

Electrically charged strange quark stars

Rodrigo Picanço Negreiros* and Fridolin Weber†

Department of Physics, San Diego State University, 5500 Campanile Drive, San Diego, California 92182, USA

Manuel Malheiro‡

Instituto Tecnológico da Aeronáutica, Praça Marechal Eduardo Gomes, 50, Vila das Acácias, São José dos Campos, São Paulo, Brazil

Vladimir Usov§

Center for Astrophysics, Weizmann Institute, Rehovot 76100, Israel

(Received 30 July 2009; published 12 October 2009)

The possible existence of compact stars made of absolutely stable strange quark matter—referred to as strange stars—was pointed out by Witten almost a quarter of a century ago. One of the most amazing features of such objects concerns the possible existence of ultrastrong electric fields on their surfaces, which, for ordinary strange matter, is around 10^{18} V/cm. If strange matter forms a color superconductor, as expected for such matter, the strength of the electric field may increase to values that exceed 10^{19} V/cm. The energy density associated with such huge electric fields is on the same order of magnitude as the energy density of strange matter itself, which, as shown in this paper, alters the masses and radii of strange quark stars at the 15% and 5% levels, respectively. Such mass increases facilitate the interpretation of massive compact stars, with masses of around $2M_{\odot}$, as strange quark stars.

DOI: [10.1103/PhysRevD.80.083006](https://doi.org/10.1103/PhysRevD.80.083006)

PACS numbers: 21.65.Qr, 26.60.Gj, 97.10.Cv, 97.60.Jd

I. INTRODUCTION

The possibility that strange quark matter (strange matter for short) [1–4], made up of roughly equal numbers of unconfined up, down, and strange quarks, may be the absolute ground state of the strong interaction is known as the strange quark matter hypothesis [5–7]. In the latter event objects made of strange matter—ranging from strangelets at the low baryon number end [1,8–15] to compact stars at the high baryon number end [2–4,16–21]—would be more stable than their nonstrange counterparts—atomic nuclei and neutron stars. On theoretical scale arguments, strange matter is as plausible a ground state as the confined state of hadrons [1,3,6,22]. Despite several decades of research, both theoretical and experimental, there is no sound scientific basis on which one could either confirm or reject the hypothesis so that it remains a serious possibility of fundamental significance for physics and astrophysics [16]. One very striking consequence of the hypothesis is the prediction of the existence of a new class of compact stars called strange (quark) stars [2–4,23]. The heavier members of this hypothetical family of compact stars have masses and radii similar to those of neutron stars. In contrast to neutron stars, however, strange stars would form a distinct and disconnected branch of compact stars and are not part of the continuum of equilibrium configurations that include white dwarfs and neutron stars [11–13].

If strange stars should exist in the Universe, they ought to be made of chemically equilibrated strange matter, which requires the presence of electrons inside strange stars. The presence of electrons plays a crucial role for strange stars, since they may cause the formation of an electric dipole layer on the surfaces of such stars leading to huge electric fields on the order of 10^{18} V/cm [2,3]. The situation is even more extreme if strange stars were made of color superconducting strange matter [24], which could be either in the color-flavor-locked (CFL) phase [25–29] or in the 2-flavor color superconducting phase [25,26,30]. In the latter event the electric fields on the surfaces of quark stars may even be on the order of 10^{19} V/cm [31,32], depending on electrostatic effects, including Debye screening, and the surface tension of the interface between vacuum and quark matter [33,34]. The energy density $E^2/8\pi$ of such tremendously large electric fields is on the same order of magnitude as the energy density of strange quark matter itself and, thus, should be incorporated in the energy-momentum tensor that is used to describe strange quark stars. This paper outlines how this is accomplished mathematically and discusses the consequences of this extra contribution for the bulk properties of strange stars.

II. GENERAL RELATIVISTIC STELLAR STRUCTURE

Our discussion is performed for spherically symmetric strange quark matter stars, whose metric is specified by the line element ($ds^2 = g_{\nu\mu} dx^\nu dx^\mu$, where $\nu, \mu = 0, 1, 2, 3$),

$$ds^2 = e^{\Phi(r)} c^2 dt^2 - e^{\Lambda(r)} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (1)$$

*negreiros@sciences.sdsu.edu

†fweber@sciences.sdsu.edu

‡malheiro@ita.br

§Vladimir.Usov@weizmann.ac.il

The properties of the stellar matter enter Einstein's field equation, $G_{\nu}^{\mu} = (8\pi/c^4)T_{\nu}^{\mu}$, through the energy-momentum tensor, T_{ν}^{μ} , which for electrically uncharged matter is given by

$$T_{\nu}^{\mu} = (P + \rho c^2)u_{\nu}u^{\mu} + P\delta_{\nu}^{\mu}, \quad (2)$$

where P is the pressure, $\epsilon = \rho c^2$ the energy density of strange matter, and u^{ν} its contravariant four velocity. The equation of state of strange matter is computed from the MIT bag model,

$$P = (\epsilon - 4B)/3, \quad (3)$$

for a bag constant of $B^{1/4} = 150$ MeV. This places the energy of strange matter at around 870 MeV, well below the energy per particle of the most stable atomic nucleus, ^{56}Fe , as well as of infinite nuclear matter [1]. The presence of strong electric stellar fields, as considered in this paper, renders the expression of the energy-momentum tensor Eq. (2) significantly more complicated [35,36],

$$T_{\nu}^{\mu} = (P + \rho c^2)u_{\nu}u^{\mu} + P\delta_{\nu}^{\mu} + \frac{1}{4\pi} \left(F^{\mu l} F_{\nu l} + \frac{1}{4\pi} \delta_{\nu}^{\mu} F_{kl} F^{kl} \right), \quad (4)$$

where $F^{\nu\mu}$ is the electromagnetic field tensor. The latter satisfies the covariant Maxwell equations,

$$[(-g)^{1/2} F^{\nu\mu}]_{,\mu} = 4\pi j^{\nu} (-g)^{1/2}, \quad (5)$$

where j^{ν} stands for the electromagnetic four-current and $g \equiv \det(g^{\nu\mu})$. For static stellar configurations, which are considered in this paper, the only nonvanishing component of the four-current is j^0 . Because of symmetry reasons, the four-current is only a function of radial distance, r , and all components of the electromagnetic field tensor vanish, with the exception of F^{01} and F^{10} , which describe the radial component of the electric field. From Eq. (5) one obtains the following expression for the electric field:

$$E(r) = F^{01}(r) = \frac{1}{r^2} e^{-(\Phi+\Lambda)/2} 4\pi \int_0^r r'^2 \rho_{\text{ch}} e^{\Lambda/2} dr', \quad (6)$$

where $\rho_{\text{ch}} = e^{\Phi/2} j^0(r)$ represents the electric charge distribution inside the star. The electric charge within a sphere of radius r is given by

$$Q(r) = 4\pi \int_0^r r'^2 \rho_{\text{ch}} e^{\Lambda/2} dr', \quad (7)$$

which can be interpreted as the relativistic version of Gauss's law. With the aid of Eqs. (5)–(7) the energy-momentum tensor can be written as

$$T_{\nu}^{\mu} = \begin{pmatrix} -(\epsilon + \frac{Q^2}{8\pi r^4}) & 0 & 0 & 0 \\ 0 & P - \frac{Q^2}{8\pi r^4} & 0 & 0 \\ 0 & 0 & P + \frac{Q^2}{8\pi r^4} & 0 \\ 0 & 0 & 0 & P + \frac{Q^2}{8\pi r^4} \end{pmatrix}, \quad (8)$$

where the electric charge is connected to the electric field through the relation $Q^2(r)/8\pi r^4 = E^2(r)/8\pi$. Substituting Eq. (8) into Einstein's field equation leads to

$$e^{-\Lambda} \left(\frac{1}{r^2} - \frac{1}{r} \frac{d\Lambda}{dr} \right) - \frac{1}{r^2} = -\frac{8\pi G}{c^4} \left(\epsilon + \frac{Q^2}{8\pi r^4} \right), \quad (9)$$

$$e^{-\Lambda} \left(\frac{1}{r} \frac{d\Phi}{dr} + \frac{1}{r^2} \right) - \frac{1}{r^2} = -\frac{8\pi G}{c^4} \left(p - \frac{Q^2}{8\pi r^4} \right). \quad (10)$$

In analogy to the electrically uncharged case, next we define

$$e^{-\Lambda}(r) \equiv 1 - \frac{Gm(r)}{rc^2} + \frac{GQ^2(r)}{r^2 c^4}. \quad (11)$$

Equations (9) and (11) then lead to [37,38]

$$\frac{dm}{dr} = \frac{4\pi r^2}{c^2} \epsilon + \frac{Q}{c^2 r} \frac{dQ}{dr}, \quad (12)$$

where $m(r)$ is the gravitational mass contained in a sphere of radius r . The first term on the right-hand side of Eq. (12) comes from the mass energy of the stellar matter (quark matter in our case), while the second term on the right-hand side has its origin in the mass energy of the electric field carried by the electrically charged quark star.

The hydrostatic equilibrium equation that determines the global structure of electrically charged quark stars is obtained by requiring the conservation of mass energy, $T_{\nu}^{\mu};_{\mu} = 0$. This leads to [37]

$$\frac{dP}{dr} = -\frac{2G(m + \frac{4\pi r^3}{c^2} (P - \frac{Q^2}{4\pi r^4 c^2}))}{c^2 r^2 (1 - \frac{2Gm}{c^2 r} + \frac{GQ^2}{r^2 c^4})} (P + \epsilon) + \frac{Q}{4\pi r^4} \frac{dQ}{dr}. \quad (13)$$

The standard stellar structure equation of electrically uncharged stars, known as the Tolman-Oppenheimer-Volkoff (TOV) equation, is obtained from Eq. (13) for $Q \rightarrow 0$. We also note that if gravity were switched off, Eq. (7) leads to

$$\nabla^2 \mu_e = 4\pi e^2 (n_q - n_e), \quad (14)$$

with n_q and n_e denoting the electric charge distributions of quarks and electrons of strange matter, respectively. For a gas of free electrons one has $n_e = \mu_e^3/3\pi^2$ so that Eq. (14) becomes the Poisson equation

$$\nabla^2 \mu_e = 4\pi e^2 (n_q - \mu_e^3/3\pi^2), \quad (15)$$

which determines the electron chemical potential for given electric quark charge densities n_q .

III. MODELING THE ELECTRIC CHARGE DISTRIBUTION

Our goal is to explore the influence of huge electric fields on the bulk properties of strange stars. As already described at the beginning of this paper, the electric charge distribution that is causing these huge electric fields on strange stars is located in the immediate surface regions of such objects. We model this distribution in terms of a Gaussian which is centralized at the surface of a strange star. This is accomplished by the following ansatz:

$$\rho_{\text{ch}}(r) = \kappa \exp(-((r - r_g)/b)^2), \quad (16)$$

where b is a charge constant describing the width of the Gaussian, and r_g is the radial distance at which the charge distribution is centralized. The quantity κ is a normalization constant to be determined such that

$$4\pi \int_{-\infty}^{+\infty} \rho_{\text{ch}}(r) r^2 dr = \sigma, \quad (17)$$

where σ is a constant proportional to the magnitude of the electric charge distribution. In flat space-time, σ would be the total electric charge of the system. This is not the case here, however, as the electric charge Q depends on the metric [see Eq. (7)]. In the latter event the total electric charge of the system can only be computed self-consistently. Substituting Eq. (16) into Eq. (17) leads to

$$8\pi\kappa = \sigma(\sqrt{\pi}b^3/4 + r_g b^2 + \sqrt{\pi}r_g^2 b/2)^{-1}, \quad (18)$$

which establishes a connection between κ and σ .

IV. RESULTS

A. Parameters and boundary conditions

Having derived the equations that describe electrically charged strange stars, we now proceed to solve them numerically and discuss their solutions. We begin with discussing the boundary conditions of the TOV equation of electrically charged stars, Eq. (13). These are (1) $\epsilon(r=0) = \epsilon_c$, which specifies the energy density at the center of the star, (2) $Q(r=0) = 0$, which ensures that the electric charge is zero at the star's center, (3) $m(r=0) = 0$, which ensures that the gravitational mass is zero at the star's center, and (4) $P(R) = 0$, which defines the surface of the star located at $r = R$. Equations (3), (7), (12), (13), and (16) can then be solved for the mass-radius relationship of electrically charged strange stars shown in Fig. 1, computed for a range of different values for σ . The curve for which $\sigma = 0$ describes electrically uncharged strange stars. For numerical reasons a sample value of 0.001 km was chosen for the width b of the electric charge distribution on the surface of a strange star. This value is many orders of magnitude greater than the true width of the electric charge distribution, which is only around $\sim 10^3$ fm. This particular

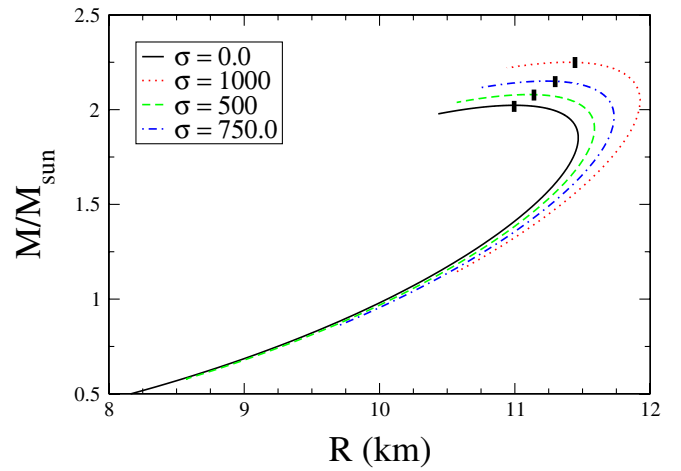


FIG. 1 (color online). Mass-radius relationship of electrically charged strange stars. Tick marks denote the maximum-mass star of each sequence, whose properties are given in Table I.

choice, however, has no influence on the numerical outcome, which can be seen by inspecting

$$\frac{dQ}{dr} = \frac{r^2 \sigma \exp(-((r - r_g)/b)^2) \exp(\Lambda/2)}{2(\sqrt{\pi}b^3/4 + r_g b^2 + \sqrt{\pi}r_g^2 b/2)}, \quad (19)$$

which follows from Eqs. (7), (16), and (18). Equation (19) reveals that $dQ/dr \rightarrow \sigma(r/r_g)^2 \exp(\Lambda/2) \delta(r - r_g)$ as the width goes to zero, $b \rightarrow 0$. Hence, the dQ/dr term in the TOV equation (13) is independent of the width of the electric charge distribution, provided it is sufficiently narrowly distributed as is the case for strange stars.

B. Sequences of electrically charged strange stars

Figure 1 shows the mass-radius relationship of electrically charged strange quark stars for different values of the electric charge constant σ . The masses and radii of such stars change at the 10% to 15% level, depending on the amount of electric charge carried by the star. Table I lists the properties of the maximum-mass star of each sequence displayed in Fig. 1. It is important to note that the electric field strengths listed in Table I are the electric fields at the surface of the quark star (where $P \rightarrow 0$). The electron layer

TABLE I. Properties of electrically charged maximum-mass strange quark stars. The quantities R and M denote their radii and gravitational masses, respectively. The stars carry given electric charges, Q , which give rise to electric stellar surface fields E .

σ	R (km)	M (M_\odot)	Q ($\times 10^{17} C$)	E (10^{19} V/cm)
0	10.99	2.02	0	0.0
500	11.1	2.07	989	7.1
750	11.2	2.15	1486	10.5
1000	11.4	2.25	1982	13.5

outside the quark star, however, nullifies the electric field, and an observer at infinity will thus not be able to detect it.

The pressure profiles of electrically charged strange quark stars shown in Fig. 2 exhibit several peculiar features which are absent in compact stars made of ordinary neutron star matter [17,18,20]. First, we note that the interior pressure profiles of quark stars are completely unaffected by the electric charge layer, since the latter is located in a thin, spherical shell near the surface of quark stars. As a consequence, the pressure decreases monotonically from the stellar center toward the surface, as it is the case for electrically uncharged quark stars and ordinary neutron stars. The situation changes drastically in the stellar surface region, however, where the electric charge distribution causes a sudden and very sharp increase in pressure, as shown in the inset of Fig. 2. This sudden increase in pressure is a result of the appearance of the ultrahigh electric fields and the drastic change in the total electric charge (dQ/dr is very high in this region) of the system, resulting in a significant positive contribution to dP/dr [see Eq. (13)]. This additional pressure, added to the system in a region that would otherwise be the surface if the star were uncharged, creates a new and qualitatively different region that surrounds any electrically charged strange quark star. Electrically charged quark stars thus possess two surfaces: a “baryonic matter surface,” where the surface of the uncharged star would be located, and an “electric surface,” where the total pressure of the star vanishes. In between these surfaces is the ultrahigh electric field region, which we will call from now on the “electrostatic layer,” since it only exists because of the electric field. These surfaces along with the electrostatic layer are shown in Fig. 3 for the sample quark star with $\sigma = 1000$ listed in Table I. The mass increase of the stars shown in Fig. 1 and Table I originates from the mass energy added to these stars by the Coulomb field.

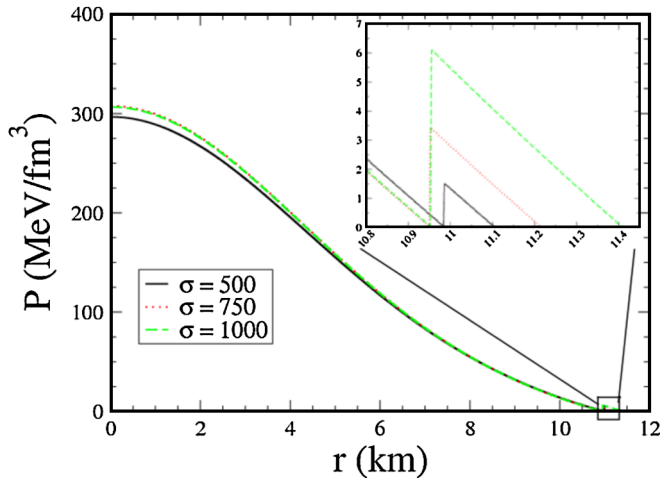


FIG. 2 (color online). Pressure profiles of electrically charged maximum-mass strange stars shown in Fig. 1.

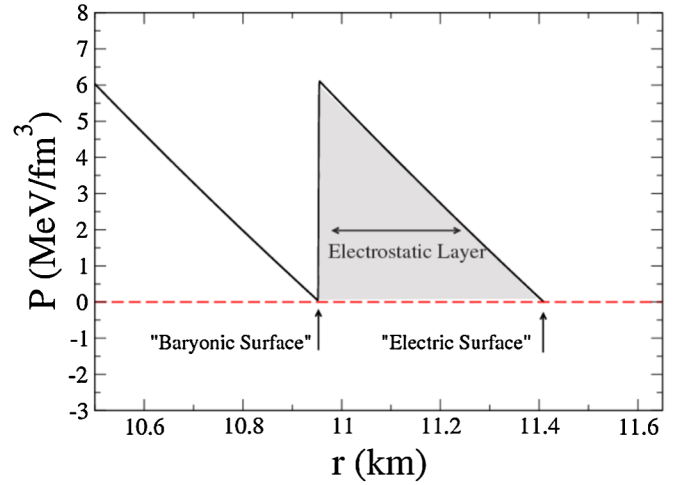


FIG. 3 (color online). Pressure profile for the maximum-mass star for the $\sigma = 1000$ sequence. We show the definitions of “baryonic surface” (where the baryon pressure goes to zero), “electric surface,” where the total pressure of the star vanishes, and the electrostatic layer between them.

Next, we discuss the electric fields of charged quark stars. The radial dependence of these fields is shown in Fig. 4 for the maximum-mass stars of Table I. As expected, the electric fields exhibit a very steep increase at the interface between the baryonic surface and the electrostatic layer. Since the electric charge is located in a thin spherical layer, the electric fields quickly weaken with increasing radial distance, and the pressure stemming from the charge contributions drops down to zero at the electric surface of the star. The electric fields are as high as 10^{19-20} V/cm. As already mentioned above, however, an observer at infinity would not be able to see these electric fields since the star is surrounded by a layer of electrons that nullifies the electric fields. An observer outside of the electric layer would thus not be able to detect it.

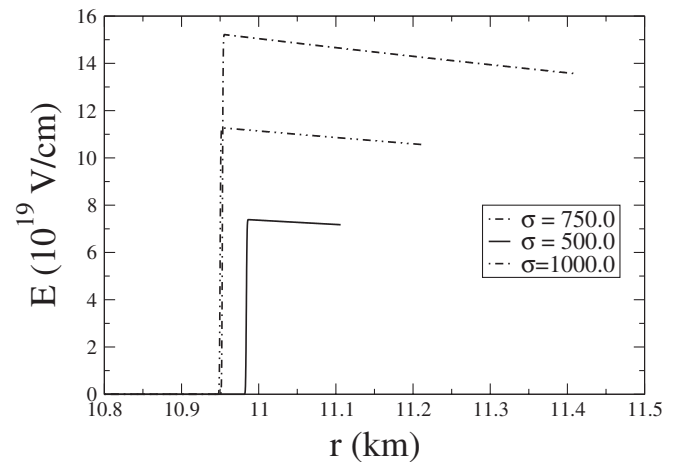


FIG. 4. Electric field profile inside maximum-mass stars (plotted in the relevant region).

V. CONCLUSIONS

Almost all of the ambient conditions that characterize compact stars tend to be extreme. This would specifically be the case for the electric fields carried by hypothetical compact stars made of absolutely stable strange quark matter (strange quark stars), which could be as high as 10^{19} – 10^{20} V/cm. In this paper, we perform a detailed investigation of the physical implications of such ultra-strong fields for the bulk properties of strange quark stars. The sources of these electric fields are electric charge distributions located on the surfaces of strange quark stars. Depending on whether these stars are made of regular (i.e. nonsuperconducting) strange quark matter or color superconducting strange quark matter, the respective charge distributions possess very different physical characteristics. CFL matter, for instance, is rigorously electrically neutral in bulk [28,39]. The situation is different for strange stars made of regular quark matter, which do possess electrons in bulk. For such stars two different electric layers would be formed [2]—a positively charged region of strange quark matter in a thin surface layer of the quark star, and an electron layer outside of the star. The core of such stars has a net positive charge and the forces acting on it are nonzero. It is this scenario that can be described by the model presented in this paper. Our study shows that the electric charge distribution can have a significant impact on the structure of quark stars. For this to happen, the pressure (or equivalently the energy density) associated with the electric field needs to amount to at least

a few percent of the typical pressure $P \sim 100$ MeV/fm³ that exists inside of quark stars. Assuming that $E^2/8\pi \sim$ a few $\times 10^{-2}P$, we thus estimate the required electric field strengths as $E \sim 10^{19-20}$ V/cm. We have shown that electric fields of this magnitude, generated by charge distributions located near the surfaces of strange quark stars, increase the stellar mass by up to 15% and the radius by up to 5%, depending on the strength of the electric field. These changes are caused by both the sudden increase of pressure in the surface regions of electrically charged quark stars, as well as by the energy density of the electric surface field which acts as an additional energy-momentum source in relativistic gravity. They facilitate the interpretation of massive compact stars, with masses of around $2M_{\odot}$, as strange quark stars.

Last but not least, another interesting feature that we have discovered is that the electric charge gradient term, dQ/dr [Eq. (19)], which emerges in the TOV equation (13) of any electrically charged compact star, is independent of the width of the electric charge distribution, provided the charge distribution is sufficiently narrowly spread over the star. This finding is not limited to strange quark stars but applies to any general relativistic stellar object that carries a narrowly concentrated electric charge distribution.

ACKNOWLEDGMENTS

This work was supported by the National Science Foundation under Grant No. PHY-55873A ARRA. M. M. acknowledges financial support from CNPq and FAPESP.

-
- [1] E. Farhi and R. L. Jaffe, Phys. Rev. D **30**, 2379 (1984).
 - [2] C. Alcock, E. Farhi, and A. V. Olinto, Astrophys. J. **310**, 261 (1986).
 - [3] C. Alcock and A. V. Olinto, Annu. Rev. Nucl. Part. Sci. **38**, 161 (1988).
 - [4] J. Madsen, Lect. Notes Phys. **516**, 162 (1999).
 - [5] A. R. Bodmer, Phys. Rev. D **4**, 1601 (1971).
 - [6] E. Witten, Phys. Rev. D **30**, 272 (1984).
 - [7] H. Terazawa, INS, Univ. of Tokyo Report No. INS-Report-338, 1979; J. Phys. Soc. Jpn. **58**, 3555 (1989); **58**, 4388 (1989); **59**, 1199 (1990).
 - [8] M. S. Berger and R. L. Jaffe, Phys. Rev. C **35**, 213 (1987).
 - [9] J. Schaffner, C. Greiner, and H. Stöcker, Phys. Rev. C **46**, 322 (1992).
 - [10] E. P. Gilson and R. L. Jaffe, Phys. Rev. Lett. **71**, 332 (1993).
 - [11] N. K. Glendenning, Ch. Kettner, and F. Weber, Astrophys. J. **450**, 253 (1995).
 - [12] N. K. Glendenning, Ch. Kettner, and F. Weber, Phys. Rev. Lett. **74**, 3519 (1995).
 - [13] Ch. Kettner, F. Weber, M. K. Weigel, and N. K. Glendenning, Phys. Rev. D **51**, 1440 (1995).
 - [14] Y. Zhang and R.-K. Su, Phys. Rev. C **67**, 015202 (2003).
 - [15] M. G. Alford and D. A. Eby, Phys. Rev. C **78**, 045802 (2008).
 - [16] F. Weber, Prog. Part. Nucl. Phys. **54**, 193 (2005).
 - [17] N. K. Glendenning, *Compact Stars, Nuclear Physics, Particle Physics, and General Relativity* (Springer-Verlag, New York, 2000), 2nd ed..
 - [18] F. Weber, *Pulsars as Astrophysical Laboratories for Nuclear and Particle Physics*, High Energy Physics, Cosmology and Gravitation Series (IOP Publishing, Bristol, 1999).
 - [19] D. Page, J. M. Lattimer, M. Prakash, and A. W. Steiner, Astrophys. J. Suppl. Ser. **155**, 623 (2004).
 - [20] A. Sedrakian, Prog. Part. Nucl. Phys. **58**, 168 (2007).
 - [21] B. Golf, J. Hellmers, and F. Weber, Phys. Rev. C **80**, 015804 (2009).
 - [22] N. K. Glendenning, Mod. Phys. Lett. A **5**, 2197 (1990).
 - [23] M. Malheiro, M. Fiolhais, and A. R. Taurines, J. Phys. G **29**, 1045 (2003).
 - [24] M. G. Alford, A. Schmitt, K. Rajagopal, and T. Schäfer, Rev. Mod. Phys. **80**, 1455 (2008).
 - [25] K. Rajagopal and F. Wilczek, *At the Frontier of Particle Physics/Handbook of QCD*, edited by M. Shifman (World Scientific, Singapore, 2001).

- [26] M. Alford, *Annu. Rev. Nucl. Part. Sci.* **51**, 131 (2001).
- [27] L.P. Linares, M. Malheiro, A.R. Taurines, and M. Fiolhais, *Braz. J. Phys.* **36**, 1391 (2006).
- [28] K. Rajagopal and F. Wilczek, *Phys. Rev. Lett.* **86**, 3492 (2001).
- [29] M. Alford and K. Rajagopal, *J. High Energy Phys.* 06 (2002) 031.
- [30] D. Page and S. Reddy, *Annu. Rev. Nucl. Part. Sci.* **56**, 327 (2006).
- [31] V. V. Usov, *Phys. Rev. D* **70**, 067301 (2004).
- [32] V. V. Usov, T. Harko, and K. S. Cheng, *Astrophys. J.* **620**, 915 (2005).
- [33] P. Jaikumar, S. Reddy, and A. W. Steiner, *Phys. Rev. Lett.* **96**, 041101 (2006).
- [34] M. Alford, K. Rajagopal, S. Reddy, and A. W. Steiner, *Phys. Rev. D* **73**, 114016 (2006).
- [35] S. Ray, A. L. Espíndola, M. Malheiro, J. P. S. Lemos, and V. T. Zanchin, *Phys. Rev. D* **68**, 084004 (2003).
- [36] M. Malheiro, R. Picanço, S. Ray, J. P. S. Lemos, and V. T. Zanchin, *Int. J. Mod. Phys. D* **13**, 1375 (2004).
- [37] J. D. Bekenstein, *Phys. Rev. D* **4**, 2185 (1971).
- [38] F. de Felice, Y. Yu, and J. Fang, *Mon. Not. R. Astron. Soc.* **277**, L17 (1995).
- [39] M. Oertel and M. Urban, *Phys. Rev. D* **77**, 074015 (2008).