

Stability and the equation of state for kinky vortonsRichard A. Battye^{1,*} and Paul M. Sutcliffe^{2,†}¹*Jodrell Bank Centre for Astrophysics, University of Manchester, Manchester M13 9PL, U.K.*²*Department of Mathematical Sciences, Durham University, Durham DH1 3LE, U.K.*

(Received 24 August 2009; published 27 October 2009)

Vortons are closed loops of superconducting strings carrying current and charge. A formalism has been developed to study vortons in terms of an elastic string approximation, but its implementation requires knowledge of the unknown equation of state, relating the string tension to the energy per unit length. Recently, a planar analogue of the vorton, known as a kinky vorton, has been introduced. In this paper we derive an exact formula for the equation of state of a kinky vorton and use it to calculate the properties of the associated elastic string, such as the transverse and longitudinal propagation speeds. In particular, the elastic string approximation predicts a complicated and highly nontrivial pattern of intervals of instability, which we are able to confirm using full field simulations. The implications of the results for vortons are also discussed.

DOI: 10.1103/PhysRevD.80.085024

PACS numbers: 11.27.+d

I. INTRODUCTION

Superconducting cosmic strings were introduced by Witten [1], who realized that if the field of a cosmic string is coupled to another complex scalar field then a non-dissipative current can flow along the string. A vorton [2] is a closed loop of superconducting cosmic string that carries both current and charge, which may provide a force to balance the string tension and prevent its collapse. As vortons have a number of possible cosmological consequences [3], it is of considerable interest to determine their stationary and dynamical properties. Vortons and related objects have also been discussed in the context of various condensed matter systems [4,5], cold atoms [6,7], and high density QCD [8].

Recently, results have been presented [9] on the first numerical construction of vortons in the simplest theory, namely, the global version of Witten's $U(1) \times U(1)$ theory. These results demonstrate that a range of stationary circular vortons exist, which are stable to axially symmetric perturbations. However, unstable modes associated with nonaxial perturbations were found to exist.

A formalism has been developed [10,11] to study vortons and their stability in terms of an elastic string approximation, but its implementation requires knowledge of the unknown equation of state, relating the string tension to the energy per unit length. Attempts have been made [12] to compute the equation of state numerically from field theory simulations using particular values of the field theory parameters, but obviously this is far from ideal.

The equation of state can be derived from the action density on the string world sheet, but one needs to know how this depends on the quantity $\chi = \omega^2 - k^2$, where ω and k are the frequency and twist rate of the phase of the

condensate field. Witten originally suggested [1] that the usual Nambu-Goto action density, which is simply a constant, could be replaced by a form linear in χ . However, this linear form produces results that are qualitatively incorrect, even for small χ , since the propagation speed of longitudinal perturbations depends on the second derivative of the action density with respect to χ . This prompted suggestions [13–15] for alternative nonlinear forms for the assumed equation of state.

Recently, a planar analogue of the vorton, known as a kinky vorton, has been introduced and shown to possess many of the features expected of a vorton [16]. A significant advantage of the kinky vorton is that several approximations required in the study of vortons can be replaced by exact results. In this paper we exploit this advantage to derive an exact formula for the kinky vorton equation of state, and make a comparison with the previously proposed approximations to the vorton equation of state. The exact equation of state is then used to calculate properties, such as the transverse and longitudinal propagation speeds, of the elastic string description of the kinky vorton. In particular, the elastic string model predicts a complicated and highly nontrivial pattern of intervals of instability, which we are able to confirm using full field simulations. These analytic results provide an understanding of similar instabilities found for vortons using numerical simulations of the full field theory dynamics [9].

II. THE ACTION DENSITY ON THE STRING WORLD SHEET

The kinky vorton Lagrangian density in $(2 + 1)$ dimensions is given by [16]

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi + \partial_\mu \sigma \partial^\mu \bar{\sigma} - \frac{\lambda_\phi}{4} (\phi^2 - \eta_\phi^2)^2 - \frac{\lambda_\sigma}{4} (|\sigma|^2 - \eta_\sigma^2)^2 - \beta \phi^2 |\sigma|^2 + \frac{\lambda_\sigma}{4} \eta_\sigma^4, \quad (2.1)$$

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where ϕ and σ are real and complex scalar fields, respectively, with $\eta_\phi, \eta_\sigma, \lambda_\phi, \lambda_\sigma, \beta$ all real positive constants.

This theory can be obtained from the global version of Witten's $U(1) \times U(1)$ theory [1] by a trivial dimensional reduction from $(3 + 1)$ dimensions to $(2 + 1)$ dimensions, followed by a restriction that one of the complex scalar fields is real.

The theory has a global $\mathbb{Z}_2 \times U(1)$ symmetry and the parameters of the model can be arranged so that in the vacuum the \mathbb{Z}_2 symmetry is broken, $\phi = \pm \eta_\phi \neq 0$, while the $U(1)$ symmetry remains unbroken, $|\sigma| = 0$. For this symmetry breaking pattern there exist kink strings constructed from the ϕ field. If the infinite kink string lies along the y axis, then it is given by the solution

$$\phi = \eta_\phi \tanh\left(\frac{\eta_\phi \sqrt{\lambda_\phi} x}{2}\right), \quad \sigma = 0. \quad (2.2)$$

The situation of interest is when a condensate of the σ field carrying current and charge forms in the core of the kink string. For the infinite string given above, such a condensate field takes the form

$$\sigma = e^{i(\omega t + ky)} |\sigma|, \quad (2.3)$$

where $|\sigma|$ is a function of x only with $|\sigma| \rightarrow 0$ as $|x| \rightarrow \infty$. The constant k describes the rate of twisting of the condensate along the string, though in the literature this is referred to as winding, rather than twisting, so we will stick to this common convention.

A nonzero value of ω induces a charge Q associated with the global $U(1)$ symmetry, and the winding k generates a current along the string. It is easy to see that charge and current will have opposite effects on the string, so it is useful to introduce the combination

$$\chi \equiv \omega^2 - k^2. \quad (2.4)$$

In the literature solutions with $\chi = 0$ are termed chiral, whereas solutions with $\chi > 0$ are referred to as electric and those with $\chi < 0$ are called magnetic [13,17].

To obtain exact solutions for the whole range of χ we set [16]

$$2\beta = \lambda_\phi = \lambda_\sigma \equiv \lambda. \quad (2.5)$$

With this choice there are three remaining parameters, which may be chosen to be η_ϕ and the two combinations

$$m_\phi^2 \equiv \frac{\lambda}{2} \eta_\phi^2, \quad \alpha \equiv \left(\frac{\eta_\sigma}{\eta_\phi}\right)^2. \quad (2.6)$$

In fact, the parameters m_ϕ and η_ϕ merely set energy and length units and can therefore be set to unity without loss of generality. To restore these parameters in the following the scalings, $L \mapsto m_\phi \eta_\phi^2 L$ and $\chi \mapsto \chi / m_\phi^2$ need to be applied.

The only significant parameter that remains is α , which must lie in the interval $\alpha \in (\frac{1}{2}, 1)$ if chiral solutions are to exist. The range of χ is

$$\chi_- \equiv \frac{1}{2} - \alpha \leq \chi \leq 1 - \alpha \equiv \chi_+, \quad (2.7)$$

and the associated exact solutions are

$$\begin{aligned} \phi &= \tanh(x\sqrt{1 - \alpha - \chi}), \\ \sigma &= e^{i(\omega t + ky)} \sqrt{2\alpha - 1 + 2\chi} \operatorname{sech}(x\sqrt{1 - \alpha - \chi}). \end{aligned} \quad (2.8)$$

Substituting the solution (2.8) into the Lagrangian density (2.1) and integrating over x yields the result

$$L = \frac{4}{3\sqrt{1 - \alpha - \chi}} \{(2\alpha + 1)(\alpha - 1) + \chi(2\chi + 4\alpha - 1)\} \quad (2.9)$$

for the action density on the string world sheet.

This should be contrasted with various proposed forms, such as the linear approximation

$$L_{\text{lin}} = -m^2 + \kappa\chi, \quad (2.10)$$

due to Witten [1], and the logarithmic approximation [13,15]

$$L_{\text{log}} = -m^2 - \frac{1}{2} m_*^2 \log(1 - \delta_*^2 \chi). \quad (2.11)$$

The parameters m^2 and κ in the linear approximation (2.10) can be determined from the exact expression (2.9) by matching the function and the derivative at $\chi = 0$. This gives

$$m^2 = \frac{4}{3} (2\alpha + 1) \sqrt{1 - \alpha}, \quad \kappa = \frac{4\alpha - 2}{\sqrt{1 - \alpha}}. \quad (2.12)$$

One approach to fixing the additional parameters m_*^2 and δ_*^2 in the logarithmic approximation is by matching the derivative at $\chi = 0$, and requiring the singularity in (2.11) to occur at the extreme electric limit $\chi = \chi_+$, which gives

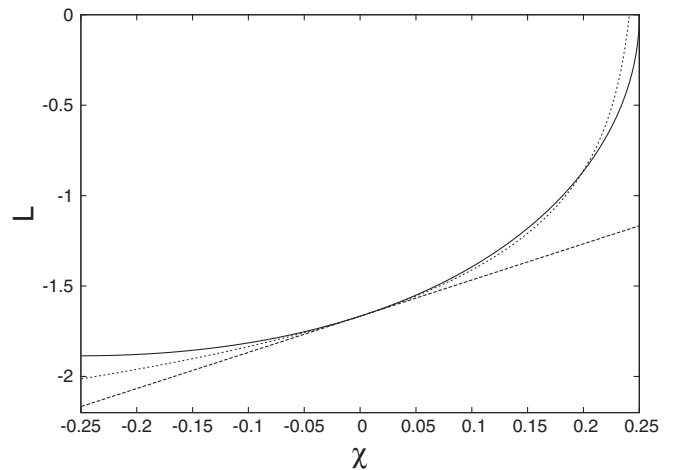


FIG. 1. The exact action density on the world sheet L (solid curve) as a function of χ , together with the linear approximation L_{lin} (dashed curve) and the logarithmic approximation L_{log} (dotted curve). This is in the theory with $\alpha = 3/4$.

$$m_*^2 = 4(2\alpha - 1)\sqrt{1 - \alpha}, \quad \delta_*^2 = 1/(1 - \alpha). \quad (2.13)$$

In Fig. 1 the exact result (2.9) is compared with the linear (2.10) and logarithmic (2.11) approximations for the parameter value $\alpha = 3/4$, with other parameter values producing similar results. This shows that the linear approximation is poor, but the logarithmic approximation is reasonably good, and therefore should produce results which are, at least, qualitatively correct.

III. THE EQUATION OF STATE

From the Lagrangian (2.9) the string tension T and energy per unit length U can be calculated as follows [10]:

The energy-momentum tensor associated with the Lagrangian density (2.1) is

$$\mathcal{T}_\nu^\mu = 2g^{\mu\alpha} \frac{\partial \mathcal{L}}{\partial g^{\alpha\nu}} - \delta_\nu^\mu \mathcal{L}. \quad (3.1)$$

Integration over the string cross section gives the macroscopic tensor

$$T^{ab} = \int_{-\infty}^{\infty} \mathcal{T}^{ab} dx, \quad (3.2)$$

where $a, b \in \{t, y\}$. The energy per unit length U and the string tension T are the eigenvalues of T^{ab} , and are given

$$T = \begin{cases} \frac{4}{3}((2\alpha + 1)(1 - \alpha) - \chi(2\chi + 4\alpha - 1))/\sqrt{1 - \alpha - \chi} & \text{if } \chi \geq 0 \\ \frac{4}{3}((2\alpha + 1)(1 - \alpha) + 2\chi(2\chi + \alpha - 1))/\sqrt{1 - \alpha - \chi} & \text{if } \chi < 0 \end{cases} \quad (3.7)$$

and

$$U = \begin{cases} \frac{4}{3}((2\alpha + 1)(1 - \alpha) + 2\chi(2\chi + \alpha - 1))/\sqrt{1 - \alpha - \chi} & \text{if } \chi \geq 0 \\ \frac{4}{3}((2\alpha + 1)(1 - \alpha) - \chi(2\chi + 4\alpha - 1))/\sqrt{1 - \alpha - \chi} & \text{if } \chi < 0. \end{cases} \quad (3.8)$$

Using these formulae it is easy to calculate the behavior of the tension and energy in the three limits $\chi = 0$ (chiral), $\chi \rightarrow \chi_+$ (extreme electric), and $\chi \rightarrow \chi_-$ (extreme magnetic).

If $\chi = 0$ then $T = U = m^2 = \frac{4}{3}(2\alpha + 1)\sqrt{1 - \alpha}$. As $\chi \rightarrow \chi_+$, then $T \rightarrow 0$ and $U \rightarrow \infty$. Finally, as $\chi \rightarrow \chi_-$ then $T \rightarrow 4\sqrt{2}/3$ and $U \rightarrow 4\sqrt{2}/3$, so again the tension and energy are equal, and independent of α .

Figure 2 displays the tension T (solid curve) and the energy per unit length U (dashed curve) as a function of χ , for the theory with $\alpha = 3/4$. This value of α has been employed in previous work since the range of χ is then symmetric around zero, explicitly $\chi_+ = 1/4$ and $\chi_- = -1/4$, allowing a range of electric and magnetic kinky vortons. Plots for other values of α share the same qualitative features. All graphs in this paper are plotted for the theory with $\alpha = 3/4$, but again the qualitative features are independent of α .

Note that the tension is never negative, which agrees with numerical computations in $(3 + 1)$ dimensions, con-

firming that there are no spring states [18], despite earlier

by $U = T^{tt}$ and $T = -T^{yy}$, in a frame in which T^{ab} is diagonal.

Applying the above procedure to the Lagrangian density (2.1) produces

$$T^{tt} = 2\omega^2 \Sigma_2 - L, \quad -T^{yy} = -2k^2 \Sigma_2 - L, \quad (3.3)$$

where

$$\Sigma_2 = \int_{-\infty}^{\infty} |\sigma|^2 dx = \frac{4\chi + 4\alpha - 2}{\sqrt{1 - \alpha - \chi}}. \quad (3.4)$$

The frame in which T^{ab} is diagonal is obtained by setting $k = 0$, if $\chi \geq 0$, or by setting $\omega = 0$ if $\chi \leq 0$.

In the electric regime, that is $\chi > 0$, setting $k = 0$ implies that $\chi = \omega^2$ and then (3.3) becomes

$$U = 2\chi \Sigma_2 - L, \quad T = -L. \quad (3.5)$$

Conversely, in the magnetic regime, that is $\chi < 0$, then setting $\omega = 0$ means that $\chi = -k^2$ and hence (3.3) gives

$$U = -L, \quad T = 2\chi \Sigma_2 - L. \quad (3.6)$$

Of course, both forms agree in the chiral limit $\chi = 0$, where $U = T = -L$.

Using the explicit expressions (2.9) and (3.4) for L and Σ_2 the above equations become

firming that there are no spring states [18], despite earlier

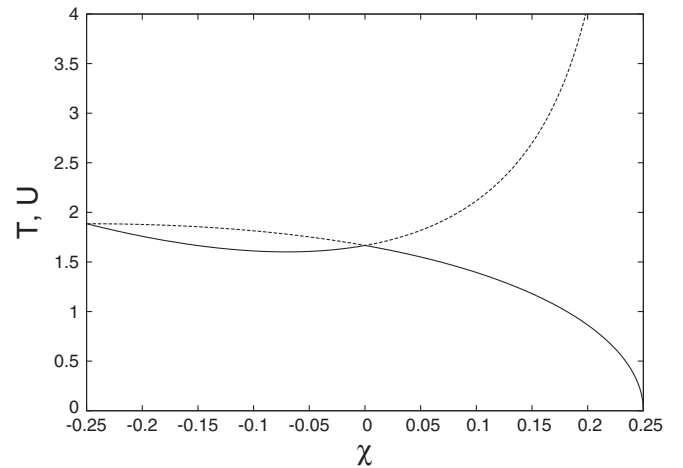


FIG. 2. The tension T (solid curve) and the energy per unit length U (dashed curve) as a function of χ , for the theory with $\alpha = 3/4$.

claims in the literature (see the discussion and references in [18]).

The equation of state is the relation between T and U , and this is given implicitly by combining the formulae (3.7) and (3.8). Figure 3 presents the equation of state in graphical form, that is, T as a function of U . For comparison the upper dashed line is the Nambu-Goto equation of state $T = U$, and the lower dotted curve is the self-dual equation of state [14] $T = m^4/U$, with the characteristic property that the equation of state is identical in the magnetic and electric regimes. The self-dual model appears to be a reasonable approximation in the electric regime, but as discussed in [13] and demonstrated below, the self-dual model does not capture the important qualitative features associated with propagation speeds, even in the electric regime.

Figure 3 has a remarkable similarity to the graphical equation of state computed numerically for superconducting cosmic strings in $(3 + 1)$ dimensions [12]. This suggests that our explicit exact formulae derived in $(2 + 1)$ dimensions also provide a good description of the $(3 + 1)$ -dimensional system, where results are only available numerically. Note that the most interesting dynamics of a string loop takes place in the plane of the loop, therefore the reduction to $(2 + 1)$ dimensions is still expected to capture the most important degrees of freedom.

String dynamics depends crucially on the propagation speeds of transverse and longitudinal (soundlike) perturbations. The equation of state allows the calculation of the transverse speed c_T and the longitudinal speed c_L via the formulae

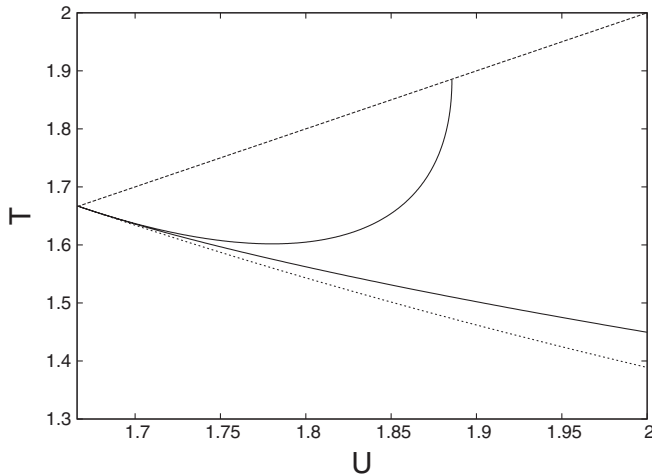


FIG. 3. The tension T as a function of the energy per unit length U (solid curve), for $\alpha = 3/4$. The upper portion of the curve is the magnetic regime ($\chi < 0$) and the lower portion is the electric regime ($\chi > 0$). For comparison the upper dashed line is the Nambu-Goto equation of state $T = U$, and the lower dotted curve is the self-dual equation of state $T = m^4/U$.

$$c_T^2 = \frac{T}{U}, \quad c_L^2 = -\frac{dT}{dU}. \quad (3.9)$$

Substitution of the tension and energy expressions (3.7) and (3.8) into the speed formulae (3.9) gives

$$c_T^{2 \operatorname{sign}(\chi)} = \frac{(2\alpha + 1)(1 - \alpha) - \chi(2\chi + 4\alpha - 1)}{(2\alpha + 1)(1 - \alpha) + 2\chi(2\chi + \alpha - 1)}, \quad (3.10)$$

$$c_L^{2 \operatorname{sign}(\chi)} = \frac{(2\alpha + 2\chi - 1)(1 - \alpha - \chi)}{(2\alpha - 1)(1 - \alpha) - 2\chi(2\chi + 3\alpha - 3)}, \quad (3.11)$$

with $c_T = c_L = 1$ if $\chi = 0$.

The string is unstable if either of the speeds are imaginary, that is, if either $c_T^2 < 0$ or $c_L^2 < 0$. It is easy to see that $c_T^2 > 0$ for all $\chi \in (\chi_-, \chi_+)$, and $c_T \rightarrow 0$ as $\chi \rightarrow \chi_+$. Similarly, it is easy to show that $c_L^2 > 0$ for all $\chi \in [0, \chi_+)$, and again $c_L \rightarrow 0$ as $\chi \rightarrow \chi_+$. However, $c_L^2 < 0$ if $\chi < \chi_c$, where the critical value for the onset of instability is given by the vanishing of the denominator in (3.11), leading to

$$\chi_c = \frac{3}{4}(1 - \alpha) - \frac{1}{4}\sqrt{(1 - \alpha)(5 - \alpha)}. \quad (3.12)$$

For the parameter value $\alpha = 3/4$ this gives $\chi_c = (3 - \sqrt{17})/16 \approx -0.07$. This is in excellent agreement with numerical field theory simulations, performed for the value $\alpha = 3/4$, which estimated the critical value to be -0.08 [16].

In Fig. 4 the speeds c_T (solid curve) and c_L (dashed curve) are plotted as a function of χ , for $\alpha = 3/4$. Plots with other values of α are qualitatively similar.

In the chiral limit $\chi = 0$, and the extreme electric limit $\chi \rightarrow \chi_+$, the string is transonic, that is, $c_T = c_L$. For all other values of χ the string is supersonic, that is, $c_T > c_L$. Such supersonic behavior has been observed in numerical

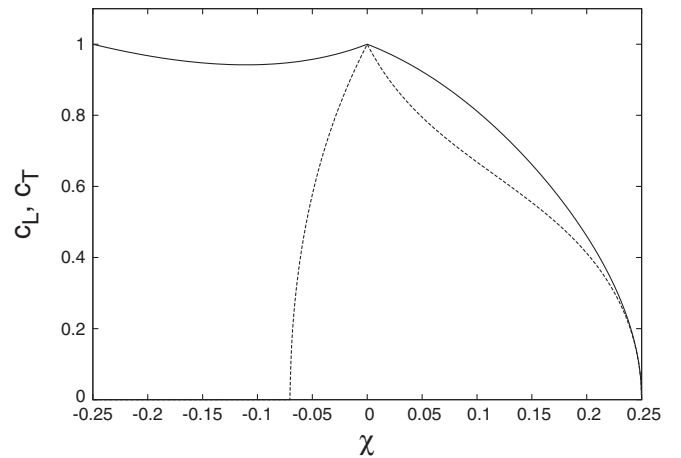


FIG. 4. The transverse speed c_T (solid curve) and the longitudinal speed c_L (dashed curve) as a function of χ , for the theory with $\alpha = 3/4$.

computations based on (3 + 1)-dimensional field theories [12].

The simple linear approximation to the action (2.10) is qualitatively incorrect, since it predicts that the string is everywhere subsonic, that is, $c_T < c_L$, which we have seen is never the case. Models based on a self-dual equation of state predict that the string is everywhere transonic, which is also false. These failures motivated attempts to construct improved actions [13–15], of which the logarithmic approximation (2.11) is an example. As we have seen, the exact action for kinky vortons has a simple form (2.9), but an action of this type does not appear to have been suggested previously. Given the remarkable similarity between the results obtained using (2.9) and numerical computations in (3 + 1) dimensions, it seems likely that an action of the form (2.9) will provide a good description for vortons.

IV. INTERVALS OF INSTABILITY

For a circular elastic string of radius R , the frequencies Ω_n , of linear perturbations with Fourier mode n , can be related to the transverse and longitudinal speeds [11]. Explicitly, the scaled frequency $\nu_n = \Omega_n R / c_T$ satisfies the cubic equation

$$a_3 \nu_n^3 + a_2 \nu_n^2 + a_1 \nu_n + a_0 = 0, \quad (4.1)$$

where

$$a_0 = 2(c_L^2 - c_T^2)(n^2 - 1)n, \quad (4.2)$$

$$a_1 = 4c_T^2(1 - c_L^2)(n^2 - 1) - (1 + c_T^2)(c_L^2 - c_T^2)(n^2 + 1), \quad (4.3)$$

$$a_2 = 2c_T^2(c_L^2 - c_T^2 - 2(1 - c_L^2 c_T^2))n, \quad (4.4)$$

$$a_3 = c_T^2(1 + c_T^2)(1 - c_L^2 c_T^2). \quad (4.5)$$

The derivation of this formula in [11] assumes that the string moves in a three-dimensional space, but the result applies equally well to a string in two-dimensional space because the modes perpendicular to the plane of the circular string decouple and are irrelevant for the stability analysis.

Instability is characterized by a complex root of the cubic (4.1). It is easy to show [11] that all roots are real for $n = 0$ and $n = 1$, simply as a consequence of the causality restrictions $0 < c_T^2 \leq 1$ and $0 < c_L^2 \leq 1$ for non-zero speeds.

For axially symmetric perturbations ($n = 0$) the solutions of the cubic are a trivial zero mode and

$$\nu_0^2 = \frac{2c_T^2(1 - c_L^2) + (c_T^2 + c_L^2)(1 - c_T^2)}{c_T^2(1 + c_T^2)(1 - c_L^2 c_T^2)}. \quad (4.6)$$

The explicit speed formulae (3.10) and (3.11) allow this frequency to be calculated for any kinky vorton given the

values of χ and R . As an example, a kinky vorton is reported in [16] with $Q = 1500$ and $N = 84$, producing the quantities $\chi = 0.0472$ and $R = 154.6$. Using these values in the above formulae gives a frequency $\Omega_0 = 0.00644$. This can be compared with the results of full field dynamics, using the numerical approach described in detail in [16]. Perturbing this kinky vorton by an axially symmetric mode that increases the radius by 1% produces an oscillation of the radius. A spectral analysis of this oscillation yields the frequency 0.00658, which is very close to the above predicted value for Ω_0 .

For a given mode n , the critical values of χ marking the limits between stability and instability correspond to the values at which the cubic (4.1) has a repeated root. This is given by the vanishing of the resultant

$$\begin{aligned} \text{Res}(a_3 \nu_n^3 + a_2 \nu_n^2 + a_1 \nu_n + a_0, 3a_3 \nu_n^2 + 2a_2 \nu_n + a_1) \\ = 27a_3^2 a_0^2 - 18a_3^2 a_0 a_1 a_2 - a_3 a_2^2 a_1^2 + 4a_2^3 a_3 a_0 \\ + 4a_1^3 a_3^2. \end{aligned} \quad (4.7)$$

In Fig. 5 we indicate the χ intervals of instability for modes up to $n = 10$, for the theory with $\alpha = 3/4$. Note that the range of χ for the plot runs from the value at which the longitudinal speed vanishes $\chi_c = (3 - \sqrt{17})/16$ (in which case there is automatically instability).

This plot reveals a complicated and highly nontrivial structure for the instability intervals as a function of mode number. In particular, if a kinky vorton is to be stable to all perturbation modes then there is an extremely limited range in the magnetic regime ($\chi < 0$).

Note that the full field dynamical simulations of kinky vortons presented in [16] included the perturbation of a solution with $\chi = -0.04$ that did not decay. However, in this case the perturbation was generated by the square boundary and therefore corresponds to modes that are a

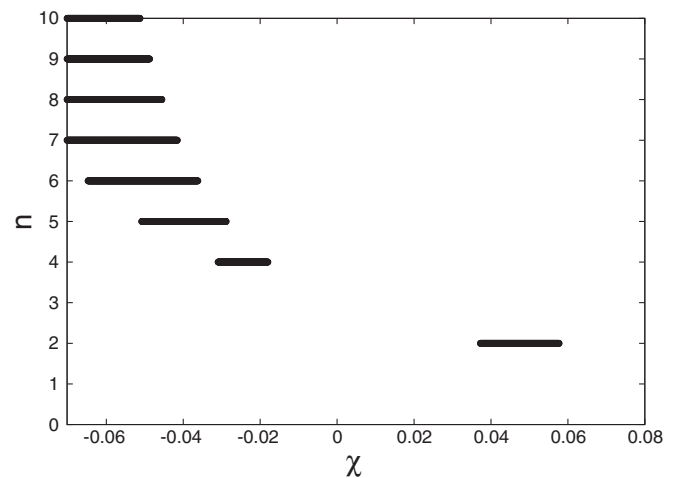


FIG. 5. The χ intervals of instability for perturbations with Fourier modes up to $n = 10$, for the theory with $\alpha = 3/4$. In this plot the lower limit of χ is equal to the value at which the longitudinal speed vanishes.

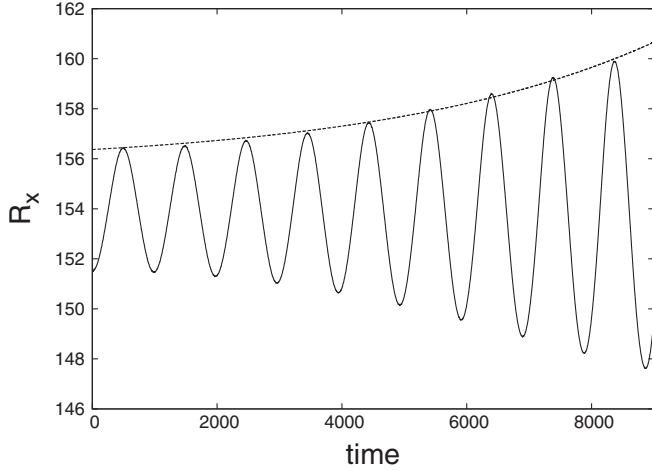


FIG. 6. The evolution of the radius, R_x (solid curve) calculated along the x axis, under an $n = 2$ perturbation, for a kinky vorton in a regime where instability is predicted. For comparison, the expected growth of the envelope is also presented (dashed curve).

multiple of 4. This result is consistent with Fig. 5 since the unstable modes for $\chi = -0.04$ are $n = 5$ and $n = 6$.

Most electric vortons ($\chi > 0$) are stable to the instabilities discussed above and the results are consistent with full field simulations [16]. The elastic string model predicts an instability for a small range of electric vortons around $\chi = 0.05$, under a perturbation with mode $n = 2$. A suitable kinky vorton solution to investigate this issue is the one discussed above with $Q = 1500$ and $N = 84$ since this has $\chi = 0.0472$ and $R = 154.6$. The elastic string model predicts an instability to $n = 2$ perturbations with the associated frequency $\Omega_2 = 0.0064 + i0.00023$. The period of kinky vorton oscillations is already large, but the imaginary part of Ω_2 is a further order of magnitude smaller than the real part. This implies that the timescale over which the instability manifests itself is very large and long simulations are required to observe the instability.

The numerical approach described in [16] has been applied to study the stability of the above kinky vorton. The perturbation consists of a squashing by 2% along the x axis and a stretching by the same factor along the y axis, in order to preserve the total charge Q . This elliptic deformation induces an $n = 2$ perturbation and the resulting evolution is presented in Fig. 6, where the solid curve represents the radius of the kinky vorton, as measured along the x axis. It is clear that the perturbation grows with time, confirming that the kinky vorton is unstable to modes with $n = 2$. The dashed curve displays the predicted growth rate of the envelope, that is, it is a curve

with the predicted growth $\exp(t\Im(\Omega_2))$. Once again, this shows an excellent agreement with the elastic string analysis.

The results described above, together with other similar simulations, including those presented previously in [16], confirm the validity of the elastic string description. The general conclusions are that most magnetic kinky vortons are unstable to generic nonaxial perturbations, whereas the stability of electric kinky vortons has a crucial and complicated dependence upon the parameters of the kinky vorton.

V. CONCLUSION

In this paper we have derived an exact formula for the action density on the string world sheet in the kinky vorton model. Using this result we have determined the equation of state and hence explicit expressions for the transverse and longitudinal propagation speeds in the elastic string description of a kinky vorton. This has allowed an explicit analytic study of the stability of kinky vortons, revealing a highly nontrivial pattern of intervals of instability. The analytic results are in excellent agreement with numerical results from full field simulations, confirming the validity of the elastic string description.

The exact kinky vorton results share the same qualitative features found in numerical computations of vortons, which suggests that the kinky vorton form of the action density on the string world sheet should provide a good approximate description for vortons.

A recent numerical investigation of vortons [9] found instabilities to nonaxial perturbations, which are very similar to the instabilities described here for kinky vortons. The kinky vorton results show that the existence of an instability is not a generic feature, but rather has a crucial and nontrivial dependence on the properties of a particular kinky vorton. Numerical simulations of vortons require considerable computational resources and unfortunately this restricts investigations to quite a limited region of parameter space. The results of the present paper suggest that the instabilities found in vorton simulations [9] may not exist for all vortons. Certainly, an important observation is that results found in a limited region of parameter space are unlikely to be generic throughout the parameter space of vorton solutions.

ACKNOWLEDGMENTS

P. M. S. thanks the STFC for support under rolling Grant No. ST/G000433/1.

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