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Evaluating decay rates and asymmetries of Λ_b into light baryons in the light-front quark model

Zheng-Tao Wei,[1] Hong-Wei Ke,[2,*] and Xue-Qian Li[1]

[1]*School of Physics, Nankai University, Tianjin, 300071, People's Republic of China*
[2]*School of Sciences, Tianjin University, Tianjin, 300072, People's Republic of China*
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In this work we calculate the branching ratios of semileptonic and nonleptonic decays of Λ_b into light baryons (p and Λ), as well as the measurable asymmetries which appear in the processes, in the light-front quark model. In the calculation, we adopt the diquark picture and discuss the justifiability of applying the picture in our case. Our result on the branching ratio of $\Lambda_b \to \Lambda + J/\psi$ is in good agreement with the data. More predictions are made in the same model and the results will be tested in the future experiments which will be conducted at the Large Hadron Collider beauty and even the International Linear Collider.

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I. INTRODUCTION

As is well known, the Λ_b weak decay gives us abundant information about Cabibbo-Kobayashi-Maskawa (CKM) elements, so that it stands as a complementary field to the meson decays. These processes are also good probes for the factorization hypothesis which has been extensively explored for dealing with hadronic transitions [1,2]. Recently many semileptonic and nonleptonic decays of Λ_b are observed and measured [3,4]. Moreover the LHCb is expected to accumulate a large data sample of b hadrons to offer a unique opportunity for studying Λ_b; thus we would like to investigate the Λ_b weak decay more systematically. As for the Λ_b decays the key is how to evaluate the form factors which parametrize the hadronic matrix elements. There are many approaches advocated to this aspect [5]. In our previous paper [6] we studied Λ_b to Λ_c weak decay in the light-front quark model (LFQM) [7] and the results seem to be quite reasonable.

The light-front quark model is a relativistic quark model based on the light-front QCD [7]. The basic ingredient is the hadron light-front wave function which is explicitly Lorentz invariant. The hadron spin is constructed using the Melosh rotation. The light-front approach has been widely applied to calculate various decay constants and form factors for the meson cases [8–12].

In our earlier work, we adopted the diquark picture for baryons [6] which is especially well explored and proved to be a good approximation for such processes where the diquarks are not broken during the transition. Indeed, it has been known for a long time that two quarks in a color-antitriplet state attract each other and may form a correlated diquark [13]. The diquark picture of baryons is considered to be appropriate for low momentum-transfer processes [14–17]. Concretely, under the diquark approximation, Λ_b and Λ_c are of the one-heavy-quark–one-light-diquark(ud) structure which is analogous to the meson case.

*Corresponding author.
khw020056@hotmail.com

In this paper we will apply these methods to Λ_b decaying into light hadrons such as proton or Λ which is made of three light quarks. These hadrons may also be regarded possessing quark-diquark structure [14].

Some authors [18–21] calculated the form factors of Λ_b decaying into light baryons and the corresponding decay rates. Reference [20] explored $\Lambda_b \to pl\bar{\nu}$ by using the method of perturbative QCD (PQCD) and they concluded the perturbative analysis is reliable only for $\rho(\equiv 2p\cdot p'/M^2_{\Lambda_b}) > 0.8$. In Ref. [21] the branching ratio of $\Lambda_b \to J/\Psi\Lambda$ in PQCD was evaluated $[(1.7\text{–}5.3)\times 10^{-4}]$ [18]; instead, Cheng used the nonrelativistic quark model to obtain this branching ratio as 1.1×10^{-4} which is lower than the experimental value $[(4.7\pm 2.8)\times 10^{-4}]$. In a recent study, the authors of [22] used the light-cone sum rules to calculate the $\Lambda_b \to p(\Lambda)$ transition form factors.

In this work, we study the form factors of $\Lambda_b \to p$ and $\Lambda_b \to \Lambda$ in the light-front model with the diquark picture, and then we calculate the rates of $\Lambda_b \to p\pi$, $\Lambda_b \to J/\Psi\Lambda$, as well as several other nonleptonic decays of Λ_b.

When Λ_b decays into light baryons, the energy of the light baryon in the Λ_b rest frame is $E = (M^2_{\Lambda_b} + m^2 - q^2)/(2M_{\Lambda_b})$ which is much larger than its mass m and the hadronic scale $\Lambda_{\rm QCD}$. One important feature of this region is that the light hadrons move nearly along the light cone. It is argued in [23] that the active quark created from the b quark by weak interaction carries most of the energy of the final light baryon. Under the large-energy limit (LEET [24]) and heavy quark limit (HQET [25]), we can obtain the relations between f_3, g_3 and f_2, g_2, which may help to achieve the orders of f_3, g_3. We write up these relations in Sec. II, and then derive the form factors (f_1, f_2, g_1, and g_2) of $\Lambda_b \to p$ and $\Lambda_b \to \Lambda$ in Sec. III. We carry out the numerical computations in Sec. IV. Finally, Sec. V is devoted to discussions from which we draw our conclusion.

II. FORMULATION

A. The form factors in the large-energy limit

The form factors for the weak transition $\Lambda_b \to H$, where H represents a light baryon (refers to p, Λ in this study), are defined in the standard way as

$$\begin{aligned}\mathcal{M}_\mu &= \langle H(P', S', S_z') \mid \bar{q}\gamma_\mu(1-\gamma_5)b \mid \Lambda_b(P, S, S_z)\rangle \\ &= \bar{u}_H(P', S_z')\Big[\gamma_\mu f_1(q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{M_{\Lambda_b}} f_2(q^2) \\ &\quad + \frac{q_\mu}{M_{\Lambda_b}} f_3(q^2)\Big]u_{\Lambda_b}(P, S_z) - \bar{u}_H(P', S_z') \\ &\quad \times \Big[\gamma_\mu g_1(q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{M_{\Lambda_b}} g_2(q^2) + \frac{q_\mu}{M_{\Lambda_b}} g_3(q^2)\Big] \\ &\quad \times \gamma_5 u_{\Lambda_b}(P, S_z), \end{aligned} \tag{1}$$

where $q \equiv P - P'$, Q and Q' denote the heavy quark and the light quark, and H stands for the light baryon, respectively. The above formulation is the most general expression with only constraints of enforcing the Lorentz invariance and parity conservation for strong interaction. There are six form factors f_i, g_i ($i = 1, 2, 3$) in total for the vector and axial-vector current $\bar{q}\gamma_\mu(1-\gamma_5)b$, where the light quark q denotes u for p and s for Λ. All the information about the strong interaction is involved in those form factors. Since $S = S' = 1/2$, we will be able to write $\mid \Lambda_b(P, S, S_z)\rangle$ as $\mid \Lambda_b(P, S_z)\rangle$ and similarly for $\bar{u}_H(P', S_z')$ in the following formulations.

Another parametrization in terms of the four-velocities is widely used and is found to be convenient for the heavy-to-heavy transitions, such as $\Lambda_b \to \Lambda_c$. But for the heavy-to-light transitions at the large recoil region where the energy of the final light baryon H is much larger than its mass, it is more convenient to use the following formulation. Analogous to heavy quark symmetry in the heavy-to-heavy case, there is a large-energy symmetry relation for the heavy-to-light at large-energy recoil [23]. For the heavy-to-light baryon transition, the symmetry has not been searched until the present. In this section, we explore the large-energy symmetry and show that they lead to a simplification of the form factors: the six form factors are reduced to three independent ones.

Let us introduce the velocity v of initial Λ_b and a light-front unit vector n by

$$v = \frac{P}{M_{\Lambda_b}}, \qquad n = \frac{P'}{E}, \tag{2}$$

where E is the energy of H. Using these vectors, the amplitude of the weak transition $\Lambda_b \to H$ is parametrized by

$$\begin{aligned}\mathcal{M}_\mu &= \langle H(n, S_z') \mid \bar{q}\gamma_\mu(1-\gamma_5)b \mid \Lambda_Q(v, S_z)\rangle \\ &= \bar{u}_H(n, S_z')[F_1(E)\gamma_\mu + F_2(E)v_\mu + F_3(E)n_\mu]u_{\Lambda_b} \\ &\quad \times (v, S_z) - \bar{u}_H(n, S_z')[G_1(E)\gamma_\mu + G_2(E)v_\mu \\ &\quad + G_3(E)n_\mu]\gamma_5 u_{\Lambda_b}(v, S_z). \end{aligned} \tag{3}$$

Up to leading order in $1/M_{\Lambda_b}$ the relation between the two parametrization schemes is

$$\begin{aligned} f_1 &= F_1 + \frac{1}{2}\left(\frac{F_2}{M_{\Lambda_b}} + \frac{F_3}{E}\right)M_{\Lambda_b}, \\ g_1 &= G_1 - \frac{1}{2}\left(\frac{G_2}{M_{\Lambda_b}} + \frac{G_3}{E}\right)M_{\Lambda_b}, \\ f_2 &= \frac{1}{2}\left(\frac{F_2}{M_{\Lambda_b}} + \frac{F_3}{E}\right)M_{\Lambda_b}, \qquad g_2 = \frac{1}{2}\left(\frac{G_2}{M_{\Lambda_b}} + \frac{G_3}{E}\right)M_{\Lambda_b}, \\ f_3 &= \frac{1}{2}\left(\frac{F_2}{M_{\Lambda_b}} - \frac{F_3}{E}\right)M_{\Lambda_b}, \qquad g_3 = \frac{1}{2}\left(\frac{G_2}{M_{\Lambda_b}} - \frac{G_3}{E}\right)M_{\Lambda_b}, \end{aligned} \tag{4}$$

where M_{Λ_b} is the mass of Λ_b. We have neglected the mass of the final light baryon compared to M_{Λ_b}.

Under the large-energy limit, the light energetic quark q is described by the two-component spinor $\xi = \frac{\not{n}\not{\bar{n}}}{4}q$, where $\bar{n} = 2 - n$ is another light-front unit vector and the heavy quark is replaced by

$$h_v = e^{im_b v\cdot x}\frac{(1+\not{v})}{2}b.$$

The weak current $\bar{q}\Gamma b$ in the full QCD is matched onto the current $\bar{\xi}\Gamma h_v$ in the effective theory at tree level. For an arbitrary matrix Γ, $\bar{\xi}\Gamma h_v$ has only three independent Dirac structures. One convenient choice is discussed in [26]: $\bar{\xi}h_v$, $\bar{\xi}\gamma_5 h_v$, and $\bar{\xi}\gamma_\perp^\mu h_v$. Thus, we have

$$\begin{aligned} \bar{q}\gamma^\mu b &= \bar{\xi}\gamma_\perp^\mu h_v + n^\mu\bar{\xi}h_v, \\ \bar{q}\gamma^\mu\gamma_5 b &= i\epsilon_\perp^{\mu\nu}\bar{\xi}\gamma_\perp^\nu h_v - n^\mu\bar{\xi}\gamma_5 h_v, \end{aligned} \tag{5}$$

where $\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}v_\alpha n_\beta$.

The three independent form factors are defined by

$$\begin{aligned} &\langle H(P', S_z') \mid \bar{\xi}h_v \mid \Lambda_b(P, S_z)\rangle \\ &\qquad = \bar{u}_H(n, S_z)u_{\Lambda_b}(v, S_z)\zeta_0(E), \\ &\langle H(P', S_z') \mid \bar{\xi}\gamma_5 h_v \mid \Lambda_b(P, S_z)\rangle \\ &\qquad = \bar{u}_H(n, S_z)\gamma_5 u_{\Lambda_b}(v, S_z)\zeta_5(E), \\ &\langle H(P', S_z') \mid \bar{\xi}\gamma_{\perp\mu} h_v \mid \Lambda_b(P, S_z)\rangle \\ &\qquad = \bar{u}_H(n, S_z)\gamma_{\perp\mu} u_{\Lambda_b}(v, S_z)\zeta_\perp(E). \end{aligned} \tag{6}$$

Then, we find

$$\begin{aligned} F_1 = G_1 &= \zeta_\perp(E), \qquad F_2 = G_2 = 0, \\ F_3 = \zeta_0(E) - \zeta_\perp(E), &\qquad G_3 = \zeta_\perp(E) - \zeta_5(E). \end{aligned} \tag{7}$$

From the above equation, we obtain the relations among the form factors:

$$f_1 + f_2 = g_1 - g_2, \qquad f_2 = -f_3, \qquad g_2 = -g_3. \quad (8)$$

This is one major result of this work. The f_3 and g_3 are not independent, but related to f_2 and g_2.

B. Vertex function in the light-front approach

In the diquark picture, the heavy baryon Λ_b is composed of one heavy quark b and a light diquark [ud]. In order to form a color singlet hadron, the diquark [ud] is in a color antitriplet. Because Λ_b is at the ground state, the diquark is a 0^+ scalar ($s = 0$, $l = 0$) and the orbital angular momentum between the diquark and the heavy quark is also zero, i.e. $L = l = 0$. However the situation is complicated for a light baryon even thought it is in the ground state. The diquark in the light baryon may be a 0^+ scalar or a 1^- vector. Fortunately the diquark is a spectator in the concerned transition and its spin is not affected so that only the scalar diquark can transit into the final baryon and one only needs to consider the scalar diquark structure of the light baryon.

In the light-front approach, the heavy baryon Λ_Q composed of only a scalar diquark with total momentum P and spin $S = 1/2$ can be written as

$$\begin{aligned}|\Lambda_Q(P, S, S_z)\rangle = &\int \{d^3 p_1\}\{d^3 p_2\} 2(2\pi)^3 \delta^3(\tilde{P} - \tilde{p}_1 - \tilde{p}_2) \\ &\times \sum_{\lambda_1} \Psi^{SS_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1) C_{\alpha\beta\gamma} F^{bc} \\ &\times |Q^\alpha(p_1, \lambda_1)[q_b^\beta q_c^\gamma](p_2)\rangle, \end{aligned} \quad (9)$$

and the light baryon (total momentum P, spin $J = 1/2$, composed of 0^+ scalar diquark and orbital angular momentum $L = 0$) has the similar form,

$$\begin{aligned}|H(P, S, S_z)\rangle = &\int d^3 p_1 d^3 p_2 2(2\pi)^3 \delta^3(\tilde{P} - \tilde{p}_1 - \tilde{p}_2) \\ &\times \sum_{\lambda_1} \Psi^{SS_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1) C_{\alpha,\beta,\gamma} F_L^{a,b,c} \\ &\times |q_a^\alpha(p_1, \lambda_1)[q_b^\beta q_c^\gamma](p_2)\rangle, \end{aligned} \quad (10)$$

where Q, $[q_b q_c]$ represent the heavy quark and the diquark, respectively, and λ denotes the helicity, where α, β, γ and a, b, c are the color and flavor indices, and p_1, p_2 are the on-mass-shell light-front momenta defined by

$$\tilde{p} = (p^+, p_\perp), \qquad p_\perp = (p^1, p^2), \qquad p^- = \frac{m^2 + p_\perp^2}{p^+}, \quad (11)$$

and

$$\begin{aligned}\{d^3 p\} &\equiv \frac{dp^+ d^2 p_\perp}{2(2\pi)^3}, \\ \delta^3(\tilde{p}) &= \delta(p^+)\delta^2(p_\perp), \\ |Q(p_1, \lambda_1)[q_1 q_2](p_2)\rangle &= b^\dagger_{\lambda_1}(p_1) a^\dagger(p_2)|0\rangle, \\ [a(p'), a^\dagger(p)] &= 2(2\pi)^3 \delta^3(\tilde{p}' - \tilde{p}), \\ \{d_{\lambda'}(p'), d^\dagger_\lambda(p)\} &= 2(2\pi)^3 \delta^3(\tilde{p}' - \tilde{p})\delta_{\lambda'\lambda}. \end{aligned} \quad (12)$$

The coefficient $C_{\alpha\beta\gamma}$ is a normalized color factor and $F^{bc}(F^{abc})$ is a normalized flavor coefficient,

$$\begin{aligned}&C_{\alpha\beta\gamma} F^{bc} C_{\alpha'\beta'\gamma'} F^{b'c'} \\ &\quad\times \langle Q^{\alpha'}(p_1', \lambda_1')[q_{b'}^{\beta'} q_{c'}^{\gamma'}](p_2')|Q^\alpha(p_1, \lambda_1)[q_b^\beta q_c^\gamma](p_2)\rangle \\ &= 2^2(2\pi)^6 \delta^3(\tilde{p}_1' - \tilde{p}_1)\delta^3(\tilde{p}_2' - \tilde{p}_2)\delta_{\lambda_1'\lambda_1}, \\ &C_{\alpha\beta\gamma} F^{abc} C_{\alpha'\beta'\gamma'} F^{a'b'c'} \\ &\quad\times \langle q_{a'}^{\alpha'}(p_1', \lambda_1')[q_{b'}^{\beta'} q_{c'}^{\gamma'}](p_2')|q_a^\alpha(p_1, \lambda_1)[q_b^\beta q_c^\gamma](p_2)\rangle \\ &= 2^2(2\pi)^6 \delta^3(\tilde{p}_1' - \tilde{p}_1)\delta^3(\tilde{p}_2' - \tilde{p}_2)\delta_{\lambda_1'\lambda_1}. \end{aligned} \quad (13)$$

In order to describe the motion of the constituents, one needs to introduce intrinsic variables $(x_i, k_{i\perp})$ with $i = 1, 2$ through

$$\begin{aligned}p_1^+ &= x_1 P^+, \qquad p_2^+ = x_2 P^+, \qquad x_1 + x_2 = 1, \\ p_{1\perp} &= x_1 P_\perp + k_{1\perp}, \qquad p_{2\perp} = x_2 P_\perp + k_{2\perp}, \\ k_\perp &= -k_{1\perp} = k_{2\perp}, \end{aligned} \quad (14)$$

where x_i's are the light-front momentum fractions satisfying $0 < x_1, x_2 < 1$. The variables $(x_i, k_{i\perp})$ are independent of the total momentum of the hadron and thus are Lorentz invariant. The invariant mass square M_0^2 is defined as

$$M_0^2 = \frac{k_{1\perp}^2 + m_1^2}{x_1} + \frac{k_{2\perp}^2 + m_2^2}{x_2}. \quad (15)$$

The invariant mass M_0 is in general different from the hadron mass M which satisfies the physical mass-shell condition $M^2 = P^2$. This is due to the fact that in the baryon, the heavy quark and the diquark cannot be on their mass shells simultaneously. We define the internal momenta as

$$\begin{aligned}k_i &= (k_i^-, k_i^+, k_{i\perp}) = (e_i - k_{iz}, e_i + k_{iz}, k_{i\perp}) \\ &= \left(\frac{m_i^2 + k_{i\perp}^2}{x_i M_0}, x_i M_0, k_{i\perp}\right). \end{aligned} \quad (16)$$

It is easy to obtain

$$\begin{aligned}M_0 &= e_1 + e_2, \\ e_i &= \frac{x_i M_0}{2} + \frac{m_i^2 + k_{i\perp}^2}{2x_i M_0} = \sqrt{m_i^2 + k_{i\perp}^2 + k_{iz}^2}, \\ k_{iz} &= \frac{x_i M_0}{2} - \frac{m_i^2 + k_{i\perp}^2}{2x_i M_0}, \end{aligned} \quad (17)$$

where e_i denotes the energy of the ith constituent. The momenta $k_{i\perp}$ and k_{iz} constitute a momentum vector $\vec{k}_i = (k_{i\perp}, k_{iz})$ and correspond to the components in the transverse and z directions, respectively.

In the momentum space, the function Ψ^{SS_z} appearing in Eq. (9) is expressed as

$$\Psi^{SS_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1) = \langle \lambda_1 | \mathcal{R}_M^\dagger(x_1, k_{1\perp}, m_1) | s_1 \rangle \times \langle 00; \tfrac{1}{2} s_1 | \tfrac{1}{2} S_z \rangle \phi(x, k_\perp), \quad (18)$$

where $\phi(x, k_\perp)$ is the light-front wave function which describes the momentum distribution of the constituents in the bound state with $x = x_2$, $k_\perp = k_{2\perp}$; $\langle 00; \frac{1}{2} s_1 | \frac{1}{2} S_z \rangle$ is the corresponding Clebsch-Gordan coefficient with total spin of the scalar diquark $s = s_z = 0$; and $\langle \lambda_1 | \mathcal{R}_M^\dagger(x_1, k_{1\perp}, m_1) | s_1 \rangle$ is the well-known Melosh transformation matrix element which transforms the conventional spin states in the instant form into the light-front helicity eigenstates,

$$\langle \lambda_1 | \mathcal{R}_M^\dagger(x_1, k_{1\perp}, m_1) | s_1 \rangle = \frac{\bar{u}(k_1, \lambda_1) u_D(k_1, s_1)}{2m_1} = \frac{(m_1 + x_1 M_0)\delta_{\lambda_1 s_1} + i\vec{\sigma}_{\lambda_1 s_1} \cdot \vec{k}_{1\perp} \times \vec{n}}{\sqrt{(m_1 + x_1 M_0)^2 + k_{1\perp}^2}}, \quad (19)$$

where $u_{(D)}$ denotes a Dirac spinor in the light-front (instant) form and $\vec{n} = (0, 0, 1)$ is a unit vector in the z direction. In practice, it is more convenient to use the covariant form for the Melosh transform matrix [8,11]

$$\langle \lambda_1 | \mathcal{R}_M^\dagger(x_1, k_{1\perp}, m_1) | s_1 \rangle \left\langle 00; \frac{1}{2} s_1 \middle| \frac{1}{2} S_z \right\rangle = \frac{1}{\sqrt{2(p_1 \cdot \bar{P} + m_1 M_0)}} \bar{u}(p_1, \lambda_1) \Gamma u(\bar{P}, S_z), \quad (20)$$

where

$$\Gamma = 1, \qquad \bar{P} = p_1 + p_2, \quad (21)$$

for the scalar diquark. If the diquark is a vector which is usually supposed to be the case for the $\Sigma_{c(b)}$ baryon, the Melosh transform matrix should be modified (since it is irrelevant to our present work, we omit the corresponding expressions).

The baryon state is normalized as

$$\langle \Lambda(P', S', S'_z) | \Lambda(P, S, S_z) \rangle = 2(2\pi)^3 P^+ \delta^3(\tilde{P}' - \tilde{P}) \delta_{S'S} \delta_{S'_z S_z}, \quad (22)$$

and the same for $H(P, S, S_z)$.

Thus, the light-front wave function obeys the constraint

$$\int \frac{dx d^2 k_\perp}{2(2\pi^3)} |\phi(x, k_\perp)|^2 = 1. \quad (23)$$

In principle, the wave functions can be obtained by solving the light-front bound state equations. However, it is too hard to calculate them based on the first principle, so that instead, we would like to adopt a phenomenological function, and obviously, a Gaussian form is most preferable,

$$\phi(x, k_\perp) = N \sqrt{\frac{\partial k_{2z}}{\partial x_2}} \exp\left(\frac{-\vec{k}^2}{2\beta^2}\right), \quad (24)$$

with

$$N = 4\left(\frac{\pi}{\beta^2}\right)^{3/4}, \qquad \frac{\partial k_{2z}}{\partial x_2} = \frac{e_1 e_2}{x_1 x_2 M_0}, \quad (25)$$

where β determines the confinement scale. The phenomenological parameters in the light-front quark model are quark masses and the hadron wave function parameter β which should be prior determined before numerical computations can be carried out, and we will do the job in the later sections.

C. $\Lambda_Q \to H$ weak transitions

Equipped with the light-front quark model description of $| \Lambda_Q(P, S_z) \rangle$ and $| H(P, S_z) \rangle$, we can calculate the weak transition matrix elements

$$\langle \Lambda_Q(P', S'_z) | \bar{q}\gamma_\mu(1 - \gamma_5) Q | H(P, S_z) \rangle = N_{IF} \int \{d^3 p_2\} \times \frac{\phi_H^{\prime *}(x', k'_\perp) \phi_{\Lambda_Q}(x, k_\perp)}{2\sqrt{p_1^+ p_1^{\prime +}(p_1 \cdot \bar{P} + m_1 M_0)(p'_1 \cdot \bar{P}' + m'_1 M'_0)}} \times \bar{u}(\bar{P}', S'_z) \bar{\Gamma}' (\not{p}'_1 + m'_1) \gamma_\mu (1 - \gamma_5)(\not{p}_1 + m_1) \times \Gamma_{Lm} u(\bar{P}, S_z), \quad (26)$$

where N_{IF} is a flavor-spin factor of I (initial particle) decaying into F (final particle). Following [14], the flavor-spin functions of Λ_b, proton and Λ take the forms in the diquark picture

$$\begin{aligned} \chi_S^{\Lambda_b} &= bS_{[u,d]}, \qquad \chi_S^p = uS_{[u,d]}, \\ \chi_V^p &= [uV_{[u,d]} - \sqrt{2} dV_{[u,u]}]/\sqrt{3} \chi_S^\Lambda \\ &= [uS_{[d,s]} - dS_{[u,s]} - 2sS_{[u,d]}]/\sqrt{6}, \\ \chi_V^\Lambda &= [uV_{[d,s]} - dV_{[u,s]}]/\sqrt{2}, \end{aligned} \quad (27)$$

where S and V denote the scalar and the axial-vector diquark. We can get $N_{\Lambda_b p} = \frac{1}{\sqrt{2}}$, $N_{\Lambda_b \Lambda} = \frac{1}{\sqrt{3}}$, which are consistent with [18], and

$$\begin{aligned} \bar{\Gamma}' &= \gamma_0 \Gamma \gamma_0 = \Gamma = 1, \qquad m_1 = m_b, \\ m'_1 &= m_q, \qquad m_2 = m_{[ud]}, \end{aligned} \quad (28)$$

with P and P' denoting the momenta of initial and final baryons, and p_1, p_1' are the momenta of b and c quarks, respectively. Because the diquark is a scalar, one does not need to deal with the spinors which make computations more complex. In this framework, at each effective vertex, only the three-momentum rather than the four-momentum is conserved; hence $\tilde{p}_1 - \tilde{p}_1' = \tilde{q}$ and $\tilde{p}_2 = \tilde{p}_2'$. From $\tilde{p}_2 = \tilde{p}_2'$, we have

$$x' = \frac{P^+}{P'^+}x, \qquad k'_\perp = k_\perp + x_2 q_\perp, \tag{29}$$

with $x = x_2$, $x' = x_2'$. Thus, Eq. (26) is rewritten as

$$\langle H(P', S_z') \mid \bar{q}\gamma^\mu(1-\gamma_5)Q \mid \Lambda_Q(P, S_z)\rangle = N_{IF}\int \frac{dxd^2k_\perp}{2(2\pi)^3}\frac{\phi_H(x', k'_\perp)\phi_{\Lambda_Q}(x, k_\perp)}{2\sqrt{x_1x_1'(p_1\cdot\bar{P} + m_1M_0)(p_1'\cdot\bar{P}' + m_1'M_0')}} \times \bar{u}(\bar{P}', S_z')(\not{p}_1' + m_1')\gamma^\mu(1-\gamma_5)(\not{p}_1 + m_1)u(\bar{P}, S_z). \tag{30}$$

Following [6,27], we get the final expressions for the $\Lambda_Q \to H$ weak transition form factors

$$\begin{aligned}
f_1(q^2) &= N_{IF}\int\frac{dxd^2k_\perp}{2(2\pi)^3}\frac{\phi_H(x', k'_\perp)\phi_{\Lambda_Q}(x, k_\perp)[k_{2\perp}\cdot k'_{2\perp} + (x_1M_0 + m_1)(x_1'M_0' + m_1')]}{\sqrt{[(m_1 + x_1M_0)^2 + k_{2\perp}^2][(m_1' + x_1M_0')^2 + k'^2_{2\perp}]}},\\
g_1(q^2) &= N_{IF}\int\frac{dxd^2k_\perp}{2(2\pi)^3}\frac{\phi_H(x', k'_\perp)\phi_{\Lambda_Q}(x, k_\perp)[-k_{2\perp}\cdot k'_{2\perp} + (x_1M_0 + m_1)(x_1'M_0' + m_1')]}{\sqrt{[(m_1 + x_1M_0)^2 + k_{2\perp}^2][(m_1' + x_1M_0')^2 + k'^2_{2\perp}]}},\\
\frac{f_2(q^2)}{M_{\Lambda_Q}} &= \frac{N_{IF}}{q_\perp^i}\int\frac{dxd^2k_\perp}{2(2\pi)^3}\frac{\phi_H(x', k'_\perp)\phi_{\Lambda_Q}(x, k_\perp)[(m_1 + x_1M_0)k'^i_{1\perp} - (m_1' + x_1'M_0')k^i_{1\perp}]}{\sqrt{[(m_1 + x_1M_0)^2 + k_{2\perp}^2][(m_1' + x_1M_0')^2 + k'^2_{2\perp}]}},\\
\frac{g_2(q^2)}{M_{\Lambda_Q}} &= \frac{N_{IF}}{q_\perp^i}\int\frac{dxd^2k_\perp}{2(2\pi)^3}\frac{\phi_H(x', k'_\perp)\phi_{\Lambda_Q}(x, k_\perp)[(m_1 + x_1M_0)k'^i_{1\perp} + (m_1' + x_1'M_0')k^i_{1\perp}]}{\sqrt{[(m_1 + x_1M_0)^2 + k_{2\perp}^2][(m_1' + x_1M_0')^2 + k'^2_{2\perp}]}}.
\end{aligned} \tag{31}$$

It is noted that the form factors f_3 and g_3 cannot be extracted in our method because we have imposed the condition $q^+ = 0$. The fact that the calculated f_2 and g_2 at $q^2 = 0$ are small compared to f_1 and g_1 and the large-energy limit relations $f_3 = -f_2$ and $g_3 = -g_2$ show that using the large-energy limit relations for f_3 and g_3 does not produce substantial theoretical errors.

III. SEMILEPTONIC AND NONLEPTONIC DECAYS OF TRANSITION $\Lambda_b \to$ LIGHT HADRONS

In this section, we obtain formulations for the rates of semileptonic and nonleptonic processes. In this work, we are concerned with only the exclusive decay modes.

A. Semileptonic decays of $\Lambda_b \to pl\bar{\nu}_l$

Generally the polarization effects may be important for testifying different theoretical models, so that we should pay more attention to the physical consequences brought up by them. The transition amplitude of $\Lambda_b \to p$ contains several independent helicity components. According to the definitions of the form factors for $\Lambda_b \to p$ given in Eq. (1), the helicity amplitudes $H^V_{i,j}$ are related to these form factors through the following expressions [28]:

$$\begin{aligned}
H^V_{1/2,0} &= \frac{\sqrt{Q_-}}{\sqrt{q^2}}\left((M_{\Lambda_b} + M_{\Lambda_c})f_1 - \frac{q^2}{M_{\Lambda_b}}f_2\right),\\
H^V_{1/2,1} &= \sqrt{2Q_-}\left(-f_1 + \frac{M_{\Lambda_b} + M_{\Lambda_c}}{M_{\Lambda_b}}f_2\right),\\
H^A_{1/2,0} &= \frac{\sqrt{Q_+}}{\sqrt{q^2}}\left((M_{\Lambda_b} - M_{\Lambda_c})g_1 + \frac{q^2}{M_{\Lambda_b}}g_2\right),\\
H^A_{1/2,1} &= \sqrt{2Q_+}\left(-g_1 - \frac{M_{\Lambda_b} - M_{\Lambda_c}}{M_{\Lambda_b}}g_2\right),
\end{aligned} \tag{32}$$

where $Q_\pm = 2(P\cdot P' \pm M_{\Lambda_b}M_p) = 2M_{\Lambda_b}M_p(\omega \pm 1)$.

The helicities of the W boson λ_W can be either 0 or 1, corresponding to the longitudinal and transverse polarizations. Following the definitions in literature, we decompose the decay width into a sum of the longitudinal and transverse parts according to the helicity states of the virtual W boson. The differential decay rate of $\Lambda_b \to pl\bar{\nu}_l$ is

$$\frac{d\Gamma}{d\omega} = \frac{d\Gamma_L}{d\omega} + \frac{d\Gamma_T}{d\omega}, \tag{33}$$

and the longitudinally (L) and transversely (T) polarized rates are, respectively [28],

$$\frac{d\Gamma_L}{d\omega} = \frac{G_F^2|V_{ub}|^2}{(2\pi)^3}\frac{q^2 p_c M_p}{12M_{\Lambda_b}}[|H_{1/2,0}|^2 + |H_{-(1/2),0}|^2],$$
$$\frac{d\Gamma_T}{d\omega} = \frac{G_F^2|V_{ub}|^2}{(2\pi)^3}\frac{q^2 p_c M_p}{12M_{\Lambda_b}}[|H_{1/2,1}|^2 + |H_{-(1/2),-1}|^2], \tag{34}$$

where $p_c = M_p\sqrt{\omega^2 - 1}$ is the momentum of the proton in the rest frame of Λ_b. The relations between $H_{i,j}$ and $H^V_{i,j}$ can be found in [28]. Integrating over the solid angle, we obtain the decay rate as

$$\Gamma = \int_1^{\omega_{\max}} d\omega \frac{d\Gamma}{d\omega}, \tag{35}$$

where the upper bound of the integration

$$\omega_{\max} = \frac{1}{2}\left(\frac{M_{\Lambda_b}}{M_p} + \frac{M_p}{M_{\Lambda_b}}\right)$$

corresponds to the maximal recoil. In order to compare our results with those in the literature, we use the variable ω in the expression for the differential decay rate.

The polarization of the cascade decay $\Lambda_b \to p + W(\to l\nu)$ is expressed by various asymmetry parameters [28,29]. Among them, the integrated longitudinal and transverse asymmetries are defined by

$$a_L = \frac{\int_1^{\omega_{\max}} d\omega q^2 p_c[|H_{1/2,0}|^2 - |H_{-1/2,0}|^2]}{\int_1^{\omega_{\max}} d\omega q^2 p_c[|H_{1/2,0}|^2 + |H_{-1/2,0}|^2]},$$
$$a_T = \frac{\int_1^{\omega_{\max}} d\omega q^2 p_c[|H_{1/2,1}|^2 - |H_{-1/2,-1}|^2]}{\int_1^{\omega_{\max}} d\omega q^2 p_c[|H_{1/2,1}|^2 + |H_{-1/2,-1}|^2]}. \tag{36}$$

The ratio of the longitudinal to transverse decay rates R is defined by

$$R = \frac{\Gamma_L}{\Gamma_T} = \frac{\int_1^{\omega_{\max}} d\omega q^2 p_c[|H_{1/2,0}|^2 + |H_{-1/2,0}|^2]}{\int_1^{\omega_{\max}} d\omega q^2 p_c[|H_{1/2,1}|^2 + |H_{-1/2,-1}|^2]}, \tag{37}$$

and the longitudinal proton polarization asymmetry P_L is given as

$$P_L = \frac{\int_1^{\omega_{\max}} d\omega q^2 p_c[|H_{1/2,0}|^2 - |H_{-(1/2),0}|^2 + |H_{1/2,1}|^2 - |H_{-1/2,-1}|^2]}{\int_1^{\omega_{\max}} d\omega q^2 p_c[|H_{1/2,0}|^2 + |H_{-(1/2),0}|^2 + |H_{1/2,1}|^2 + |H_{-1/2,-1}|^2]} = \frac{a_T + Ra_L}{1 + R}. \tag{38}$$

B. Nonleptonic decay of $\Lambda_b \to p + M$

From the theoretical aspects, the nonleptonic decays are much more complicated than the semileptonic ones because of the strong interaction. Generally, the present theoretical framework is based on the factorization assumption, where the hadronic matrix element is factorized into a product of two matrix elements of single currents. One can be written as a decay constant while the other is expressed in terms of a few form factors according to the Lorentz structure of the current. For the weak decays of mesons, such a factorization approach is verified to work very well for the color-allowed processes and the nonfactorizable contributions are negligible.

For the nonleptonic decays $\Lambda_b^0 \to p + M$, the effective interaction at the quark level is $b \to u\bar{q}_1 q_2$. The relevant Hamiltonian is

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}} V_{ub}V^*_{q_1q_2}(c_1O_1 + c_2O_2),$$
$$O_1 = (\bar{u}b)_{V-A}(\bar{q}_2q_1)_{V-A},$$
$$O_2 = (\bar{q}_2b)_{V-A}(\bar{u}q_1)_{V-A}, \tag{39}$$

where c_i denotes the short-distance Wilson coefficient, $V_{ub}(V_{q_1q_2})$ is the CKM matrix elements, q_1 stands for u, and q_2 for d in the context. Then one needs to evaluate the hadronic matrix elements

$$\langle pM|\mathcal{H}_W|\Lambda_b\rangle = \frac{G_F}{\sqrt{2}} V_{ub}V^*_{q_1q_2}\sum_{i=1,2} c_i\langle pM|O_i|\Lambda_b\rangle. \tag{40}$$

Under the factorization approximation, the hadronic matrix element is reduced to

$$\langle pM|O_i|\Lambda_b\rangle = \langle p|J_\mu|\Lambda_b\rangle\langle M|J'^\mu|0\rangle, \tag{41}$$

where $J(J')$ is the $V - A$ weak current. The first factor $\langle p|J_\mu|\Lambda_b\rangle$ is parametrized by six form factors as was done in Eq. (1). The second factor defines the decay constants as follows:

$$\langle P(P)|A_\mu|0\rangle = f_P P_\mu, \quad \langle S(P)|V_\mu|0\rangle = f_S P_\mu,$$
$$\langle V(P,\epsilon)|V_\mu|0\rangle = f_V M_V \epsilon^*_\mu, \quad \langle A(P,\epsilon)|A_\mu|0\rangle = f_V M_A \epsilon^*_\mu, \tag{42}$$

where $P(V)$ denotes a pseudoscalar (vector) meson, and $S(A)$ denotes a scalar (axial-vector) meson. In the definitions, we omit a factor $(-i)$ for the pseudoscalar meson decay constant.

In general, the transition amplitude of $\Lambda_b \to p\pi^-$ can be written as

$$\mathcal{M}(\Lambda_b \to pP) = \bar{u}_p(A + B\gamma_5)u_{\Lambda_b},$$
$$\mathcal{M}(\Lambda_b \to pV) = \bar{u}_p\epsilon^{*\mu}[A_1\gamma_\mu\gamma_5 + A_2(p_{\Lambda_c})_\mu\gamma_5 + B_1\gamma_\mu + B_2(p_{\Lambda_c})_\mu]u_{\Lambda_b}, \tag{43}$$

where ϵ^μ is the polarization vector of the final vector or axial-vector mesons. Including the effective Wilson coefficient $a_1 = c_1 + c_2/N_c$, the decay amplitudes under the factorization approximation are [30,31]

$$\begin{aligned} A &= \lambda f_P(M_{\Lambda_b} - M_{\Lambda_c}) f_1(M^2), \\ B &= \lambda f_P(M_{\Lambda_b} + M_{\Lambda_c}) g_1(M^2), \\ A_1 &= -\lambda f_V M\left[g_1(M^2) + g_2(M^2)\frac{M_{\Lambda_b} - M_{\Lambda_c}}{M_{\Lambda_b}}\right], \\ A_2 &= -2\lambda f_V M\frac{g_2(M^2)}{M_{\Lambda_b}}, \\ B_1 &= \lambda f_V M\left[f_1(M^2) - f_2(M^2)\frac{M_{\Lambda_b} + M_{\Lambda_c}}{M_{\Lambda_b}}\right], \\ B_2 &= 2\lambda f_V M\frac{f_2(M^2)}{M_{\Lambda_b}}, \end{aligned} \tag{44}$$

where $\lambda = \frac{G_F}{\sqrt{2}} V_{ub} V^*_{q_1 q_2} a_1$ and M is the π mass. Replacing P, V by S and A in the above expressions, one can easily obtain similar expressions for scalar and axial-vector mesons.

The decay rates of $\Lambda_b \to p\pi^-$ and up-down asymmetries are [31]

$$\begin{aligned} \Gamma &= \frac{p_c}{8\pi}\left[\frac{(M_{\Lambda_b} + M_p)^2 - M^2}{M^2_{\Lambda_b}}|A|^2 + \frac{(M_{\Lambda_b} - M_p)^2 - M^2}{M^2_{\Lambda_b}}|B|^2\right], \\ \alpha &= -\frac{2\kappa\,\mathrm{Re}(A^* B)}{|A|^2 + \kappa^2|B|^2}, \end{aligned} \tag{45}$$

where p_c is the proton momentum in the rest frame of Λ_b and $\kappa = p_c/(E_p + M_p)$. For $\Lambda_b \to \Lambda_c V(A)$ decays, the decay rates and up-down asymmetries are

$$\begin{aligned} \Gamma &= \frac{p_c(E_p + M_p)}{8\pi M_{\Lambda_b}}\left[2(|S|^2 + |P_2|^2) + \frac{E^2}{M^2}(|S + D|^2 + |P_1|^2)\right], \\ \alpha &= \frac{4M^2\,\mathrm{Re}(S^* P_2) + 2E^2\,\mathrm{Re}(S + D)^* P_1}{2M^2(|S|^2 + |P_2|^2) + E^2(|S + D|^2 + |P_1|^2)}, \end{aligned} \tag{46}$$

where E is the energy of the vector (axial-vector) meson, and

$$\begin{aligned} S &= -A_1, \qquad P_1 = -\frac{p_c}{E}\left(\frac{M_{\Lambda_b} + M_p}{E_p + M_p}B_1 + B_2\right), \\ P_2 &= \frac{p_c}{E_p + M_p}B_1, \qquad D = -\frac{p_c^2}{E(E_p + M_p)}(A_1 - A_2). \end{aligned} \tag{47}$$

C. Nonleptonic decay $\Lambda_b \to \Lambda + M$

Theses decays proceed only via the internal W emission. With the factorization assumption, the amplitude is

$$\begin{aligned} A(\Lambda_b \to \Lambda M) &= \frac{G_F}{\sqrt{2}} V_{qb} V^*_{q's} a_2 \langle M|\bar{q}'\gamma_\mu(1 - \gamma_5)q|0\rangle \\ &\quad\times \langle\Lambda|\bar{s}\gamma^\mu(1 - \gamma_5)b|\Lambda_b\rangle. \end{aligned} \tag{48}$$

In general, we can use the same formula [Eqs. (42)–(47)] to obtain the decay rates and up-down asymmetries of $\Lambda_b \to \Lambda + M$. Note that (1) at this time λ is replaced by $\frac{G_F}{\sqrt{2}} V_{ub} V^*_{q_1 q_2} a_2$, and (2) when q and $\bar{q}'$ are u and $\bar{u}$, respectively, the final meson may be π^0, η, or η'.

For the decay constants of π^0, η, and η', we have

$$\begin{aligned} &\langle\pi^0|\bar{u}\gamma_\mu\gamma_5 u|0\rangle = f^u_{\pi^0}P_\mu, \qquad \langle\eta|\bar{u}\gamma_\mu\gamma_5 u|0\rangle = f^u_\eta P_\mu, \\ &\langle\eta'|\bar{u}\gamma_\mu\gamma_5 u|0\rangle = f^u_{\eta'}P_\mu, \end{aligned} \tag{49}$$

where $f^u_{\pi^0} = \frac{f_\pi}{\sqrt{2}}$, f^u_η, and $f^u_{\eta'}$ can be gotten from [1].

IV. NUMERICAL RESULTS

In this section we perform the numerical computations of the form factors for $\Lambda_b \to p$ and $\Lambda_b \to \Lambda$; then using them we estimate the rates of $\Lambda_b \to p + l\nu$, $\Lambda_b \to p + M$, and $\Lambda_b \to \Lambda + M$ where M stands for various mesons.

In our calculation, the quark masses of m_b and m_s are taken from [27]; m_u is set to be 0.3 GeV; the mass of diquark [ud], parameters $\beta_{b,[ud]}$, $\beta_{s,[ud]}$, and $\beta_{u,[ud]}$ are chosen from [6,11,27]. The baryon masses $M_{\Lambda_b} = 5.624$ GeV, $M_p = 0.938$ GeV, and $\Lambda = 1.116$ GeV come from [4]. The input parameters are collected in Table I.

A. Form factor

In LFQM, the calculation of form factors is performed in the frame $q^+ = 0$ with $q^2 = -q^2_\perp \leq 0$, and only the values of the form factors in the spacelike region can be obtained. The advantage of this choice is that the so-called Z-graph contribution arising from the nonvalence quarks vanishes. In order to obtain the physical form factors, an extrapolation from the spacelike region to the timelike region is required. Following [27], the form factors in the spacelike region can be parametrized in a three-parameter form as

$$F(q^2) = \frac{F(0)}{(1 - \frac{q^2}{M^2_{\Lambda_b}})[1 - a(\frac{q^2}{M^2_{\Lambda_b}}) + b(\frac{q^2}{M^2_{\Lambda_b}})^2]}, \tag{50}$$

where F represents the form factors $f_{1,2}$ and $g_{1,2}$. The parameters a, b, and $F(0)$ are fixed by performing a three-parameter fit to the form factors in the spacelike region which were obtained in previous sections. We then use these parameters to determine the physical form

TABLE I. Input parameters in LFQM (in units of GeV).

m_b	m_s	m_u	$m_{[ud]}$	$\beta_{u,[ud]}$	$\beta_{b,[ud]}$	$\beta_{s,[ud]}$
4.4	0.45	0.3	0.5	0.3	0.4	0.3

TABLE II. The $\Lambda_b \to p$ form factors in the three-parameter form.

F	$F(0)$	a	b
f_1	0.1131	1.70	1.60
f_2	−0.0356	2.50	2.57
g_1	0.1112	1.65	1.60
g_2	−0.0097	2.80	2.70

TABLE III. The $\Lambda_b \to \Lambda$ form factors in the three-parameter form.

F	$F(0)$	a	b
f_1	0.1081	1.70	1.60
f_2	−0.0311	2.50	2.50
g_1	0.1065	1.70	1.40
g_2	−0.0064	2.70	2.70

factors in the timelike region. The fitted values of a, b, and $F(0)$ for different form factors $f_{1,2}$ and $g_{1,2}$ are given in Tables II and III. The q^2 dependence of the form factors is plotted in Fig. 1.

From Fig. 1, we can see that there is only a tiny difference between f_1 and g_1, i.e. they are close to each other. g_2 is small compared to f_1 and g_1. This is the same as the conclusion of [6,32]. But the difference between f_2 and g_2 increases as q^2 increases. This will break the large-energy limit relation $f_1 + f_2 = g_1 - g_2$ proposed in Sec. II A.

Our method of smooth extrapolation of form factors from space- to timelike momentum regions is by no means an analytical continuation in the rigorously mathematical sense but an extension, although it is used in many phenomenological analyses. In [33], the authors suggest to write the form factor as a dispersion relation in q^2 with a lowest-lying pole plus a contribution from multiparticle states. We follow this scheme and use a parametrization method adopted in [34]

TABLE IV. The $\Lambda_b \to p$ form factors in the form of Eq. (51).

F	r_1	r_2	$M_{\rm fit}$
f_1	−0.183	0.295	6.8
f_2	−0.176	0.287	6.8
g_1	0.080	−0.114	6.8
g_2	0.023	−0.032	6.8

$$F(q^2) = \frac{r_1}{(1 - \frac{q^2}{M_{\rm fit}^2})} + \frac{r_2}{(1 - (\frac{q^2}{M_{\rm fit}^2})^2)}. \quad (51)$$

The parameters r_1, r_2, and $M_{\rm fit}$ are fixed in the spacelike regions for the transition of $\Lambda_b \to p$. The results are presented in Table IV. We also plot the form factors in the new parametrization method in Fig. 1 for comparison. From Fig. 1, we can find there is a little difference between the form factors fitted by the above two methods. In particular, the f_1 and g_1 in the two methods are nearly the same. The difference of f_2 and g_2 in the methods increases when q^2 increases, but due to the smallness of their values, they will not produce substantial errors to our predictions.

B. Semileptonic decay of $\Lambda_b \to p + l\bar{\nu}_l$

With the form factors given in the previous section, we are able to calculate the branching ratio and various asymmetries of $\Lambda_b \to pl\bar{\nu}_l$ decay. Table V presents our numerical predictions. The ratio of longitudinal to transverse rates $R > 1$ implies that the longitudinal polarization dominates.

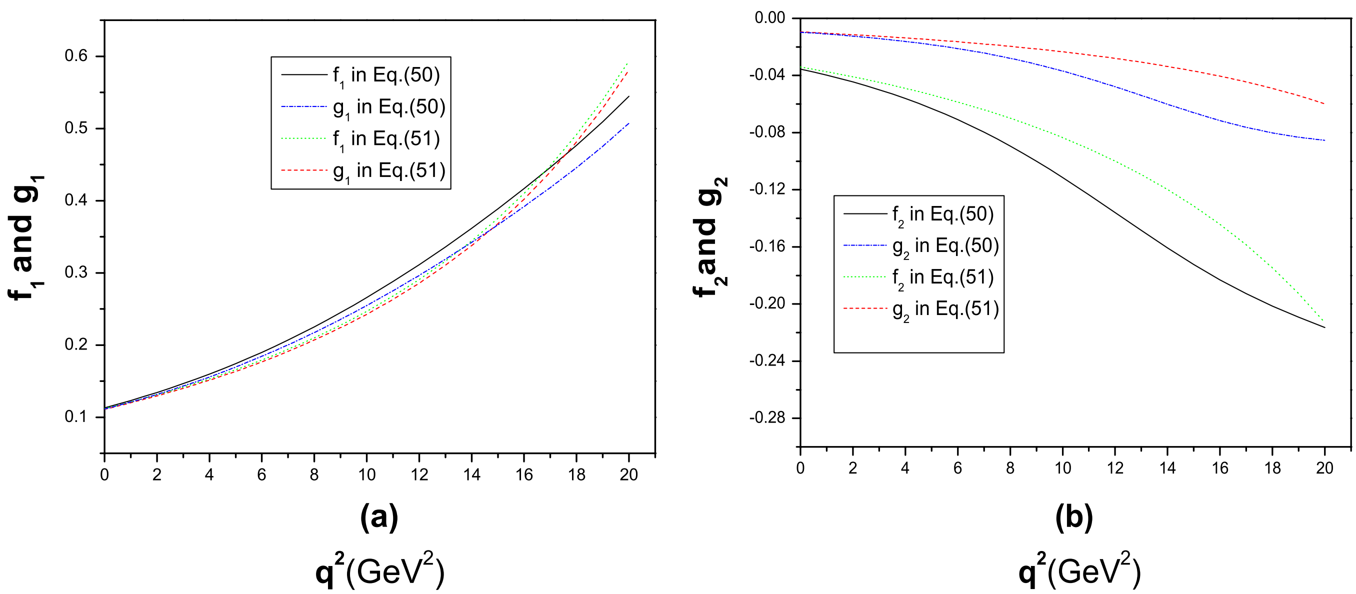

FIG. 1 (color online). (a) Form factors f_1 and g_1 of $\Lambda_b \to p$. (b) Form factors f_2 and g_2 of $\Lambda_b \to p$.

TABLE V. The branching ratios and polarization asymmetries of $\Lambda_b \to p l \bar{\nu}_l$.

BR	a_L	a_T	R	P_L
2.54×10^{-4}	-0.99	-0.96	1.11	-0.97

Nonleptonic decays of $\Lambda_b \to p + M$ and $\Lambda_b \to \Lambda + M$

The nonleptonic decays $\Lambda_b \to p(\Lambda) + M$ in the factorization approach have been studied in the previous section. Now, we present our numerical predictions on the decay rates and relevant measurable quantities. The CKM matrix elements take the values [4]

$$V_{ud} = 0.97377, \qquad V_{us} = 0.2257,$$
$$V_{cd} = 0.230, \qquad V_{cs} = 0.957, \qquad (52)$$
$$V_{cb} = 0.0416, \qquad V_{ub} = 0.00413,$$

and the effective Wilson coefficient $a_1 = 1$ [27], $a_2 = 0.23$ [18]. The meson decay constants are shown in Table VI.

The predictions for $\Lambda_b^0 \to p + M$ are provided in Table VII. Table VIII demonstrates a comparison of our result with other approaches and experimental data for $\Lambda_b^0 \to \Lambda J/\psi$. In Table IX we give predictions on the rates of $\Lambda_b^0 \to \Lambda +$ meson.

TABLE VI. Meson decay constants f (in units of MeV) [11,18].

Meson	π	ρ	K	K^*	D	D^*	D_s	D_s^*	a_1	J/ψ
f	131	216	160	210	200	220	230	230	203	395

TABLE IX. Branching ratios and up-down asymmetries for nonleptonic decay $\Lambda_b^0 \to \Lambda + M$ with different theoretical approaches.

	Branching ratios	Up-down asymmetries
$\Lambda_b^0 \to \Lambda + \pi^0$	7.49×10^{-8}	-1
$\Lambda_b^0 \to \Lambda + \eta$	5.46×10^{-8}	-1
$\Lambda_b^0 \to \Lambda + \eta'$	2.29×10^{-8}	-1
$\Lambda_b^0 \to \Lambda + D^0$	4.54×10^{-5}	-0.998
$\Lambda_b^0 \to \Lambda + D^{0*}$	4.78×10^{-5}	-0.551
$\Lambda_b^0 \to \Lambda + \bar{D}^0$	8.76×10^{-6}	-0.998
$\Lambda_b^0 \to \Lambda + \bar{D}^{0*}$	5.08×10^{-6}	-0.551

From Table VIII we can find that there are some differences among the predictions by various theoretical approaches. In our calculation, $f_1(m_{J/\psi}^2)$, $g_1(m_{J/\psi}^2)$ are nearly equal; however, $g_1(m_{J/\psi}^2)$ is bigger than $f_1(m_{J/\psi}^2)$ in [18,31].

V. CONCLUSION

In this work, we carefully investigate the processes where a heavy baryon decays into a light baryon plus a lepton pair (semileptonic decay) or a meson (nonleptonic decay) in terms of the LFQM. Besides the regular input parameters such as the quark masses and well-measured decay constants of various mesons, there is only one free parameter to be determined, that is β in the light-front wave function. In our earlier work [6], by fitting the data of the semileptonic decays $\Lambda_b \to \Lambda_c + l + \bar{\nu}$, we obtained the values of $\beta_{b[ud]}$. Similarly, we fix the values $\beta_{u,[ud]}$ for proton and $\beta_{s,[ud]}$ for Λ.

Our numerical results are shown in corresponding tables and some measurable quantities such as the up-down asymmetries are also evaluated. A clear comparison of

TABLE VII. Branching ratios and up-down asymmetries for nonleptonic decay $\Lambda_b^0 \to p + M$.

	Branching ratios	Up-down asymmetries	Exp.
$\Lambda_b^0 \to p + \pi^-$	3.15×10^{-6}	-1	$(3.5 \pm 0.6(\text{stat}) \pm 0.9(\text{syst})) \times 10^{-6}$
$\Lambda_b^0 \to p + \rho$	6.12×10^{-6}	-0.873	...
$\Lambda_b^0 \to p + a_1$	4.08×10^{-6}	-0.741	...
$\Lambda_b^0 \to p + D^-$	5.75×10^{-7}	-0.998	...
$\Lambda_b^0 \to p + D^{*-}$	6.05×10^{-7}	-0.546	...
$\Lambda_b^0 \to p + D_s$	1.36×10^{-5}	-0.997	...
$\Lambda_b^0 \to p + D_s^*$	6.70×10^{-6}	-0.514	...
$\Lambda_b^0 \to p + K$	2.58×10^{-7}	-1	$(5.8 \pm 0.8(\text{stat}) \pm 1.5(\text{syst})) \times 10^{-6}$
$\Lambda_b^0 \to p + K^*$	3.21×10^{-7}	-0.850	...

TABLE VIII. Branching ratio and up-down asymmetry for nonleptonic decay $\Lambda_b^0 \to \Lambda J/\psi$ within different theoretical approaches and data from experiment.

	This work	[21]	[31]	[35]	[36]	Exp. [4]
Br($\times 10^{-4}$)	3.94	$1.65–5.271.65 \sim 5.27$	1.6	2.55	6.037	4.7 ± 2.8
α	-0.204	$-0.17– -0.14$	-0.1	-0.208	-0.18	...

our prediction on the decay rate of $\Lambda_b \to \Lambda + J/\psi$ with the results predicted by other models and as well as the experimental data is also explicitly presented. One can notice that our result for $\Lambda_b \to \Lambda + J/\psi$ is 3.94×10^{-4} which is in good agreement with the data. The success is not too surprising even though the model we adopt is much simplified. Definitely this value obtained in this work is closer to the central value of measurement than the previous evaluations, but since there is a large uncertainty in the data, one still cannot justify which model is more preferable than the others because within 2 standard deviations, all the numerical results achieved with all the approaches listed in the tables are consistent with data. The asymmetry parameter which may be important for determining the applicability of the adopted model is estimated as -0.204, which is generally consistent with that obtained in other models and approaches. Of course, the details, especially the branching ratios, will be further tested by more accurate experiments in the future.

Besides the semileptonic decays, we also estimate the branching ratios of several nonleptonic decay modes which are listed in Table VIII. Recently the CDF Collaboration [37] measured the branching ratios of $\Lambda_b \to p + \pi^-$ and $\Lambda_b \to p + K^-$ as $\mathrm{BR}(\Lambda_b \to p + \pi^-) = (3.5 \pm 0.6(\mathrm{stat}) \pm 0.9(\mathrm{syst})) \times 10^{-6}$ and $\mathrm{BR}(\Lambda_b \to p + K^-) = (5.8 \pm 0.8(\mathrm{stat}) \pm 1.5(\mathrm{syst})) \times 10^{-6}$. Our prediction on $\mathrm{BR}(\Lambda_b \to p + \pi^-) = (3.15 \times 10^{-6})$ is consistent with the measurement of the CDF within 1 standard deviation, but for $\mathrm{BR}(\Lambda_b \to p + K^-)$ our value is 2.58×10^{-7}, 1 order smaller than the data of the CDF Collaboration. Following the literature, in our calculation, we employ the factorization scheme where the emitted pseudoscalar meson (π or K) is factorized out and described by the common-accepted form factor $\langle 0|A_\mu|M\rangle = if_M p_\mu$, where A_μ, f_M, and p_μ are the corresponding axial current, decay constant of meson M, and its four-momentum, respectively. It is noticed that in the case $\Lambda_b \to p + \pi^-$, at the vertex $W^-\bar{u}d$ the Cabibbo-Kabayashi-Maskawa entry is approximately $\cos\theta_C \approx 1$, whereas for the case $\Lambda_b \to p + K^-$, the CKM entry is $\sin\theta_C \approx 0.22$; thus comparing with $\Lambda_b \to p + \pi^-$, the amplitude of the process $\Lambda_b \to p + K^-$ is suppressed by a factor $\frac{f_K}{f_\pi}\sin\theta_C \sim 0.27$. Thus besides a small difference between the final phase spaces of the two reactions, one can roughly estimate that $\mathrm{BR}(\Lambda_b \to p + K^-)/\mathrm{BR}(\Lambda_b \to p + \pi^-) \sim 0.07$, and this estimate is consistent with our numerical results. Therefore the smallness of $\mathrm{BR}(\Lambda_b \to p + K^-)$ seems reasonable. However the data of CDF show completely different results where $\mathrm{BR}(\Lambda_b \to p + K^-)$ is anomalously larger than $\mathrm{BR}(\Lambda_b \to p + \pi^-)$.

In fact, in our calculations on the nonleptonic decays, we only consider the contributions from the tree diagrams and neglect the penguin-loop effects. For the mode of $\Lambda_b \to p + \pi^-$, the penguin contribution can be safely neglected compared to the tree level. However, for the mode of $\Lambda_b \to p + K^-$, the tree level contribution is suppressed by the Cabibbo-Kabayashi-Maskawa entry $V_{ub}V_{us}^*$, while for the penguin diagram, the main contribution comes from the loop where the top quark is the intermediate fermion. In the case, the CKM entry would be $V_{tb}V_{ts}^*$ which is almost 2 orders larger than $V_{ub}V_{us}^*$. Thus even though there is a loop suppression of order $\alpha_s/4\pi$, it is compensated for by the much larger CKM entry. This situation was discussed in [38] where the authors used the pQCD method to carry out the calculations. In fact, we make a rough estimation of the contribution from the top penguin, and the result is almost 5 times larger than the contribution from the tree diagram given above.

However, from another aspect, when the penguin diagram is taken into account, the factorization is dubious. That is why we do not include the loop contributions in this present work, but will make a detailed discussion in our coming paper.

Actually, even including the penguin contribution, the theoretically estimated branching ratio of $\mathrm{BR}(\Lambda_b \to p + K^-)$ is still below the data and obviously smaller than that of $\mathrm{BR}(\Lambda_b \to p + \pi^-)$. If this measurement is valid and approved by further experiments, it would be a new anomaly which may hint at an unknown mechanism which dominates the transition or new physics beyond the standard model[1] and it is also consistent with the result of [38].

The good agreement of our results on the semileptonic decays of Λ_b to light baryon and several nonleptonic decay modes with data indicates the following points.

First, the diquark picture: as we know, two quarks in a color-antitriplet attract each other and constitute a Cooper-pairlike subject of spin 0 or 1. However, until now, many theorists still doubted the justifiability of the diquark picture. It is true that even though the diquark structure was raised almost as early as the birth of the quark model, its validity or reasonability of application is still in sharp dispute. In fact, it should be rigorously testified by experiments. We have argued that for some processes, the diquark picture may be more applicable than in the others. Actually, in our case, we can convince ourselves that the picture should apply. As aforementioned, the diquark is only a spectator in the transitions which we are concerned with in this work; therefore its inner structure may not affect the numerical results much. Secondly, the produced baryon is very relativistic, i.e. very close to the light cone; generally the details of the inner structure of the spectator diquark may not be important. This interpretation is somehow similar to the parton picture which was conceived by Feynman and Bjorken a long time ago. Namely, at very high energy collisions, the interaction among partons can be ignored at the leading order. thus in our case the

[1]We thank Tonelli [37] for bringing our attention to the new measurements of the CDF Collaboration on $\mathrm{BR}(\Lambda_b \to p + \pi^-)$ and $\mathrm{BR}(\Lambda_b \to p + K^-)$.

interaction between the quark which undergoes a transition and the spectator diquark should be weak and negligible. Third is that the small effects caused by the inner structure of the diquark may be partly included in the parameter β of the light-front wave function. The agreement with data indicates that the diquark picture and the light-front quark model indeed apply in the analysis of the heavy baryon transiting into a light one.

Moreover, since we employ the factorization scheme to deal with the nonleptonic decays, we find that to some modes, it works well, but to some modes where loop contributions may dominate or just are comparable to the tree contributions, the scenario encounters serious challenges [39].

We further investigate the measurable polarization asymmetries. Because the information on the polarization asymmetries may be more sensitive to the model adopted in the theoretical calculations than the decay width, accurate measurements would discriminate against various models and indicate how to improve the details of the models.

Moreover, we also predict the rates and asymmetries of several similar modes of Λ_b nonleptonic decays in the same model, and the results are listed in Tables VII and VIII. The numbers will be tested in the future.

Fortunately, the high luminosity at LHCb can provide a large database on Λ_b and moreover, with great improvements of the experimental facility and the detection technique, we expect that more and more accurate measurements will be carried out in the near future and theorists will be able to further testify, improve, or even negate our present models. Indeed, the baryons are much more complicated than mesons, but careful studies on the processes where baryons are involved would be very beneficial for getting better insight into the hadron structure and underlying principles, especially the nonperturbative QCD effects including the factorization and plausibility of the diquark picture. The LHCb will be an ideal place to do the job.

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