

Possibility of catastrophic black hole growth in the warped brane-world scenario at the LHCRoberto Casadio,^{1,*} Sergio Fabi,^{2,†} and Benjamin Harms^{2,‡}¹*Dipartimento di Fisica, Università di Bologna and INFN, Sezione di Bologna, via Irnerio 46, 40126 Bologna, Italy*²*Department of Physics and Astronomy, The University of Alabama, Box 870324, Tuscaloosa, Alabama 35487-0324, USA*

(Received 24 February 2009; published 26 October 2009)

In this paper, we present the results of our analysis of the growth and decay of black holes possibly produced at the Large Hadron Collider, based on our previous study of black holes in the context of the warped brane-world scenario. The black hole mass accretion and decay is obtained as a function of time, and the maximum black hole mass are obtained as a function of a critical mass parameter. The latter occurs in our expression for the luminosity and is related to the size of extra-dimensional corrections to Newton's law. Based on this analysis, we argue against the possibility of catastrophic black hole growth at the LHC.

DOI: 10.1103/PhysRevD.80.084036

PACS numbers: 04.70.Dy, 04.50.-h, 14.80.-j

I. INTRODUCTION

The hypotheses [1,2] that there exist extra spatial dimensions with length scales large enough to be probed by the Large Hadron Collider (LHC) lead to the possibility that quantum gravity can be investigated via the production and detection of microscopic black holes [3–5]. In particular, in the Randall-Sundrum (RS) brane-world of Ref. [2], our world is a four-dimensional brane (with coordinates x^μ , $\mu = 0, \dots, 3$) embedded in a five-dimensional manifold whose metric, without other sources present, is given by

$$ds^2 = e^{-\kappa|y|} g_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (1)$$

where y parametrizes the fifth dimension. In the above, κ is a deformation parameter, with units of inverse length, determined by the brane tension (for bounds on κ , see, e.g., [6] and references therein). It also relates the four-dimensional Planck mass M_p to the five-dimensional gravitational mass $M_{(5)}$ and one can, thus, have $M_{(5)} \simeq 1 \text{ TeV}/c^2$. This, in turn, allows for the existence of black holes with mass in the TeV range. In order to be phenomenologically viable, the brane must also have a thickness, which we denote by L , below which deviations from the four-dimensional Newton law occur. Current precision experiments require that $L \lesssim 44 \mu\text{m}$ [7]. In the analysis below, the parameters κ and L are assumed to be independent of one another.

Describing black holes in the presence of extra dimensions (for some reviews, see Ref. [5]) has proven a rather difficult and stimulating topic. In fact, to date, only approximate black hole metrics are known on the brane [8,9]. Since the standard model interactions are confined to the brane, and gravity is the only force which acts in the bulk as a whole, when a black hole decays, the decay products are

confined to the brane except for gravitons. In a previous publication [4], we showed that, using the metric of Ref. [8], and depending upon the choice of parameter values, black hole lifetimes can be very long. If the RS model is a valid representation of the physical world, then the black holes created on the brane can live long enough to escape the LHC and penetrate into Earth. As they travel through matter, the black holes can accrete by absorbing nuclei, electrons, or any other matter which comes within their capture radii [10]. A conjecture has been made [11] that, for the longer-lived black holes predicted by the model discussed in [4], accretion by the black holes might bring them to rest within Earth and allow them to grow to sizes at which their radiation would be catastrophic. This conjecture was criticized in Ref. [10], where it was argued that the black hole energy release conjectured in [11] was greatly overestimated. In this paper, we analyze this conjecture by solving the system of equations which describes the mass of a black hole and its momentum as functions of time for various initial conditions and various values of the parameters which occur in the model developed in Ref. [4]. Based on the results of these calculations, we comment on the possibility of catastrophic black hole growth on Earth within the RS scenario.

We shall use units with $1 = c = \hbar = M_p \ell_p$, where $M_p \simeq 2.2 \times 10^{-8} \text{ kg}$ and $\ell_p \simeq 1.6 \times 10^{-35} \text{ m}$ are the Planck mass and length related to the four-dimensional Newton constant $G_N = \ell_p/M_p$. The corresponding constants in $D = 4 + d$ dimensions are denoted by $M_{(D)}$ and $\ell_{(D)}$, respectively. In our analysis, we shall consider only the D equals five-dimensional RS scenario with $M_{(5)} \simeq M_{\text{ew}} \simeq 1 \text{ TeV}$ ($\simeq 1.8 \times 10^{-24} \text{ kg}$), the electroweak scale, and $\ell_{(5)} \simeq 2.0 \times 10^{-19} \text{ m}$.

II. BLACK HOLE DECAY

In order to determine the black hole mass as a function of time, the accretion and decay rates must be expressed in terms of the dynamical quantities of the system. The decay

*casadio@bo.infn.it

†fabi001@bama.ua.edu

‡bharms@bama.ua.edu

rate is determined by multiplying the luminosity by the horizon area. The luminosity is calculated by means of either the canonical or microcanonical ensemble [4, 12, 13], depending upon the relative sizes of the black hole mass M and the energy scale for the emitted particles.

A. Canonical ensemble

If the mass of the black hole is much larger than the typical energy scale of the emitted particles, one can safely introduce a black hole temperature $T_H = \beta_H^{-1}$ and the canonical ensemble is then appropriate to calculate the luminosity. The luminosity per unit area in D space-time dimensions in this case is given by

$$\mathcal{L}_{(D)} \simeq \sum_s \int_0^\infty \Gamma_s \frac{\omega^{D-1} d\omega}{e^{\beta_H \omega} \mp 1} = f_{(D)} T_H^D, \quad (2)$$

where Γ_s denotes grey-body factors and f is a coefficient which depends upon the number of available particle species s with energy smaller than the Hawking temperature T_H . In general, the decay rate of the black hole is then given by

$$\frac{dM}{d\tau} = -\mathcal{A}_{(D)} \mathcal{L}_{(D)}, \quad (3)$$

where τ is the black hole proper time, $\mathcal{A}_{(D)} = S_{(D)} R_H^{D-2}$ the horizon area, $S_{(D)}$ the area of a unit sphere, and R_H the horizon radius in D space-time dimensions. For example, for a $D = 4$ Schwarzschild black hole the relevant energy scale is the Planck mass M_p and the well-known result [14]

$$\frac{dM}{d\tau} \simeq f_{(4)} \frac{M_p}{\ell_p} \left(\frac{M_p}{M} \right)^2, \quad (4)$$

is obtained when the black hole mass $M \gg M_p$.

B. Microcanonical ensemble

When the black hole mass M is on the order of the energy scale for the emitted particles, the appropriate ensemble to use is the microcanonical ensemble. The occupation number density for the Hawking particles in the microcanonical ensemble is given by [12]

$$n_{(D)}(\omega) = B \sum_{l=1}^{[[M/\omega]]} \frac{\exp\{4\pi \frac{(M-l\omega)^{(D-2)/(D-3)}\}}{\exp\{4\pi \frac{M^{(D-2)/(D-3)}\}}, \quad (5)$$

where $[[X]]$ denotes the integer part of X and $B = B(\omega)$ encodes deviations from the area law [15] (in the following, we shall also assume B is constant in the range of interesting values of M). In the limit $M \rightarrow \infty$, $n_{(D)}(\omega)$ mimics the canonical ensemble (Planckian) number density and Eq. (2) is recovered.

C. Metric for a RS black hole

Since gravity propagates in the bulk, a black hole confined to a brane will produce a backreaction on the brane

itself. This effect can likely be described in the form of a tidal ‘‘charge’’ q , and the effective four-dimensional metric for such a system is thus given by [8]

$$ds^2 = -Adt^2 + A^{-1}dr^2 + r^2(d\theta^2 + \sin(\theta)^2 d\phi^2), \quad (6)$$

where

$$A = 1 - \frac{2M\ell_p}{M_p r} - \frac{qM_p^2 \ell_p^2}{M_{(5)}^2 r^2}, \quad (7)$$

with $M_{(5)} \simeq M_{\text{ew}}$ as the fundamental mass scale. A dimensional analysis (and plausibility arguments) shows that the (dimensionless) tidal charge q must depend upon the black hole mass and, at least for $M \sim M_{\text{ew}}$, it is given by

$$q \sim \left(\frac{M_p}{M_{\text{ew}}} \right)^\alpha \frac{M}{M_{\text{ew}}}. \quad (8)$$

In the following analysis, we shall assume that $\alpha = 0$ [16]. Since $M_{\text{ew}} \simeq 1 \text{ TeV}/c^2$, the tidal term in the metric dominates over the usual general relativistic $1/r$ term for black hole masses up to extremely high values. The range of values for which the tidal term dominates is determined by the critical mass parameter, M_c , which is the mass analogue of the length L mentioned in the Introduction and which we discuss below. In this range of values the outer horizon radius is given by

$$R_H \simeq \ell_p \frac{M_p}{M_{(5)}} \sqrt{\frac{M}{M_{(5)}}}, \quad (9)$$

which, for $M > M_{(5)} \simeq M_{\text{ew}}$, is larger than the usual four-dimensional expression

$$R_h = 2\ell_p \frac{M}{M_p}. \quad (10)$$

The number density used to calculate the luminosity is determined by the effective four-dimensional Euclidean action [4, 17]

$$S_{(4)}^E = \frac{M_p}{16\pi\ell_p} 4\pi R_H^2 = \frac{M_p \ell_p M}{4M_{\text{eff}}}, \quad (11)$$

where the effective energy scale M_{eff} is defined by

$$M_{\text{eff}} = \left(\frac{M_{\text{ew}}}{M_p} \right)^2 M_{\text{ew}}. \quad (12)$$

Since $M_{\text{ew}} \ll M_p$, the effective energy scale is very small compared to the electroweak energy scale near which the black holes are created. For black hole masses much larger than the effective energy scale, the microcanonical and canonical ensembles give the same expression for the luminosity.

Using Eq. (11), we showed in Ref. [4] that the decay rate can be written as

$$\frac{dM}{d\tau} = -C \left(\frac{M_p}{M_{\text{ew}}} \right)^2 \frac{M}{M_{\text{ew}}}, \quad (13)$$

where C is a numerical constant which can be obtained by equating the above decay rate to the four-dimensional Hawking decay rate (4) for $M \simeq M_c$, that is,

$$C = \frac{g_{\text{eff}} M_p}{960 \pi \ell_p} \left(\frac{M_{\text{ew}}}{M_c} \right)^3, \quad (14)$$

where M_c is again the critical mass and g_{eff} the number of degrees of freedom into which the black hole can decay.

The value of M_c depends upon the defining condition used. One possible choice is to require that the black hole horizon radius equal the four-dimensional expression (10) for sufficiently large mass, that is,

$$R_h \simeq L, \quad (15)$$

for $M \simeq M_c$. In this case, one has

$$M_c \simeq M_p \left(\frac{L}{\ell_p} \right) \equiv M_c^h. \quad (16)$$

Another possibility is to require that the effective horizon radius (9) not exceed L ,

$$R_H \simeq L, \quad (17)$$

for $M \simeq M_c$, which yields

$$M_c \simeq M_{\text{ew}} \left(\frac{L M_{\text{ew}}}{\ell_p M_p} \right)^2 \equiv M_c^H. \quad (18)$$

Since $M_{\text{ew}} \ll M_p$ and $L \lesssim 44 \mu\text{m}$, this represents a stronger constraint on the possible value of the critical mass, namely, $M_c^H \ll M_c^h$. For example, setting $L \simeq 1 \mu\text{m}$ and $M_{\text{ew}} = 1 \text{ TeV}/c^2$ gives

$$M_c^H \simeq 10^{26} \text{ TeV}/c^2 \simeq 10^2 \text{ kg}. \quad (19)$$

It is important to remark that to a given value of M_c , there correspond two different critical values of the horizon radius, namely, $R_H(M_c) \sim \sqrt{M_c}$ and $R_h(M_c) \sim M_c$. This means that the horizon radius of an accreting [18] five-dimensional black hole, necessarily starting with a mass $M < M_c$, can first be approximated by R_H in Eq. (9) with $M = M(t)$. If M reaches the critical mass M_c , the black hole becomes four dimensional and the expression of its horizon radius afterward changes from R_H to R_h given Eq. (10). The difference $R_h(M_c) - R_H(M_c)$ is always negative (meaning the horizon should shrink at the transition) and can be rather large in magnitude. For example, on using the value of $M_c^H \simeq 10^2 \text{ kg}$, $R_H(M_c^H) \simeq 10^{-6} \text{ m} \gg R_h(M_c^H) = 10^{-25} \text{ m}$. Such a huge (19 orders of magnitude) jump in the horizon radius likely signals that we need a better description of RS black holes near the dimensional transition scale (that is, for $M \simeq M_c$). However, we shall show in Sec. IV that $M \ll M_c$ at all times for black holes produced at the LHC, and the approximation outlined above is therefore adequate for the present analysis.

In any case, the radius of the five-dimensional black hole cannot exceed the current experimental limit on the size of corrections to Newton law's, that is, $R_H(M_c) \ll 44 \mu\text{m}$, which, for $M_{(5)} \simeq M_{\text{ew}} = 1 \text{ TeV}/c^2$, implies $M_c \ll 10^3 \text{ kg}$ [from Eq. (9)] [19]. In the following, we shall provide an argument which actually places a stronger bound on M_c , namely, $M_c \ll M_c^H \simeq 10^2 \text{ kg}$.

III. BLACK HOLE ACCRETION

After the black holes are created at the LHC, they can, depending on the value of M_c , live long enough in the RS scenario to escape into the atmosphere or into Earth. They can grow in mass and therefore in horizon radius by absorbing anything which comes within their capture radii. There are two basic mechanisms by which the black holes, in general, might accrete: one due to their collisions with the atomic and subatomic particles they encounter as they sweep through matter, and one due to the gravitational force the black holes exert on surrounding matter once they come to rest. The latter form is known as Bondi accretion and is appreciable only when the black holes have horizon radii greater than atomic size.

A. Capture radius

The accretion rate due to collisions is given by

$$\frac{dM}{dt} \Big|_{\text{acc}} = \pi v \rho R_{\text{eff}}^2, \quad (20)$$

where ρ is the density of the material through which the black hole is moving, and v is the relative velocity of the black hole and the surrounding matter, while t is the time of observers at rest with respect to the medium. The effective radius R_{eff} depends upon the details of the accretion mechanism and, for the problem at hand, of the extra-dimensional scenario.

For sufficiently small horizon radius R_H , the capture radius R_{eff} can be determined by simple Newtonian arguments. In particular, we can assume that it is given by the range over which the gravitational force of the black hole can overcome the electromagnetic force which binds the nucleus of an atom to the surrounding medium. For a black hole in motion through matter, accretion will then occur when the impact parameter is small enough for the gravitational field of the black hole to overcome the electromagnetic binding force.

An expression for the electromagnetic (EM) capture radius in D dimensions was obtained in Ref. [10] and reads

$$R_{\text{EM}} = \ell_p \frac{M_p}{M_{(D)}} \left(\frac{\beta_D M}{M_{(D)}} \right)^{1/(D-1)}, \quad (21)$$

where

$$\beta_D = \frac{(D-1)^{D-1} \bar{k}_D M_{(D)}^2 m}{(D-2)^{D-2} M_p^2 \ell_p^2 K}, \quad (22)$$

\bar{k}_D and K are constants and m is the mass of the absorbed nucleus. For $D = 5$ and using Eq. (9), we can rewrite the capture radius as

$$R_{\text{EM}} = C_{\text{EM}} M^{1/4} \quad (23)$$

in which C_{EM} is a constant which depends on K and m .

Of course, the above expressions are meaningful only if $R_{\text{EM}} \gg R_H$, otherwise the Newtonian argument leading to Eq. (21) fails. As we shall show in Sec. IV C, black holes created at the LHC indeed have $R_H \ll R_{\text{EM}}$. It will therefore be consistent to set $R_{\text{eff}} = R_{\text{EM}}$ in the accretion rate (20) in Sec. IV.

The above expression for R_{EM} also allows us to restrict the possible values of M_c even further. Since Eq. (21) follows from a metric of the form (6), it will hold only at distances shorter than $L \simeq 44 \mu\text{m}$, that is, for

$$R_{\text{EM}} \ll L, \quad (24)$$

which yields the bound

$$M_c \simeq \left(\frac{L}{C_{\text{EM}}} \right)^4. \quad (25)$$

This is a stronger bound than that following from Eq. (17) for $L \lesssim C_{\text{EM}}^2 M_{\text{ew}} / \ell_p M_p$. For example, with $L \simeq 1 \mu\text{m}$, $D = 5$, $M_{(5)} \simeq M_{\text{ew}}$, $\bar{k} = 2/3\pi$, $K = 224 \text{ J/m}^2$, and $m \simeq 9 \times 10^{-27} \text{ kg}$, one obtains

$$C_{\text{EM}} \simeq 1.1 \times 10^{-6} \text{ m/kg}^{1/4} \quad (26)$$

and

$$M_c \simeq 0.6 \text{ kg} \ll M_c^H. \quad (27)$$

In order to properly describe accretion, we next need to consider two relevant length scales, namely, the typical size of an atom, that is, 1 \AA , and the length $L \lesssim 44 \mu\text{m}$ associated with extra-dimensional corrections to Newton's law. Since we assume $L \gtrsim 1 \text{ \AA}$, a black hole of subatomic size will always be described as five dimensional.

B. Subatomic accretion

For $R_H \ll R_{\text{EM}} \ll 1 \text{ \AA}$, the relevant accretion rate is given by Eq. (20) with $R_{\text{eff}} \simeq R_{\text{EM}}$. When such a subatomic black hole absorbs a nucleus or electrons, it acquires the charge and spin of the absorbed particles as well as their masses. In this analysis the assumption is made that the spins of the absorbed particles are randomly directed so there is no net angular momentum of the black hole as it accretes. Also, the charge on the absorbed nuclei is assumed to be neutralized by the absorption of valence electrons, which will be drawn into the black hole by the Coulomb force until the black hole is neutralized. A further assumption is that Earth's density is uniform, although the conclusions would not be significantly altered by considering a more accurate model of Earth's interior.

For the capture radius $R_{\text{EM}} \simeq 1 \text{ \AA}$, the mass of the five-dimensional black hole is on the order of kg's [from Eqs. (23) and (26)]. If the black hole were to reach this size before traversing Earth's diameter, most likely, it would have ceased moving and begun accreting by absorbing any matter which came within its horizon radius. Further, in the RS scenario, a black hole with such a mass has an horizon radius of $R_H \simeq 10^{-6} \text{ m}$ [from Eq. (9)], which is on the order of the assumed thickness of the brane L . The black hole will therefore accrete like a four-dimensional one beyond this point.

C. Bondi accretion

If a black hole comes to rest inside Earth, it will continue to accrete according to the same basic formula as in Eq. (20). Now, however, the relative velocity is due to the motion of particles in the surrounding medium. A particle whose velocity v is less than the escape velocity at a particular distance R_{eff} from the black hole will be absorbed. The accretion rate can thus be written as

$$\left. \frac{dM}{dt} \right|_{\text{acc}} = \frac{4\pi\rho\ell_p^2}{v^3} \left(\frac{M}{M_p} \right)^2. \quad (28)$$

This type of accretion is known as Bondi accretion (see [20,21] and references therein) and holds for any massive object, e.g., a star, which is accreting any surrounding material which is free to move. As will be shown below, Bondi accretion never becomes effective for all values of the critical mass parameter and initial conditions of interest for the LHC.

IV. BLACK HOLE EVOLUTION

The growth and decay of microscopic black holes created at the LHC are described by the expressions given above for the evaporation rate and the accretion rate along with the expression for the rate of change of the black hole's momentum. The solution of this system of equations gives the time evolution of the mass and momentum [23].

A. System of equations

The time evolution of the black hole mass is obtained by summing the evaporation and accretion expressions

$$\frac{dM}{dt} = \left. \frac{dM}{dt} \right|_{\text{evap}} + \left. \frac{dM}{dt} \right|_{\text{acc}}. \quad (29)$$

The decay rate in the reference frame of Earth is obtained from Eq. (13),

$$\left. \frac{dM}{dt} \right|_{\text{evap}} \simeq -\frac{1}{\gamma} \left. \frac{dM}{d\tau} \right|_{\text{evap}}, \quad (30)$$

where γ is the relativistic factor for a point particle of mass M and three momentum of magnitude p ,

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} = \frac{\sqrt{M^2 + p^2}}{M}. \quad (31)$$

Inserting this expression for γ and that for C in terms of M_c [see Eq. (14)] into Eq. (30) gives

$$\left. \frac{dM}{dt} \right|_{\text{evap}} \simeq - \frac{g_{\text{eff}}}{960\pi\ell_p} \left(\frac{M_p}{M_c} \right)^3 \frac{M^2(t)}{\sqrt{M^2(t) + p^2(t)}}. \quad (32)$$

The accretion rate is given in general by Eq. (20). For subatomic growth and $p > 0$, the accretion rate is

$$\left. \frac{dM}{dt} \right|_{\text{acc}} = \pi\rho v(t) R_{\text{EM}}^2, \quad (33)$$

where $\rho \simeq 5.5 \times 10^3 \text{ kg/m}^3$ is Earth's mean density and R_{EM} is given by Eq. (23) expressed in terms of a time-dependent mass $M = M(t)$.

Finally, the time evolution of the momentum in the Earth frame is described by the equation

$$\begin{aligned} \left. \frac{dp}{dt} \right|_{\text{evap}} &= \frac{p(t)}{M(t)} \left. \frac{dM}{dt} \right|_{\text{evap}} \\ &\simeq - \frac{g_{\text{eff}}}{960\pi\ell_p} \left(\frac{M_p}{M_c} \right)^3 \frac{M(t)p(t)}{\sqrt{M^2(t) + p^2(t)}}. \end{aligned} \quad (34)$$

The net change of mass with respect to time (29) and Eq. (34) for the time evolution of the momentum form a system of equations which can be solved numerically to obtain $M(t)$ and $p(t)$. Note that accretion dominates only if the momentum is larger than the critical value

$$p_c = \frac{g_{\text{eff}} M_{\text{ew}}^{3/2}}{960\pi^2 \ell_p \rho C_{\text{EM}}^2} \left(\frac{M_p}{M_c} \right)^3 \left(\frac{M}{M_{\text{ew}}} \right)^{3/2}, \quad (35)$$

in which we again used $R_{\text{eff}} = R_{\text{EM}}$.

B. Time evolution of mass and momentum

The solutions to the above system of equations show a rather simple qualitative behavior: for a given value of the critical mass M_c , there are initial conditions for which the black hole never accretes ($p < p_c$ from the outset). Otherwise, it first accretes and then begins to evaporate since, for large black hole mass, the evaporation rate in the laboratory frame grows like M whereas the accretion rate decreases like $M^{-1/2}$.

A typical example of the first kind of evolution for the mass and momentum is shown in Fig. 1 for a black hole with initial mass $M(0) = 10 \text{ TeV}/c^2 \simeq 1.8 \times 10^{-23} \text{ kg}$ and momentum $p(0) = 5 \text{ TeV}/c \simeq 2.7 \times 10^{-15} \text{ kg m/sec}$, with $\bar{k} = 2/3\pi$, $K = 224 \text{ J/m}^2$, and $m = 9.0 \times 10^{-26} \text{ kg}$ [yielding $C_{\text{EM}} \simeq 1.1 \times 10^{-6} \text{ m/kg}^{1/4}$, $R_H(0) \simeq 6.1 \times 10^{-19} \text{ m}$, and $R_{\text{EM}}(0) \simeq 2.3 \times 10^{-12} \text{ m}$] and $M_c = 10 \text{ kg}$ [22,24]. With this choice of parameters, $p(0) < p_c(0)$ and the black hole just evaporate. Note, though, that the mass plot does not resemble the

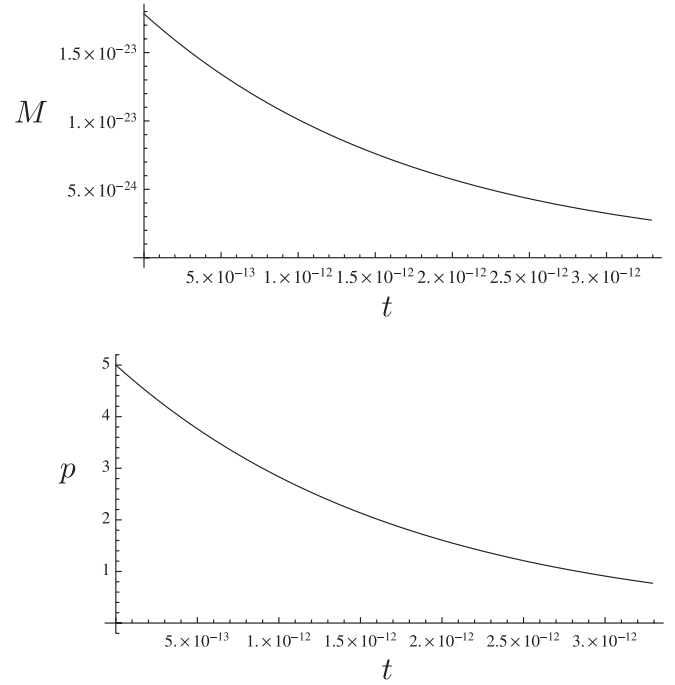


FIG. 1. Mass (in kg; upper panel) and momentum in (TeV/c; lower panel) of a black hole as a function of time (in sec) when $p(0) < p_c(0)$.

usual Hawking behavior but rather a more conventional (quasiexponential) decay, with a much longer decay time [4].

The second kind of evolution [$p(0) > p_c(0)$] is displayed in Fig. 2 with the same initial mass and momentum and $M_c = 10^3 \text{ kg}$. The maximum mass $M_{\text{max}} \simeq 1 \times 10^{-21} \text{ kg}$ is reached about $1 \times 10^{-9} \text{ sec}$ after production and corresponds to a horizon radius $R_H \simeq 5 \times 10^{-18} \text{ m}$ and capture radius $R_{\text{EM}} \simeq 7 \times 10^{-12} \text{ m}$. Subsequently, evaporation dominates and the decay time of such a black hole is about $3 \times 10^{-7} \text{ sec}$, corresponding to a traveled distance of around $7 \times 10^{-3} \text{ m}$. Note that the momentum behaves the same in both cases, but the time scales are significantly different.

C. Impact of the critical mass scale

Since the choice of the critical mass is not unique, it is of interest to study the effect of the value of the critical mass M_c on the essential quantities associated with the time evolution of a black hole produced at the LHC.

For the same initial conditions $M(0) = 10 \text{ TeV}/c^2$ and $p(0) = 5 \text{ TeV}/c$ used in Fig. 2, one can evolve the system considering different values of M_c (see Figs. 3–5). Although we showed in previous sections that a reasonable upper bound for $M_c \simeq 1 \text{ kg}$, we evolved the system in the range $1 \text{ kg} \leq M_c \leq 10^4 \text{ kg}$. One first finds that the black hole accretes [$p(0) > p_c(0)$] only when $M_c \geq 10^2 \text{ kg}$. The maximum black hole mass then very closely follows the scaling law (see Fig. 3)

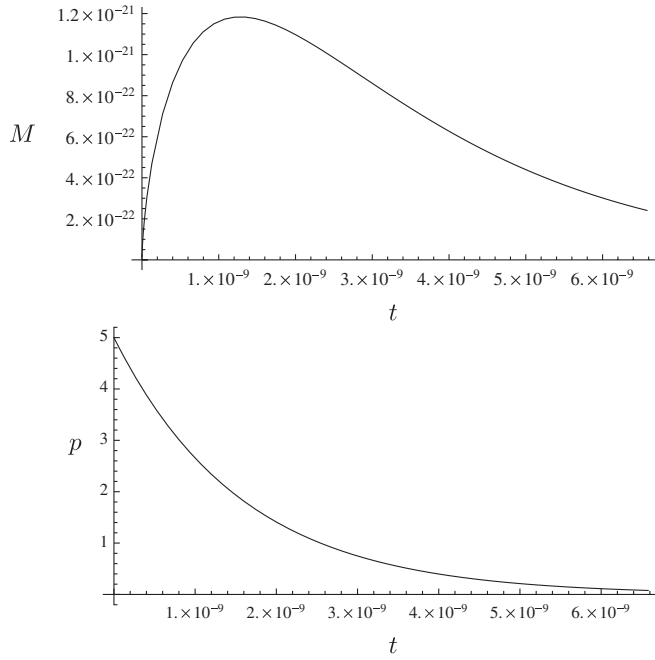


FIG. 2. Mass (in kg; upper panel) and momentum (in TeV/ c ; lower panel) of a black hole as a function of time (in sec) when $p(0) > p_c(0)$.

$$M_{\max} \propto M_c^2, \quad (36)$$

for $M_c > 10^2$ kg. Note that $M_{\max} \ll M_c$ and the black holes remain five dimensional in all cases considered. According to Eq. (23), the above scaling law implies that a similar law also holds for the maximum value of the capture radius, that is,

$$R_{\text{EM}} \propto M_c^{1/2}. \quad (37)$$

The decay time t and the time T to reach M_{\max} are shown in Fig. 4, the total traveled distance s and the distance S traveled to reach M_{\max} are shown in Fig. 5. Note that s grows more slowly with M_c for the cases in which accretion is significant.

D. Impact of the initial conditions

In order to complete our analysis, we next show the results for different values of $M(0)$ and $p(0)$. In Tables I, II, and III, M_{\max} is the maximum value of the mass attained by the black hole, R_{EM} is the value of the electromagnetic capture radius and R_H of the horizon radius when $M = M_{\max}$, S is the distance traveled by the black hole when it reaches M_{\max} , and T is the time elapsed before attaining

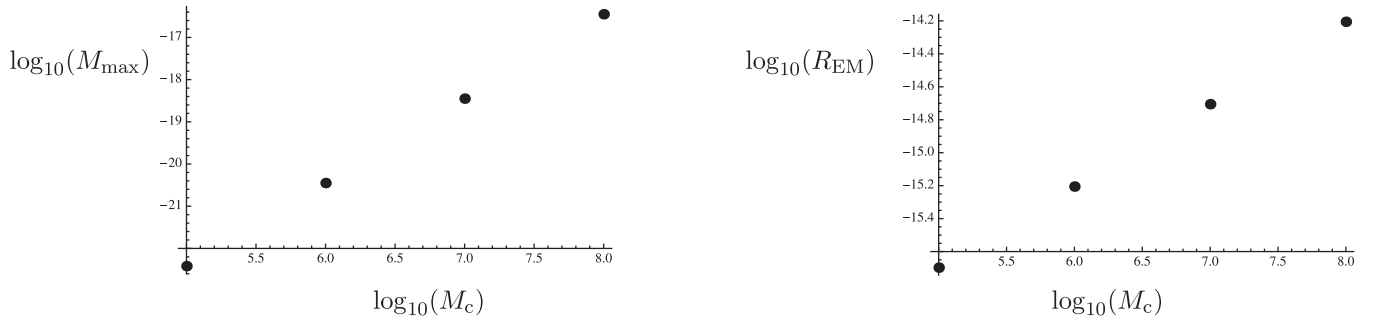


FIG. 3. Black hole maximum mass (in kg; left panel) and maximum capture radius (in m; right panel) as a function of the critical mass M_c for growing black holes (10^2 kg $\leq M_c \leq 10^4$ kg, $M(0) = 10$ TeV/ c^2 and $p(0) = 5$ TeV/ c).

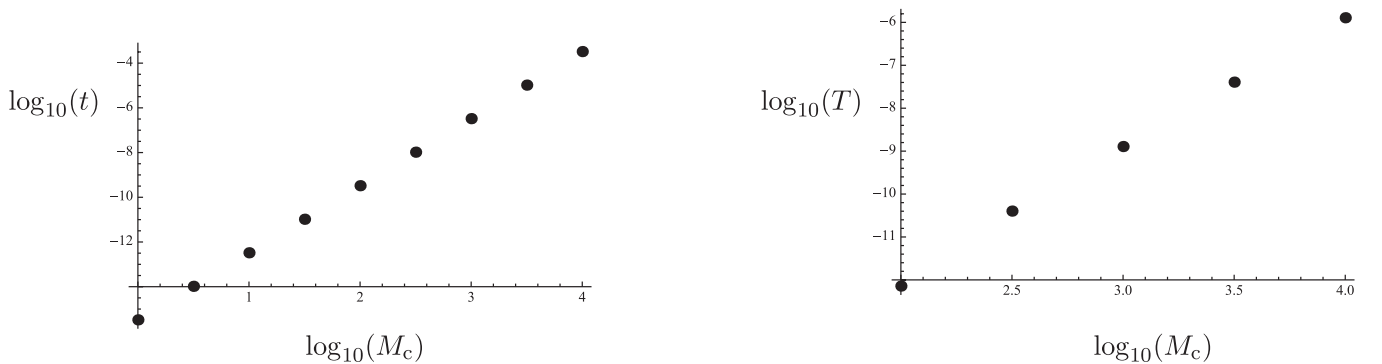


FIG. 4. Decay time (in sec; left panel, for 1 kg $\leq M_c \leq 10^4$ kg) and time to mass peak (in sec; right panel, for 10^2 kg $\leq M_c \leq 10^4$ kg) as a function of the critical mass M_c ($M(0) = 10$ TeV/ c^2 and $p(0) = 5$ TeV/ c).

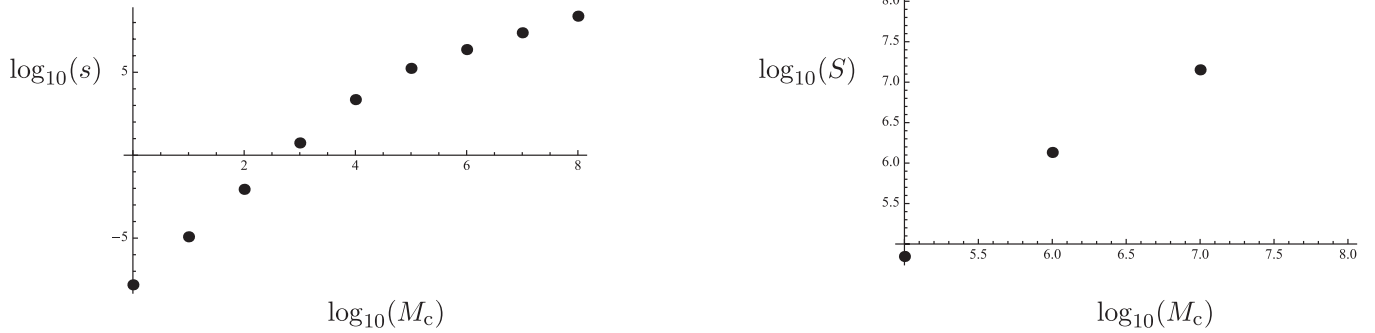


FIG. 5. Maximum traveled distance (in m; left panel, for $1 \text{ kg} \leq M_c \leq 10^4 \text{ kg}$) and traveled distance to the mass peak (in m; right panel, for $10^2 \text{ kg} \leq M_c \leq 10^4 \text{ kg}$) as a function of the critical mass M_c ($M(0) = 10 \text{ TeV}/c^2$ and $p(0) = 5 \text{ TeV}/c$).

M_{max} . The computed quantities are displayed with two digits for convenience, although it is just the order of magnitude that should be considered.

Comparing the entries between Tables I and II shows that, within the range of values that could be of interest for the LHC, these quantities do not depend significantly on the initial mass. On the other hand, lowering the initial momentum increases the value of the critical mass above which the black holes can grow [since $p_c \propto M_c^{-3}$ from Eq. (35)]. Further, comparing the entries between Table II and III shows that the maximum black hole mass actually decreases for decreasing $p(0)$ at a given critical mass.

Since the maximum value of the capture radius R_{EM} stays below 1 \AA , even for the physically unreasonable case of $M_c \approx 10^4 \text{ kg}$, Bondi accretion never becomes effective. Finally, it is worth noting that, on using the values for the black hole initial velocity and the maximum values of R_H and R_{EM} given in this section, one always obtains $R_H \ll R_{\text{EM}}$ for all the allowed initial conditions.

V. CONCLUSIONS

We have studied the evolution in time of microscopic black holes that could be produced at the LHC based on our previous paper [4] and the description of brane-world black holes given in Ref. [8]. With respect to Ref. [4], accretion

TABLE I. Data in this table are for initial conditions: $M(0) = 1 \text{ TeV}/c^2$ ($= 1.8 \times 10^{-24} \text{ kg}$) and $p(0) = 5 \text{ TeV}/c$.

M_c (kg)	M_{max} (kg)	R_{EM} (m)	R_H (m)	S (m)	T (sec)
10^2	1.2×10^{-23}	2.1×10^{-12}	5.1×10^{-19}	3.1×10^{-4}	1.7×10^{-12}
10^3	1.2×10^{-21}	6.6×10^{-12}	5.1×10^{-18}	4.2×10^{-3}	1.3×10^{-9}
10^4	1.2×10^{-19}	2.1×10^{-11}	5.1×10^{-17}	4.3×10^{-2}	1.3×10^{-6}

TABLE II. Data in this table are for the initial conditions: $M(0) = 10 \text{ TeV}/c^2$ ($= 1.8 \times 10^{-23} \text{ kg}$) and $p(0) = 5 \text{ TeV}/c$.

M_c (kg)	M_{max} (kg)	R_{EM} (m)	R_H (m)	S (m)	T (sec)
10^2	1.8×10^{-23}	2.3×10^{-12}	6.3×10^{-19}	3.7×10^{-5}	3.0×10^{-13}
10^3	1.2×10^{-21}	6.5×10^{-12}	5.0×10^{-18}	3.9×10^{-3}	1.3×10^{-9}
10^4	1.2×10^{-19}	2.1×10^{-11}	5.1×10^{-17}	4.3×10^{-2}	1.3×10^{-6}

TABLE III. Data in this table are for the initial conditions: $M(0) = 10 \text{ TeV}/c^2$ ($= 1.8 \times 10^{-23} \text{ kg}$) and $p(0) = 1 \text{ TeV}/c$.

M_c (kg)	M_{max} (kg)	R_{EM} (m)	R_H (m)	S (m)	T (sec)
3×10^2	4.3×10^{-23}	2.9×10^{-12}	9.7×10^{-19}	4.1×10^{-4}	3.6×10^{-11}
10^3	4.1×10^{-22}	5.0×10^{-12}	3.0×10^{-18}	2.1×10^{-3}	1.3×10^{-9}
10^4	4.0×10^{-20}	1.6×10^{-11}	3.0×10^{-17}	2.5×10^{-2}	1.3×10^{-6}

has been now included in the analysis and all of the parameters have been chosen so as to cover a fairly comprehensive range of possible outcomes. In particular, our model contains a critical mass scale, M_c , which is related to the transition from the five-dimensional behavior, effectively described by the metric (6), to the usual four-dimensional description of a black hole.

As shown in the previous section, in particular, in Tables I, II, and III, the maximum black hole mass never reaches catastrophic size before leaving Earth. The black hole mass remains at microscopic values for a wide range of acceptable initial conditions and for a wide range of critical masses, M_c . In order for the black holes created at the LHC to grow at all, the critical mass should be $M_c \geq 10^2$ kg. This value is already larger than the maximum compatible with experimental tests of Newton's law (and we further relaxed it to $M_c = 10^4$ kg in our analysis). For smaller values of M_c , the black holes cannot accrete fast enough to overcome the decay rate. Furthermore, the larger M_c is taken to be, the longer a black hole takes to reach its maximum value.

The data in Table III show that, within the warped brane-world scenario, the maximum masses reached by black holes produced at the LHC are about 4 orders of magnitude

greater than that of a nucleon. If these black holes or their remnants come to rest in Earth, they will begin to Bondi accrete. Assuming the extremal conditions that the accreted matter is mostly free iron nuclei at low energy ($T \approx 300$ K), the rate of Bondi accretion is [see Eq. (28)] 1.9×10^{-64} kg/s. For a black hole at rest to accrete, even the mass of a single nucleon would thus require a time interval many orders of magnitude larger than the age of the Universe.

We conclude that, for the RS scenario and black holes described by the metric (6), the growth of black holes to catastrophic size is not possible. Nonetheless, it remains true that the expected decay times are significantly longer than is typically predicted by other models, as was first shown in Ref. [4].

ACKNOWLEDGMENTS

R. C. and B. H. would like to thank S. Giddings and R. Plaga for many discussions on this topic. R. C. would also like to thank R. Maartens and the members of the ICG, Portsmouth, and O. Micu and the members of the theory group at Dortmund University.

-
- [1] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, *Phys. Lett. B* **429**, 263 (1998); *Phys. Rev. D* **59**, 086004 (1999); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, *Phys. Lett. B* **436**, 257 (1998).
 - [2] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 4690 (1999).
 - [3] P. C. Argyres, S. Dimopoulos, and J. March-Russell, *Phys. Lett. B* **441**, 96 (1998); S. Dimopoulos and G. L. Landsberg, *Phys. Rev. Lett.* **87**, 161602 (2001); S. B. Giddings and S. D. Thomas, *Phys. Rev. D* **65**, 056010 (2002); C. M. Harris, P. Richardson, and B. R. Webber, *J. High Energy Phys.* 08 (2003) 033; G. L. Alberghi, R. Casadio, and A. Tronconi, *J. Phys. G* **34**, 767 (2007); M. Cavaglia, R. Godang, L. Cremaldi, and D. Summers, *Comput. Phys. Commun.* **177**, 506 (2007); D. C. Dai, G. Starkman, D. Stojkovic, C. Issever, E. Rizvi, and J. Tseng, *Phys. Rev. D* **77**, 076007 (2008).
 - [4] R. Casadio and B. Harms, *Int. J. Mod. Phys. A* **17**, 4635 (2002).
 - [5] M. Cavaglia, *Int. J. Mod. Phys. A* **18**, 1843 (2003); P. Kanti, *Int. J. Mod. Phys. A* **19**, 4899 (2004);
 - [6] C. G. Boehmer, T. Harko, and F. S. N. Lobo, *Classical Quantum Gravity* **25**, 045015 (2008).
 - [7] D. J. Kapner, T. S. Cook, E. G. Adelberger, J. H. Gundlach, B. R. Heckel, C. D. Hoyle, and H. E. Swanson, *Phys. Rev. Lett.* **98**, 021101 (2007).
 - [8] N. Dadhich, R. Maartens, P. Papadopoulos, and V. Rezanian, *Phys. Lett. B* **487**, 1 (2000).
 - [9] R. Gregory, *Lect. Notes Phys.* **769**, 259 (2009); R. Whisker, arXiv:0810.1534; R. Emparan, G. T. Horowitz, and R. C. Myers, *J. High Energy Phys.* 01 (2000) 007; T. Shiromizu and M. Shibata, *Phys. Rev. D* **62**, 127502 (2000); A. Chamblin, H. S. Reall, H. a. Shinkai, and T. Shiromizu, *Phys. Rev. D* **63**, 064015 (2001); R. Casadio, A. Fabbri, and L. Mazzacurati, *Phys. Rev. D* **65**, 084040 (2002); P. Kanti and K. Tamvakis, *Phys. Rev. D* **65**, 084010 (2002); R. Casadio and L. Mazzacurati, *Mod. Phys. Lett. A* **18**, 651 (2003); H. Kudoh, T. Tanaka, and T. Nakamura, *Phys. Rev. D* **68**, 024035 (2003); S. Creek, R. Gregory, P. Kanti, and B. Mistry, *Classical Quantum Gravity* **23**, 6633 (2006).
 - [10] S. B. Giddings and M. L. Mangano, *Phys. Rev. D* **78**, 035009 (2008).
 - [11] R. Plaga, arXiv:0808.1415.
 - [12] R. Casadio and B. Harms, *Phys. Rev. D* **64**, 024016 (2001).
 - [13] R. Casadio, B. Harms, and Y. Leblanc, *Phys. Rev. D* **58**, 044014 (1998).
 - [14] S. W. Hawking, *Nature (London)* **248**, 30 (1974); *Commun. Math. Phys.* **43**, 199 (1975); **46**, 206(E) (1976).
 - [15] B. Harms and Y. Leblanc, *Phys. Rev. D* **46**, 2334 (1992); **47**, 2438 (1993); *Europhys. Lett.* **27**, 557 (1994); *Ann. Phys. (N.Y.)* **242**, 265 (1995); **244**, 262 (1995); **244**, 272 (1995); P. H. Cox, B. Harms, and Y. Leblanc, *Europhys. Lett.* **26**, 321 (1994).
 - [16] This choice was first made in Ref. [4]. Other cases will be considered elsewhere.
 - [17] L. A. Gergely, N. Pidokrajt, and S. Winitzki, arXiv:

0811.1548.

- [18] The opposite picture would arise for an evaporating black hole which is four dimensional (with initial mass $M > M_c$) and reaches the transition stage from above.
- [19] Larger values of M_c would be allowed for $M_{(5)} \gg M_{\text{ew}}$, but then black holes could not be produced at the LHC.
- [20] H. Bondi and F. Hoyle, *Mon. Not. R. Astron. Soc.* **104**, 273 (1944).
- [21] H. Bondi, *Mon. Not. R. Astron. Soc.* **112**, 195 (1952).
- [22] These values correspond to a black hole energy of about 11 TeV in the laboratory and were chosen considering the LHC total collision energy of 14 TeV and the fact that a black hole cannot be the only product of a collision.
- [23] S. B. Giddings and M. L. Mangano, arXiv:0808.4087; B. Koch, M. Bleicher, and H. Stoecker, *Phys. Lett. B* **672**, 71 (2009).
- [24] The actual decay of a black hole is a discrete process which causes jumps in both M and p [3]. The evolution equations. (29) and (34) as well as the continuous solutions displayed in the figures, represent valid approximations only for M sufficiently larger than the mass of the decay products. The actual decay time is also affected by these considerations.