CP violation in B_q decays and final state strong phases

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Using the unitarity, $SU(2)$ and C invariance of hadronic interactions, the bounds on final state phases are derived. It is shown that values obtained for the final state phases relevant for the direct CP asymmetries $A_{CP}(B^0 \to K^+\pi^-, K^0\pi^0)$ are compatible with experimental values for these asymmetries. For the decays $B^0 \to D^{(*)-}\pi^+(D^{(*)+}\pi^-)$ described by two independent single amplitudes A_f and $A'_{\bar{f}}$ with different weak phases (0 and γ) it is argued that the C invariance of hadronic interactions implies the equality of the final state phase δ_f and $\delta'_{\bar{f}}$. This in turn implies that the CP asymmetry $\frac{S_+ + S_-}{2}$ is determined by weak phase $(2\beta + \gamma)$ only, whereas $\frac{S_+ - S_-}{2} = 0$. Assuming factorization for tree graphs, it is shown that the $B \to D^{(*)}$ form factors are in excellent agreement with the heavy quark effective theory. From the experimental value for $(\frac{S_+ + S_-}{2})_{D^*\pi}$, the bound $\sin(2\beta + \gamma) \geq 0.69$ is obtained and $(\frac{S_+ + S_-}{2})_{D_s^{*-}K^+} \approx -(0.41 \pm 0.08)\sin\gamma$ is predicted. For the decays described by the amplitudes $A_f \neq A_{\bar{f}}$ such as $B^0 \to \rho^+\pi^-$: $A_{\bar{f}}$ and $B^0 \to \rho^-\pi^+$: A_f where these amplitudes are given by tree and penguin diagrams with different weak phases, it is shown that in the limit $\delta^T_{f,\bar{f}} \to 0$, $r_{f,\bar{f}}\cos\delta_{f,\bar{f}} = \cos\alpha$ and $\frac{S_{\bar{f}}}{S_f} = \frac{S+\Delta S}{S-\Delta S} = -\frac{\sqrt{1-C^2_{\bar{f}}}}{\sqrt{1-C^2_f}}$.

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I. INTRODUCTION

The CP asymmetries in the hadronic decays of B and K mesons involve strong final state phases. Thus, strong interactions in these decays play a crucial role. The short distance strong interactions effects at quark level are taken care of by perturbative QCD in terms of Wilson coefficients. The Cabbibo-Kobayashi-Maskawa matrix, which connects the weak eigenstates with mass eigenstates, is another aspect of strong interactions at the quark level. In the case of semileptonic decays, the long distance strong interaction effects manifest themselves in the form factors of final states after hadronization. Likewise, the strong interaction final state phases are long distance effects. These phase shifts essentially arise in terms of the S matrix, which changes an "in" state into an "out" state viz.

$$|f\rangle_{\rm in} = S|f\rangle_{\rm out} = e^{2i\delta_f}|f\rangle_{\rm out}. \quad (1)$$

In fact, the CPT invariance of weak interaction Lagrangian gives for the weak decay $B(\bar{B}) \to f(\bar{f})$

$$\bar{A}_{\bar{f}} \equiv_{\rm out} \langle\bar{f}|\mathcal{L}_w|\bar{B}\rangle = \eta_f e^{2i\delta_f} A_f*. \quad (2)$$

Taking out the weak phase ϕ, the amplitude A_f can be written as

$$A_f = e^{i\phi}F_f = e^{i\phi}e^{i\delta_f}|F_f|. \quad (3)$$

Then Eq. (2) implies

$$\bar{A}_{\bar{f}} = e^{-i\phi}e^{2i\delta_f}F_f^* = e^{-i\phi}F_f.$$

It is difficult to reliably estimate the final state strong phase shifts. It involves the hadronic dynamics. However, using isospin, C invariance of S matrix and unitarity, we can relate these phases. In this regard, the following cases are of interest:

Case (i) The decays $B^0 \to f$, $\bar{f}$ are described by two independent single amplitudes A_f and $A'_{\bar{f}}$ with different weak phases:

$$A_f = \langle f|\mathcal{L}_W|B^0\rangle = e^{i\phi}F_f = e^{i\phi}e^{i\delta_f}|F_f|$$
$$A'_{\bar{f}} = \langle\bar{f}|\mathcal{L}'_W|B^0\rangle = e^{i\phi'}F'_{\bar{f}} = e^{i\phi'}e^{i\delta'_{\bar{f}}}|F'_{\bar{f}}|,$$

where the states $|\bar{f}\rangle$ and $|f\rangle$ are the C conjugate of each other such as states $D^{(*)-}\pi^+(D^{(*)+}\pi^-)$, $D_s^{(*)-}K^+(D_s^{(*)+}K^-)$, $D^-\rho^+(D^+\rho^-)$.

For case (i), there is an added advantage that the decay amplitudes A_f and $A_{\bar{f}}$ are given by tree graphs. Assuming factorization for tree amplitudes, it is shown that the form factors $f_0^{B-D}(m_\pi^2)$, $A_0^{B-D^*}(m_\pi^2)$, $f_+^{B-D}(m_\rho^2)$ obtained from the experimental branching ratios are in excellent agreement with the heavy quark effective theory (HQET). Hence, factorization assumption is experimentally on sound footing for these decays.Case (ii) The weak amplitudes $A_f \neq A_{\bar{f}}$,

$$A_f = \langle f|\mathcal{L}_W|B^0\rangle = [e^{i\phi_1}F_{1f} + e^{i\phi_2}F_{2f}]$$
$$A_{\bar{f}} = \langle\bar{f}|\mathcal{L}_W|B^0\rangle = [e^{i\phi_1}F_{1\bar{f}} + e^{i\phi_2}F_{2\bar{f}}]$$

as is the case for the following decays:

$$B^0 \to \rho^-\pi^+(f){:}\ A_f, \qquad B^0 \to \rho^+\pi^-(\bar{f}){:}\ A_{\bar{f}}$$
$$B_s^0 \to K^{*-}K^+, \qquad B_s^0 \to K^{*+}K^- \qquad B^0 \to D^{*-}D^+,$$
$$B^0 \to D^{*+}D^- \qquad B_s^0 \to D_s^{*-}D_s^+, \qquad B_s^0 \to D_s^{*+}D_s^-.$$

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The C invariance of the S matrix gives $S_{\bar{f}} = S_f$, which implies

$$\delta_f = \delta'_{\bar{f}}, \qquad \delta_{1f} = \delta_{1\bar{f}}, \qquad \delta_{2f} = \delta_{2\bar{f}}.$$

II. UNITARITY AND FINAL STATE STRONG PHASES

The time reversal invariance gives

$$F_f \underset{\text{out}}{=} \langle f|\mathcal{L}_W|B\rangle \underset{\text{in}}{=} \langle f|\mathcal{L}_W|B\rangle^*, \tag{4}$$

where $\mathcal{L}_W$ is the weak interaction Lagrangian without the Cabbibo-Kobayashi-Maskawa factor such as $V^*_{ud}V_{ub}$. From Eq. (4), we have

$$F^*_f \underset{\text{out}}{=} \langle f|S^\dagger \mathcal{L}_W|B\rangle = \sum_n S^*_{nf}F_n. \tag{5}$$

It is understood that the unitarity equation that follows from time reversal invariance holds for each amplitude with the same weak phase. The above equation can be written in two equivalent forms:

(1) Exclusive version of unitarity [1,2]
Writing

$$S_{nf} = \delta_{nf} + iM_{nf} \tag{6}$$

we get from Eq. (5),

$$\mathrm{Im}\, F_f = \frac{1}{2}\sum_n M^*_{nf}F_n, \tag{7}$$

where M_{nf} is the scattering amplitude for $f \to n$ and F_n is the decay amplitude for $B \to n$. In this version, the sum is over all allowed exclusive channels. This version is more suitable in a situation where a single exclusive channel is dominant one. To get the final result, one uses the dispersion relation. In the dispersion relation two particle unitarity gives a dominant contribution. From Eq. (7), using the two particle unitarity, we get [1],

$$\text{Disc } F(B \to f') \approx \frac{1}{16\pi s}\int_{-\infty}^{0} M^*_{f'f}F(B \to f)dt, \tag{8}$$

where $t = -2\vec{p}^2(1-\cos\theta)$, $|\vec{p}| \approx \frac{1}{2}\sqrt{s}$. Equation (8) is especially suitable to calculate rescattering corrections to color suppressed T amplitude in terms of color favored T amplitude as, for example, rescattering correction to color suppressed decay $B^0 \to \pi^0\bar{D}^0(f)$ in terms of dominant decay mode $B^0 \to \pi^+D^-(f)$. Before using two particle unitarity in this form, it is essential to consider two particle scattering processes.

$SU(3)$ or $SU(2)$ and the C invariance of the S matrix can be used to express scattering amplitudes in terms of two amplitudes M^+ and M^-, which in terms of Regge trajectories are given by [3–5]

$$\begin{aligned} M^{(+)} &= P + f + A_2 \\ &= -C_P\frac{e^{-i\pi\alpha_p(t)/2}}{\sin\pi\alpha_p(t)/2}(s/s_0)^{\alpha(t)} \\ &\quad - 2C_\rho\frac{1+e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}(s/s_0)^{\alpha(t)}, \end{aligned} \tag{9}$$

$$M^{(-)} = \rho + \omega = 2C_\rho\frac{1-e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}(s/s_0)^{\alpha(t)}. \tag{10}$$

For linear Regge trajectories, using exchange degeneracy, we have

$$\begin{aligned} &\alpha_\rho(t) = \alpha_{A_2}(t) = \alpha_\omega(t) = \alpha_f(t) = \alpha^0 + \alpha' t, \\ &\alpha_p(t) = \alpha_p(0) + \alpha'_p(t), \qquad C_f = C_\omega; \\ &C_{A_2} = C_\rho; \qquad C_\omega = C_\rho. \end{aligned} \tag{11}$$

We take $\alpha_0 \approx 1/2$, $\alpha' \approx 1\ \text{GeV}^{-2}$, $\alpha_p(0) \approx 1$, $\alpha'_p \approx 0.25\ \text{GeV}^{-2}$. Using $SU(3)$ and taking $\gamma_{\rho D^+D^-} = \gamma_{\rho K^+K^-}$, we get $C_\rho = \gamma_{\rho\pi^+\pi^-}\gamma_{\rho K^+K^-} = \gamma_{\rho\pi^+\pi^-}\gamma_{\rho D^+D^-} = \frac{1}{2}\gamma_0^2$, $\gamma_0 = \gamma_{\rho\pi^+\pi^-}$; $\gamma_0^2 \approx 72$ [3]. Hence, for π^+D^- or π^-K^+ scattering we get

$$\begin{aligned} M &= M^{(+)} + M^{(-)} \\ &= iC_P e^{bt}(s/s_0) + 2\gamma_0^2 i e^{\alpha'(\ln(s/s_0)-i\pi)t}(s/s_0)^{1/2}, \end{aligned} \tag{12}$$

where $b = \alpha'_P\ln(s/s_0)$.
For $\pi^0\bar{D}^0 \to \pi^+D^-$, $\pi^0K^0 \to \pi^-K^+$

$$M = \pm\sqrt{2}M^{(-)} = \pm i2\sqrt{2}C_\rho\frac{e^{-i\pi\alpha(t)/2}}{\cos\alpha(t)/2}(s/s_0)^{\alpha(t)}. \tag{13}$$

From Eqs. (8) and (13) with the use of dispersion relation, we obtain

$$\begin{aligned} A(B^0 \to \pi^0\bar{D}^0)_{\text{FSI}} &= \frac{\sqrt{2}\gamma_0^2(1-i)}{16\pi}\frac{A(B^0 \to \pi^+D^-)}{[\ln\frac{m_B^2}{s_0} + \frac{i\pi}{2}]} \\ &\quad \times\frac{1}{\pi}\int_{(m_B+m_D)^2}^{\infty}\frac{ds}{s-m_B^2}(s/s_0)^{\alpha(t)} \\ &= -\sqrt{2}\epsilon A(B^0 \to \pi^+D^-)e^{i\theta}. \end{aligned} \tag{14}$$

We get $\epsilon \approx 0.06$, $\theta \approx 33°$ by putting $s \approx m_B^2$ in $\ln(s/s_0)$. Now $A(B^0 \to \pi^+D^-) = T$. Hence, with rescattering correction [6]

$$\begin{aligned} A(B^0 \to \pi^0\bar{D}^0) &= -\frac{1}{\sqrt{2}}C - \sqrt{2}\epsilon T e^{i\theta} \\ &= -\frac{C}{\sqrt{2}}\left[1 + \frac{\epsilon}{b}e^{i\theta}\right], \end{aligned} \tag{15}$$

where $2b = C/T$. Hence, the final state phase shift δ_C for the color suppressed amplitude induced by the final state interaction is given by

$$\tan\delta_C = \frac{\epsilon/b\sin\theta}{1+\epsilon/b\cos\theta} \to \delta_C \approx 8° \tag{16}$$

with $b \approx 0.174$, which we get from

$$\frac{\Gamma(B^0 \to \pi^+ D^-)}{\Gamma(B^+ \to \pi^+ \bar{D}^0)} = \frac{1}{(1+2b)^2} \approx 0.55 \pm 0.03. \tag{17}$$

For $B^0 \to \pi^0 K^0$, the color suppressed T-amplitude with rescattering correction is given by

$$-\frac{1}{\sqrt{2}}C + \sqrt{2}\epsilon T e^{i\theta} = -\frac{1}{\sqrt{2}}C\left[1 - \frac{\epsilon}{b}e^{i\theta}\right], \tag{18}$$

where $2b = C/T \approx 0.37$ [7]. Hence, δ_C generated by the final state interaction is given by

$$\tan\delta_C = \frac{-\epsilon/b\sin\theta}{1-\epsilon/b\cos\theta} \to \delta_C \approx -8°. \tag{19}$$

To conclude, the scattering amplitude $M(s,t)$ for the two particle final state obtained in Eq. (13) is used in the unitarity equation to generate the final state strong phase by rescattering for the color suppressed tree amplitude.

(2) Inclusive version of unitarity [2]
This version is more suitable for our analysis. For this case, we write Eq. (5) in the form

$$F_f^* - S_{ff}^* F_f = \sum_{n\neq f} S_{nf}^* F_n. \tag{20}$$

Parametrizing the S matrix as $S_{ff} \equiv S = \eta e^{2i\Delta}$ [5], $0 \le \eta \le 1$, we get after taking the absolute square of both sides of Eq. (20)

$$|F|^2[(1+\eta^2) - 2\eta\cos2(\delta_f - \Delta)] = \sum_{n,n'} F_n S_{nf}^* F_{n'}^* S_{n'f}. \tag{21}$$

The above equation is an exact equation. In the random phase approximation [2], we can put

$$\sum_{n',n\neq f} F_n S_{nf}^* F_{n'} S_{n'f} = \sum_{n\neq f} |F_n|^2 |S_{nf}|^2 = |\bar{F}_n|^2(1-\eta^2). \tag{22}$$

We note that in a single channel description [5,8]

$$(\mathrm{Flux})_{\mathrm{in}} - (\mathrm{Flux})_{\mathrm{out}} = 1 - |\eta e^{2i\Delta}|^2 = 1 - \eta^2 = \text{Absorption}.$$

The absorption takes care of all the inelastic channels.
Similarly for the amplitude $F_{\bar{f}}$, we have

$$F_{\bar{f}}^* - S_{\bar{f}\bar{f}}^* F_{\bar{f}} = \sum_{\bar{n}\neq\bar{f}} S_{\bar{n}\bar{f}}^* F_{\bar{n}}. \tag{23}$$

The C invariance of the S matrix gives

$$S_{fn} = \langle f|S|n\rangle = \langle f|C^{-1}CSC^{-1}C|n\rangle = \langle\bar{f}|S|\bar{n}\rangle = S_{\bar{f}\bar{n}}. \tag{24}$$

Thus, in particular, the C invariance of the S matrix gives

$$S_{\bar{f}\bar{f}} = S_{ff} = \eta e^{2i\Delta}. \tag{25}$$

Hence, from Eq. (21), using Eqs. (22)–(25), we get

$$\frac{1}{1-\eta^2}[(1+\eta^2) - 2\eta\cos2(\delta_{f,\bar{f}} - \Delta)] = \rho^2, \bar{\rho}^2, \tag{26}$$

where

$$\rho^2 = \frac{\overline{|F_n|^2}}{|F_f|^2}, \qquad \bar{\rho}^2 = \frac{\overline{|F_{\bar{n}}|^2}}{|F_{\bar{f}}|^2}. \tag{27}$$

From Eq. (26), we get

$$\sin(\delta_{f,\bar{f}} - \Delta) = \pm\sqrt{\frac{1-\eta^2}{4\eta}}\left[\rho^2, \bar{\rho} - \frac{1-\eta}{1+\eta}\right]^{1/2}. \tag{28}$$

The maximum value for ρ^2, $\bar{\rho}^2$ is 1, and the minimum value for them is $\frac{1-\eta}{1+\eta}$. Hence, we get the following bounds:

$$\frac{1-\eta}{1+\eta} \le \rho^2, \bar{\rho}^2 \le 1 \qquad 0 \le \delta_{f,\bar{f}} - \Delta \le \theta \qquad -\theta \le \delta_f - \Delta \le 0, \tag{29}$$

$$\theta = \sin^{-1}\sqrt{\frac{1-\eta}{2}}. \tag{30}$$

From now on, we will confine our self to positve square root in Equation (28).
The strong interaction parameter Δ and η in the above bounds can be obtained from the scattering amplitude $M(s,t)$ given in Eq. (12) obtained from the Regge pole analysis. The s-wave scattering amplitude f is given by

$$f \approx \frac{1}{16\pi s}\int_{-s}^{0} M(s,t). \tag{31}$$

For the scattering amplitude $M = M^+ + M^-$ relevant for π^+D^-, π^-K^+, and $\pi^+\pi^-$, we obtain from Eq. (31) using Eq. (12)

$$f = f_P + f_\rho = \frac{1}{16\pi s}\frac{iC_P}{b}\left(\frac{s}{s_0}\right) + 2\frac{\gamma_0^2}{16\pi}\frac{1}{\ln(s/s_0) - i\pi} \times (s/s_0)^{-1/2}, \tag{32}$$

$$= \begin{bmatrix} 0.12i + (-0.08 + 0.08i) \\ 0.17i + (-0.08 + 0.08i) \\ 0.16i + (-0.16 \pm 0.16i) \end{bmatrix}, \quad (33)$$

where we have used $s \approx m_B^2 \approx (5.27)^2$ GeV^2. For C_P we have used the values of Ref. [2], whereas for $C_\rho = \gamma_{\rho\pi^+\pi^-}\gamma_{\rho K^+K^-} = \gamma_{\rho\pi^+\pi^-}\gamma_{\rho D^+D^-} = \frac{1}{2}\gamma_0^2$ and $C_\rho = \gamma_{\rho\pi^+\pi^-}\gamma_{\rho\pi^+\pi^-} = \gamma_0^2 \approx 72$ for πD, πK and $\pi\pi$, respectively.
Using the relation $S = \eta e^{2i\Delta} = 1 + 2if$, where f is given by Eq. (33), the phase shift Δ, the parameter η and the phase angle θ can be determined. One gets

$$\pi^+D^-(\pi^-D^+)\colon \Delta \approx -7°, \eta \approx 0.62, \theta \approx 26°$$
$$\pi^-K^+ \quad \text{or} \quad \pi^0K^0\colon \Delta \approx -9°, \eta \approx 0.52, \theta \approx 29°$$
$$\pi^+\pi^-\colon \Delta \approx -21°, \eta \approx 0.48, \theta \approx 31°. \quad (34)$$

Hence, we get the following bounds:

$$\pi^+D^-(\pi^-D^+)\colon 0 \le \delta_{f,\bar f} - \Delta \le 26°$$
$$\pi^-K^+ \quad \text{or} \quad \pi^0K^0\colon 0 \le \delta_f - \Delta \le 29°$$
$$\pi^+\pi^-\colon 0 \le \delta_f - \Delta \le 31°. \quad (35)$$

Further we note that for these decays, the b quark is converted into the c or the u quark $b \to c(u) + \bar u + d(s)$. In particular, for the tree graph, the configuration is such that $\bar u$ and $d(s)$ essentially go together into a color singlet state with the third quark $c(u)$ recoiling; there is a significant probability that the system will hadronize as a two body final state [9]. This physical picture has been put on the strong theoretical basis [10,11], where in these references the QCD factorization have been proven. For the tree amplitude, factorization implies $\delta_f^T = 0$. We, therefore take the point of view that the effective final state phase shift is given by $\delta_f - \Delta$. We take the lower bound for the tree amplitude so that the final state effective phase shift $\delta_f^T = 0$. Thus, for $\pi^+D^-(\pi^-D^+)$, $\delta_f^T = \delta_f'^T = 0$.
The decay $B^0 \to \pi^-K^+$ is described by two amplitudes [7]

$$A(B^0 \to \pi^-K^+) = -[P + e^{i\gamma}T] = |P|[1 - re^{i(\gamma+\delta_{+-})}], \quad (36)$$

where

$$P = -|P|e^{-i\delta_P}, \qquad T = |T|e^{i\delta_T},$$
$$\delta_{+-} = \delta_P, \qquad r = \frac{|T|}{|P|}.$$

The decay $B^0 \to \pi^0K^0$ is described by the two amplitudes [7]

$$A(B^0 \to \pi^0K^0) = -\frac{1}{\sqrt{2}}|P|[1 + r_0e^{i(\gamma+\delta_{00})}], \quad (37)$$

where

$$C = |C|e^{i\delta_C}, \qquad \delta_{00} = \delta_C + \delta_P, \qquad r_0 = \frac{|C|}{|P|}.$$

For these decays, we use the lower bounds in Eq. (35) for the tree amplitude so that the effective final state phase $\delta_T = 0$. The phase δ_C is generated by a rescattering correction, and its value is $-8°$. For the direct CP asymmetries, the relevant phases are δ_{+-} and δ_{00}. For the penguin amplitude, we assume that the effective final state phase δ_P has the value near the upper bound. Thus, we have $\delta_{+-} \approx 29°$, $\delta_{00} \approx 21°$.
Now [7]

$$A_{CP}(B^0 \to \pi^-K^+) = -\frac{2r\sin\gamma\sin\delta_{+-}}{R}$$
$$R = 1 - 2r\cos\gamma\cos\delta_{+-} + r_{+-}^2. \quad (38)$$

Neglecting the terms of order r^2, we have

$$\tan\gamma\tan\delta_{+-} = \frac{-A_{CP}(B^0 \to \pi^-K^+)}{1 - R}. \quad (39)$$

For $B^0 \to \pi^0K^0$

$$A_{CP}(B^0 \to \pi^0K^0) = (R_0 - 1)\tan\gamma\tan\delta_{00}$$
$$R_0 = 1 + 2r_0\cos\gamma\cos\delta_{00} + r_{00}^2. \quad (40)$$

Now the experimental values of A_{CP}, R, and R_0 are [12]

$$A_{CP}(B^0 \to \pi^-K^+) = -0.101 \pm 0.015 \times(-0.097 \pm 0.012)$$
$$A_{CP}(B^0 \to \pi^0K^0) = -0.14 \pm 0.11(-0.00 \pm \pm 0.10)$$
$$R = 0.899 \pm 0.048$$
$$R_0 = 0.908 \pm 0.068,$$

where the numerical values in the bracket are the latest experimental values as given in Ref. [7]. With $\delta_{+-} \approx 29°$, we get from Eq. (39), $\gamma = (60 \pm 3)°$. However, for $\delta_{+-} \approx 20°$ which one gets from Eq. (28) for $\rho^2 = 0.65$, $\gamma = (69 \pm 3)°$. We obtain the following values for $A_{CP}(B^0 \to \pi^0K^0)$ from Eqs. (39) and (40):

$$A_{CP}(B^0 \to \pi^0K^0) = \frac{(1-R_0)\tan\delta_{00}}{(1-R)\tan\delta_{+-}}A_{CP}(B^0 \to \pi^-K^+) = \begin{Bmatrix} -0.06 \pm 0.01, & \delta_{+-} = 29° \\ & \delta_{00} = 21° \\ -0.05 \pm 0.01, & \delta_{+-} = 20° \\ & \delta_{00} = 12° \end{Bmatrix}.$$

We conclude that the phase shift $\delta_{+-} \approx (20\text{–}29)°$

for $\pi^- K^+$ is compatible with the experimental value of the direct CP asymmetry for the $\pi^- K^+$ decay mode. For $\pi^+\pi^-$, $\delta_{+-} \sim 31°$ is compatible with the value $(33 \pm 7^{+8}_{-10})°$ obtained by the authors of Ref. [7]. Finally, we note that the actual value of the effective phase shift $(\delta_f - \Delta)$ depends on one free parameter ρ, factorization implies $\delta_f^T = 0$, i.e. $\delta_f - \Delta = 0$ for the tree amplitude; for the penguin amplitude, δ_f^P depends on ρ. However, from the experimental values of the direct CP violation for $\pi^- K^+$, $\pi^-\pi^+$, it is near the upper bound.

Finally, we note that $\pi^+ D^-(\pi^- D^+)$, $\pi^- K^+$, $\pi^-\pi^+$ decays are s-wave decay, whereas $B^0 \to \rho^+\pi^-(\rho^-\pi^+)$ decays are p-wave decays. For the p wave, the decay amplitude

$$\begin{aligned}
f &= \frac{1}{16\pi s}\int_{-s}^{0} M(s,t)\left(1+\frac{2t}{s}\right)dt \\
&= \frac{1}{16\pi s} iC_P\left[\frac{1}{b}+\frac{2}{b^2}\frac{1}{s}\right](s/s_0) \\
&\quad + \frac{2\gamma_0^2}{16\pi} i\left[\frac{1}{\ln(s/s_0)-i\pi} - \frac{2}{s}\frac{1}{[\ln(s/s_0)-i\pi]^2} \times (s/s_0)^{-1/2}\right] \\
&\approx \frac{1}{16\pi s} iC_P\frac{1}{b}(s/s_0) + \frac{2\gamma_0^2}{16\pi} i\frac{1}{\ln(s/s_0)-i\pi} \times (s/s_0)^{-1/2} + O\left(\frac{1}{s}\right)
\end{aligned}$$

is to be compared with Eq. (32). Now for the $B \to \rho\pi$ decay, only the longitudinal polarization of ρ is effectively involved. Since the longitudinal ρ meson emulates a pseudoscalar meson and if we assume the same couplings as for pions, we conclude that the final state phase for $\rho\pi$ should be of the order 30°; in any case, it should not be greater than 30°. The upper bound $\delta_f \leq 30°$ can be used to select the several possible solutions in Table II (Sec. IV) obtained from the analysis of weak decays $B \to \rho^+\pi^-(\rho^-\pi^+)$.

III. CP ASYMMETRIES AND STRONG PHASES

In this section, we discuss the experimental tests to verify the equality (implied by the C invariance of the S matrix) of phase shifts δ_f and $\delta_{\bar f}$ for the weak decays of B mesons mentioned in Sec. I.

It is convenient to write the time-dependent decay rates in the form [6,13]

$$\begin{aligned}
&[\Gamma_f(t) - \bar\Gamma_{\bar f}(t)] + [\Gamma_{\bar f} - \bar\Gamma_f(t)] \\
&\quad = e^{-\Gamma t}\{\cos\Delta mt[(|A_f|^2 - |\bar A_{\bar f}|^2) + (|A_{\bar f}|^2 - |\bar A_f|^2)] \\
&\qquad + 2\sin\Delta mt[\mathrm{Im}(e^{2i\phi_M}A_f^*\bar A_f) + \mathrm{Im}(e^{2i\phi_M}A_{\bar f}^*\bar A_{\bar f})]\},
\end{aligned} \tag{41}$$

$$\begin{aligned}
&[\Gamma_f(t) + \bar\Gamma_{\bar f}(t)] - [\Gamma_{\bar f}(t) + \bar\Gamma_f(t)] \\
&\quad = e^{-\Gamma t}\{\cos\Delta mt[(|A_f|^2 + |\bar A_{\bar f}|^2) - (|A_{\bar f}|^2 + |\bar A_f|^2)] \\
&\qquad + 2\sin\Delta mt[\mathrm{Im}(e^{2i\phi_M}A_f^*\bar A_f) - \mathrm{Im}(e^{2i\phi_M}A_{\bar f}^*\bar A_{\bar f})]\}.
\end{aligned} \tag{42}$$

Case (i): Eqs. (41) and (42) give

$$\begin{aligned}
\mathcal{A}(t) &\equiv \frac{[\Gamma_f(t) - \bar\Gamma_{\bar f}(t)] + [\Gamma_{\bar f}(t) - \bar\Gamma_f(t)]}{[\Gamma_f(t) + \bar\Gamma_{\bar f}(t)] + [\Gamma_{\bar f}(t) + \bar\Gamma_f]} \\
&= \frac{2|F_f||F'_{\bar f}|}{|F_f|^2 + |F'_{\bar f}|^2}\sin\Delta mt\sin(2\phi_M - \phi - \phi') \\
&\quad \times \cos(\delta_f - \delta'_{\bar f}),
\end{aligned} \tag{43}$$

$$\begin{aligned}
\mathcal{F}(t) &\equiv \frac{[\Gamma_f(t) + \bar\Gamma_{\bar f}] - [\Gamma_{\bar f}(t) + \bar\Gamma_f]}{[\Gamma_f(t) + \bar\Gamma_{\bar f}] + [\Gamma_{\bar f}(t) + \bar\Gamma_f]} \\
&= \frac{|F_f|^2 - |F'_{\bar f}|^2}{|F_f|^2 + |F'_{\bar f}|^2}\cos\Delta mt - \frac{2|F_f||F'_{\bar f}|}{|F_f|^2 + |F'_{\bar f}|^2} \\
&\quad \times \sin\Delta mt\cos(2\phi_M - \phi - \phi')\sin(\delta_f - \delta'_{\bar f}).
\end{aligned} \tag{44}$$

The effective Lagrangians $\mathcal{L}_W$ and $\mathcal{L}'_W$ are given by $(q = d, s)$

$$\mathcal{L}_W = V_{cb}V_{uq}^*[\bar q\gamma^\mu(1-\gamma^5)u][\bar c\gamma_\mu(1-\gamma_5)b], \tag{45}$$

$$\mathcal{L}'_W = V_{ub}V_{cq}^*[\bar q\gamma^\mu(1-\gamma^5)c][\bar u\gamma_\mu(1-\gamma_5)b]. \tag{46}$$

Hence, for these decays

$$\phi = 0, \qquad \phi' = \gamma$$

and

$$\phi_M = \begin{cases} -\beta, & \text{for } B^0 \\ -\beta_s, & \text{for } B_s^0 \end{cases} \tag{47}$$

$$\begin{aligned}
A_f &= \langle D^-\pi^+|\mathcal{L}_W|B^0\rangle = F_f \\
{}'A_{\bar f} &= \langle D^+\pi^-|\mathcal{L}_W{}'|B^0\rangle = e^{i\gamma'}F'_{\bar f} \\
A_{f_s} &= \langle K^+D_s^-|\mathcal{L}_W|B_s^0\rangle = F_{f_s} \\
{}'A_{\bar f_s} &= \langle K^-D_s^+|\mathcal{L}_W{}'|B_s^0\rangle = e^{i\gamma'}F'_{\bar f_s}.
\end{aligned} \tag{48}$$

Thus, we get from Eqs. (43)–(48) for B^0 decays,

$$\begin{aligned}
\mathcal{A}(t) &= -\frac{2r_D}{1+r_D^2}\sin\Delta m_B t\sin(2\beta+\gamma)\cos(\delta_f - \delta'_{\bar f}) \\
\mathcal{F}(t) &= \frac{1-r_D^2}{1+r_D^2}\cos\Delta m_B t - \frac{2r_D}{1+r_D^2}\sin\Delta m_B t\cos(2\beta+\gamma) \\
&\quad \times \sin(\delta_f - \delta'_{\bar f}),
\end{aligned} \tag{49}$$

$$\mathcal{A} = \frac{-2r_D}{1+r_D^2}\sin(2\beta+\gamma)\frac{(\Delta m_B/\Gamma)}{1+(\Delta m_B/\Gamma)^2}\cos(\delta_f-\delta'_{\bar{f}}), \tag{50}$$

where

$$r_D = \lambda^2 R_b \frac{|F'_{\bar{f}}|}{|F_f|}. \tag{51}$$

For the decays,

$$\bar{B}_s^0(B_s^0) \to D_s^+K^-(D_s^-K^+)$$
$$\bar{B}_s^0(B_s^0) \to D_s^-K^+(D_s^+K^-),$$

we get

$$\mathcal{A}_s(t) = -\frac{2r_{D_s}}{1+r_{D_s}^2}\sin\Delta m_{B_s}t\sin(2\beta_s+\gamma)\cos(\delta_{f_s}-\delta'_{\bar{f}_s})$$
$$\mathcal{F}_s(t) = \frac{1-r_{D_s}^2}{1+r_{D_s}^2}\cos\Delta m_{B_s}t - \frac{2r_{D_s}}{1+r_{D_s}^2}\sin\Delta m_{B_s}t \times\cos(2\beta_s+\gamma)\sin(\delta_{f_s}-\delta'_{\bar{f}_s}), \tag{52}$$

where

$$r_{D_s} = R_b\frac{|F'_{\bar{f}_s}|}{|F_{f_s}|}. \tag{53}$$

We note that for time integrated CP asymmetry,

$$\mathcal{A}_s \equiv \frac{\int_0^\infty[\Gamma_{fs}(t)-\bar{\Gamma}_{fs}(t)]dt}{\int_0^\infty[\Gamma_{fs}(t)+\bar{\Gamma}_{fs}(t)]dt} = -\frac{2r_{D_s}}{1+r_{D_s}^2}\sin(2\beta_s+\gamma) \times\frac{\Delta m_{B_s}/\Gamma_s}{1+(\Delta m_{B_s}/\Gamma_s)^2}\cos(\delta_{f_s}-\delta'_{\bar{f}_s}). \tag{54}$$

The experimental results for the B decays are as follows: [12]

	$D^-\pi^+$	$D^{*-}\pi^+$	$D^-\rho^+$
$\frac{S_-+S_+}{2}$:	-0.046 ± 0.023	-0.037 ± 0.012	$-0.024\pm0.031\pm0.009$,
$\frac{S_--S_+}{2}$:	-0.022 ± 0.021	-0.006 ± 0.016	$-0.098\pm0.055\pm0.018$

(55)

where

$$\frac{S_-+S_+}{2} \equiv -\frac{2r_D}{1+r_D^2}\sin(2\beta+\gamma)\cos(\delta_f-\delta'_{\bar{f}})$$
$$\frac{S_--S_+}{2} \equiv -\frac{2r_D}{1+r_D^2}\cos(2\beta+\gamma)\sin(\delta_f-\delta'_{\bar{f}}). \tag{56}$$

For $B_s^0\to D_s^{*-}K^+$, $D_s^-K^+$, $D_s^-K^{*+}$, replace $r_D\to r_{Ds}$, $\beta\to\beta_s$, $\delta_f\to\delta_{f_s}$, $\delta'_{\bar{f}}\to\delta'_{\bar{f}_s}$ in Eq. (56).

Since for B_s^0, in the standard model, with three generations, gives $\beta_s=0$, so we have for the CP asymmetries $\sin\gamma$ or $\cos\gamma$ instead of $\sin(2\beta+\gamma)$, $\cos(2\beta+\gamma)$. Hence. B_s^0 decays are more suitable for testing the equality of phase shifts δ_{f_s} and $\delta'_{\bar{f}_s}$ as for this case neither r_s nor $\cos\gamma$ is suppressed as compared to the corresponding quantities for B^0. To conclude, for B_q^0 decays, the equality of phases δ_f and $\delta'_{\bar{f}}$ for B_d^0 gives

$$-\frac{S_-+S_+}{2} = 2r_D\sin(2\beta+\gamma), \qquad -\frac{S_--S_+}{2} = 0, \tag{57}$$

whereas for B_s^0 decays, we get

$$-\frac{S_-+S_+}{2} = \frac{2r_{D_s}}{1+r_{D_s}^2}\sin(2\beta_s+\gamma)$$
$$-\frac{S_--S_+}{2} = 0. \tag{58}$$

Corresponding to the decays $B_s^0\to D_s^-K^+$, $D_s^+K^-$ described by the tree diagrams, we have the color suppressed decays $B^0\to\bar{D}^0K^0$, D^0K^0. For these decays,

$$-\frac{S_-+S_+}{2} = \frac{2r_{DK}}{1+r_{DK}^2}\sin(2\beta+\gamma)\cos(\delta_{\bar{D}^0K_s^0}-\delta'_{D^0\bar{K}_s^0})$$
$$-\frac{S_--S_+}{2} = \frac{2r_{DK}}{1+r_{DK}^2}\cos(2\beta+\gamma)\sin(\delta_{\bar{D}^0K_s^0}-\delta'_{D^0\bar{K}_s^0})$$
$$r_{DK} = R_b\frac{|C'_{D^0K_s}|}{|C_{\bar{D}^0K_s}|}$$

and the corresponding expression for $B_s^0\to\bar{D}^0\phi$, $D^0\phi$. For the color suppressed decays $B^0\to\bar{D}^0\pi^0$, $D^0\pi^0$, we get a similar expression as for $B^0\to D^-\pi^+$, $D^+\pi^-$, with

$$r_D\equiv r_{D^-\pi^+}, \quad \delta_{D^-\pi^+}, \quad \delta'_{D^+\pi^-} \text{ replaced by } r_{D^0\pi^0}, \quad \delta_{\bar{D}^0\pi^0}, \quad \delta'_{D^0\pi^0}.$$

To determine the parameter r_D or r_{D_s}, we assume factorization for the tree amplitude [7]. Factorization gives for the decays $\bar{B}^0\to D^+\pi^-$, $D^{*+}\pi^-$, $D^+\rho^-$, $D^+a_1^-$:

$$|\bar{F}_{\bar{f}}| = |\bar{T}_{\bar{f}}| = G[f_\pi(m_B^2 - m_D^2)f_0^{B-D}(m_\pi^2), 2f_\pi m_B|\vec{p}|A_0^{B-D^*} \times (m_\pi^2), 2f_\rho m_B|\vec{p}|f_+^{B-D}(m_\rho^2), 2f_{a_1}m_B|\vec{p}|f_+^{B-D}(a_1^2)], \quad (59)$$

$$|\bar{F}'_f| = |\bar{T}'_f| = G'[f_D(m_B^2 - m_\pi^2)f_0^{B-\pi}(m_D^2), 2f_{D^*}m_B|\vec{p}|f^{B-\pi}(m_{D^*}^2), 2f_D m_B|\vec{p}|A_0^{B-\rho}(m_D^2), 2f_D m_B|\vec{p}|A_0^{B-a_1}(m_B^2)], \quad (60)$$

$$G = \frac{G_F}{\sqrt{2}}|V_{ud}||V_{cb}|a_1, \qquad G' = \frac{G_F}{\sqrt{2}}|V_{cd}||V_{ub}|. \quad (61)$$

The decay widths for the above channels are given in the Table I, where we have used

$$a_1^2|V_{ud}|^2 \approx 1, \qquad f_\pi = 131\text{ MeV},$$
$$f_\rho = 209\text{ MeV}, \qquad f_{a_1} = 229\text{ MeV}.$$

Using the experimental branching ratios and [12]

$$|V_{cb}| = (38.3 \pm 1.3) \times 10^{-3}, \quad (62)$$

we obtain the corresponding form factors given in Table I.

In terms of variables [14,15]

$$\omega = v \cdot v', \qquad v^2 = v'^2 = 1,$$
$$t = q^2 = m_B^2 + m_{D^{(*)}}^2 - 2m_B m_{D^{(*)}}\omega, \quad (63)$$

the form factors can be put in the following form:

$$f_+^{B-D}(t) = \frac{m_B + m_D}{2\sqrt{m_B m_D}}h_+(\omega),$$
$$f_0^{B-D}(t) = \frac{\sqrt{m_B m_D}}{m_B + m_D}(1+\omega)h_0(\omega)$$
$$A_2^{B-D^*}(t) = \frac{m_B + m_{D^*}}{2\sqrt{m_B m_{D^*}}}(1+\omega)h_{A_2}(\omega), \quad (64)$$
$$A_0^{B-D^*}(t) = \frac{m_B + m_{D^*}}{2\sqrt{m_B m_{D^*}}}h_{A_0}(\omega)$$
$$A_1^{B-D^*}(t) = \frac{\sqrt{m_B m_{D^*}}}{m_B + m_{D^*}}(1+\omega)h_{A_1}(\omega).$$

HQET gives [14,15]:

$$h_+(\omega) = h_0(\omega) = h_{A_0}(\omega) = h_{A_1}(\omega) = h_{A_2}(\omega) = \zeta(\omega),$$

where $\zeta(\omega)$ is the form factor, with normalization $\zeta(1) = 1$. For

$$t = m_\pi^2, m_\rho^2, m_{a_1}^2 \qquad \omega^{(*)} = 1.589(1.504), 1.559, 1.508. \quad (65)$$

In Ref. [16], the value quoted for $h_{A_1}(\omega^*_{\max})$ is

$$|h_{A_1}(\omega^*_{\max})| = 0.52 \pm 0.03. \quad (66)$$

Since $\omega^*_{\max} = 1.504$, the value for $|h_{A_0}(\omega^*_{\max})|$ obtained in Table I is in remarkable agreement with the value given in Eq. (66) showing that factorization assumption for $B^0 \to \pi D^{(*)}$ decays is experimentally on solid footing and is in agreement with HQET.

From Eqs. (56) and (60), we obtain

$$r_D = \lambda^2 R_b \frac{|\bar{T}'_f|}{|\bar{T}_{\bar{f}}|} = \lambda^2 R_b\left[\frac{f_D(m_B^2 - m_\pi^2)f_0^{B-\pi}(m_D^2)}{f_\pi(m_B^2 - m_D^2)f_0^{B-D}(m_\pi^2)}, \frac{f_{D^*}f_+^{B-\pi}(m_{D^*}^2)}{f_\pi A_0^{B-D}(m_\pi^2)}, \frac{f_D A_0^{B-\rho}(m_D^2)}{f_\rho f_+^{B-D}(m_{\rho^2})}\right], \quad (67)$$

where

$$\frac{|V_{ub}||V_{cd}|}{|V_{cb}||V_{ud}|} = \lambda^2 R_b \approx (0.227)^2(0.40) \approx 0.021. \quad (68)$$

To determine r_D, we need information for the form factors $f_0^{B-\pi}(m_D^2)$, $f_+^{B-\pi}(m_D^2)$, $A_0^{B-\rho}(m_D^2)$. For these form factors, we use the following values [17,18]:

$$A_0^{B-\rho}(0) = 0.30 \pm 0.03,$$
$$A_0^{B-\rho}(m_D^2) = 0.38 \pm 0.04$$
$$f_+^{B-\pi}(0) = f_0^{B-\pi}(0) = 0.26 \pm 0.04,$$
$$f_+^{B-\pi}(m_{D^*}^2) = 0.32 \pm 0.05,$$
$$f_0^{B-D}(m_D^2) = 0.28 \pm 0.04.$$

Along with the values of remaining form factors given in Table I, we obtain

$$r_{D^{(*)}} = [0.018 \pm 0.002, \quad 0.017 \pm 0.003, \quad 0.012 \pm 0.002]. \quad (69)$$

TABLE I. Form factors.

Decay	Decay width (10^{-9} MeV $\times |V_{cb}|^2$)	Form factor	Form factors $h(w^{(*)})$
$\bar{B}^0 \to D^+\pi^-$	$(2.281)|f_0^{B-D}(m_\pi^2)|^2$	0.58 ± 0.05	0.51 ± 0.03
$\bar{B}^0 \to D^{*+}\pi^-$	$(2.129)|A_0^{B-D^*}(m_\pi^2)|^2$	0.61 ± 0.04	0.54 ± 0.03
$\bar{B}^0 \to D^+\rho^-$	$(5.276)|f_+^{B-D}(m_\rho^2)|^2$	0.65 ± 0.11	0.57 ± 0.10
$\bar{B}^0 \to D^+a_1^-$	$(5.414)|f_+^{B-D}(m_{a_1}^2)|^2$	0.57 ± 0.31	0.50 ± 0.27

The above value for r_D^* gives

$$-\left(\frac{S_+ + S_-}{2}\right)_{D^*\pi} = 2(0.017 \pm 0.003)\sin(2\beta + \gamma). \tag{70}$$

The experimental value of the CP asymmetry for $B^0 \to D^*\pi$ decay has the least error. Hence, we obtain the following bounds:

$$\sin(2\beta + \gamma) > 0.69, \tag{71}$$

$$44° \le (2\beta + \gamma) \le 90°, \tag{72}$$

$$\text{or} \quad 90° \le (2\beta + \gamma) \le 136°. \tag{73}$$

Selecting the second solution, and using $2\beta \approx 43°$, we get

$$\gamma = (70 \pm 23)°. \tag{74}$$

Further, we note that the factorization for the decay $\bar{B}^0 \to D_s^{*-}\pi^+$ gives

$$\bar{T} = |V_{ub}||V_{cs}|f_{D_s^*}2m_B|\vec{p}|f_+^{B-\pi}(m_{D_s^*}^2). \tag{75}$$

Using the experimental branching ratio for this decay, we get

$$\left(\frac{f_{D_s^*}}{f_\pi}\right)^2 \left|\frac{f_+^{B-\pi}(m_{D_s^*}^2)}{f_+^{B-\pi}(0)}\right|^2 = 7.7 \pm 1.9. \tag{76}$$

On using

$$\frac{f_+^{B-\pi}(0)}{f_+^{B-\pi}(m_{D_s^*}^2)} = 0.77 \pm 0.09, \tag{77}$$

we get

$$f_{D_s^*} = 279 \pm 79 \text{ MeV}. \tag{78}$$

Similar analysis for $\bar{B}^0 \to D_s^-\pi^+$ gives

$$\left(\frac{f_{D_s}}{f_\pi}\right)^2 \left|\frac{f_0^{B-\pi}(m_{D_s}^2)}{f_0^{B-\pi}(0)}\right|^2 = 2.72 \pm 0.64. \tag{79}$$

On using

$$\frac{f_0^{B-\pi}(0)}{f_0^{B-\pi}(m_{D_s^2})} = 0.93 \pm 0.05, \tag{80}$$

we get

$$f_{D_s} = 201 \pm 47 \text{ MeV}. \tag{81}$$

Finally, from the experimental branching ratio for the decay $\bar{B}_s^0 \to D_s^+\pi^-$, we obtain

$$f_0^{B_s-D_s}(0) = 0.62 \pm 0.18, \tag{82}$$

$$h_0(1.531) = 0.55 \pm 0.16. \tag{83}$$

In ending this section, we discuss the decays $\bar{B}_s^0 \to D_s^+K^-$, $D_s^{*+}K^-$ for which no experimental data are available. However, using factorization, we get

$$\Gamma(\bar{B}_s^0 \to D_s^+K^-) = (1.75 \times 10^{-10})|V_{cb}f_0^{B_s-D_s}(m_K^2)|^2 \text{ MeV}, \tag{84}$$

$$\Gamma(\bar{B}_s^0 \to D_s^{*+}K^-) = (1.57 \times 10^{-10})|V_{cb}A_0^{B_s-D_s^*}(m_K^2)|^2 \text{ MeV}. \tag{85}$$

$SU(3)$ gives

$$\begin{aligned} |V_{cb}f_0^{B_s-D_s}(m_K^2)|^2 &\approx |V_{cb}||f_0^{B-D}(m_\pi^2)|^2 \\ &= (0.50 \pm 0.04) \times 10^{-3} \\ |V_{cb}A_0^{B_s-D_s^*}(m_K^2)|^2 &\approx |V_{cb}||A_0^{B-D^*}(m_\pi^2)|^2 \\ &= (0.56 \pm 0.04) \times 10^{-3}. \end{aligned} \tag{86}$$

From the above equations, we get the following branching ratios:

$$\frac{\Gamma(\bar{B}_s{}^0 \to D_s^{(*)+}K^-)}{\Gamma_{\bar{B}_s^0}} = (1.94 \pm 0.07) \times 10^{-4}[(1.96 \pm 0.07) \times 10^{-4}]. \tag{87}$$

For $\bar{B}_s^0 \to D_s^{*+}K^-$

$$r_{D_s} = R_b\left[\frac{f_{D_s^*}f_+^{B_s-K}(m_{D_s^*}^2)}{f_KA_0^{B_s-D_s^*}(m_K^2)}\right]. \tag{88}$$

Hence, we get

$$\begin{aligned} -\left(\frac{S_+ + S_-}{2}\right)_{D_s^*K} &= (0.41 \pm 0.08)\sin(2\beta_s + \gamma) \\ &= (0.41 \pm 0.08)\sin\gamma, \end{aligned} \tag{89}$$

where we have used

$$R_b = 0.40, \qquad \frac{f_{D_s}}{f_K} = \frac{f_{D_s^*}}{f_K} = 1.75 \pm 0.06,$$

$$f_+^{B_s-K}(m_{D_s^*}^2) = 0.34 \pm 0.06$$

$$\begin{aligned} A_0^{B_s-D_s^*}(m_K^2) = A_0^{B_s-D_s^*}(0) &= \frac{m_{B_s} + m_{D_s^*}}{2\sqrt{m_{B_s m_{D_s^*}}}}[h_0(\omega_s^* = 1.453) \\ &= 0.52 \pm .03] = 0.58 \pm 0.03. \end{aligned} \tag{90}$$

IV. CP ASYMMETRIES FOR $A_f \neq A_{\bar{f}}$

We now discuss the decays listed in case (ii), where $A_f \neq A_{\bar{f}}$. Subtracting and adding Eqs. (41) and (42), we get

$$\frac{\Gamma_f(t)-\bar{\Gamma}_f(t)}{\Gamma_f(t)+\bar{\Gamma}_f(t)} = C_f\cos\Delta mt + S_f\sin\Delta mt = (C-\Delta C)\cos\Delta mt + (S-\Delta S)\sin\Delta mt, \tag{91}$$

$$\frac{\Gamma_{\bar f}(t)-\bar{\Gamma}_{\bar f}(t)}{\Gamma_{\bar f}(t)+\bar{\Gamma}_{\bar f}(t)} = C_{\bar f}\cos\Delta mt + S_{\bar f}\sin\Delta mt = (C+\Delta C)\cos\Delta mt + (S+\Delta S)\sin\Delta mt, \tag{92}$$

where

$$C_{\bar f,f} = (C\pm\Delta C) = \frac{|A_{\bar f,f}|^2-|\bar A_{\bar f,f}|^2}{|A_{\bar f,f}|^2+|\bar A_{\bar f,f}|^2} = \frac{\Gamma_{\bar f,f}-\bar\Gamma_{\bar f,f}}{\Gamma_{\bar f,f}+\bar\Gamma_{\bar f,f}} = \frac{R_{\bar f,f}(1-A_{CP}^{\bar f,f})-R_{\bar f,f}(1+A_{CP}^{\bar f,f})}{\Gamma(1\pm A_{CP})}, \tag{93}$$

$$S_{\bar f,f} = (S\pm\Delta S), \tag{94}$$

$$= \frac{2\,\mathrm{Im}[e^{2i\phi_M}A^*_{\bar f,f}\bar A_{\bar f,f}]}{\Gamma_{\bar f,f}+\bar\Gamma_{\bar f,f}}, \tag{95}$$

$$A_{CP}^{\bar f} = \frac{\bar\Gamma_f-\Gamma_{\bar f}}{\Gamma_{\bar f}+\bar\Gamma_f} \qquad A_{CP}^{f} = \frac{\bar\Gamma_{\bar f}-\Gamma_f}{\Gamma_f+\bar\Gamma_{\bar f}}, \tag{96}$$

$$A_{CP} = \frac{(\Gamma_{\bar f}+\bar\Gamma_{\bar f})-(\bar\Gamma_f+\Gamma_f)}{(\Gamma_{\bar f}-\bar\Gamma_{\bar f})-(\bar\Gamma_f+\Gamma_f)}, \tag{97}$$

$$= \frac{R_f A_{CP}^f - R_{\bar f}A_{CP}^{\bar f}}{\Gamma}, \tag{98}$$

where

$$R_f = \tfrac{1}{2}(\Gamma_f+\bar\Gamma_{\bar f}), \qquad R_{\bar f} = \tfrac{1}{2}(\Gamma_{\bar f}+\bar\Gamma_f) \qquad \Gamma = R_f + R_{\bar f}. \tag{99}$$

The following relations are also useful and can be easily derived from above equations:

$$\frac{R_{\bar f,f}}{R_f+R_{\bar f}} = \frac{1}{2}[(1\pm\Delta C)\pm A_{CP}C], \tag{100}$$

$$\frac{R_{\bar f}-R_f}{R_f+R_{\bar f}} = [\Delta C + A_{CP}C], \tag{101}$$

$$\frac{R_{\bar f}A_{CP}^{\bar f}+R_fA_{CP}^f}{R_f+R_{\bar f}} = [C + A_{CP}\Delta C]. \tag{102}$$

For these decays, the decay amplitudes can be written in terms of tree amplitude $e^{i\phi_T}T_f$ and the penguin amplitude $e^{i\phi_P}P_f$:

$$\begin{aligned} A_f &= e^{i\phi_T}e^{i\delta_f^T}|T_f|[1+r_fe^{i(\phi_P-\phi_T)}e^{i\delta_f}] \\ A_{\bar f} &= e^{i\phi_T}e^{i\delta_{\bar f}^T}|T_{\bar f}|[1+r_{\bar f}e^{i(\phi_P-\phi_T)}e^{i\delta_{\bar f}}], \end{aligned} \tag{103}$$

where $r_{f,\bar f} = \frac{|P_{f,\bar f}|}{|T_{f,\bar f}|}$, $\delta_{f,\bar f} = \delta^P_{f,\bar f} - \delta^T_{f,\bar f}$

$$\begin{aligned} \bar A_{\bar f} &= e^{-i\phi_T}e^{i\delta_f^T}|T_f|[1+r_fe^{-i(\phi_P-\phi_T)}e^{i\delta_f}] \\ \bar A_f &= e^{-i\phi_T}e^{i\delta_{\bar f}^T}|T_{\bar f}|[1+r_{\bar f}e^{-i(\phi_P-\phi_T)}e^{i\delta_{\bar f}}], \end{aligned} \tag{104}$$

$$\begin{aligned} &\text{For } B^0\to\rho^-\pi^+: A_f; \qquad B^0\to\rho^+\pi^-: A_{\bar f}; \\ &\phi_T=\gamma, \qquad \phi_P=-\beta. \end{aligned} \tag{105}$$

$$\begin{aligned} &\text{For } B^0\to D^{*-}D^+: A_f^D; \qquad B^0\to D^{*+}D^-: A_{\bar f}^D; \\ &\phi_T=0, \qquad \phi_P=-\beta. \end{aligned} \tag{106}$$

Hence, for $B^0\to\rho^-\pi^+$, $B^0\to\rho^+\pi^-$, we have

$$\begin{aligned} A_f &= |T_f|e^{i\gamma}e^{i\delta_f^T}[1-r_fe^{i(\alpha+\delta_f)}] \\ A_{\bar f} &= |T_{\bar f}|e^{i\gamma}e^{i\delta_{\bar f}^T}[1-r_{\bar f}e^{i(\alpha+\delta_{\bar f})}], \end{aligned} \tag{107}$$

$$\text{where}\quad r_{f,\bar f} = \frac{|V_{tb}||V_{td}|}{|V_{ub}||V_{ud}|}\frac{|P_{f,\bar f}|}{|T_{f,\bar f}|} = \frac{R_t}{R_b}\frac{|P_{f,\bar f}|}{|T_{f,\bar f}|}, \tag{108}$$

and for $B^0\to D^{*-}D^+$, $B^0\to D^{*+}D^-$, we have

$$\begin{aligned} A_f^D &= |T_f^D|e^{i\delta_f^{TD}}[1-r_f^De^{i(-\beta+\delta_f^D)}] \\ A_{\bar f}^D &= |T_{\bar f}^D|e^{i\delta_{\bar f}^{TD}}[1-r_{\bar f}^De^{i(-\beta+\delta_{\bar f}^D)}], \end{aligned} \tag{109}$$

$$\text{where}\quad r_{f,\bar f} = R_t\frac{|P^D_{f,\bar f}|}{|T^D_{f,\bar f}|}.$$

We now confine ourselves to $B^0(\bar B^0)\to\rho^-\pi^+$, $\rho^+\pi^-(\rho^+\pi^-, \rho^-, \pi^+)$ decays only [19,20]. The experimental results for these decays are [12] as follows:

$$\Gamma = R_f + R_{\bar f} = (22.8\pm 2.5)\times 10^{-6}, \tag{110}$$

$$A_{CP}^f = -0.16\pm 0.23, \qquad A_{CP}^{\bar f} = 0.08\pm 0.12, \tag{111}$$

$$C = 0.01\pm 0.14, \qquad \Delta C = 0.37\pm 0.08, \tag{112}$$

$$S = 0.01\pm 0.09, \qquad \Delta S = -0.05\pm 0.10. \tag{113}$$

With the above values, it is hard to draw any reliable conclusion. Neglecting the term $A_{CP}C$ in Eqs. (100) and (101), we get

$$R_{\bar f,f} = \tfrac{1}{2}\Gamma(1\pm\Delta C), \tag{114}$$

$$R_{\bar f} - R_f = \Delta C.$$

Using the above value for ΔC, we obtain

$$R_{\bar{f}} = (15.6 \pm 1.7) \times 10^{-6} \qquad R_f = (7.2 \pm 0.8) \times 10^{-6}. \tag{115}$$

We analyze these decays by assuming factorization for the tree graphs [10,11]. This assumption gives

$$T_{\bar{f}} = \bar{T}_f \sim 2m_B f_\rho |\vec{p}| f_+(m_\rho^2), \tag{116}$$

$$T_f = \bar{T}_{\bar{f}} \sim 2m_B f_\pi |\vec{p}| A_0(m_\pi^2). \tag{117}$$

Using $f_+(m_\rho^2) \approx 0.26 \pm 0.04$ and $A_0(m_\pi^2) \approx A_0(0) = 0.29 \pm 0.03$ and $|V_{ub}| = (3.5 \pm 0.6) \times 10^{-3}$, we get the following values for the tree amplitude contribution to the branching ratios:

$$\Gamma_{\bar{f}}^{\text{tree}} = (15.6 \pm 1.1) \times 10^{-6} \equiv |T_{\bar{f}}|^2, \tag{118}$$

$$\Gamma_f^{\text{tree}} = (7.6 \pm 1.4) \times 10^{-6} \equiv |T_f|^2, \tag{119}$$

$$t = \frac{T_f}{T_{\bar{f}}} = \frac{f_\pi A_0(m_\pi^2)}{f_\rho f_+(m_\rho^2)} = 0.70 \pm 0.12. \tag{120}$$

Now,

$$B_{\bar{f}} = \frac{R_{\bar{f}}}{|T_{\bar{f}}|^2} = 1 - 2r_{\bar{f}} \cos\alpha \cos\delta_{\bar{f}} + r_{\bar{f}}^2, \tag{121}$$

$$B_f = \frac{R_f}{|T_f|^2} = 1 - 2r_f \cos\alpha \cos\delta_f + r_f^2. \tag{122}$$

Hence, from Eqs. (115) and (119), we get

$$B_{\bar{f}} = 1.00 \pm 0.12 \qquad B_f = 0.95 \pm 0.11. \tag{123}$$

In order to take into account the contribution of penguin diagram, we introduce the angles $\alpha_{\text{eff}}^{f,\bar{f}}$ [21], defined as follows:

$$e^{i\beta} A_{f,\bar{f}} = |A_{f,\bar{f}}| e^{-i\alpha_{\text{eff}}^{f,\bar{f}}} \qquad e^{-i\beta} \bar{A}_{\bar{f},f} = |\bar{A}_{f,\bar{f}}| e^{i\alpha_{\text{eff}}^{f,\bar{f}}}. \tag{124}$$

With this definition, we separate out tree and penguin contributions:

$$\begin{aligned} e^{i\beta} A_{f,\bar{f}} - e^{-i\beta} \bar{A}_{f,\bar{f}} &= |A_{f,\bar{f}}| e^{-i\alpha^{f,\bar{f}}} - |\bar{A}_{\bar{f},f}| e^{i\alpha^{f,\bar{f}}} \\ &= 2iT_{f,\bar{f}} \sin\alpha, \end{aligned} \tag{125}$$

$$\begin{aligned} e^{i(\alpha+\beta)} A_{f,\bar{f}} - e^{-i(\alpha+\beta)} \bar{A}_{\bar{f},f} &= |\bar{A}_{f,\bar{f}}| e^{i(\alpha_{\text{eff}}^{f,\bar{f}} - \alpha) -} \\ &= (2iT_{f,\bar{f}} \sin\alpha) r_{f,\bar{f}} e^{i\delta_{f,\bar{f}}} \\ &= 2iP_{f,\bar{f}} \sin\alpha. \end{aligned} \tag{126}$$

From Eq. (125), we get

$$2\frac{|T_{f,\bar{f}}|^2}{R_{f,\bar{f}}} \sin^2\alpha \equiv \frac{2\sin^2\alpha}{B_{f,\bar{f}}} = 1 - \sqrt{1 - A_{CP}^{f,\bar{f}2}} \cos 2\alpha_{\text{eff}}^{f,\bar{f}}, \tag{127}$$

$$\sin 2\delta_{f,\bar{f}}^T = -A_{CP}^{f,\bar{f}} \frac{\sin 2\alpha_{\text{eff}}^{f,\bar{f}}}{1 - \sqrt{1 - A_{CP}^{f,\bar{f}2}} \cos 2\alpha_{\text{eff}}^{f,\bar{f}}}, \tag{128}$$

$$\cos 2\delta_{f,\bar{f}}^T = \frac{\sqrt{1 - A_{CP}^{f,\bar{f}2}} - \cos 2\alpha_{\text{eff}}^{f,\bar{f}}}{1 - \sqrt{1 - A_{CP}^{f,\bar{f}2}} \cos 2\alpha_{\text{eff}}^{f,\bar{f}}}. \tag{129}$$

From Eqs. (125) and (126), we get

$$r_{f,\bar{f}}^2 = \frac{1 - \sqrt{1 - A_{CP}^{f,\bar{f}2}} \cos(2\alpha_{\text{eff}}^{f,\bar{f}} - 2\alpha)}{1 - \sqrt{1 - A_{CP}^{f,\bar{f}2}} \cos 2\alpha_{\text{eff}}^{f,\bar{f}}}, \tag{130}$$

$$r_{f,\bar{f}} \cos\delta_{f,\bar{f}} = \frac{\cos\alpha - \sqrt{1 - A_{CP}^{f,\bar{f}2}} \cos(2\alpha_{\text{eff}}^{f,\bar{f}} - \alpha)}{1 - \sqrt{1 - A_{CP}^{f,\bar{f}2}} \cos 2\alpha_{\text{eff}}^{f,\bar{f}}}, \tag{131}$$

$$r_{f,\bar{f}} \sin\delta_{f,\bar{f}} = \frac{-\frac{A_{CP}^{f,\bar{f}}}{\sin\alpha}}{1 - \sqrt{1 - A_{CP}^{f,\bar{f}2}} \cos 2\alpha_{\text{eff}}^{f,\bar{f}}}. \tag{132}$$

Now factorization implies [22]

$$\delta_f^T = 0 = \delta_{\bar{f}}^T. \tag{133}$$

Thus, in the limit $\delta_f^T \to 0$, we get for Eq. (129)

$$\cos 2\alpha_{\text{eff}}^{f,\bar{f}} = -1, \qquad \alpha_{\text{eff}}^{f,\bar{f}} = 90°, \tag{134}$$

$$r_{f,\bar{f}} \cos\delta_{f,\bar{f}} = \cos\alpha, \tag{135}$$

$$r_{f,\bar{f}} \sin\delta_{f,\bar{f}} = \frac{-A_{CP}^{f,\bar{f}}/\sin\alpha}{1 + \sqrt{1 - A_{CP}^{f,\bar{f}2}}}, \tag{136}$$

$$r_{f,\bar{f}}^2 = \frac{1 + \sqrt{1 - A_{CP}^{f,\bar{f}2}} \cos 2\alpha}{1 + \sqrt{1 - A_{CP}^{f,\bar{f}2}}}, \tag{137}$$

$$\approx \cos^2\alpha + \tfrac{1}{4} A_{CP}^{f,\bar{f}2} \sin^2\alpha. \tag{138}$$

The solution of Eq. (135) is graphically shown in Fig. 1 for α in the range of $80° \leq \alpha < 103°$ for $r_{f,\bar{f}} = 0.10, 015, 0.20, 0.25, 0.30$. From the figure, the final state phases $\delta_{f,\bar{f}}$ for various values of $r_{f,\bar{f}}$ can be read for each value of α in the above range. A few examples are given in Table II.

For a $\alpha > 90°$, change $\alpha \to \pi - \alpha$, $\delta_f \to \pi - \delta_f$. For example, for $\alpha = 103°$

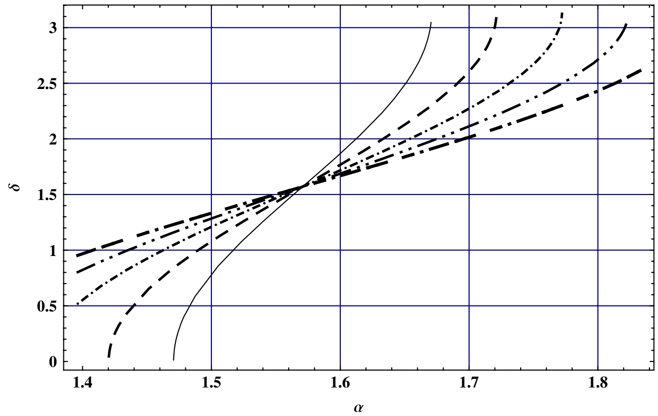

FIG. 1 (color online). Plot of equation $r_f \cos\delta_{(f)} = \cos\alpha$ for different values of r. For $80° \leq \alpha \leq 103°$. Where the solid curve, dashed curve, dashed dotted curve, dashed double dotted, and double dashed dotted curve correspond to $r = 0.1$, $r = 0.15$, $r = 0.2$, $r = 0.25$, and $r = 0.3$, respectively.

$$r_f = 0.25, \qquad \delta_f = 154°, \qquad A^f_{CP} \approx -0.22$$
$$r_f = 0.30, \qquad \delta_f = 138°, \qquad A^f_{CP} \approx -0.40.$$

These examples have been selected keeping in view that final state phases $\delta_{f,\bar{f}}$ are not too large. For $A^{f,\bar{f}}_{CP}$, we have used Eq. (136) neglecting the second order term. An attractive option is $A^f_{CP} = A^{\bar{f}}_{CP}$ for each value of α; although $A^f_{CP} \neq A^{\bar{f}}_{CP}$ is also a possibility. $A^f_{CP} = A^{\bar{f}}_{CP}$ implies $r_f = r_{\bar{f}}$, $\delta_f = \delta_{\bar{f}}$.

Neglecting terms of order $r^2_{f,\bar{f}}$, we have

$$A_{CP} \approx \frac{2\sin\alpha(r_{\bar{f}}\sin\delta_{\bar{f}} - t^2 r_f \sin\delta_f)}{1+t^2} = -\frac{A^{\bar{f}}_{CP} - t^2 A^f_{CP}}{1+t^2}, \tag{139}$$

$$C \approx -\frac{2t^2}{(1+t)^2}(A^{\bar{f}}_{CP} + A^f_{CP}), \tag{140}$$

TABLE II. Selected examples obtained from Fig 1 for $\delta f < 70°$.

α	r_f	δ_f	$A^f_{CP} \approx -2r_f \sin\delta_f \sin\alpha$
80°	0.20	29°	−0.19
	0.25	46°	−0.36
82°	0.15	22°	−0.11
	0.20	46°	−0.28
85°	0.10	29°	−0.10
	0.15	54°	−0.24
86°	0.10	46°	−0.14
	0.15	62°	−0.26
88°	0.10	70°	−0.19

$$\Delta C \approx \frac{1-t^2}{1+t^2} - \frac{4t^2\cos\alpha}{(1+t^2)^2}(r_{\bar{f}}\cos\delta_{\bar{f}} - r_f\cos\delta_f). \tag{141}$$

Now the second term in Eq. (141) vanishes and using the value of t given in Eq. (120), we get

$$\Delta C \approx 0.34 \pm 0.06. \tag{142}$$

Assuming $A^{\bar{f}}_{CP} = A^f_{CP}$, we obtain

$$A_{CP} = -\frac{1-t^2}{1+t^2}A^{\bar{f}}_{CP} = (0.34 \pm 0.06)(-A^{\bar{f}}_{CP}), \tag{143}$$

$$C \approx -\frac{4t^2}{(1+t^2)^2}A^{\bar{f}}_{CP} \approx -(0.88 \pm 0.14)A^{\bar{f}}_{CP}. \tag{144}$$

Finally, the *CP* asymmetries in the limit $\delta^T_{f,\bar{f}} \to 0$

$$S_{\bar{f}} = S + \Delta S = \frac{2\,\mathrm{Im}[e^{2i\phi_M}A_{\bar{f}^*}\bar{A}_{\bar{f}}]}{\Gamma(1+A_{CP})} = \sqrt{1-C^2_{\bar{f}}}\sin(2\alpha^{\bar{f}}_{\mathrm{eff}} + \delta) = -\sqrt{1-C^2_{\bar{f}}}\cos\delta, \tag{145}$$

$$S_f = S - \Delta S = \frac{2\,\mathrm{Im}[e^{2i\phi_M}A^*_f\bar{A}_f]}{\Gamma(1-A_{CP})} = \sqrt{1-C^2_f}\sin(2\alpha^f_{\mathrm{eff}} - \delta) = \sqrt{1-C^2_f}\cos\delta. \tag{146}$$

The phase δ is defined as

$$\bar{A}_{\bar{f}} = \frac{|\bar{A}_{\bar{f}}|}{|\bar{A}_f|}\bar{A}_f e^{i\delta}. \tag{147}$$

To conclude, the final state strong phases essentially arise in terms of the S matrix, which converts an in state into an out state. The isospin, C invariance of hadronic dynamics and the unitarity together with two particle scattering amplitudes in terms of Regge trajectories are used to get information about these phases. In particular, two body unitarity is used to calculate the final state phase δ_C generated by rescattering for the color suppressed decays in terms of the color favored decays. In the inclusive version of unitarity, the information obtained for s-wave scattering from Regge trajectories is used to derive the bounds on the final state phases. In particular, the value obtained for the final state phases $\delta_{+-} = \delta^P \approx 29° - 20°$ and $\delta_{00} = \delta^C + \delta^P \approx 20°, 12°$ is found to be compatible with the experimental values for direct *CP* asymmetries $A_{CP}(B^0 \to \pi^- K^+, \pi^0 K^0)$. For $B^0 \to D^{(*)-}\pi^+(D^{(*)+}\pi^-)$, $B^0_s \to D^{(*)-}_s K^+(D^{(*)+}_s K^-)$ decays described by two independent single amplitudes A_f, $A'_{\bar{f}}$ and A_{f_s}, $A'_{\bar{f}_s}$ with different weak phases viz. 0 and γ, the equality of phases $\delta_f = \delta'_{\bar{f}}$ implies, the time-dependent *CP* asymmetries

$$-\left(\frac{S_+ + S_-}{2}\right) = \frac{2r_{D^{(*)}_{(s)}}}{1+r^2_{D^{(*)}_{(s)}}}\sin(2\beta_{(s)} + \gamma), \tag{148}$$

$$\frac{S_+ - S_-}{2} = 0. \quad (149)$$

An added advantage is that these decays are described by tree graphs. Assuming factorization, the decay amplitude A_f can be determined in terms of the form factors $f_0^{B-D}(m_\pi^2)$ and $A_0^{B-D^*}(m_\pi^2)$. The parameter $r_{D^{(*)}}$ can be expressed in terms of the ratios of the form factors $f_D f_0^{B-\pi}(m_D^2)/f_\pi f_0^{B-D}(m_\pi^2)$ and $f_{D^*} f_+^{B-\pi}(m_{D^*}^2)/f_\pi A_0^{B-D^*}(m_\pi^2)$. From the experimental branching ratios, we have obtained the form factors $f_0^{B-D}(m_\pi^2)$ and $A_0^{B-D^*}(m_\pi^2)$, which are in excellent agreement with the prediction of HQET. We have also determined r_{D^*}. For r_{D^*} we get the value $r_{D^*} = 0.017 \pm 0.003$. Using this value we get the following bound from the experimental value of $\frac{S_+ + S_-}{2}$ for $B^0 \to D^{*-}\pi^+$ decay:

$$\sin(2\beta + \gamma) > 0.69.$$

Using $SU(3)$, for the form factors for $B_s^0 \to D_s^{*-}K^+ (D_s^{*+}K^-)$ decays, we predict

$$-\left(\frac{S_+ + S_-}{2}\right) = (0.41 \pm 0.08)\sin(2\beta + \gamma)$$
$$= (0.41 \pm 0.08)\sin\gamma$$

in the standard model.

In Sec. IV, the decays $B \to \rho^+\pi^-(\rho^-\pi^+)$ for which decay amplitudes $A_{\bar f}$ and A_f are given in terms of tree and penguin diagrams are discussed. We have analyzed these decays assuming factorization for the tree graph. Factorization implies $\delta_f^T = \delta_{\bar f}^T$. In the limit $\delta_{f,\bar f}^T \to 0$, we have shown that

$$r_{f,\bar f}\cos\delta_{f,\bar f} = \cos\alpha \qquad r_{f,\bar f}^2 \approx \cos^2\alpha + A_{CP}^{f,\bar f 2}\sin^2\alpha.$$

The first equation has been solved graphically, from which the final state phases $\delta_{f,\bar f}$ corresponding to various values of $r_{f,\bar f}$ can be found for a particular value of α. The upper bound $\delta_{f,\bar f} \leq 30°$ obtained in Sec. II, using unitarity and strong interaction dynamics based on Regge pole phenomonalogy can be used to select the solutions given in Table II. Neglecting the terms of order $r_{f,\bar f}^2$, we get using factorization

$$\Delta C = 0.34 \pm 0.06.$$

Finally, in the limit $\delta_{f,\bar f}^T \to 0$, we get

$$\frac{S_{\bar f}}{S_f} = \frac{S + \Delta S}{S - \Delta S} = -\frac{\sqrt{1 - C_{\bar f}^2}}{\sqrt{1 - C_f^2}}.$$

With the present experimental data, it is hard to draw any definite conclusion.

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[1] J. F. Donoghue *et al.*, Phys. Rev. Lett. **77**, 2178 (1996).
[2] M. Suzuki and L. Wofenstein, Phys. Rev. D **60**, 074019 (1999).
[3] A. Falk *et al.*, Phys. Rev. D **57**, 4290 (1998).
[4] I. Caprini, L. Micer, and C. Bourrely, Phys. Rev. D **60**, 074016 (1999).
[5] Fayyazuddin, J. High Energy Phys. 09 (2002) 055.
[6] Fayyazuddin, Phys. Rev. D **70**, 114018 (2004).
[7] M. Gronau and J. L. Rosner, Phys. Lett. B **666**, 467 (2008).
[8] L. Wolfenstein, arXiv:hep-ph/0407344v1; N. Spokvich, Nuovo Cimento **26**, 186 (1962); K. Gottfried and J. D. Jackson, Nuovo Cimento **34**, 735 (1964); see also, Fayyazuddin and Riazuddin, *Quantum Mechanics* (World Scientific, Singapore, 1990), p. 140.
[9] J. D. Bjorken, Nucl. Phys. B, Proc. Suppl. **11**, 325 (1989).
[10] M. Beneke, G. Buchalla, M. Neubart, and C. T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999); Nucl. Phys. **B591**, 313 (2000).
[11] C. W. Bauer, D. Pirjol, and I. W. Stewart, Phys. Rev. Lett. **87**, 201806 (2001).
[12] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).
[13] For a review, see for example *CP-Violation*, edited by C. Jarlskog (World Scientific, Singapore, 1989); H. Quinn, arXiv:hep-ph/0111174; Fayyazuddin and Riazuddin, *A Modern Introduction to Particle Physics* (World Scientific, Singapore, 2000), 2nd ed..
[14] S. Balk, J. G. Korner, G. Thompson, and F. Hussain, Z. Phys. C **59**, 283 (1993).
[15] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989); N. Isgur and M. B. Wise*ibid.***237**, 527 (1990).
[16] S. Faller *et al.*, Eur. Phys. J. C **60**, 603 (2009).
[17] P. Ball, R. Zweicky, and W. I. Fine, Phys. Rev. D **71**, 014029 (2005).
[18] G. Duplancic *et al.*, J. High Energy Phys. 04 (2008) 014.
[19] V. Page and D. London, Phys. Rev. D **70**, 017501 (2004).
[20] M. Gronau and J. Zupan, Phys. Rev. D **70**, 074031 (2004), 2004 references to earlier literature can be found in this reference.
[21] Y. Grossman and H. R. Quinn, Phys. Rev. D **58**, 017504 (1998); J. Charles, Phys. Rev. D **59**, 054007 (1999); M. Gronau *et al.*, Phys. Lett. B **514**, 315 (2001).
[22] M. Beneke and M. Neubert, Nucl. Phys. **B675**, 333 (2003).