PHYSICAL REVIEW D **80,** 084033 (2009)

de Sitter spacetimes with torsion in the model of de Sitter gauge theory of gravity

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(Received 31 March 2009; published 23 October 2009)

In the model of the de Sitter gauge theory of gravity, the empty homogenous and isotropic spacetimes with constant curvature scalar and nonvanishing homogenous and isotropic torsion must have de Sitter metrics. The static de Sitter spacetime with static, $O(3)$-symmetric, vector torsion is the only spherically symmetric, vacuum solution with the metric of the form $g_{\mu\nu} = \mathrm{diag}(A^2(r), -B^2(r), -r^2, -r^2\sin^2\theta)$. The expressions of the torsion for different de Sitter spacetimes are presented. They are different from one to another. The properties of different de Sitter spacetimes with torsion are also studied.

DOI: 10.1103/PhysRevD.80.084033 PACS numbers: 04.50.−h, 04.20.Jb, 04.90.+e

I. INTRODUCTION

The astronomical observations show that our Universe is probably an asymptotically de Sitter (dS) one [1,2]. It raises the interest on dS gauge theories of gravity. There is a model of dS gravity,[1] which was first proposed in the 1970's [3,4]. The dS gravity can be stimulated from dS invariant special relativity [5–7] and the principle of localization—the full symmetry of the special relativity as well as the laws of dynamics are both localized [8–10]—and realized in terms of the dS connection on a kind of totally umbilical submanifold (under the dS-Lorentz gauge) and Yang-Mills–type of action [3,8,10].

It has been shown that all vacuum solutions of Einstein field equations with a cosmological constant are the vacuum solutions of the set of field equations without torsion [8,9]. In particular, Schwarzschild-dS and Kerr-dS metrics are two solutions. On the other hand, it can also been shown that the vacuum solutions of the set of field equations without torsion must be the vacuum solutions of Einstein field equations with the same cosmological constant [11]. Therefore, one may expect that the dS gravity may pass all solar-system-scale observations and experimental tests for general relativity (GR).[2] It has also been shown [10] that the dS gravity may explain the accelerating expansion and supply a natural transit from decelerating expansion to accelerating expansion without the help of the introduction of matter fields in addition to dust.

The present paper aims at finding the dS spacetimes with torsion in the dS gravity and studying their properties. The $k = 0$ de Sitter spacetime with constant torsion [13,14] and the static de Sitter spacetime with spherical torsion, which satisfy the double duality ansatz [15,16], have been presented for some gauge theories of gravity. Other de Sitter solutions with nonvanishing torsion are also given for other theories [17,18]. But, all of the theories are different from the dS gravity. We present the de Sitter spacetimes with homogenous and isotropic torsion for spatial curvature $k = 0, \pm 1$ and static de Sitter spacetime with static, $O(3)$-symmetric, vector torsion. The formers are the only vacuum solutions in the dS gravity for the empty, homogenous, isotropic, constant-curvature-scalar universe. The latter is the only spherically symmetric, vacuum solution in the dS gravity for a large class of spacetimes.

The paper is arranged as follows. We first review the model of the dS gravity in the next section. In the third section, we study the dS solutions with homogeneous and isotropic torsion. In Sec. IV, we solve the $O(3)$-symmetric, static, vacuum field equations. We shall give some concluding remarks in the final section.

II. DE SITTER GAUGE THEORY OF GRAVITY

The dS gauge theory of gravity is established based on the following consideration. The nongravitational theory is de Sitter invariant special relativity. The theory of gravity should follow the principle of localization, which says that the *full symmetry*, as well as the *laws of dynamics*, are both localized, and the gravitational action takes Yang-Mills–type.

A model of dS gauge theory of gravity has been constructed [3,4,8–10] in terms of the de Sitter connection in the dS-Lorentz frame, which reads[3]

$$(\mathcal{B}^{AB}{}_{\mu}) = \begin{pmatrix} B^{ab}{}_{\mu} & R^{-1}e^a_{\mu} \\ -R^{-1}e^b_{\mu} & 0 \end{pmatrix} \in \mathfrak{so}(1,4), \tag{2.1}$$

where $\mathcal{B}^{AB}{}_{\mu} = \eta^{BC}\mathcal{B}^{A}{}_{C\mu}$, in which η^{AB} is the inverse of $\eta_{AB} = \mathrm{diag}(\eta_{ab}, -1) = \mathrm{diag}(1, -1, -1, -1, -1)$, and $e^a{}_{\mu}$ is the tetrad field. Its curvature is then

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[1]Hereafter, the model of the dS gauge theory of gravity is called the dS gravity for short in this paper.

[2]The problem of matching the exterior solution with an interior solution has been studied in [12].

[3]The same connection with different gravitational dynamics has also been studied (See, e.g. [19–27])

$$(\mathcal{F}^{AB}{}_{\mu\nu}) = \begin{pmatrix} F^{ab}{}_{\mu\nu} + R^{-2}e^{ab}{}_{\mu\nu} & R^{-1}T^{a}{}_{\mu\nu} \\ -R^{-1}T^{b}{}_{\mu\nu} & 0 \end{pmatrix} \in \mathfrak{so}(1,4), \tag{2.2}$$

where $e^a{}_{b\mu\nu} = e^a_\mu e_{b\nu} - e^a_\nu e_{b\mu}$, $e_{a\mu} = \eta_{ab}e^b_\mu$, $F^{ab}{}_{\mu\nu}$, and $T^a{}_{\mu\nu}$ are the curvature and torsion of the Lorentz connection:

$$\begin{aligned} \mathbf{\Omega}^a &= d\boldsymbol{\vartheta}^a + \boldsymbol{\omega}^a{}_b \wedge \boldsymbol{\vartheta}^b = \tfrac{1}{2}T^a{}_{\mu\nu}dx^\mu \wedge dx^\nu, \\ T^a{}_{\mu\nu} &= \partial_\mu e^a_\nu - \partial_\nu e^a_\mu + B^a{}_{c\mu}e^c_\nu - B^a{}_{c\nu}e^c_\mu, \end{aligned} \tag{2.3}$$

$$\begin{aligned} \mathbf{\Omega}^a{}_b &= d\boldsymbol{\omega}^a{}_b + \boldsymbol{\omega}^a{}_c \wedge \boldsymbol{\omega}^c{}_b = \tfrac{1}{2}F^a{}_{b\mu\nu}dx^\mu \wedge dx^\nu, \\ F^a{}_{b\mu\nu} &= \partial_\mu B^a{}_{b\nu} - \partial_\nu B^a{}_{b\mu} + B^a{}_{c\mu}B^c{}_{b\nu} - B^a{}_{c\nu}B^c{}_{b\mu}, \end{aligned} \tag{2.4}$$

where $\boldsymbol{\vartheta}^a = e^a_\mu dx^\mu$ is the coframe, and $\boldsymbol{\omega}^a{}_b = B^a{}_{b\mu}dx^\mu$ is the connection 1 form.

The action for the model of the de Sitter gauge theory of gravity with sources takes the form of

$$S_{\rm T} = S_{\rm GYM} + S_{\mathbf{M}}, \tag{2.5}$$

where

$$\begin{aligned} S_{\rm GYM} &= \frac{1}{4g^2}\int_{\mathcal{M}} d^4x e(\mathcal{F}^{AB}{}_{\mu\nu}\mathcal{F}_{BA}{}^{\mu\nu}) \\ &= -\int_{\mathcal{M}} d^4x e\bigg[\frac{1}{4g^2}F^{ab}{}_{\mu\nu}F_{ab}{}^{\mu\nu} - \chi(F - 2\Lambda) \\ &\quad - \frac{\chi}{2}T^a{}_{\mu\nu}T_a{}^{\mu\nu}\bigg] \end{aligned} \tag{2.6}$$

is the gravitational Yang-Mills action, and S_M is the action of sources with minimum coupling. In Eq. (2.6), $g = (R\sqrt{\chi})^{-1} \sim 10^{-61}$ is the dimensionless gravitational coupling constant, $e = \det(e^a_\mu)$, $\Lambda = 3/R^2$, $\chi = 1/(16\pi G)$, $g^{-2} = 3\chi\Lambda^{-1}$, G is the Newtonian gravitational coupling constant, and $F = -\frac{1}{2}F^{ab}{}_{\mu\nu}e_{ab}{}^{\mu\nu}$ is the scalar curvature of the Cartan connection ($c = 1$, $\hbar = 1$).

The field equations can be given via the variational principle with respect to $e^a{}_\mu$, $B^{ab}{}_\mu$,

$$\begin{aligned} \mathcal{E}_a{}^\mu &= T_a{}^{\mu\nu}{}_{\|\nu} - F^\mu{}_a + \tfrac{1}{2}Fe^\mu_a - \Lambda e^\mu_a \\ &\quad - 8\pi G(T_{{\rm M}a}{}^\mu + T_{{\rm G}a}{}^\mu) = 0, \end{aligned} \tag{2.7}$$

$$\mathcal{Y}_{ab}{}^\mu = F_{ab}{}^{\mu\nu}{}_{\|\nu} - R^{-2}(16\pi G S_{{\rm M}ab}{}^\mu + S_{{\rm G}ab}{}^\mu) = 0. \tag{2.8}$$

$\|$ represents the covariant derivative defined by the Christoffel symbol

$$\begin{Bmatrix} \mu \\ \nu\kappa \end{Bmatrix}$$

and Lorentz connection $B^a{}_{b\mu}$, $F_a{}^\mu = -F_{ab}{}^{\mu\nu}e^b_\nu$, $F = F_a{}^\mu e^a_\mu$;

$$T_{{\rm M}a}{}^\mu = -\frac{1}{e}\frac{\delta S_{\rm M}}{\delta e^a_\mu}, \qquad S_{{\rm M}ab}{}^\mu = \frac{1}{2\sqrt{-g}}\frac{\delta S_{\rm M}}{\delta B^{ab}{}_\mu} \tag{2.9}$$

are the tetrad forms of the stress-energy tensor and spin current for matter, respectively.

$$T_{{\rm G}a}{}^\mu = g^{-2}T_{{\rm F}a}{}^\mu + 2\chi T_{{\rm T}a}{}^\mu \tag{2.10}$$

is the tetrad form of the stress-energy tensor of the gravitational field, which can be split into the curvature part

$$T_{{\rm F}a}{}^\mu = e^\kappa_a {\rm Tr}(F^{\mu\lambda}F_{\kappa\lambda}) - \tfrac{1}{4}e^\mu_a {\rm Tr}(F^{\lambda\sigma}F_{\lambda\sigma}) \tag{2.11}$$

and the torsion part

$$T_{{\rm T}a}{}^\mu = e^\kappa_a T_b{}^{\mu\lambda}T^b{}_{\kappa\lambda} - \tfrac{1}{4}e^\mu_a T_b{}^{\lambda\sigma}T^b{}_{\lambda\sigma}. \tag{2.12}$$

Similarly, the gravitational spin current

$$S_{{\rm G}ab}{}^\mu = S_{{\rm F}ab}{}^\mu + 2S_{{\rm T}ab}{}^\mu \tag{2.13}$$

can also be divided into two parts:

$$S_{{\rm F}ab}{}^\mu = -e_{ab}{}^{\mu\nu}{}_{\|\nu} = Y^\mu{}_{\lambda\nu}e_{ab}{}^{\lambda\nu} + Y^\nu{}_{\lambda\nu}e_{ab}{}^{\mu\lambda}, \tag{2.14}$$

$$S_{{\rm T}ab}{}^\mu = T_{[a}{}^{\mu\lambda}e_{b]\lambda}, \tag{2.15}$$

where

$$Y^\lambda{}_{\mu\nu} = \tfrac{1}{2}(T^\lambda{}_{\nu\mu} + T_\mu{}^\lambda{}_\nu + T_\nu{}^\lambda{}_\mu) \tag{2.16}$$

is the contortion.

III. DS SOLUTIONS WITH HOMOGENEOUS AND ISOTROPIC TORSION

First of all, there is no dS solution with SO(1,4) symmetric torsion in the model of the dS gauge theory of gravity.

For the homogeneous and isotropic universe, the metric of spacetime takes the Friedmann-Robertson-Walker form

$$ds^2 = dt^2 - a^2(t)\bigg[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\bigg], \tag{3.1}$$

where $k = 0, \pm 1$, and there are 6 Killing vector fields $\boldsymbol{\xi}_{(I)}$ ($I = 1\cdots 6$) for each k. To keep the homogeneity and isotropy of the Universe, the torsion is also required to be homogeneous and isotropic. In other words, the torsion should satisfy

$$\mathcal{L}_{\boldsymbol{\xi}_{(I)}}\boldsymbol{T}^a = 0, \qquad I = 1\cdots 6. \tag{3.2}$$

Furthermore, we require that the torsion be invariant under space inversion. Then, for any k, the torsion always takes the form [14]

$$\begin{aligned} \boldsymbol{T}^0 &= 0, \\ \boldsymbol{T}^1 &= T_+(t)\boldsymbol{\vartheta}^0 \wedge \boldsymbol{\vartheta},^1 \\ \boldsymbol{T}^2 &= T_+(t)\boldsymbol{\vartheta}^0 \wedge \boldsymbol{\vartheta},^2 \\ \boldsymbol{T}^3 &= T_+(t)\boldsymbol{\vartheta}^0 \wedge \boldsymbol{\vartheta}^3, \end{aligned} \tag{3.3}$$

where $\boldsymbol{\vartheta}^0 = dt$, $\boldsymbol{\vartheta}^1 = \frac{a(t)}{\sqrt{1-kr^2}}dr$, $\boldsymbol{\vartheta}^2 = a(t)rd\theta$, $\boldsymbol{\vartheta}^3 = a(t)r\sin\theta d\phi$.

The reduced vacuum Einstein-like equations and Yang-like equations are

$$\begin{aligned} &-\frac{\ddot{a}^2}{a^2} - \left(\dot{T}_+ + 2\frac{\dot{a}}{a}T_+ - 2\frac{\ddot{a}}{a}\right)\dot{T}_+ + T_+^4 - 4\frac{\dot{a}}{a}T_+^3 \\ &+ \left(5\frac{\dot{a}^2}{a^2} + 2\frac{k}{a^2} - \frac{3}{R^2}\right)T_+^2 + 2\frac{\dot{a}}{a}\left(\frac{\ddot{a}}{a} - 2\frac{\dot{a}^2}{a^2} - 2\frac{k}{a^2} + \frac{3}{R^2}\right)T_+ \\ &+ \frac{\dot{a}^2}{a^2}\left(\frac{\dot{a}^2}{a^2} + 2\frac{k}{a^2} - \frac{2}{R^2}\right) + \frac{k^2}{a^4} - \frac{2}{R^2}\frac{k}{a^2} + \frac{2}{R^4} = 0, \end{aligned} \tag{3.4}$$

$$\begin{aligned} &\frac{\ddot{a}^2}{a^2} + \left(\dot{T}_+ + 2\frac{\dot{a}}{a}T_+ - 2\frac{\ddot{a}}{a} + \frac{6}{R^2}\right)\dot{T}_+ - T_+^4 + 4\frac{\dot{a}}{a}T_+^3 \\ &- \left(5\frac{\dot{a}^2}{a^2} + 2\frac{k}{a^2} + \frac{3}{R^2}\right)T_+^2 - 2\frac{\dot{a}}{a}\left(\frac{\ddot{a}}{a} - 2\frac{\dot{a}^2}{a^2} - 2\frac{k}{a^2} - \frac{6}{R^2}\right)T_+ \\ &- \frac{4}{R^2}\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\left(\frac{\dot{a}^2}{a^2} + 2\frac{k}{a^2} + \frac{2}{R^2}\right) - \frac{k^2}{a^4} - \frac{2}{R^2}\frac{k}{a^2} + \frac{6}{R^4} = 0, \end{aligned} \tag{3.5}$$

$$\begin{aligned} &\ddot{T}_+ + 3\frac{\dot{a}}{a}\dot{T}_+ - \left(2T_+^2 - 6\frac{\dot{a}}{a}T_+ - \frac{\ddot{a}}{a} + 5\frac{\dot{a}^2}{a^2} + 2\frac{k}{a^2} - \frac{3}{R^2}\right)T_+ \\ &- \frac{\dddot{a}}{a} - \frac{\dot{a}\,\ddot{a}}{a^2} + 2\frac{\dot{a}^3}{a^3} + 2\frac{\dot{a}}{a}\frac{k}{a^2} = 0. \end{aligned} \tag{3.6}$$

The trace of Einstein-like equations give rise to

$$\frac{\ddot{a}}{a} = -\left(\frac{\dot{a}}{a}\right)^2 - \frac{k}{a^2} + \frac{2}{R^2} + \frac{3}{2}\left(\dot{T}_+ + 3\frac{\dot{a}}{a}T_+ - T_+{}^2\right). \tag{3.7}$$

A direct calculation shows the curvature scalar in this case is

$$F = 6\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 - \frac{3\dot{a}}{a}T_+ - \dot{T}_+ + T_+^2 + \frac{k}{a^2}\right]. \tag{3.8}$$

In terms of the expression, Eq. (3.6) can be written in a very simple form,

$$\dot{F} + 2T_+(F - 9/R^2) = 0. \tag{3.9}$$

In particular, its constant-curvature-scalar solutions require either $T_+ = 0$ or

$$F - 9/R^2 = 0. \tag{3.10}$$

The former is the torsion-free case, which has been discussed in [9]. For the latter case, the combinations of Eqs. (3.7) and (3.8) with (3.10) give rise to

$$\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{1}{2R^2}, \tag{3.11}$$

$$\dot{T}_+ + \frac{3\dot{a}}{a}T_+ - T_+^2 + \frac{1}{R^2} = 0. \tag{3.12}$$

With the help of Eqs. (3.11) and (3.12), Eq. (3.4) reduces to

$$\dot{a}^2 + k = a^2/(4R^2). \tag{3.13}$$

Equations (3.11), (3.12), and (3.13) constitute a (over-determinant) system of equations for a and T_+.

One can immediately solve Eq. (3.13),

$$a(t) = \begin{cases} H^{-1}\cosh(Ht) & \text{for } k = 1 \\ H^{-1}e^{Ht} & \text{for } k = 0 \\ H^{-1}\sinh(Ht) & \text{for } k = -1, \end{cases} \tag{3.14}$$

where $H = \pm 1/2R$. They are de Sitter solutions and satisfy Eq. (3.11), too. The remaining task is to solve Eq. (3.12) for different de Sitter spacetimes, which can be written in the dimensionless form,

$$y' + 3\frac{a'}{a}y - y^2 + 4 = 0, \tag{3.15}$$

where $y = T_+/H$, $x = Ht$, and a prime represents the derivative with respect to x.

For $k = 0$ de Sitter spacetime, Eq. (3.15) reads

$$y' + 3y - y^2 + 4 = 0. \tag{3.16}$$

It has the general solutions

$$y = \frac{4 + Ce^{5x}}{1 - Ce^{5x}} \quad \text{or} \quad T_+ = \frac{H(4 + Ce^{5Ht})}{1 - Ce^{5Ht}}, \tag{3.17}$$

where C is a constant of integration. In particular, when $C = \infty$, $y = -1$, $T_+ = \mp\frac{1}{2R}$. When $C = 0$, $y = 4$, $T_+ = \pm\frac{2}{R}$. Both are constant solutions. They are also the asymptotic states at $x \to \pm\infty$ for a generic C. When $x \to -\frac{1}{5}\ln C$, y and thus T_+ tends to ∞. Figure 1 plots the dimensionless torsion y versus the dimensionless time x for $C = 1$.

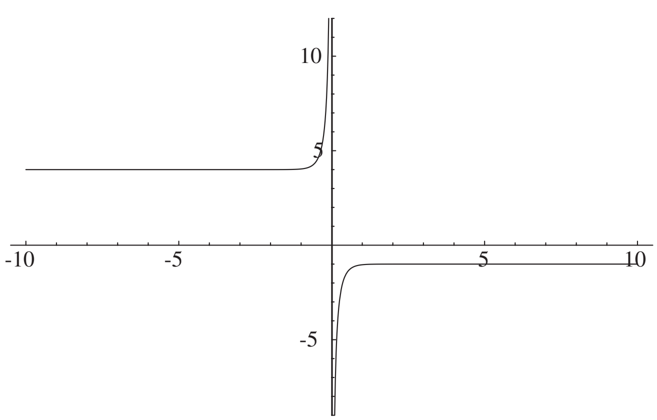

FIG. 1. The evolution of dimensionless torsion y for $k = 0$ de Sitter spacetime and $C = 1$.

The stress-energy tensors and the spin currents of the gravitational field are

$$T_{\mathrm{F}}{}^{a}{}_{b} = 12H^2(T_+^2 - 2HT_+ - 2H^2)\mathrm{diag}(3,-1,-1,-1) = 12H^4\frac{(6+18Ce^{5Ht}+C^2e^{10Ht})}{(1-Ce^{5Ht})^2}\mathrm{diag}(3,-1,-1,-1), \tag{3.18}$$

$$T_{\mathrm{T}}{}^{a}{}_{b} = \frac{T_+^2}{2}\mathrm{diag}(3,-1,-1,-1) = \frac{H^2}{2}\frac{(4+Ce^{5Ht})^2}{(1-Ce^{5Ht})^2}\mathrm{diag}(3,-1,-1,-1), \tag{3.19}$$

$$S_{\mathrm{F}ab}{}^{c} = 4S_{\mathrm{T}ab}{}^{c} = -2T_+(\delta_a^c\delta_b^0 - \delta_a^0\delta_b^c) = -2H\frac{4+Ce^{5Ht}}{1-Ce^{5Ht}}(\delta_a^c\delta_b^0 - \delta_a^0\delta_b^c), \tag{3.20}$$

respectively. In particular, they reduce to

$$T_{\mathrm{F}}{}^{a}{}_{b} = \frac{\Lambda^2}{12}\mathrm{diag}(3,-1,-1,-1), \tag{3.21}$$

$$T_{\mathrm{T}}{}^{a}{}_{b} = \frac{\Lambda}{24}\mathrm{diag}(3,-1,-1,-1), \tag{3.22}$$

$$S_{\mathrm{F}ab}{}^{c} = 4S_{\mathrm{T}ab}{}^{c} = \pm\sqrt{\Lambda/3}(\delta_a^c\delta_b^0 - \delta_a^0\delta_b^c) \tag{3.23}$$

for the case $C=\infty$, $T_+ = -H = \mp\frac{1}{2}\sqrt{\Lambda/3}$, and

$$T_{\mathrm{F}}{}^{a}{}_{b} = \frac{\Lambda^2}{2}\mathrm{diag}(3,-1,-1,-1), \tag{3.24}$$

$$T_{\mathrm{T}}{}^{a}{}_{b} = \frac{2\Lambda}{3}\mathrm{diag}(3,-1,-1,-1), \tag{3.25}$$

$$S_{\mathrm{F}ab}{}^{c} = 4S_{\mathrm{T}ab}{}^{c} = \mp 4\sqrt{\Lambda/3}(\delta_a^c\delta_b^0 - \delta_a^0\delta_b^c) \tag{3.26}$$

for the case $C=0$, $T_+ = 4H = \pm 2\sqrt{\Lambda/3}$, which are all finite everywhere. For other C, the stress-energy tensors and the spin currents will be divergent at $t = -\frac{1}{5H}\ln C$.

For the $k=+1$ de Sitter spacetime, Eq. (3.15) reads

$$y'(x) + 3\tanh(x)y - y^2 = -4. \tag{3.27}$$

Figure 2 presents the finite, nonoscillatory, numerical solutions $y(x)$ with $T_+ \sim o(H)$.

The stress-energy tensors of gravitational fields

$$T_{\mathrm{F}}{}^{a}{}_{b} = 12H^2[T_+^2 - 2H\tanh(Ht)T_+ - 2H^2]\mathrm{diag}(3,-1,-1,-1), \tag{3.28}$$

$$T_{\mathrm{T}}{}^{a}{}_{b} = \frac{T_+^2}{2}\mathrm{diag}(3,-1,-1,-1), \tag{3.29}$$

and the spin currents of gravitational fields

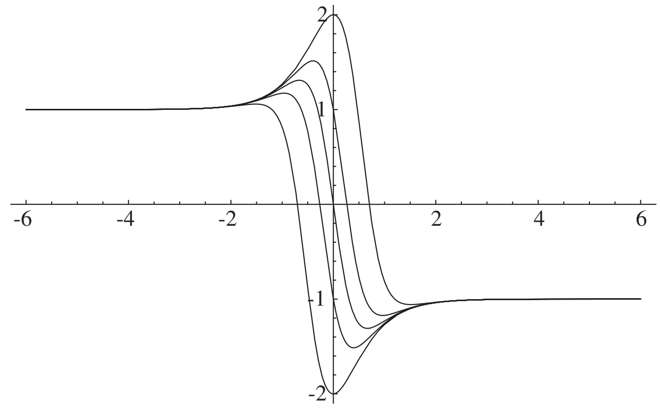

FIG. 2. Finite, nonoscillatory, numerical solution $y(x)$ with $T_+ \sim o(H)$.

$$S_{\mathrm{F}ab}{}^{c} = 4S_{\mathrm{T}ab}{}^{c} = -2T_+(\delta_a^c\delta_b^0 - \delta_a^0\delta_b^c) \tag{3.30}$$

are all finite everywhere.

For the $k=-1$ de Sitter spacetime, Eq. (3.15) reduces to

$$y'(x) + 3\coth(x)y - y^2 = -4, \tag{3.31}$$

which has the asymptotical solution $y=-1$ as $x\to\infty$ and the asymptotical solution

$$y \to 2\mathrm{csch}^2 x\left\{\coth x + \left[\log\left(\tanh\frac{x}{2}\right) - C\right]\sinh x\right\}^{-1} \tag{3.32}$$

when $x\to 0$. Therefore, $T_+(t)$ should be initially huge and decay to $T_+ = -H$ as $t\to\infty$. Figure 3 sketches out the numerical solutions for Eq. (3.31).

In this case, the stress-energy tensors of gravitational fields

$$T_{\mathrm{F}}{}^{a}{}_{b} = 12H^2[T_+^2 - 2H\coth(Ht)T_+ - 2H^2] \times \mathrm{diag}(3,-1,-1,-1), \tag{3.33}$$

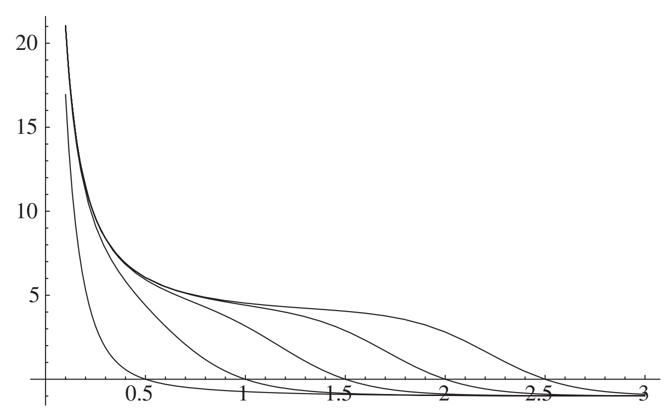

FIG. 3. The numerical solution for Eq. (3.31).

$$T_{\mathrm{T}}{}^{a}{}_{b} = \frac{T_{+}^{2}}{2}\mathrm{diag}(3, -1, -1, -1), \tag{3.34}$$

and the spin currents of gravitational fields

$$S_{\mathrm{F}ab}{}^{c} = 4S_{\mathrm{T}ab}{}^{c} = -2T_{+}(\delta_{a}^{c}\delta_{b}^{0} - \delta_{a}^{0}\delta_{b}^{c}) \tag{3.35}$$

are all initially divergent.

IV. DS SOLUTIONS WITH STATIC, SPHERICALLY SYMMETRIC TORSION

To find the static dS solutions, we suppose that the metric takes the form

$$ds^2 = A^2(r)dt^2 - B^2(r)dr^2 - r^2 d\Omega^2. \tag{4.1}$$

On the same reason as the cosmological case above, the torsion should be static and $O(3)$ invariant, namely,

$$\mathcal{L}_{\boldsymbol{\xi}_{(I)}}\boldsymbol{T}^{a} = 0, \qquad I = 1\cdots 4, \tag{4.2}$$

where $\boldsymbol{\xi}_{(I)}$ $(I = 1\cdots 4)$ are the timelike Killing vector fields and three rotation Killing vector fields, and $\boldsymbol{T}^{a}$ is invariant under the space inversion. Generally, the static spherically symmetric torsion can be taken as the forms in the papers [28]. Furthermore, the torsion can be irreducibly decomposed as trace-vector, axial-vector, and tensor pieces under the Lorentz group. For static and $O(3)$-symmetric torsion, the axial-vector piece automatically vanishes. For simplicity, we consider the trace-vector piece

$$\begin{aligned}
\boldsymbol{T}^{0} &= T_{0}(r)\boldsymbol{\vartheta}^{0}\wedge\boldsymbol{\vartheta},^{1}\\
\boldsymbol{T}^{1} &= T_{1}(r)\boldsymbol{\vartheta}^{0}\wedge\boldsymbol{\vartheta},^{1}\\
\boldsymbol{T}^{2} &= T_{1}(r)\boldsymbol{\vartheta}^{0}\wedge\boldsymbol{\vartheta}^{2} - T_{0}(r)\boldsymbol{\vartheta}^{1}\wedge\boldsymbol{\vartheta},^{2}\\
\boldsymbol{T}^{3} &= T_{1}(r)\boldsymbol{\vartheta}^{0}\wedge\boldsymbol{\vartheta}^{3} - T_{0}(r)\boldsymbol{\vartheta}^{1}\wedge\boldsymbol{\vartheta}^{3},
\end{aligned} \tag{4.3}$$

where $\boldsymbol{\vartheta}^{0} = A(r)dt$, $\boldsymbol{\vartheta}^{1} = B(r)dr$, $\boldsymbol{\vartheta}^{2} = rd\theta$, and $\boldsymbol{\vartheta}^{3} = r\sin\theta d\phi$.

By substituting the coframe and torsion forms into Eqs. (2.7) and (2.8), we can get 9 independent equations, 5 for the Einstein-like equation and 4 for the Yang-like equation, which are listed in the Appendix for completion. Below, we can simplify the field equations to get the de Sitter solutions.

First, $\mathcal{E}_{1}{}^{\mu}e_{\mu}^{0} - \mathcal{E}_{0}{}^{\mu}e_{\mu}^{1} = 0$ [i.e. (A2) − (A3)] gives rise to

$$A(r)T_{1}(r) = \bar{C}, \tag{4.4}$$

where $\bar{C}$ is an arbitrary constant with the dimension of the inverse of length.

Then with the help of Eq. (4.4), $\mathcal{Y}_{01}{}^{\mu}e_{\mu}^{1} + \mathcal{Y}_{20}{}^{\mu}e_{\mu}^{2} = 0$ [i.e. (A7) + (A8)] leads to

$$T_{1}'' = T_{1}'\left(\frac{2T_{1}'}{T_{1}} + \frac{B'}{B} + \frac{1}{r}\right) + \frac{1}{r}T_{1}\left(\frac{B'}{B} + \frac{1 - B^{2}}{r}\right). \tag{4.5}$$

Additionally, one can obtain from $\mathcal{Y}_{01}{}^{\mu}e_{\mu}^{1} = 0$ [i.e. Eq. (A7)],

$$T_{0}' = \frac{1}{rB}\left(\frac{T_{1}'}{T_{1}} + \frac{B'}{B}\right) + \frac{T_{0}}{T_{1}}T_{1}' + B\left(T_{1}^{2} - T_{0}^{2} - \frac{\Lambda}{2}\right) - \frac{2}{r}T_{0}. \tag{4.6}$$

The trace of Einstein-like equations, namely $\mathcal{E}_{a}{}^{\mu}e_{\mu}^{a} = 0$, then gives rise to

$$\frac{T_{1}'}{T_{1}} = -\frac{B'}{B} + \frac{\Lambda}{6}rB^{2}. \tag{4.7}$$

Then, the system of Eqs. (A1)–(A9) reduce to the differential equation

$$2r\frac{B'}{B} - \frac{\Lambda}{4}r^{2}B^{2} + B^{2} - 1 = 0 \tag{4.8}$$

and the algebraic equation

$$\Lambda r^{2}B^{2} - 12B^{2} + 12 = 0, \tag{4.9}$$

plus Eqs. (4.4), (4.5), (4.6), and (4.7). From Eq. (4.9), one immediately finds

$$B^{2} = \frac{1}{1 - H^{2}r^{2}}, \tag{4.10}$$

where $H^{2} = \Lambda/12$. It also solves Eq. (4.8) obviously. The integration of Eq. (4.7) shows

$$T_{1}^{2} = \frac{C_{1}^{2}H^{2}}{1 - H^{2}r^{2}}, \tag{4.11}$$

where C_{1} is a dimensionless, integration constant. Equation (4.11) is consistent with Eq. (4.5) as it should be. Further,

$$A^{2} = \frac{\bar{C}^{2}}{H^{2}C_{1}^{2}}(1 - H^{2}r^{2}). \tag{4.12}$$

Without loss of generality, one can choose $\bar{C}^{2} = H^{2}C_{1}^{2} = H^{2}C^{2}$ by the rescale of the time t. Then,

$$A^{2} = (1 - H^{2}r^{2}). \tag{4.13}$$

Equation (4.6) becomes

$$\begin{aligned}
T_{0}' + \frac{2 - 3H^{2}r^{2}}{r(1 - H^{2}r^{2})}T_{0} + \frac{1}{(1 - H^{2}r^{2})^{1/2}}T_{0}^{2}\\
= -\frac{4H^{2}}{(1 - H^{2}r^{2})^{1/2}} + \frac{C^{2}H^{2}}{(1 - H^{2}r^{2})^{3/2}}.
\end{aligned} \tag{4.14}$$

It can be written as

$$\frac{dy}{d\zeta} + \frac{2 - \tan^{2}\zeta}{\tan\zeta}y + y^{2} = C^{2}\sec^{2}\zeta - 4, \tag{4.15}$$

where $y = T_{0}/H$, $r = H^{-1}\sin\zeta$. The general solution of the equation is a function of hypergeometric functions with an integration of constant C_{0}. The reality of both y and C_{0} requires C_{0} to be zero. Thus, the solution takes the form

$$y(\zeta) = \left[4 - \frac{C^2-16}{7}\sec^2\zeta \frac{F(\frac{6-C}{2}, \frac{6+C}{2}, \frac{9}{2}, \sec^2\zeta)}{F(\frac{4-C}{2}, \frac{4+C}{2}, \frac{7}{2}, \sec^2\zeta)}\right]\tan\zeta. \tag{4.16}$$

As a special case, $C = 4$, the solution reduces to

$$y(\zeta) = 4\tan\zeta \quad \text{or} \quad T_0 = \frac{4H^2 r}{\sqrt{1-H^2r^2}}, \tag{4.17}$$

which has the asymptotic behavior

$$T_0 \rightarrow \frac{CH}{\sqrt{1-H^2r^2}} = T_1 \tag{4.18}$$

as $\zeta \rightarrow \pi/2$ or $r \rightarrow H^{-1}$. In fact, the general solution (4.16) also shares the same asymptotic property. The behavior of $T_0(r)$ is shown in Fig. 4, which is not sensitive to $T_0(0)$.

In brief, the static dS space

$$ds^2 = (1-H^2r^2)dt^2 - \frac{dr^2}{1-H^2r^2} - r^2 d\Omega^2$$

with the static torsion T_0 and T_1, given by Eqs. (4.16) and (4.11), respectively, is the only solution of the vacuum field equations for the Ansatz (4.1) and (4.3).

The nonzero tetrad components of the stress-energy tensor of gravitational fields are

$$\begin{aligned}
T_{\rm F}{}^0{}_0 &= \Lambda\left[T_0^2 + T_1^2 + 2H^2 - \frac{2H^2 r}{\sqrt{1-H^2r^2}}T_0\right],\\
T_{\rm F}{}^1{}_0 &= -T_{\rm F}{}^0{}_1 = 2T_1\Lambda\left[T_0 - \frac{H^2 r}{\sqrt{1-H^2r^2}}\right],\\
T_{\rm F}{}^1{}_1 &= \Lambda\left[T_1^2 - 3T_0^2 - 6H^2 - \frac{2}{r}\frac{2-3H^2r^2}{\sqrt{1-H^2r^2}}T_0\right],\\
T_{\rm F}{}^2{}_2 &= T_{\rm F}{}^3{}_3 = \Lambda\left[T_0^2 - T_1^2 + 2H^2 + \frac{2}{r}T_0\sqrt{1-H^2r^2}\right],
\end{aligned} \tag{4.19}$$

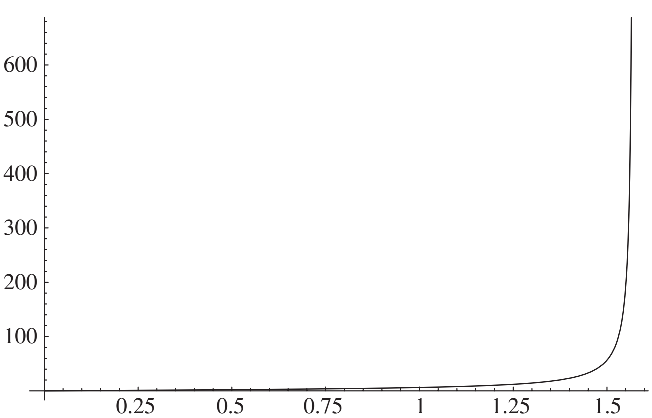

FIG. 4. y thus T_0 is divergent at $\zeta = \pi/2$ or $r = H^{-1}$.

$$\begin{aligned}
T_{\rm T}{}^0{}_0 &= \frac{1}{2}(T_0^2 + 3T_1^2),\\
T_{\rm T}{}^1{}_0 &= -T_{\rm T}{}^0{}_1 = 2T_0T_1,\\
T_{\rm T}{}^1{}_1 &= -\frac{1}{2}(3T_0^2 + T_1^2),\\
T_{\rm T}{}^2{}_2 &= T_{\rm T}{}^3{}_3 = \frac{1}{2}(T_0^2 - T_1^2).
\end{aligned} \tag{4.20}$$

The spin currents of gravitational fields are

$$\begin{aligned}
S_{{\rm F}ab}{}^c &= 4S_{{\rm T}ab}{}^c\\
&= -2T_1(\delta_a^c\delta_b^0 - \delta_a^0\delta_b^c) + 2T_0(\delta_a^c\delta_b^1 - \delta_a^1\delta_b^c).
\end{aligned} \tag{4.21}$$

The straightforward calculation shows that

$$\begin{aligned}
F_{a\mu}F^{a\mu} = 4\Bigg(&3T_0^4 + T_1^4 - 4T_0^2T_1^2 + \frac{4}{r}\frac{2-3H^2r^2}{\sqrt{1-H^2r^2}}T_0^3\\
&- \frac{4}{r}\frac{1-2H^2r^2}{\sqrt{1-H^2r^2}}T_0T_1^2 + \frac{2}{r^2}\frac{3-2H^2r^2}{1-H^2r^2}T_0^2 - 2H^2\\
&\times \frac{2-H^2r^2}{1-H^2r^2}T_1^2 + \frac{8H^2}{r}\frac{2-3H^2r^2}{\sqrt{1-H^2r^2}}T_0 + 93H^4\Bigg),
\end{aligned} \tag{4.22}$$

$$\begin{aligned}
F_{a\mu}F_{b\nu}e^a_\nu e^b_\mu = 4\Bigg\{&\frac{H^4r^4}{(1-H^2r^2)^2}(\sqrt{1-H^2r^2} - 3)T_0^4 + 3T_0^4\\
&+ T_1^4 - 4T_0^2T_1^2 + \frac{4}{r}\sqrt{1-H^2r^2}\\
&\times[2T_0^2 - (1-2H^2r^2)T_1^2]T_0 + \frac{4}{r\sqrt{1-H^2r^2}}\\
&\times[2H^2(2-3H^2r^2) - H^2r^2T_0^2]T_0\\
&+ \frac{2}{1-H^2r^2}\bigg[\frac{3-2H^2r^2}{r^2}T_0^2\\
&- H^2(2-H^2r^2)T_1^2\bigg] - 93\frac{H^4}{1-H^2r^2}\Bigg\}.
\end{aligned} \tag{4.23}$$

Obviously, they are divergent when $r \rightarrow 1/H$.

V. CONCLUDING REMARKS

The vacuum equations of the dS gravity have been solved, and several important dS solutions have been obtained. The $k = 0$ dS spacetime with constant torsion and $k = +1$ dS spacetime have no singularity in whole spacetimes. In contrast, the $k = 0$ dS spacetime with varying torsion, $k = -1$ dS spacetime, and the static dS spacetime have singularity.

The most important feature of our solutions is that the different de Sitter metrics describe different geometries because of the existence of the nonzero torsions and their dependence on different coordinates in different manners. Although the metric admits 10 Killing vector fields for each case, the torsion does not possess as high of symmetry. For different metrics, the torsion has different symmetry. Even for the 3 homogeneous and isotropic cases, the symmetries are different. When $k = 0, \pm 1$, the torsion is ISO(3), SO(4), SO(3,1)symmetric, respectively. Therefore, they are not equivalent to each other.

Another important feature of our solutions is that for the static de Sitter spacetime with torsion, the horizon is no longer a coordinate singularity. Since the torsion, gravitational stress-energy tensor, and invariant curvature scalars become divergent at singularities, these singularities are intrinsic ones in the dS gravity. In GR, the horizon singularity is a kind of coordinate singularity which can be removed by coordinate transformation. Here, these singularities at horizon are not coordinate singularities and cannot be removed, so the Riemann-Cartan spacetime can not be extended to pass through the horizon. Some properties about horizon in GR, like Hawking radiation and horizon entropy, should be reconsidered in this kind of theory of gravity.

ACKNOWLEDGMENTS

This work is supported by NSFC under Grant No. 10775140 and Knowledge Innovation Funds of CAS (KJCX3-SYW-S03).

APPENDIX: EXPLICIT FIELD EQUATIONS FOR STATIC, SPHERICALLY SYMMETRIC ANSATZ

The explicit expressions of the field equations for Ansatz (4.1) and (4.3) are as follows. The independent Einstein-like equations $(T_a{}^{\mu\nu}{}_{\|\nu} e^b_\mu + \cdots)$ are $(ab{:})$ 00, 10, 01, 11, and 22 component equations. They are

$$\begin{aligned}
&-3\frac{A''}{A}\left[\frac{1}{2}\frac{A''}{A}-\frac{A'}{A}\frac{B'}{B}+B\left(\frac{A'}{A}T_0+T_0'\right)\right]+3\frac{(A')^2}{A^2}\left(B^2T_1^2-\frac{1}{2}\frac{(B')^2}{B^2}-\frac{1}{r^2}\right)+3\frac{(A')^2}{A^2}\left(B'-\frac{3}{2}B^2T_0-\frac{2}{r}B\right)T_0\\
&+3\frac{A'}{A}\left(B'T_0'-\frac{2}{r^2}BT_0\right)-3\frac{A'}{A}B^2T_0\left(T_0'+2BT_0^2-2BT_1^2+\frac{4}{r}T_0\right)-\frac{1}{r}\left(6B'T_0'-\frac{3}{r}\frac{(B')^2}{B^2}+2\Lambda BB'\right)+\frac{6}{r}B'\left(BT_1^2-\frac{1}{r}T_0\right)\\
&+\frac{3}{2}B^2T_0'\left(T_0'-4BT_1^2+\frac{4}{r}T_0+2\Lambda B\right)-3B^2T_1'(T_1'-2BT_0T_1)+\frac{3}{2}B^4\left(3T_1^4-T_0^4-2T_0^2T_1^2+\Lambda T_0^2-3\Lambda T_1^2-\frac{2}{r^2}T_0^2\right.\\
&\left.+\frac{2}{r^2}T_1^2+\frac{1}{r^4}-\frac{2\Lambda}{3r^2}+\frac{2}{3}\Lambda^2\right)-\frac{6}{r}B^3T_0\left(2T_1^2+\frac{1}{r^2}-\Lambda\right)+\frac{1}{r^2}B^2\left(9T_0^2-3T_1^2-\frac{3}{r^2}+\Lambda\right)+\frac{3}{2r^3}\left(4BT_0+\frac{1}{r}\right)=0,
\end{aligned}\tag{A1}$$

$$\begin{aligned}
&\frac{A'}{A}\left(B(T_1T_0'-T_0T_1')-\frac{1}{r}T_1'+B^2T_1({T_0}^2-{T_1}^2)+\frac{2}{r}BT_1T_0-\frac{1}{r}\frac{B'}{B}T_1\right)\\
&\quad-\frac{1}{2}\Lambda B^2T_1'+BT_0\left(B(T_1T_0'-T_0T_1')-\frac{1}{r}T_1'+B^2T_1\left({T_0}^2-{T_1}^2+\frac{1}{2}\Lambda\right)+\frac{2}{r}BT_0T_1-\frac{1}{r}\frac{B'}{B}T_1\right)=0,
\end{aligned}\tag{A2}$$

$$\left(BT_0+\frac{A'}{A}\right)\left(B(T_1T_0'-T_0T_1')-\frac{1}{r}T_1'+B^2T_1\left(T_0^2-T_1^2+\frac{\Lambda}{2}\right)+\frac{2}{r}BT_0T_1-\frac{1}{r}\frac{B'}{B}T_1\right)=0,\tag{A3}$$

$$\begin{aligned}
&-\frac{1}{2}\frac{A''^2}{A^2}+\frac{A''}{A}\left(\frac{A'}{A}\left(\frac{B'}{B}-BT_0\right)-BT_0'\right)-\frac{A'^2}{A^2}\left(\frac{1}{2}\frac{B'^2}{B^2}-T_0B'-B^2\left(\frac{1}{2}T_0^2-T_1^2\right)-\frac{2}{r}BT_0-\frac{1}{r^2}\right)\\
&+B\frac{A'}{A}\left(\left(\frac{B'}{B}T_0'-BT_0T_0'+\frac{2}{r^2}T_0\right)+2B^2T_0\left(T_0^2-T_1^2+\frac{\Lambda}{2}\right)+B\left(\frac{4}{r}T_0^2+\frac{2\Lambda}{3r}\right)\right)-\frac{1}{r^2}\frac{B'^2}{B^2}+\frac{2}{r}B'\left(T_0'-BT_1^2+\frac{1}{r}T_0\right)\\
&+B^2\left(T_1'^2-\frac{3}{2}T_0'^2+2BT_1(T_1T_0'-T_0T_1')-\frac{2}{r}T_0T_0'\right)+B^4\left(\frac{3}{2}T_0^4-\frac{1}{2}T_1^4-T_0^2T_1^2-\frac{1}{r^2}(T_0^2-T_1^2)+\frac{\Lambda}{2}(3T_0^2-T_1^2)\right.\\
&\left.+\frac{1}{2r^4}-\frac{\Lambda}{3r^2}+\frac{1}{3}\Lambda^2\right)+\frac{2}{r}B^3T_0\left(2T_0^2+\Lambda-\frac{1}{r^2}\right)+\frac{1}{r^2}B^2\left(3T_0^2-T_1^2+\frac{\Lambda}{3}-\frac{1}{r^2}\right)+\frac{1}{r^3}\left(2BT_0+\frac{1}{2r}\right)=0,
\end{aligned}\tag{A4}$$

$$\begin{aligned}
&\frac{1}{2}\frac{A''^2}{A^2}+\frac{A''}{A}\left(\frac{A'}{A}\left(BT_0-\frac{B'}{B}\right)+BT_0'+\frac{\Lambda}{3}B^2\right)+\frac{A'^2}{A^2}\left(\frac{1}{2}\frac{B'^2}{B^2}-T_0B'+\frac{1}{2}B^2T_0^2\right)-B\frac{A'}{A}\left(\left(\frac{B'}{B}T_0'-\frac{\Lambda}{3}B'-BT_0T_0'\right)\right.\\
&\quad\left.-\Lambda B^2T_0-\frac{\Lambda}{3r}B\right)-\frac{\Lambda}{3}\frac{1}{r}BB'+B^2T_0'\left(\frac{1}{2}T_0'+\Lambda B\right)+B^4\left(-\frac{3}{2}(T_0^4+T_1^4)+T_0^2T_1^2+\frac{1}{r^2}(T_0^2-T_1^2)+\frac{\Lambda}{2}(T_0^2-3T_1^2)\right.\\
&\quad\left.+\frac{1}{2r^4}+\frac{1}{3}\Lambda^2\right)+\frac{2}{r}B^3T_0\left(T_1^2-T_0^2+\frac{\Lambda}{2}+\frac{1}{r^2}\right)-\frac{1}{r^2}B^2\left(3T_0^2-T_1^2-\frac{1}{r^2}\right)-\frac{1}{r^3}\left(2BT_0+\frac{1}{2r}\right)=0.
\end{aligned}\tag{A5}$$

The independent Yang-like equations ($F_{ab}{}^{\mu\nu}{}_{\|\nu} e^c_\mu + \cdots$) are ($abc$:) 010, 011, 202, and 122 component equations.

$$\frac{A'''}{A} - \frac{A''}{A}\left(\frac{A'}{A} + 3\frac{B'}{B} - BT_0 - \frac{2}{r}\right) - \frac{B''}{B}\frac{A'}{A} + \frac{A'^2}{A^2}\left(\frac{B'}{B} - BT_0\right) + \frac{A'}{A}\frac{B'}{B}\left(3\frac{B'}{B} - BT_0 - \frac{2}{r}\right)$$
$$+ \frac{A'}{A}\left(BT_0' - 2B^2T_0^2 + 2B^2T_1^2 - \frac{2}{r}BT_0 - \frac{2}{r^2}\right) + BT_0'' - B'T_0' + \frac{2}{r}BT_0' + 2B^3T_0\left(T_1^2 - T_0^2 - \frac{\Lambda}{2}\right) - \frac{2}{r}BT_0\left(2BT_0 + \frac{1}{r}\right) = 0, \tag{A6}$$

$$2B^2(T_1'T_0 - T_0'T_1) + \frac{2}{r}(BT_1' + B'T_1) + 2B^3T_1\left(T_1^2 - T_0^2 - \frac{1}{2}\Lambda\right) - \frac{4}{r}B^2T_0T_1 = 0, \tag{A7}$$

$$B\left(\frac{A'^2}{A^2}T_1 - \frac{A'}{A}T_1' + \frac{B'}{B}T_1' - T_1'' + \frac{1}{r^2}T_1(1 - B^2)\right) + B^2T_0\left(3\frac{A'}{A}T_1 + T_1'\right) + 2B^2T_0'T_1 - \frac{1}{r}(B'T_1 + BT_1')$$
$$- 2B^3T_1\left(T_1^2 - T_0^2 - \frac{1}{2}\Lambda\right) + \frac{4}{r}B^2T_0T_1 = 0, \tag{A8}$$

$$\frac{A'^2}{A^2}\left(BT_0 + \frac{1}{r}\right) - \frac{A'}{A}\left(BT_0' - 2B^2T_0^2 - B^2T_1^2 + \frac{1}{r}BT_0 - \frac{1}{r}\frac{B'}{B}\right) + \frac{1}{r}\left(\frac{B''}{B} - 3\frac{B'^2}{B^2}\right) - \left(BT_0'' - B'T_0' - \frac{1}{r}B'T_0 + \frac{2}{r}BT_0'\right)$$
$$+ 3B^2T_1T_1' + B^3T_0\left(2T_0^2 - 2T_1^2 - \frac{1}{r^2} + \Lambda\right) + \frac{1}{r}B^2\left(4T_0^2 - \frac{1}{r^2}\right) + \frac{3}{r^2}BT_0 + \frac{1}{r^3} = 0. \tag{A9}$$

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