

Dijet imbalance in hadronic collisionsDaniël Boer,^{1,2,*} Piet J. Mulders,^{1,†} and Cristian Pisano^{1,3,‡}¹*Department of Physics and Astronomy, Vrije Universiteit Amsterdam, NL-1081 HV Amsterdam, The Netherlands*²*Theory Group, KVI, University of Groningen, Zernikelaan 25, NL-9747 AA Groningen, The Netherlands*³*Dipartimento di Fisica, Università di Cagliari, and INFN, Sezione di Cagliari, I-09042 Monserrato (CA), Italy*

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The imbalance of dijets produced in hadronic collisions has been used to extract the average transverse momentum of partons inside the hadrons. In this paper we discuss new contributions to the dijet imbalance that could complicate or even hamper this extraction. They are due to polarization of initial state partons inside unpolarized hadrons that can arise in the presence of nonzero parton transverse momentum. Transversely polarized quarks and linearly polarized gluons produce specific azimuthal dependences of the two jets that in principle are not suppressed. Their effects cannot be isolated just by looking at the angular deviation from the back-to-back situation; rather they enter jet broadening observables. In this way they directly affect the extraction of the average transverse momentum of unpolarized partons that is thought to be extracted. We discuss appropriately weighted cross sections to isolate the additional contributions.

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I. INTRODUCTION

Event shape observables have been widely studied for various reasons. In e^+e^- annihilation, observables such as the thrust and jet broadening have been studied primarily to extract $\alpha_s(M_z)$ (cf. for instance Refs. [1–6] for theoretical studies and Refs. [7–15] for experimental studies). In the center-of-mass system (cms) of the e^+e^- collisions at lowest order in α_s , the produced quark-antiquark pair is exactly back-to-back leading for two-jet events to a thrust T equal to unity. Gluon radiation, i.e. order α_s corrections, gives rise to nonzero $1 - T$ and also to nonzero jet broadening. In the perturbative regime these observables can be used to extract α_s , which has been done recently at next-to-next-to-leading order [16–18]. The results compare very well with those obtained by other means of extraction. In the nonperturbative regime, hadronization will also lead to nonzero event or jet shapes. This is characterized by a mean transverse momentum $\langle k_\perp \rangle$, leading in general to a contribution suppressed by a power of the large scale, the cms energy Q . For example, in the nonperturbative regime $1 - T \propto \langle k_\perp \rangle / Q$. It has been suggested that this contains universal information on α_s in the infrared regime. We refer to Ref. [19] for a review on this topic.

Event and jet shapes have also been studied in hadronic collisions. Compared to e^+e^- annihilation, here the additional complication of initial parton transverse momenta arises. Another difference is that instead of the thrust axis, it is common to use the transverse thrust axis n_t , which is the axis in the transverse plane having maximum transverse energy flow. The corresponding transverse thrust is

defined as [20]

$$T_t = \max \sum_{i=1}^n \frac{|\mathbf{p}_T^i \cdot \mathbf{n}_t|}{E_T}, \quad (1)$$

where \mathbf{p}_T^i is the transverse momentum of the outgoing hadron i , $E_T = \sum_i |\mathbf{p}_T^i|$ is the total transverse energy (neglecting masses), and the transverse thrust axis is the transverse unit vector \mathbf{n}_t that maximizes T_t . Here we use the notation of Ref. [20], where also the jet broadening variable Q_t is defined as

$$Q_t = \sum_{i=1}^n |\mathbf{p}_T^i \times \mathbf{n}_t|. \quad (2)$$

An experimental investigation of the average Q_t as a function of E_T in $p\bar{p}$ collisions has been reported in Ref. [21]. Higher order perturbative corrections to the transverse thrust and jet broadening are discussed in e.g. Refs. [22,23].

Assuming collinear factorization and ignoring broadening from hadronization, Q_t will be zero for $2 \rightarrow 2$ partonic subprocesses and only sensitive to $2 \rightarrow 3$ processes, like for e^+e^- annihilation except that there are more subprocesses to consider in hadronic collisions. Extraction of α_s in hadronic collisions [24,25] is however complicated due to the presence of parton transverse momenta and the transverse momentum distribution of hadrons inside the jet. The former effect one can minimize by considering events with at least three pronounced jets, which means considering only large values of Q_t , whereas the latter effect could be minimized by considering Q_t for jets, instead of hadrons. In fact, the quantity Q_t for two-jet events, where i now denotes the i th jet and $n = 2$, has been used to study and extract the average parton transverse momentum. This has been done for instance in

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Refs. [26–35]. As can be seen from those results, the average parton transverse momentum extracted from the data increases with energy (\sqrt{s}) and is in general much too large to be attributable to “intrinsic” transverse momentum. This is a consequence of soft parton radiation, similar to what happens for the transverse momentum distribution of vector boson production in hadronic processes [36–39] (see also the instructive discussion in Ref. [40]). Resummation of soft radiation effectively broadens the transverse momentum dependence of the parton distributions, increasingly so with increasing center-of-mass energy.

In this paper we point out that besides initial parton transverse momentum and soft parton radiation, there are additional contributions to Q_t , even for the simplest two-jet case. These are contributions due to the transverse polarization of quarks and the linear polarization of gluons inside the initial unpolarized hadrons. These contributions can arise for nonzero initial parton transverse momentum. We will show how these effects contribute to Q_t and discuss that besides complicating the extraction of the average parton transverse momentum from Q_t , they may even hamper that extraction altogether depending on their magnitude.

In Ref. [20] collinear factorization was assumed, making the observable $\langle Q_t \rangle$ only sensitive to $2 \rightarrow 3$ subprocesses. In reality collinear factorization is not always applicable, due to the partonic transverse momentum effects. In a simple picture of a Gaussian distribution of intrinsic parton momentum p_t , the average value $\langle p_t \rangle$ can be extracted from $\langle Q_t \rangle$, but in fact, no factorization theorem has been established for two-jet or two-hadron production in pp or $p\bar{p}$ collisions for observables that are sensitive to parton transverse momenta. To make matters worse, in the framework of transverse momentum dependent parton distribution functions, nowadays commonly referred to as TMDs, it even seems that factorization cannot be established for this particular type of process when taking into account nontrivial effects of gauge links [41–45]. This would cast doubt on any conclusion drawn from $\langle Q_t \rangle$ in hadronic collisions, except for large Q_t where collinear factorization can be applied. But even if factorization will work out in some as yet unknown way, the additional contributions from spin-dependent TMDs may complicate matters considerably. Schematically this can be seen as follows.

Consider the process $h_1 h_2 \rightarrow j_1 j_2 X$, where j_i stands for produced jet i . In the plane transverse to the collision axis, $\delta\phi$ denotes the deviation of the (azimuthal) angle between the two jets from π , i.e. $\delta\phi = \phi_{j_1} - \phi_{j_2} - \pi$. It is sometimes referred to as the dijet imbalance. Let us consider only $2 \rightarrow 2$ subprocesses. In collinear factorization the $\delta\phi$ dependence of the cross section will then only receive a contribution at $\delta\phi = 0$. Allowing for parton transverse momentum in the initial hadrons leads to a smearing of the $\delta\phi$ distribution. For the idealized case of equal jet

transverse momenta (both equal to $E_T/2$) the differential cross section takes the form

$$\frac{d\sigma}{dE_T d\delta\phi} = A(Q_t^2) + B(Q_t^2)Q_t^2 + C(Q_t^2)Q_t^4, \quad (3)$$

where $Q_t = E_T |\sin(\delta\phi/2)|$ is equal to the absolute value of the transverse momentum of the two-jet system. A , B , C are functions of Q_t^2 , which do not need to vanish at $Q_t^2 = 0$. The terms B and C appear from spin effects inside the initial hadrons h_i , for which expressions will be presented in this paper. In general these spin-dependent contributions are not suppressed by powers of $1/E_T$, also not when arising from polarized gluons as claimed in Ref. [46]. A result for B has recently been obtained in [46] following a calculation similar to the one for $p\bar{p} \rightarrow \gamma j X$ presented in [47]. This contribution arises from the quark TMD $h_1^{\perp q}$ [48], which represents the distribution of transversely polarized quarks inside an unpolarized hadron. The new result in this paper is the contribution from $h_1^{\perp g}$ [49], the distribution of linearly polarized gluons inside an unpolarized hadron, which gives rise to C . Upon ignoring these spin effects, only the term A remains, and the average Q_t value in that case will indeed be directly related to the average transverse momentum that is thought to be extracted in Refs. [26–35]. Our results in principle cast doubt on whether the actual value of $\langle k_t^2 \rangle$ has been extracted in those cases. In practice, it all depends on the magnitude of B and C . We will present a simple Gaussian model to illustrate the generic shape of the modification of the dijet imbalance distribution by B and C terms.

The paper is organized as follows. First we will present the calculation and expressions for the cross section in Eq. (3), assuming factorization in terms of transverse momentum dependent correlators and ignoring the possible effects from gauge links. We will actually discuss the more general case in which the two-jet transverse momenta are not equal, but differ by a small amount with respect to E_T . In that case the angular dependence is more involved than given in Eq. (3), even upon expansion in the small transverse momentum difference of the two jets with respect to their sum. We will first express the cross section in terms of the individual jet momenta through their sum and difference [Sec. II, in particular, Eq. (16)] and subsequently in terms of the sum and difference of the lengths of the jet momenta in order to arrive at the dijet imbalance distribution expressed in more standard variables [Sec. III, in particular, Eq. (51)]. In Sec. IV we discuss angular-projected asymmetries, such as $\langle \cos\delta\phi \rangle$ and the ones that can be used to extract B and C . After that we consider the consequences of nonzero h_1^{\perp} functions for the jet broadening quantity Q_t , in particular, for the averages $\langle Q_t \rangle$ and $\langle Q_t^2 \rangle$. Finally (Sec. VI) we briefly address the open issues of factorization (breaking) and color flow dependence upon inclusion of gauge links. We end with conclusions and two appendixes, one on relations among

various variables in the transverse plane and one on photon-jet production that completes the treatment given in Ref. [47].

II. THEORETICAL FRAMEWORK: CALCULATION OF THE CROSS SECTION

We consider the process

$$h_1(P_1) + h_2(P_2) \rightarrow \text{jet}(K_1) + \text{jet}(K_2) + X, \quad (4)$$

where the four-momenta of the particles are given within brackets, and the jet-jet pair in the final state is almost back-to-back in the plane perpendicular to the direction of the incoming hadrons. Along the lines of Ref. [47], we will instead of collinear factorization consider a generalized factorization scheme taking into account partonic transverse momenta. We make a light cone decomposition of the two incoming hadronic momenta in terms of the light-like Sudakov vectors n_+ and n_- , satisfying $n_+^2 = n_-^2 = 0$ and $n_+ \cdot n_- = 1$:

$$P_1^\mu = P_1^+ n_+^\mu + \frac{M_1^2}{2P_1^+} n_-^\mu, \quad \text{and} \quad P_2^\mu = \frac{M_2^2}{2P_2^-} n_+^\mu + P_2^- n_-^\mu. \quad (5)$$

The partonic momenta (p_1, p_2) can be expressed in terms of the light cone momentum fractions (x_1, x_2) and the intrinsic transverse momenta (p_{1T}, p_{2T}) , as follows:

$$p_1^\mu = x_1 P_1^+ n_+^\mu + \frac{p_{1+}^2 + p_{1T}^2}{2x_1 P_1^+} n_-^\mu + p_{1T}^\mu, \quad \text{and} \quad (6)$$

$$p_2^\mu = \frac{p_{2+}^2 + p_{2T}^2}{2x_2 P_2^-} n_+^\mu + x_2 P_2^- n_-^\mu + p_{2T}^\mu.$$

In general n_+ and n_- will define the light cone components of every vector a as $a^\pm \equiv a \cdot n_\pm$, while perpendicular vectors a_\perp will always refer to the components of a orthogonal to both incoming hadronic momenta, P_1 and P_2 . Therefore in Eq. (6), if we neglect hadron masses, $p_{1T}^\mu = p_{1\perp}^\mu$ and $p_{2T}^\mu = p_{2\perp}^\mu$. We denote with s the total energy squared in the hadronic cms frame, $s = (P_1 + P_2)^2 = E_{\text{cms}}^2$, and with η_i the pseudorapidities of the outgoing partons, i.e. $\eta_i = -\ln(\tan(\frac{1}{2}\theta_i))$, θ_i being the polar angles of the outgoing partons in the same frame. Finally, we introduce the partonic Mandelstam variables

$$\hat{s} = (p_1 + p_2)^2, \quad \hat{t} = (p_1 - K_1)^2, \quad \hat{u} = (p_1 - K_2)^2, \quad (7)$$

which satisfy the relations

$$-\frac{\hat{t}}{\hat{s}} \equiv y = \frac{1}{e^{\eta_1 - \eta_2} + 1}, \quad \text{and} \quad -\frac{\hat{u}}{\hat{s}} = 1 - y. \quad (8)$$

Following Refs. [47,50] we assume that at sufficiently high energies the hadronic cross section factorizes in a soft parton correlator for each observed hadron and a hard part:

$$d\sigma^{h_1 h_2 \rightarrow \text{jet jet } X} = \frac{1}{2s} \frac{d^3 K_1}{(2\pi)^3 2E_1} \frac{d^3 K_2}{(2\pi)^3 2E_2} \\ \times \int dx_1 d^2 p_{1T} dx_2 d^2 p_{2T} (2\pi)^4 \\ \times \delta^4(p_1 + p_2 - K_1 - K_2) \\ \times \sum_{a,b,c,d} \Phi_a(x_1, \mathbf{p}_{1T}) \otimes \Phi_b(x_2, \mathbf{p}_{2T}) \\ \otimes |H_{ab \rightarrow cd}(p_1, p_2, K_1, K_2)|^2. \quad (9)$$

This form assumes the simplest possible factorization omitting any gauge link dependence in the correlators, which can modify or even break the factorization (see Sec. VI for a discussion of these open issues).

In Eq. (9) the sum runs over all the incoming and outgoing partons taking part in the reaction. The convolutions \otimes indicate the appropriate traces over Dirac indices and $|H|^2$ is the hard partonic squared amplitude. The parton correlators are defined on the light front LF ($\xi \cdot n \equiv 0$, with $n \equiv n_-$ for parton 1 and $n \equiv n_+$ for parton 2); they describe the hadron \rightarrow parton transitions and can be parametrized in terms of TMD distribution functions. In particular, the quark content of an unpolarized hadron is at leading twist (omitting gauge links) described by the correlator [48]

$$\Phi_q(x, \mathbf{p}_T) = \int \frac{d(\xi \cdot P) d^2 \xi_T}{(2\pi)^3} e^{i p \cdot \xi} \langle P | \bar{\psi}(0) \psi(\xi) | P \rangle_{\text{LF}} \\ = \frac{1}{2} \left\{ f_1^q(x, \mathbf{p}_T^2) \not{P} + i h_1^{\perp q}(x, \mathbf{p}_T^2) \frac{[\not{p}_T, \not{P}]}{2M} \right\}, \quad (10)$$

where $f_1^q(x, \mathbf{p}_T^2)$ is the unpolarized quark distribution, which integrated over \mathbf{p}_T gives the familiar light cone momentum distribution $f_1^q(x)$. The time-reversal (T) odd function $h_1^{\perp q}(x, \mathbf{p}_T^2)$ is interpreted as the quark transverse spin distribution in an unpolarized hadron [48]. Analogously, for an antiquark,

$$\bar{\Phi}_q(x, \mathbf{p}_T) = \int \frac{d(\xi \cdot P) d^2 \xi_T}{(2\pi)^3} e^{-i p \cdot \xi} \langle P | \bar{\psi}(0) \psi(\xi) | P \rangle_{\text{LF}} \\ = \frac{1}{2} \left\{ f_1^{\bar{q}}(x, \mathbf{p}_T^2) \not{P} + i h_1^{\perp \bar{q}}(x, \mathbf{p}_T^2) \frac{[\not{p}_T, \not{P}]}{2M} \right\}. \quad (11)$$

The gluon correlator (omitting gauge links) is given by [49]

$$\Phi_g^{\mu\nu}(x, \mathbf{p}_T) = \frac{n_\rho n_\sigma}{(p \cdot n)^2} \int \frac{d(\xi \cdot P) d^2 \xi_T}{(2\pi)^3} \\ \times e^{i p \cdot \xi} \langle P | \text{Tr}[F^{\mu\rho}(0) F^{\nu\sigma}(\xi)] | P \rangle_{\text{LF}} \\ = \frac{1}{2x} \left\{ -g_T^{\mu\nu} f_1^g(x, \mathbf{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M^2} + g_T^{\mu\nu} \frac{\mathbf{p}_T^2}{2M^2} \right) \right. \\ \left. \times h_1^{\perp g}(x, \mathbf{p}_T^2) \right\}, \quad (12)$$

with $g_T^{\mu\nu}$ being a transverse tensor defined as

$$g_T^{\mu\nu} = g^{\mu\nu} - n_+^\mu n_-^\nu - n_-^\mu n_+^\nu. \quad (13)$$

The function $f_1^g(x, \mathbf{p}_T^2)$ represents the unpolarized gluon distribution, while the T -even function $h_1^{\perp g}(x, \mathbf{p}_T^2)$ is the distribution of linearly polarized gluons in an unpolarized hadron.

In order to derive an expression for the cross section in terms of parton distributions, we insert the parametrizations in Eqs. (10)–(12) of the TMD correlators into Eq. (9). Furthermore, utilizing the decompositions of the parton momenta in Eq. (6), the δ function in Eq. (9) can be rewritten as

$$\begin{aligned} & \delta^4(p_1 + p_2 - K_1 - K_2) \\ &= \frac{2}{s} \delta\left(x_1 - \frac{1}{\sqrt{s}}(|\mathbf{K}_{1\perp}|e^{\eta_1} + |\mathbf{K}_{2\perp}|e^{\eta_2})\right) \\ & \quad \times \delta\left(x_2 - \frac{1}{\sqrt{s}}(|\mathbf{K}_{1\perp}|e^{-\eta_1} + |\mathbf{K}_{2\perp}|e^{-\eta_2})\right) \\ & \quad \times \delta^2(\mathbf{p}_{1T} + \mathbf{p}_{2T} - \mathbf{K}_{1\perp} - \mathbf{K}_{2\perp}), \end{aligned} \quad (14)$$

with corrections of order $\mathcal{O}(1/s)$. After integration over x_1 and x_2 , from the first two δ functions on the right-hand side of Eq. (14), one obtains

$$\begin{aligned} x_1 &= \frac{1}{\sqrt{s}}(|\mathbf{K}_{1\perp}|e^{\eta_1} + |\mathbf{K}_{2\perp}|e^{\eta_2}), \\ x_2 &= \frac{1}{\sqrt{s}}(|\mathbf{K}_{1\perp}|e^{-\eta_1} + |\mathbf{K}_{2\perp}|e^{-\eta_2}), \end{aligned} \quad (15)$$

which relates the partonic momentum fractions x_1, x_2 to the rapidities and the transverse momenta of the jets. These basic tree-level relations will be used in our treatment. We will not consider several other effects that need to be accounted for in practice such as the actually used jet definition and higher order corrections that affect the above relations and cause additional smearing.

The hadronic cross section can be written in the form

$$\begin{aligned} \frac{d\sigma^{h_1 h_2 \rightarrow \text{jet jet } X}}{d\eta_1 d\eta_2 d^2\mathbf{K}_{1\perp} d^2\mathbf{K}_{2\perp}} &= \frac{\alpha_s^2}{s\mathbf{K}_\perp^2} [A(\mathbf{q}_T^2) + B(\mathbf{q}_T^2)\mathbf{q}_T^2 \\ & \quad \times \cos 2(\phi_T - \phi_\perp) \\ & \quad + C(\mathbf{q}_T^2)\mathbf{q}_T^4 \cos 4(\phi_T - \phi_\perp)], \end{aligned} \quad (16)$$

where $\mathbf{q}_T \equiv \mathbf{K}_{1\perp} + \mathbf{K}_{2\perp}$ and $\mathbf{K}_\perp \equiv (\mathbf{K}_{1\perp} - \mathbf{K}_{2\perp})/2$. The sum momentum \mathbf{q}_T is useful as an in principle accessible experimental observable momentum which in our calculations via the delta function in Eq. (14) is related to intrinsic transverse momenta, $\mathbf{q}_T = \mathbf{p}_{1T} + \mathbf{p}_{2T}$. We denote with ϕ_T and ϕ_\perp the azimuthal angles of \mathbf{q}_T and \mathbf{K}_\perp , respectively. Besides \mathbf{q}_T^2 , the terms A, B , and C depend on other kinematic variables often not explicitly indicated, namely, y, x_1, x_2 , and contain convolutions of the various parton distributions. These are discussed separately in the following three subsections, where explicit expressions for them can be found, calculated at leading order in pertur-

bative QCD. In deriving these expressions we will often employ the approximation $|\mathbf{q}_T| \ll |\mathbf{K}_{1\perp}| \approx |\mathbf{K}_{2\perp}| \approx |\mathbf{K}_\perp|$ which is applicable in the situation in which the two jets are almost back-to-back in the transverse plane. However, in deriving Eq. (16) we must be particularly careful with the angular dependence, because approximations in the angular dependence that boil down to approximating $\phi_1 \approx \phi_2 + \pi$ [such as in Eq. (21) of Ref. [47]] will of course not give the proper dependence of the dijet imbalance angle $\delta\phi = \phi_1 - \phi_2 + \pi$. In Eq. (16) the combination $\phi_T - \phi_\perp$ appears, which will allow isolating the terms B and C by \mathbf{q}_T^2 -weighted integration over \mathbf{q}_T (cf. Sec. IV). However, in order to arrive at the $\delta\phi$ distribution discussed in the introduction, it is more convenient to express the cross section in terms of the combination $\phi_T - \phi_j$, where ϕ_j is the average jet-direction angle, i.e. $\phi_j = (\phi_1 + \phi_2 - \pi)/2$ with ϕ_1 and ϕ_2 the azimuthal angles of the two outgoing jets in the transverse plane. In the present case where $|\mathbf{K}_\perp| \gg (|\mathbf{K}_{1\perp}| - |\mathbf{K}_{2\perp}|)$, it holds that $\phi_\perp \approx \phi_j$ allowing the two angles to be identified to good approximation for all values of $\delta\phi$ [cf. Eq. (A8)]. In the limiting case when $|\mathbf{K}_{1\perp}| = |\mathbf{K}_{2\perp}|$, the angles ϕ_\perp and ϕ_j exactly coincide and the T and \perp directions are orthogonal, so we have exactly $\cos 2(\phi_T - \phi_\perp) = -1$ [note that this will lead to Eq. (3) with a minus sign in front of B , but that is of course only a matter of definition] and $\cos 4(\phi_T - \phi_\perp) = 1$. This implies that all angular dependence then resides in \mathbf{q}_T^2 , which in that case solely depends on the off-collinearity of the jets through the dijet imbalance angle $\delta\phi$ (discussed in Sec. III).

A. Angular independent part of the cross section

The term A in Eq. (16) is the angular independent part of the cross section and is given by the sum of several contributions $\mathcal{A}^{ab \rightarrow cd}$ coming from the partonic subprocesses $ab \rightarrow cd$ underlying the reaction $h_1 h_2 \rightarrow \text{jet jet } X$:

$$A(y, x_1, x_2, \mathbf{q}_T^2) = \sum_{a,b,c,d} \mathcal{A}^{ab \rightarrow cd}(y, x_1, x_2, \mathbf{q}_T^2), \quad (17)$$

with $a, \dots, d = q, q', \bar{q}, \bar{q}', g$. We denote with q and q' two quarks having different flavors, and similar notation holds for the antiquarks. Furthermore, the following convolutions of unpolarized parton distributions are defined

$$\begin{aligned} \mathcal{F}^{ab}(x_1, x_2, \mathbf{q}_T^2) &\equiv \int d^2\mathbf{p}_{1T} d^2\mathbf{p}_{2T} \delta^2(\mathbf{p}_{1T} + \mathbf{p}_{2T} - \mathbf{q}_T) \\ & \quad \times f_1^a(x_1, \mathbf{p}_{1T}^2) f_1^b(x_2, \mathbf{p}_{2T}^2), \end{aligned} \quad (18)$$

where a sum over all (anti)quark flavors is understood. Our results for the terms $\mathcal{A}^{ab \rightarrow cd}$ in Eq. (17) are listed below, starting from the ones corresponding to the (anti)quark induced processes,

$$\begin{aligned} \mathcal{A}^{qq' \rightarrow qq'} &= a(y) \mathcal{F}^{qq'}(x_1, x_2, \mathbf{q}_T^2) \\ & \quad + a(1-y) \mathcal{F}^{q'q}(x_1, x_2, \mathbf{q}_T^2), \end{aligned} \quad (19)$$

$$\begin{aligned} \mathcal{A}^{q\bar{q}'\rightarrow q\bar{q}'} &= a(y)\mathcal{F}^{q\bar{q}'}(x_1, x_2, \mathbf{q}_T^2) \\ &+ a(1-y)\mathcal{F}^{\bar{q}'q}(x_1, x_2, \mathbf{q}_T^2), \end{aligned} \quad (20)$$

$$\begin{aligned} \mathcal{A}^{qq\rightarrow qq} &= \frac{N^2-1}{2N^2}y(1-y)\left[\frac{1+(1-y)^2}{y^2} + \frac{1+y^2}{(1-y)^2}\right. \\ &\left. - \frac{2}{N}\frac{1}{y(1-y)}\right]\mathcal{F}^{qq}(x_1, x_2, \mathbf{q}_T^2), \end{aligned} \quad (21)$$

$$\begin{aligned} \mathcal{A}^{q\bar{q}\rightarrow q\bar{q}} &= b(y)\mathcal{F}^{q\bar{q}}(x_1, x_2, \mathbf{q}_T^2) \\ &+ b(1-y)\mathcal{F}^{\bar{q}q}(x_1, x_2, \mathbf{q}_T^2), \end{aligned} \quad (22)$$

$$\begin{aligned} \mathcal{A}^{q\bar{q}\rightarrow q'\bar{q}'} &= \frac{N^2-1}{2N^2}y(1-y)[y^2 + (1-y)^2] \\ &\times [\mathcal{F}^{q\bar{q}}(x_1, x_2, \mathbf{q}_T^2) + \mathcal{F}^{\bar{q}q}(x_1, x_2, \mathbf{q}_T^2)], \end{aligned} \quad (23)$$

$$\begin{aligned} \mathcal{A}^{q\bar{q}\rightarrow gg} &= \frac{N^2-1}{N}\left(y^2 + (1-y)^2 - \frac{1}{N^2}\right)\frac{y^2 + (1-y)^2}{2} \\ &\times [\mathcal{F}^{q\bar{q}}(x_1, x_2, \mathbf{q}_T^2) + \mathcal{F}^{\bar{q}q}(x_1, x_2, \mathbf{q}_T^2)], \end{aligned} \quad (24)$$

with N being the number of colors and

$$\begin{aligned} a(y) &= \frac{N^2-1}{2N^2}(1-y)\frac{1+(1-y)^2}{y}, \\ b(y) &= a(y) + \frac{N^2-1}{2N^2}y(1-y)\left[y^2 + (1-y)^2\right. \\ &\left. + \frac{2}{N}\frac{(1-y)^2}{y}\right]. \end{aligned} \quad (25)$$

Analogously, from the gluon induced processes, one has

$$\begin{aligned} \mathcal{A}^{qg\rightarrow qg} &= c(y)\mathcal{F}^{qg}(x_1, x_2, \mathbf{q}_T^2) \\ &+ c(1-y)\mathcal{F}^{gq}(x_1, x_2, \mathbf{q}_T^2), \end{aligned} \quad (26)$$

$$\mathcal{A}^{gg\rightarrow gg} = 4\frac{N^2}{N^2-1}\frac{(1-y(1-y))^3}{y(1-y)}\mathcal{F}^{gg}(x_1, x_2, \mathbf{q}_T^2), \quad (27)$$

$$\begin{aligned} \mathcal{A}^{gg\rightarrow q\bar{q}} &= \frac{N}{N^2-1}\left(y^2 + (1-y)^2 - \frac{1}{N^2}\right) \\ &\times \frac{y^2 + (1-y)^2}{2}\mathcal{F}^{gg}(x_1, x_2, \mathbf{q}_T^2), \end{aligned} \quad (28)$$

where

$$c(y) = \frac{1+(1-y)^2}{2}\left[\frac{1+(1-y)^2}{y} - \frac{y}{N^2}\right]. \quad (29)$$

Agreement is found between the results given in the present subsection and the explicit expressions of the partonic cross sections published, for example, in [51–55]. However, with respect to Ref. [46] we find agreement

with the expression for the unpolarized qq production subprocess, but not for the $q\bar{q}$ production subprocesses. In particular, we find differences as compared with their Eqs. (24) and (33).

B. The $\cos 2(\phi_T - \phi_\perp)$ angular distribution of the dijet

In Ref. [46] it is shown that the subprocesses $qq \rightarrow qq$ and $q\bar{q} \rightarrow q\bar{q}$ contribute not only to the angular independent part of the cross section, according to Eqs. (21) and (22), but also to an azimuthal asymmetry of the dijet arising from the product of two T -odd functions, $h_1^{\perp q} h_1^{\perp \bar{q}}$ or $h_1^{\perp q} h_1^{\perp \bar{q}}$. Such an asymmetry is similar to the one calculated in the Drell-Yan [56] and in the photon-jet production [47] processes. We refer to [47] for the details of the derivation and present here only our final results. In analogy to Eq. (17), we write

$$B(y, x_1, x_2, \mathbf{q}_T^2) = \sum_{a,b,c,d} \mathcal{B}^{ab\rightarrow cd}(y, x_1, x_2, \mathbf{q}_T^2), \quad (30)$$

with

$$\mathcal{B}^{qq\rightarrow qq} = \frac{N^2-1}{N^3}y(1-y)\mathcal{H}^{qq}(x_1, x_2, \mathbf{q}_T^2), \quad (31)$$

$$\begin{aligned} \mathcal{B}^{q\bar{q}\rightarrow q\bar{q}} &= d(y)\mathcal{H}^{q\bar{q}}(x_1, x_2, \mathbf{q}_T^2) \\ &+ d(1-y)\mathcal{H}^{\bar{q}q}(x_1, x_2, \mathbf{q}_T^2), \end{aligned} \quad (32)$$

and

$$d(y) = \frac{N^2-1}{N^3}y(1-y)^2(1+Ny). \quad (33)$$

The following convolution of (transversely polarized) quark and antiquark distributions has been introduced,

$$\begin{aligned} \mathbf{q}_T^2 \mathcal{H}^{q\bar{q}}(x_1, x_2, \mathbf{q}_T^2) &\equiv \frac{1}{M_1 M_2} \sum_{\text{flavors}} \int d^2 \mathbf{p}_{1T} d^2 \mathbf{p}_{2T} \\ &\times \delta^2(\mathbf{p}_{1T} + \mathbf{p}_{2T} - \mathbf{q}_T)(2(\hat{\mathbf{h}} \cdot \mathbf{p}_{1T}) \\ &\times (\hat{\mathbf{h}} \cdot \mathbf{p}_{2T}) - (\mathbf{p}_{1T} \cdot \mathbf{p}_{2T})) \\ &\times h_1^{\perp q}(x_1, \mathbf{p}_{1T}^2) h_1^{\perp \bar{q}}(x_2, \mathbf{p}_{2T}^2), \end{aligned} \quad (34)$$

with $\hat{\mathbf{h}} \equiv \mathbf{q}_T/|\mathbf{q}_T|$, and a similar definition holds for \mathcal{H}^{qq} upon replacement of $\bar{q} \rightarrow q$ in Eq. (34). The small- \mathbf{q}_T behavior of \mathcal{H} is regular provided the integrations over $\mathbf{p}_T^4 h_1^{\perp q}(x, \mathbf{p}_T^2)$ converge. In addition to Eqs. (31) and (32), we find that the subprocesses $q\bar{q} \rightarrow gg$ and $q\bar{q} \rightarrow q'\bar{q}'$, not considered in [46], also show a $\cos 2(\phi_T - \phi_\perp)$ angular dependence, leading, respectively, to

$$\begin{aligned} \mathcal{B}^{q\bar{q}\rightarrow gg} &= \frac{N^2-1}{N}\left(y^2 + (1-y)^2 - \frac{1}{N^2}\right)y(1-y) \\ &\times [\mathcal{H}^{q\bar{q}}(x_1, x_2, \mathbf{q}_T^2) + \mathcal{H}^{\bar{q}q}(x_1, x_2, \mathbf{q}_T^2)], \end{aligned} \quad (35)$$

and

$$\mathcal{B}^{q\bar{q}\rightarrow q'\bar{q}'} = \frac{N^2 - 1}{N^2} y^2 (1-y)^2 [\mathcal{H}^{q\bar{q}}(x_1, x_2, \mathbf{q}_T^2) + \mathcal{H}^{\bar{q}q}(x_1, x_2, \mathbf{q}_T^2)]. \quad (36)$$

Agreement is found between the results given in the present subsection and the explicit expressions of the polarized partonic cross sections published in [54,55,57,58]. For the polarized qq production subprocess we find agreement with Ref. [46], but again not for the $q\bar{q}$ production subprocesses [in particular, we find a difference compared to their Eq. (26)].

C. The $\cos 4(\phi_T - \phi_\perp)$ angular distribution of the dijet

The $\cos 4(\phi_T - \phi_\perp)$ angular distribution of the dijet is related to the presence of linearly polarized gluons in unpolarized hadrons. This being a new result of the present paper, its derivation will be discussed in some more detail. The gluon-gluon induced part of the reaction under study, to lowest order in pQCD, is described in terms of the partonic two-to-two subprocesses

$$\begin{aligned} g(p_1) + g(p_2) &\rightarrow g(K_1) + g(K_2), \quad \text{and} \\ g(p_1) + g(p_2) &\rightarrow q(K_1) + \bar{q}(K_2). \end{aligned} \quad (37)$$

The corresponding cross sections are given by

$$\begin{aligned} \frac{d\sigma^{gg\rightarrow gg}}{d\eta_1 d\eta_2 d^2\mathbf{K}_\perp d^2\mathbf{q}_T} &= \frac{\alpha_s^2}{s\mathbf{K}_\perp^2} \left[\mathcal{A}^{gg\rightarrow gg}(y, x_1, x_2, \mathbf{q}_T^2) \right. \\ &+ \int d^2\mathbf{p}_{1T} d^2\mathbf{p}_{2T} \delta^2(\mathbf{p}_{1T} + \mathbf{p}_{2T} - \mathbf{q}_T) \\ &\times \frac{N^2}{N^2 - 1} y(1-y)(1-y(1-y)) \\ &\times \mathcal{P}^{gg}(\mathbf{p}_{1T}, \mathbf{p}_{2T}, \mathbf{K}_{1\perp}, \mathbf{K}_{2\perp}) \left. \right], \quad (38) \end{aligned}$$

and

$$\begin{aligned} \frac{d\sigma^{gg\rightarrow q\bar{q}}}{d\eta_1 d\eta_2 d^2\mathbf{K}_\perp d^2\mathbf{q}_T} &= \frac{\alpha_s^2}{s\mathbf{K}_\perp^2} \left[\mathcal{A}^{gg\rightarrow q\bar{q}}(y, x_1, x_2, \mathbf{q}_T^2) \right. \\ &- \int d^2\mathbf{p}_{1T} d^2\mathbf{p}_{2T} \delta^2(\mathbf{p}_{1T} + \mathbf{p}_{2T} - \mathbf{q}_T) \\ &\times \frac{N}{N^2 - 1} \frac{y(1-y)}{4} \left(y^2 + (1-y)^2 - \frac{1}{N^2} \right) \\ &\times \mathcal{P}^{gg}(\mathbf{p}_{1T}, \mathbf{p}_{2T}, \mathbf{K}_{1\perp}, \mathbf{K}_{2\perp}) \left. \right], \quad (39) \end{aligned}$$

where

$$\begin{aligned} \mathcal{P}^{gg}(\mathbf{p}_{1T}, \mathbf{p}_{2T}, \mathbf{K}_{1\perp}, \mathbf{K}_{2\perp}) &= \left[-\mathbf{p}_{1T}^2 \mathbf{p}_{2T}^2 + \frac{2}{\mathbf{K}_\perp^4} ((\mathbf{K}_{1\perp} \cdot \mathbf{K}_{2\perp})(\mathbf{p}_{1T} \cdot \mathbf{p}_{2T}) \right. \\ &- (\mathbf{K}_{1\perp} \cdot \mathbf{p}_{1T})(\mathbf{K}_{2\perp} \cdot \mathbf{p}_{2T}) \\ &- (\mathbf{K}_{1\perp} \cdot \mathbf{p}_{2T})(\mathbf{K}_{2\perp} \cdot \mathbf{p}_{1T}))^2 \left. \right] \\ &\times \frac{1}{M_1^2 M_2^2} h_1^{\perp g}(x_1, \mathbf{p}_{1T}^2) h_1^{\perp g}(x_2, \mathbf{p}_{2T}^2). \quad (40) \end{aligned}$$

The functions $\mathcal{A}^{gg\rightarrow gg}$ and $\mathcal{A}^{gg\rightarrow q\bar{q}}$, given in Eqs. (27) and (28), contain the convolution of unpolarized gluon distribution functions \mathcal{F}^{gg} defined in Eq. (18). In order to show that the two cross sections in Eqs. (38) and (39) can be written in the same form as Eq. (16), we introduce the functions

$$\begin{aligned} \mathbf{q}_T^4 I^{gg}(x_1, x_2, \mathbf{q}_T^2) &\equiv \frac{1}{M_1^2 M_2^2} \int d^2\mathbf{p}_{1T} d^2\mathbf{p}_{2T} \\ &\times \delta^2(\mathbf{p}_{1T} + \mathbf{p}_{2T} - \mathbf{q}_T) (2(\hat{\mathbf{h}} \cdot \mathbf{p}_{1T}) \\ &\times (\hat{\mathbf{h}} \cdot \mathbf{p}_{2T}) - (\mathbf{p}_{1T} \cdot \mathbf{p}_{2T}))^2 \\ &\times h_1^{\perp g}(x_1, \mathbf{p}_{1T}^2) h_1^{\perp g}(x_2, \mathbf{p}_{2T}^2), \quad (41) \end{aligned}$$

where again $\hat{\mathbf{h}} \equiv \mathbf{q}_T/|\mathbf{q}_T|$, and

$$\begin{aligned} \mathbf{q}_T^4 \mathcal{L}^{gg}(x_1, x_2, \mathbf{q}_T^2) &\equiv \frac{1}{M_1^2 M_2^2} \int d^2\mathbf{p}_{1T} d^2\mathbf{p}_{2T} \\ &\times \delta^2(\mathbf{p}_{1T} + \mathbf{p}_{2T} - \mathbf{q}_T) \mathbf{p}_{1T}^2 \mathbf{p}_{2T}^2 \\ &\times h_1^{\perp g}(x_1, \mathbf{p}_{1T}^2) h_1^{\perp g}(x_2, \mathbf{p}_{2T}^2). \quad (42) \end{aligned}$$

The small- \mathbf{q}_T behavior of I and \mathcal{L} are regular provided the integrations over $\mathbf{p}_T^g h_1^{\perp g}(x, \mathbf{p}_T^2)$ converge. Hence we have

$$\begin{aligned} &\int d^2\mathbf{p}_{1T} d^2\mathbf{p}_{2T} \delta^2(\mathbf{p}_{1T} + \mathbf{p}_{2T} - \mathbf{q}_T) \mathcal{P}^{gg} \\ &= \mathbf{q}_T^4 \sum_{i,j,l,m=1}^2 \frac{\mathbf{K}_{1\perp}^{\{i} \mathbf{K}_{2\perp}^{\{j} \mathbf{K}_{1\perp}^{\{l} \mathbf{K}_{2\perp}^{\{m}}}{4\mathbf{K}_\perp^4} \left[2(\delta^{il} \delta^{jm} - \delta^{ij} \delta^{lm}) \right. \\ &+ \delta^{im} \delta^{jl} (\mathcal{L}^{gg} - I^{gg}) - \delta^{il} \delta^{jm} \mathcal{L}^{gg} \\ &+ 2 \left(\frac{\mathbf{q}_T^i \mathbf{q}_T^j}{\mathbf{q}_T^2} - \delta^{ij} \right) \left(\frac{\mathbf{q}_T^l \mathbf{q}_T^m}{\mathbf{q}_T^2} - \delta^{lm} \right) (2I^{gg} - \mathcal{L}^{gg}) \left. \right] \\ &= \mathbf{q}_T^4 \cos 2(\phi_T - \phi_1 - \phi_2) (2I^{gg} - \mathcal{L}^{gg}). \quad (43) \end{aligned}$$

The difference between the angular dependence $\cos 2(\phi_T - \phi_1 - \phi_2) = \cos 4(\phi_T - \phi_j)$ and $\cos 4(\phi_T - \phi_\perp)$ is of order $\mathbf{q}_T^2/\mathbf{K}_\perp^2$ (cf. Appendix A). Substituting Eq. (43) into Eqs. (38) and (39), and defining $d\sigma^{gg} \equiv d\sigma^{gg\rightarrow gg} + d\sigma^{gg\rightarrow q\bar{q}}$, we finally obtain

$$\frac{d\sigma^{gg}}{d\eta_1 d\eta_2 d^2\mathbf{K}_\perp d^2\mathbf{q}_T} = \frac{\alpha_s^2}{s\mathbf{K}_\perp^2} [\mathcal{A}^{gg}(y, x_1, x_2, \mathbf{q}_T^2) + \mathcal{C}^{gg}(y, x_1, x_2, \mathbf{q}_T^2) q_T^4 \cos 4(\phi_T - \phi_\perp)], \quad (44)$$

where $\mathcal{A}^{gg} \equiv \mathcal{A}^{gg \rightarrow gg} + \mathcal{A}^{gg \rightarrow q\bar{q}}$, $\mathcal{C}^{gg} \equiv \mathcal{C}^{gg \rightarrow gg} + \mathcal{C}^{gg \rightarrow q\bar{q}}$, with

$$\mathcal{C}^{gg \rightarrow gg} = \frac{N^2}{N^2 - 1} y(1-y)(1-y(1-y)) \times [2\mathcal{I}^{gg}(x_1, x_2, \mathbf{q}_T^2) - \mathcal{L}^{gg}(x_1, x_2, \mathbf{q}_T^2)], \quad (45)$$

and

$$\mathcal{C}^{gg \rightarrow q\bar{q}} = -\frac{N}{N^2 - 1} \frac{y(1-y)}{4} \left(y^2 + (1-y)^2 - \frac{1}{N^2} \right) \times [2\mathcal{I}^{gg}(x_1, x_2, \mathbf{q}_T^2) - \mathcal{L}^{gg}(x_1, x_2, \mathbf{q}_T^2)]. \quad (46)$$

It turns out that the two subprocesses $gg \rightarrow gg$ and $gg \rightarrow q\bar{q}$ are the only ones that determine the $\cos 4(\phi_T - \phi_\perp)$ dependence of the cross section. Therefore in Eq. (16)

$$C(y, x_1, x_2, \mathbf{q}_T^2) = \mathcal{C}^{gg} = \mathcal{C}^{gg \rightarrow gg} + \mathcal{C}^{gg \rightarrow q\bar{q}}, \quad (47)$$

which, together with Eqs. (41), (42), (45), and (46), leads to

$$C = \frac{N}{N^2 - 1} y(1-y) \left[N(1-y(1-y)) - \frac{1}{4} \left(y^2 + (1-y)^2 - \frac{1}{N^2} \right) \right] \times [2\mathcal{I}^{gg}(x_1, x_2, \mathbf{q}_T^2) - \mathcal{L}^{gg}(x_1, x_2, \mathbf{q}_T^2)], \quad (48)$$

showing how the azimuthal asymmetry under investigation is related to the T -even, spin and transverse momentum dependent parton distribution function $h_1^{\perp g}(x, \mathbf{p}_T^2)$.

III. DIJET IMBALANCE DISTRIBUTIONS

In this section we study the cross section for the process $h_1 h_2 \rightarrow \text{jet jet } X$ in terms of the total transverse energy E_T and the dijet imbalance $\delta\phi \equiv \phi_1 - \phi_2 - \pi$, which are the kinematic variables commonly used in the experiments. The dijet imbalance angle describes the deviation of the two jets from a back-to-back configuration (see Fig. 2 in Appendix A).

The transverse energy is the sum of the transverse energies of the two jets, $E_T = |\mathbf{K}_{1\perp}| + |\mathbf{K}_{2\perp}|$, and the difference is defined as $\Delta K_\perp = |\mathbf{K}_{1\perp}| - |\mathbf{K}_{2\perp}|$. In our basic expression for the cross section in Eq. (16) we have traded $\mathbf{K}_{1\perp}$ and $\mathbf{K}_{2\perp}$ for \mathbf{q}_T and \mathbf{K}_\perp , but we can also trade the variables ($|\mathbf{K}_{1\perp}|, |\mathbf{K}_{2\perp}|$) for ($E_T, \Delta K_\perp$) and (ϕ_1, ϕ_2) for ($\phi_j, \delta\phi$). We find in the back-to-back approximation

$$\mathbf{q}_T^2 = \Delta K_\perp^2 \cos^2\left(\frac{\delta\phi}{2}\right) + E_T^2 \sin^2\left(\frac{\delta\phi}{2}\right) \approx \Delta K_\perp^2 + E_T^2 \sin^2\left(\frac{\delta\phi}{2}\right), \quad (49)$$

$$4\mathbf{K}_\perp^2 = E_T^2 \cos^2\left(\frac{\delta\phi}{2}\right) + \Delta K_\perp^2 \sin^2\left(\frac{\delta\phi}{2}\right) \approx E_T^2. \quad (50)$$

In the first expression we cannot drop the term proportional to ΔK_\perp^2 because it is not a good approximation for $\delta\phi \approx 0$, which is most relevant. Note also that this implies $\mathbf{q}_T^2 \geq \Delta K_\perp^2$, i.e. ΔK_\perp^2 sets a lower bound on the \mathbf{q}_T^2 values probed, which may be very relevant if the functions A, B, C are steeply falling functions with increasing \mathbf{q}_T^2 .

The cross section in Eq. (16) rewritten yields

$$\frac{d\sigma^{h_1 h_2 \rightarrow \text{jet jet } X}}{d\eta_1 d\eta_2 dE_T d\Delta K_\perp d\phi_j d\delta\phi} = \frac{\alpha_s^2}{2s} [A(\mathbf{q}_T^2) + B(\mathbf{q}_T^2) \mathbf{q}_T^2 \cos 2(\phi_T - \phi_j) + C(\mathbf{q}_T^2) \mathbf{q}_T^4 \cos 4(\phi_T - \phi_j)], \quad (51)$$

with \mathbf{q}_T^2 given in the unapproximated first part of Eq. (49) and

$$\mathbf{q}_T^2 \cos 2(\phi_T - \phi_j) = \Delta K_\perp^2 \cos^2\left(\frac{\delta\phi}{2}\right) - E_T^2 \sin^2\left(\frac{\delta\phi}{2}\right), \quad (52)$$

$$\mathbf{q}_T^4 \cos 4(\phi_T - \phi_j) = \left[E_T^2 \sin^2\left(\frac{\delta\phi}{2}\right) - \Delta K_\perp^2 \cos^2\left(\frac{\delta\phi}{2}\right) \right]^2 - E_T^2 \Delta K_\perp^2 \sin^2(\delta\phi). \quad (53)$$

In this way we have arrived at an expression that is amenable to phenomenological studies, approximating $|\mathbf{K}_{1\perp}| \approx |\mathbf{K}_{2\perp}| \approx E_T/2$ only in places where the difference is negligible for all values of $\delta\phi$.

For $\Delta K_\perp = 0$ we obtain

$$\frac{d\sigma^{h_1 h_2 \rightarrow \text{jet jet } X}}{d\eta_1 d\eta_2 dE_T d\Delta K_\perp d\phi_j d\delta\phi} = \frac{\alpha_s^2}{2s} [A(\mathbf{q}_T^2) - B(\mathbf{q}_T^2) \mathbf{q}_T^2 + C(\mathbf{q}_T^2) \mathbf{q}_T^4], \quad (54)$$

with in that case exactly $\mathbf{q}_T^2 = E_T^2 \sin^2(\delta\phi/2)$, and in essence recovering Eq. (3) (the sign in front of B is just a matter of definition).

To illustrate the effect of nonzero B and C terms, we will make a Gaussian ansatz for these functions of \mathbf{q}_T^2 . We will take

$$A(\mathbf{q}_T^2) = \frac{R_A^2}{\pi} \exp(-\mathbf{q}_T^2 R_A^2), \quad B(\mathbf{q}_T^2) = \frac{R_B^4}{c\pi} \exp(-\mathbf{q}_T^2 R_B^2), \quad C(\mathbf{q}_T^2) = \frac{R_C^6}{2c^2\pi} \exp(-\mathbf{q}_T^2 R_C^2), \quad (55)$$

normalized such that

$$\int d^2\mathbf{q}_T A(\mathbf{q}_T^2) = 1, \quad \int d^2\mathbf{q}_T \mathbf{q}_T^2 B(\mathbf{q}_T^2) = 1/c, \quad (56)$$

$$\int d^2\mathbf{q}_T \mathbf{q}_T^4 C(\mathbf{q}_T^2) = 1/c^2.$$

Figure 1 shows a plot of the cross section in Eq. (51) as a function of $\delta\phi$ for the arbitrary, but perhaps realistic, choices $|\mathbf{K}_{1\perp}| = 30$ GeV, $|\mathbf{K}_{2\perp}| = 31$ GeV, $R_A = 0.5$ GeV $^{-2}$, $R_B = 2R_A$, $R_C = 3R_A$, and $c = 3$. For smaller ΔK_\perp the shoulders become more pronounced, but already for $\Delta K_\perp = 2$ GeV the shoulders are hardly distinguishable anymore. In general, B and C have to be significant in size and broad enough to generate an observable effect, i.e. for the $\delta\phi$ distribution to deviate visibly from a Gaussian distribution.

Although the B and C terms were not considered before in experimental analyses of dijet imbalance measurements in hadronic collisions, experimental data are available that have some bearing on the size of B compared to A . They come from the measurement of the violation of the Lam-Tung relation in the Drell-Yan process. As shown in Ref. [56], this violation κ is given by the ratio $\mathbf{q}_T^2 \mathcal{H}^{q\bar{q}} / \mathcal{F}^{q\bar{q}}$ [Eq. (34) divided by the angular averaged result in Eq. (18)] but with the sums over flavors weighted with a factor e_q^2 , the quark charge squared. This has the effect of emphasizing the contribution from up quarks. In the present two-jet production case, the ratio $\mathbf{q}_T^2 B/A$ in the midrapidity region ($\eta_1 \approx \eta_2 \approx 0$) and for large N can be approximated by $\mathbf{q}_T^2 \mathcal{H}^{q\bar{q}} / \mathcal{F}^{q\bar{q}}$. The size of κ in Drell-Yan may thus be expected to give some indication of the size of $\mathbf{q}_T^2 B/A$. The violation of the Lam-Tung relation in Drell-Yan has recently been measured in pp and $p\bar{d}$ collisions [59]. It is consistent with no violation, but with sizeable errors. Small violation would be in line with the expectation that h_1^\perp for antiquarks inside a proton is considerably smaller than for quarks. For $p\bar{p}$ one however expects a large violation, as observed in πp collisions [60–62]. So the effect of a nonzero h_1^\perp for *quarks* may be mostly relevant for jet broadening studies in $p\bar{p}$ [21].

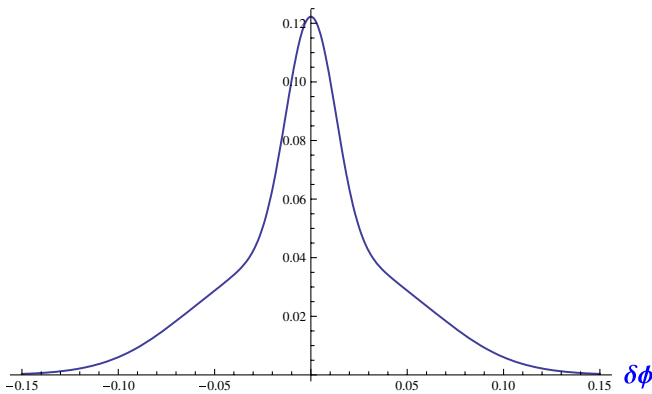


FIG. 1 (color online). An illustration of the effect of sizeable B and C terms on the $\delta\phi$ distribution of the cross section in Eq. (51).

IV. WEIGHTED CROSS SECTIONS

Apart from the fact that nonzero h_1^\perp functions for quarks and gluons modify the $\delta\phi$ distribution and hence affect the extraction of the average initial parton transverse momentum from this dijet imbalance distribution, it would in principle be of interest to extract these functions themselves from it. Therefore, the question arises whether one can project out the B and C terms separately. In Ref. [46] this is discussed for B only, but there are some problems with the proposed method. It was suggested that $\langle \cos\delta\phi \rangle$, i.e. the cross section integrated over $\delta\phi$ weighted with an additional factor of $\cos\delta\phi$, projects out a contribution from $h_1^{\perp q}$ exclusively.¹ However, our result in Eq. (51) shows that $\langle \cos\delta\phi \rangle$ does not project out B nor a part of B exclusively, not even in the idealized case when $|\mathbf{K}_{1\perp}| = |\mathbf{K}_{2\perp}|$, as can be seen from Eq. (54). To see the appropriate weighting, we return to the form in Eq. (16) and note that B is projected out by

$$\langle \cos 2(\phi_T - \phi_\perp) \rangle \equiv \int \frac{d\phi_T}{2\pi} \cos 2(\phi_T - \phi_\perp) \times \frac{d\sigma^{h_1 h_2 \rightarrow \text{jet jet } X}}{d\eta_1 d\eta_2 d^2\mathbf{K}_\perp d^2\mathbf{q}_T} = \frac{1}{2} \frac{\alpha_s^2}{s \mathbf{K}_\perp^2} \mathbf{q}_T^2 B(y, x_1, x_2, \mathbf{q}_T^2). \quad (57)$$

Integrating over the length of \mathbf{q}_T gives with possible inclusion of additional weighting with powers of \mathbf{q}_T^2 ,

$$\pi \int d\mathbf{q}_T^2 \left(\frac{\mathbf{q}_T^2}{M_1 M_2} \right)^M \langle \cos 2(\phi_T - \phi_\perp) \rangle = \int d^2\mathbf{q}_T \left(\frac{\mathbf{q}_T^2}{M_1 M_2} \right)^M \cos 2(\phi_T - \phi_\perp) \times \frac{d\sigma^{h_1 h_2 \rightarrow \text{jet jet } X}}{d\eta_1 d\eta_2 d^2\mathbf{K}_\perp d^2\mathbf{q}_T}, \quad (58)$$

in which we get for $M = 1$ the factorized result

$$\pi \int d\mathbf{q}_T^2 \left(\frac{\mathbf{q}_T^2}{M_1 M_2} \right) \mathbf{q}_T^2 \mathcal{H}^{qq}(x_1, x_2, \mathbf{q}_T^2) = 8 \sum_{\text{flavors}} h_1^{\perp q(1)}(x_1) h_1^{\perp q(1)}(x_2), \quad (59)$$

in the B contributions. For $M = 0$, the expression does not deconvolute. In that case usable but model dependent expressions may be obtained by making a Gaussian ansatz for the transverse momentum shape of h_1^q . For the type of convolution that appears in B this has been done in the literature (see for instance Ref. [63]).

¹In Ref. [46] actually $\langle P_\perp^2 / M^2 \cos\delta\phi \rangle$ was considered, where $P_\perp \approx |\mathbf{K}_{1\perp}| \approx |\mathbf{K}_{2\perp}|$, despite the fact that $\mathbf{K}_{2\perp}$ was integrated over. The factor P_\perp^2 / M^2 artificially enhances the weighted asymmetry if not divided by $\langle P_\perp^2 / M^2 \rangle$.

Next, we will analyze in some more detail the weighted asymmetry that projects out C . A measurement of the weighted cross section

$$\begin{aligned} \langle \cos 4(\phi_T - \phi_\perp) \rangle &\equiv \int \frac{d\phi_T}{2\pi} \cos 4(\phi_T - \phi_\perp) \\ &\times \frac{d\sigma^{h_1 h_2 \rightarrow \text{jet jet } X}}{d\eta_1 d\eta_2 d^2 \mathbf{K}_\perp d^2 \mathbf{q}_T} \\ &= \frac{1}{2} \frac{\alpha_s^2}{s \mathbf{K}_\perp^2} \mathbf{q}_T^4 C(y, x_1, x_2, \mathbf{q}_T^2), \end{aligned} \quad (60)$$

with C given in Eq. (48), would give access to the linearly polarized gluon distribution of a hadron. After integration over the length of \mathbf{q}_T with possible inclusion of additional weighting with \mathbf{q}_T^2 , we obtain

$$\begin{aligned} &\pi \int d\mathbf{q}_T^2 \left(\frac{\mathbf{q}_T^2}{M_1 M_2} \right)^M \langle \cos 4(\phi_T - \phi_\perp) \rangle \\ &= \int d^2 \mathbf{q}_T \left(\frac{\mathbf{q}_T^2}{M_1 M_2} \right)^M \cos 4(\phi_T - \phi_\perp) \\ &\times \frac{d\sigma^{h_1 h_2 \rightarrow \text{jet jet } X}}{d\eta_1 d\eta_2 d^2 \mathbf{K}_\perp d^2 \mathbf{q}_T}. \end{aligned} \quad (61)$$

In this case we get for $M = 2$ the deconvoluted result

$$\begin{aligned} &\pi \int d\mathbf{q}_T^2 \left(\frac{\mathbf{q}_T^2}{M_1 M_2} \right)^2 \mathbf{q}_T^4 (2I^{gg} - \mathcal{L}^{gg}) \\ &= 96 h_1^{\perp g(2)}(x_1) h_1^{\perp g(2)}(x_2), \end{aligned} \quad (62)$$

in the C contributions. In order to evaluate the integral in Eq. (60) without weights or study the explicitly \mathbf{q}_T^2 dependence, one can employ a Gaussian model for $h_1^{\perp g}$, of which the easiest choice has a factorized x and \mathbf{p}_T dependence, that is, neglecting the dependence on the factorization scale,

$$h_1^{\perp g}(x, \mathbf{p}_T^2) = \frac{R_h^2}{\pi} h_1^{\perp g}(x) e^{-R_h^2 \mathbf{p}_T^2}, \quad (63)$$

$$\begin{aligned} h_1^{\perp g(n)}(x) &= \int d^2 \mathbf{p}_T \left(\frac{\mathbf{p}_T^2}{2M^2} \right)^n h_1^{\perp g}(x, \mathbf{p}_T^2) \\ &= \frac{n!}{(2M^2 R_h^2)^n} h_1^{\perp g}(x), \end{aligned} \quad (64)$$

where R_h is a size parameter related to the average partonic \mathbf{p}_T^2 by the relation $R_h^2 = 1/\langle \mathbf{p}_T^2 \rangle$. For incoming (anti)protons, $R_p = R_{\bar{p}} \equiv R$, so one has

$$\begin{aligned} &\int d^2 \mathbf{q}_T \mathbf{q}_T^4 (2I^{gg} - \mathcal{L}^{gg}) \\ &= \frac{1}{M_1^2 M_2^2} \int d^2 \mathbf{q}_T d^2 \mathbf{p}_{1T} d^2 \mathbf{p}_{2T} \delta^2(\mathbf{p}_{1T} + \mathbf{p}_{2T} - \mathbf{q}_T) \\ &\times [2(2(\hat{\mathbf{h}} \cdot \mathbf{p}_{1T})(\hat{\mathbf{h}} \cdot \mathbf{p}_{2T}) - (\mathbf{p}_{1T} \cdot \mathbf{p}_{2T}))^2 - \mathbf{p}_{1T}^2 \mathbf{p}_{2T}^2] \\ &\times \frac{R^4}{\pi^2} h_1^{\perp g}(x_1) h_1^{\perp g}(x_2) e^{-R^2(\mathbf{p}_{1T}^2 + \mathbf{p}_{2T}^2)}. \end{aligned} \quad (65)$$

Using the \mathbf{p}_{2T} integration to eliminate the delta function in Eq. (65) and shifting the integration variable $\mathbf{p}_{1T} \rightarrow \mathbf{p}'_{1T} = \mathbf{p}_{1T} - \frac{1}{2} \mathbf{q}_T$, one arrives at

$$\begin{aligned} \int d^2 \mathbf{q}_T \mathbf{q}_T^4 (2I^{gg} - \mathcal{L}^{gg}) &= \frac{1}{M_1^2 M_2^2} \frac{R^4}{16\pi} \int d^2 \mathbf{q}_T d\mathbf{p}'_T{}^2 \mathbf{q}_T^4 \\ &\times e^{-R^2(2\mathbf{p}'_T{}^2 + (1/2)\mathbf{q}_T^2)} \\ &\times h_1^{\perp g}(x_1) h_1^{\perp g}(x_2) \\ &= \frac{1}{2M_1^2 M_2^2} \frac{1}{R^4} h_1^{\perp g}(x_1) h_1^{\perp g}(x_2). \end{aligned} \quad (66)$$

Substituting Eq. (66) into Eq. (60) shows that for a Gaussian shape one finds for the unweighted average the final result

$$\begin{aligned} &\pi \int d\mathbf{q}_T^2 \langle \cos 4(\phi_T - \phi_\perp) \rangle \\ &= \frac{\alpha_s^2}{s \mathbf{K}_\perp^2} \frac{N}{N^2 - 1} y(1-y) \left[N(1-y(1-y)) \right. \\ &\quad \left. - \frac{1}{4} (y^2 + (1-y)^2 - \frac{1}{N^2}) \right] h_1^{\perp g(1)}(x_1) h_1^{\perp g(1)}(x_2). \end{aligned} \quad (67)$$

V. JET BROADENING

In our almost back-to-back jet situation, the jet-direction j coincides with the transverse thrust axis, and the jet broadening variable Q_t defined in Eq. (2) is given by

$$\begin{aligned} Q_t &= E_T \left| \sin\left(\frac{\delta\phi}{2}\right) \right| = |\mathbf{q}_T| |\sin(\phi_T - \phi_j)| \\ &\approx |\mathbf{q}_T| |\sin(\phi_T - \phi_\perp)|, \end{aligned} \quad (68)$$

for which we refer to Eq. (A6) in Appendix A and one needs to use Eqs. (A7) and (A8) to check the validity of the approximation. Using this expression we can now turn to the evaluation of the average jet broadening $\langle Q_t \rangle$ as a function of $|\mathbf{K}_\perp|$,

$$\langle Q_t \rangle \propto \int \frac{d\phi_\perp}{2\pi} d^2 \mathbf{q}_T Q_t(|\mathbf{q}_T|, \phi_T, \phi_\perp) \frac{d\sigma^{h_1 h_2 \rightarrow \text{jet jet } X}}{d^2 \mathbf{K}_\perp d^2 \mathbf{q}_T}. \quad (69)$$

The differential cross section in the integrand is obtained

from Eq. (16) and contains, besides the well-known, angular independent term A , also the terms B (due to the transverse polarization of quarks and antiquarks in the colliding hadrons) and C (related to the linear polarization of gluons). The following integrals,

$$\int_0^{2\pi} d\phi_T |\sin(\phi_T - \phi_\perp)| = 4, \quad (70)$$

$$\int_0^{2\pi} d\phi_T |\sin(\phi_T - \phi_\perp)| \cos 2(\phi_T - \phi_\perp) = -\frac{4}{3}, \quad (71)$$

$$\int_0^{2\pi} d\phi_T |\sin(\phi_T - \phi_\perp)| \cos 4(\phi_T - \phi_\perp) = -\frac{4}{15}, \quad (72)$$

are all different from zero, meaning that the A , B , and C terms contribute to $\langle Q_i \rangle$:

$$\langle Q_i \rangle \propto \int d^2 \mathbf{q}_T |\mathbf{q}_T| \left[A(\mathbf{q}_T^2) - \frac{1}{3} B(\mathbf{q}_T^2) \mathbf{q}_T^2 - \frac{1}{15} C(\mathbf{q}_T^2) \mathbf{q}_T^4 \right]. \quad (73)$$

In order to calculate $\langle Q_i^2 \rangle$, one needs to evaluate the integrals

$$\int_0^{2\pi} d\phi_T \sin^2(\phi_T - \phi_\perp) = \pi, \quad (74)$$

$$\int_0^{2\pi} d\phi_T \sin^2(\phi_T - \phi_\perp) \cos 2(\phi_T - \phi_\perp) = -\frac{\pi}{2}, \quad (75)$$

$$\int_0^{2\pi} d\phi_T \sin^2(\phi_T - \phi_\perp) \cos 4(\phi_T - \phi_\perp) = 0, \quad (76)$$

which show that only the terms A and B enter in the estimate of $\langle Q_i^2 \rangle$:

$$\langle Q_i^2 \rangle \propto \int d^2 \mathbf{q}_T \mathbf{q}_T^2 \left[A(\mathbf{q}_T^2) - \frac{1}{2} B(\mathbf{q}_T^2) \mathbf{q}_T^2 \right]. \quad (77)$$

VI. COLOR FLOW DEPENDENCE AND FACTORIZATION

In our treatment in this paper we have simply convoluted the quark and gluon correlators with the hard partonic cross sections, without worrying about possible nontrivial effects arising from the gauge link structure in these correlators. The proper gauge invariant definitions of TMDs as well as collinear correlators involve nonlocal operators containing path-ordered exponentials, the gauge links. The gauge link is the result of resumming all gluons with polarizations along the momentum of a particular hadron into the soft parts. In the case of TMDs the path of the gauge links generally depends on the process. The path dependence disappears after integration over transverse momenta. In the collinear correlators, one can usually choose a gauge that makes the gauge link unity, but the same procedure for

TMDs can leave transverse pieces that are situated at light cone infinity. These links can have physical effects, for instance in single transverse spin asymmetries that arise from the Sivers effect, which is described by a T -odd TMD. The Sivers asymmetries in semi-inclusive deep inelastic scattering and the Drell-Yan process are predicted to differ by a sign as a consequence of the gauge links [64].

In the more complicated processes $h_1^\dagger h_2 \rightarrow \gamma/\text{jet} + \text{jet} + X$ the single spin asymmetries involving the Sivers function [50,65,66] come from correlators with more complex paths in the gauge links. This causes deviations that are more involved than a simple sign change with respect to e.g. semi-inclusive deep inelastic scattering. But also in this case, calculable process-dependent ‘‘color flow’’ factors can be obtained which may be different for each hard partonic subprocess. In this way they allow for the calculation of particular weighted cross sections in dijet production, resulting in a small asymmetry [66–68], as also shown by the data [69]. However, claims of possible factorization breaking have been put forward for this process [43,44] and this remains an open question.

For observables involving a product of two T -odd TMDs, such as the one discussed in the present paper, the situation is less clear. For $\cos 2\phi$ asymmetries in Drell-Yan [56] and $h_1 h_2 \rightarrow \gamma \text{jet} X$ the effects of nontrivial gauge links were included in Ref. [47] following the methods outlined in Refs. [41,42,55,70]. In both cases the color flow factor obtained was $+1$. However, since the methods used were developed for observables involving a single noncontracted transverse momentum p_T for a T -odd TMD for one of the hadrons in the process, the extension to cases in which noncontracted transverse momenta of partons in two different hadrons are involved certainly needs careful study. The p_T dependence for h_1^\dagger in the correlators for gluons, moreover, has a rank two tensor structure in the noncontracted transverse momentum, although it is T even. For the present case of dijet production (for which nontrivial color flow factors were presented in Ref. [46]), which is necessarily more complicated and for which doubts about factorization have been put forward, at this stage we do not include any color flow factors. Since we have presented the expressions for each partonic subprocess separately, it is possible to include the correct factors at a later stage, once they have been firmly established. If factorization cannot be proven for the process of interest, however, this implies not only that the functions h_1^\dagger cannot be extracted but neither can in that case $\langle p_T^2 \rangle$ be obtained.

VII. SUMMARY AND CONCLUSIONS

In this paper we study the effects of transverse momenta of the initial state hadrons in hadronic dijet production. The transverse momentum produces an imbalance in the dijets in the transverse plane. In the usual treatments the effects are attributed to gluon radiation and to the transverse momentum dependence of the unpolarized quark distribu-

tions. We look at the effects of two additional TMD functions that enter in the scattering of unpolarized hadrons, the distribution of transversely polarized quarks ($h_1^{\perp q}$) and the distribution of linearly polarized gluons ($h_1^{\perp g}$). They produce specific azimuthal dependences of the two jets, which are not suppressed *a priori*. The effects cannot be isolated by only looking at the angular deviation from the back-to-back situation, but depend on the jet transverse energy and the contributions to it of the two jets. We have discussed appropriate weighting to isolate the specific additional contributions. We also pointed out their effect on the jet broadening quantities $\langle Q_i \rangle$ and $\langle Q_i^2 \rangle$, which we considered for the simplest two-jet case, but the conclusion that h_1^{\perp} functions contribute to them also affects the more general cases containing a sum over hadrons. This in principle complicates the extraction of the average initial parton transverse momentum from the jet broadening, but possibly even hampers it altogether if factorization of the (sufficiently sizeable) spin-dependent contributions indeed turns out to be broken.

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APPENDIX A: TRANSVERSE PLANE VARIABLES

In the transverse plane we have the two-jet momenta $\mathbf{K}_{1\perp}$ and $\mathbf{K}_{2\perp}$ defining azimuthal angles ϕ_1 and ϕ_2 . From them one can construct the sum and difference angles,

$$\phi_j = (\phi_1 + \phi_2 - \pi)/2, \quad (\text{A1})$$

$$\delta\phi = \phi_1 - \phi_2 - \pi. \quad (\text{A2})$$

The sum and difference of the transverse energies of the two jets, $|\mathbf{K}_{1\perp}|$ and $|\mathbf{K}_{2\perp}|$, define

$$E_T = |\mathbf{K}_{1\perp}| + |\mathbf{K}_{2\perp}|, \quad (\text{A3})$$

$$\Delta K_{\perp} = |\mathbf{K}_{1\perp}| - |\mathbf{K}_{2\perp}|. \quad (\text{A4})$$

One can use $d^2\mathbf{K}_{1\perp}d^2\mathbf{K}_{2\perp} = \frac{1}{8}(E_T^2 - \Delta K_{\perp}^2)dE_T \times d\Delta K_{\perp}d\phi_j d\delta\phi$ for the phase space or go to the sum and difference momenta and their angles as shown in Fig. 2. In that case one has $d^2\mathbf{K}_{1\perp}d^2\mathbf{K}_{2\perp} = d^2\mathbf{K}_{\perp}d^2\mathbf{q}_T = |\mathbf{q}_T||\mathbf{K}_{\perp}|d|\mathbf{q}_T|d|\mathbf{K}_{\perp}|d\phi_T d\phi_{\perp}$. We have the following exact relations:

$$|\mathbf{q}_T| \cos(\phi_T - \phi_j) = \Delta K_{\perp} \cos(\delta\phi/2), \quad (\text{A5})$$

$$|\mathbf{q}_T| \sin(\phi_T - \phi_j) = E_T \sin(\delta\phi/2), \quad (\text{A6})$$

$$2|\mathbf{K}_{\perp}| \cos(\phi_{\perp} - \phi_j) = E_T \cos(\delta\phi/2), \quad (\text{A7})$$

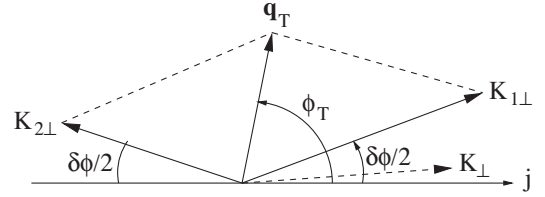


FIG. 2. The transverse plane is defined as orthogonal with respect to the two incoming hadrons. The jet direction (j) is defined as $\phi_j = (\phi_1 + \phi_2 - \pi)/2$. The momenta $\mathbf{q}_T = \mathbf{K}_{1\perp} + \mathbf{K}_{2\perp}$ and $\mathbf{K}_{\perp} = (\mathbf{K}_{1\perp} - \mathbf{K}_{2\perp})/2$ define the azimuthal angles ϕ_T and ϕ_{\perp} .

$$2|\mathbf{K}_{\perp}| \sin(\phi_{\perp} - \phi_j) = \Delta K_{\perp} \sin(\delta\phi/2), \quad (\text{A8})$$

$$\begin{aligned} \mathbf{q}_T^2 &= \Delta K_{\perp}^2 \cos^2(\delta\phi/2) + E_T^2 \sin^2(\delta\phi/2) \\ &= \Delta K_{\perp}^2 + (E_T^2 - \Delta K_{\perp}^2) \sin^2(\delta\phi/2), \end{aligned} \quad (\text{A9})$$

$$\begin{aligned} 4\mathbf{K}_{\perp}^2 &= E_T^2 \cos^2(\delta\phi/2) + \Delta K_{\perp}^2 \sin^2(\delta\phi/2) \\ &= E_T^2 - (E_T^2 - \Delta K_{\perp}^2) \sin^2(\delta\phi/2), \end{aligned} \quad (\text{A10})$$

$$2|\mathbf{q}_T||\mathbf{K}_{\perp}| \cos(\phi_T - \phi_{\perp}) = E_T \Delta K_{\perp}, \quad (\text{A11})$$

$$2|\mathbf{q}_T||\mathbf{K}_{\perp}| \sin(\phi_T - \phi_{\perp}) = (E_T^2 - \Delta K_{\perp}^2)/2 \sin(\delta\phi). \quad (\text{A12})$$

We note that the order of the momenta is $|\mathbf{q}_T| \sim \Delta K_{\perp} \sim M$ (hadronic scale) while $E_T \sim 2|\mathbf{K}_{\perp}| \sim \sqrt{s}$, so we see from Eq. (A6) that $\delta\phi \sim M/\sqrt{s}$ and from Eq. (A8) that $\phi_{\perp} - \phi_j \sim M^2/s$. Further useful relations are

$$\begin{aligned} |\mathbf{q}_T|^2 \cos 2(\phi_T - \phi_{\perp}) &\approx |\mathbf{q}_T|^2 \cos 2(\phi_T - \phi_j) \\ &= \Delta K_{\perp}^2 \cos^2(\delta\phi/2) \\ &\quad - E_T^2 \sin^2(\delta\phi/2), \end{aligned} \quad (\text{A13})$$

$$\begin{aligned} |\mathbf{q}_T|^2 \sin 2(\phi_T - \phi_{\perp}) &\approx |\mathbf{q}_T|^2 \sin 2(\phi_T - \phi_j) \\ &= E_T \Delta K_{\perp} \sin(\delta\phi), \end{aligned} \quad (\text{A14})$$

$$\begin{aligned} |\mathbf{q}_T|^4 \cos 4(\phi_T - \phi_{\perp}) &\approx |\mathbf{q}_T|^4 \cos 4(\phi_T - \phi_j) \\ &\approx \Delta K_{\perp}^4 + E_T^4 \sin^4(\delta\phi/2) \\ &\quad - 6E_T^2 \Delta K_{\perp}^2 \sin^2(\delta\phi/2). \end{aligned} \quad (\text{A15})$$

APPENDIX B: PHOTON-JET PRODUCTION

In this appendix we include expressions for the terms A and B for the photon-jet production case, because in Ref. [47] we only considered approximate angular dependence. Similarly to Eq. (16), one can write

$$\frac{d\sigma^{h_1 h_2 \rightarrow \text{jet } X}}{d\eta_\gamma d\eta_j d^2\mathbf{K}_{\gamma\perp} d^2\mathbf{K}_{j\perp}} = \frac{\alpha\alpha_s}{s\mathbf{K}_\perp^2} [A(\mathbf{q}_T^2) + B(\mathbf{q}_T^2)\mathbf{q}_T^2 \times \cos 2(\phi_T - \phi_\perp)], \quad (\text{B1})$$

with

$$\begin{aligned} A(y, x_1, x_2, \mathbf{q}_T^2) &= \mathcal{A}^{qg \rightarrow \gamma q} + \mathcal{A}^{q\bar{q} \rightarrow \gamma g}, \\ B(y, x_1, x_2, \mathbf{q}_T^2) &= \mathcal{B}^{q\bar{q} \rightarrow \gamma g}. \end{aligned} \quad (\text{B2})$$

By comparison with Eqs. (15), (16), and (19) in Ref. [47], we find the following expressions:

$$\begin{aligned} \mathcal{A}^{qg \rightarrow \gamma q} &= \sum_q e_q^2 [h(y)\mathcal{F}^{qg}(x_1, x_2, \mathbf{q}_T^2) \\ &+ h(1-y)\mathcal{F}^{gq}(x_1, x_2, \mathbf{q}_T^2)], \end{aligned} \quad (\text{B3})$$

with

$$h(y) = \frac{1}{N}(1-y)(1+y^2), \quad (\text{B4})$$

$$\begin{aligned} \mathcal{A}^{q\bar{q} \rightarrow \gamma g} &= \frac{N^2 - 1}{N^2} (y^2 + (1-y)^2) \sum_q e_q^2 [\mathcal{F}^{q\bar{q}}(x_1, x_2, \mathbf{q}_T^2) \\ &+ \mathcal{F}^{\bar{q}q}(x_1, x_2, \mathbf{q}_T^2)], \end{aligned} \quad (\text{B5})$$

and

$$\begin{aligned} \mathcal{B}^{q\bar{q} \rightarrow \gamma g} &= 2 \frac{N^2 - 1}{N^2} y(1-y) \sum_q e_q^2 [\mathcal{H}^{q\bar{q}}(x_1, x_2, \mathbf{q}_T^2) \\ &+ \mathcal{H}^{\bar{q}q}(x_1, x_2, \mathbf{q}_T^2)], \end{aligned} \quad (\text{B6})$$

in agreement with the results in Refs. [54,57,58].

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