Anisotropic cosmologies in warped DGP braneworld

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The DGP braneworld scenario explains accelerated expansion of the Universe via leakage of gravity to extra dimensions without any need for dark energy. We study the behavior of homogeneous and anisotropic cosmologies on a warped DGP brane with perfect fluid as a matter source. Taking a conformally flat bulk, we obtain the general solutions of the field equations in an exact parametric form for Bianchi type I space-time with a pressureless fluid. Finally, the behavior of the observationally important parameters like shear, anisotropy, and the deceleration parameter is considered in detail. We find that isotropization can proceed slower in the warped DGP model than the generalized Randall-Sundrum II model.

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I. INTRODUCTION

Measurements of the apparent brightnesses of type Ia supernovae [1,2] and independent observations of the cosmic microwave background (CMB) by the WMAP satellites [3] and other CMB experiments [4] provide strong evidence that the Universe is accelerating in its expansion. Although general relativity with a nonzero cosmological constant can accommodate these observations, it suffers from serious problems such as the fine-tuning problem and the coincidence problem [5,6]. This cosmological constant problem has motivated the theoretical physics community to explore alternative descriptions of gravity that can account for the accelerating Universe without the need for dark energy. An interesting way of explaining the observed acceleration of the late time universe is to modify gravity at large scale.

The idea that our familiar 4-dimensional (4D) space-time is a hypersurface (brane) in a 5D space-time (bulk) [7] has been under detailed elaboration during the last decade. According to this braneworld scenario, all matter and gauge interactions reside on the brane, while gravity can propagate in the 5D space-time. There are two main pictures in the braneworld scenario. In the first picture, which we refer to as the Randall-Sundrum type II brane model (RS II), a positive tension 3-brane embedded in the 5D anti–de Sitter (AdS) bulk and the crossover between 4D and 5D gravity is set by the AdS radius [8,9]. In this case, the extra dimension has a finite size. For reviews of the dynamics and geometry of the brane universes, as well as for the discussions of the cosmological implications, see [10]. In another picture which was proposed by Dvali, Gabadadze, and Porrati (DGP) [11,12], a 3-brane is embedded in a 5D Minkowski space-time with an infinite-sized extra dimension, with the hope that this picture could shed new light on the standing problem of the cosmological constant as well as on supersymmetry breaking [11,13]. While the RS II model produces the high energy modifications to general relativity, the DGP model leads to a low energy modification. This proposal rests on the key assumption of the presence of a 4D Ricci scalar in the bulk action. There are two main reasons that make the DGP model phenomenologically appealing. First, it predicts that 4D Newtonian gravity on a braneworld is regained at distances shorter than a given crossover scale r_c (high energy limit), whereas 5D effects become manifest above that scale (low energy limit) [14]. Second, the model can explain late time acceleration without having to invoke a cosmological constant or quintessential matter [15,16]. For a recent and comprehensive review of the phenomenology of DGP cosmology, the reader is referred to [17].

In a series of papers, a number of authors [18] have presented detailed descriptions of the dynamics of homogeneous and anisotropic braneworlds. By making an assumption about the Weyl term on the brane, the dynamics of a Bianchi type I brane in the presence of a scalar field were studied in [19], and it was shown that high energy effects from extra-dimensional gravity remove the anisotropic behavior near the singularity that is found in general relativity. The shear dynamics in the Bianchi type I cosmological model on a brane with perfect fluid has been studied by Toporensky [20]. An exact new solution of the gravitational field equation in the braneworld model for anisotropy Bianchi types I and V geometries for a conformally flat bulk has been studied in [21]. In [22], an anisotropic braneworld model with Bianchi types I and V geometries was studied in which matter is confined to the brane by the action of a confining potential without using any junction conditions, offering a geometrical explanation for the accelerated expansion of the Universe. All the above studies mainly focused on anisotropy in the RS braneworld. In the framework of the braneworld models Maeda, Mizuno, and Torii constructed a brane scenario which combines the RS II model and the DGP model [23]. In this combination, an induced curvature term appears on the brane in the RS II model. This model has been called

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the warped DGP braneworld in the literature [24]. It would therefore be interesting to study anisotropic Bianchi models in the context of a warped DGP braneworld model. In the framework of general relativity, Wald showed that initially expanding homogeneous cosmological Bianchi models isotropize in the presence of a positive cosmological constant [25]. Kobayashi extended Wald's result in DGP gravity in [26] along the same line. The study of the asymptotic behavior of anisotropic DGP branes has shown that all Bianchi models except type IX isotropize if the $\mathcal{E}_{\mu\nu}$ term satisfies some energy condition, $\mathcal{U} \leq 0$, and isotropization proceeds slower in the DGP gravity than in general relativity [26]. To check that the anisotropic brane metric with $\mathcal{U} \leq 0$ leads to a consistent physical bulk metric, one needs to solve the 5D field equations, which is difficult work.

In the present paper, we obtain the gravitational field equations in a warped DGP braneworld model for an anisotropic Bianchi type I geometry for a conformally flat bulk (with vanishing Weyl tensor). For a perfect fluid as a confined matter source, the general solution of the field equations can be obtained in a parametric form of the volume scale factor. The behavior of the mean anisotropy parameter, shear, and deceleration parameters is considered in detail. For a special case, the field equations reduce to the generalized RS model and allow one to compare its results to the warped DGP braneworld model.

II. EFFECTIVE FIELD EQUATIONS ON THE WARPED DGP BRANE

In this section we present a brief review of the model proposed in [23]. Consider a 5D space-time with a 4D brane, located at $Y(X^A) = 0$, where X^A ($A = 0, 1, 2, 3,$ and 4) are the 5D coordinates. The effective action is given by

$$\mathcal{S} = \mathcal{S}_{\text{bulk}} + \mathcal{S}_{\text{brane}}, \tag{1}$$

where

$$\mathcal{S}_{\text{bulk}} = \int d^5X\sqrt{-\mathcal{G}}\left[\frac{1}{2\kappa_5^2}\mathcal{R} + \mathcal{L}_m^{(5)}\right], \tag{2}$$

and

$$\mathcal{S}_{\text{brane}} = \int_{Y=0} d^4x\sqrt{-g}\left[\frac{1}{\kappa_5^2}K^{\pm} + \mathcal{L}_{\text{brane}}(g_{\alpha\beta}, \psi)\right], \tag{3}$$

where $\kappa_5^2 = 8\pi G_5$ is the 5D gravitational constant, and $\mathcal{R}$ and $\mathcal{L}_m^{(5)}$ are the 5D scalar curvature and the matter Lagrangian in the bulk, respectively. Also, x^μ ($\mu = 0, 1, 2,$ and 3) are the induced 4D coordinates on the brane, $K^{\pm}$ is the trace of extrinsic curvature on either side of the brane [27], and $\mathcal{L}_{\text{brane}}(g_{\alpha\beta}, \psi)$ is the effective 4D Lagrangian, which is given by a generic functional of the brane metric $g_{\alpha\beta}$ and matter fields.

The 5D Einstein field equations are given by

$$\mathcal{R}_{AB} - \tfrac{1}{2}\mathcal{R}\mathcal{G}_{AB} = \kappa_5^2[T_{AB}^{(5)} + \delta(Y)\tau_{AB}], \tag{4}$$

where

$$T_{AB}^{(5)} \equiv -2\frac{\delta\mathcal{L}_m^{(5)}}{\delta\mathcal{G}^{AB}} + \mathcal{G}_{AB}\mathcal{L}_m^{(5)}, \tag{5}$$

and

$$\tau_{\mu\nu} \equiv -2\frac{\delta\mathcal{L}_{\text{brane}}}{\delta g^{\mu\nu}} + g_{\mu\nu}\mathcal{L}_{\text{brane}}. \tag{6}$$

We study the case where the induced gravity on the brane is due to the quantum corrections. The interaction between the bulk gravity and local matter induces gravity on the brane through its quantum effects. If we take into account quantum effects of the matter fields confined to the brane, the gravitational action on the brane is modified as

$$\mathcal{L}_{\text{brane}}(g_{\alpha\beta}, \psi) = \frac{\mu^2}{2}R - \lambda + \mathcal{L}_m, \tag{7}$$

where μ is a mass scale which may correspond to the 4D Planck mass, λ is the tension of the brane, and $\mathcal{L}_m$ presents the Lagrangian of the matter fields on the brane. We note that for $\lambda = 0$ and $\Lambda^{(5)} = 0$, Eq. (1) gives the DGP model and the RS II model if $\mu = 0$.

We obtain the gravitational field equations on the braneworld as

$$G_{\mu\nu} = \frac{2\kappa_5^2}{3}\left[T_{AB}^{(5)}g_\mu^A g_\nu^B + g_{\mu\nu}\left(T_{AB}^{(5)}n^A n^B - \frac{1}{4}T^{(5)}\right)\right] + \kappa_5^4\pi_{\mu\nu} - \mathcal{E}_{\mu\nu}, \tag{8}$$

$$\nabla_\nu \tau_\mu^\nu = -2T_{AB}^{(5)}n^A g_\mu^B, \tag{9}$$

where ∇_ν is the covariant derivative with respect to $g_{\mu\nu}$ and the quadratic correction has the form

$$\pi_{\mu\nu} = -\frac{1}{4}\tau_{\mu\alpha}\tau_\nu^\alpha + \frac{1}{12}\tau\tau_{\mu\nu} + \frac{1}{8}g_{\mu\nu}\tau^{\alpha\beta}\tau_{\alpha\beta} - \frac{1}{24}g_{\mu\nu}\tau^2, \tag{10}$$

and the projection of the bulk Weyl tensor to the surface orthogonal to n^A is given by

$$\mathcal{E}_{\mu\nu} = C_{ABCD}^{(5)}n^A n^B g^C{}_\mu g^D{}_\nu. \tag{11}$$

In order to find the basic field equations on the brane with induced gravity, we have to obtain the energy-momentum tensor of the brane $\tau_{\mu\nu}$, given by definition (14) from the Lagrangian (7), yielding

$$\tau_\nu^\mu = -\lambda\delta_\nu^\mu + T_\nu^\mu - \mu^2 G_\nu^\mu, \tag{12}$$

where

$$T_{\mu\nu} \equiv -2\frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}} + g_{\mu\nu}\mathcal{L}_m \tag{13}$$

is the energy-momentum tensor of the matter fields on the brane and $G_{\mu\nu}$ is defined as

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu}. \tag{14}$$

Assuming that the 5D bulk space includes only a cosmological constant $\Lambda^{(5)}$ and inserting Eq. (12) into Eq. (8), we find the effective field equations for the 4D metric $g_{\mu\nu}$ as

$$\begin{aligned}\left(1 + \frac{\lambda}{6}\kappa_5^4\mu^2\right)G_{\mu\nu} = {}& \frac{\lambda}{6}\kappa_5^4 T_{\mu\nu} - \Lambda g_{\mu\nu} \\ & - \kappa_5^4\mu^2\mathcal{K}_{\mu\nu\alpha\beta}G^{\alpha\beta} \\ & + \kappa_5^4[\pi_{\mu\nu}^{(T)} + \mu^4\pi_{\mu\nu}^{(G)}] - \mathcal{E}_{\mu\nu},\end{aligned} \tag{15}$$

where

$$\begin{aligned}\mathcal{K}_{\mu\nu\rho\sigma} = {}& \frac{1}{4}(g_{\mu\nu}T_{\rho\sigma} - g_{\mu\rho}T_{\nu\sigma} - g_{\nu\sigma}T_{\mu\rho}) + \frac{1}{12} \\ & \times [T_{\mu\nu}g_{\rho\sigma} + T(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\nu}g_{\rho\sigma})],\end{aligned} \tag{16}$$

$$\begin{aligned}\pi_{\mu\nu}^{(T)} = {}& -\frac{1}{4}T_{\mu\alpha}T_{\alpha\nu} + \frac{1}{12}TT_{\mu\nu} + \frac{1}{8}g_{\mu\nu}T_{\alpha\beta}T^{\alpha\beta} \\ & - \frac{1}{24}g_{\mu\nu}T^2,\end{aligned} \tag{17}$$

$$\begin{aligned}\pi_{\mu\nu}^{(G)} = {}& -\frac{1}{4}G_{\mu\alpha}G_{\alpha\nu} + \frac{1}{12}GG_{\mu\nu} + \frac{1}{8}g_{\mu\nu}G_{\alpha\beta}G^{\alpha\beta} \\ & - \frac{1}{24}g_{\mu\nu}G^2,\end{aligned} \tag{18}$$

and the effective cosmological constant on the brane is given by

$$\Lambda = \frac{\kappa_5^2}{2}\left[\Lambda^{(5)} + \frac{1}{6}\kappa_5^2\lambda^2\right]. \tag{19}$$

Note that for $\mu = 0$ and $\kappa_4^2 = \frac{\kappa_5^4\lambda}{6}$, these equations are exactly the same effective equations as in Ref. [28].

III. COSMOLOGY WITH BIANCHI TYPE I BRANE

In the following we will investigate the influence of the induced gravity on the anisotropic universe described by Bianchi type I geometry. We reduce the model to the generalized RS II model and compare its results to the warped DGP braneworld model.

The line element of a Bianchi type I space-time, which generalizes the flat Robertson-Walker metric to the anisotropic case, is described by

$$ds^2 = -dt^2 + a_1^2(t)dx^2 + a_2^2(t)dy^2 + a_3^2(t)dz^2, \tag{20}$$

where $a_i(t)$, $i = 1$, 2, and 3 are the expansion factors in different spatial directions. For later convenience we define the following variables:

$$\upsilon = \prod_{i=1}^3 a_i, \qquad H_i = \frac{\dot{a}_i}{a_i}, \quad i = 1, 2, 3, \qquad 3H = \sum_{i=1}^3 H_i,$$
$$\Delta H_i = H_i - H, \quad i = 1, 2, 3. \tag{21}$$

In the above equation, υ is the volume scale factor, H_i, $i = 1$, 2, and 3 are the directional Hubble parameters, and H is the mean Hubble parameter. The physical quantities of observational importance in cosmology are the expansion scalar Θ, the mean anisotropy parameter A, the shear scalar σ^2, and the deceleration parameter q, which are defined as

$$\Theta = 3H, \tag{22}$$

$$A = \frac{1}{3}\sum_{i=1}^3\left(\frac{\Delta H_i}{H}\right)^2, \tag{23}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\sum_{i=1}^3 H_i^2 - 3H^2, \tag{24}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -\frac{1}{H^2}(\dot{H} + H^2). \tag{25}$$

The sign of the deceleration parameter indicates how the Universe expands. A positive sign for q corresponds to the standard decelerating models, whereas a negative sign indicates inflation. We also note that $A = 0$ for an isotropic expansion.

We assume that the confined matter source on the brane is the perfect fluid in which its pressure obeys a linear barotropic equation of state $p = (\gamma - 1)\rho$ with $1 \leq \gamma \leq 2$, where $\gamma = 2$ represents the stiff cosmological fluid. In this paper we also restrict our analysis to a conformally flat bulk geometry with $C_{ABCD} = 0$.

With use of the Codazzi equation and the Bianchi identity, from Eq. (12) we obtain

$$\nabla^\nu T_{\mu\nu} = 0, \tag{26}$$

which gives

$$\dot{\rho} + 3\gamma H\rho = 0. \tag{27}$$

Thus the time evolution of the energy density of the matter is given by

$$\rho = \rho_0\upsilon^{-\gamma}. \tag{28}$$

Using the variables (21) the gravitational field equations on the brane take the form

$$\left(3\dot{H}+\sum_{i=1}^{3}H_i^2\right)\left[1+\frac{\lambda}{6}\kappa_5^4\mu^2+\frac{1}{6}\kappa_5^4\mu^2\rho+\frac{1}{3}\kappa_5^4\mu^4(H_1H_2+H_1H_3+H_2H_3)\right]$$
$$-\frac{1}{6}\kappa_5^4\mu^4[H_1^2H_2^2+H_1^2H_3^2+H_2^2H_3^2+2H_1^2H_2H_3+2H_2^2H_1H_3+2H_3^2H_1H_2]=\Lambda-\frac{(3\gamma-2)}{2}\frac{\lambda\kappa_5^4}{6}\rho-\frac{(3\gamma-1)\kappa_5^4\rho^2}{12}, \tag{29}$$

and

$$\left[1+\frac{\lambda}{6}\kappa_5^4\mu^2+2\kappa_5^4\mu^4(\dot{H}_2+H_2^2+\dot{H}_3+H_3^2+H_2H_3)\right]\frac{1}{v}\frac{d}{dt}(vH_1)+\frac{1}{12}\kappa_5^4\mu^2\rho\left[\frac{1}{v}\frac{d}{dt}(vH_2+vH_3)\right]$$
$$=\Lambda-\frac{(\gamma-2)}{2}\frac{\lambda\kappa_5^4}{6}\rho-\frac{(\gamma-1)}{12}\kappa_5^4\rho^2+\kappa_5^4\mu^4(\dot{H}_2+H_2^2+\dot{H}_3+H_3^2+H_2H_3)^2$$
$$-\frac{\kappa_5^4\mu^2}{12}p[\dot{H}_2+H_2^2+\dot{H}_3+H_3^2-2(\dot{H}_1+H_1^2)-(H_1H_2+H_1H_3-2H_2H_3)], \tag{30}$$

$$\left[1+\frac{\lambda}{6}\kappa_5^4\mu^2+2\kappa_5^4\mu^4(\dot{H}_1+H_1^2+\dot{H}_3+H_3^2+H_1H_3)\right]\frac{1}{v}\frac{d}{dt}(vH_2)+\frac{1}{12}\kappa_5^4\mu^2\rho\left[\frac{1}{v}\frac{d}{dt}(vH_1+vH_3)\right]$$
$$=\Lambda-\frac{(\gamma-2)}{2}\frac{\lambda\kappa_5^4}{6}\rho-\frac{(\gamma-1)}{12}\kappa_5^4\rho^2+\kappa_5^4\mu^4(\dot{H}_1+H_1^2+\dot{H}_3+H_3^2+H_1H_3)^2$$
$$-\frac{\kappa_5^4\mu^2}{12}p[\dot{H}_1+H_1^2+\dot{H}_3+H_3^2-2(\dot{H}_2+H_2^2)-(H_1H_2+H_2H_3-2H_1H_3)], \tag{31}$$

$$\left[1+\frac{\lambda}{6}\kappa_5^4\mu^2+2\kappa_5^4\mu^4(\dot{H}_1+H_1^2+\dot{H}_2+H_2^2+H_1H_2)\right]\frac{1}{v}\frac{d}{dt}(vH_3)+\frac{1}{12}\kappa_5^4\mu^2\rho\left[\frac{1}{v}\frac{d}{dt}(vH_1+vH_2)\right]$$
$$=\Lambda-\frac{(\gamma-2)}{2}\frac{\lambda\kappa_5^4}{6}\rho-\frac{(\gamma-1)}{12}\kappa_5^4\rho^2+\kappa_5^4\mu^4(\dot{H}_1+H_1^2+\dot{H}_2+H_2^2+H_1H_2)^2$$
$$-\frac{\kappa_5^4\mu^2}{12}p[\dot{H}_1+H_1^2+\dot{H}_2+H_2^2-2(\dot{H}_3+H_3^2)-(H_1H_3+H_2H_3-2H_1H_2)]. \tag{32}$$

By summing Eqs. (30)–(32) we find

$$\left[1+\frac{\lambda}{6}\kappa_5^4\mu^2+2\kappa_5^4\mu^4(\dot{H}_2+H_2^2+\dot{H}_3+H_3^2+H_2H_3)\right]\frac{1}{v}\frac{d}{dt}(vH_1)$$
$$+\left[1+\frac{\lambda}{6}\kappa_5^4\mu^2+2\kappa_5^4\mu^4(\dot{H}_1+H_1^2+\dot{H}_3+H_3^2+H_1H_3)\right]\frac{1}{v}\frac{d}{dt}(vH_2)$$
$$+\left[1+\frac{\lambda}{6}\kappa_5^4\mu^2+2\kappa_5^4\mu^4(\dot{H}_1+H_1^2+\dot{H}_2+H_2^2+H_1H_2)\right]\frac{1}{v}\frac{d}{dt}(vH_3)+\frac{1}{6}\kappa_5^4\mu^2\rho\left[\frac{1}{v}\frac{d}{dt}(vH_1+vH_2+vH_3)\right]$$
$$=3\Lambda-\frac{(\gamma-2)}{4}\lambda\kappa_5^4\rho-\frac{(\gamma-1)}{4}\kappa_5^4\rho^2+\kappa_5^4\mu^4[(\dot{H}_2+H_2^2+\dot{H}_3+H_3^2+H_2H_3)^2$$
$$+(\dot{H}_1+H_1^2+\dot{H}_3+H_3^2+H_1H_3)^2+(\dot{H}_1+H_1^2+\dot{H}_2+H_2^2+H_1H_2)^2]. \tag{33}$$

As we noted before, assuming $\mu=0$ and $\kappa_4^2=\frac{\kappa_5^4\lambda}{6}$, the effective field equation (15) reduces to the gravitational field equations in the generalized Randall-Sundrum II model; thus we find that Eqs. (30)–(33) reduce to the following equations, respectively [29]:

$$\frac{1}{v}\frac{d}{dt}(vH_i)=\Lambda-\frac{(\gamma-2)}{2}\kappa_4^2\rho-\frac{(\gamma-1)}{12}\kappa_5^4\rho^2,\quad i=1,2,3, \tag{34}$$

$$\frac{1}{v}\frac{d}{dt}(3vH)=3\Lambda-\frac{3(\gamma-2)}{2}\kappa_4^2\rho-\frac{(\gamma-1)}{4}\kappa_5^4\rho^2. \tag{35}$$

Now, by substituting Eq. (35) into Eq. (34), we obtain

$$H_i=H+\frac{K_i}{v},\qquad i=1,2,3, \tag{36}$$

with K_i being constants of integration satisfying the consistency condition $\sum_{i=1}^{3}K_i=0$. By using relation $H=\frac{\dot{v}}{3v}$, Eq. (35) describing the dynamics of the anisotropic brane-

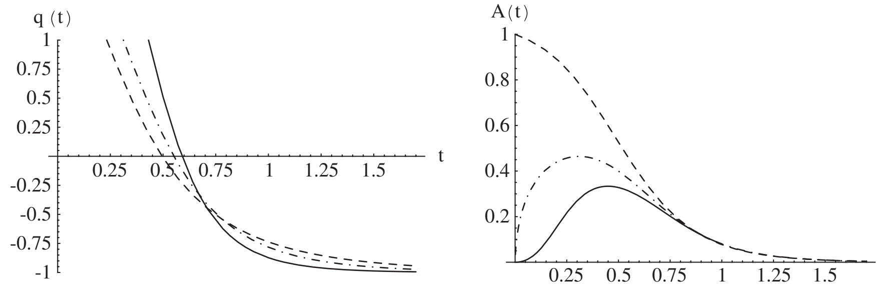

FIG. 1. (Left panel) The deceleration parameter q of the Bianchi type I brane universe with confined perfect cosmological fluid. (Right panel) The anisotropy parameter A of the Bianchi type I brane universe for $\gamma = 2$ (solid curve), $\gamma = 4/3$ (dot-dashed curve), and $\gamma = 1$ (dashed curve) as a function of time in the generalized RS II model.

world can be written as

$$\ddot{v} = 3\Lambda v - \frac{3(\gamma - 2)}{2}\kappa_4^2\rho_0 v^{1-\gamma} - \frac{(\gamma - 1)}{4}\kappa_5^4\rho_0^2 v^{1-2\gamma}. \tag{37}$$

In this case the behavior of the expansion scalar, mean anisotropy parameter, shear scalar, and deceleration parameter in the Bianchi type I universe can be expressed in the following exact parametric form, with $v \geq 0$ taken as a parameter:

$$\Theta = (3\Lambda + 3\kappa_4^2\rho_0 v^{-\gamma} + \tfrac{1}{4}\kappa_5^4\rho_0^2 v^{-2\gamma} + Cv^{-2})^{1/2}, \tag{38}$$

$$A = 3K^2(3\Lambda v^2 + 3\kappa_4^2\rho_0 v^{2-\gamma} + \tfrac{1}{4}\kappa_5^4\rho_0^2 v^{2-2\gamma} + C)^{-1}, \tag{39}$$

$$\sigma^2 = \frac{K^2}{2v^2}, \tag{40}$$

$$q = 2 - \frac{36\Lambda v^2 + 18(2-\gamma)\kappa_4^2\rho_0 v^{2-\gamma} + 3(1-\gamma)\kappa_5^4\rho_0^2 v^{2-2\gamma}}{12\Lambda v^2 + 12\kappa_4^2\rho_0 v^{2-\gamma} + \kappa_5^4\rho_0^2 v^{2-2\gamma} + 4C}, \tag{41}$$

where $K^2 = \sum_{i=1}^3 K_i^2$. The behavior of the deceleration parameter of the Bianchi type I geometry is illustrated, for different values of γ, in Fig. 1. In the initial stage the evolution of the Bianchi type I brane universe is noninflationary, but in the late time limit the brane universe ends in an accelerating stage. In Fig. 1 we have also plotted the mean anisotropy parameter for different values of γ. The behavior of the mean anisotropy parameter shows that at high densities for $\gamma = 2$ and $\gamma = 4/3$, due to the presence of the quadratic terms in the energy-momentum tensor, the brane universe starts its evolutions from an isotropic state with $A(t_0) = 0$ and ends up in an isotropic de Sitter inflationary phase at late time. For $\gamma = 1$ the quadratic contribution to the energy-momentum tensor in Eq. (37) vanishes and so the universe is born in a state of maximum anisotropy. The study of anisotropic homogeneous braneworld cosmological models has shown an important difference between these models and standard 4D general relativity, namely, that brane universes are born in an isotropic state [29]. The time variation of the shear parameter is represented, for different values of γ, in Fig. 2.

Now, we are going to express the full warped DGP braneworld model in a parametric form of the volume scale factor. Using relation $2r_c = \frac{\kappa_5^2}{\kappa_4^2} \equiv \kappa_5^2\mu^2$ and assuming that the directional Hubble parameters is much smaller than the inverse of the crossover scale[1] $r_c H_i \ll 1$ and $r_c \dot{H}_i \ll 1$, $i = 1, 2$, and 3, from Eq. (33) we obtain

$$\frac{1}{v}\frac{d}{dt}(3vH)\left(1 + \frac{\lambda}{6}\kappa_5^4\mu^2 + \frac{1}{6}\kappa_5^4\mu^2\rho\right) = 3\Lambda - \frac{(\gamma - 2)}{4}\lambda\kappa_5^4\rho - \frac{(\gamma - 1)}{4}\kappa_5^4\rho^2. \tag{42}$$

Using relation $H = \frac{\dot{v}}{3v}$ and the energy density of the matter, we can rewrite Eq. (42) in the form

$$\ddot{v}\left(1 + \frac{\lambda}{6}\kappa_5^4\mu^2 + \frac{1}{6}\kappa_5^4\mu^2\rho_0 v^{-\gamma}\right) = 3\Lambda v - \frac{3(\gamma - 2)}{2}\frac{\lambda\kappa_5^4}{6}\rho_0 v^{1-\gamma} - \frac{(\gamma - 1)}{4}\kappa_5^4\rho_0^2 v^{1-2\gamma}. \tag{43}$$

The general solution of Eq. (43) becomes

$$t - t_0 = \int [F(v, \gamma) + C]^{-1/2} dv, \tag{44}$$

where

[1] It was shown that the expansion of the Universe enters the 5D regime when the Hubble expansion rate is much smaller than the inverse of the critical length scale, r_c^{-1} [30].

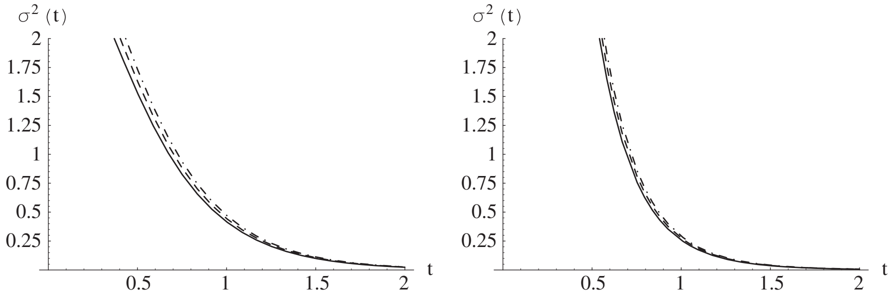

FIG. 2. (Left panel) The shear scalar σ^2 for the Bianchi type I brane universe with confined perfect cosmological fluid for $\gamma = 2$ (solid curve), $\gamma = 4/3$ (dot-dashed curve), and $\gamma = 1$ (dashed curve) as a function of time in the generalized RS II model. (Right panel) The same parameter for $\gamma = 2$ (solid curve), $\gamma = 4/3$ (dot-dashed curve), and $\gamma = 1$ (dashed curve) as a function of time in the warped DGP braneworld model.

$$\begin{aligned} F(v, \gamma) = {}& \frac{36}{\kappa_5^8 \mu^4 \rho_0^2} v^{-\gamma} \Big[\frac{\kappa_5^4 \rho_0^2}{4} \Big(1 + \frac{\lambda}{6} \kappa_5^4 \mu^2 \Big) (\gamma - 1) v^\gamma \\ & - \mu^2 \frac{\lambda \kappa_5^8 \rho_0^2}{24} (\gamma - 2) v^\gamma + \mu^2 \frac{\kappa_5^8 \rho_0^3}{12} \frac{(\gamma - 1)}{(\gamma - 2)} \Big] \\ & - \frac{36}{\kappa_5^8 \mu^4 \rho_0^2} v^{-\gamma} {}_2F_1 \Big(-\frac{2}{\gamma}, 1, \frac{\gamma - 2}{\gamma}, \\ & - \frac{\kappa_5^4 \mu^2 \rho_0}{6(1 + \frac{\lambda}{6} \kappa_5^4 \mu^2)} v^{-\gamma} \Big) \Big[\frac{\kappa_5^4 \rho_0^2}{4} \Big(1 + \frac{\lambda}{6} \kappa_5^4 \mu^2 \Big) \\ & \times (\gamma - 1) v^\gamma - \mu^2 \frac{\lambda \kappa_5^8 \rho_0^2}{24} (\gamma - 2) v^\gamma \\ & - \frac{\Lambda \kappa_5^8 \mu^4 \rho_0^2}{12(1 + \frac{\lambda}{6} \kappa_5^4 \mu^2)} v^\gamma \Big], \end{aligned} \tag{45}$$

and here C is a constant of integration. The time variations of the physically important parameters described above in the exact parametric form for $\gamma = 1$, with v taken as a parameter, are given by

$$\Theta = [f(v) v^{-2} + C v^{-2}]^{1/2}, \tag{46}$$

$$A = \frac{3h^2}{[f(v) + C]}, \tag{47}$$

$$\sigma^2 = \frac{h^2}{2v^2}, \tag{48}$$

$$\begin{aligned} q = {}& 2 \\ & - \frac{3v[3\Lambda v - \frac{3(\gamma-2)}{2} \frac{\lambda \kappa_5^4}{6} \rho_0 v^{1-\gamma} - \frac{(\gamma-1)}{4} \kappa_5^4 \rho_0^2 v^{1-2\gamma}]}{[f(v) + C][1 + \frac{\lambda}{6} \kappa_5^4 \mu^2 + \frac{1}{6} \kappa_5^4 \mu^2 \rho_0 v^{-\gamma}]}, \end{aligned} \tag{49}$$

where h^2 is a constant of integration and $f(v) \equiv F(v, \gamma = 1)$. We consider $\lambda = 0$ and show that the extra terms in the warped DGP braneworld model can be used to account for the accelerated expansion of the anisotropic universe. In Fig. 3 we have plotted the dynamics of the deceleration parameter for $\gamma = 1$. As mentioned before, $q(t) > 0$ corresponds to the standard decelerating models, whereas $q(t) < 0$ indicates an accelerating expansion in late times. Therefore, the Universe undergoes an accelerated expansion at late times in the absence of a positive cosmological constant.

The behavior of the mean anisotropy parameter of the Bianchi type I geometry is illustrated, for $\gamma = 1$, in Fig. 3. The behavior of this parameter shows that the Universe starts from a singular state with maximum anisotropy and ends up in an isotropic de Sitter inflationary phase at late time. In Fig. 4, we compare the behavior of the anisotropy parameter for the generalized RS II model and warped

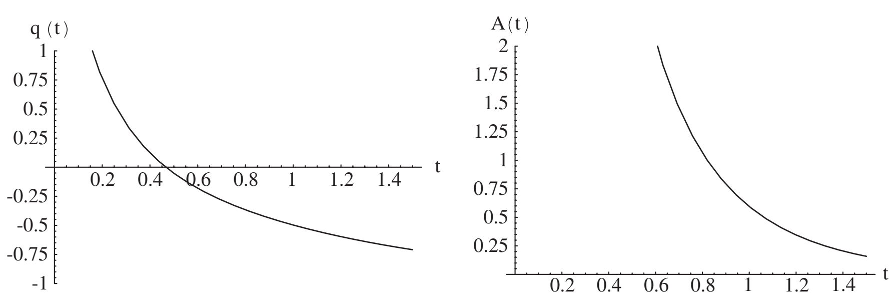

FIG. 3. (Left panel) The deceleration parameter q of the Bianchi type I brane universe with confined perfect cosmological fluid. (Right panel) The anisotropy parameter A of the Bianchi type I brane universe for $\gamma = 1$ as a function of time in the warped DGP braneworld model with $\lambda = 0$.

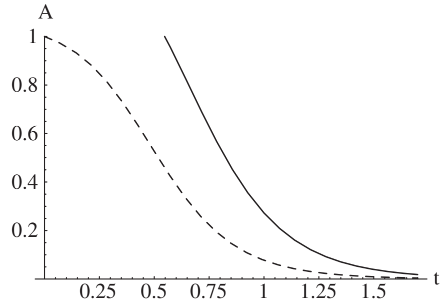

FIG. 4. The anisotropy parameter A of the Bianchi type I brane universe for $\gamma = 1$ as a function of time in the generalized RS II model (dashed curve) and the warped DGP braneworld model (solid curve).

DGP braneworld model. Also, the time variation of the shear parameter is represented, for different values of γ, in Fig. 2.

IV. CONCLUSIONS

The braneworld model of DGP provides an interesting alternative to a positive cosmological constant by modifying gravity at a large distance. In this paper we have studied the time variation of the physically important parameters on the Bianchi type I cosmological model based on the warped DGP braneworld scenario. We have shown that the extra terms in the effective gravitational field equations on the warped DGP brane can be used to account for the accelerated expansion of an anisotropic universe.

The study of the behavior of the anisotropy parameter shows that in a DGP model, the brane universe starts its evolution from a singular state with maximum anisotropy and reaches, for Bianchi type I space-time, an isotropic state in the late time limit. It has been found that isotropization can proceed slower in the warped DGP model than the generalized RS II model.

Finally, we mention that the self-accelerating branch of the original DGP model is unstable due to the existence of a ghost [31]. There were attempts to merge the DGP proposal and models with warped backgrounds to get a long distance modification of gravity. But it turned out that it is unlikely to cure the problems of the DGP gravity by invoking branes of nonzero tension and warped bulk space[2] [32,33]. However, as has been argued in [34], the generalized version of the DGP scenario can be ghost free and can give rise to transient acceleration [35]. It needs further justification which is beyond the scope of the present paper.

[2]For a comprehensive study of the general behavior of the self-accelerating warped braneworlds and a compression of the original DGP model, see [32].

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