

Holographic approach to finite temperature QCD: The case of scalar glueballs and scalar mesons

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We study scalar glueballs and scalar mesons at $T \neq 0$ in the soft wall holographic QCD model. We find that, using the anti-de Sitter–black-hole metric for all values of the temperature, the masses of the hadronic states decrease and the widths become broader when T increases, and there are temperatures for which the states disappear from the scalar glueball and scalar meson spectral functions. However, the values of the temperatures in correspondence of which such phenomena occur are low, of the order of 40–60 MeV. A consistent holographic description of in-medium effects on hadron properties should include the Hawking-Page transition, which separates the phase with the anti-de Sitter metric at small temperatures from the phase with anti-de Sitter–black-hole metric at high temperatures.

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I. INTRODUCTION

Finite temperature effects on hadron properties are currently investigated in heavy ion collision experiments, such as the ones at the Relativistic Heavy Ion Collider at Brookhaven, and will be the subject of analyses at the CERN Large Hadron Collider [1]. It is presently believed that the hot and dense medium where the hadrons are created modifies masses and widths, and that this distortion can be observed reconstructing the states produced in the hadron decays. A significant example is represented by heavy mesons like J/ψ and the radial and orbital $c\bar{c}$ excitations: it is expected that these mesons have different behavior when the temperature of the medium varies, with the lowest lying state (J/ψ) surviving in the medium after the deconfinement transition, up to temperatures $T \simeq 1.5T_c$, while the excited states melt at temperatures close to the deconfinement temperature T_c [2].

The theoretical tools used to describe these phenomena are lattice QCD, QCD Sum Rules, effective QCD theories, and models of QCD [3]. Recently, in the framework of the gauge/string duality approach [4–6], it has been suggested that a description of a strongly coupled quantum gauge theory at finite temperature can be obtained from a semi-classical theory formulated into a higher dimensional anti-de Sitter (AdS) space-time containing a black hole (BH) [7]. In order to achieve smoothness and completeness of the metric, the time direction must also be compact [7]; the temperature is inversely proportional to the distance z_h of the black-hole horizon to the AdS boundary: $T = 1/(\pi z_h)$.

Therefore, the smallest is the system, the highest is the effect of the thermal fluctuations.

Another way to describe finite temperature effects in a holographic description is to consider a thermal AdS space, i.e. an anti-de Sitter space with a compact Wick-rotated time, with no black hole. In this case, as in standard finite temperature field theory, the temperature is given by the inverse dimension of the compactified time direction.

In Ref. [7], the gravity dual of $\mathcal{N} = 4$ super Yang-Mills theory on $S^3 \times S^1$, with $N \rightarrow \infty$, has been studied, showing the existence of two phases. A critical temperature was found at which a first order Hawking-Page (HP) transition [8] occurs between the thermal AdS and the anti-de Sitter–black-hole (AdS-BH) geometries, due to an inversion of the hierarchy of the free energies relative to the two metrics [9]. The temperature at which this transition occurs has been identified with the deconfinement temperature of the gauge theory on the boundary [10], which in this way receives a remarkable geometric interpretation.

A description inspired to gauge/string duality has been applied to QCD considering, in particular, two holographic models constructed in the (phenomenological) so-called bottom-up approach [12]: the hard wall (HW) and the soft wall (SW) model. In these two models, QCD is the gauge theory defined on the boundary of a five-dimensional holographic space. Since gauge/gravity [anti-de Sitter/conformal field theory (AdS/CFT)] correspondence has been conjectured for a conformal theory defined on the boundary, the generalization of duality to QCD (which is not a conformal theory) requires a mechanism to break conformal invariance. The HW and SW models differ in the way this invariance is broken: a sharp cutoff of the holographic space, up to a maximum value of the fifth coordinate z_m , is imposed in the HW model, with $z_m = \mathcal{O}(\Lambda_{\text{QCD}}^{-1})$ [14–18]; a

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background dilatonlike field, vanishing at the AdS boundary and involving a dimensionful scale $c = \mathcal{O}(\Lambda_{\text{QCD}})$, is included in the soft wall model, implementing a smooth cutoff of the holographic space [19,20]. At $T = 0$, the two models allow computation of QCD quantities such as the spectrum and decay constants of light mesons and glueballs, meson form factors and strong couplings, and QCD condensates; in many cases the agreement with the experiment is noticeable. In those models, the metric is not a dynamical field; hence, when generalized to the case of finite temperature, there is no reason to justify the occurrence of a Hawking-Page phase transition of the kind described in [7]. However, the existence of such a transition has been inferred [21,22]: considering the free energies of the thermal AdS and AdS-BH configurations, in both of the models, it was found that the stable metric is thermal AdS at low temperatures and AdS-BH at high temperatures. The temperature at which the transition occurs depends on the model: $T_{\text{HP}} = \frac{2^{1/4}}{\pi z_m}$ in the case of the hard wall model, with z_m as the position of the hard wall; $T_{\text{HP}} = \frac{1}{\pi} \frac{c}{0.647}$ in the case of the soft wall model, with c as the dimensionful constant introduced by the background dilaton field in the model. Using the value for c determined at zero temperature, in the SW model, the HP temperature is $T_{\text{HP}} = \frac{1}{2\pi} \frac{m_\rho}{0.647} \simeq 192$ MeV, with m_ρ as the mass of the ρ meson [21]. Therefore, the Hawking-Page transition temperature T_{HP} is close to the QCD deconfinement temperature obtained, e.g., by lattice QCD simulations. The presence of the HP transition has been found also in different holographic models of QCD [23].

An analysis of vector mesons at $T \neq 0$ has been carried out in the SW model with an AdS-BH metric for all values of T , and the temperature dependence of vector meson masses and widths has been investigated [24]. In this analysis, the scale fixing the physical temperature $c_{J/\psi}$, which appears once again in the dilaton field, has been suitably chosen to obtain the masses of hidden charm vector mesons (J/ψ and radial excitations) at $T = 0$. Thus, it is assumed that the model describes $c\bar{c}$ in a thermalized medium, and a critical temperature has been identified, where the lightest charmed vector meson disappears from the two-point spectral function of the retarded two-point Green's function of the vector operator $J_\mu = \bar{c}\gamma_\mu c$ in the boundary theory [24].

Here, we consider an analogous problem: we investigate the SW model with an AdS-BH metric for all values of T , thus including the effects of the temperature with no reference to the HP transition, focusing on the cases of scalar glueballs and scalar mesons. We show that, in both cases, the qualitative dependence on the temperature of the hadron masses and widths follows general expectations, with the masses becoming lighter and the widths broader than at $T = 0$. However, the physical values of the temperature at which such phenomena occur are lower than found in determinations based, e.g., on lattice QCD simulations,

and they occur in the confined phase of QCD, so the use of an AdS-BH metric for all values of temperature is inappropriate. In order to describe QCD at finite T by this model, the Hawking-Page transition must be taken into account, using the thermal AdS metric for $T < T_{\text{HP}}$ and the AdS-BH metric for $T > T_{\text{HP}}$. We consider this case, finding a simple behavior of the hadron masses and widths versus T , different from the behavior found using AdS-BH for all T .

The plan of the paper is the following. In Sec. II, we consider the sector of scalar glueballs at $T \neq 0$, with the AdS-BH metric in the soft wall model. We discuss the temperature dependence of the mass and width, finding the values of the temperatures where the states dissolve in the equilibrated medium. In Sec. III, we find the same features in the sector of light scalar mesons described in the SW AdS-BH model, finding that the scalar meson spectral function does not reveal resonances already at low temperatures. In Sec. IV, we discuss the model including the Hawking-Page transition: now the spectral functions do not display peaks at $T \geq T_{\text{HP}}$, therefore hadronic states do not survive the deconfinement transition. In the Appendix, we discuss the scalar glueball in the hard wall model at $T \neq 0$.

II. SCALAR GLUEBALL AT $T \neq 0$: SOFT WALL MODEL WITH ADS-BH METRIC

We start our discussion ignoring the issue of the stability of the metric in different ranges of the temperature, and we consider the case of a dual QCD model described, for all values of T , by an anti-de Sitter–black-hole metric, defined by the metric tensor

$$g_{MN} = \left(\frac{R}{z}\right)^2 \text{diag}(f(z), -1, -1, -1, -1/f(z)) \quad (1)$$

($M, N = 0, 1, 2, 3, 5$), where R is the AdS radius, and

$$f(z) = 1 - \left(\frac{z}{z_h}\right)^4. \quad (2)$$

z_h is the position of the black-hole horizon along the holographic axis z and is related to the Hawking temperature T : $T = 1/(\pi z_h)$. The fifth coordinate z varies in the range $0 < z < z_h$. In the five-dimensional (5D) space, we define a field $X(x, z)$ dual to the QCD operator $\mathcal{O}_G = \beta(\alpha_s) G_{\mu\nu}^a G^{\mu\nu a}$ (a is color index) with $J^{\text{PC}} = 0^{++}$ and conformal dimension $\Delta = 4$; $\beta(\alpha_s)$ is the Callan-Symanzik function. This operator is defined in the field theory living on the four-dimensional boundary. According to the AdS/CFT correspondence, the conformal dimension of a (p -form) operator on the boundary is related to the AdS mass m_5 of the dual field in the bulk by the relation [5]

$$m_5^2 R^2 = (\Delta - p)(\Delta + p - 4). \quad (3)$$

Following the AdS/QCD correspondence dictionary, the dual $X(x, z)$ of the scalar ($p = 0$) field \mathcal{O}_G is massless: $m_5^2 = 0$ [25]. It can be described, in the soft wall model, by

the 5D action S_G :

$$S_G = \frac{1}{2k} \int d^4x dz e^{-\phi(z)} \sqrt{g} g^{MN} (\partial_M X(x, z)) (\partial_N X(x, z)), \quad (4)$$

where k is a parameter rendering S_G dimensionless, and g is the determinant of the metric tensor.

The field ϕ in Eq. (4) characterizes the holographic model: it represents a background dilaton field depending on the coordinate z only and vanishing at the AdS boundary $z = 0$. It introduces a dimensionful parameter c producing a breaking of conformal invariance. In the model proposed in [19], ϕ is given by the expression

$$\phi(z) = (cz)^2 \quad (5)$$

and allows us to obtain, at $T = 0$, linear Regge trajectories for vector and axial-vector mesons [19], scalar glueballs [25], and light scalar mesons [26]. The parameter c fixes the hadronic scale; at $T = 0$, it can be determined from the spectrum of the vector mesons, in particular, from the ρ mass and is given by [19]

$$c \equiv c_\rho = \frac{m_\rho}{2} = 390 \text{ MeV}, \quad (6)$$

since the relation found for the masses of the vector mesons is $m^2 = 4(n+1)c^2$, with $n = 0, 1, \dots$. We keep the value (6) in the discussion below.

In order to determine the T dependence of the mass spectrum associated to the operator \mathcal{O}_G , we follow the method used in [24] and consider the equation of motion for the field X obtained from (4)

$$e^{-\phi(z)} \sqrt{g} g^{\mu\nu} \partial_\mu \partial_\nu X(x, z) + \partial_z (e^{-\phi(z)} \sqrt{g} g^{zz} \partial_z X(x, z)) = 0 \quad (7)$$

($\mu, \nu = 0, 1, 2, 3$). According to the AdS/CFT dictionary, field/operator duality implies that the function $X_0(x)$ is associated to the field $X(x, z)$ in the AdS space, such that $X_0(x)$ acts, in the generating functional of the boundary theory, as the source of the four-dimensional (gauge invariant) local operator $\mathcal{O}_G(x)$ dual to X . This implies the definition of the bulk-to-boundary propagator K relating the field $X(x, z)$ on the bulk to its value on the boundary $X_0(x')$:

$$X(x, z) = \int d^4x' K(x, z; x', 0) X_0(x') \quad (8)$$

with the condition $K(x, z; x', 0) \rightarrow_{z \rightarrow 0} \delta^4(x - x')$. In the momentum space, the relation involving the bulk-to-boundary propagator \tilde{K} , the field \tilde{X} , and the source \tilde{X}_0 is

$$\tilde{X}(q, z) = \tilde{K}(q, z) \tilde{X}_0(q) \quad (9)$$

with $\tilde{K}(q, 0) = 1$.

From Eqs. (7) and (9), one finds that $\tilde{K}(q, z)$ satisfies the equation

$$\tilde{K}''(q, z) - \frac{4 - f(z) + 2c^2 z^2 f(z)}{z f(z)} \tilde{K}'(q, z) + \left(\frac{q_0^2}{f(z)^2} - \frac{\bar{q}^2}{f(z)} \right) \tilde{K}(q, z) = 0, \quad (10)$$

where $q = (q_0, \bar{q})$ and the primes denote derivatives with respect to the holographic variable z . This equation must be solved for different values of q_0 and \bar{q} , and we first consider the case of vanishing three-momentum $\bar{q} = 0$. Putting $\omega = q_0$, Eq. (10) can be written in terms of the dimensionless variable $u = z/z_h$:

$$\tilde{K}''(\omega^2, u) - \frac{3 + u^4 + 2c^2 z_h^2 u^2 (1 - u^4)}{u(1 - u^4)} \tilde{K}'(\omega^2, u) + \frac{\omega^2 z_h^2}{(1 - u^4)^2} \tilde{K}(\omega^2, u) = 0 \quad (11)$$

with the primes now denoting derivatives with respect to u .

Let us specify the boundary conditions for (11). The solution for $u \rightarrow 0$ reads

$$\tilde{K}(\omega^2, u) \xrightarrow{u \rightarrow 0} A(\omega^2) \left(1 + \frac{\omega^2 z_h^2}{4} u^2 + \dots \right) + B(\omega^2) \left(\frac{c^4 z_h^4}{2} u^4 + \dots \right), \quad (12)$$

so the condition $\tilde{K}(\omega^2, 0) = 1$ fixes the coefficient A : $A(\omega^2) = 1$. For the second boundary condition, we look at the solutions of (11) near the horizon $u = 1$:

$$\tilde{K}_\mp(\omega^2, u) = (1 - u)^{\mp(i\sqrt{\omega^2 z_h^2}/4)}. \quad (13)$$

As discussed in [27], the choice of the boundary condition at the horizon selects the Green function obtained using the AdS/CFT procedure in the Minkowskian space-time. We impose, as a boundary condition, the matching of \tilde{K} with the *in falling* solution of (11) near the black-hole horizon:

$$\tilde{K} \xrightarrow{u \rightarrow 1} \tilde{K}_-(\omega^2, u) \sim (1 - u)^{-(i\sqrt{\omega^2 z_h^2}/4)}, \quad (14)$$

so the matching of the boundary conditions of \tilde{K} for $u \rightarrow 0$ and $u \rightarrow 1$ fixes the coefficient function $B(\omega^2)$, which can be determined numerically [28].

To get the glueball masses, we consider the relation allowing us to compute, within the AdS/CFT correspondence, correlation functions in the gauge theory defined on the boundary of the AdS space starting from the effective action in the 5D bulk theory [5,6] extended to the Minkowskian space-time:

$$\langle e^{i \int d^4x X_0(x) \mathcal{O}_G(x)} \rangle_{\text{CFT}} = e^{i S_{\text{5D}}[X(x, z)]}. \quad (15)$$

In our case, $S_{\text{5D}}[X(x, z)]$ is the action (4) of the bulk field $X(x, z)$ dual to $\mathcal{O}_G(x)$, with $X_0(x)$ as a source term. As discussed in [27], in case of Minkowskian correlators, the retarded two-point Green's function Π_G^R can be derived by differentiating the right-hand side of Eq. (15) with respect

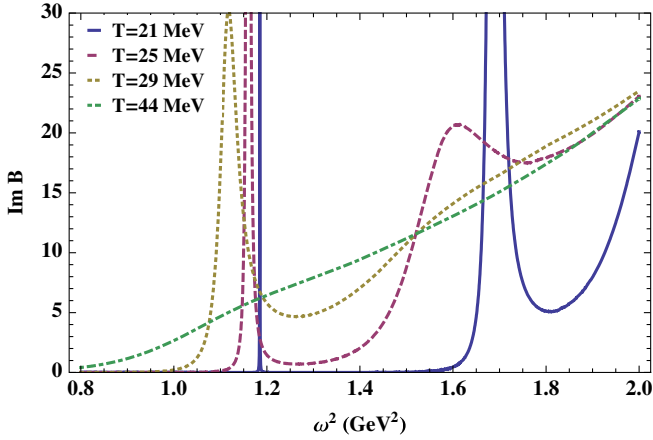


FIG. 1 (color online). Imaginary part of the coefficient $B(\omega^2)$, proportional to the spectral function $\text{Im}\Pi_G^R(\omega^2)$ of Eq. (16), at different temperatures T , in the case of the scalar glueball and using the AdS-BH metric.

to the source and by imposing the boundary conditions discussed above. In terms of the bulk-to-boundary propagator \tilde{K} , Π_G^R reads:

$$\Pi_G^R(\omega^2) = \frac{1}{2k} \frac{R^3 f(u)}{u^3 z_h^4} e^{-\phi(u)} \tilde{K}(\omega^2, u) \partial_u \tilde{K}(\omega^2, u) \Big|_{u=0}. \quad (16)$$

Substituting Eq. (12) into Eq. (16), we determine the spectral function, the imaginary part of $\Pi^R(\omega^2)$, which is proportional to the imaginary part of the coefficient $B(\omega^2)$. The resulting spectral function (modulo a numerical factor) is depicted in Fig. 1 for several values of the physical temperature obtained from the position z_h of the BH horizon, using the value of c in the dilaton background field fixed in (6).

For each value of the temperature, the spectral function in Fig. 1 displays various peaks, which become broader as

T increases. We identify the position of each peak with the mass of scalar glueballs, in particular, the lowest lying state and the first excitation. At small values of T , $T < 20$ MeV, and the results in [25] for the mass spectrum are recovered: $m_G^2 = (4n + 8)c^2$, i.e. $m_G^2 = 1.217 \text{ GeV}^2$ and $m_G^2 = 1.825 \text{ GeV}^2$ for the first two states. By increasing the temperature T , the position of the peaks is shifted toward smaller values, and the widths become broader. Both of these quantities can be determined by fitting the spectral function with a Breit-Wigner form [24]:

$$\rho(\omega^2) = \frac{am\Gamma\omega^b}{(\omega^2 - m^2)^2 + m^2\Gamma^2} \quad (17)$$

with parameters a and b . The results of m^2 and Γ from the fit are shown in Fig. 2(a) for the ground state and for the first excited state. At temperatures below $T \sim 20$ MeV ($T \sim 17$ MeV for the excited state), the horizon of the black hole is far enough and the eigenfunctions vanish before reaching it, so in this range of temperatures, it is possible to determine glueball masses as the eigenvalues of the equation

$$\begin{aligned} & -H''(m^2, u) + \left(\frac{15}{4u^2} + 2c^2 z_h^2 + c^4 z_h^4 u^2 \right) H(m^2, u) \\ & = m^2 z_h^2 H(m^2, u) \end{aligned} \quad (18)$$

coming from a Bogoliubov transformation of Eq. (11) as at $T = 0$ [25]. In the range $T = 20$ – 22 MeV, the results obtained solving the eigenvalue problem and fitting the spectral function coincide.

At $T = 25$ – 30 MeV, the squared glueball mass is reduced to about 80% of its value at $T = 0$; the second peak disappears at $T \sim 29$ MeV, while the peak corresponding to the lowest lying state persists until $T \sim 45$ MeV. Let us discuss in more detail the structure corresponding to the lightest scalar glueball. At $T = 28$ MeV, the width of the first peak in the spectral function becomes sizeable, i.e.

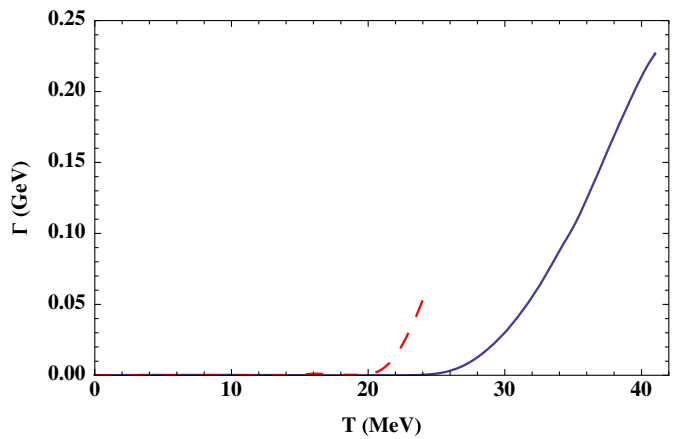
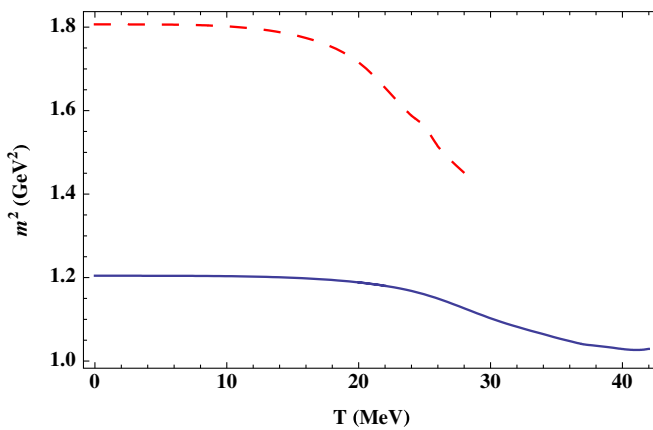


FIG. 2 (color online). Squared mass (left) and width (right) of the lightest glueball state (continuous lines) and of the first excited state (dashed lines) as a function of the temperature T (MeV) in the SW model with AdS-BH metric. Each curve ends at the temperature where the corresponding peak disappears from the spectral function.

larger than 1% of the corresponding mass. We fit the spectral function by a Breit-Wigner term plus a function, which we interpret as representing a continuum, of the simple form $P(x) = a + bx + cx^d$, with $1 < d < 2$ and $x = \omega^2$. We define a melting temperature as the one at which the height of the Breit-Wigner peak obtained after subtracting the continuum contribution from the spectral function is less than 0.05 times the value at $T = 28$ MeV. In this way, we find the temperature $T \approx 45$ MeV. The result of subtracting the continuum is depicted in Fig. 3, which shows the broadening and the shift of the mass of the resonance. Near the melting temperature, we also observe a slight rising of the mass, analogously to what found in Ref. [24]. Using the same criterion, the melting temperature of the first excited state is $T \approx 29$ MeV.

The behavior of the width of the first two peaks with respect to T is shown in Fig. 2. It is qualitatively analogous to the behavior of the scalar glueball mass and width observed in lattice studies [29]; however, the temperature scale is very different.

In [24], in the vector meson channel, a range of temperatures was found where Δm^2 [with $\Delta m = m(T=0) - m(T)$] is linearly related to the width $\Gamma(T)$; in the case of the lowest lying scalar glueball, we find an approximately linear dependence of Δm^2 on $\Gamma(T)$ in the range $T = 20$ –45 MeV, as depicted Fig. 4. Such a relation could represent a benchmark for other approaches to finite T QCD.

For nonvanishing values of the three-momentum, $\vec{q} \neq 0$, the results are similar. The spectral function can be obtained from Eq. (10), imposing the same boundary conditions as for $\vec{q} = 0$. As depicted in Fig. 5 at $T = 30$ MeV and for values of \vec{q}^2 in the range $\vec{q}^2 = 0 - 0.8$ GeV², by increasing \vec{q}^2 , the peaks of the spectral function are shifted towards higher values of q_0^2 and become broader. The difference $q_0^2 - \vec{q}^2$ is not constant, as Lorentz invariance is violated in the finite temperature theory. The same effect,

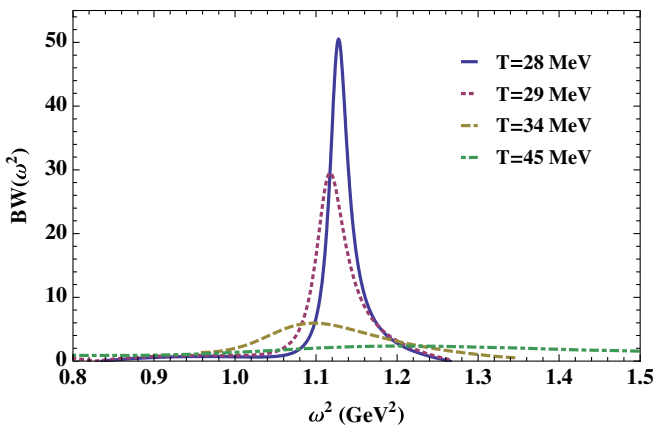


FIG. 3 (color online). Lowest lying resonance in Fig. 1, after subtracting from the spectral function a term representing a continuum; several temperatures are considered.

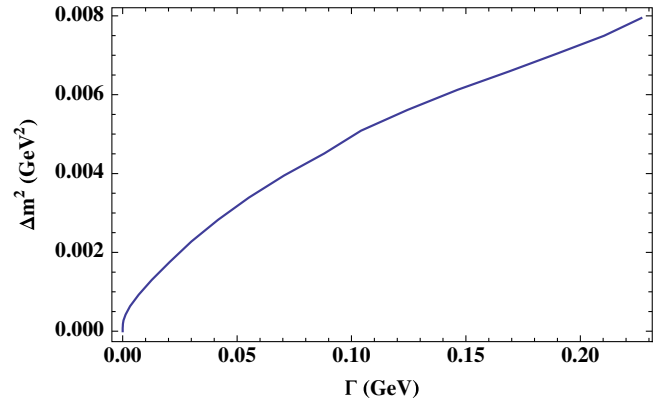


FIG. 4 (color online). Difference $\Delta m^2(T) = (m(T=0) - m(T))^2$ (GeV²) versus the width $\Gamma(T)$ (GeV) varying the temperature T in the range $T = 20$ –45 MeV, in the case of the lightest scalar glueball.

shown in Fig. 5, was found in the case of vector mesons [24].

The qualitative behavior of the mass and width of scalar glueballs in the soft wall model with an AdS-BH metric is similar to the one found in [24] for vector mesons: by increasing T continuously from $T = 0$, the mass of the various states is shifted to lower values while the width increases, and at some critical value of T , the peaks disappear from the spectral function. The broadening of the states and their disappearance can inspire the picture of their melting in the thermalized medium. Moreover, the temperature at which, e.g., the first excitation disappears from the spectral function (dissolves) is lower than the temperature at which the lowest lying state disappears, thus suggesting that lightest resonances persist in the medium at higher temperatures also in the case of glueballs.

However, the physical temperatures at which such phenomena occur are low (30–40 MeV), as the scale is determined by the dimensionful constant c in the background dilaton field (5) and fixed at $T = 0$ to (6). Such low

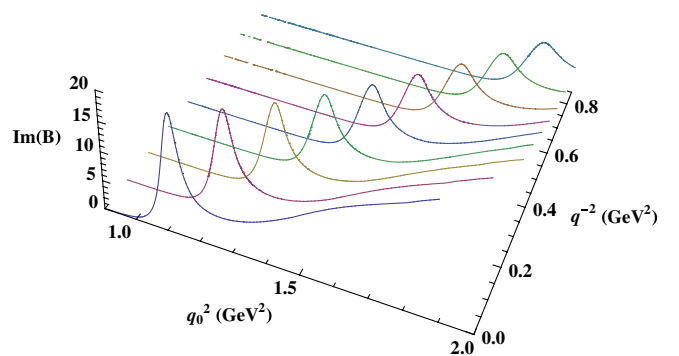


FIG. 5 (color online). Imaginary part of the coefficient $B(q_0^2, \vec{q}^2)$, proportional to the spectral function $\text{Im}\Pi_G^R(q_0^2, \vec{q}^2)$, for the scalar glueball at $T = 30$ MeV, for different values of the three-momentum squared \vec{q}^2 in the range $\vec{q}^2 = 0$ –0.8 GeV², in the SW model with AdS-BH metric.

temperatures are inconsistent with those found, e.g., in lattice calculations of the glueball masses, in which a decrease of about 20% is observed for the mass of the lightest scalar glueball in the range of temperatures $0.81T_c < T < T_c$, with $T_c \simeq 260$ MeV [29]. In the case of vector mesons, in [24], a further assumption was adopted: the scale fixing the temperature in this sector, called $c_{J/\psi}$, is different from the scale $c = c_\rho$ fixed from the ρ vector meson spectrum, considering mesons of $c\bar{c}$ type in the holographic approach. The resulting melting temperature of vectors can be compared to the deconfinement temperature in QCD: since the ratio of the masses of ρ and J/ψ is recovered for $c_{J/\psi} \simeq 4c_\rho$, a value $c_{J/\psi} \simeq 1.56$ GeV produces values of the J/ψ dissociation temperature of about 230 MeV [24].

In the case of glueballs, however, it is unclear how an analogous assumption could be adopted. There are no other scales in terms of which the physical temperature can be expressed, and the only possibility is $c = c_\rho$. Hence, the low physical dissociation temperatures inferred from Figs. 1–5 is not avoidable.

The problem finds a solution if one considers the stability of the metric, i.e. the fact that at a low temperature, the stable metric is thermal AdS, not AdS-BH. However, before considering this issue, let us analyze another sector of QCD, the light scalar mesons.

III. SCALAR MESONS AT $T \neq 0$: SOFT WALL MODEL WITH ADS-BH METRIC

The analysis of the scalar meson sector at $T \neq 0$ in the AdS-BH soft wall model produces results analogous to the case of scalar glueballs. We generalize the 5D action studied in [19] and also considered in [24]:

$$S_{\text{eff}} = \frac{1}{k'} \int d^4x dz e^{-\phi(z)} \sqrt{g} \text{Tr} \times \left\{ |DY|^2 - m_5^2 Y^2 - \frac{1}{4g_5^2} (F_L^2 + F_R^2) \right\}, \quad (19)$$

which includes fields dual to QCD operators defined at the boundary $z = 0$. There is a scalar bulk field Y of mass m_5^2 , written as

$$Y = (Y_0 + S) e^{2i\pi} \quad (20)$$

in terms of a background field $Y_0(z)$, of the scalar field $S(x, z)$, and of the chiral field $\pi(x, z)$. Y_0 is dual to $\langle \bar{q}q \rangle$ (q are light quarks) and represents the term responsible for the chiral symmetry breaking [15,16,19]. The scalar bulk field S includes singlet $S_1(x, z)$ and octet $S_8^a(x, z)$ components, gathered into the multiplet

$$S = S^A T^A = S_1 T^0 + S_8^a T^a, \quad (21)$$

with $T^0 = 1/\sqrt{2n_F} = 1/\sqrt{6}$ and T^a as the generators of $SU(3)_F$ [with normalization $\text{Tr}(T^A T^B) = \frac{\delta^{AB}}{2}$, where $A = 0, a$, and $a = 1, \dots, 8$]. S^A is dual to the QCD operator

$\mathcal{O}_S^A(x) = \bar{q}(x) T^A q(x)$, therefore $m_5^2 R^2 = -3$ from Eq. (3). The action (19) also involves the fields $A_{L,R}^a(x, z)$ introduced to gauge the chiral symmetry in the 5D space; they are dual to the QCD operators $\bar{q}_{L,R} \gamma_\mu T^a q_{L,R}$ (defining $q_{L,R} = \frac{1 \mp \gamma_5}{2} q$), with field strengths

$$F_{L,R}^{MN} = F_{L,R}^{MNa} T^a = \partial^M A_{L,R}^N - \partial^N A_{L,R}^M - i[A_{L,R}^M, A_{L,R}^N]. \quad (22)$$

The gauge fields enter into the covariant derivative: $D^M Y = \partial^M Y - iA_L^M Y + iY A_R^M$. Writing $A_{L,R}$ in terms of vector V and axial-vector A fields, $V^M = \frac{1}{2}(A_L^M + A_R^M)$ and $A^M = \frac{1}{2}(A_L^M - A_R^M)$, the action (19) can be written as

$$S_{\text{eff}} = \frac{1}{k'} \int d^4x dz e^{-\phi(z)} \sqrt{g} \text{Tr} \times \left\{ |DY|^2 - m_5^2 Y^2 - \frac{1}{2g_5^2} (F_V^2 + F_A^2) \right\} \quad (23)$$

with

$$F_V^{MN} = \partial^M V^N - \partial^N V^M - i[V^M, V^N] - i[A^M, A^N], \quad (24)$$

$$F_A^{MN} = \partial^M A^N - \partial^N A^M - i[V^M, A^N] - i[A^M, V^N],$$

and $D^M Y = \partial^M Y - i[V^M, Y] - i\{A^M, Y\}$.

The quadratic part in the field S^A of this 5D action

$$S_S^{(2)} = \frac{1}{2k'} \int d^4x dz e^{-\phi(z)} \sqrt{g} (g^{MN} \partial_M S^A(x, z) \partial_N S^A(x, z) - m_5^2 S^A(x, z) S^A(x, z)) \quad (25)$$

has been studied at $T = 0$ [26] and can be used to analyze the thermal dependence of the mass of the scalar mesons. The dilaton field is given in (5), and the AdS-BH metric is used for all temperatures.

Also in this case, we define the bulk-to-boundary propagator $\tilde{S}(q, z)$, which satisfies the equation of motion

$$\tilde{S}''(q, z) - \frac{2c^2 z^2 f(z) + 3 + \frac{z}{z_h}}{z f(z)} \tilde{S}'(q, z) + \frac{3}{z^2 f(z)} \tilde{S}(q, z) + \left(\frac{q_0^2}{f(z)^2} - \frac{\bar{q}^2}{f(z)} \right) \tilde{S}(q, z) = 0 \quad (26)$$

with $q = (q_0, \bar{q})$.

For $\bar{q} = 0$, this equation, written in terms of the variable $u = \frac{z}{z_h}$, becomes

$$\tilde{S}''(q_0^2, u) - \frac{2c^2 z_h^2 u^2 (1 - u^4) + 3 + u^4}{u(1 - u^4)} \tilde{S}'(q_0^2, u) + \frac{3}{u^2 (1 - u^4)} \tilde{S}(q_0^2, u) + \frac{q_0^2 z_h^2}{(1 - u^4)^2} \tilde{S}(q_0^2, u) = 0 \quad (27)$$

with the primes denoting again the derivative with respect to u . At the horizon $u \rightarrow 1$, the independent solutions of (27) are the same functions as in Eq. (13):

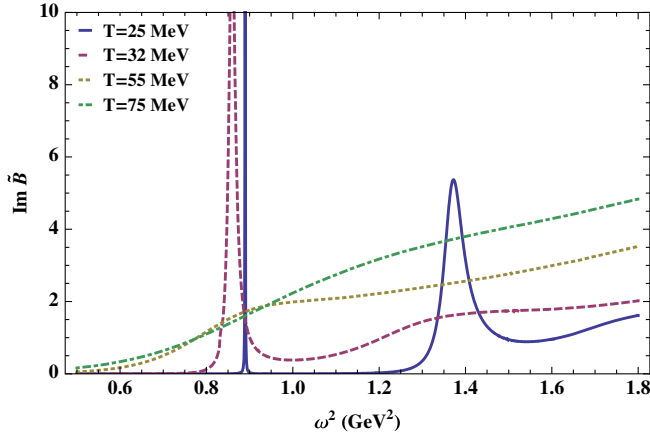


FIG. 6 (color online). Imaginary part of the coefficient $\tilde{B}(\omega^2)$, proportional to the spectral function $\text{Im}\Pi_S^R(\omega^2)$, for several temperatures in the case of scalar mesons, using the AdS-BH metric.

$$\tilde{S}_{\mp}(\omega^2, u) = (1-u)^{\mp} i\sqrt{\omega^2 z_h^2}/4, \quad (28)$$

so we can choose the in falling solution as a boundary condition at the horizon, as in (14). For $u \rightarrow 0$, Eq. (27) admits two independent solutions:

$$\begin{aligned} \tilde{S}_1(\omega^2, u) &= uU\left(\frac{2c^2 - \omega^2}{4c^2}, 0, c^2 z_h^2 u^2\right), \\ \tilde{S}_2(\omega^2, u) &= uL\left(-\frac{2c^2 - \omega^2}{4c^2}, -1, c^2 z_h^2 u^2\right), \end{aligned} \quad (29)$$

where U is the Tricomi confluent hypergeometric function, and L is the generalized Laguerre function. The boundary condition at $u \rightarrow 0$, $\tilde{S}(u) \sim u$ [26], allows us to write the solution

$$\tilde{S}(\omega^2, u) = \tilde{S}_1(\omega^2, u) + \tilde{B}(\omega^2)\tilde{S}_2(\omega^2, u) \quad (30)$$

with \tilde{B} , a function of $\omega^2 = q_0^2$, numerically determined as in the case of the scalar glueball.

The retarded two-point Green's function of the operator $\mathcal{O}_S^A(x)$ can be obtained in terms of \tilde{S} , and the spectral function $\text{Im}\Pi_S^R(\omega^2)$ is proportional to the imaginary part of $\tilde{B}(\omega^2)$. This spectral function (modulo a numerical overall factor) is depicted in Fig. 6 for several values of the temperature.

As in the case of scalar glueballs, the spectral function displays peaks becoming broader when the temperature increases. For low T , the positions of the peaks correspond to the spectral condition $m_S^2 = (4n+6)c^2$ obtained in [26], i.e. $m_S^2 = 0.913 \text{ GeV}^2$ and $m_S^2 = 1.521 \text{ GeV}^2$ for the lightest states. By increasing T , the masses are shifted toward smaller values, and the widths become broader, as shown in Figs. 7(a) and 7(b) for the first two states. At particular values of the temperature, the peaks disappear from the spectral function. At odds with the case of the scalar glueball, the temperature dependence of the mass of the lowest lying state is milder, while the dependence on T of the width is visible from $T \approx 30 \text{ MeV}$, with an abrupt increase with the temperature. For the first excitation, the width starts increasing at $T \approx 25 \text{ MeV}$, and for $T \geq 35 \text{ MeV}$, the peak disappears from the spectral function.

The discussion of these results follows that presented in the previous section. The qualitative dependence of masses and widths versus the temperature T agrees with general expectations, since the particle masses decrease and the widths increase with the temperature. At particular values of T , the peaks disappear from the spectral function (melt); the lightest state survives after the dissolution of the excited states, a behavior which seems universal in all sectors considered so far. However, also in this case, such phenomena occur at a low temperature ($T \approx 40\text{--}60 \text{ MeV}$), unless one once again invokes the presence of a different scale ($c_{\text{scalar}} \neq c_{\rho}$) to fix the physical temperatures. Without such an assumption, scalar meson dissociation

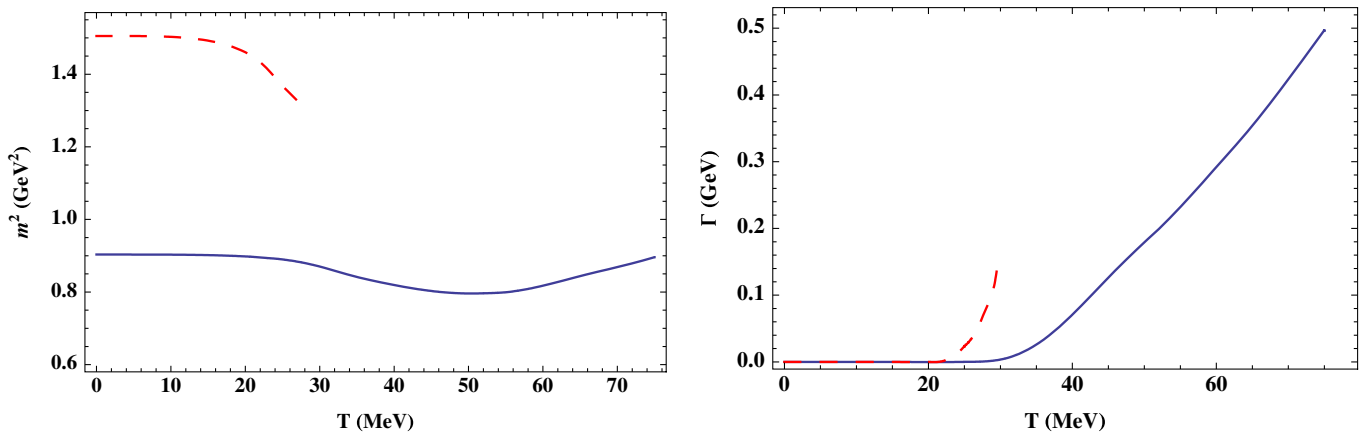


FIG. 7 (color online). Squared mass (left) and width (right) of the lightest scalar meson (continuous lines) and of the first excited state (dashed lines) as a function of the temperature T (MeV) in the SW model with AdS-BH metric. Each curve ends at the temperature where the corresponding peak disappears from the spectral function.

occurs in the QCD confined phase, far from the deconfinement transition.

IV. MODELS WITH THE HAWKING-PAGE TRANSITION

According to the analysis of the minimum of the free energy, in the soft wall model, the AdS-BH metric is stable only at high temperatures, $T \gtrsim 192$ MeV [21]. At low temperatures, the stable metric is thermal AdS, and the first order Hawking-Page transition to the AdS-BH metric is associated to the deconfinement transition in QCD [7,21]. Following these hints, the temperature dependence of hadron properties, such as the mass, must be evaluated, at a low temperature, using thermal AdS metric, while for $T \gtrsim T_{\text{HP}}$, one should use the holographic model based on the AdS-BH metric described in Secs. II and III.

In the case of thermal AdS metric, the equations of motion are the same as at $T = 0$. From the calculation of the two-point Green's functions and of their spectral functions at $T \neq 0$, one obtains the same masses as at $T = 0$. Therefore, for temperature T up to T_{HP} , the scalar glueball and scalar meson mass are given by the spectral formulae derived in [25,26].

On the other hand, for $T \gtrsim T_{\text{HP}}$, the results in Secs. II and III show that in both the scalar glueball and scalar meson sectors, no peaks appear in the spectral functions; dissociation has already occurred at these temperatures. At $T = T_{\text{HP}}$, when the black hole appears in the metric, the masses jump from $m^2 \neq 0$ to $m^2 = 0$; dissociation occurs together with deconfinement, as it could be expected in a discontinuous transition. This is shown in Fig. 8. As observed in [21], the temperature independence of the mass spectrum below T_c is consistent with large N_c expectations and is supported by chiral perturbation theory analyses [30].

The same conclusion holds for the vector mesons considered in [24]: the temperature dependence of the vector

meson mass and the broadening of the width in a metastable phase, such as AdS-BH at $T \leq T_{\text{HP}}$, are different than in the stable phase, and a model aimed at describing QCD should take the difference into account. However, in modifications of the soft wall model as proposed, e.g., in [31], the possibility of the persistence of some hadron resonances above T_c is not excluded, and needs to be investigated by a dedicated study.

V. CONCLUSIONS

Without considering the existence of the HP critical point, the SW model is able to reproduce only qualitatively some commonly expected features of finite temperature QCD, like in-medium mass shifts and width broadening, but at temperatures different from those found by lattice QCD simulations. Using the AdS-BH for all values of T would imply that scalar glueballs and scalar mesons disappear from the spectral functions (melt) at temperatures of about 40–60 MeV. On the other hand, in a holographic description based on SW with thermal AdS geometry below the Hawking-Page transition temperature, and AdS-BH geometry above this temperature, the hadronic states are found to persist in the confined phase and melt at deconfinement. This suggests that a more refined dual model of finite temperature QCD could be found modifying the background dilaton, so the qualitative behavior is preserved while the temperature scale is enlarged.

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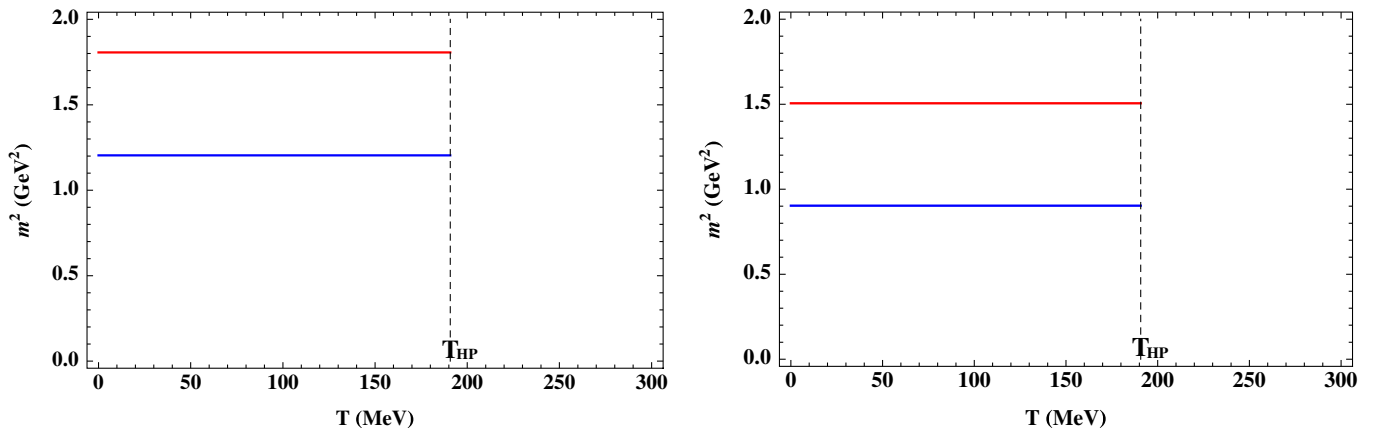


FIG. 8 (color online). Squared mass of the two lightest scalar glueballs (left) and of the two lightest scalar mesons (right) as a function of the temperature T in the soft wall model with HP transition.

APPENDIX: SCALAR GLUEBALL AT $T \neq 0$ IN THE HARD WALL MODEL

It is interesting to consider scalar glueballs at finite T in the hard wall holographic model of QCD. In this model, an AdS slice is used, up to a maximum value of z , z_m , and there is no dilatonlike background field. Therefore, it is sufficient to put $c = 0$ in the equations obtained in Sec. II, imposing suitable boundary conditions. Using the AdS-BH metric for all values of T , as done in an analysis of the static potential in QCD [32], two cases are possible. In the first one, the black-hole horizon is located beyond the IR cutoff z_m : $z_m < z_h$; in the second one $z_h < z_m$. In the first case, regardless of position of the horizon, the eigenfunctions can be obtained by solving the equation of motion with boundary conditions:

$$X(m^2, 0) = 0, \quad X'(m^2, z_m) = 0,$$

hence determining the mass squared of the scalar glueballs.

When $z_m > z_h$, the horizon position z_h becomes the only mass scale, and the equation of motion reads

$$\begin{aligned} \tilde{K}''(m_h^2, u) - \frac{3 + u^4}{u(1 - u^4)} \tilde{K}'(m_h^2, u) \\ + \frac{m_h^2}{(1 - u^4)^2} \tilde{K}(m_h^2, u) = 0, \quad 0 < u < 1, \end{aligned} \quad (\text{A1})$$

where $m_h = mz_h$. The masses can be obtained, imposing that the solution of (A1) is the in falling solution into the black hole at $u \rightarrow 1$ and considering the spectral function of the retarded Green's function. Once having obtained m_h^2 , the glueball mass is given by $m^2 = m_h^2 \pi^2 T^2$, hence non-vanishing values of m linearly increase with the temperature T . However, the spectral function depicted in Fig. 9 has only one peak at $m_h^2 = 0$; therefore, when $z_h < z_m$ [or $T \geq 1/(\pi z_h)$] the only value found for the scalar glueball mass is $m^2 = 0$. The resulting plot of the squared masses at various temperatures T is shown in Fig. 10: dissociation occurs when $z_m = z_h$. This corresponds to $T \approx 103$ MeV, using the value of z_m fixed from the mass of the ρ meson in the HW model: $z_m = \frac{1}{323} \text{ MeV}^{-1}$ [15]. Analogous results hold for mesons in HW at $T \neq 0$ [33].

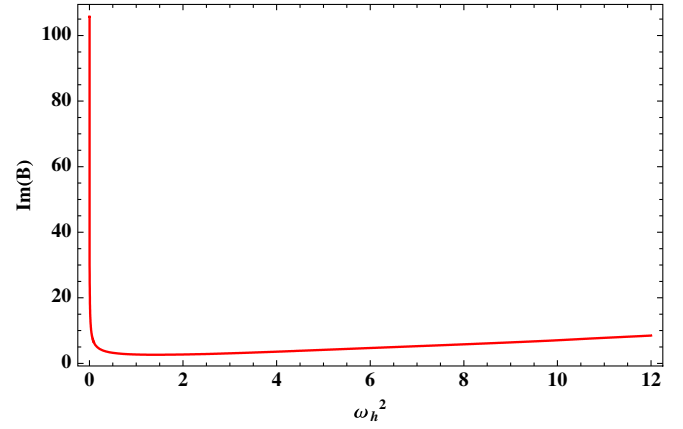


FIG. 9 (color online). Imaginary part of the coefficient $B(\omega_h^2)$, proportional to the spectral function $\text{Im}\Pi_G^R(\omega_h^2)$, in the hard wall model with AdS-BH metric, for $z_h < z_m$. A peak is found only at $\omega_h^2 = 0$.

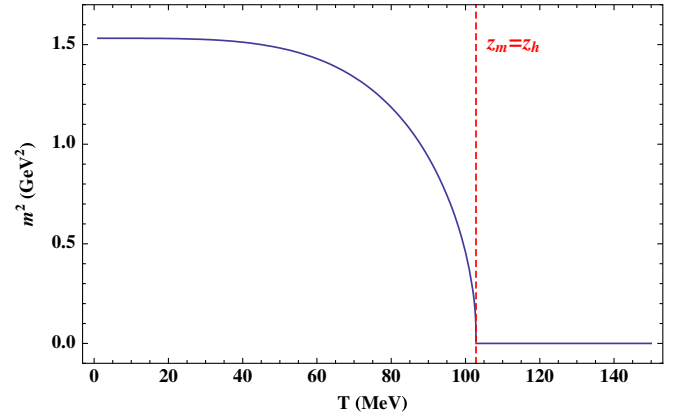


FIG. 10 (color online). Squared mass of the lightest scalar glueball as a function of the temperature T in the hard wall model with AdS-BH.

Imposing the presence of the Hawking-Page transition, which occurs at $T_{\text{HP}} = 2^{1/4}/(\pi z_m) \sim 122$ MeV, i.e. for $z_h < z_m$, the masses do not vary up to the critical temperature T_{HP} , at which they jump to $m^2 = 0$.

After completing this work, we noticed the preprint [34], where an analysis of glueballs at finite temperature similar to the one here has been carried out.

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