Vacuum energy density kicked by the electroweak crossover

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Using q-theory, we show that the electroweak crossover can generate a remnant vacuum energy density $\Lambda \sim E_{\rm ew}^8/E_{\rm Planck}^4$, with effective electroweak energy scale $E_{\rm ew} \sim 10^3$ GeV and reduced Planck-energy scale $E_{\rm Planck} \sim 10^{18}$ GeV. The obtained expression for the effective cosmological constant Λ may be a crucial input for the suggested solution by Arkani-Hamed *et al.* of the triple cosmic coincidence puzzle (why the orders of magnitude of the energy densities of vacuum, matter, and radiation are approximately the same in the present Universe).

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I. INTRODUCTION

The q-theory description of the quantum vacuum provides a natural cancellation mechanism for the vacuum energy density [1–3]. The basic idea is to consider the macroscopic equations of a *conserved* microscopic variable q, whose precise nature need not be known. For a particular realization of q, it was found [2] that, if the vacuum energy density has initially a large Planck-scale value, $\rho_V \sim E_{\rm Planck}^4$, it relaxes according to the following power-law modulation:

$$\rho_V(t)|^{\rm nondissipative} \propto \frac{\omega^2}{t^2}\sin^2\omega t, \tag{1.1a}$$

with $\hbar = c = k = 1$ in natural units and a frequency ω of the order of the reduced Planck-energy scale $E_{\rm Planck} \equiv 1/\sqrt{8\pi G_N} \approx 2.44 \times 10^{18}$ GeV. Quantum dissipative effects have not been taken into account in the above result. Indeed, matter field radiation (matter quanta emission) by the oscillations of the vacuum can be expected to lead to faster relaxation [4,5],

$$\rho_V(t)|^{\rm dissipative} \propto \Gamma^4 \exp(-\Gamma t), \tag{1.1b}$$

with a decay rate $\Gamma \sim \omega \sim E_{\rm Planck}$.

In the present article, we consider what happens during the electroweak crossover [6] of a spatially flat Friedmann-Robertson-Walker (FRW) universe [7] at cosmic time

$$t_{\rm ew} \sim E_{\rm Planck}/E_{\rm ew}^2, \tag{1.2}$$

where $E_{\rm ew} \sim 10^3$ GeV is the effective electroweak energy scale. In the epoch before the crossover, the vacuum energy density has already relaxed to zero, according to (1.1b). The classical equations of q-theory demonstrate that during the epoch when only ultrarelativistic matter (“radiation”) is present, i.e., when the matter equation-of-state (EOS) parameter $w_M \equiv P_M/\rho_M$ is exactly $1/3$, the vacuum energy density remains strictly zero. But $w_M(t)$ deviates from $1/3$ during the electroweak crossover and the subsequent period when massive particles annihilate. This implies, as will be shown in the present article, that the vacuum energy density moves away from zero and acquires, at $t \sim t_{\rm ew}$, a positive value of order

$$\rho_{V,0}(t) \sim (w_M(t) - 1/3)^2 H^4(t), \tag{1.3}$$

where the suffix 0 will be explained later and $H(t)$ is the Hubble parameter of the spatially flat FRW universe considered.

After the electroweak crossover, the value $w_M = 1/3$ is restored and, if no other effects are operative, the vacuum energy density smoothly returns to a zero value. If, however, quantum relaxation effects are taken into account, the vacuum energy density does not return to zero, but approaches a constant value, which is of the order of the vacuum energy density (1.3) at $t \sim t_{\rm ew}$. This remnant vacuum energy density corresponds to the measured value of the cosmological constant (see, e.g., Refs. [7,8] and other references therein):

$$\Lambda \equiv \lim_{t\to\infty}\rho_V(t) \sim \rho_{V,0}(t_{\rm ew}) \sim H^4(t_{\rm ew}) \sim t_{\rm ew}^{-4} \sim (E_{\rm ew}^2/E_{\rm Planck})^4 \sim (10^{-3}\ {\rm eV})^4, \tag{1.4}$$

for the energy scales $E_{\rm Planck}$ and $E_{\rm ew}$ defined under (1.1a) and (1.2), respectively. The several steps in (1.4) will be detailed in the following, with the most important intermediate steps collected in (3.5) and (4.5).

The scenario outlined above differs from that of a cosmological phase transition, for which the vacuum energy density may only decrease (changing to a negative value if it was originally zero), and resembles the scenario in which the vacuum energy density is generated by the conformal anomaly. In fact, it has been suggested in Refs. [9,10] that

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the conformal anomaly of quantum chromodynamics (QCD) gives rise to the vacuum energy density $\rho_V(t) \propto |H(t)| E_{\rm QCD}^3$, where $E_{\rm QCD} \sim 10^2$ MeV is the QCD energy scale (see also Ref. [11] for related remarks). The rigorous microscopic derivation of this nonanalytic term has not yet been given, as it requires the detailed behavior of QCD in the infrared. For the moment, the main motivation of this particular nonanalytic term is that it naturally provides the correct order of magnitude for the present vacuum energy density and appears to give a good description of the late evolution of the Universe [12]. We remark also that part of the contribution of the conformal anomaly to the vacuum energy density has been estimated [13] as $\rho_V(t) \propto H^4(t)$, which has the same H dependence as (1.3). But the mechanisms of Ref. [13] and the present article are different, as will be explained later.

The scenario with the emergence of a positive vacuum energy density (1.4) triggered by the electroweak crossover confirms the earlier suggestion by Arkani-Hamed *et al.* [14] that electroweak physics is at the origin of a "triple cosmic coincidence" for the matter, radiation, and vacuum energy densities in the present Universe (see also the general discussion in Ref. [15]). While the coincidence among the matter and radiation energy densities appears to be justified by the electroweak scenario [14], the coincidence of these two ingredients with the remnant vacuum energy density (effective cosmological constant) Λ requires a particular relation in terms of the electroweak energy scale $E_{\rm ew}$ and the ultraviolet energy scale $E_{\rm Planck}$, namely, $\Lambda \sim E_{\rm ew}^8/E_{\rm Planck}^4$. In order to explain this particular relation, the authors of Ref. [14] suggested a phenomenological model but had to assume (page 4436, right column of the cited reference) that "an unknown mechanism canceled the vacuum energy density at the global minimum of the potential." In our scenario, this mechanism is natural.

II. DYNAMICAL EQUATIONS

The present discussion starts from the theory outlined in Ref. [2]. We introduce a special conserved quantity, the vacuum "charge" q, to describe the statics and dynamics of the quantum vacuum. An example of this vacuum variable is given by the four-form field strength [16–23], expressed in terms of q as $F_{\alpha\beta\gamma\delta}(x) = q(x)\sqrt{-g(x)}\epsilon_{\alpha\beta\gamma\delta}$. But the dynamic equations for the vacuum variable q and the metric $g_{\alpha\beta}$ are *universal*, that is, they do not depend on the particular realization of q. For example, in the four-form realization, the generalized Maxwell equation for the F field is reduced to the following generic equation for the charge q:

$$\frac{\partial \epsilon(q)}{\partial q} + R\frac{\partial K(q)}{\partial q} = \mu, \tag{2.1}$$

where $\epsilon(q)$ is the vacuum energy density expressed in terms of q (the possible dependence on other fields is kept implicit), R the Ricci curvature scalar, $K(q)$ the gravitational coupling parameter which depends on the vacuum state, and μ an integration constant. The latter quantity μ plays the role of a Lagrange multiplier related to the conservation of the charge q and corresponds to the chemical potential in thermodynamics [1,2].

The metric field $g_{\alpha\beta}$ obeys the generalized Einstein equation

$$2K(R_{\alpha\beta} - g_{\alpha\beta}R/2) = -2(\nabla_\alpha\nabla_\beta - g_{\alpha\beta}\Box)K(q) + \rho_V(q) g_{\alpha\beta} - T_{\alpha\beta}, \tag{2.2a}$$

$$\rho_V(q) \equiv \epsilon(q) - \mu q, \tag{2.2b}$$

where the metric has signature $(-, +, +, +)$ and $T_{\alpha\beta}$ is the matter energy-momentum tensor with vanishing covariant divergence $\nabla_\alpha T^{\alpha\beta} = 0$ from general coordinate invariance. The particular combination (2.2b), and not $\epsilon(q)$, is seen to determine the cosmological term in (2.2a), which is perhaps the most important characteristic of our approach.

In what follows, we choose a value μ_0 of the integration constant μ in such a way that, in the absence of matter or other types of perturbations, the solution of the equations corresponds to the full-equilibrium Minkowski-spacetime vacuum. The actual value μ_0 and corresponding charge q_0 of the equilibrium vacuum are determined by two equations:

$$[d\epsilon(q)/dq - \mu]_{\mu=\mu_0, q=q_0} = 0, \tag{2.3a}$$

$$[\epsilon(q) - \mu q]_{\mu=\mu_0, q=q_0} = 0, \tag{2.3b}$$

which follow from (2.1) and (2.2), respectively, for $R_{\alpha\beta} = T_{\alpha\beta} = 0$ and spacetime-independent q_0. The equilibrium conditions (2.3) are supplemented by the following stability condition:

$$(\chi_0)^{-1} \equiv q^2 \frac{d^2\epsilon(q)}{dq^2}\bigg|_{q=q_0} > 0, \tag{2.4}$$

where χ corresponds to the vacuum compressibility [1].

The "cosmological constant problem" would be completely solved if we could explain the origin of this particular value μ_0 for the integration constant μ appearing in (2.1) and (2.2). Here, our assumption is that the Minkowski-spacetime vacuum is a *self-sustained* system, i.e., an isolated system that can exist without external pressure, at $P = 0$. In general, the vacuum pressure P and the vacuum energy density ϵ are related by the thermodynamic Gibbs-Duhem equation [1], $P = -\epsilon + \mu q$. The vanishing pressure P allowed for a self-sustained system (from the assumed absence of external pressure) then gives the additional condition (2.3b), which fixes μ to the value μ_0. From this viewpoint, cosmology corresponds to the dynamic process of approach to the equilibrium state with $q = q_0$, which is natural for any system isolated from the external environment.

Close to equilibrium, at $|q - q_0| \ll |q_0|$, the dynamics of the system is determined by the coefficients in the Taylor expansion of $\epsilon(q)$ and $K(q)$ near the equilibrium point q_0:

$$K(q) = K(q_0) + K'(q_0)(q - q_0) + O((q - q_0)^2), \tag{2.5a}$$

$$\epsilon(q) - \mu_0 q = \epsilon''(q_0)(q - q_0)^2/2 + \epsilon'''(q_0)(q - q_0)^3/6 + O((q - q_0)^4). \tag{2.5b}$$

All coefficients in these expansions have Planck-scale values, for example, $K(q_0) = 1/(16\pi G_N) = (1/2)E^2_{\text{Planck}}$ in terms of Newton's constant G_N and the energy scale E_{Planck} defined under (1.1a).

We now consider the spatially flat FRW universe [7] described by the Hubble expansion parameter $H \equiv (da/dt)/a$ for scale factor $a(t)$ and use the dimensionless variables $y \propto (q - q_0)$ and $h \propto H$, which have been rescaled with the Planck-scale parameters of the theory. These two variables $y(\tau)$ and $h(\tau)$ are governed by the following two coupled ordinary differential equations (ODEs):

$$\ddot{y} - \dot{y}h + 2(1 + y)\dot{h} = -3(1 + w_M)[\dot{y}h + (1 + y)h^2 - r_V], \tag{2.6a}$$

$$\dot{h} + 2h^2 = r'_V, \tag{2.6b}$$

with the prime standing for differentiation with respect to y and the overdot for differentiation with respect to dimensionless cosmic time τ (cosmic time t in the corresponding Planckian units). In the derivation of the above ODEs, the function $K(q)$ has been assumed [2] to be linear in q for simplicity [in terms of the coefficients of (2.5a), one has $q_0K'(q_0) = K(q_0)$ and $K^{(n)}(q_0) = 0$ for $n \geq 2$].

The dimensionless vacuum energy density r_V (vacuum energy density ρ_V in Planckian units) is taken to be given by

$$r_V(y) = \tfrac{1}{2}\, y^2 + \tfrac{2}{3}\, y^3 + \tfrac{1}{6}\, y^4, \tag{2.7}$$

which vanishes in the equilibrium state $y = 0$, having chosen $\mu = \mu_0$ in (2.1) and (2.2). Later on, only the quadratic part of $r_V(y)$ will be relevant. Equations (2.6) and (2.7) lead to the rapid relaxation (1.1a), if the Universe starts out with a nonequilibrium value of the charge, $q_{\text{initial}} \neq q_0$ or $y_{\text{initial}} \neq 0$. These Eqs. (2.6) and (2.7) are, in fact, identical to Eqs. (5.2) and (5.3) in Ref. [2], to which the reader is referred for all details.

For the present analysis, it turns out to be useful to define the following matter EOS parameter:

$$\kappa_M \equiv 4 - 3(1 + w_M), \tag{2.8}$$

where $\kappa_M = 0$ corresponds to having matter with $T_\alpha{}^\alpha = 0$, for example, electromagnetic radiation (photons) or ultrarelativistic massive particles (e.g., electrons and positrons). Then, (2.6a) and (2.6b) can be written as

$$\ddot{y} + 3\dot{y}h + 2(1 + y)r'_V = 4r_V + \kappa_M[\dot{y}h + (1 + y)h^2 - r_V], \tag{2.9a}$$

$$\dot{h} + 2h^2 - r'_V = 0. \tag{2.9b}$$

The crucial observation, now, is that, for $\kappa_M(\tau) = 0$, there is a solution of the ODEs (2.9a) and (2.9b), where the vacuum energy density is exactly zero. This solution corresponds to an FRW universe with ultrarelativistic matter present but dark energy and cold dark matter (CDM) absent:

$$y(\tau) = 0, \tag{2.10a}$$

$$h(\tau) = 1/(2\tau), \tag{2.10b}$$

which, as said, holds for $\kappa_M(\tau) = 0$.

Next, consider what happens when the model universe described by (2.10) enters a phase at $t \sim t_{\text{kick}}$ for which $\kappa_M(t) \neq 0$. Then, the vacuum variable y becomes nonzero and a nonzero value of the vacuum energy density emerges continuously. Specifically, we consider a time $t_{\text{kick}} \gg t_{\text{Planck}}$, so that the corresponding dimensionless time is large, $\tau_{\text{kick}} \gg 1$. At large τ, the variable $y(\tau)$ is always small and one can make an expansion in terms of powers of y. To first order in y and h^2, one obtains the following ODEs from (2.9a) and (2.9b):

$$\ddot{y} + 3h\dot{y} + \omega^2 y = \kappa_M h^2, \tag{2.11a}$$

$$\dot{h} + 2h^2 - y = 0, \tag{2.11b}$$

with an implicit τ dependence for all three functions y, h, and κ_M. Here, ω is the natural frequency of the microscopic oscillations [2], which is given by $\omega = \sqrt{2}$ in Planckian units.

III. ELECTROWEAK KICK

There are different regimes for the behavior of the vacuum energy density obtained from (2.11), depending on the sharpness of the profile of the transition, i.e., the width $\Delta\tau_\kappa$ of the function $\kappa_M(\tau)$. For the case of a smooth transition (that is, smooth on microscopic time scales, $\Delta\tau_\kappa \gg 1/\omega \sim 1$), one may neglect the time derivatives of y in (2.11a) to obtain

$$y = \kappa_M h^2/2, \tag{3.1a}$$

$$y = \dot{h} + 2h^2, \tag{3.1b}$$

where the specific value $\omega^2 = 2$ has been reinstated in the first equation. Eliminating y from the above equations gives immediately the following solution for $h(\tau)$:

$$h(\tau) = \left[2\int_0^\tau d\tau'(1 - \kappa_M(\tau')/4)\right]^{-1}, \tag{3.2}$$

which holds for an arbitrary (smooth) function $\kappa_M(\tau)$ and has boundary condition $1/h(0) = 0$, appropriate for the standard hot big bang universe. Taking the square of (3.2), the solution for $y(\tau)$ follows from (3.1a).

Now apply this result to the cosmological epoch of the electroweak crossover [6]. During the crossover, the standard model particles acquire masses and, as a result, $w_M(t)$ deviates from $1/3$. In principle, this deviation may be enhanced by "new physics" at the TeV energy scale, which might be responsible for the observed cold-dark-matter component of the present Universe by providing a TeV-scale WIMP (weakly interacting massive particle). According to the electroweak scenario of Ref. [14], this new physics may have many particles ($n = 1, \ldots, N$) with masses $M_n \sim E_{\rm ew} \sim 1$ TeV, which are created before and during the electroweak epoch. Perhaps we will know from future particle-collider experiments (for example, at the Large Hadron Collider of CERN) whether or not there exists a TeV-scale WIMP responsible for the observed CDM.

Anyway, massive standard model particles (and possible additional massive particles of new TeV-scale physics) annihilate during the electroweak-crossover period and, afterwards, the EOS parameter returns to its standard radiation-dominated value $w_M = 1/3$ (or $\kappa_M = 0$), with the result that the vacuum energy density is no longer perturbed. In the epoch after the electroweak period when all perturbations have ceased, the Hubble parameter (3.2) is given by

$$h(\tau) \approx \frac{1}{2(\tau - \tau_0)}, \tag{3.3a}$$

$$\tau_0 \equiv \frac{1}{4}\int_0^\infty d\tau' \kappa_M(\tau'), \tag{3.3b}$$

for $\tau \gg \tau_0 \sim \tau_{\rm ew} \sim E^2_{\rm Planck}/E^2_{\rm ew} \sim 10^{30}$.

From (2.7) and (3.1a), the dimensionless and dimensionful vacuum energy densities during the electroweak crossover behave as follows:

$$r_V(\tau) = (1/8)\kappa_M^2(\tau) h^4(\tau), \tag{3.4a}$$

$$\rho_V(t) \propto \kappa_M^2(t) H^4(t), \tag{3.4b}$$

where only the quadratic part of (2.7) has been kept as $|y| \ll 1$ and where the precise numerical constant in (3.4b) depends on the microphysics but can be expected to be of order unity [2].

Even though result (3.4b) is similar to the vacuum energy density estimate [13] from the conformal anomaly, $\rho_V(t) \sim \langle T_\alpha{}^\alpha \rangle \sim H^4(t)$, the mechanism of the emerging vacuum energy density in (3.4) is different. The underlying theory [2] of result (3.4) has, in fact, a gravitational coupling parameter K that depends on the vacuum variable, $K = K(q)$, with Newton's constant recovered in the $q = q_0$ equilibrium state, $G_N = 1/(16\pi K(q_0))$. Precisely this variability $K(q)$ allows for a time-dependent vacuum energy density, $\dot\rho_V \propto \dot K(\dot H + 2H^2)$, provided the expansion differs from that of a radiation-dominated FRW universe with $H(t) = 1/(2t)$ and $\dot H + 2H^2 = 0$.

From (3.4b), the magnitude of the vacuum energy density at the crossover time (1.2) is given by

$$\rho_{V,0}(t_{\rm ew}) \sim H^4(t_{\rm ew}) \sim t_{\rm ew}^{-4} \sim E^8_{\rm ew}/E^4_{\rm Planck}, \tag{3.5}$$

where $\kappa_M(t_{\rm ew})$ has been assumed to be of order unity and where, for later use, a suffix 0 has been appended to distinguish the "classical" result. This completes the first step toward establishing a nonzero cosmological constant of the present Universe. The second step is to make sure that the vacuum energy density generated at $t \sim t_{\rm ew} \sim 10^{-12}$ s is not lost during the remaining 10^{10} yr.

IV. SUBSEQUENT EVOLUTION

The typical value of the vacuum energy density (3.5) emerging from the electroweak crossover is comparable to the presently observed value [7,8] of the vacuum energy density.[1] As mentioned in Sec. I, this suggests a possible explanation of the triple cosmic coincidence according to the electroweak scenario discussed in Ref. [14]. But, for this explanation to work, we need a mechanism to stabilize the vacuum energy density after the electroweak crossover.

At the moment, we do not have a complete theory which describes the *irreversible* dynamics of the quantum vacuum. The classical equations of q-theory [1] describe only the *reversible* classical dynamics of the vacuum. One needs to extend q-theory to the quantum domain, in order to incorporate the dissipative relaxation of the vacuum energy density due to the quantum effect of matter field radiation (matter quanta emission).

Awaiting the definite theory of the quantum vacuum, the following model equation can be used for a rough estimate:

$$\dot\rho_V = -\Gamma(t)[\rho_V(t) - \rho_{V,0}(t)]. \tag{4.1}$$

Here, $\rho_{V,0}(t)$ is the "bare" vacuum energy density driven by the kick, which, according to result (3.4b) of the classical q-theory, is given by

$$\rho_{V,0}(t) \propto \kappa_M^2(t) H^4(t), \tag{4.2}$$

and $\Gamma(t) \geq 0$ in (4.1) is the rate at which the "surplus" vacuum energy density is dissipated into particles.

Particle production occurs when the background spacetime is changing on a time scale comparable with the particle Compton time [24], which implies different particle production rates for different cosmological epochs. In the epoch before the electroweak crossover, matter consists of ultrarelativistic particles (radiation) with EOS parameter $\kappa_M = 0$ and, thus, there is no "external force" to drive the vacuum energy density. Rapid oscillations with frequency $\omega \sim E_{\rm Planck}$ lead to the decay of the vacuum energy density with the rate $\Gamma \sim \omega \sim E_{\rm Planck}$ [4,5]. As a result, (4.1) gives

[1] An excellent description of the currently available data is, in fact, given by the flat-ΛCDM model (cf. Refs. [7,8]), with an inhomogeneous cold-dark-matter component (EOS parameter $w_{\rm CDM} = 0$) and a perfectly homogeneous and time-independent vacuum energy density component ($w_V = -1$), which corresponds to Einstein's cosmological constant Λ.

exponential decay (1.1b) of the vacuum energy density to a zero value. The model universe rapidly approaches the stage with pure radiation, evolving as in (2.10).

During the electroweak crossover, the EOS parameter $\kappa_M(t)$ in (4.2) deviates from zero, which drives the vacuum energy density (4.1) away from zero toward a positive value. The change of the vacuum energy density during the crossover results in the emission of particles. The radiation rate $\Gamma(t)$ is concentrated in the crossover period, because after the crossover the model universe returns to radiation-dominated expansion without particle production. The decay rate $\Gamma(t)$ is, therefore, peaked at $t \sim t_{\rm ew}$,

$$\Gamma(t)|_{t \ll t_{\rm ew} \vee t \gg t_{\rm ew}} \ll \Gamma(t_{\rm ew}) \sim 1/t_{\rm ew}, \tag{4.3}$$

where the maximal value $1/t_{\rm ew}$ will be derived shortly. Note that the maximal rate $\Gamma(t_{\rm ew}) \sim E_{\rm ew}^2/(\hbar E_{\rm Planck})$ goes to infinity for $\hbar \to 0$ and fixed energy $E_{\rm ew}^2/E_{\rm Planck}$, so that (4.1) reproduces the classical result, $\rho_V(t) \to \rho_{V,0}(t)$. In fact, this particular classical limit corresponds to the hydrodynamic limit in fluid dynamics; cf. the section on "second viscosity" in Ref. [25]. Further remarks on the heuristics of the vacuum dynamics equation (4.1) will be presented in the paragraph starting a few lines after (4.5).

The estimate for the maximal value of the decay rate in (4.3) can be obtained as follows. Start from the observation [26] that, for an FRW universe with appropriate boundary conditions [27], the number of particles created per unit of time and per unit of volume is given by $\dot{n} \propto R^2$, where R is the Ricci curvature scalar. For an FRW universe with pure radiation, the Ricci scalar $R \propto (\dot{H} + 2H^2)$ vanishes and there is no particle production. As mentioned before, this is the reason why the radiation rate $\Gamma(t)$ is peaked in the crossover period.[2] In the period of the electroweak crossover, one has $R^2(t_{\rm ew}) \sim \dot{H}^2(t_{\rm ew}) \sim H^4(t_{\rm ew}) \sim \rho_V(t_{\rm ew})$. Particles created [24] during this period have a Compton time of order $t_{\rm ew}$ and, thus, a characteristic energy of order $E \propto 1/t_{\rm ew}$. The only known elementary particles whose energy E can be of order $1/t_{\rm ew} \sim E_{\rm ew}^2/E_{\rm Planck} \sim$ meV are massless gravitons and massive neutrinos, some of whose masses [34] may be comparable with $1/t_{\rm ew}$ (all the other particles of the standard model have larger masses, including the photon which gets an effective mass in the cosmic plasma). During the electroweak-crossover period, the radiated energy per unit of time and per unit of volume is then $\dot{\rho}_V \propto -E\dot{n} \propto -\rho_V/t_{\rm ew}$, giving $\Gamma(t_{\rm ew}) \sim 1/t_{\rm ew}$ for the decay rate entering (4.1) and delivering the announced estimate (4.3).

[2]Matter radiation must also vanish in a de Sitter spacetime, where no relaxation of the vacuum energy density is expected. For a discussion of the controversies concerning the stability of de Sitter spacetime, see, e.g., Refs. [28–33].

Now, the solution of (4.1) is given by

$$\rho_V(t) = \int_0^t dt' \Gamma(t') \rho_{V,0}(t') \exp\left[- \int_{t'}^t dt'' \Gamma(t'') \right], \tag{4.4}$$

for boundary condition $\rho_V(0) = 0$, which is reasonable for times t well after the Planck era. Since $\Gamma(t)$ is concentrated in the crossover period and has peak value (4.3), the solution (4.4) gives $\lim_{t\to\infty} \rho_V(t) \sim \rho_{V,0}(t_{\rm ew})$. For very late times, $t \gg t_{\rm ew}$, one thus obtains that the vacuum energy density approaches the following positive and time-independent value:

$$\rho_V(t)|_{t \gg t_{\rm ew}} \sim \rho_V(t_{\rm ew}) \sim \rho_{V,0}(t_{\rm ew}) \sim E_{\rm ew}^8/E_{\rm Planck}^4, \tag{4.5}$$

where (3.5) has been used in the last step. The final result (4.5) is comparable to the measured value of the cosmological constant, as shown in (1.4).

The heuristics of the obtained nonzero remnant vacuum energy density is as follows. The quantity $\Gamma(t)$ in (4.1) can be interpreted as the inverse of the instantaneous response time $\theta(t)$ of the vacuum energy density $\rho_V(t)$ to an "external perturbation." Here, the external perturbation (4.2) comes from the "kick" in $\kappa_M(t)$, which is assumed to happen at $t \sim t_{\rm ew}$ and to have a full width at half maximum $\Delta t_\kappa \sim t_{\rm ew}$. Moreover, $\Gamma(t) \equiv 1/\theta(t)$ is taken to have a width Δt_Γ, which is comparable to or larger than the duration of the kick, $\Delta t_\Gamma \gtrsim \Delta t_\kappa$. *A priori*, there are then two possibilities. First, the typical response time θ is short ($\theta \ll \Delta t_\kappa$), which implies that the vacuum energy density $\rho_V(t)$ can follow the kick in $\kappa_M(t)$ and that $\rho_V(t)$ can recover a near-zero value, as $\kappa_M(t)$ drops to zero for $t \gg t_{\rm ew}$. Second, the typical response time θ is relatively long ($\theta \gtrsim \Delta t_\kappa$), which implies that the vacuum energy density $\rho_V(t)$ cannot keep up with $\kappa_M(t)$, as the latter drops to zero, and that a nonzero asymptotic value of ρ_V remains. According to (4.3), this second type of behavior occurs for the case considered, with $\theta \sim \Delta t_\kappa \sim t_{\rm ew}$, and a nonvanishing asymptotic value of $\rho_V(t)$ follows from the general solution (4.4). In short, the nonzero remnant vacuum energy density (4.5) is a *time-lag effect*, because the response (relaxation) time of the vacuum energy density is of the same order of magnitude as the duration of the kick.[3]

After the electroweak crossover, further perturbations of the vacuum energy density occur during the QCD confinement transition at a typical temperature

[3]In principle, the same time-lag (freezing) mechanism may work for the scenario of Ref. [13], where a vacuum energy density $\rho_V \propto H^4(t)$ emerges due to the conformal anomaly. During the electroweak crossover, the number of massless fields contributing to the anomaly changes, which results in a kick of the vacuum energy density. In turn, this gives rise to matter radiation, which leads to the stabilization of the vacuum energy density at a value of the order of (4.5).

$T \sim E_{\rm QCD} \sim 10^2$ MeV and the epoch following the moment of radiation-matter equality, when the radiation-dominated effective EOS parameter $w_M = 1/3$ changes to the matter-dominated parameter $w_M = 0$. (The moment of radiation-matter energy density equality happens to be close to the epoch of recombination with $T \sim T_{\rm rec} \sim 10^{-1}$ eV and this energy scale will be used for definiteness.) The first-mentioned perturbation of the vacuum energy density by the QCD confinement transition (see, e.g., Fig. 19.3 in Ref. [34] for the change in the number of relativistic degrees of freedom) can be expected to give a change of the order of $H^4(t_{\rm QCD}) \sim (E^2_{\rm QCD}/E_{\rm Planck})^4$, which is negligible compared to the present value of Λ according to (1.4). The second perturbation of the vacuum energy density acts during the whole matter-dominated era. However, the resulting change of the vacuum energy density can be expected not to exceed a value of order $H^4(t_{\rm rec}) \sim (T^2_{\rm rec}/E_{\rm Planck})^4$, which is, again, many orders of magnitude smaller than the present value of Λ and can be neglected.

Turning the argument of the preceding paragraph around, it would seem that the suggested electroweak explanation (1.4) of the present value of Λ would rule out (leave no room for) similar crossover effects at much higher temperature $T_\star \gg E_{\rm ew} \sim$ TeV, the expected remnant vacuum energy density $H^4(t_\star)$ being much larger than $H^4(t_{\rm ew})$. This conclusion, if correct, may be consistent with the picture [14] of having only two fundamental energy scales, $E_{\rm ew}$ and $E_{\rm Planck}$, without unification of the standard model gauge group at an intermediate grand-unification energy scale [35,36].

V. DISCUSSION

The q-theory approach [1] to gravitational effects of the quantum vacuum suggests at least two types of behavior for the evolution of the vacuum energy density, each based on solutions of the q-theory dynamical equations and their modifications due to dissipative effects from matter radiation. For the first type of solution [2,3], the model universe is vacuum dominated with, according to (1.1a), the vacuum energy density $\rho_V(t)$ relaxing as $1/t^2$ from its natural Planck-scale value at early times when the system is far from equilibrium to a naturally small value at late times when the system is close to equilibrium. [Quantum effects (e.g., the emission of matter quanta caused by the rapid oscillations of the vacuum state) make the relaxation even faster, as shown by (1.1b).] This essentially solves the main cosmological problem (but with the *caveat* mentioned in Sec. II): the present vacuum energy density is small compared to Planck-scale values simply because the age of our Universe happens to be large compared to Planck-scale values. However, it leaves the following question: why does the vacuum energy density not relax completely to zero as $t \to \infty$?

In order to answer this last question, we presented a second type of solution in which the vacuum energy density has already relaxed to zero after the initial disturbance in the very early universe and a nonzero value reemerges only after a kick generated by nonrelativistic matter during the epoch of the electroweak crossover. (These nonrelativistic particles consist of standard model particles and possibly thermal relics from new physics at the TeV scale, as discussed in Sec. III.) In the process, a nonoscillating vacuum energy density is generated, which starts to decay after the kick. Such a behavior emerges during the electroweak period, because in this epoch the matter EOS parameter $w_M(t)$ deviates from the radiative value $w_M = 1/3$. Quantum effects now lead to a stabilization of the vacuum energy density at the level indicated by (4.5), which reproduces the expression suggested previously by Arkani-Hamed *et al.* [14].

It was assumed in the reasoning leading up to (4.5) that there was no real phase transition at cosmic time $t_{\rm ew}$. Instead, there was taken to be a crossover at a temperature $T_{\rm ew} = O(10^2\ {\rm GeV})$, which does not give a change of order $T^4_{\rm ew}$ in the vacuum energy density as a genuine phase transition would do. The absence of a real electroweak phase transition is by now well established [6], at least, in the framework of the standard model of elementary particle physics (the numerical value of the crossover temperature is estimated [6] as $T_{\rm ew} \sim 300$ GeV for $m_{\rm Higgs} \sim 150$ GeV). The new physics at the TeV scale mentioned in the previous paragraph and Sec. III is assumed not to affect the nature of the electroweak crossover. But the massive relic particles of the new physics can make a significant contribution to the EOS parameter $\kappa_M(t)$ and can also increase the numerical value of the effective energy scale $E_{\rm ew}$, thereby augmenting the magnitude of the estimated dark energy (4.2), (4.3), and (4.4) and bringing the theoretical value (1.4) closer to the observed value [7,8] of approximately $(2\ {\rm meV})^4$.

The electroweak scenario of Ref. [14] may solve part of the triple cosmic coincidence puzzle, as the same order of magnitude follows naturally for the cold-dark-matter density and the radiation density in the present epoch. Combined with the argument for the effective cosmological constant (1.4) of the present article, this suggests that TeV-scale physics may be responsible for the triple coincidence of vacuum, matter, and radiation energy densities in the present Universe (perhaps even a quintuple coincidence if also the baryon and neutrino energy densities are considered [14]).

For the present epoch, the vacuum energy density would be essentially time independent according to (4.4) and, observationally, the corresponding universe would be indistinguishable from the one of the ΛCDM model (cf. footnote 1). But, theoretically, we would have gained in understanding the magnitude of the cosmological "constant" Λ as given by (1.4), in addition to explaining the triple or quintuple cosmic coincidence mentioned above.

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