New perspective on galaxy clustering as a cosmological probe: General relativistic effects

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We present a general relativistic description of galaxy clustering in a Friedmann-Lemaître-Robertson-Walker universe. The observed redshift and position of galaxies are affected by the matter fluctuations and the gravity waves between the source galaxies and the observer, and the volume element constructed by using the observables differs from the physical volume occupied by the observed galaxies. Therefore, the observed galaxy fluctuation field contains additional contributions arising from the distortion in observable quantities and these include tensor contributions as well as numerous scalar contributions. We generalize the linear bias approximation to relate the observed galaxy fluctuation field to the underlying matter distribution in a gauge-invariant way. Our full formalism is essential for the consistency of theoretical predictions. As our first application, we compute the angular auto correlation of large-scale structure and its cross correlation with CMB temperature anisotropies. We comment on the possibility of detecting primordial gravity waves using galaxy clustering and discuss further applications of our formalism.

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I. INTRODUCTION

Galaxies are known to trace the underlying matter distribution on large scales and galaxy redshift measurements can therefore provide crucial information about the time evolution of large-scale structure of the universe. Over the past decade rapid progress has been made in this field, following the advent of large galaxy redshift surveys such as the Sloan Digital Sky Survey (SDSS; [1]) and the Two Degree Field Galaxy Redshift Survey (2dFGRS; [2]), and much higher precision measurements with large survey area have opened a new horizon for the role of galaxy clustering as a cosmological probe (see, e.g., [3,4]).

However, a critical question naturally arises: is the Newtonian description of galaxy clustering sufficiently accurate on large scales, close to the horizon scale at high redshift? General relativity provides a natural framework for cosmology, and the general relativistic description is essential to understand the formation of CMB anisotropies, since the horizon size at the recombination epoch is of order three degrees on the sky (see, e.g., [5]). Therefore, one would naturally expect that a similar relativistic treatment of galaxy clustering needs to be considered when using galaxy clustering as a cosmological probe.

Galaxies are measured by observing photons emitted from them and the photon path is distorted by the matter fluctuations and the gravity waves between the source galaxy and the observer. The volume element constructed using the observed redshift and observed angle is different from the real physical volume that the observed galaxies occupy, and the observed flux and redshift of the source galaxies are also different from their intrinsic properties. Therefore, the observed galaxy number density is affected by the same perturbations given the total number of observed galaxies, and it contains additional contributions from the distortion in the observable quantities, compared to the standard description that galaxies simply trace the underlying matter distribution $\delta_g = b\delta_m$.

Furthermore, perturbations are gauge-dependent quantities and hence they are not directly observable. For example, the matter fluctuation δ_m computed in the synchronous gauge is different from the matter fluctuation δ_m computed in the conformal Newtonian gauge, which diverges on large scales. The observable quantities such as observed galaxy clustering should be independent of a choice of the gauge condition, and this implies that the standard description is incomplete. Related to this problem is the linear bias approximation. A scale-independent galaxy bias factor b assumed in one gauge appears as a scale-dependent galaxy bias factor $b(k)$ in another gauge. Galaxy formation is a local process and its relation to the underlying matter density should be well defined and gauge invariant.

These issues are naturally resolved when we construct theoretical predictions in terms of observable quantities. Here we provide a fully general relativistic description of galaxy clustering in a general Friedmann-Lemaître-Robertson-Walker (FLRW) universe, and as our first application we compute the cross correlation of CMB temperature anisotropies with large-scale structure. Tracers of large-scale structure contain additional contributions from relativistic effects and these effects on the cross correlation

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progressively become significant at low angular multipoles at high redshift, since the relativistic effects are significant at the horizon scale and the horizon size decreases with redshift. For a photometric quasar sample from the SDSS, we find that the predicted signals are larger than the standard method would predict at low angular multipoles and its deviation is larger than the estimated cosmic variance limit.

The organization of this paper is as follows. In Sec. II we describe our notation for a general FLRW metric and solve the geodesic equation for photons. In Sec. III we discuss the fluctuation in luminosity distance that affects the observed flux of source galaxies, and we present our main results on the general relativistic description of galaxy clustering in Sec. IV. In Sec. V we compute the angular correlation of large-scale structure and its cross correlation with CMB anisotropies with the main emphasis on the systematic errors. Finally, we discuss the implication of our new results and conclude with a discussion of further applications in Sec. VI.

II. GEODESIC EQUATION

We present our notation for the background metric in an inhomogeneous universe and solve the geodesic equation for photons emitted from galaxies to derive the relation between the source galaxies and the observer.

A. FLRW metric

We assume that the background universe is well described by the FLRW metric with a constant spatial curvature,

$$ds^2 = g_{ab}dx^a dx^b = -dt^2 + a^2(t)\bar{g}_{\alpha\beta}dx^\alpha dx^\beta, \tag{1}$$

where $a(t)$ is the scale factor and $\bar{g}_{\alpha\beta}$ is the metric tensor for a three-space. The conformal time τ is defined as $a d\tau = dt$ with the speed of light $c \equiv 1$, and it is related to the comoving line-of-sight distance,

$$r(\tau) = a(\tau_0)(\tau_0 - \tau) = a(\tau_0)\int_t^{t_0}\frac{dt}{a(t)} = \int_0^z \frac{dz}{H(z)}, \tag{2}$$

where $H = \mathcal{H}/a = \dot{a}/a^2$ is the Hubble parameter and the dot denotes the derivative with respect to the conformal time. The subscript 0 represents that the quantities are computed at origin in a homogeneous universe. In a flat universe, the comoving line-of-sight distance is coincident with the comoving angular diameter distance. From now on we set $a(\tau_0) \equiv 1$.

The metric tensor can be expanded to represent its perturbations for the spacetime geometry and to describe the departure from the homogeneity and isotropy,

$$ds^2 = -a^2(1+2A)d\tau^2 - 2a^2 B_\alpha d\tau dx^\alpha + a^2[(1+2D)\bar{g}_{\alpha\beta} + 2E_{\alpha\beta}]dx^\alpha dx^\beta. \tag{3}$$

We can further decompose the perturbation variables depending on their spatial transformation properties as $B_\alpha = BQ_\alpha$ and $E_{\alpha\beta} = EQ_{\alpha\beta} + E^{\mathrm{T}}_{\alpha\beta}$, where $E^{\mathrm{T}}_{\alpha\beta}$ is the divergenceless tensor. We adopted the convention [6] for the eigenmode Q_α and $Q_{\alpha\beta}$ of the Helmholtz equations and assumed there is no vector mode. Throughout the paper we use Greek indices to represent the 3D spatial components, running from 1 to 3, while Latin indices are used to represent the 4D spacetime components with 0 being the conformal time component.

Here we will work with the general representation of the metric without fixing gauge conditions (see, e.g., [6–8]), but it often proves convenient to understand our general formulas in conjunction with other gauges such as the conformal Newtonian gauge and the synchronous gauge. The metric in the conformal Newtonian gauge (see, e.g., [9]) is

$$ds^2 = -a^2(1+2\psi)d\tau^2 + a^2[(1+2\phi)\bar{g}_{\alpha\beta} + 2E^{\mathrm{T}}_{\alpha\beta}]dx^\alpha dx^\beta, \tag{4}$$

and the metric in the synchronous gauge (see, e.g., [10]) is

$$ds^2 = -a^2 d\tau^2 + a^2[\bar{g}_{\alpha\beta} + h_{\alpha\beta}]dx^\alpha dx^\beta. \tag{5}$$

Throughout the paper, we adopt as our fiducial model a flat ΛCDM universe with the matter density $\Omega_m = 0.24$ ($\Omega_m h^2 = 0.128$), the baryon density $\Omega_b = 0.042$ ($\Omega_b h^2 = 0.0224$), the Hubble constant $h = 0.73$, the spectral index $n_s = 0.954$, the optical depth to the last scattering surface $s = 0.09$, and the primordial curvature perturbation amplitude $\Delta^2_\phi = 2.38 \times 10^{-9}$ at $k = 0.05$ Mpc^{-1} ($\sigma_8 = 0.81$), consistent with the recent results (e.g., [4,11]). We use the Boltzmann code CMBFAST [12] to obtain the transfer functions of perturbation variables.

B. Temporal component: Sachs-Wolfe effect

The photon geodesic $x^a(\lambda)$ can be parametrized by an affine parameter λ, and its propagation direction is then $k^a = dx^a/d\lambda$, subject to the null equation ($ds^2 = k^a k_a = 0$). We choose the normalization of the affine parameter, such that the time component of the null vector represents the photon frequency $\bar{\nu}$, measured by an observer in a homogeneous universe [13]. The null vector is therefore

$$k^0 = \frac{\bar{\nu}}{a}(1+\delta\nu), \qquad k^\alpha = -\frac{\bar{\nu}}{a}(e^\alpha + \delta e^\alpha), \tag{6}$$

where the unit vector e^α is the photon propagation direction seen from the observer. The spatial component of the null vector is obtained by the null condition and we expanded the null vector to the first order in perturbations to represent its dimensionless temporal and spatial perturbations $\delta\nu$ and δe^α.

To the zeroth order in perturbations, the photon frequency is redshifted as $\bar{\nu} \propto 1/a$ in an expanding universe, and the geodesic path is described by $d/d\chi \equiv (a/\bar{\nu}) \times (d/d\lambda) = \partial_\tau - e^\alpha \partial_\alpha = -d/dr$. Equivalently the affine parameter χ describes the same geodesic path $x^a(\chi)$, but in a conformally transformed metric $\tilde{g}_{ab} = (\bar{\nu}/a)g_{ab}$ (see, e.g., [14] for conformal transformation). We will put tilde to represent quantities in the conformally transformed metric.

The temporal component of the null vector can be integrated to obtain the relation between τ and χ as

$$\tau - \tau_o = \chi - \chi_o + \int_{\chi_o}^{\chi} d\chi' \delta\nu(\chi'), \tag{7}$$

where the subscript o indicates that the affine parameter is computed at origin in an inhomogeneous universe. The perturbations of the null vector are related to the metric perturbations as

$$e^\alpha \delta e_\alpha = \delta\nu + A - B_\alpha e^\alpha - D - E_{\alpha\beta} e^\alpha e^\beta, \tag{8}$$

by the null equation, and as

$$\frac{d}{d\chi}(\delta\nu + 2A) = (\dot{A} - \dot{D}) - (B_{\alpha|\beta} + \dot{E}_{\alpha\beta}) e^\alpha e^\beta, \tag{9}$$

by the temporal component of the geodesic equation ($k^0{}_{;b} k^b = 0$). The vertical bar and the semicolon represent the covariant derivatives with respect to $\bar{g}_{\alpha\beta}$ and g_{ab}, respectively.

Consider a comoving observer of which the rest frame has vanishing total three momentum. Its four velocity is $u^a = [(1 - A)/a, v^\alpha/a]$ and the observer measures the redshift parameter of a source,

$$1 + z_s = \frac{(k^a u_a)_s}{(k^a u_a)_o} = \left(\frac{a_o}{a_s}\right)\{1 + [\delta\nu + A + (v_\alpha - B_\alpha)e^\alpha]_o^s\}, \tag{10}$$

with the spacetime of the source indicated by the subscript s and the bracket representing a difference of the quantities at two spacetime points. Using Eq. (9), this relation can be further simplified [15] as

$$1 + z_s = \left(\frac{a_o}{a_s}\right)\bigg\{1 + [(v_\alpha - B_\alpha)e^\alpha - A]_o^s - \int_0^{r_s} dr[(\dot{A} - \dot{D}) - (B_{\alpha|\beta} + \dot{E}_{\alpha\beta})e^\alpha e^\beta]\bigg\}, \tag{11}$$

where $r_s = r(z_s)$ is the comoving line-of-sight distance to the source galaxies at z_s and $v_\alpha e^\alpha$ is the line-of-sight peculiar velocity. Equation (11) in the conformal Newtonian gauge is known as the Sachs-Wolfe effect [16]. The first square bracket represents the redshift-space distortion by peculiar velocities, frame dragging, and gravitational redshift, respectively. The first round bracket in the integral also represents the gravitational redshift, arising from the net difference in gravitational potential due to its time evolution for the duration of photon propagation, and this effect is referred to as the integrated Sachs-Wolfe effect. The last terms in the integral represent the tidal effect from the frame dragging and the integrated Sachs-Wolfe effect from the time evolution of the primordial gravity waves.

Since the redshift parameter in a homogeneous universe is defined as $1/a$, we define a quantity δz that relates the observed redshift z_s of the source and the redshift of the source that would be measured in a homogeneous universe as $1/a_s \equiv (1 + z_s)(1 - \delta z)$, and note that $a_o = 1 + \mathcal{H}_o \delta\tau_o$. The redshift $1/a_s$ of the source in a homogeneous universe is not directly measurable and hence δz is gauge-dependent. One can easily verify that for a coordinate transformation $\tau \to \tau' = \tau + T$, the perturbation in the observed redshift transforms as $\delta z \to \delta z' = \delta z + \mathcal{H} T$, while the observed redshift z_s is gauge invariant.[1]

C. Spatial components: Gravitational lensing effect

Metric perturbations, sourced by matter fluctuations and gravity waves along the line-of-sight, deflect the photon propagation direction emitted from galaxies and displace their observed position on the sky. This effect, known as the gravitational lensing effect, is described by the spatial components of the geodesic equation ($k^\alpha{}_{;b} k^b = 0$) as

$$\frac{d}{d\chi}(\delta e^\alpha + B^\alpha + 2De^\alpha + 2E^\alpha_\beta e^\beta) = \delta e^\beta e^\alpha{}_{|\beta} - \delta\nu \dot{e}^\alpha + A^{|\alpha} - B_\beta{}^{|\alpha} e^\beta - D^{|\alpha} - E^{|\alpha}_{\beta\gamma} e^\beta e^\gamma. \tag{12}$$

Noting that $(d/d\chi)\delta x^\alpha = -\delta e^\alpha$, the spatial components of the geodesic equation can be integrated and expressed in spherical coordinates to obtain the angular displacements

$$\delta\theta = -\int_0^{r_s} dr \bigg\{\frac{[(B^\alpha - B^\alpha_o) + 2(E^{\alpha\beta} - E^{\alpha\beta}_o)e_\beta]e^\theta_\alpha}{r_s} + \left(\frac{r_s - r}{r r_s}\right)\frac{\partial}{\partial\theta}(A - D - B_\alpha e^\alpha - E_{\alpha\beta} e^\alpha e^\beta)\bigg\}, \tag{13}$$

and

$$\delta\phi = -\int_0^{r_s} dr \bigg\{\frac{[(B^\alpha - B^\alpha_o) + 2(E^{\alpha\beta} - E^{\alpha\beta}_o)e_\beta]e^\phi_\alpha}{r_s \sin\theta} + \left(\frac{r_s - r}{r r_s \sin^2\theta}\right)\frac{\partial}{\partial\phi}(A - D - B_\alpha e^\alpha - E_{\alpha\beta} e^\alpha e^\beta)\bigg\}. \tag{14}$$

Apart from the frame distortion described by $\mathcal{I}^\alpha \equiv (B^\alpha - B^\alpha_o) + 2(E^{\alpha\beta} - E^{\alpha\beta}_o)e_\beta$, the gravitational lensing dis-

[1] Particular attention needs to be paid to the difference between z and $1/a$ in conjunction with Eq. (11). Throughout the paper the redshift parameter z refers to the "observed" redshift, which is different from the gauge-dependent redshift parameter z_h in a homogeneous and isotropic universe, defined as $1 + z_h = 1/a$.

placement depends only on the spatial derivative of the metric perturbations, i.e., a constant gravitational potential results in no observable effect.

Since the comoving line-of-sight distance to the source in an inhomogeneous universe is $(\tau_0 - \tau_s)$ and the source position τ_s is related to the observed redshift z_s through a_s in Eq. (11), it can be expressed in terms of $r(z)$ and $H(z)$ in a homogeneous universe as

$$\bar{r} \equiv \tau_0 - \tau_s = r[z_s - (1+z_s)\delta z] = r_s\left(1 - \frac{1+z_s}{H_s r_s}\delta z\right), \tag{15}$$

where $H_s = H(z_s)$. Note that we have expanded the argument of $r(x)$ in the square bracket around the observed redshift z_s of the source. Finally, using the null equation, the radial displacement is then obtained as

$$\begin{aligned}\delta r &= \chi_o - \chi_s + e_\alpha \delta x^\alpha - \bar{r} \\ &= \delta\tau_o + \int_0^{r_s} dr (A - D - B_\alpha e^\alpha + E_{\alpha\beta} e^\alpha e^\beta).\end{aligned} \tag{16}$$

With the full solution of the geodesic equation, the angular position $\hat{\mathbf{s}}$ of the source galaxies can be obtained by tracing backward the photon path and expressed in terms of observed angle $\hat{\mathbf{n}} = (\theta, \phi \sin\theta)$ as $\hat{\mathbf{s}} = [\theta + \delta\theta, (\phi + \delta\phi)\sin(\theta + \delta\theta)]$. Because of the lensing displacement a unit solid angle $|d^2\hat{\mathbf{s}}|$ in the source plane is distorted to a unit solid angle $|d^2\hat{\mathbf{n}}|$ in the image plane. The amplitude of this distortion is described by the convergence κ as

$$\left|\frac{d^2\hat{\mathbf{n}}}{d^2\hat{\mathbf{s}}}\right| = 1 - \frac{\partial}{\partial\phi}\delta\phi - \left(\cot\theta + \frac{\partial}{\partial\theta}\right)\delta\theta \equiv 1 + 2\kappa, \tag{17}$$

and therefore

$$\begin{aligned}\kappa = \int_0^{r_s} dr \bigg\{ &\frac{\csc\theta \partial_\phi (e_\alpha^\phi I^\alpha) + \partial_\theta (e_\alpha^\theta I^\alpha) + \cot\theta e_\alpha^\theta I^\alpha}{2r_s} \\ &+ \left(\frac{r_s - r}{2rr_s}\right)\hat{\nabla}^2 (A - D - B_\alpha e^\alpha - E_{\alpha\beta} e^\alpha e^\beta)\bigg\},\end{aligned} \tag{18}$$

where $\hat{\nabla}$ is the differential operator in two dimensional unit sphere. In the literature Eq. (17) is often referred to as the gravitational lensing magnification μ. However, the angular position $\hat{\mathbf{s}}$ of the source galaxies is not observable; its coordinate value depends on the choice of gauge condition, while the spacetime of the source position is physical. Consequently, the convergence κ in Eq. (18) is gauge-dependent, whereas magnification should be a gauge-invariant quantity. In Sec. III we provide a correct gauge-invariant expression for magnification μ. Note that the gravitational lensing displacements δr, $\delta\theta$, and $\delta\phi$ are also gauge-dependent.

The standard Newtonian expression for the convergence can be obtained with a few approximations: When the Newtonian potential and curvature are constant in time as in an Einstein-de Sitter universe, we can replace the total derivative d/dr by the partial derivative ∂_r. Integrating by part and ignoring the boundary terms yield the standard form [17,18] as

$$\begin{aligned}\kappa &= \int_0^{r_s} dr \frac{(r_s - r)r}{2r_s}\left[\nabla^2 - \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right)\right](\psi - \phi) \\ &= \frac{3H_0^2}{2}\Omega_m \int_0^{r_s} dr \frac{\delta_m}{a}\frac{(r_s - r)r}{r_s},\end{aligned} \tag{19}$$

where we have used the Newtonian Poisson equation $k^2\phi \simeq -k^2\psi \simeq 4\pi G a^2 \delta\rho$. Deep inside the horizon where the Newtonian approximation is accurate, there is no gauge ambiguity and the gravitational lensing magnification is $\mu \simeq |d^2\hat{\mathbf{n}}/d^2\hat{\mathbf{s}}| = 1 + 2\kappa$.

III. OBSERVED LUMINOSITY DISTANCE

The observed position and the redshift of source galaxies are affected by the matter fluctuations and the gravity waves between the source galaxies and the observer, and this relation is described by the geodesic equation in Sec. II. The observed flux of the source galaxies is also affected by the same fluctuations and this relation is described by the fluctuations in the luminosity distance. Here we derive the observed luminosity distance in an inhomogeneous universe (see [19,20] for earlier derivations).

Consider a source with intrinsic luminosity L and proper radius ΔR_s. The flux $\mathcal{F}_o$ and redshift z_s of the source are measured at origin and the observed luminosity distance is defined as

$$\mathcal{D}_L(z_s) \equiv \sqrt{\frac{L}{4\pi\mathcal{F}_o}} = \sqrt{\frac{\mathcal{F}_s}{\mathcal{F}_o}}\Delta R_s = \frac{\mathcal{A}_s \nu_s}{\mathcal{A}_o \nu_o}\Delta R_s, \tag{20}$$

where $\mathcal{A}$ is the scalar amplitude of the four potential of the photons and we have used $\mathcal{F} \propto \mathcal{A}^2\nu^2$. When the wavelength of the photons is shorter than the curvature scale, the propagation of light rays can be locally described by Maxwell's equations, and the governing equations are known as the geometric optics in curved spacetime (see, e.g., [21,22]).

The optical scalar equations are the propagation equations of the scalar amplitude

$$\frac{d}{d\chi}(\mathcal{A}a) + \frac{1}{2}\mathcal{A}a\vartheta = 0, \tag{21}$$

and the expansion of the wave vector $\vartheta = \tilde{k}^a{}_{;a}$

$$\frac{d}{d\chi}\vartheta + \frac{1}{2}\vartheta^2 = -\tilde{R}_{ab}\tilde{k}^a\tilde{k}^b, \tag{22}$$

where $\tilde{R}_{ab}$ is the Ricci tensor in the conformally transformed metric $\tilde{g}_{ab} = (\bar{\nu}/a)g_{ab}$. To the zeroth order in perturbations Eq. (22) has no source term in a flat universe and it can be integrated to obtain the expansion of the wave vector $\vartheta = 2/(\chi - \chi_s - \Delta\chi_s)$, where $\Delta\chi_s$ is related to the size of the source. Since the proper radius of the source is $\Delta R_s = |dt|$ in a local Lorentz frame, it can be expressed in terms of the affine parameter χ by considering the photon frequency at the source as

$$-\nu_s = (k^a u_a)_s = -\frac{\bar{\nu}_s}{a_s}\frac{dt}{d\chi}. \tag{23}$$

Note that dt in Eq. (23) is defined in the local Lorentz frame of the source. Solving Eq. (21) for $\mathcal{A}a$ and using $\Delta R_s = a_s\nu_s|\Delta\chi_s|/\bar{\nu}_s$ yields the observed luminosity distance as

$$\mathcal{D}_L(z_s) = (1+z_s)\Delta R_s\frac{a_o}{a_s}\exp\left[-\int_o^s d\chi\frac{\vartheta}{2}\right] = a_o(1+z_s)(\chi_o-\chi_s)\frac{\nu_s}{\bar{\nu}_s}\left(1-\int_o^s d\chi\frac{\delta\vartheta}{2}\right), \tag{24}$$

in the limit $\Delta\chi_s \to 0$, and $\delta\vartheta$ is the first order perturbation of the expansion of the wave vector that can be obtained by expanding Eq. (22).

Now to solve for $\delta\vartheta$ we integrate Eq. (22) along the zeroth order solution ϑ,

$$\int_o^s d\chi\frac{\delta\vartheta}{2} = \int_0^{r_s} dr\frac{(r_s-r)r}{2r_s}\delta(\tilde{R}_{ab}\tilde{k}^a\tilde{k}^b), \tag{25}$$

with the source term in the integral

$$\begin{aligned}\delta(\tilde{R}_{ab}\tilde{k}^a\tilde{k}^b) = {}& -k^2\left[A - \left(D+\frac{E}{3}\right) + \left(\frac{\dot{B}}{k} - \frac{\ddot{E}}{k^2}\right)\right] - 2\left(\ddot{D}+\frac{\ddot{E}}{3}\right) + 4\left(\dot{D}+\frac{\dot{E}}{3}\right)_{|\alpha}e^\alpha \\ & - \left[A + \left(D+\frac{E}{3}\right) + \left(\frac{\dot{B}}{k} - \frac{\ddot{E}}{k^2}\right)\right]_{|\alpha\beta}e^\alpha e^\beta + (\ddot{E}^T_{\alpha\beta} + k^2E^T_{\alpha\beta})e^\alpha e^\beta. \end{aligned} \tag{26}$$

Noting that the luminosity distance in a homogeneous universe is $D_L(z) = (1+z)r(z)$ and the comoving line-of-sight distance $\bar{r}$ of the source is related to the affine parameter via Eqs. (7) and (15), the observed luminosity distance $\mathcal{D}_L(z_s)$ can be written as [19]

$$\begin{aligned}\frac{\mathcal{D}_L(z_s)}{D_L(z_s)} = {}& 1 + (v_\alpha - B_\alpha)_s e^\alpha - A_s - \frac{1+z_s}{H_s r_s}\delta z + 2\int_0^{r_s} dr\frac{A}{r_s} - \int_0^{r_s} dr\frac{r}{r_s}[(\dot{A}-\dot{D}) \\ & - (B_{\alpha|\beta} + \dot{E}_{\alpha\beta})e^\alpha e^\beta] - \int_0^{r_s} dr\frac{(r_s-r)r}{2r_s} \times \delta(\tilde{R}_{ab}\tilde{k}^a\tilde{k}^b) + \left(\mathcal{H}_o + \frac{1}{r_s}\right)\delta\tau_o. \end{aligned} \tag{27}$$

With the full expression for luminosity distance, the magnification of a source at observed redshift z is defined as the ratio of the observed flux $\mathcal{F}_o$ to the flux of the source that would be measured in a homogeneous universe:

$$\mu = \mathcal{F}_o\left(\frac{L}{4\pi D_L^2}\right)^{-1} = \left(\frac{D_L}{\mathcal{D}_L}\right)^2 = 1 - 2\delta\mathcal{D}_L. \tag{28}$$

We have defined the perturbations in Eq. (27) as $\mathcal{D}_L(z) \equiv D_L(z)(1+\delta\mathcal{D}_L)$, and note that written in terms of observable variables $\delta\mathcal{D}_L$ is gauge invariant and both $\mathcal{D}_L(z)$ and $D_L(z)$ are evaluated at the observed redshift z. In the Newtonian limit, Eq. (25) becomes the convergence κ and it is the dominant factor for $\delta\mathcal{D}_L$. Therefore, we recover the Newtonian expressions $\delta\mathcal{D}_L \simeq -\kappa$ and $\mu \simeq 1 + 2\kappa$.

IV. OBSERVED GALAXY FLUCTUATION FIELD

Drawing on the formalism developed in Secs. II and III we present the expression for the observed galaxy fluctuation field $\delta_{\rm obs}$, accounting for all the relativistic effects to the linear order. Our formalism is crucial for the theoretical consistency and the gauge invariance of the predictions using galaxy clustering as a cosmological probe. To construct the observed galaxy overdensity field we start by considering a gauge-invariant quantity, the total number $N_{\rm tot}$ of observed galaxies. The total number of observed galaxies in a small volume described by observed redshift z and observed angle $\hat{\mathbf{n}}$ can be formulated in terms of a covariant volume integration in a four-dimensional spacetime manifold [23], and it is related to the photon geodesic $x^a(\chi)$ via

$$N_{\rm tot} = \int\sqrt{-g}\,n_p\varepsilon_{abcd}u^d\frac{\partial x^a}{\partial z}\frac{\partial x^b}{\partial\theta}\frac{\partial x^c}{\partial\phi}dzd\theta d\phi, \tag{29}$$

where n_p is the physical number density of the source galaxies, the metric determinant is $\sqrt{-g} = a^4(1+A+3D)$, and $\varepsilon_{abcd} = \varepsilon_{[abcd]}$ is the Levi-Civita symbol.

To the linear order in perturbations the photon geodesic is a straight path with small distortion and, with the geodesic path $x^a(\chi)$ in Sec. II we obtain

$$N_{\rm tot} = \int n_p\frac{r^2\sin\theta}{(1+z)^3H}dzd\theta d\phi\left[1 + 3D + v^\alpha e_\alpha + 2\frac{\delta r}{r} + H\frac{\partial}{\partial z}\delta r + \left(\cot\theta + \frac{\partial}{\partial\theta}\right)\delta\theta + \frac{\partial}{\partial\phi}\delta\phi + \frac{\bar{r}^2}{r^2}H\frac{\partial\bar{r}}{\partial z}\right]. \tag{30}$$

The observed galaxy number density n_g is then defined in relation to the total number of observed galaxies and the observed volume element as

$$N_{\rm tot} \equiv \int n_g\frac{r^2\sin\theta}{(1+z)^3H}dzd\theta d\phi, \tag{31}$$

and therefore the observed galaxy number density is

$$n_g = n_p\bigg[1 + A + 2D + (v^\alpha - B^\alpha)e_\alpha + E_{\alpha\beta}e^\alpha e^\beta - (1+z)\frac{\partial}{\partial z}\delta z - 2\frac{1+z}{Hr}\delta z - \delta z - 2\kappa + \frac{1+z}{H}\frac{dH}{dz}\delta z + 2\frac{\delta r}{r}\bigg]. \quad (32)$$

Given the total number of observed galaxies, the observed galaxy number density is affected by the matter fluctuations and the gravity waves, since the volume element is constructed in terms of observed redshift and observed angle. Equation (29) automatically takes into account the full effects of the volume distortion described by the photon geodesic equation in Sec. II.

However, additional distortions arise due to the intrinsic luminosity function dn_p/dL of the source galaxies. As described in Sec. III the observed flux of the source galaxies is affected by the matter fluctuations and the gravity waves between the source galaxies and the observer, and magnification of the source galaxy flux changes the observed galaxy number density. Given an observational threshold $\mathcal{F}_{\rm thr}$ in flux at origin, the physical number density n_p in the above equations should be modified as

$$n_p \to \int_{\mathcal{F}_{\rm thr}}^\infty d\mathcal{F}_o \frac{dL}{d\mathcal{F}_o}\frac{dn_p}{dL} = n_p[L_{\rm thr}(1 + 2\delta\mathcal{D}_L)], \quad (33)$$

where $n_p(L)$ is the cumulative (physical) number density of the source galaxies brighter than L and $L_{\rm thr} = 4\pi D_L^2(z)\mathcal{F}_{\rm thr}$ is the inferred luminosity threshold for the source galaxy sample. For a galaxy sample with $dn_p/dL \propto L^{-s}$, the cumulative number density can be expanded as $n_p(L_{\rm thr})(1 - 5p\delta\mathcal{D}_L)$, and $p = 0.4\ (s-1)$ is the slope of the luminosity function in magnitude.

Furthermore, since we observe galaxies rather than the underlying matter distribution, we need to relate the physical number density n_p of the source galaxies to the matter density ρ_m. In the simplest model of galaxy formation, the galaxy number density is simply proportional to the underlying matter density ρ_m, when ρ_m is above some threshold ρ_t dictated by complicated but local process involving atomic physics. The matter density at the source galaxy position is related to the mean matter density at the observed redshift z as[2]

$$\rho_m(x^a) = \frac{\bar\rho_m(\tau_0)}{a^3}(1 + \delta_m) = \bar\rho_m(z)[1 + \delta_m - 3\delta z], \quad (34)$$

and the mean matter density at the observed redshift is $\bar\rho_m(z) = (3H_0^2/8\pi G)\Omega_m(1+z)^3$. The combination $(\delta_m - 3\delta z)$ is gauge invariant and is proportional to the matter density at the source galaxy position. Within the linear bias approximation, the long wavelength fluctuations of the matter density ρ_m at a given point effectively lower the threshold for galaxy formation and the galaxy number density can be written as

$$n_p = \bar n_p(z)[1 + b(\delta_m - 3\delta z)], \quad (35)$$

and b is a scale-independent linear bias factor.[3] Equation (35) can be contrasted with the gauge-dependent relation $\delta_g = b\delta_m$.

Finally, putting all the ingredients together the observed galaxy fluctuation field can be written as

$$\delta_{\rm obs} = b(\delta_m - 3\delta z) + A + 2D + (v^\alpha - B^\alpha)e_\alpha + E_{\alpha\beta}e^\alpha e^\beta - (1+z)\frac{\partial}{\partial z}\delta z - 2\frac{1+z}{Hr}\delta z - \delta z - 5p\delta\mathcal{D}_L - 2\kappa + \frac{1+z}{H}\frac{dH}{dz}\delta z + 2\frac{\delta r}{r}, \quad (36)$$

where δz, δr, κ, and $\delta\mathcal{D}_L$ are given in Eqs. (11), (16), (18), and (27), respectively. This equation is the main result of our paper. Constructed from the gauge-invariant expressions and expressed in terms of observables, this result is gauge invariant. Note that in addition to the scalar contributions, Eq. (36) includes tensor contributions from the primordial gravity waves, mainly from the integrated Sachs-Wolfe effect in δz.

One remaining ambiguity in computing $\delta_{\rm obs}$ is the time lapse $\delta\tau_o$ at origin, representing the departure from τ_0 in a homogeneous universe. However, this quantity is independent of the position and angle of the source galaxies; In practice the mean number density $\bar n_p(z)$ of the observed galaxies is obtained by averaging n_g over observed angle $\hat{\mathbf{n}}$ at a fixed observed redshift z and $\delta\tau_o$ is absorbed in the monopole set equal $\bar n_p(z)$. In the Newtonian limit the dominant contribution in δz is the peculiar velocity V and Eq. (36) reduces to the standard relation for redshift-space distortions [26] and magnification bias [27] as

$$\delta_{\rm std} = b\delta_m + (5p - 2)\kappa - \frac{1+z}{H}\frac{\partial V}{\partial r}. \quad (37)$$

Figure 1 illustrates the theoretical inconsistency in the standard method by showing the power spectra of perturbation variables computed at $z = 0$ in the conformal Newtonian and the synchronous gauges. The power spectra of matter fluctuations in two gauges (solid line; synchronous, short dot-dashed line; conformal Newtonian) noticeably deviate from each other well before they reach the horizon scale (dark gray), reflecting that theoretical predictions in the standard method depend on the choice of

[2]The observed redshift z is related to the expansion parameter a of the source galaxy as $1 + z = (1 + \delta z)/a$.

[3]More general ansatz for Eq. (35) can be obtained by generalizing the earlier approach [24,25] as $n_p = \bar n_p(z) \times \exp[b_L \int \sqrt{-g}d^4y(\delta_m - 3\delta z)(y)\mathcal{W}(x-y)]$, where $\mathcal{W}$ is a local filter function that cuts off small scale fluctuations, and the Lagrangian bias b_L is related to the bias in Eulerian space as $b = b_L + 1$.

gauge conditions. In particular, as we observe higher redshift, larger comoving scales (light gray) are accessible and the horizon scale is smaller, and therefore the systematic errors in the standard methods start to become significant on progressively smaller scales. The infrared divergence shown as the dot-dashed line on large scales is an artifact in the conformal Newtonian gauge, while the matter fluctuation δ_m (solid line) in the synchronous gauge is also gauge-dependent. Theoretical quantities plotted in Fig. 1 are not directly observable.

V. CROSS CORRELATION OF CMB ANISOTROPIES WITH LARGE-SCALE STRUCTURE

As the first application of our formalism, we compute the angular correlation of large-scale structure and its cross correlation with CMB anisotropies. In the standard approach, the observed galaxy fluctuation field is written in the Newtonian limit, and neglecting the additional contributions to the observed galaxy fluctuation field results in systematic errors in the theoretical predictions. We first introduce the formalism for computing the angular correlations in Sec. VA, and present the angular auto and cross correlations with the main emphasis on the systematic errors in Sec. V B.

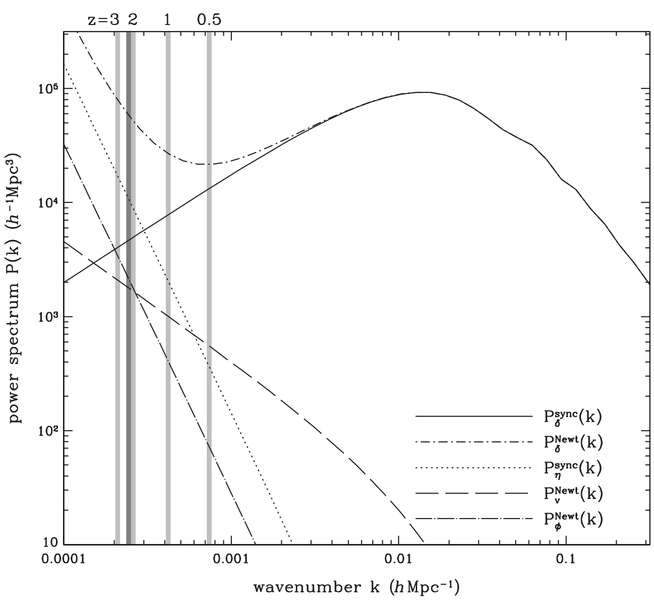

FIG. 1. Power spectra of perturbation variables computed at $z = 0$ in the conformal Newtonian and the synchronous gauges. Vertical lines show the comoving line-of-sight distance ($k = 1/r(z)$; light gray) at each redshift indicated in the legend and the horizon scale ($k = H_0$; dark gray) today. Near the horizon scale, even power spectra of matter fluctuations in two gauges differ dramatically, showing that gauge effects are substantial and it is nontrivial to relate perturbation variables to observable quantities. Two distinct choices of gauge conditions cannot be used simultaneously around the horizon scale (e.g., Newtonian gauge equations with synchronous gauge transfer function outputs from CMBFast or CAMB).

A. Observed angular fluctuation field

The observed angular fluctuation field can be obtained by integrating Eq. (36) along the line-of-sight as

$$\delta_{\rm obs}^{\rm 2D}(\hat{\mathbf{n}}) = \int dz P(z) \delta_{\rm obs}(z, \hat{\mathbf{n}}), \tag{38}$$

with the normalized selection function $P(z)$ of the galaxy sample. The selection function $P(z)$ can be obtained by averaging the observed galaxy number density n_g at each observed redshift slice. Since $\delta_{\rm obs}$ in Eq. (36) is a linear combination of perturbation variables T_i with different weight function $W_i(r, \hat{\mathbf{n}}, \hat{\mathbf{k}})$, it proves convenient to further decompose their functional dependence by

$$\delta_{\rm obs}(z, \hat{\mathbf{n}}) = \sum_i \int_0^{r_s} dr \int \frac{d^3\mathbf{k}}{(2\pi)^3} W_i(r, \hat{\mathbf{n}}, \hat{\mathbf{k}}) T_i(\mathbf{k}, r) e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{39}$$

and $\mathbf{x} = (r, \hat{\mathbf{n}})$ in the spherical coordinate. The angular fluctuation field is often expanded as a function of spherical harmonics and the observed angular component is then

$$\begin{aligned} a_{lm} &= \int d^2\hat{\mathbf{n}} \delta_{\rm obs}^{\rm 2D}(\hat{\mathbf{n}}) Y_{lm}^*(\hat{\mathbf{n}}) \\ &= \sum_i \int \frac{d^3\mathbf{k}}{(2\pi)^3} \int dz P(z) \\ &\quad \times \int_0^{r_s} dr \int d^2\hat{\mathbf{n}} Y_{lm}^*(\hat{\mathbf{n}}) W_i(r, \hat{\mathbf{n}}) T_i(\mathbf{k}, r) e^{i\mathbf{k}\cdot\mathbf{x}}. \end{aligned} \tag{40}$$

For most of the perturbation variables such as δ_m, A, and D in Eq. (36), the weight function takes the simple form $W(r) = \delta^D(r - r_s)$, because they are independent of the photon propagation direction and its path. The angular dependence of the integrand is then carried by the plane wave and this functional dependence can be further separated by using the partial wave expansion

$$e^{i\mathbf{k}\cdot\mathbf{x}} = 4\pi \sum_{lm} i^l j_l(kx) Y_{lm}^*(\hat{\mathbf{k}}) Y_{lm}(\hat{\mathbf{n}}). \tag{41}$$

The line-of-sight velocity $V = v^\alpha e_\alpha$ is independent of the photon path, but depends on the photon propagation direction; the weight function is

$$W(r) = \delta^D(r - r_s)(-i\hat{\mathbf{n}} \cdot \hat{\mathbf{k}}) = -\delta^D(r - r_s)\left(\frac{1}{k}\frac{\partial}{\partial r}\right), \tag{42}$$

and now it is an operator, acting upon the radial part of the plane wave. Note that we have explicitly removed the dependence on the photon propagation direction by making the weight function an operator. The weight function for the weak lensing convergence κ depends on both the photon path and its propagation direction, and it is therefore another operator acting upon the angular part of the plane wave:

$$W(r) = \left(\frac{r_s - r}{2r_s r}\right) \qquad \hat{\nabla}^2 = -l(l+1)\left(\frac{r_s - r}{2r_s r}\right), \qquad (43)$$

where we have used the relation $\hat{\nabla}^2 Y_{lm}(\hat{\mathbf{n}}) = -l(l+1)Y_{lm}(\hat{\mathbf{n}})$.

Finally, for the initial conditions described by a Gaussian random distribution with $\Delta^2_\phi(k) \propto k^{n_s-1}$, the auto correlation of large-scale structure can be written as

$$C_l = \langle a^*_{lm} a_{lm}\rangle = 4\pi \int \frac{dk}{k} \Delta^2_\phi(k) \mathcal{T}^2_l(k), \qquad (44)$$

and the cross correlation of CMB anisotropies with large-scale structure is

$$C^\times_l = \langle a^{\mathrm{cmb}*}_{lm} a_{lm}\rangle = 4\pi \int \frac{dk}{k} \Delta^2_\phi(k) \Theta^*_l(k) \mathcal{T}_l(k), \qquad (45)$$

where we have defined the angular multipole function of large-scale structure in Fourier space

$$\mathcal{T}_l(k) = \sum_i \int dz P(z) \int_0^{r_s} dr T_i(k, r) W_i(r) j_l(kr), \qquad (46)$$

and Θ_l is the angular multipole function of CMB anisotropies (see, e.g., [12,28]).

B. Angular correlations

Here we consider a quasar sample without spectroscopic redshift measurements used for the cross correlation analysis, such as the photometric quasar (QSO) sample [29] obtainable from the SDSS. The redshift distribution of the sample is assumed to have the standard functional form

$$P(z)dz \propto z^\alpha \exp\left[-\left(\frac{z}{z_0}\right)^\beta\right] dz, \qquad (47)$$

with $(\alpha, \beta, z_0) = (3, 13, 3.4)$. The mean and the peak redshifts of the sample are 2.7 and 3, respectively.

Figure 2 shows the systematic errors in theoretical predictions of the auto correlation (left) of the QSO sample and its cross correlation (right) with CMB temperature anisotropies, when the relativistic effects are ignored. Compared to our full expression in Eq. (36), the theoretical predictions in the standard method are computed by using $\delta_{\mathrm{std}} = b\delta_m + (5p - 2)\kappa$, where δ_m is the matter fluctuation in the synchronous gauge and κ is the convergence in the conformal Newtonian gauge. We have assumed $b = 2$ and $(5p - 2) = 0.1$ for the QSO sample [29], and the full sky coverage of the survey is assumed for comparison.

In the standard approach to modeling the cross correlation of CMB anisotropies with large-scale structure, the matter fluctuation in large-scale structure correlates with the integrated Sachs-Wolfe effect in CMB anisotropies. However, the observed fluctuation field δ_{obs} in Eq. (36)

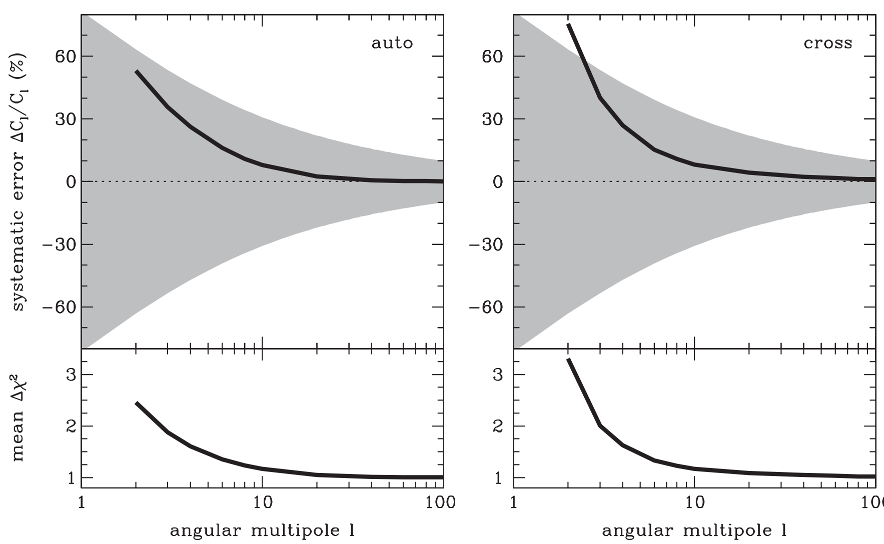

FIG. 2. Systematic errors in theoretical predictions of the auto correlation (left) of the QSO sample and its cross correlation (right) with CMB temperature anisotropies. Attached bottom panels show the mean $\Delta\chi^2$ of the measurements, when only the cosmic variance is considered. As in the standard practice, the theoretical predictions of the angular correlations are computed by using $\delta_{\mathrm{std}} = b\delta_m + (5p - 2)\kappa$ with δ_m in the synchronous gauge and κ in the conformal Newtonian gauge, and the angular correlations computed with our full expression for δ_{obs} in Eq. (36) are compared to the predictions with δ_{std}. Projection along the line-of-sight suppresses the large-scale modes, where the matter fluctuations in two gauges in Fig. 1 differ substantially. Note that at $l = 2$ the correct theoretical prediction is larger by a factor of 1.8 than that one would incorrectly predict in the standard method, and it is 1.2-σ away from the estimated cosmic variance shown as shaded regions. Since the signal-to-noise ratio of the cross correlation measurements is largest at low angular multipoles (measurements uncertainties are large at $l > 10$), the systematic errors in the standard method could bias the inferred cosmology.

contains numerous new contributions, including the peculiar velocity, the gravitational potential, and the integrated Sachs-Wolfe effect, when written in the conformal Newtonian gauge. Therefore, when computing the cross correlation with the observed galaxy fluctuation field, the correlations of the new contributions are required to be considered in addition to the matter correlation. For example, the integrated Sachs-Wolfe effect present in both CMB anisotropies and large-scale structure directly correlates with each other.

In the conformal Newtonian gauge, $\delta z \ll \delta_m$ for the QSO sample, since the peculiar velocity, the gravitational potential, and the integrated Sachs-Wolfe effect are of the same order. Therefore, when $\delta_{\rm obs}$ is computed in the conformal Newtonian gauge, the correlation of the matter fluctuation δ_m contributes most to C_l and $C_l^\times$ compared to the other numerous contributions, and the systematic errors in Fig. 2 arise mainly from the difference in δ_m of the conformal Newtonian and the synchronous gauges seen in Fig. 1. However, in the synchronous gauge, δz simply results from the integrated Sachs-Wolfe effect due to the absence of the peculiar velocity and the gravitational potential, and therefore without accounting for δz the theoretical predictions in the standard method are underestimated. For the tensor-to-scalar ratio $r = 0.1$ at $l = 2$, the tensor contribution is $\sim 1\%$ of the matter fluctuation. We emphasize again that $\delta_{\rm obs}$ is gauge invariant and it can be computed in any gauges.

As opposed to the dramatic contrast seen in Fig. 1 the systematic errors in the theoretical predictions seem relatively small in Fig. 2. The main reason is the projection effect in the angular correlation: each Fourier mode is projected along the line-of-sight and the amplitude of C_l is largely determined by the mode $k \simeq l/r_s$, and slightly larger scale mode for the cross correlation $C_l^\times$ due to the cancellation of two spherical Bessel functions with different distance scales. This projection effect highly suppresses the largest scale modes $k \sim 1/r_s$ of the sample, reducing the dramatic difference in the matter fluctuations. For computing the cross correlation with CMB temperature anisotropies, one would in practice need to compute Eq. (37) with δ_m replaced by the combination $(\delta_m - 3\delta z)$ computed in the conformal Newtonian gauge to be consistent with the calculation of the convergence κ.

Finally, we comment on the impact of the systematic errors. At the lowest angular multipole the accurate theoretical prediction is about a factor of 1.8 larger than the standard method predicts at the 1.2-σ confidence level with the estimated cosmic variance limit. Since the cross correlation signals decline rapidly with angular multipole l, the signal-to-noise ratio of the measurements is determined by the estimated cosmic variance at $l < 10$. The lower theoretical predictions in the standard method underestimate the cosmic variance, resulting in additional $\Delta\chi^2$ of a few of the measurements. Considering that the current detection significance is at the 3-σ level for each galaxy sample [29], these systematic errors could bias the inferred cosmology.

VI. DISCUSSION

We have developed a fully general relativistic description of galaxy clustering as a cosmological probe—we have derived a covariant expression for the observed galaxy fluctuation field in a general Friedmann-Lemaître-Robertson-Walker metric without fixing a gauge condition and our formalism includes tensor contributions from primordial gravity waves. The observed volume element is constructed by using the observed redshift and observed angle in a homogeneous universe, while the real physical volume element given the observables needs to be constructed by tracing backward the photon geodesic in an inhomogeneous universe. This discrepancy in the observable quantities results in a significant modification of the observed galaxy fluctuation field and provides a key clue for understanding gauge issues related to the observables.

As our first application, we have computed the angular auto correlation of the photometric QSO sample from the SDSS and its cross correlation with CMB anisotropies. The cross correlation in the standard method arises from the correlation of the integrated Sachs-Wolfe effect in CMB anisotropies and the underlying matter fluctuation of the QSO sample. However, since there are numerous additional contributions to the observed QSO fluctuation field, the correlations of the additional terms with CMB anisotropies need to be considered. The dominant contribution to the cross correlation still arises from the matter fluctuation and the correct theoretical predictions are larger at low angular multipoles. The systematic errors in theoretical predictions are highly suppressed in the angular correlations due to the projection effect, but they can result in $\Delta\chi^2$ of a few at low angular multipoles.

We comment on the possibility to detect primordial gravity waves using galaxy samples. Primordial gravity waves affect the observed redshift and position of galaxies and therefore its effect is imprinted in observed galaxy fluctuation fields. We find that the tensor contribution in the cross correlation is 1% of the scalar contribution from the matter fluctuation at the lowest angular multipoles, when the tensor-to-scalar ratio is assumed to be $r = 0.1$ at $l = 2$. In general, it is extremely difficult to isolate tensor contributions in galaxies, because they are completely swamped by scalar contributions. One possibility is to cross correlate CMB B-mode polarization anisotropies with large-scale structure at low angular multipoles as they should be uncorrelated in the absence of primordial gravity waves. However, it may not be feasible in practice, as the parity odd quantities need to be constructed from the observed galaxy samples.

The next application of our formalism is to investigate the effect on the three-dimensional power spectrum of galaxy samples. Recently, Dalal *et al.* [30] showed that the primordial non-Gaussianity feature can be probed by the scale-dependence of galaxy bias on large scales and this new development has spurred an extensive theoretical and observational investigation [31–35]. However, at this large scale, where the primordial non-Gaussianity feature can be most sensitively probed, relativistic effects become substantial and observed quantities are significantly different from simple theoretical predictions. Therefore, without proper theoretical modeling of observables, cosmological interpretation of these measurements would be significantly biased with the current data, and even more so with galaxy samples from future dark energy surveys. Correct modeling of the observed power spectrum would not only need to account for the discrepancy in the observable quantities, but also need to account for additional anisotropies arising from the angle dependence of the observable quantities [36].

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