplets.

In a previous paper [20], two of us considered a dark matter sector containing heavy stable and lighter unstable particles, and discussed how the PAMELA [21] cosmic ray positron spectrum could result from annihilation of the heavy stable dark matter into the lighter particles, which then decay into electron positron pairs. This model exhibits all the necessary ingredients of such a dark matter model, if the heavy stable particles are identified as dark matter, and the scale Λ is above 80 GeV. In addition, the dark matter states come in multiplets with small splittings, which may allow for the inelastic dark matter explanation [22] of the DAMA/LIBRA experiment [23] and/or the exciting dark matter [24] explanation of the integral low energy positron excess [25].

One final comment concerns the LSP, or lightest superpartner. As in all gauge mediated models, the LSP will be an ultralight gravitino, and the next lightest superpartner, or NLSP, will decay into the gravitino. Typically in gauge mediated models the NLSP is one of the MSSM states, e.g. the stau or the lightest neutralino. In this model, the NLSP will be a fermion in the hidden sector, e.g. a bound state of a scalar and a fermion with mass of order f_s, times small couplings. The lightest MSSM state will therefore decay into the hidden sector, implying that every collider event that produces superpartners will produce hidden sector unparticles. The NLSP in turn will decay into a gravitino and one or more pseudoscalar mesons. Some of the latter particles will decay back into the visible sector via mixing with the pseudoscalar Higgs, making some NLSP decays potentially visible.

C. Comparing to naive dimensional analysis (NDA)

We summarize here some of the arguments in [12,26], in order to clarify the differences with the setup of this paper. In a generic realization of the unparticle scenario, the couplings between the standard model and the unparticle sector can be described in the far UV by a set of higher-order operators:

$$c_n^i \frac{O_{UV} O_n^i}{M^{d_{UV}+n-4}}, \tag{22}$$

where we used the same notation as in [26], according to which O_{UV} is an operator of dimension d_{UV} in the hidden sector, O_n^i is a standard-model operator of dimension n, c_n^i is a dimensionless coupling and M some very high scale. The latter may be thought of as the mass of the fields responsible for mediating the interactions between the two sectors (in our setup, this would be related to the mass scale μ').

Below the scale Λ_U at which the hidden sector enters its conformal phase, the operator O_{UV} flows onto a d-dimensional operator O, resulting in the effective coupling

$$c_n^i \frac{O O_N^i}{\Lambda_n^{d+n-4}}, \tag{23}$$

where $\Lambda_n^{d+n-4} \equiv M^{d_{UV}+n-4}/\Lambda_U^{d_{UV}-d}$. The problem highlighted in [26] is that one of the operators of the standard model is $O_2 = H^\dagger H$, the $n = 2$ dimensional bilinear in the Higgs sector. Also, the operators O that are most interesting phenomenologically, and most often considered in the literature, are unparticle operators with $1 < d < 2$. This results in a relevant effective interaction from Eq. (23). Furthermore, below the electroweak scale, it induces the breaking of conformal symmetry in the unparticle sector by a highly relevant operator.

The conclusion is that a generic unparticle model cannot yield sizable phenomenological signals unless fine-tuning is introduced in order to protect the unparticle sector from the effect of the scale of electroweak symmetry breaking. Only at large scales can the unparticle description hold, while at scales parametrically lower than the electroweak scale the unparticle sector turns into a hidden sector of massive, neutral particles with conventional phenomenology.

In this paper we took a closer look at the mechanism generating the couplings in Eq. (22), in order to gauge the generality of the conclusion summarized above. Instead of allowing for generic unconstrained couplings, we explicitly built a model both for the unparticle sector [based on a $\mathcal{N} = 1$ $SU(N_c)$ gauge theory] and for the mediators (the S and T superfields), which, in order to preserve supersymmetry and hence allow for calculability, we coupled to the MSSM in the visible sector, with a very specific choice of superpotential $\mathcal{W}$. Soft terms in the MSSM are the only explicit source of SUSY breaking.

In comparison with [26], we find three important differences.

(i) Supersymmetry, and hence a degree of calculability, is preserved in the hidden sector down to a scale f_s that is parametrically lower than the electroweak scale.

(ii) The scale Λ induced in the hidden sector by the Higgs VEV in the visible sector is itself parametrically suppressed in respect to the electroweak scale.

(iii) Below the latter scale, conformal symmetry is broken only in the sense that the hidden sector flows away from the IR fixed point it is approaching below

Λ_U, but is recovered asymptotically as in the far IR the renormalization group flow approaches a different fixed point.

We now present some important physical implications of these three results. First of all, the analysis we carried out is reliable enough that we do not need to introduce generic naive dimensional analysis (NDA) arguments. A large energy range over which the hidden sector behaves as a genuine unparticle sector exists, down to energies well below the electroweak scale. At even lower energies, not all of the hidden sector degrees of freedom become just ordinary massive particles, but a new conformal regime appears.

We draw three final lessons. First of all, the analysis of [26] is correct for generic models, but being only based on NDA arguments it cannot be applied blindly to all possible models: our work shows that there are specific realizations of the mediation mechanism that evade the bounds in [26] without fine-tuning.

On the other hand, the specific example we constructed is very peculiar: besides being supersymmetric (which in our context is only a technical requirement), it requires the interplay of two singlet fields S and T, with a very specific W, and cannot work in the presence of only one of these mediators. In other words: it is not easy to construct counterexamples to the pessimistic conclusions of [26]. The good implication is that if experimental data were to discover unparticle-type signatures, these would yield interesting information not only about the unparticle sector itself, but also about the mechanism that couples it to the visible sector at high energy.

Finally, the third lesson is that at low energies the phenomenology of these models is very rich, consisting of an admixture of particle- and unparticlelike signatures, where part of the new particle content is determined by the transition between two different IR fixed points induced indirectly by the electroweak symmetry breaking VEV.

III. OBSERVING UNPARTICLES

A. Higgs-to-unparticle couplings

When the theory described in previous sections is at its fixed point, the quark and meson fields of the hidden sector attain fixed anomalous dimensions, and we obtain couplings to fields recently described as unparticles. At tree level, the only coupling of the MSSM to this new physics is between the Higgs and unparticle operators. In this section we will consider the leading order couplings to neutral Higgs. We will take some number n of nonzero couplings, that is

$$g_{ii} \neq 0 \tag{24}$$

where $(N_f - n + 1) \leq i \leq N_f$, and we take g_{ii} to be perturbative.

We now determine how unparticles couple to scalar Higgs mass eigenstates. In the MSSM, the mixing of the Higgs fields H_u and H_d into these fields can be described by

$$H_u = \left(\frac{\sqrt{2}v_u + R_u}{2}\right)\exp\left\{i\frac{I_u}{\sqrt{2}v_u}\right\}, \tag{25}$$

$$H_d = \left(\frac{\sqrt{2}v_d + R_d}{2}\right)\exp\left\{i\frac{I_d}{\sqrt{2}v_d}\right\} \tag{26}$$

where

$$R_u = \cos\alpha h^0 + \sin\alpha H^0, \tag{27}$$

$$R_d = -\sin\alpha h^0 + \cos\alpha H^0, \tag{28}$$

$$I_u = \cos\beta A^0, \tag{29}$$

$$I_d = \sin\beta A^0, \tag{30}$$

where we have gone to unitary gauge. The fields h^0 and H^0 are the light and heavy Higgs scalars, and A^0 is the pseudoscalar Higgs. To leading order the mixing angles α and β are determined by the parameters of the Higgs sector, and match the MSSM values in the absence of the hidden sector. The terms v_u and v_d are the expectations values for H_u and H_d, and are also equivalent to the typical MSSM values up to small corrections.

We will work with the effective potential which has only the heavy scalars S and T integrated out, described in previous sections as $\mathcal{K}_{\mathrm{eff}}$ and $\mathcal{W}_{\mathrm{eff}}$. We will make explicit the presence of quantum corrections in the Kahler potential, and include effects of the running of g_{ij} to scale μ. This leaves us with an effective Kahler potential of the form

$$\mathcal{K} \supset \frac{\lambda_S^2}{\mu'^2}|H_uH_d|^2 + \frac{\lambda_S\mu^\gamma}{\mu'^{1+2\gamma}}\left(H_uH_d\sum_i g_{ii}A_{ii}^\dagger + \mathrm{H.c.}\right) + K(A_{ij}, q_i, \bar{q}_i, A_{ij}^\dagger, q_i^\dagger, \bar{q}_i^\dagger). \tag{31}$$

Here K is the Kahler potential of the hidden sector, including quantum corrections which may, in general, be a complex function of the fields. Subscripts on K refer to derivatives with respect to the field given in the subscript.

Upon integration of the superspace variables, the cross terms in the full potential $\mathcal{K}$ generate a number of Higgs-unparticle couplings in the Lagrangian. These have the schematic form

$$\mathcal{L} \supset \mathcal{L}_{Hq} + \mathcal{L}_{HA} \tag{32}$$

where the first term contains couplings to the hidden quark fields, and the second to the hidden meson fields.

The coupling between Higgs and quarks is generated by

$$\mathcal{L}_{Hq} = \sum_i \frac{g_{ii}\lambda_s \mu^\gamma}{\mu'^{1+2\gamma}} F_{A_{ii}A_{ii}}(H_u^* F_{H_d}^* + H_d^* F_{H_u}^* + 2\Psi_{H_u}^\dagger \Psi_{H_d}^\dagger) + \text{H.c.} \tag{33}$$

where the H fields are now scalars, and the Ψ fields are fermionic. Solving the equation of motion for the A_{ii} auxiliary field yields the solution

$$F_{A_{ii}} = -\sum_j K^{-1}_{\Phi_j^* A_{ii}} W^*_{\Phi_j} \equiv \mathcal{O}_i \tag{34}$$

where the sum runs over all fields Φ_j^* on which K has functional dependence. The dimension of this unparticle operator, $\mathcal{O}_i$, matches the dimension of the A_{ii} field F term, which is $2+\gamma$. Inserting these solutions into the Lagrangian yields the following couplings,

$$\mathcal{L}_{Hq} = \sum_i \frac{g_{ii}\lambda_s \mu_W \mu^\gamma}{\mu'^{1+2\gamma}}(|H_u|^2 + |H_d|^2)(\mathcal{O}_i + \text{H.c.}) + \sum_i \frac{2g_{ii}\lambda_s\mu^\gamma}{\mu'^{1+2\gamma}}\Psi_{H_u}^\dagger\Psi_{H_d}^\dagger \mathcal{O}_i + \text{H.c.} \tag{35}$$

Expanding this to Higgs mass states at leading order provides the Higgs-unparticle couplings,

$$\mathcal{L}_{Hq} = \frac{\lambda_s \mu_W \mu^\gamma}{4\mu'^{1+2\gamma}}\{h^{02} + H^{02} + 2\sqrt{2}H^0 v\cos(\alpha-\beta) - 2\sqrt{2}h^0 v \sin(\alpha-\beta)\}\sum_i (g_{ii}\mathcal{O}_i + \text{H.c.}) + \sum_i \frac{2g_{ii}\lambda_s\mu^\gamma}{\mu'^{1+2\gamma}}\Psi_{H_u}^\dagger\Psi_{H_d}^\dagger\mathcal{O}_i + \text{H.c.} \tag{36}$$

where $v^2 = v_u^2 + v_d^2$.

There are also interactions between Higgs and the hidden meson fields that come from the A cross terms in the Kahler potential. Integrating over superspace yields 3-scalar terms and fermion-fermion-scalar terms

$$\begin{aligned}\mathcal{L}_{HA} \supset \sum_i \frac{\lambda_s g_{ii}\mu^\gamma}{\mu'^{1+2\gamma}}\Big\{&-\Big(\frac{3}{2}H_u^* H_d^* \partial^2 A_{ii} + \frac{1}{2}A_{ii}\sigma^\mu\partial_\mu H_d^*\sigma^\nu\partial_\nu H_u^*\Big) \\ &+ (2i\sigma^\mu\partial_\mu H_u^*\Psi_{H_d}^\dagger\Psi_{A_{ii}} + 2i\sigma^\mu\partial_\mu H_d^*\Psi_{H_u}^\dagger\Psi_{A_{ii}}) \\ &+ (iH_u^*\Psi_{H_d}^\dagger\partial_\mu\Psi_{A_{ii}}\sigma^\mu + iH_d^*\Psi_{H_u}^\dagger\partial_\mu\Psi_{A_{ii}}\sigma^\mu) \\ &- (H_u^*\sigma^\mu\partial_\mu\Psi_{H_d}^\dagger\Psi_{A_{ii}} + H_d^*\sigma^\mu\partial_\mu\Psi_{H_u}^\dagger\Psi_{A_{ii}})\Big\} + \text{H.c.}\end{aligned} \tag{37}$$

after combining some terms using integration by parts.

B. Decays to unparticles

We now briefly examine a few of the processes that are generated by the couplings described above. Our goal is to point out a few examples of processes with unparticlelike properties, rather than exhaust the space of interesting decay topologies. These processes occur when the Higgs couples to effectively conformal fields, that is at energies above Λ, as discussed in previous sections. As before, we focus on the neutral Higgs scalars which have interesting phenomenological potential at the LHC. For simplicity of presentation, in this section we will set $n = 1$, and define $\mathcal{O} \equiv \mathcal{O}_{N_f}$, $g \equiv g_{NfNf}$, $\epsilon \equiv \epsilon_{NfNf}$ and $A \equiv A_{NfNf}$.

The calculations of rates involving unparticles differs somewhat from the standard particle cases due to the unparticle phase space. Following the notation of [3] the phase space for scalar unparticles is

$$\delta\Phi = A_{d_u}\theta(p^0_{\mathcal{U}})\theta(p^2_{\mathcal{U}})(p^2_{\mathcal{U}})^{d_u-2} \tag{38}$$

and for fermionic unparticles (see [27–29]) is

$$\delta\Phi = \frac{3}{2}A_{2d_u/3}\theta(p^0_{\mathcal{U}})\theta(p^2_{\mathcal{U}})(p^2_{\mathcal{U}})^{d_u-5/2}. \tag{39}$$

In each case, d_u is the dimension of an unparticle operator $\mathcal{O}_u$, and A_n is a normalization constant given by

$$A_n = \frac{16\pi^{5/2}\Gamma(n+\frac{1}{2})}{(2\pi)^{2n}\Gamma(n-1)\Gamma(2n)}, \tag{40}$$

which should not be confused with the meson field A.

As a simple example, for a coupling of a scalar and unparticles of the generic form

$$ch\mathcal{O}_u, \tag{41}$$

where h is scalar with mass M and c is a dimensionful coupling constant, the total decay rate for H into unparticle stuff is given by

$$\Gamma = \frac{|c|^2}{2}A_{d_u}M^{2d_u-5}. \tag{42}$$

Note that this approaches the standard decay rate to two massless particles as d_u goes to 2. If the unparticle operator is derivatively coupled to the scalar, $ch\partial^2\mathcal{O}_u$, the decay rate is

$$\Gamma = \frac{|c|^2}{2}A_{du}M^{2d_u-1}. \tag{43}$$

An alternate method is to use the optical theorem in a $1 \to 1$ scattering, and find the width from the imaginary part of the propagators, after including the mixing in the unparticle-Higgs system, as suggested in [30]. Our procedure is equivalent, and convenient for the present purposes.

The first process we consider is the decay of the light Higgs of the MSSM directly into unparticles. This process is generated by the terms of the form $h^0\mathcal{O}$ in Eq. (36), and of the form $h^0\partial^2 A_{ii}$ from terms in Eq. (37). After expanding in terms of mass eigenstates, the relevant coupling terms are

$$\mathcal{L} \supset -\frac{g\lambda_s \mu_W \mu^\gamma}{\sqrt{2}\mu'^{1+2\gamma}} v \sin(\alpha - \beta) h^0 \mathcal{O} - \frac{3\sqrt{2}}{8} \times \frac{g\lambda_s \mu^\gamma}{\mu'^{1+2\gamma}} v \cos(\alpha + \beta) h^0 (\partial^2 A + \text{H.c.}). \quad (44)$$

Using the decay rates from above, we find the total decay rate to unparticles is given by

$$\begin{aligned}\Gamma_{h\to\text{unp}} &= \frac{g^2\lambda_s^2 v^2}{4\mu'^{2+4\gamma}} \Big\{ A_{2+\gamma}\mu_W^2 \sin^2(\alpha - \beta) m_h^{4\gamma-1} + \frac{9}{16} A_{1+\gamma} \cos^2(\alpha + \beta) m_h^{4\gamma+1} \Big\} \\ &= \frac{\epsilon^2}{4}\left(\frac{m_h}{v}\right)^{4\gamma} \Big\{ A_{2+\gamma}\mu_W^2 \sin^2(\alpha - \beta) m_h^{-1} + \frac{9}{16} A_{1+\gamma} \cos^2(\alpha + \beta) m_h \Big\} \end{aligned} \quad (45)$$

in which the ϵ is now evaluated at the Higgs mass rather than the weak scale, as had been written in Eq. (11).

In order to present numerical results, we need to choose a set of MSSM parameters. We will take $\tan\beta \approx 10$ and the decoupling limit $m_{A^0} \gg m_Z$. Specifically, we set $m_{A^0} \approx 800$ GeV. At this point, the tree-level value of the light Higgs mass is 89.3 GeV, but will receive large corrections at one-loop from couplings to the quarks. We will take the appropriate squark masses so that the physical value of the light Higgs mass is 120 GeV. The rate for this decay is shown in Fig. 1, in which we vary the dimensionless combination of parameters described by ϵ at several fixed values of γ.

At this point in the MSSM parameter space, the couplings of the light Higgs are standard-model-like, and the dominant decay mode for h^0 is to $b\bar{b}$. The relatively weak coupling of the Higgs and b means that couplings to new sectors do not need to be particularly strong to compete with standard model channels (a concise summary of the situation is presented in [31]). The decay rate for the light Higgs to $b\bar{b}$ at tree level is given by

$$\Gamma_{h\to b\bar{b}} = \frac{3}{\pi}\frac{m_b^5}{v^2 m_h^2}\left(\frac{m_h^2}{4m_b^2} - 1\right)^{3/2}. \quad (46)$$

Decays to unparticles will dominate decays to b quarks for certain choices of hidden sector parameters. We plot this as a bound on the dimensionless constant g for a particular choice of parameters in Fig. 2.

The heavy Higgs also decays through the same channels. The calculation for this rate is the same as the calculation for the light Higgs, with parameters adjusted to match the coefficients in the Lagrangian,

$$\mathcal{L} \supset \frac{g\lambda_s \mu_w \mu^\gamma}{\sqrt{2}\mu'^{1+2\gamma}} v \cos(\alpha - \beta) H^0 \mathcal{O} - \frac{3\sqrt{2}}{8} \times \frac{g\lambda_s \mu^\gamma}{\mu'^{1+2\gamma}} v \sin(\alpha + \beta) H^0 (\partial^2 A + \text{H.c.}). \quad (47)$$

Next we turn to the decay of the LSP of the MSSM. For concreteness, we will take the LSP to be the stau. This decay may be of particular interest, as it has a distinct signature of a single lepton and large missing energy. Moreover, the kinematics of the decay would reveal the unparticle phase space. In our model, the stau can decay to a tau and a virtual Higgsino. The Higgsino then goes to fermionic hidden sector mesons through the derivative couplings in Eq. (37).

In general, the momentum structure of phase space does not lend itself to integration. Closed form solutions exists for certain limits, including the large Higgsino mass case. In this approximation, the rate is given by

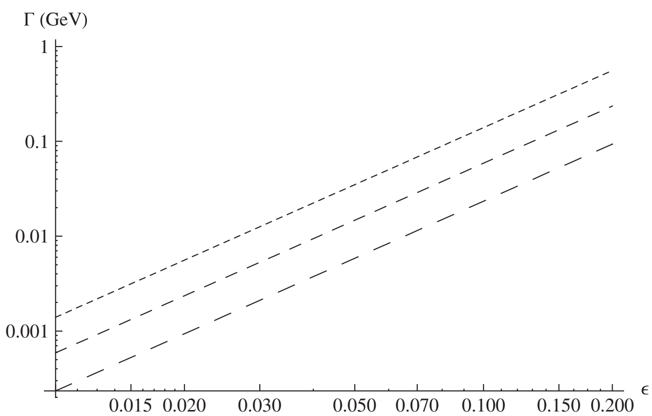

FIG. 1. The decay rate for the light Higgs to unparticles for the parameters described in the text. Three values of the anomalous dimension γ are shown. From shortest to longest dashes, these are $\gamma = 0.1$, 0.3 and 0.5. The value of the Higgs mass is fixed at $m_h = 120$ GeV.

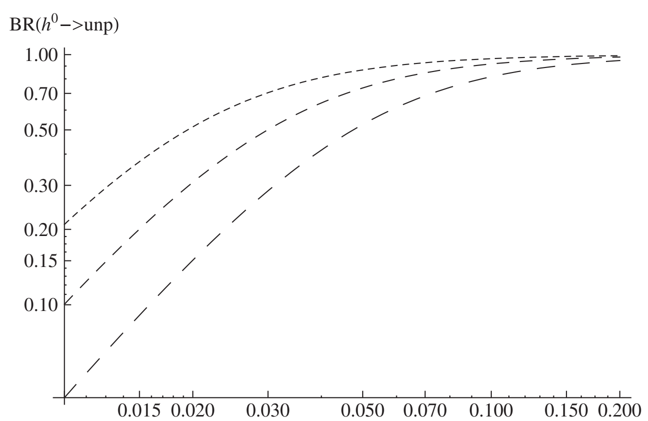

FIG. 2. The branching ratio for Higgs decays to the unparticles. Three values of γ are shown. From shortest to longest dashes, these are $\gamma = 0.1$, 0.3 and 0.5. The light Higgs mass is fixed at $m_h = 120$ GeV.

$$\Gamma_{\tilde{\tau}\mathrm{dec}} \approx c_{\tilde{\tau}} \frac{(m_{\tilde{\tau}}^2 - m_\tau^2)^{4+2\gamma}}{m_{\tilde{\tau}}^{3-2\gamma} m_{\tilde{H}}^2 (m_{\tilde{\tau}}^2 + m_\tau^2)^{2+\gamma}} (2(m_{\tilde{\tau}}^2 + m_\tau^2) N_1 - (m_{\tilde{\tau}}^2 - m_\tau^2) N_2) \quad (48)$$

where N_1 and N_2 are dimensionless numbers of $O(1)$ across most of parameter space, and M is the mass of the LSP. The overall coupling constant $c_{\tilde{\tau}}$ of dimension -4γ is given by

$$c_{\tilde{\tau}} = \frac{3(y_\tau g \lambda_s v \sin(\beta))^2}{8\pi^{3/2} \mu'^{2+4\gamma}} \quad (49)$$

where y_τ is the τ Yukawa coupling. The total decay rate, including the correct numerical factors, is plotted in Fig. 3. Here we have chosen a Higgsino with mass $m_{\tilde{H}} = 800$ GeV, an LSP mass of $m_{\tilde{\tau}} = 500$ GeV and vary ϵ as we had for the Higgs decay above. All other parameters are the same as in the previous cases.

Next we will briefly comment on the relevance for this theory to the widely studied properties of mesons in theories with unparticles [32–40]. These papers generally consider the effects of the coupling of the standard model quarks to the unparticle sector

$$\frac{1}{\Lambda^{d_u - 1}} \bar{q}(c_1 \gamma_\mu - c_2 \gamma_\mu \gamma_5) q \mathcal{O}_U^\mu \quad (50)$$

where c_1 and c_2 are unknown constants. Here we simply wish to comment that this coupling does not arise at tree level in our theory as the unparticles only couple directly to the Higgs. Quark-unparticle couplings occur effectively in penguin diagrams in which the loop quark emits a Higgs, which in turn decays to unparticles through the coupling in Eq. (36). These coefficients scale as

$$c_{1,2} \sim N c_{q_i q_j} \frac{v \lambda_s \mu_w}{\mu'^2} \quad (51)$$

where N is a small number containing the loop factor and phase space normalization, and $c_{q_i q_j}$ is a value from the Cabbibo-Kobayashi-Maskawa matrix that couples quarks of type q_i and q_j to the Higgs. The suppression by μ' pushes these coefficients below all established bounds without requiring unnaturally small coefficients.

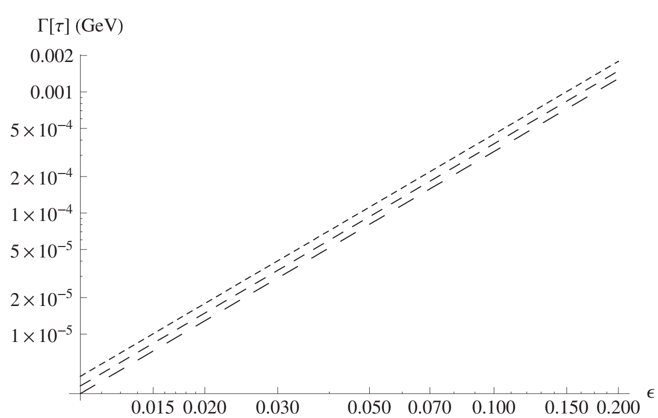

FIG. 3. The decay rate for the LSP $\tilde{\tau}$ to a τ and unparticles for the parameters described in the text. As before, the values for γ from shortest to longest dashes are 0.1, 0.3 and 0.5.

Finally, we consider the eventual fate of the unparticle stuff that would be produced in the preceding examples. As discussed previously, below the supersymmetry breaking scale f_s we assume that the unparticle sector becomes confining and forms bound meson and baryon states. Since there remain strong forces within the hidden sector, we expect that the states which are produced undergo cascade decays and populate stable baryon states, and light meson states. These latter pseudoscalar particle mix with the pseudoscalar Higgs, A^0, which allow them to decay to standard model fermions. This may produce either interesting topologies or displaced vertices if the pseudoscalar decays occur within the detector.

To estimate the lifetime of these particles, we need to make a small modification to the above assumptions to provide the light states with some mass. We now assume that all unparticle fields have some small coupling to the Higgs, that is $g_{ii} \neq 0$ for all i. However, we may still have hierarchical couplings so that the processes discussed above are not significantly modified. We will take the lightest meson state to correspond to A_{11}.

The mixing between the hidden pseudoscalar and A^0 is given by the first term in Eq. (37). Upon electroweak symmetry break, this gives rise to a coupling

$$\frac{3\sqrt{2} i \lambda_S g_{11} v \mu^\gamma}{8\mu'^{1+2\gamma}} A^0 \partial^2 A_{11}. \quad (52)$$

Integrating out the A^0, we find an effective coupling between the hidden meson and the standard model fermions.

We take the mass of the lightest state to be 5 GeV, just above the τ production threshold. The decay to τ's dominates. There will also be subdominant tree-level decays to lighter fermions and loop-suppressed decays to gauge bosons that are not considered here. The effective term that is generated is

$$c_{\pi_A} \pi_A \bar{\tau} \gamma_5 \tau \quad (53)$$

below the scale f_s, in which π_A is the meson state corresponding to the operator A_{11}. The coupling constant c_{π_A} is

$$c_{\pi_A} = -C\left(\frac{3\sqrt{2} i \lambda_S g_{11} v f_s^{2\gamma}}{8\mu'^{1+2\gamma}} \frac{m_\tau \tan\beta}{v}\right) \frac{m_{\pi_A}^2}{m_{A^0}^2} = -C \frac{3\sqrt{2} i \epsilon_{11} m_\tau \tan\beta}{8 f_s} \frac{m_{\pi_A}^2}{m_{A^0}^2} \quad (54)$$

where π_A is the meson state corresponding to the operator $A_1 1$ and C is an unknown constant resulting from the strong dynamics (analogous to the shift in F_π from the strong scale in the effective couplings of standard model pions). Here we have run the coupling down to the scale f_s,

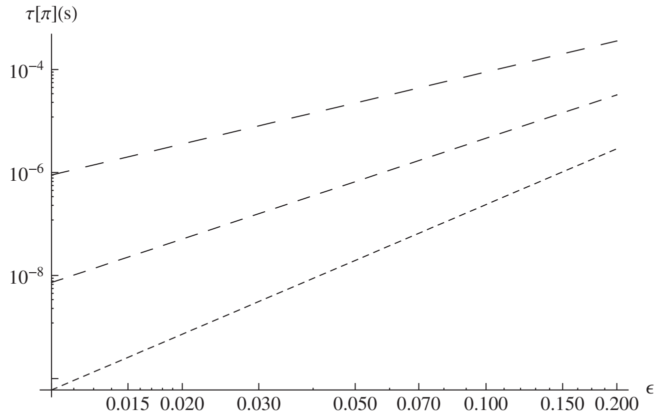

FIG. 4. The rest frame lifetime for the particlelike lightest hidden meson state, for the simple example discussed in the text, with the overall numerical constant set to 1. The mass of the state is fixed at 5 GeV so that decays are dominantly to taus. The values for γ from shortest to longest dashes are 0.1, 0.3 and 0.5.

and included additional powers of f_s to get the correct dimensions. The resulting rest frame decay rate for the lightest hidden meson is

$$\Gamma_{\pi_A \mathrm{dec}} = \frac{|c_{\pi_A}|^2 m_{\pi_A}}{8\pi}\left(1 - \frac{4m_\tau^2}{m_{\pi_A}^2}\right)^{1/2}. \quad (55)$$

To get a sense for the lifetime of these light particles, we plot the rest frame lifetime for the parameters described above in Fig. 4.

IV. SUMMARY

General dimensional analysis arguments indicate that the coupling between the standard model and a light hidden sector should be dominated by low dimension standard model gauge invariant operators, most likely involving the Higgs. Previous work has considered couplings between the Higgs and an unparticle sector that induce a gap into the unparticle spectrum. In contrast, this model illustrates a different possibility, where the coupling between the Higgs and the unparticle sector drives the unparticle sector to a new conformal fixed point in the infrared. We illustrate how to couple the unparticle sector to the Higgs through an effective Giudice-Masiero mechanism, which shields the unparticles from the effects of supersymmetry breaking, so that the supersymmetry breaking scale is low and supersymmetry may be used for theoretical control over the computations. Below the supersymmetry breaking scale we must make dynamical assumptions about the strongly coupled theory. We assume the very low energy theory is confining.

There are several ways in which the hidden sector may be produced and observed in colliders. The Higgs may dominantly decay into unparticles. The exotic kinematics of unparticle decays are conceivably observable through the decays of the lightest MSSM superpartner into a visible particle and unparticles. Once the unparticles are produced, at long distances they hadronize. There are a multitude of stable and metastable particles with a broad spectrum of masses in the unparticle sector, providing interesting possibilities for dark matter. As in ''hidden valley'' models of strongly coupled hidden sectors, some of the hidden hadrons decay promptly, some with displaced vertices, and some are stable. These models require further study to understand their implications for detection of the Higgs, supersymmetry and dark matter.

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APPENDIX: LARGE-*N* COUNTING

In order to provide a set of estimates for the masses of the particles that decouple from the unparticle sector in the hidden sector when symmetry breaking takes place (at the scale Λ), it is useful to consider the limiting case in which $\gamma \simeq 0$, so that perturbative techniques can be used, and study the large-N behavior of the physical quantities.

The masses of the heavy vector superfields resulting from symmetry breaking in the magnetic sector are

$$m_V^2 \simeq \frac{1}{2} g_s^2 m^2, \quad \text{(A1)}$$

where

$$m^2 = \langle \bar{q}_i q_i \rangle. \quad \text{(A2)}$$

In order for these masses to be finite, one has to assume that

$$m^2 \propto N_c, \quad \text{(A3)}$$

since the gauge coupling itself $g_s^2 \propto 1/N_c$.

The masses of the chiral (meson) superfields corresponding to the pseudo-Goldstone bosons carrying one unbroken flavor index I and one index along the broken directions N are

$$m_{A_{NI}}^2 \simeq \lambda^2 m^2, \quad \text{(A4)}$$

which implies that

$$\lambda^2 \propto \frac{1}{N_c}. \quad \text{(A5)}$$

As a consequence, the scaling of m^2 requires that $\epsilon_{ii}/\lambda \propto N_c$, and hence $\epsilon_{ii} \propto g_{ii} \propto \sqrt{N_c}$. The relevant couplings g_{ii} grow with N_c as

$$g_{ii}^2 \propto N_c. \quad \text{(A6)}$$

The massive meson superfields carrying only indexes on the broken directions A'_{NN} have mass

$$m_{A_{NN}}^2 \simeq \frac{2\lambda^2 m^2}{1 + g_{ii}^2}. \tag{A7}$$

This is suppressed in the large-N_c limit.

Also the mass of linear combination of quarks and antiquarks carrying both flavor and color indexes in the broken directions is suppressed as $m_{q_N}^2 \simeq m_{A_{NN}}^2 \propto 1/N_c$.

This fixes the large-N_c scaling of all the parameters and resulting masses. Notice how the same scalings would have been obtained by rescaling $A \to \tilde{A}/g$ for all the magnetic fields, but not for the MSSM and messenger fields, and requiring that all terms we wrote in the superpotential appear at the same order in the $1/N$ expansion. Interestingly, besides expected results, we find that in the large-N_c limit the fields corresponding to perturbations in the transverse directions of the potential become light.

All of these expressions generalize to the case where $\gamma > 0$ by replacing $g_{ii} \to g_{ii}(v/\mu')^\gamma$, while m^2 has to be evaluated at the electroweak scale (as done in the body of the paper). The couplings λ and g_s do not run between v and μ'.

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