PHYSICAL REVIEW D **80,** 083003 (2009)

Quintessence and phantom dark energy from ghost D-branes

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(Received 2 July 2009; published 6 October 2009)

We present a novel dark-energy candidate, based upon the existence and dynamics of ghost D-branes in a warped compactification of type IIB string theory. Gp-branes cancel the combined BPS sectors of the Dp-branes, while they preserve the same supersymmetries. We show that this scenario can naturally lead to either quintessence or phantomlike behaviors, depending on the form of the involved potentials and brane tension. As a specific example we investigate the static, dark-energy dominated solution subclass.

DOI: 10.1103/PhysRevD.80.083003 PACS numbers: 95.36.+x, 11.25.Uv, 98.80.−k

I. INTRODUCTION

The theoretical description of the observed universe acceleration [1] is one of the challenges of current research. The simplest way to explain this remarkable behavior (apart from the sole cosmological constant which leads to the corresponding problem) is to construct various ‘‘field’’ models of dark energy, using a canonical scalar field (quintessence) [2], a phantom field, that is a scalar field with a negative sign of the kinetic term [3,4], or the combination of quintessence and phantom in a unified model named quintom [5]. However, the arbitrary consideration of additional scalar fields (which may even have nonconventional kinetic terms inserted by hand) should be constrained by the fact these extra scalars to be neutral under all the standard model symmetries, and thus not introducing additional fifth forces. This nontrivial requirement led many authors to the alternative direction of modifying gravity itself [6], with a promising attempt in these lines being perhaps the recent developments in Hořava-Lifshitz gravity [7], a power-counting renormalizable, ultraviolet (UV) complete gravitational theory (although there may well be problems with the theory due to additional degrees of freedom becoming strongly coupled in the infrared).

On the other hand, constructions arising from string theory are hard to result in dark-energy phenomenology consistent with observations, purely using the closed-string sector. For more details see the review [8]. However, cosmological dynamics driven by the open-string sector through dynamical Dp-branes, which is the basic idea of the so-called Dirac Born Infeld (DBI) formalism, has led to interesting successes, mainly in inflationary paradigms [9–11]. After inflation the universe lives on branes that wrap various cycles within the compact space, and in this sense the GUT or electroweak phase transition can be manifested through a geometric fashion. Thus, dark energy does present a dynamical nature, retaining additionally a form of geometric origin. Quantitatively, the tight constraints from WMAP five-year data set [12] on the model parameters have led DBI models to more complex versions, including multiple fields [13], multiple branes [14–16], wrapped branes [17], or monodromies [18]. Finally, the phase-space analysis of a solitary $D3$-brane moving through a particular warped compactification of type IIB was done [19], while the generalization to multiple and partially wrapped branes has been performed in [20].

In the present work we are interested in constructing a DBI scenario based on ghost D-branes, that is Dp-branes that have a $\mathbb{Z}_2$ symmetry acting to flip the signs of the NS-NS and RR sectors. Such a consideration is more robust than the naive and ambiguous use of the prototypical non-BPS $D3$-brane action, albeit with the wrong sign kinetic term. In addition, although in a typical flux compactification of type II string theory down to four dimensions one must introduce negative tension objects called orientifolds (in order to cancel the $D3$-brane charge associated with the closed-string fluxes, and to project out various string states breaking half of the bulk supersymmetry so that the vacuum retains an $\mathcal{N} = 1$ structure), the existence of ghost branes could possibly negate the need for orientifolds. As we show, in such a ghost D-brane scenario we can naturally acquire an effective dark energy behaving either as quintessence or as phantom.

The plan of the work is as follows: In Sec. II we present the formalism of Dp-branes, used in dark-energy scenarios. In Sec. III we extend it, introducing the concept of ghost D-branes, and we extract the dark-energy equation-of-state parameter. In Sec. IV we investigate its general features, examining the conditions for the appearance of quintessence or phantom behavior, while in Sec. V we perform an explicit phase-space analysis of the static, dark-energy dominated, solution subclass. Finally, our results are summarized in Sec. VI.

II. Dp-BRANE ACTION

The open-string sector of type II string theory is usually governed by the DBI action, governing the low energy

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fluctuations of such strings attached to a Dp-brane. For N coincident branes, the world-volume symmetry is enhanced from $U(1)$ to $U(N)$, and the scalar fluctuations are then promoted to matrices obeying a Lie structure. This is similar to the induced noncommutative world-volume theory on a single Dp-brane when we turn on a nontrivial B field.

String scattering calculations indicate that there is a particular trace prescription required in order to account for the full string cross section, which is given by the symmetrized average over all possible orderings of the Lie-algebra valued objects [21]. Much like the single Dp-brane action, one can sum the relevant terms into a nonlinear form [although only valid up to $\mathcal{O}(\alpha')^3$] to partially reconstruct the non-Abelian theory using the effective action [22,23]

$$S = -T_p \int d^{p+1}\xi \,\mathrm{STr}\Big(e^{-\phi}\sqrt{-\det(\mathcal{P}[E_{ab} + E_{ai}(Q^{-1} - \delta)^{ij}E_{jb} + \lambda F_{ab})])}\sqrt{\det Q^i_j}\Big) \pm \mu_p \int \mathrm{STr}\mathcal{P}[e^{i\lambda i_\phi i_\phi} \sum C^{(n)} e^{B}]e^{\lambda F}, \tag{1}$$

where

$$\lambda = 2\pi\alpha' \qquad E_{ab} = G_{ab} + B_{ab} \qquad Q^i_j = \delta^i_j + i\lambda[\psi^i, \psi^k]E_{kj}. \tag{2}$$

In the expressions above, λ is the inverse of the F-string tension, and α' is the square of the string length—the fundamental length scale in our theory. The scalar ψ^i is related the space-time embedding $X^i = \lambda\psi^i$ and finally $B^{(2)}$ is the NS-NS gauge potential, which we will ignore in the following. Moreover $\mathcal{P}$ denotes the pullback operator acting on the bulk space-time tensor fields, and ψ^i are the scalar field fluctuations where $i = (p+1), \ldots, 9$. In the RR sector we see the introduction of the interior derivative i_ψ, whose action on an n form is

$$i_\psi i_\psi C^{(n)} = \tfrac{1}{2}[\psi^i, \psi^j]C^{(n)}_{ji}. \tag{3}$$

The presence of the interior rather than exterior derivative allows the Dp-brane to couple to gauge potentials of higher order, such as the $(p+3)$, $(p+5)$ forms. This suggests that there is a transmutation (or dielectric) effect where the Dp-brane can blow up into a $D(p+2)$-brane through higher order terms in the expansion of the Chern-Simons action [22,24]. A concrete example of this effect is when N $D3$-branes blow up into a solitary $D5$-brane via the formation of a fuzzy S^2, more commonly referred to as the Myers effect. If the scalar fields transform under an appropriate representation of a higher-dimensional gauge group, then the $D3$-branes can be polarized into higher-dimensional branes in an analogous fashion through the extended Myers effect [22,25].[1] For example, if the scalars transform under irreducible representations of the n-fold tensor product of $SO(5)$, then the branes orient themselves along a fuzzy S^4 to form a configuration of n $D7$-branes [27]. The construction of odd-dimensional fuzzy sphere solutions is actually nontrivial and requires the introduction of spinorial representations [28] of $SO(2k)$, where $k \in \mathbb{Z}$.

One important simplification to the above action is when we consider the large-N limit, as the STr operation reduces to a trace (up to $1/N$ corrections). The reason why this limit is important can be understood when one considers the dual description of the brane configuration. Recall that the Myers effect describes lower-dimensional branes being polarized into higher-dimensional configurations via a fuzzy sphere. This means that there is a dual description of the Myers effect in terms of a higher-dimensional (spherical) brane with world-volume flux. More concretely we see that [assuming the scalars lying in irreducible representations of $SO(3)$] N $D3$-branes are dual to a single $D5$-brane wrapped on $S^2 \times R^3$ with N units of flux through the S^2 [29]. Duality in this sense actually means that the effective actions are identical, provided that the $U(1)$ flux on the $D5$-brane is large. The above statements are all assumed to be true in a curved background, although the required string scattering calculations are difficult to be computed to the necessary order and therefore a direct check is not possible. However, given the prevalence of such dualities in string theory, one can be reasonably confident that the statement is correct.

The most general cosmological backgrounds in type II string theory can be written in the following form [30]:

$$ds^2 = h^2(\rho)ds_4^2 + h^{-2}(\rho)(d\rho^2 + \rho^2 ds^2_{X_5}), \tag{4}$$

where h is the warp factor, which is a function of ρ—a warped throat that is fibered over some five-dimensional manifold X_5, and the four-dimensional metric takes the usual Friedmann-Robertson-Walker form. For concreteness we will specialize to the case of type IIB string theory, where the throat can be generated by threading $D3$-brane flux through a compact three-cycle. Moreover, since the dilaton is constant in these backgrounds, the Einstein frame and string frames coincide. We will also assume that our theory consists of N $D3$-branes which are oriented parallel to the $(3+1)$-large dimensions and that the scalars are

[1] Although there exists a different action, proposed by Tseytlin [26], which does not admit such an effect.

homogeneous, transforming under irreducible representations of $SO(3) \sim SU(2)$. The resulting action for N coincident $D3$-branes can be written as [16][2]

$$S = -T \int d^4\xi N\sqrt{-g_4}[h^4\sqrt{1 - h^{-4}\lambda^2\hat{C}\dot{R}^2} \times \sqrt{1 + 4\lambda^2\hat{C}h^{-4}R^4} - h^4 + V(R)], \tag{5}$$

with T the warped, positive-definite brane tension. The radius of the fuzzy sphere is defined in terms of the geometric radius ρ via

$$R^2 = \frac{\rho^2}{\lambda^2\hat{C}}, \tag{6}$$

and $\hat{C}$ is the quadratic Casimir of $SU(2)$, namely $\hat{C} = N^2 - 1$. We have also included a scalar potential contribution arising from the interaction of the $D3$-branes with the closed-string background. We remind the reader that at large N this action is precisely the same as that arising from a single wrapped $D5$-brane with N units of $U(1)$ flux.

It is convenient to use the field redefinition $\phi = \rho/\sqrt{T}$, with ρ the induced world-volume scalar coming from the background in the string frame [16]. However, it has (mass) dimension -1 and thus since we desire to write all the fields with canonical mass terms we have to redefine the involved world-volume scalars. Following these lines, the action (5) can be rewritten in the generalized form

$$S = -\int d^4\xi\sqrt{-g_4}[T(\phi)W(\phi)\gamma^{-1} - T(\phi) + V(\phi)], \tag{7}$$

where $\gamma = [1 - \dot{\phi}^2/T(\phi)]^{-1/2}$ is the usual generalization of the relativistic factor. The cosmological consequences of such an action have been discussed elsewhere [16] and we refer the interested reader there for more details.

The appearance of the positive-definite function $W(\phi)$, which generalizes the aforementioned action comparing to the usual $W(\phi) \equiv 1$ case, can be theoretically justified [20], since if N multiple coincident branes are present then the world-volume field theory is a $U(N)$ non-Abelian gauge theory and this "potential" term is simply a reflection of the additional degrees of freedom [21]. Additionally, this configuration is related to a $D5$-brane, wrapping a two-cycle within the compact space and carrying a nonzero magnetic flux along this cycle. On the other hand, the positive-definite effective potential $V(\phi)$ accounts for the possible open or closed-string interactions. Its precise form depends upon the number of additional branes and geometric moduli, the number of nontrivial cycles in the compact space, the choice of embedding for branes on these cycles, the coupling of the brane to any background RR-form fields, the contribution from higher-dimensional bulk forms [31], etc. Finally, note that using the above generalized form for the action allows us to interpolate between a single $D3$-brane [taking $W(\phi) \to 1$] and the multibrane, or wrapped $D5$-brane [where $W(\phi) > 1$] solutions.

[2]For $\bar{D}3$-branes we would require the second term to take the positive sign.

III. GHOST D-BRANE COSMOLOGY

It is well established that Dp-branes are not the only hypersurfaces within string theory. There are also orientifold Op-planes which have negative tension and reduced charge (compared to the Dp-branes) [32]. Their role is vital in flux compactifications of type II string theory, since they cancel global flux tadpoles and also break one-half of the residual supersymmetries. There exists another type of extended object, which has been dubbed a ghost brane [33], that we will briefly describe using the boundary state formalism, which is the most appropriate for the CFT description of Dp-branes.

The bosonic sector of a BPS Dp-brane is represented by a boundary state of the form

$$|D\rangle = |D\rangle_{NSNS} + |D\rangle_{RR}, \tag{8}$$

where $|D\rangle$ represents the full Dp-brane state. It was shown in [33,34] that one can define an analogous (BPS) ghost-brane state (which we will denote by a Gp-brane) through the introduction of an operator g:

$$|G\rangle = |gD\rangle = -|D\rangle, \tag{9}$$

such that the ghost state precisely cancels the combined BPS sectors of the Dp-brane. Since the Gp-brane state preserves the overall relative sign of the two different sectors, it must also preserve the same supersymmetries as the Dp-brane. This makes it a distinct object, and it should not be confused with the $\bar{D}p$-brane—which has a boundary state of the form

$$|\bar{D}\rangle = |D\rangle_{NSNS} - |D\rangle_{RR}. \tag{10}$$

The formalism implies that the Gp-sector exactly cancels the Dp-sector. This means that a theory consisting of N coincident Dp-branes and M coincident Gp-branes can be described in two equivalent ways; either by $(N - M)$ Dp-branes or by $(M - N)$ Gp-branes. The corresponding world-volume theory is given by a $U(N) \times U(M)$ gauge theory, which is enhanced to $U(N|M)$ as the two groups of branes are brought together. Importantly, this means that when $N = M$ the resulting solution is simply space-time with no Dp-branes. In this way we see that the ghost brane can screen the Dp-brane, and a useful consequence of this screening was employed in AdS/CFT framework in [35].

Since the Gp-brane is simply minus the standard Dp-brane state, one sees that the effective world-volume theory for the Gp-brane is also of DBI form, albeit with an additional sign change in the definition of the tension. Thus, for multiple coincident $G3$-branes, we expect the effective theory to be well described by the action

$$S = \int d^4\xi\sqrt{-g_4}[T(\phi)W(\phi)\gamma^{-1} - T(\phi) - V(\phi)], \quad (11)$$

where we have embedded the branes in the warped background (4). Note that in the nonrelativistic expansion of this action, the kinetic term will have the wrong sign, implying phantomlike behavior for the scalar fluctuations. This suggests that the world-volume theory tends to antigravitate, rather than gravitate.

We stress that the $G3$-brane theory is different from previously proposed phantom models based on the DBI action [36], which have been constructed in the light of the non-BPS action proposed by Sen as an effective description of tachyon condensation [37]. The models in this class have the wrong sign kinetic term *inside* the square-root structure, in contrast to that of the ghost action. Furthermore, that sign change can only be inserted by hand [38], since the world-volume metric is induced from the background geometry, and it is unlikely to contain a submanifold where the sign changes in a continuous fashion.[3] Additionally, the non-BPS action can only couple to any of the bulk RR-form fields through terms of the form $dT \wedge C^{(3)}$, which are typically zero according to our assumptions. Therefore, it seems unlikely that such boundary states can be stable within the full theory. On the other hand, the ghost branes are supersymmetric and do couple to the bulk form fields, suggesting that they constitute actually stable states within the theory.

Let us now focus on the cosmological consequences of the scenario at hand. Assuming that the scalar is time dependent, one reads off the diagonal components of the energy momentum tensor in the usual fashion:

$$\rho_\phi = T(\phi)[1 - W(\phi)\gamma] + V(\phi) \quad (12)$$

$$p_\phi = T(\phi)[W(\phi)\gamma^{-1} - 1] - V(\phi). \quad (13)$$

Thus, since in DBI constructions ϕ is responsible for dark energy, we can define its equation-of-state parameter as

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{T(\phi)[W(\phi)\gamma^{-1} - 1] - V(\phi)}{T(\phi)[1 - W(\phi)\gamma] + V(\phi)}. \quad (14)$$

As can be deduced from expression (14), the dark-energy equation-of-state parameter can present quintessence or phantom behavior, depending on the choice of scalar potential and background.

The Friedmann equations arising from action (11) write

$$H^2 = \frac{1}{3M_p^2}(\rho_M + \rho_\phi), \quad (15)$$

$$\dot{H} = -\frac{1}{2M_p^2}[\rho_M + p_M + \rho_\phi + p_\phi] = -\frac{1}{2M_p^2}[\rho_M + p_M - \gamma W(\phi)\dot{\phi}^2], \quad (16)$$

with $H \equiv \dot{a}/a$ the Hubble parameter. Additionally, variation of the action (11) with respect to ϕ leads to the equation of motion for the scalar field, namely,

$$3HW(\phi)\gamma\dot{\phi} + W(\phi)\gamma^3\ddot{\phi} - V_\phi(\phi) + W_\phi(\phi)T(\phi)\gamma + \frac{T_\phi(\phi)}{2}[W(\phi)\gamma(3-\gamma^2) - 2] = 0, \quad (17)$$

where the subscript ϕ denotes differentiation with respect to ϕ. This equation is the generalization of the Klein-Gordon one in the DBI framework, and using (12) and (13) it can be written in the usual form $\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = 0$. Finally, the corresponding equation of motion for matter writes $\dot{\rho}_M + 3H(\rho_M + p_M) = 0$.

IV. COSMOLOGICAL IMPLICATIONS: THE GENERAL CASE

In the previous section we introduced the concept of ghost D-branes, and we extracted the dark-energy equation-of-state parameter of this ghost version of DBI scenario. Thus, we can now investigate the various cosmological possibilities, trying to remain sufficiently general. We mention that we desire to explore the general features of w_ϕ for possible forms of the involved tension and potentials, without examining in detail the equations of motion. As we see, although a full dynamical investigation would be interesting, this basic "kinematical" study is sufficient to qualitatively reveal the novel features of the ghost D-brane model.

Let us first consider the scenario where no scalar potential is present, that is study solely the brane action. In this case the equation of state reduces to

$$w_\phi = \frac{1}{\gamma}\left[\frac{W(\phi) - \gamma}{1 - W(\phi)\gamma}\right]. \quad (18)$$

For a general $W(\phi)$, in the regime where $\gamma \gg 1$ we see that the equation of state is typically zero, unless there are divergences in W, which is not the case if we desire our model to be physical. On the other hand, in regions where $W(\phi)$ is dominant we find that $w_\phi \sim -1/\gamma^2$ and therefore it is negative definite (although small). Similarly, if $W(\phi) = 1$ then $w_\phi = 1/\gamma$ which is positive definite although typically small. Note that physical solutions imply $W(\phi) \geq 1$, however if we treat the action phenomenologically and assume smaller values for $W(\phi)$ then we find solutions where $w_\phi \to 0$ from above after being initially large. In conclusion, we observe that the possible solution space is quite large even without a scalar potential.

[3] Leaving aside any issues concerning type II* string theory [39] for the moment.

This preliminary phenomenology suggests that w_ϕ could cross the -1 bound. In particular, the equation of state would become phantom if

$$\frac{W(\phi)[1-\gamma^2]}{[1-W(\phi)\gamma]}<0. \tag{19}$$

However, this condition cannot be met physically, and thus we conclude that the brane action alone cannot generate phantom dynamics.

Let us now turn on the scalar potential term $V(\phi)$. A first simple solution subclass would be to consider $T(\phi)=0$, where we obtain $w_\phi=-1$ recovering the case of pure de-Sitter expansion. In the general case of nonzero potential and tension terms, but with $V(\phi)\gg T(\phi)$, we can expand (14) in Taylor series acquiring

$$w_\phi\approx -1+\frac{T(\phi)}{V(\phi)}\frac{W(\phi)(1-\gamma^2)}{\gamma}+\cdots, \tag{20}$$

neglecting higher order terms. Therefore, in the relativistic regime ($\gamma^2\gg 1$) the correction term will be negative definite, leading to the realization of the phantom phase. We mention that this phantom realization is obtained naturally from a large solution subclass of the model. Additionally, it is not the only combination of possibilities which lead to phantom behavior, but just a simple example. These features reveal that the use of ghost D-branes does lead to quintessence and phantom realization, depending on the specific forms of the potential-like terms and of the tension in the effective action.

Another class of solutions will occur when we have $T(\phi)\gg V(\phi)$, which upon performing the Taylor expansion leads to

$$w_\phi\approx\frac{\gamma-W(\phi)}{\gamma[\gamma W(\phi)-1]}\bigg\{1+\frac{V(\phi)}{T(\phi)}\times\frac{W(\phi)(\gamma^2-1)}{[\gamma W(\phi)-1][\gamma-W(\phi)]}\bigg\} \tag{21}$$

at leading order, and therefore it is highly dependent on the particular background field parametrization. For initially static configurations ($\dot\phi=0$, i.e., $\gamma=1$), we recover the usual result $w_\phi\approx -1$, and therefore the static brane mimics the cosmological constant. As the velocity of the brane increases we again find that $w_\phi\to 0$ along the asymptotic branch. On the other hand, if $W(\phi)\gg 1$ then the equation of state tends to $-1/\gamma^2$ and therefore will relax to zero from below. Finally, imposing $W(\phi)=1$, that is considering the single brane case, the resulting equation-of-state parameter tends to zero from above as the velocity term increases, as can be seen from (21). Note that since $\gamma\geq 1$, all the cases of the regime $T(\phi)\gg V(\phi)$ present a quintessence behavior with $w_\phi\geq -1$.

As we have mentioned, in the present work we are interested in exploring the general qualitative features of the equation-of-state parameter of ghost D-brane scenario. We have not extracted the equations of motion, studying just w_ϕ as a function of $T(\phi)$, $W(\phi)$, $V(\phi)$, and γ, which is itself a function of ϕ and $\dot\phi$. Therefore, for given $T(\phi)$, $W(\phi)$, $V(\phi)$, the dependence of w_ϕ on γ provides qualitative information about the phase-space structure. A first observation is that (14) possesses a singularity at

$$\gamma_c=\frac{1}{W(\phi)}\bigg[1+\frac{V(\phi)}{T(\phi)}\bigg]. \tag{22}$$

According to the specific choice of $T(\phi)$, $W(\phi)$, $V(\phi)$, and of initial conditions, a particular universe evolution [i.e., a particular orbit of $\gamma(\phi,\dot\phi)$ in the $(\phi,\dot\phi)$ plane] can remain either to one or to the other regime, tend asymptotically into the singularity, or even cross it. Such singularities are common in field dark-energy models, especially in phantom ones, and they correspond to big rip [3,40] or to realization of a cosmological bounce [41]. Finally, note that if γ_c turns out to be less than 1, that is unphysical, then the specific model is free of such behaviors, independently of the initial conditions.

In order to provide a more transparent picture of the obtained cosmological behavior, in Fig. 1 we present the solution space for the simple scenario of fixed $V(\phi)/T(\phi)$, imposing $W(\phi)=1$ (corresponding to the single brane model).

As we observe, for $V(\phi)/T(\phi)\ll 1$ there is a singularity at $\gamma_c=1.1$, thus in almost the whole phase space the equation-of-state parameter is quintessencelike and, in particular, it is positive definite and tends to zero asymptotically. For larger values of $V(\phi)/T(\phi)$ (i.e., when the scalar potential effect is enhanced comparing to that of the tension), we observe the singularity at a specific γ_c,

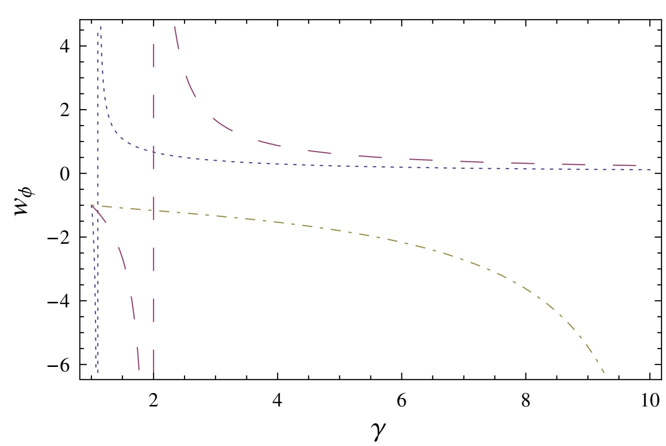

FIG. 1 (color online). The dark-energy equation-of-state parameter w_ϕ as a function of the generalized relativistic factor γ, for fixed $W(\phi)=1$. The curves correspond to $V(\phi)/T(\phi)=0.1$ (dotted), 1 (dashed), and 10 (dot-dashed), respectively.

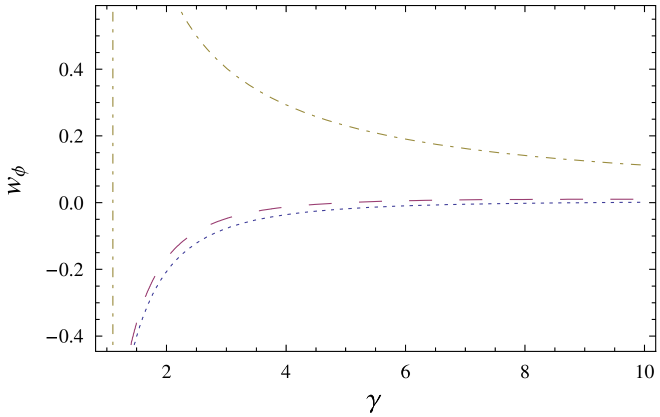

FIG. 2 (color online). The dark-energy equation-of-state parameter w_ϕ as a function of the generalized relativistic factor γ, for fixed $W(\phi) = 10$. The curves correspond to $V(\phi)/T(\phi) = 0.1$ (dotted), 1 (dashed), and 10 (dot-dashed), respectively.

which in this special subclass [fixed $V(\phi)/T(\phi)$ and ϕ-independent W] is constant, i.e., ϕ independent. For values of $\gamma < \gamma_c$ phantom behavior is realized, while for $\gamma > \gamma_c$ one finds quintessencelike evolution with $w_\phi \to 0$ from above.

Let us now consider the same subclass of fixed $V(\phi)/T(\phi)$, but setting $W(\phi) = 10$. This scenario can be obtained in a class of string theory backgrounds with $G3$-branes or a $G5$-brane with flux. In Fig. 2 we depict the corresponding w_ϕ behavior as a function of γ.

In this case, for small values of $V(\phi)/T(\phi)$ the value of γ_c is unphysical. Therefore, the resulting trajectories are quintessencelike, and the model is singularity-free independently of the initial conditions. For larger values of $V(\phi)/T(\phi)$, γ_c becomes physical, with $\gamma < \gamma_c$ leading to phantom behavior and $\gamma > \gamma_c$ to a quintessencelike one with $w_\phi \to 0_+$ or $w_\phi \to 0_-$.

In summary, from this simple solution subclass we observe an interesting w_ϕ behavior. We mention that in principle, $W(\phi)$ and $T(\phi)$ are determined by the supergravity background and can have various forms, while $V(\phi)$ can be more arbitrary since it arises from the interactions of the open/closed-string sector which are difficult to compute in general. Clearly, considering more general scenarios, with various $T(\phi)$ and $V(\phi)$ and/or not constant $W(\phi)$, the resulting cosmological behavior can be significantly richer.

V. STATIC DARK-ENERGY-DOMINATED SOLUTIONS

Having discussed qualitatively the cosmological behavior of the model at hand, it would be interesting to perform a systematic investigation of the various cosmological solution subclasses. In particular, we desire to study the cosmologically important scenario of static solutions characterized by complete dark-energy domination. We examine whether there exist late-time attractor solutions, and if they do exist to determine their observable features, that is the dark-energy equation-of-state parameter and density parameter. Furthermore, we want to extract information about the intermediate-time behavior, that is the evolution towards the aforementioned late-time attractors, since such an investigation could also leave imprints in observables related to the cosmological past.

As usual, we will first transform the cosmological system into its autonomous form [42]: $\dot{\mathbf{X}} = \mathbf{f}(\mathbf{X})$, where $\mathbf{X}$ is the column vector constituted by the (suitably defined) dimensionless variables and $\mathbf{f}(\mathbf{X})$ the corresponding column vector of the autonomous equations, and we extract its critical points $\mathbf{X_c}$ satisfying $\dot{\mathbf{X}} = 0$. Then, in order to determine the stability properties of these critical points, we expand around $\mathbf{X_c}$, setting $\mathbf{X} = \mathbf{X_c} + \mathbf{U}$ with $\mathbf{U}$ the perturbations of the variables considered as a column vector. Thus, for each critical point we expand the equations for the perturbations up to the first order as $\dot{\mathbf{U}} = \mathbf{\Xi} \cdot \mathbf{U}$, where the matrix $\mathbf{\Xi}$ contains the coefficients of the perturbation equations. Thus, for each critical point, the eigenvalues of $\mathbf{\Xi}$ determine its type and stability.

Defining the dimensionless variables

$$X = \frac{\phi}{M_p}, \qquad Y = \frac{\dot{\phi}}{\sqrt{T}}, \tag{23}$$

the equations of motion reduce to the following set of equations:

$$\dot{X} = \frac{Y\sqrt{T}}{M_p} \tag{24}$$

$$\dot{Y} = \frac{V_\phi}{W\gamma^3\sqrt{T}} - \frac{T_\phi}{\sqrt{T}}\left[\frac{(3-\gamma^2)}{2\gamma^2} + \frac{Y^2}{2} - \frac{1}{W\gamma^3}\right] - \frac{W_\phi}{W}\frac{\sqrt{T}}{\gamma^2} - \frac{\sqrt{3}Y}{\gamma^2 M_p}\sqrt{T(1-W\gamma)+V}, \tag{25}$$

where we have set $\rho_M = 0 = p_M$ since we are investigating the complete dark-energy dominated scenario. Furthermore, in terms of the dimensionless variables, the dark-energy equation-of-state parameter (14) writes

$$w_\phi = \sqrt{1-Y^2}\left[\frac{W(X)\sqrt{1-Y^2} - 1}{\sqrt{1-Y^2} - W(X)}\right]. \tag{26}$$

Since we are interested in static late-time attractors, that is possessing $\dot{\phi} = 0$, the corresponding critical points are of the form $(X_c, 0)$. Thus, linearized perturbations ($X = X_c + \delta X$, $Y = 0 + \delta Y$) lead to

$$\dot{\delta X} = \frac{\sqrt{T}\delta Y}{M_p}$$
$$\begin{aligned}\dot{\delta Y} = \frac{\delta X}{W\sqrt{T}}\bigg\{&V_\phi\bigg[\frac{V_{\phi\phi}}{V_\phi} - \frac{W_\phi}{W} - \frac{T_\phi}{2T}\bigg] \\ &- W_\phi T\bigg[\frac{W_{\phi\phi}}{W_\phi} - \frac{W_\phi}{W}\bigg] - \frac{T_\phi}{2}\bigg[\frac{T_{\phi\phi}}{T_\phi} + \frac{W_\phi}{W} - \frac{T_\phi}{2T}\bigg]\bigg\} \\ &+ \delta Y\bigg\{-3H_0 - \frac{T_\phi}{\sqrt{T}} + \frac{3V_\phi}{2W\sqrt{T}} - \frac{W_\phi\sqrt{T}}{W}\bigg[1 - \frac{T_\phi}{2T}\bigg] \\ &- \frac{T_\phi(W-2)}{4W\sqrt{T}}\bigg\} = \alpha\delta X + \beta\delta Y, \end{aligned} \tag{27}$$

where all the derivative terms on the right-hand side are evaluated at $X = X_c$, and H_0 stands for the value of the Hubble parameter [given by (12) and (15)] calculated at $X = X_c$. Thus, the corresponding stability matrix reads

$$\Xi = \begin{bmatrix} 0 & \frac{\sqrt{T}}{M_p} \\ \alpha & \beta \end{bmatrix},$$

and its eigenvalues are $\lambda_\pm = \frac{1}{2}(\beta \pm \sqrt{\beta^2 + 4\frac{\alpha\sqrt{T}}{M_p}})$. Requiring negativity of the eigenvalue real part (which corresponds to stability of the corresponding fixed point) we result to the constraint

$$\begin{aligned}&V_\phi\bigg[\frac{V_{\phi\phi}}{V_\phi} - \frac{W_\phi}{W} - \frac{T_\phi}{2T}\bigg] - W_\phi T\bigg[\frac{W_{\phi\phi}}{W_\phi} - \frac{W_\phi}{W}\bigg] \\ &- \frac{T_\phi}{2}\bigg[\frac{T_{\phi\phi}}{T_\phi} + \frac{W_\phi}{W} - \frac{T_\phi}{2T}\bigg] < 0. \end{aligned} \tag{28}$$

In the following we explore the general features of this stability condition, for various cases of the involved potentials and tension.

We first consider a solution where $W(\phi) = T(\phi) =$ const, and therefore condition (28) reduces to

$$V_{\phi\phi} < 0, \tag{29}$$

evaluated at the critical value of $X = X_c$. This expression (together with the potential positivity) imposes tight restrictions on the form of the potential, if we desire to obtain a late-time attractor. In particular, it requires potentials where the field is initially localized near $\phi \sim 0$ and rolls to larger values (analogous to the small-field models of inflation). Candidates are therefore $V \sim V_0/\cosh(\xi\phi)$, $V = V_0\cos(\upsilon\phi)$, and $V \sim V_0 - m^2\phi^2$.

In order to provide an explicit example of this subclass, we consider the potential $V \sim V_0\phi^2/\phi_0^2$. Transforming to the variables $\phi = \phi_0\delta X$, $t = \phi_0 s$, where ϕ_0 is a reference field position, we can write the equations of motion (27) in dimensionless form

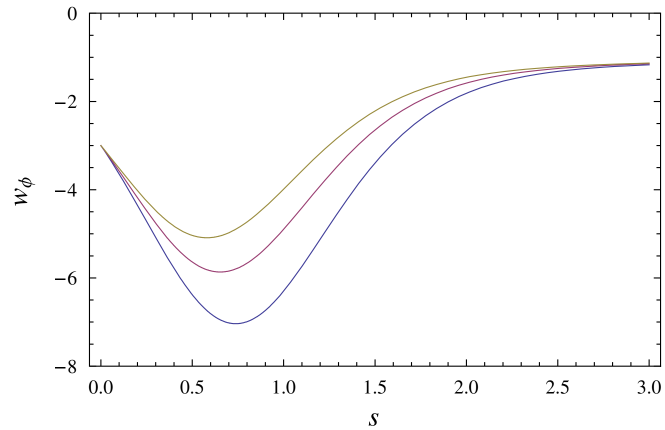

FIG. 3 (color online). w evolution for the potential $V \sim V_0\phi^2/\phi_0^2$, in terms of the variable $s = t/\phi_0$, for $M_p = V_0 = 1$ and $W(\phi) = T(\phi) = 1$. The top curve corresponds to $\phi_0 = 300$, the middle curve to $\phi_0 = 250$, and the bottom curve to $\phi_0 = 200$.

$$\begin{aligned}\frac{d(\delta X)}{ds} &= \delta Y\sqrt{T} \\ \frac{d(\delta Y)}{ds} &= \frac{2\delta X}{\gamma^3\sqrt{T}} - \frac{\sqrt{3}\delta Y}{\gamma^2}\sqrt{T(1-\gamma) + (\delta X)^2}. \end{aligned} \tag{30}$$

The usual approach is to depict the phase-space plots in the (X, Y) plane, and show the convergence to a stable fixed point. However, it is more transparent to depict the evolution of w in terms of the variable s. Thus, convergence of the system to a static late-time attractor $(X_c, 0)$ means convergence of w to -1 [as can be immediately seen from (26) setting $Y \equiv Y_c = 0$]. In Fig. 3 we depict w evolution, for $M_p = V_0 = 1$, $W(\phi) = T(\phi) = 1$, and for various ϕ_0 values.

As we can see, the system presents phantom behavior, going asymptotically to the cosmological-constant uni-

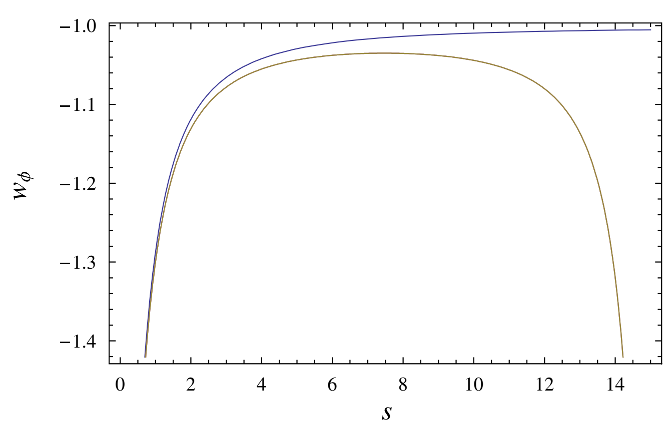

FIG. 4 (color online). w evolution for the potential to $V = V_0/\cosh(\phi/\phi_0)$, in terms of the variable $s = t/\phi_0$, for $M_p = V_0 = 1$ and $W(\phi) = T(\phi) = 1$. The top curve corresponds to $\phi_0 = 1$ and the lower one to $\phi_0 = 10$.

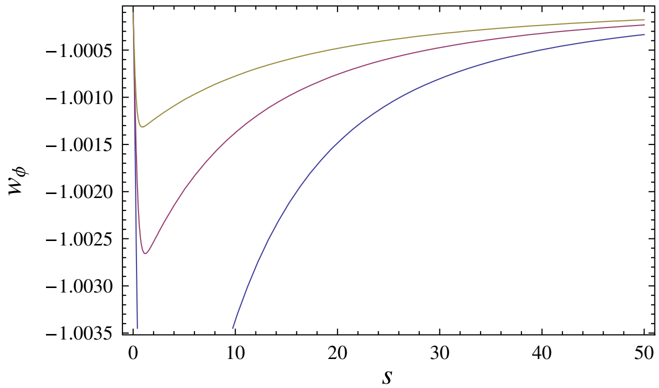

FIG. 5 (color online). w evolution for $T(\phi) = (\phi/L)^\kappa$, $V(\phi) = (\phi/\phi_0)^\lambda$, and $W(\phi) = 1$, in terms of the variable $s = t/\phi_0$, for $\epsilon = \phi_0/L = 0.1$ and $\kappa = \lambda = 2$. The curves, from bottom to top, correspond to $\phi_0 = 10$, 20, and 30, respectively.

verse. Additionally, we see that the location of the (global) minimum is shifted to earlier values of s as we increase ϕ_0, and it also is closer to the cosmological-constant equation of state.

For completeness, in Fig. 4 we depict another example of this solution subclass, namely, corresponding to $V = V_0/\cosh(\phi/\phi_0)$, with $M_p = V_0 = 1$ and $W(\phi) = T(\phi) = 1$.

As we can see the solution only appears to converge for $\phi_0 = 1$, and diverges for larger values.

Let us now examine a more complicated case, considering $T(\phi) = (\phi/L)^\kappa$ and $V(\phi) = (\phi/\phi_0)^\lambda$, accompanied with $W(\phi) = 1$. This choice introduces a new mass scale, which combined with ϕ_0 allows us to write $\epsilon = \phi_0/L$. If we desire to provide a theoretical justification through supergravity solutions then L is typically large since it governs the radius of an AdS space-time, and thus ϵ will be small. In Fig. 5 we present the corresponding $w(s)$. One notices that as ϕ_0 is increased, the equation of state tends to -1 from below, however it is never too far away from -1. Finally, numerical investigation reveals that the solution is sensitive to the ϵ value, with smaller ϵ leading $w_{\min}$ to larger time scales.

VI. CONCLUSIONS

In this work we have introduced a novel mechanism for realizing either quintessence or phantom dark-energy-dominated phases, within a string-theoretical context. This mechanism is based upon the existence and subsequent dynamics of ghost Gp-branes in a warped compactification of the type IIB theory, which cancel the combined BPS sectors of the Dp-brane, preserving the same supersymmetries as their Dp counterparts.

The scenario at hand admits a wide range of cosmological behavior, depending on the various terms arising from the supergravity background. In the simplest case, consisting of a single $G3$-brane and with $V(\phi)/T(\phi)$ being constant, we see that phantom behavior will dominate the phase-space dynamics for sufficiently large $V(\phi)/T(\phi)$, γ, since the phase-space singularity γ_c is pushed to larger values. Beyond the singularity, one finds a quintessence solution, asymptotically tending towards $w_\phi = 0$. Although these features arise from this particular model subclass, it is clear that more complicated behavior can be revealed considering more general $W(\phi)$, $T(\phi)$, and $V(\phi)$ cases, with a natural realization of quintessence and phantom behavior, of the -1 crossing and of a big rip.

Surprisingly enough, although the corresponding Dp-brane scenario experiences only quintessence-type solutions [20], the present Gp-model may lead to both quintessence or phantom cosmology. One can proceed to a more detailed investigation of the phase-space behavior of Gp-brane cosmological scenarios, for various cases of the involved tension and potentials [43]. Alternatively one can impose the desired cosmological evolution, and reconstruct the corresponding aforementioned functions. Since in this work we desire to remain general, exploring the qualitative kinematic features of the Gp-brane model, we do not proceed to such extensions, leaving them for a future investigation [43].

One remaining issue pertains to the quantum stability of such a construction. As it is typical for phantom models, the energy is unbounded from below leading to potential problems upon quantization. However, since the Gp-brane is treated semiclassically, we may hope that quantizing the open-string modes with the appropriate boundary conditions may regularize the theory. In particular, since we are dealing with a phantom scalar field, all of the relevant energy conditions are violated. This feature suggests that the phantom may be an unstable mode. To verify the stability, one must resort to a quantum field-theoretical analysis. However, developing a quantum theory of world-volume open-string modes using the DBI action has proven to be difficult, since the D-brane itself is a nonperturbative state with regards to the string coupling. A boundary state analysis may be possible, but it is beyond the scope of the current work. Finally, we mention that, since the usual phantom models are robust only for small momenta (because at larger momenta the higher-derivative terms dominate), one could estimate the quasistable lifetime of the phantom field, provided the momentum cutoff is fine-tuned and the phantom decays solely into gravitons. Similarly, we could follow this line of reasoning for the model at hand, although this means that the field should decay to the closed-string vacuum in a way that the open-string modes give rise to gravitons. For the static case, where the $G3$-branes are localized, they play a similar role to orientifold planes. In conclusion, we stress that the quantum stability of such negatively charged objects within string theory is still an open question, but one that is ripe for future exploration.

We end this work referring to an additional advantage of the model at hand, namely, that it possesses a concrete UV completion. Therefore, it would be interesting to try to embed such branes into full stringy compactifications, particularly if they could serve as replacements for orientifold planes. Obviously, the underlying theory would then be $\mathcal{N} = 2$, which is phenomenologically unfavored, however there might be another mechanism in the bulk which breaks half of this residual supersymmetry. Exploring the nature of such scenarios is something we leave for future endeavors.

ACKNOWLEDGMENTS

J. W. is partly supported by NSERC of Canada. E. N. S. wishes to thank Institut de Physique Théorique, CEA, for the hospitality during the preparation of the present work.

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