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Automatized full one-loop renormalization of the MSSM. II. The chargino-neutralino sector, the sfermion sector, and some applications

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An on-shell renormalization program for the chargino/neutralino and the sfermion sectors within the minimal supersymmetric standard model as implemented in a fully automated code, SLOOPS, for the calculation of one-loop processes at the colliders and in astrophysics, is presented. This is a sequel to our previous study where an on-shell renormalization of the Higgs (and the gauge/fermion) sector is performed. The issue of mixing is treated in a unified and coherent manner in all these sectors, in particular, we give some new insight into the renormalization of the mixing angle in the sfermion sector. Like with the issue of $\tan\beta$ in the Higgs sector, we discuss different schemes for the mixing in the sfermion sector. We also perform numerical comparisons between our code SLOOPS and different results found in the literature. In particular, we consider loop corrections to the neutralino and sfermion masses, chargino pair production and stau pair production in e^+e^- colliders as well as a few decays of the heavier chargino. For all these observables, we analyze the $\tan\beta$ scheme dependence using different definitions of this parameter and comment on the impact of using different renormalization of the mixing parameter in the sfermion sector.

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I. INTRODUCTION

The description of the Higgs within the standard model is unsatisfactory as it poses the problem of naturalness. Besides, the Higgs particle is still missing. Moreover, there is overwhelming evidence that there is a large amount of dark matter that can not be accounted for by any of the particles of the standard model (SM). All this points to new physics. The best motivated model of this new physics is undoubtedly supersymmetry that rests on solid theoretical grounds and allows for full calculability and therefore predictions. Full calculability is not, by itself, a sacrosanct virtue but it must be admitted that supersymmetry addresses some of the problems of the SM. Indeed, although the primary motivation for supersymmetry as implemented in the MSSM, minimal supersymmetric standard model, was to solve the hierarchy and naturalness problem, it was soon realized that the model contained an excellent candidate for cold dark matter beside incorporating almost naturally the gauge unification. However, predictions of the MSSM based on tree-level calculations predict a Higgs that is lighter than the Z mass. By now, this is ruled out. It is only through radiative corrections that the MSSM has survived. Radiative corrections are therefore essential. Moreover, the next generation of experiments at the colliders will reach unsurpassed precision which will need computations beyond the tree approximation. Extraction of the cosmological parameters that are used to measure the relic density of cold dark matter have recently reached an accuracy that will also soon compete with the accuracy we have been accustomed to from the LEP era. Precision loop calculations within the MSSM are therefore a must. It must be said that quite a lot of these calculations have been performed, even though the bulk of these have been made for observables at the colliders and indirect precision measurements such $(g-2)_\mu$, $b \to s\gamma, \ldots$. Very little has been done concerning the cross sections relevant for dark matter annihilation that enter, for example, a precise prediction of the relic density. It rests that these calculations have been done piecemeal and quite often within different renormalization schemes.

One of the reasons that these calculations have been done piecemeal is that the MSSM, though minimal, still contains a large number of particles and a very large number of parameters especially through the soft-supersymmetry (SUSY) breaking terms, for example. This explains why different groups have concentrated on different sectors of the model. Performing loop calculations with so large a number of parameters and huge numbers of interactions is an almost untractable task, especially if one has to be ready to perform precision predictions for any process or at least a large number of processes as it occurs, for example, with the calculation of the relic density where many processes and subprocesses are at play for a particular choice of parameters. One has to rely on a fully automatized code for such calculations. SLOOPS is such a code with an automatization starting already from the implementation of the model file. Instead of coding by hand all the Feynman rules, which usually constitute the model file and realizing that for one-loop applications one needs to also enter the full set of counterterms, SLOOPS relies on a much improved version of LANHEP [1] to automatically generate the model file. Through LANHEP, one writes the Lagrangian in a compact

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form through multiplets and the use of the superpotential. The improved version of LANHEP has built-in rules for shifting fields and parameters thus easily generating the set of counterterms. This approach therefore takes care of generating the few thousand Feynman rules for all the vertices needed for the calculations of any one-loop process in the MSSM.

The model file thus generated is interfaced to the bundle of packages FEYNARTS [2], FORMCALC [3], and LOOPTOOLS [4] that we will refer to as FFL for short. This code has recently been used very successfully for the first calculation of a number of processes that enter the prediction of the relic density of dark matter [5] as well as some one-loop induced processes of relevance for indirect detection [6].

The aim of the present paper is to first give some details on the renormalization scheme that is implemented in SLOOPSand, in particular, how the sfermion sector and the neutralino/chargino sector are treated. This is a follow up to our paper detailing the renormalization of the Higgs sector, where apart from the implementation of the scheme we brought up crucial issues related to the definition of $\tan\beta$, the issue of gauge invariance, and the impact of different schemes on observables in the Higgs sector. The present paper will also compare one-loop predictions in the sfermion and chargino/neutralino sector based on different schemes for $\tan\beta$. We will also make some interesting observations and analyses concerning the treatment of mixing in these sectors, especially how one could define a process independent mixing angle in the sfermion sector.

The paper is structured as follows. In Sec. II, we give a brief summary of the renormalization scheme used in the code for the Higgs sector and the SM-like sector that includes the gauge and fermion parts. In the same section we also present a general overview of our approach. Section III deals with the sfermion sector, both squarks and sleptons that we use in SloopS. In Sec. IV, we detail our on-shell renormalization scheme in the chargino/neutralino sector and comment on some alternatives for the choice of the input parameters. Section V illustrates the use of the code for some applications. We will give results for the one-loop corrections to the masses of the heavier neutralinos and the sfermions that are not used as input in our schemes. We also present results for the one-loop calculation of chargino pair production and sfermion pair production at a linear collider $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$ and $e^+e^- \to \tilde{\tau}_i\bar{\tilde{\tau}}_j$, comparing whenever possible with existing calculations. Finally, we compare our results with those of GRACE-SUSY [7], taking as examples a few decay channels of the heavier chargino for a certain choice of parameters. In all these examples, the $\tan\beta$-scheme dependence is also studied thus complementing the scheme dependence that we studied for observables within the Higgs sector and for annihilation processes of interest for the relic density computations. Section VI gives a brief summary and outlook.

II. RENORMALIZATION: THE GENERAL APPROACH, THE GAUGE, THE FERMION, AND THE HIGGS SECTOR

Our renormalization of the MSSM, with *CP* conservation with all parameters taken real, follows the same strategy and the same procedure that we adopted for the renormalization of the standard model, see [8]. In particular, we strive for an on-shell renormalization of the physical parameters. Counterterms to these parameters are gauge independent. Wave function renormalization is introduced in order that the residue of the two-point function, the propagator, is unity for the *physical* state on its mass shell as well as to eliminate any mixing between the physical fields when these are on shell so that the qualification as a physical field is maintained order by order. Naturally, these field renormalization constants are not needed if one only requires that the observables of the *S*-matrix are finite, but one does not insist that all the Green's function to be finite, see [8]. On the technical side, this field renormalization avoids that one included in the calculation of matrix elements loop corrections on the external legs. Moreover, there is no need to consider field renormalization for the unphysical fields, like the Goldstones bosons, or on the current fields before mixing. Talking about the Goldstone fields, a very powerful feature of SLOOPS is the use and implementation of a nonlinear gauge-fixing condition [8–10]. The gauge-fixing condition furnishes eight gauge parameters ($\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\omega}, \tilde{\kappa}, \tilde{\rho}, \tilde{\epsilon}, \tilde{\gamma}$) on which we could perform gauge parameter independence checks, beside the ultraviolet finiteness checks. The gauge fixing writes

$$\begin{aligned}
\mathcal{L}^{GF} &= -\frac{1}{\xi_W} F^+ F^- - \frac{1}{2\xi_Z}|F^Z|^2 - \frac{1}{2\xi_\gamma}|F^\gamma|^2, \quad \text{with} \\
F^+ &= \Big(\partial_\mu - ie\tilde{\alpha}\gamma_\mu - ie\frac{c_W}{s_W}\tilde{\beta}Z_\mu\Big)W^{\mu +} \\
&\quad + i\xi_W \frac{e}{2s_W}(v + \tilde{\delta}h^0 + \tilde{\omega}H^0 + i\tilde{\rho}A^0 + i\tilde{\kappa}G^0)G^+, \\
F^Z &= \partial^\mu Z^0_\mu + \xi_Z \frac{e}{s_{2W}}(v + \tilde{\epsilon}h^0 + \tilde{\gamma}H^0)G^0, \\
F^\gamma &= \partial_\mu \gamma^\mu.
\end{aligned} \tag{2.1}$$

As extensively stressed in [8,9], the gauge-fixing term is considered *renormalized*. h^0 and H^0 are, respectively, the lightest and heaviest *CP*-even Higgses, A^0 is the *CP*-even Higgs, $G^{0,\pm}$ are the Goldstone bosons, and $W^\pm$, Z, γ are, with obvious notations, the gauge fields. We have $c_W \equiv \cos\theta_W = M_W/M_Z^0$.[1] We work with $\xi_{W,Z,\gamma} = 1$ in order not to have to deal with too high a rank tensors concerning the loop libraries, see [9].

[1]To avoid clutter, we use some abbreviations for the trigonometric functions. For example, for an angle θ, $\cos\theta$ will be abbreviated as c_θ, *etc*... t_β will then stand for $\tan\beta$.

Another crucial feature of our renormalization program is our treatment of the mixing which occurs in all sectors of the MSSM. In general, fields are expressed in the current basis. They, however, mix. Physical mass eigenstates fields are obtained from these current fields through some rotation matrix at tree level. We consistently take, in all sectors, this matrix to be renormalized and therefore no extra counterterm is introduced to this matrix. At one loop, this will still leave some transitions between fields, however, field renormalization is defined to precisely get rid of any residual mixing when the physical particles are on shell. Therefore inducing counterterms for the rotation matrix is redundant and not helpful.

Let us now briefly recap the renormalization of the gauge, fermion, and Higgs sector.

A. The fermion and gauge sector

The fermion sector as well as the gauge sector are renormalized on shell. It means, for example, that the gauge boson masses $M_{W^\pm}$ and M_{Z^0} are defined from the pole mass, imposing the one-loop on-shell condition on the mass counterterms as

$$\begin{aligned} \delta M^2_{W^\pm} &= -\mathrm{Re}\Sigma^T_{W^\pm W^\pm}(M^2_{W^\pm}), \\ \delta M^2_{Z^0} &= -\mathrm{Re}\Sigma^T_{Z^0 Z^0}(M^2_{Z^0}). \end{aligned} \tag{2.2}$$

The electric charge e is defined in the Thomson limit. Since the MSSM processes and parameters are taking place at the weak scale, the effective gauge coupling constant is of order $\alpha(M^2_{Z^0})$ which includes large logarithms from the very light standard model charged fermion masses. It is useful to reparametrize the one-loop corrections in terms of this effective coupling in order to absorb these large logarithms, as we will see later.

B. The Higgs sector

The renormalization scheme and renormalization procedure at one loop in the Higgs sector that we adopt in the code is detailed in Ref. [9]. The only ingredient that makes its way from the Higgs sector and the Higgs observables to the chargino/neutralino sector and the sfermion sector is the ubiquitous t_β and its renormalization. We use the same notation as in [8]. At tree level, t_β is defined by the ratio of the two vacuum expectation values $t_\beta = v_2/v_1$. At one loop, as pointed out in Refs. [9,11], it is difficult to find a proper definition for t_β. In [9], we critically discussed the issue of gauge invariance as regards to a different definition of t_β and looked quantitatively at the scheme dependence introduced by t_β in some Higgs observables. We will extend this investigation in our applications to observables involving the sfermions and the chargino/neutralinos. We therefore consider 4 definitions which are detailed in [9].

(i) $A_{\tau\tau}$ scheme.
t_β is extracted from the decay $A^0 \to \tau^+\tau^-$ to which the QED corrections have been subtracted. This leads to a gauge-independent counterterm. In Ref. [12], the decay of the charged Higgs boson H^+ into τ^+ and associated neutrino ν_τ has been suggested. This would qualify as a gauge-independent definition, the advantage of our $A^0 \to \tau^+\tau^-$ is that the full QED corrections can be extracted most unambiguously.

(ii) MH scheme.
Here, the heaviest CP-even Higgs mass M_{H^0} is taken as input. This definition is obviously gauge independent and process independent, but unfortunately we remarked that it induces large corrections in many cases.

(iii) $\overline{\mathrm{DR}}$ (dimensional reduction) scheme.
Here, only the ultraviolet part of an observable such as $A^0 \to \tau^+\tau^-$ (or any other definition but within the linear gauge, see [9]) is extracted. In this scheme, the counterterm depends explicitly on a scale $\bar{\mu}$. This scale $\bar{\mu}$ is fixed at M_{A^0}.

(iv) DCPR (Dabelstein, Chankowski, Pokorski, and Rosiek) scheme [13].
δt_β is extracted from the A^0-Z^0 transition at $q^2 = M^2_{A^0}$,

$$\frac{\delta t_\beta^{\mathrm{DCPR}}}{t_\beta} = -\frac{1}{M_Z s_{2\beta}} \mathrm{Re}\Sigma_{A^0 Z^0}(M^2_{A^0}). \tag{2.3}$$

The self energy of the A^0-Z^0 transition at large t_β is dominated by the bottom/tau loops because of the $A^0 bb$ vertex which is proportional to $m_b t_\beta$ and thus enhanced when t_β becomes large,

$$\begin{aligned} \frac{\delta t_\beta^{\mathrm{DCPR}}}{t_\beta} \simeq -\frac{t_\beta}{s_{2\beta}} \frac{g^2}{c_W^2 M_Z^2} \frac{1}{4\pi^2} (3m_b^2 B_0(M^2_{A^0}, m_b^2, m_b^2) \\ + m_\tau^2 B_0(M^2_{A^0}, m_\tau^2, m_\tau^2)). \end{aligned} \tag{2.4}$$

The loop functions B_0 is defined in [14]. At large t_β, $s_{2\beta} \sim 2/t_\beta$, the finite part of $\delta t_\beta/t_\beta$ in the DCPR scheme is of order t_β^2. This scheme is not gauge independent and would depend on some parameter of the nonlinear gauge, for example. When comparing the results of observables within this scheme, we will set all nonlinear gauge parameters to zero, i.e. we will be specializing to the linear gauge.

III. THE SFERMION SECTOR AND ITS RENORMALIZATION

The sfermion sector comprises the superpartners of the fermions of the standard model where the interaction fields are the *chiral* left and right states. We do not consider generation mixing. For each generation, the field content is therefore the doublet $\tilde{Q}_L = (\tilde{u}_L, \tilde{d}_L)$ and singlets $\tilde{u}_R$ and $\tilde{d}_R$ for the squarks. For the sleptons, we have $\tilde{E}_L = (\tilde{\nu}_L, \tilde{e}_L)$ and $\tilde{e}_R$. In case the corresponding Yukawa coupling is zero with vanishing fermion masses, we expect no $\tilde{u}_L$-$\tilde{u}_R$ and $\tilde{d}_L$-$\tilde{d}_R$ mixing so that the physical fields are $\tilde{u}_L$, $\tilde{u}_R$, and $\tilde{d}_L$, $\tilde{d}_R$ in the squark sector. Let us briefly recall where the mass parameters of the sfermion sector originate from and how many can be identified and defined solely within the sfermion sector, once, for example, the Higgs sector and gauge sector have been identified.

(i) The soft supersymmetry breaking terms

$$\mathcal{L}^{\tilde{f}}_{\text{soft}} = -\sum_{\tilde{f}_i} M^2_{\tilde{f}_i}\tilde{f}^*_i\tilde{f}_i \qquad \tilde{f}_i = \tilde{Q}_L, \tilde{L}_L, \tilde{u}_R, \tilde{d}_R, \tilde{e}_R \tag{3.1}$$

$$- \epsilon_{ij}\bigg(\frac{\sqrt{2}m_u}{v_2}A_u H^i_2 \tilde{Q}^j_L \tilde{u}^*_R + \frac{\sqrt{2}m_d}{v_1}A_d H^i_1 \tilde{Q}^j_L \tilde{d}^*_R + \frac{\sqrt{2}m_e}{v_1}A_e H^i_1 \tilde{L}^j_L \tilde{e}^*_R + \text{H.c.}\bigg). \tag{3.2}$$

Our conventions for the Higgs doublet and the vacuum expectation values of these are defined in [9]. Supersymmetry breaking therefore provides the soft scalar masses $M^2_{\tilde{f}_i}$ in Eq. (3.1) and the trilinear scalar coupling A_f parameters in Eq. (3.2) for $f = e, u, d$ of one generation. The contribution of the latter vanishes in the chiral limit where the mass of the fermion m_f vanishes. The latter generates not only a contribution to the mass of the different sfermions but also contributes to the coupling of the sfermions to Higgses and Goldstones. As known, because of the $SU(2)$ symmetry, there is only one soft mass parameter for the up and down left component of the scalars.

(ii) Sfermion masses get also a contribution from the usual Yukawa mass terms; these are proportional to the corresponding m^2_f.

(iii) We also get contributions from the supersymmetry conserving F terms. The $F(f)$ contribution does not mix left and right explicitly (though it is proportional to the corresponding fermion masses, m^2_f). This only generates couplings to Higgses. The $F(H_{1,2})$ involve the μ parameter and generate supersymmetry conserving trilinear scalar coupling. They lead to left-right mixing which is proportional to $m_f\mu$.

(iv) There are also D term contributions, chirality conserving, proportional to the gauge boson masses. These give contributions to the sfermion mass terms, $\tilde{f}\tilde{f}$, Higgs couplings $\tilde{f}\tilde{f}H$, G, and quartic scalar couplings: $\tilde{f}\tilde{f}\tilde{f}\tilde{f}$ and $\tilde{f}\tilde{f}HH$. Once the gauge and Higgs sector have been renormalized, these contributions are also.

These simple observations show that since the A_f terms and μ contributions do not act similarly on the mass term and the Higgs couplings of sfermions, renormalization of the sfermion two-point functions (mass, mixing, and wave function renormalization) is not enough to completely renormalize processes with ordinary standard particles and sfermions. One needs also to define a renormalization to the μ parameter. This is most conveniently done from the chargino/neutralino sector. Note, however, that the Higgs coupling to sfermions can provide an alternative definition to μ.

A. Renormalization of the squark sector

We show in detail the different steps specializing to those squarks with mixing. The case with no mixing is then trivial.

1. Fields and parameters at tree level

The tree-level kinetic and mass term for the squarks $\tilde{q} = \tilde{u}, \tilde{d}$ are given by

$$\mathcal{L}^{\tilde{q}} = -\frac{1}{2}\begin{pmatrix} \partial_\mu \tilde{q}^*_L & \partial_\mu \tilde{q}^*_R \end{pmatrix}\begin{pmatrix} \partial^\mu \tilde{q}_L \\ \partial^\mu \tilde{q}_R \end{pmatrix} + \begin{pmatrix} \tilde{q}^*_L & \tilde{q}^*_R \end{pmatrix}\mathcal{M}^2_{\tilde{q}}\begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}, \tag{3.3}$$

with the 2×2 nondiagonal mass matrix

$$\mathcal{M}^2_{\tilde{q}} = \begin{bmatrix} M^2_{\tilde{q}LL} & M^2_{\tilde{q}LR} \\ M^2_{\tilde{q}LR} & M^2_{\tilde{q}RR} \end{bmatrix}. \tag{3.4}$$

The different components of this matrix are

$$M^2_{\tilde{q}LL} = M^2_{\tilde{Q}_L} + m^2_q + c_{2\beta}(T^3_q - Q_q s^2_W)M^2_Z, \tag{3.5}$$

$$M^2_{\tilde{q}RR} = M^2_{\tilde{q}_R} + m^2_q + c_{2\beta}Q_q s^2_W M^2_Z, \tag{3.6}$$

$$M^2_{\tilde{q}LR} = m_q(A_q - \mu t_\beta^{-2T^3_q}). \tag{3.7}$$

$M^2_{\tilde{Q}_L}$ is the soft-supersymmetry-breaking mass parameter of the $SU(2)_L$ doublet, whereas $M^2_{\tilde{q}_R}$ is the soft-SUSY-breaking mass parameter of the singlet. T^3_q and Q_q are the third component of the isospin and the electric charge, respectively. $M^2_{\tilde{q}LR}$ is the mixing parameter that has contributions from both the Higgsino supersymmetry conserving mass parameters and the trilinear supersymmetry breaking term. This induces mixing between the left and right components. This mixing vanishes for sfermions associated to massless quarks but is important especially for the third family squarks. Note that this mixing can also vanish at tree level, even for massive quarks for exceptional $A_t = \mu/t_\beta$ for stops and $A_b = \mu t_\beta$ for sbottoms.

If μ is to be determined from the chargino/neutralino sector, this sector involves 5 new parameters, $M_{\tilde{Q}_L}$, $M_{\tilde{u}_R}$, $M_{\tilde{d}_R}$, A_u, A_d and thus requires 5 renormalization conditions. For a physical on-shell renormalization, this requires trading these Lagrangian parameters with 5 physical parameters. Owing to $SU(2)$ invariance, the soft-breaking mass parameters $M_{\tilde{Q}_L}$ of the left-chiral scalar fermions of each isospin doublet are identical. Thus, one of the physical squark masses, say $\tilde{u}_1$, could be expressed in terms of the other masses which will be used as input. The mass of the $\tilde{u}_1$ would then receive a finite shift at the one-loop level. In order to find the physical fields $\tilde{q}_{1,2}$, we introduce a rotation matrix $R_{\tilde{q}}$ such as

$$\begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = R_{\tilde{q}} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}, \qquad R_{\tilde{q}} = \begin{pmatrix} c_{\theta_q} & s_{\theta_q} \\ -s_{\theta_q} & c_{\theta_q} \end{pmatrix}. \tag{3.8}$$

This transformation diagonalizes the mass matrix $\mathcal{M}^2_{\tilde{q}}$,

$$M^2_{\tilde{q}} = R_{\tilde{q}} \mathcal{M}^2_{\tilde{q}} R^\dagger_{\tilde{q}} = \mathrm{diag}(m^2_{\tilde{q}_1}, m^2_{\tilde{q}_2}), \qquad m^2_{\tilde{q}_1} > m^2_{\tilde{q}_2}. \tag{3.9}$$

The physical masses are expressed in terms of the soft-SUSY mass terms as

$$m^2_{\tilde{q}_{1,2}} = \frac{1}{2}\left(M^2_{\tilde{q}LL} + M^2_{\tilde{q}RR}) \pm \frac{1}{2} \times \sqrt{(M^2_{\tilde{q}LL} - M^2_{\tilde{q}RR})^2 + 4(M^2_{\tilde{q}LR})^2}\right). \tag{3.10}$$

For further reference, it is useful to express $s_{2\theta_q}$, the parameter that measures the amount of mixing, in terms of the Lagrangian mixing parameter and the physical masses

$$s_{2\theta_q} = \frac{2M^2_{\tilde{q}LR}}{m^2_{\tilde{q}_1} - m^2_{\tilde{q}_2}}. \tag{3.11}$$

Note, also the trivial fact that $s_{2\theta_q}$ as expressed through Eq. (3.11) is regular in the limit $m^2_{\tilde{q}_1} \to m^2_{\tilde{q}_2}$ since this necessarily corresponds to no mixing with $M^2_{\tilde{q}LR} = 0$. In this limit, we can take $\theta_q = 0$. At tree-level, $s_{2\theta_q}$ can be accessed directly through the $\tilde{q}_1 \to \tilde{q}_2 Z^0$ (or $Z^{0*} \to \tilde{q}_1 \bar{\tilde{q}}_2$) which is described by the Lagrangian

$$\mathcal{L}_{\tilde{q}_1\tilde{q}_2 Z} = i g_Z T^3_f \frac{s_{2\theta_f}}{2} ((\tilde{f}^*_1 \overleftrightarrow{\partial} \tilde{f}_2 + \tilde{f}^*_2 \overleftrightarrow{\partial} \tilde{f}_1) Z^0_\mu). \tag{3.12}$$

Provided both parameters $\theta_{u,d}$ have been determined along side the physical masses of $\tilde{u}_2$, $\tilde{d}_2$, $\tilde{d}_1$ one determines the tree-level $\tilde{u}_1$ mass

$$m^2_{\tilde{u}_1} = \frac{1}{c^2_{\theta_u}} (c^2_{\theta_d} m^2_{\tilde{d}_1} + s^2_{\theta_d} m^2_{\tilde{d}_2} - s^2_{\theta_u} m^2_{\tilde{u}_2} + m^2_u - m^2_d + c_{2\beta} M^2_W). \tag{3.13}$$

In principle, we could also use all four masses as input and trade this input with one of the mixing parameters, leading to

$$s^2_{\theta_u} = \frac{c^2_{\theta_d} m^2_{\tilde{d}_1} + s^2_{\theta_d} m^2_{\tilde{d}_2} - m^2_{\tilde{u}_1} + m^2_u - m^2_d + c_{2\beta} M^2_W}{m^2_{\tilde{u}_2} - m^2_{\tilde{u}_1}}. \tag{3.14}$$

Note, however, that the appearance of $(m^2_{\tilde{u}_2} - m^2_{\tilde{u}_1})$ in the denominator makes this definition subject to large uncertainties especially for nearly degenerate masses of $\tilde{u}_1$ and $\tilde{u}_2$. The definition from a decay such as $\tilde{u}_1 \to \tilde{u}_2 Z^0$, if open, is more direct. This is reminiscent of our discussion about the choice of a good definition of the parameter $\tan\beta$ in [9]. Compared to the case of the neutralino/chargino system, the extraction of the underlying parameters in terms of the physical mass parameters is rather trivial. In fact, the most important underlying parameter to extract here is A_f as this will be needed for the coupling to Higgses.

2. Counterterms

So far all fields and the parameters of the Lagrangian should be considered as bare quantities. The bare parameters, for example, labeled as $\mathcal{P}_0$ will now be split into a renormalized parameter $\mathcal{P}$ and its counterterm $\delta\mathcal{P}_0$.

It is very important to stress that the rotation matrix is defined as renormalized in our approach. This we have pursued consistently throughout all the sectors. Therefore from Eq. (3.8)

$$\begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix}_0 = R_{\tilde{q}} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}_0, \quad \text{implies} \begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = R_{\tilde{q}} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}. \tag{3.15}$$

This allows us to introduce the wave function renormalization directly on the "physical" fields after rotation to the mass basis. These field renormalization constants will be chosen so that one gets rid of the mixing introduced by the mass shifts, at least one of these physical particles are on their mass shell. We therefore introduce the following counterterms

$$\tilde{q}_{i0} = \left(\delta_{ij} + \frac{1}{2}\delta Z^{\tilde{q}}_{ij}\right)\tilde{q}_j, \tag{3.16}$$

$$\mathcal{M}^2_{\tilde{q}0} = \mathcal{M}^2_{\tilde{q}} + \delta\mathcal{M}^2_{\tilde{q}}. \tag{3.17}$$

The shifts on the parameters induce

$$\delta\mathcal{M}^2_{\tilde{q}} = \begin{bmatrix} \delta M^2_{\tilde{Q}_L} + \delta(m^2_q + c_{2\beta}(T^3_q - Q_q s^2_W) M^2_Z) & \delta(m_q A_q) - \delta(m_q \mu t_\beta^{-2T^3_q}) \\ \delta(m_q A_q) - \delta(m_q \mu t_\beta^{-2T^3_q}) & \delta M^2_{\tilde{q}_R} + \delta(m^2_q + c_{2\beta} Q_q s^2_W M^2_Z) \end{bmatrix}. \tag{3.18}$$

After shifting the parameters and the fields, the renormalized self energies for the squarks are given by

$$\hat{\Sigma}_{\tilde{q}_i\tilde{q}_j}(q^2) = \Sigma_{\tilde{q}_i\tilde{q}_j}(q^2) + \delta m^2_{\tilde{q}_{ij}} - \frac{1}{2}\delta Z^{\tilde{q}}_{ij}(q^2 - m^2_{\tilde{q}_i}) - \frac{1}{2}\delta Z^{\tilde{q}}_{ji}(q^2 - m^2_{\tilde{q}_j}). \quad (3.19)$$

The counterterm $\delta m^2_{\tilde{q}_{ij}}$ is connected to the counterterm $\delta\mathcal{M}^2_{\tilde{q}_{ij}}$ through the relation

$$\delta m^2_{\tilde{q}_{ij}} = (R_{\tilde{q}}\delta\mathcal{M}^2_{\tilde{q}}R^\dagger_{\tilde{q}})_{ij}. \quad (3.20)$$

3. Constraining the wave function renormalization constants

The residue condition at the pole for the diagonal self-energy propagator imposes 4 conditions on the diagonal wave function renormalization constants, for $\tilde{q} = (\tilde{u}, \tilde{d})$,

$$\delta Z^{\tilde{q}}_{11} = \mathrm{Re}\Sigma'_{\tilde{q}_1\tilde{q}_1}(m^2_{\tilde{q}_1}), \qquad \delta Z^{\tilde{q}}_{22} = \mathrm{Re}\Sigma'_{\tilde{q}_2\tilde{q}_2}(m^2_{\tilde{q}_2}). \quad (3.21)$$

We impose that no mixing occurs between the two squarks $\tilde{q}_1$ and $\tilde{q}_2$ when on shell, constraining the nondiagonal wave function renormalization constants accordingly,

$$\begin{aligned}\delta Z^{\tilde{q}}_{12} &= \frac{2}{m^2_{\tilde{q}_2} - m^2_{\tilde{q}_1}}(\mathrm{Re}\Sigma_{\tilde{q}_1\tilde{q}_2}(m^2_{\tilde{q}_2}) + \delta m^2_{\tilde{q}_{12}}),\\ \delta Z^{\tilde{q}}_{21} &= \frac{2}{m^2_{\tilde{q}_1} - m^2_{\tilde{q}_2}}(\mathrm{Re}\Sigma_{\tilde{q}_1\tilde{q}_2}(m^2_{\tilde{q}_1}) + \delta m^2_{\tilde{q}_{12}}).\end{aligned} \quad (3.22)$$

In our approach, the nondiagonal wave functions are not completely determined at this stage because the mixing counterterm $\delta m^2_{\tilde{q}_{12}}$ appears in their definitions. It is also important to point out that unless $\delta m^2_{\tilde{q}_{12}}$ is chosen judiciously these nondiagonal wave functions are ill-defined in the limit $m^2_{\tilde{q}_1} \to m^2_{\tilde{q}_2}$. For further reference it is interesting to define

$$\begin{aligned}\delta Z^{\tilde{q}S}_{12} &= \frac{1}{m^2_{\tilde{q}_2} - m^2_{\tilde{q}_1}}(\mathrm{Re}\Sigma_{\tilde{q}_1\tilde{q}_2}(m^2_{\tilde{q}_2}) - \mathrm{Re}\Sigma_{\tilde{q}_1\tilde{q}_2}(m^2_{\tilde{q}_1})),\\ \delta Z^{\tilde{q}A}_{12} &= \frac{1}{m^2_{\tilde{q}_2} - m^2_{\tilde{q}_1}}(\mathrm{Re}\Sigma_{\tilde{q}_1\tilde{q}_2}(m^2_{\tilde{q}_2}) + \mathrm{Re}\Sigma_{\tilde{q}_1\tilde{q}_2}(m^2_{\tilde{q}_1}) + 2\delta m^2_{\tilde{q}_{12}}),\end{aligned} \quad (3.23)$$

such that

$$\delta Z^{\tilde{q}}_{12,21} = \delta Z^{\tilde{q}S}_{12} \pm \delta Z^{\tilde{q}A}_{12}. \quad (3.24)$$

Only $\delta Z^{\tilde{q}A}_{12}$ is now potentially singular in the limit $m^2_{\tilde{q}_2} \to m^2_{\tilde{q}_1}$. We will come back to this issue when fixing a renormalization for $\delta m^2_{\tilde{q}_{12}}$.

4. Renormalization of the mass parameters, physical masses as input

The default scheme in SLOOPS takes $m_{\tilde{d}_1}$, $m_{\tilde{d}_2}$, and $m_{\tilde{u}_2}$ (the lightest up-type squark) as input parameters considered to be the physical masses of $\tilde{d}_1$, $\tilde{d}_2$, and $\tilde{u}_2$, respectively. This fixes 3 counterterms

$$\delta m^2_{\tilde{d}_{11}} = -\mathrm{Re}\Sigma_{\tilde{d}_1\tilde{d}_1}(m^2_{\tilde{d}_1}), \qquad \delta m^2_{\tilde{d}_{22}} = -\mathrm{Re}\Sigma_{\tilde{d}_2\tilde{d}_2}(m^2_{\tilde{d}_2}), \qquad \delta m^2_{\tilde{u}_{22}} = -\mathrm{Re}\Sigma_{\tilde{u}_2\tilde{u}_2}(m^2_{\tilde{u}_2}). \quad (3.25)$$

5. Renormalization of the mass parameters, the issue of the mixing parameter at one loop

To complete the renormalization of the squark sector for each generation, as we need 5 renormalization conditions, we have to impose two additional conditions on what measures the mixing in the up squarks and down squarks and therefore fixes $\delta m^2_{\tilde{q}_{12}}$ for $\tilde{q} = \tilde{u}, \tilde{d}$. Once this is fixed, the remaining heaviest up squark $\tilde{u}_1$ mass receives a *finite* correction at one loop. One possibility is to define these mixing parameters through physical observables. One can, for example, choose the two decays $\tilde{d}_1 \to \tilde{d}_2 Z$ and $\tilde{d}_1 \to \tilde{u}_2 W^-$ as inputs, provided they are open. This is within the spirit we have followed to define a gauge-invariant $\tan\beta$ from the decay $A^0 \to \tau^+\tau^-$ [9]. This will then define A_d and A_u at one loop, respectively. The one-loop radiative corrections to sfermions into gauge bosons have been studied in previous work [15,16]. Since the issue of mixing is quite subtle with many definitions based on two-point functions being rather *ad hoc*, we look at the problem rather afresh. Moreover, the discussion is the same for sleptons with mixing, we therefore generalize this for sfermions in general and consider that the counterterm $\delta m^2_{\tilde{f}_{12}}$ absorbs the ultraviolet divergence of the decay $\tilde{f}_1 \to \tilde{f}_2 Z^0$. We have just seen, for example, that at tree level this coupling is a direct measure of the mixing. Taking a physical observable will unravel how to possibly extract a gauge-invariant universal definition based on the two-point functions.

With $\mathcal{M}_0^{\tilde{f}_1\tilde{f}_2Z^0}$ representing the tree-level amplitude, $\mathcal{M}_0^{\tilde{f}_1\tilde{f}_2Z^0} = ig_Z T^3_f s_{2\theta_f}/2$, where $g_Z = g/c_W$, the one-loop correction can be written as

$$\begin{aligned}\mathcal{M}_1^{\tilde{f}_1\tilde{f}_2Z^0} = {}& \mathcal{M}_0^{\tilde{f}_1\tilde{f}_2Z^0}\left(1 + \delta_{V_1}^{\tilde{f}_1\tilde{f}_2Z^0} + \frac{\delta e}{e} - \frac{c_{2W}}{c^2_W}\frac{\delta s_W}{s_W} + \frac{1}{2}\delta Z_{ZZ} + \frac{1}{2}\delta Z^{\tilde{f}}_{11} + \frac{1}{2}\delta Z^{\tilde{f}}_{22}\right) + ig_Z T^3_f \delta_{V_2}^{\tilde{f}_1\tilde{f}_2Z^0} + ig_Z T^3_f(1 - 4s^2_W|Q_f|)\\ &\times\left(\frac{\mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_2}) - \mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_1})}{m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2}}\right) + ig_Z T^3_f c_{2\theta_f}\left(\frac{2\delta m^2_{\tilde{f}_{12}} + \mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_1}) + \mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_2})}{m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2}}\right).\end{aligned} \quad (3.26)$$

The first part of the correction proportional to the tree-level contribution is due to diagonal wave function renormalization and renormalization of the gauge parameters. Just like the tree-level contribution, this part is regular in the limit $(m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2}) \to 0$, see the trivial remark we made after Eq. (3.11). $\delta^{\tilde{f}_1\tilde{f}_2Z^0}_{V_2}$ represents purely one-loop virtual corrections which do not necessarily vanish in the limit of a vanishing tree-level mixing with $\theta_f = 0$ much like the one-loop induced $\tilde{f}_1 \to \tilde{f}_2\gamma$. The corrections in the third and fourth line of Eq. (3.26) are due to $\tilde{f}_1 \leftrightarrow \tilde{f}_2$ transitions triggered from the diagonal couplings $\tilde{f}_i\tilde{f}_iZ$.

$\mathcal{M}^{\tilde{f}_1\tilde{f}_2Z^0}_1$ contains pure QED corrections that can be unambiguously extracted, these contain infrared singularities that need to be combined with the bremsstrahlung corrections. Subtracting these pure QED virtual corrections and the corresponding gluonic QCD corrections defines a gauge-invariant infrared safe observable that does not depend on any experimental cutoff on the energy of the bremsstrahlung photon or gluon. Let us define this observable as $\bar{\mathcal{M}}^{\tilde{f}_1\tilde{f}_2Z^0}_1$. $\delta m^2_{\tilde{f}_{12}}$ defined from $\bar{\mathcal{M}}^{\tilde{f}_1\tilde{f}_2Z^0}_1$ by requiring that the one-loop correction $(\bar{\mathcal{M}}^{\tilde{f}_1\tilde{f}_2Z^0}_1 - \mathcal{M}^{\tilde{f}_1\tilde{f}_2Z^0}_0)$ vanishes, constitutes a fully gauge invariant, although process dependent definition of $\delta m^2_{\tilde{f}_{12}}$. In this definition, process dependent vertex corrections combine with self-energy contributions leading to a gauge-independent definition. Equation (3.26) is also instructive in that it reveals how to extract a process and gauge-independent definition of $\delta m^2_{\tilde{f}_{12}}$. Indeed, Eq. (3.26) exhibits a specific pole structure in $(m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2})$. The residue of the pole in $(m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2})$ must be gauge independent. Therefore considering a Laurent series of the amplitude in the pole $(m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2})$,[2] a gauge and process independent definition based on two-point functions can be defined as

$$\begin{aligned}\delta m^2_{\tilde{f}_{12}} &= -\frac{1}{2}\lim_{m^2_{\tilde{f}_1}\to m^2_{\tilde{f}_2}} (\mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(p^2 = m^2_{\tilde{f}_1}) \\ &\quad + \mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(p^2 = m^2_{\tilde{f}_2})) \\ &\equiv -\mathrm{Re}\Sigma^{\mathcal{P}}_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_1}, m^2_{\tilde{f}_2}). \end{aligned} \quad (3.27)$$

It is important to note that $\delta m^2_{\tilde{f}_{12}}$ is expressed in terms of the nondiagonal self energy which is a function of the *two* variables $(m^2_{\tilde{f}_1}, m^2_{\tilde{f}_2})$. To make things clearer, let us keep in mind that in general $\Sigma_{\tilde{f}_1\tilde{f}_2}(p^2)$ calculated at some external momenta p^2 is in fact a function of p^2, $m^2_{\tilde{f}_1}$, $m^2_{\tilde{f}_2}$ beside some other masses involved in the loop. One usually just writes $\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_1})$ for $\Sigma_{\tilde{f}_1\tilde{f}_2}(p^2 = m^2_{\tilde{f}_1})$, and at the same time suppressing the dependence in $m^2_{\tilde{f}_2}$ in the writing of $\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_1})$. As an example, the neutral Goldstone coupling to the sfermions $\tilde{f}_1\tilde{f}_2G^0$ that contributes to the sfermion self energies is proportional to the off-diagonal element of the sfermion mass matrix [see Eq. (3.7)]. The latter can be written directly in terms of $(m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2})$ through Eq. (3.11). This shows explicitly the dependence of $\Sigma_{\tilde{f}_1\tilde{f}_2}(p^2)$ in the two sfermion masses for any p^2 beside the possible contribution coming from the mass of the internal sfermion line. For what concerns us here, one should always keep in mind that the function we are dealing with is a function $\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_1}, m^2_{\tilde{f}_2})$. The value at the pole in $\mathrm{Re}\Sigma^{\mathcal{P}}_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_1}, m^2_{\tilde{f}_2})$ is gauge invariant and universal. All the remaining contributions in Eq. (3.26) are then regular in the limit $(m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2}) \to 0$ and, in particular, the contribution in the third line of Eq. (3.26).

Care should be taken in defining these limits. To take the limit in Eq. (3.27), it is useful to express $m^2_{\tilde{f}_{1,2}}$ in terms of $m^2_{\tilde{f}_\pm}$

$$m^2_{\tilde{f}_\pm} = \frac{m^2_{\tilde{f}_1} \pm m^2_{\tilde{f}_2}}{2}, \quad (3.28)$$

in order to make the dependence in the pole $m^2_{\tilde{f}_-}$ explicit. Then $\Sigma_{\tilde{f}_1\tilde{f}_2}(p^2 = m^2_{\tilde{f}_i}) = \Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_+}, \pm m^2_{\tilde{f}_-})$ so that $\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_i})$ is a function of these two variables. These functions should be expanded in $m^2_{\tilde{f}_-}$ such that

$$\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_+}, \pm m^2_{\tilde{f}_-}) = \Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_+}, 0) \pm m^2_{\tilde{f}_-}\frac{\partial\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_+}, 0)}{\partial m^2_{\tilde{f}_-}} + \cdots. \quad (3.29)$$

We then have

$$\begin{aligned}&\frac{\mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_1}) + \mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_2})}{m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2}} \\ &\quad = \frac{\mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_+}, 0)}{m^2_{\tilde{f}_-}} + \frac{m^2_{\tilde{f}_-}}{2}\mathrm{Re}\frac{\partial^2\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_+}, 0)}{(\partial m^2_{\tilde{f}_-})^2} \\ &\qquad + \mathcal{O}((m^2_{\tilde{f}_-})^3), \\ &\frac{\mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_1}) - \mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_2})}{m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2}} \\ &\quad = \mathrm{Re}\frac{\partial\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_+}, 0)}{\partial m^2_{\tilde{f}_-}} + \mathcal{O}((m^2_{\tilde{f}_-})^2).\end{aligned} \quad (3.30)$$

We can identify

$$\mathrm{Re}\,\Sigma^{\mathcal{P}}_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_1}, m^2_{\tilde{f}_2}) = \mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_+}, 0). \quad (3.31)$$

By looking at the pole structure of the amplitude it is now clear that $\mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_+}, 0)$ is gauge independent. However, $\mathrm{Re}\partial\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_+}, 0)/\partial m^2_{\tilde{f}_-}$ in Eq. (3.30), for example, is not

[2] This is in line with the definition of the Z^0 mass from $e^+e^- \to \mu^+\mu^-$ through a Laurent series based on analyticity properties of the S matrix, see [17].

guaranteed to be gauge independent. Its gauge-dependent part cancels against those contained in the vertex corrections.

One should be aware of the following:

$$\mathrm{Re}\Sigma^{\mathcal{P}}_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_1}, m^2_{\tilde{f}_2}) = \mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_+}, 0) \neq \mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(p^2 = m^2_{\tilde{f}_+}). \tag{3.32}$$

In $\Sigma_{\tilde{f}_1\tilde{f}_2}(p^2 = m^2_{\tilde{f}_+}) \equiv \Sigma_{\tilde{f}_1\tilde{f}_2}(p^2 = m^2_{\tilde{f}_+}, m^2_{\tilde{f}_-})$, $m^2_{\tilde{f}_-}$ is still undefined. Whereas, we should have

$$\begin{aligned}\mathrm{Re}\Sigma^{\mathcal{P}}_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_1}, m^2_{\tilde{f}_2}) &= \mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_+}, 0)\\ &= \mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(p^2 = m^2_{\tilde{f}_+}, m^2_{\tilde{f}_-} \to 0)\\ &= \mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(p^2 = m^2_{\tilde{f}_+}, m^2_{\tilde{f}_-} = 0).\end{aligned} \tag{3.33}$$

Indeed, a naive replacement $\mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(p^2 = m^2_{\tilde{f}_+})$ may still give extra contributions that are of order $m^2_{\tilde{f}_-}$. This is exactly what happens when we calculate $\mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(p^2)$ in a gauge which is not the Feynman gauge with $\xi_{W,Z} \neq 1$. One finds that the gauge dependent parts of the quantity $\mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(p^2 = m^2_{\tilde{f}_+})$ proportional to $(1 - \xi_{W,Z})$ are of order $m^2_{\tilde{f}_-}$, see [18,19]. To make this point more transparent, let us borrow from [18,19] the expression for the two-point function in the stop sector for a general $\xi_{W,Z}$ for the W and Z gauge bosons

$$\begin{aligned}\Sigma_{\tilde{t}_1\tilde{t}_2}(p^2) = {}& \Sigma_{\tilde{t}_1\tilde{t}_2}(p^2)|_{\xi_Z=\xi_W=1} + \frac{g_Z^2}{16\pi^2}(1-\xi_Z)\chi^Z_{1k}\chi^Z_{2k}\\ &\times[-f_{12}(p^2)\alpha_Z + g_{12}(p^2, m^2_{\tilde{t}_k})\beta^{(0)}_{Z\tilde{t}_k}(p^2)]\\ &+\frac{g^2}{8\pi^2}(1-\xi_W)\chi^W_{1k}\chi^W_{2k}[-f_{12}(p^2)\alpha_W\\ &+g_{12}(p^2, m^2_{\tilde{b}_k})\beta^{(0)}_{W\tilde{b}_k}(p^2)].\end{aligned} \tag{3.34}$$

The first term in Eq. (3.34) represents the result in the Feynman gauge. χ^Z_{ik}, χ^W_{ik} describe the mixing involved in the coupling of the stops with the Z and the W. $\alpha_{Z,W}$ and $\beta^{(0)}_{Z\tilde{t}_k}(p^2)$, $\beta^{(0)}_{W\tilde{b}_k}(p^2)$ are scalar loop functions, see [18,19]. The important factors that concern us here are the functions

$$f_{12}(p^2) \equiv (p^2 - m^2_{\tilde{t}_+}), \tag{3.35}$$

$$\begin{aligned}g_{12}(p^2, m^2) \equiv {}& 2(p^2 - m^2)(p^2 - m^2_{\tilde{t}_+})\\ &-(p^2 - m^2_{\tilde{t}_+} - m^2_{\tilde{t}_-})(p^2 - m^2_{\tilde{t}_+} + m^2_{\tilde{t}_-}).\end{aligned} \tag{3.36}$$

We see that although $f_{12}(p^2 = m^2_{\tilde{t}_+}) = 0$, $g_{12}(p^2 = m^2_{\tilde{t}_+}) = \mathcal{O}(m^2_{\tilde{t}_-})^2$ is indeed of order $m^2_{\tilde{t}_-}$ and does not vanish, meaning that these two-point functions are indeed gauge dependent. As we have advocated, in a gauge-invariant definition, we should set $p^2 = m^2_{\tilde{t}_+}$ with $m^2_{\tilde{t}_-} = 0$. This example also confirms that the gauge dependence is necessarily proportional to $m^2_{\tilde{t}_-}$. Indeed, $\Sigma_{\tilde{t}_1\tilde{t}_2}(p^2 = m^2_{\tilde{t}_+})$ has a part of proportional to $(1-\xi_{W,Z})(m^2_{\tilde{t}_-})^2$, whereas $\Sigma_{\tilde{t}_1\tilde{t}_2}(p^2 = m^2_{\tilde{t}_+}, m^2_{\tilde{t}_-} = 0) = \Sigma_{\tilde{t}_1\tilde{t}_2}(p^2 = m^2_{\tilde{t}_+}, m^2_{\tilde{t}_-} = 0)|_{\xi_Z=\xi_W=1}$.

Let us mention that the choice based on $\mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(p^2 = m^2_{\tilde{f}_+})$ had been advocated to improve the scale independence of the mixing angle [20].

Note that after the renormalization of the mixing has been set according to Eqs. (3.27) and (3.31), the last term in Eq. (3.26) contributes an ultraviolet finite part. This, on the other hand, is not the case of the contribution from the third line in Eq. (3.26). Indeed, its $\mathrm{Re}\Sigma'_{\tilde{f}_1\tilde{f}_2}(p^2 = m^2_{\tilde{f}_+})$ might still be needed to absorb possible infinities from the vertex virtual corrections, for example.

In SLOOPS, we work in the Feynman gauge with $\xi_W = \xi_Z = 1$. At one loop, $\Sigma_{\tilde{f}_i\tilde{f}_j}$ is insensitive to the nonlinear gauge parameters in Eq. (2.1). We therefore obtain the same result for $\Sigma_{\tilde{f}_i\tilde{f}_j}$ as in the usual linear gauge within the Feynman gauge. Therefore one can *afford* using Eq. (3.32). Taking this into account with Eqs. (3.27) and (3.31), the default scheme in SLOOPS is

$$\delta m^2_{\tilde{f}_{12}} = -\mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(p^2 = m^2_{\tilde{f}_+}). \tag{3.37}$$

To compare with results in the literature, we have also implemented the prescription

$$\delta m^2_{\tilde{f}_{12}} = -\frac{1}{2}(\mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_1}) + \mathrm{Re}\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_2})), \tag{3.38}$$

which is equivalent to the condition introduced in Ref. [21]. In the Feynman gauge, the difference with the default scheme is ultraviolet safe and numerically small, see the examples in Secs. V A and V E.

As we stressed repeatedly, in our approach we do not introduce counterterms to the rotation matrices since nondiagonal wave function renormalization is necessary in any case. For the sfermions, this more easily reveals the correct prescription to take for the renormalization of the mixing parameter. In practically all other approaches, counterterms to mixing matrices are introduced and therefore $\theta_f \to \theta_f + \delta\theta_f$. We can recover these approaches by, for example, looking at the example of $\tilde{f}_1 \to \tilde{f}_2 Z^0$ and considering the shift to the angle, rather than introducing the shift $\delta m^2_{\tilde{f}_{12}}$ indirectly through the nondiagonal wave function renormalization constants. From $\delta s_{2\theta_f} = 2c_{2\theta_f}\delta\theta_f$, we make the identification

$$\delta\theta_f = \frac{\delta m^2_{\tilde{f}_{12}}}{m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2}}. \tag{3.39}$$

6. SUSY QCD corrections and the squark mixing angle

There have been many proposals in defining this angle or, alternatively, the mixing parameter when considering purely supersymmetric QCD corrections. The different proposals relied on constraining the mixing angle Eq. (3.39) through a combination of two-point functions in order that some specific observable be finite. This rather *ad hoc* approach would, of course, guarantee finiteness for that observable but does not necessarily guarantee that this observable or quantity is gauge invariant with this choice of counterterm. What is worse, is that if one uses the same prescription when considering one-loop electroweak corrections to the same quantity even finiteness is lost. The prescription based on the residue of the pole would have given the correct procedure. The aim of this subsection is to understand why finiteness is obtained in the case of supersymmetric QCD corrections.

Pure QCD contributions to $\Sigma_{\tilde{q}_1\tilde{q}_2}$ are from the gluino $\tilde{g}$ exchange self energies and the tadpole squark exchange. The results can be written in a very compact form, see, for example, [22]

$$\Sigma^{\tilde{g}}_{\tilde{q}_1\tilde{q}_2}(p^2) = \frac{4\alpha_s}{3\pi} m_{\tilde{g}} m_q c_{2\theta_q} B_0(p^2, m_{\tilde{g}}, m_q),$$
$$\Sigma^{\tilde{q}}_{\tilde{q}_1\tilde{q}_2}(p^2) = \frac{\alpha_s}{3\pi} c_{2\theta_q} s_{2\theta_q}(A_0(m^2_{\tilde{q}_2}) - A_0(m^2_{\tilde{q}_1})). \tag{3.40}$$

The loop functions A_0 and B_0 are as defined in [14]. It is evident that $\Sigma^{\tilde{q}}_{\tilde{q}_1\tilde{q}_2}(p^2)$ is of order $m^2_{\tilde{q}_2} - m^2_{\tilde{q}_1}$. It independently vanishes for $s_{2\theta_q} \to 0$. Note that the QCD contribution of the gluino does not depend on the squark masses for a general p^2. Therefore $\Sigma^{\tilde{g},\tilde{q}}_{\tilde{q}_1\tilde{q}_2}(m^2_{\tilde{q}_1}) - \Sigma^{\tilde{g},\tilde{q}}_{\tilde{q}_1\tilde{q}_2}(m^2_{\tilde{q}_2})$ is finite. This explains why different schemes work fine, in the sense of leading to finite results, for SUSY QCD corrections to processes involving squarks. One of the most complicated is based on tuning combinations of $\Sigma_{\tilde{q}_1\tilde{q}_2}$ such that a finite result for $e^+e^- \to \tilde{q}_1\bar{\tilde{q}}_2$ obtains as far as QCD corrections are concerned [23]. With the coupling of the Z to squarks defined as c_{ij} for $Z\tilde{q}_i\tilde{q}_j$, the following combination is used to define the counterterm:

$$\frac{c_{22}\,\mathrm{Re}\Sigma_{\tilde{q}_1\tilde{q}_2}(m^2_{\tilde{q}_1}) - c_{11}\,\mathrm{Re}\Sigma_{\tilde{q}_1\tilde{q}_2}(m^2_{\tilde{q}_1})}{c_{22} - c_{11}}. \tag{3.41}$$

This can be rewritten as

$$\frac{c_{22}\,\mathrm{Re}\Sigma_{\tilde{q}_1\tilde{q}_2}(m^2_{\tilde{q}_1}) - c_{11}\,\mathrm{Re}\Sigma_{\tilde{q}_1\tilde{q}_2}(m^2_{\tilde{q}_1})}{c_{22} - c_{11}} = \frac{\mathrm{Re}\Sigma_{\tilde{q}_1\tilde{q}_2}(m^2_{\tilde{q}_1}) + \mathrm{Re}\Sigma_{\tilde{q}_1\tilde{q}_2}(m^2_{\tilde{q}_2})}{2} + \frac{c_{22} + c_{11}}{c_{22} - c_{11}} \times \frac{\mathrm{Re}\Sigma_{\tilde{q}_1\tilde{q}_2}(m^2_{\tilde{q}_1}) - \mathrm{Re}\Sigma_{\tilde{q}_1\tilde{q}_2}(m^2_{\tilde{q}_2})}{2}. \tag{3.42}$$

The much simpler scheme based on the use of $\mathrm{Re}\Sigma_{\tilde{q}_1\tilde{q}_2}(m^2_{\tilde{q}_1})$ [24] is, in fact, a very special case of the scheme in Eq. (3.42), we can see that it is obtained as $c_{11} \to 0$ in Eq. (3.42). For the electroweak case, the extra terms proportional to $\mathrm{Re}\Sigma_{\tilde{q}_1\tilde{q}_2}(m^2_{\tilde{q}_1}) - \mathrm{Re}\Sigma_{\tilde{q}_1\tilde{q}_2}(m^2_{\tilde{q}_2})$ in Eq. (3.42) are not finite apart from the gauge invariance issue. However, as we have seen, the ultraviolet divergent part can be canceled in $(\mathrm{Re}\Sigma_{\tilde{q}_1\tilde{q}_2}(m^2_{\tilde{q}_1}) + \mathrm{Re}\Sigma_{\tilde{q}_1\tilde{q}_2}(m^2_{\tilde{q}_2}))/2$ as suggested in [25]. However, this suggestion was not based on a very strong theoretical or physical argument apart from it being more symmetric or *democratic* in the two squarks.

7. Deriving the counterterms

We are now in a position to derive all the needed counterterms. First, with both prescriptions for $\delta m^2_{\tilde{q}_{12}}$ either based on Eq. (3.37) or the naive Eq. (3.38), the nondiagonal wave function renormalization constants $\delta Z^{\tilde{u}}_{ij}$ and $\delta Z^{\tilde{d}}_{ij}$ are now regular in the limit $m^2_{\tilde{q}_1} \to m^2_{\tilde{q}_2}$, where any potential ultraviolet divergence is contained in $\delta Z^{\tilde{q},S}_{12}$. In fact, in the scheme of Eq. (3.38), only this part remains and therefore $\delta Z^{\tilde{q}}_{12} = \delta Z^{\tilde{q}}_{21} = \delta Z^{\tilde{q},S}_{12}$.

The remaining counterterm $\delta m_{\tilde{u}_{11}}$ is completely constrained

$$\delta m^2_{\tilde{u}_{11}} = \frac{1}{c^2_{\theta_u}}\Big(c^2_{\theta_d}\delta m^2_{\tilde{d}_{11}} + s^2_{\theta_d}\delta m^2_{\tilde{d}_{22}} - s_{2\theta_d}\delta m^2_{\tilde{d}_{12}} - s^2_{\theta_u}\delta m^2_{\tilde{u}_{22}} + s_{2\theta_u}\delta m^2_{\tilde{u}_{12}} + \delta m^2_u - \delta m^2_d + M^2_W\Big(c_{2\beta}\frac{\delta M^2_W}{M^2_W} - s^2_{2\beta}\frac{\delta t_\beta}{t_\beta}\Big)\Big). \tag{3.43}$$

For $c^2_{\theta_u} \ll 1$, this scheme is not appropriate as it will induce large radiative corrections. One should prefer the use of $m^2_{\tilde{u}_1}$ as input parameter *in lieu* of $m^2_{\tilde{u}_2}$. With $m^2_{\tilde{u}_2}$ as input parameter, the physical mass of $\tilde{u}_1$ will then receive a finite correction at one loop

$$m^{\mathrm{phys}}_{\tilde{u}_1} = m^2_{\tilde{u}_1} + \delta m^2_{\tilde{u}_{11}} + \mathrm{Re}\Sigma_{\tilde{u}_1\tilde{u}_1}(m^2_{\tilde{u}_1}). \tag{3.44}$$

Alternatively, we can use $m_{\tilde{u}_1}$ as input like we have done with the other squark masses. This will allow us to define t_β from the sfermion sector through

$$\frac{\delta t_\beta}{t_\beta} = \frac{1}{s^2_{2\beta}M^2_W}(c^2_{\theta_d}\delta m^2_{\tilde{d}_{11}} + s^2_{\theta_d}\delta m^2_{\tilde{d}_{22}} - s_{2\theta_d}\delta m^2_{\tilde{d}_{12}} - \delta m^2_d - c^2_{\theta_u}\delta m^2_{\tilde{u}_{11}} + s^2_{\theta_u}\delta m^2_{\tilde{u}_{22}} - s_{2\theta_u}\delta m^2_{\tilde{u}_{12}} + \delta m^2_u + c_{2\beta}\delta M^2_W). \tag{3.45}$$

Using Eq. (3.20), we find the relations between the counterterms $\delta m^2_{\tilde{q}_{ij}}$ and the counterterms $\delta M_{\tilde{Q}_L}$, $\delta M_{\tilde{u}_R}$, $\delta M_{\tilde{d}_R}$, δA_u, and δA_d of the underlying parameters at the Lagrangian level

$$\delta M_{\tilde{Q}_L} = \frac{1}{2M_{\tilde{Q}_L}}\left(c^2_{\theta_d}\delta m^2_{\tilde{d}_{11}} + s^2_{\theta_d}\delta m^2_{\tilde{d}_{22}} - s_{2\theta_d}\delta m^2_{\tilde{d}_{12}} - \delta m^2_d - M^2_Z\left(-\frac{1}{2}+\frac{1}{3}s^2_W\right)\left(c_{2\beta}\frac{\delta M^2_Z}{M^2_Z} - s^2_{2\beta}\frac{\delta t_\beta}{t_\beta}\right) - c_{2\beta}\frac{1}{3}M^2_Z\delta s^2_W\right),$$
$$\delta M_{\tilde{u}_R} = \frac{1}{2M_{\tilde{u}_R}}\left(s^2_{\theta_u}\delta m^2_{\tilde{u}_{11}} + c^2_{\theta_u}\delta m^2_{\tilde{u}_{22}} + s_{2\theta_u}\delta m^2_{\tilde{u}_{12}} - \delta m^2_u - \frac{2}{3}s^2_W M^2_Z\left(c_{2\beta}\left(\frac{\delta M^2_Z}{M^2_Z}+\frac{\delta s^2_W}{s^2_W}\right) - s^2_{2\beta}\frac{\delta t_\beta}{t_\beta}\right)\right),$$
$$\delta M_{\tilde{d}_R} = \frac{1}{2M_{\tilde{d}_R}}\left(s^2_{\theta_d}\delta m^2_{\tilde{d}_{11}} + c^2_{\theta_d}\delta m^2_{\tilde{d}_{22}} + s_{2\theta_d}\delta m^2_{\tilde{d}_{12}} - 2m_d\delta m_d + \frac{1}{3}s^2_W M^2_Z\left(c_{2\beta}\left(\frac{\delta M^2_Z}{M^2_Z}+\frac{\delta s^2_W}{s^2_W}\right) - s^2_{2\beta}\frac{\delta t_\beta}{t_\beta}\right)\right),$$
$$\delta(m_u A_u) = \frac{s_{2\theta_u}}{2}(\delta m^2_{\tilde{u}_{11}} - \delta m^2_{\tilde{u}_{22}}) + c_{2\theta_u}\delta m^2_{\tilde{u}_{12}} + \frac{m_u}{t_\beta}\left(\delta\mu + \mu\frac{\delta m_u}{m_u} - \mu\frac{\delta t_\beta}{t_\beta}\right),$$
$$\delta(m_d A_d) = \frac{s_{2\theta_d}}{2}(\delta m^2_{\tilde{d}_{11}} - \delta m^2_{\tilde{d}_{22}}) + c_{2\theta_d}\delta m^2_{\tilde{d}_{12}} + m_d t_\beta\mu\left(\frac{\delta\mu}{\mu} + \frac{\delta m_d}{m_d} + \frac{\delta t_\beta}{t_\beta}\right). \tag{3.46}$$

B. Slepton sector

After having shown in detail how the squark sector is renormalized in the case of mixing, it is straightforward to treat the slepton sector. Again for the sleptons, the case with mixing is, for all practical purposes, only relevant for the $\tilde{\tau}$. In the code, we have implemented mixing for all generations, in the first and second generation this is used only in to conduct high precision checks on the results, for applications the unmixed case is used. Here, we will show only the case with mixing, the unmixed case is then trivial.

Compared to the squark sector, as seen from Eq. (3.2), one has, for each generation, only 3 parameters: $M_{\tilde{L}_L}$, $M_{\tilde{e}_R}$, A_e and one field is missing $\tilde{\nu}_R$. $\tilde{e}_L$ and $\tilde{e}_R$ will mix, leading to the physical fields $\tilde{e}_1$ and $\tilde{e}_2$. In the unmixed case, we associate $\tilde{e}_1$ with $\tilde{e}_L$. The mixing matrix is described in exactly the same way as in Eq. (3.4) with $\tilde{q}\to\tilde{e}$ and the different components given by Eqs. (3.5), (3.6), and (3.7) with $\tilde{Q}\to\tilde{L}$ with the corresponding quantum charges. Shifting the fields and parameters, we can write the self-energies (in the case of Σ diagonal and nondiagonal) as

$$\hat{\Sigma}_{\tilde{e}_i\tilde{e}_j}(q^2) = \Sigma_{\tilde{e}_i\tilde{e}_j}(q^2) + \delta m^2_{\tilde{e}_{ij}} - \frac{1}{2}\delta Z^{\tilde{e}}_{ij}(q^2 - m^2_{\tilde{e}_i}) - \frac{1}{2}\delta Z^{\tilde{e}}_{ji}(q^2 - m^2_{\tilde{e}_j}),$$
$$\hat{\Sigma}_{\tilde{\nu}}(q^2) = \Sigma_{\tilde{\nu}}(q^2) + \delta m^2_{\tilde{\nu}} - \delta Z^{\tilde{\nu}}(q^2 - m^2_{\tilde{\nu}}). \tag{3.47}$$

We take the physical selectron masses as input parameters through the usual on shell condition. We require the residue of the propagators of $\tilde{e}_i$ and $\tilde{\nu}$ to be equal to unity and no mixing between $\tilde{e}_1$ and $\tilde{e}_2$ when these are on shell. These conditions imply

$$\delta m^2_{\tilde{e}_{ii}} = -\mathrm{Re}\Sigma_{\tilde{e}_i\tilde{e}_i}(m^2_{\tilde{e}_i}), \qquad \delta Z^{\tilde{e}}_{ii} = \mathrm{Re}\Sigma'_{\tilde{e}_i\tilde{e}_i}(m^2_{\tilde{e}_i}),$$
$$\delta Z^{\tilde{\nu}} = \mathrm{Re}\Sigma'_{\tilde{\nu}}(m^2_{\tilde{\nu}}),$$
$$\delta Z^{\tilde{e}}_{12} = \frac{2}{m^2_{\tilde{e}_2} - m^2_{\tilde{e}_1}}(\mathrm{Re}\Sigma_{\tilde{e}_1\tilde{e}_2}(m^2_{\tilde{e}_2}) + \delta m^2_{\tilde{e}_{12}}),$$
$$\delta Z^{\tilde{e}}_{21} = \frac{2}{m^2_{\tilde{e}_1} - m^2_{\tilde{e}_2}}(\mathrm{Re}\Sigma_{\tilde{e}_1\tilde{e}_2}(m^2_{\tilde{e}_1}) + \delta m^2_{\tilde{e}_{12}}). \tag{3.48}$$

The remaining parameter $\delta m^2_{\tilde{e}_{12}}$ describing mixing is fixed analogously as in the squark sector. The default scheme in SLOOPS is

$$\delta m^2_{\tilde{e}_{12}} = -\mathrm{Re}\Sigma_{\tilde{e}_1\tilde{e}_2}((m^2_{\tilde{e}_1} + m^2_{\tilde{e}_2})/2). \tag{3.49}$$

As in the squark sector, a better definition would be to relate this counterterm to a physical observable like the slepton decay $\tilde{e}_1\to\tilde{e}_2 Z^0$, for example, see (3.26). The naive scheme

$$\delta m^2_{\tilde{e}_{12}} = -\frac{1}{2}(\mathrm{Re}\Sigma_{\tilde{e}_1\tilde{e}_2}(m^2_{\tilde{e}_1}) + \mathrm{Re}\Sigma_{\tilde{e}_1\tilde{e}_2}(m^2_{\tilde{e}_2})) \tag{3.50}$$

is also implemented. Another possible scheme uses the mass of the sneutrino as an input parameter such that

$$\delta m^2_{\tilde{\nu}} = -\mathrm{Re}\Sigma_{\tilde{\nu}}(m^2_{\tilde{\nu}}), \tag{3.51}$$

and the counterterm $\delta m^2_{\tilde{e}_{12}}$ is given by

$$\delta m^2_{\tilde{e}_{12}} = \frac{1}{s_{2\theta_e}}\left(c^2_{\theta_e}\delta m^2_{\tilde{e}_{11}} + s^2_{\theta_e}\delta m^2_{\tilde{e}_{22}} - \delta m^2_{\tilde{\nu}} - \delta m^2_e + M^2_W\left(c_{2\beta}\frac{\delta M^2_W}{M^2_W} - s^2_{2\beta}\frac{\delta t_\beta}{t_\beta}\right)\right). \tag{3.52}$$

However, this definition is to be avoided since the mixing in the slepton sector is usually very small $s_{2\theta_e}\sim 0$, even for τ's which would lead to large corrections.

The extraction of the counterterms of the parameters at the Lagrangian follows:

$$\delta M_{\tilde{L}_L} = \frac{1}{2M_{\tilde{L}_L}}\left(c^2_{\theta_e}\delta m^2_{\tilde{e}_{11}} + s^2_{\theta_e}\delta m^2_{\tilde{e}_{22}} - s_{2\theta_e}\delta m^2_{\tilde{e}_{12}} - \delta m^2_e - M_Z^2\left(-\frac{1}{2}+s_W^2\right)\left(c_{2\beta}\frac{\delta M_Z^2}{M_Z^2} - s^2_{2\beta}\frac{\delta t_\beta}{t_\beta}\right) - c_{2\beta}M_Z^2\delta s_W^2\right),$$

$$\delta M_{\tilde{e}_R} = \frac{1}{2M_{\tilde{e}_R}}\left(s^2_{\theta_e}\delta m^2_{\tilde{e}_{11}} + c^2_{\theta_e}\delta m^2_{\tilde{e}_{22}} + s_{2\theta_e}\delta m^2_{\tilde{e}_{12}} - \delta m^2_e + s_W^2 M_Z^2\left(c_{2\beta}\left(\frac{\delta M_Z^2}{M_Z^2} + \frac{\delta s_W^2}{s_W^2}\right) - s^2_{2\beta}\frac{\delta t_\beta}{t_\beta}\right)\right),$$

$$\delta(m_e A_e) = \frac{s_{2\theta_e}}{2}(\delta m^2_{\tilde{e}_{11}} - \delta m^2_{\tilde{e}_{22}}) + c_{2\theta_e}\delta m^2_{\tilde{e}_{12}} + m_e\mu t_\beta\left(\frac{\delta\mu}{\mu} + \frac{\delta m_e}{m_e} + \frac{\delta t_\beta}{t_\beta}\right). \qquad (3.53)$$

If the sneutrino mass is not used as input then it is predicted with a finite correction from its value at tree level

$$m_{\tilde{\nu}}^{1\text{-loop}\,2} = m_{\tilde{\nu}}^{\text{tree}\,2} + (\text{Re}\Sigma_{\tilde{\nu}}(m_{\tilde{\nu}}^{\text{tree}\,2}) - \text{Re}\Sigma_{\tilde{e}_1\tilde{e}_1}(m^2_{\tilde{e}_1})) + M_W^2\left(c_{2\beta}\frac{\delta M_W^2}{M_W^2} - s^2_{2\beta}\frac{\delta t_\beta}{t_\beta}\right) + s^2_{\theta_e}(\text{Re}\Sigma_{\tilde{e}_1\tilde{e}_1}(m^2_{\tilde{e}_1}) - \text{Re}\Sigma_{\tilde{e}_2\tilde{e}_2}(m^2_{\tilde{e}_2})) - s_{2\theta_e}\delta m^2_{\tilde{e}_{12}} - \delta m^2_e. \qquad (3.54)$$

In the limit of massless fermions, the term in the second line vanishes and we identify, as said earlier, $\tilde{e}_1$ with $\tilde{e}_L$. This is a very good limit for the selectron and smuon sector but we have to consistently take the electron and muon Yukawa couplings to zero.

It is important to note that the renormalization of the trilinear coupling both for the sleptons [$\delta(m_e A_e)$ in Eq. (3.53)] and the squarks [$\delta(m_d A_d)$ in Eq. (3.46)] may introduce a large $\tan\beta$ scheme dependence for large values of $\tan\beta$. As underlined in the introduction of this section, contrary to the cases of sfermion masses, this trilinear coupling enters directly in the coupling of the Higgses to sfermions and therefore this might lead to possible uncertainty in the prediction at one loop of these couplings in this regime.

IV. THE CHARGINO/NEUTRALINO SECTOR AND ITS RENORMALIZATION

A. Fields and parameters

The charginos and neutralinos are mixtures of the spin-1/2 fermions which are part, on the one hand, of the two Higgses chiral multiplets $\hat{H}_{1,2}$ which constitute the Higgsinos, and, on the other hand, the electroweak gauginos within the gauge supermultiplet for the $U(1)$ and $SU(2)$ gauge groups of the standard model. In terms of the two-component (left-handed) Weyl spinors, the two Higgsino doublets, in accordance with our definition in the Higgs sector [9], are $\tilde{H}_1 = (\tilde{H}_1^0, \tilde{H}_1^-)$ and $\tilde{H}_2 = (\tilde{H}_2^+, \tilde{H}_2^0)$. We denote the $U(1)$ gaugino (bino) as $\tilde{B}^0$ and the $SU(2)$ one as $\tilde{W}^i$, $i = 0, 1, 2$ with $\tilde{W}^\pm = \frac{1}{2}(\tilde{W}^1 \mp i\tilde{W}^2)$. Because of electroweak symmetry breaking, the electrically charged components will mix and lead to the charginos that will be collected as Dirac spinors $\tilde{\chi}^\pm_{1,2}$, while the electrically neutral ones will mix leading to the neutralinos that will be described as Majorana fermions, $\tilde{\chi}^0_{1,2,3,4}$. In this sector, soft masses enter only through the soft masses of the gauginos

$$\mathcal{L}^{\tilde{V}}_{\text{soft}} = -\frac{1}{2}\left(M_1\tilde{B}^0\tilde{B}^0 + M_2\sum_i \tilde{W}^i\tilde{W}^i\right), \qquad (4.1)$$

which is the only source of mass for the gauginos before electroweak symmetry breaking. The Higgsinos get a mass from the supersymmetry, preserving μ term in the superpotential

$$\mathcal{L}^{\tilde{H}}_\mu = \mu\epsilon_{ij}\tilde{H}_1^i\tilde{H}_2^j + \text{H.c.} \qquad (4.2)$$

Supersymmetric gauge matter interactions lead to mass mixing terms between these states after symmetry breaking through

$$\mathcal{L}^{\tilde{H},\tilde{V}}_{\text{mix}} = -\frac{1}{\sqrt{2}}(H_1^\dagger(g\tilde{W}^i\tau^i - g'\tilde{B}^0)\tilde{H}_1 + H_2^\dagger(g\tilde{W}^i\tau^i + g'\tilde{B}^0)\tilde{H}_2 + \text{H.c.}), \qquad (4.3)$$

with τ^i the Pauli matrices. At this point, let us give our convention on the sign of the parameters μ, M_1, M_2. We can always take $M_2 > 0$ since any other phase can be transformed away by a field redefinition in Eq. (4.1), however, because of the mixing term in Eq. (4.3), we loose the freedom to redefine the phases of the Higgsino and bino fields and hence the sign of μ and M_1.

The kinetic term in terms of the current fields writes as

$$\mathcal{L}_{\text{kin}} = i\bar{\tilde{W}}^i\bar{\sigma}^\mu(\partial_\mu\tilde{W}^i) + i\bar{\tilde{B}}^0\bar{\sigma}^\mu(\partial_\mu\tilde{B}^0) + i\bar{\tilde{H}}_1\bar{\sigma}^\mu\partial_\mu\tilde{H}_1 + i\bar{\tilde{H}}_2\bar{\sigma}^\mu\partial_\mu\tilde{H}_2. \qquad (4.4)$$

Collecting all terms in the chargino mass matrix and defining

$$\psi_R^c = \begin{pmatrix}\tilde{W}^- \\ \tilde{H}_1^-\end{pmatrix}, \qquad \psi_L^c = \begin{pmatrix}\tilde{W}^+ \\ \tilde{H}_2^+\end{pmatrix}, \qquad (4.5)$$

leads to

$$\mathcal{L}^c = i[\psi_R^{c\,t}\sigma^\mu\partial_\mu\bar{\psi}_R^c + \bar{\psi}_L^{c\,t}\bar{\sigma}^\mu\partial_\mu\psi_L^c] - [\psi_R^{c\,t}X\psi_L^c + \bar{\psi}_L^{c\,t}X^\dagger\bar{\psi}_R^c]. \qquad (4.6)$$

^{t} stands for the transpose operation and the mass mixing matrix is given by

$$X = \begin{pmatrix} M_2 & \sqrt{2}M_W s_\beta \\ \sqrt{2}M_W c_\beta & \mu \end{pmatrix}. \qquad (4.7)$$

The system can be diagonalized by two unitary matrices U and V that define the physical (Weyl) fields as

$$\chi_R^c = U\psi_R^c, \qquad \chi_L^c = V\psi_L^c. \tag{4.8}$$

In the case of CP conservation that we will cover here, we can take both U and V as real. We write the diagonalized mass matrix $\tilde{X}$

$$\tilde{X} = UXV^t = \tilde{X}^t = VX^tU = \begin{pmatrix} m_{\tilde{\chi}_1^\pm} & 0 \\ 0 & m_{\tilde{\chi}_2^\pm} \end{pmatrix}. \tag{4.9}$$

$m_{\tilde{\chi}_{1,2}^\pm}$ are the (positive) eigenvalues of the Hermitian matrix $XX^\dagger$ with $m_{\tilde{\chi}_1^\pm} < m_{\tilde{\chi}_2^\pm}$. In our implementation in order to have positive eigenvalues, we take

$$\det U = +1 \quad \text{and} \quad \det V = \text{sign}(\det X) = \epsilon_\mu \quad \text{with} \\ \det X = M_2\mu - M_W^2 s_{2\beta}. \tag{4.10}$$

The physical masses are also defined from the invariant basis independent quantities that are the trace and the determinant of the square matrice XX^t, which give

$$m^2_{\tilde{\chi}_1^\pm} + m^2_{\tilde{\chi}_2^\pm} = M_2^2 + \mu^2 + 2M_W^2, \\ m^2_{\tilde{\chi}_1^\pm} m^2_{\tilde{\chi}_2^\pm} = (\det X)^2, \tag{4.11}$$

and

$$m^2_{\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm} = \frac{1}{2}(M_2^2 + \mu^2 + 2M_W^2 \mp [(M_2^2 - \mu^2)^2 + 4M_W^4 c_{2\beta}^2 + 4M_W^2(M_2^2 + \mu^2 + 2\mu M_2 s_{2\beta})]^{1/2}). \tag{4.12}$$

The corresponding chargino Dirac spinor $\tilde{\chi}_i^c$ ($i = 1, 2$) is constructed as

$$\tilde{\chi}_i^+ = \begin{pmatrix} \chi_{Li}^c \\ \bar{\chi}_{Ri}^c \end{pmatrix} \Rightarrow \bar{\tilde{\chi}}_i^+ = (\chi_{Ri}^{ct}, \bar{\chi}_{Li}^{ct}) = \tilde{\chi}_i^{-t} \qquad i = 1, 2. \tag{4.13}$$

Similarly, the Lagrangian for neutralinos writes

$$\mathcal{L}^n = \frac{i}{2}[\psi^{nt}\sigma^\mu\partial_\mu\bar{\psi}^n + \bar{\psi}^{nt}\bar{\sigma}^\mu\partial_\mu\psi^n] - \frac{1}{2}[\psi^{nt}Y\psi^n + \bar{\psi}^{nt}Y^\dagger\bar{\psi}^n], \tag{4.14}$$

where

$$\psi^n = \begin{pmatrix} \tilde{B}^0 \\ \tilde{W}^0 \\ \tilde{H}_1^0 \\ \tilde{H}_2^0 \end{pmatrix}. \tag{4.15}$$

The mass matrix Y

$$Y = \begin{pmatrix} M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta \\ 0 & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta \\ -M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu \\ M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu & 0 \end{pmatrix} \tag{4.16}$$

can be diagonalized by an unitary complex matrix with the physical states being

$$\chi^n = N\psi^n. \tag{4.17}$$

We will refer to the diagonal matrix as

$$\tilde{Y} = N^*YN^\dagger = \text{diag}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\chi_3^0}, m_{\chi_4^0}), \\ 0 < m_{\chi_1^0} < m_{\chi_2^0} < m_{\chi_3^0} < m_{\chi_4^0}. \tag{4.18}$$

Note that N can be written as $J\hat{N}$, where $\hat{N}$ is real and $J = \text{diag}(j_1, j_2, j_3, j_4)$. $\hat{N}$ diagonalizes Y but leads to masses that are not necessarily positive. A positive mass obtained with $\hat{N}$ corresponds to $j_i = 1$, a negative mass corresponds to $j_i = i$. The corresponding neutralino (4-component) Majorana spinor $\tilde{\chi}_i^0$ ($i = 1, 2, 3, 4$) is given by

$$\tilde{\chi}_i^0 = \begin{pmatrix} \chi_i^n \\ \bar{\chi}_i^n \end{pmatrix}. \tag{4.19}$$

B. Renormalization: Counterterms and self-energies

We could have treated the chargino and neutralino system that we have just exposed within a common notation, deriving in a compact form the neutralino sector on the basis of its Majorana nature. This could have been done through a mass matrix M that stands for either X (charginos) or Y of the neutralinos and the two fields $\psi_{R,L}$ that represent either $\psi_{R,L}^c$ or the single Majorana field ψ^n. To make our renormalization procedure of this sector as transparent as possible, we will take this common approach to show that the approach in renormalizing the chargino and neutralino sector is exactly the same and that it corresponds to the approach that we have taken in the Higgs sector and the sfermion sector as concerns the issue of mixing. In particular, we stress that we do not renormalize the rotation matrices that express the mass eigenstates from the current eigenstates. Summarizing what we have just seen in the sfermion sector and splitting, as usual, the bare Lagrangian (denoted by $_0$) into a renormalized Lagrangian and counterterms, the kinetic term and the mass term of a fermion field ψ with an arbitrary number of components can be written as

$$\mathcal{L}_0^f = i[\psi_{R0}^t\sigma^\mu\partial_\mu\bar{\psi}_{R0} + \bar{\psi}_{L0}^t\bar{\sigma}^\mu\partial_\mu\psi_{L0}] - [\psi_{R0}^t M_0\psi_{R0} + \bar{\psi}_{L0}^t M_0^\dagger\bar{\psi}_{R0}], \tag{4.20}$$

where $\psi_{R/L0}$ represents the fermion field and M_0 the non-diagonal mass matrix at bare level. At tree level, this mass matrix is diagonalized by rotating the fields with two unitary matrices D_R and D_L which define the current fields so that at bare level we write these fields as

$$\chi_{R0} = D_R\psi_{R0}, \qquad \chi_{L0} = D_L\psi_{L0}. \tag{4.21}$$

The corresponding diagonal mass matrix $\tilde{M}$ is then given by

$$\tilde{M} = D_R^* M D_L^\dagger = \tilde{M}^\dagger = D_L M^t D_R^t = \mathrm{diag}(m_{\tilde{\chi}_1}, m_{\tilde{\chi}_2}, \ldots) \quad (4.22)$$

and gives the physical masses $m_{\tilde{\chi}_i}$. The ensuing Dirac/Majorana spinors $\tilde{\chi}_{i0}$ are constructed with these Weyl spinors

$$\tilde{\chi}_{i_0} = \begin{pmatrix} \chi_{Li} \\ \bar{\chi}_{Ri} \end{pmatrix}_0. \quad (4.23)$$

After the diagonalization is performed, the counterterms for the different parameters involved in the mass matrix are introduced

$$M_0 = M + \delta M \quad (4.24)$$

and also the wave function renormalization constants $\delta Z_{ij}^{R,L}$ for each chiral physical field $\chi_{R/L}$,

$$\chi_{R,Li}|_0 = \left(\delta_{ij} + \frac{1}{2}\delta Z_{ij}^{R,L}\right)\chi_{R,Lj}. \quad (4.25)$$

These transformations for the chiral fields are equivalent to the following transformation for the four-component spinor $\tilde{\chi}_i$,

$$\tilde{\chi}_{i_0} = \tilde{\chi}_i + \frac{1}{2}[\delta Z_{ij}^L P_L + \delta Z_{ij}^{R*} P_R]\tilde{\chi}_j. \quad (4.26)$$

We stress again that in our renormalization scheme, we do not use the extra shifts on the diagonalization matrices $D_{L,R}$, $D_{L,R} \to D_{L,R} + \delta D_{L,R}$, in other words, $\delta D_{L,R} = 0$ as done in Ref. [26]. This is in the same spirit as within the Higgs sector and the sfermion sector. So we consider that the diagonalization matrices $D_{L,R}$ at tree level and at the one loop level are the same, $D_{L,R}$ are renormalized. With the renormalization counterterms (4.24) and (4.25) and

$$\delta\tilde{M} = D_R^* \delta M D_L^\dagger, \quad (4.27)$$

the renormalized self-energies $\hat{\Sigma}_{\tilde{\chi}_i\tilde{\chi}_j}$ can be cast into

$$\begin{aligned}\hat{\Sigma}_{\tilde{\chi}_i\tilde{\chi}_j}(q) = {} & \Sigma_{\tilde{\chi}_i\tilde{\chi}_j}(q) - P_L\delta\tilde{M}_{ij} - P_R\delta\tilde{M}_{ji}^* \\ & + \frac{1}{2}(\not{q} - m_{\tilde{\chi}_i})[\delta Z_{ij}^L P_L + \delta Z_{ij}^{R*} P_R] \\ & + \frac{1}{2}[\delta Z_{ji}^{L*} P_R + \delta Z_{ji}^R P_L](\not{q} - m_{\tilde{\chi}_j}). \end{aligned} \quad (4.28)$$

Equation (4.28) shows clearly that the wave function renormalization constants are not involved in the renormalization of the Lagrangian parameters contained in the mass matrices $\tilde{M}$, which in our case involve M_1, M_2, μ.

It is useful to decompose the self-energy into the independent Lorentz structures through the projectors $P_{L,R} = \frac{1\mp\gamma_5}{2}$,

$$\begin{aligned}\Sigma_{\tilde{\chi}_i\tilde{\chi}_j}(q) = {} & P_L\Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{LS}(q^2) + P_R\Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{RS}(q^2) + \not{q}P_L\Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{LV}(q^2) \\ & + \not{q}P_R\Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{RV}(q^2). \end{aligned} \quad (4.29)$$

Hermiticity imposes the following constraints on the elements of the Lorentz decomposition

$$\Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{RS}(q^2) = \Sigma_{\tilde{\chi}_j\tilde{\chi}_i}^{LS*}(q^2), \qquad \Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{LV}(q^2) = \Sigma_{\tilde{\chi}_j\tilde{\chi}_i}^{LV*}(q^2),$$
$$\Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{RV}(q^2) = \Sigma_{\tilde{\chi}_j\tilde{\chi}_i}^{RV*}(q^2). \quad (4.30)$$

These are also satisfied by the corresponding covariants of the renormalized self-energies in Eq. (4.28). For a Majorana fermion (like a neutralino in the following), the additional Majorana symmetry imposes

$$\Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{RS}(q^2) = \Sigma_{\tilde{\chi}_j\tilde{\chi}_i}^{RS}(q^2), \qquad \Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{LS}(q^2) = \Sigma_{\tilde{\chi}_j\tilde{\chi}_i}^{LS}(q^2),$$
$$\Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{LV}(q^2) = \Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{RV*}(q^2) = \Sigma_{\tilde{\chi}_j\tilde{\chi}_i}^{RV}(q^2). \quad (4.31)$$

Some of these properties are used in our code as an extra test.

To fix the wave function renormalization constants $\delta Z_{ij}^{R,L}$, we require that

(i) The propagators of all the charginos and neutralinos are properly normalized with residue of 1 at the pole mass. This pole mass may get one-loop correction. For our treatment at one loop, it is sufficient to impose the residue condition by taking the tree-level mass. Taking the one-loop mass is a higher order effect, see Sec. 4.7 of [9] of our treatment in the Higgs sector. This condition implies

$$\lim_{q^2\to m_{\tilde{\chi}_i}^2} \frac{\not{q} + m_{\tilde{\chi}_i}}{q^2 - m_{\tilde{\chi}_i}^2}\widetilde{\mathrm{Re}}\hat{\Sigma}_{\tilde{\chi}_i\tilde{\chi}_i}(q)u_{\chi_i}(q) = u_{\chi_i}(q) \quad \text{and}$$
$$\lim_{q^2\to m_{\tilde{\chi}_i}^2} \bar{u}_{\chi_i}(q)\widetilde{\mathrm{Re}}\hat{\Sigma}_{\tilde{\chi}_i\tilde{\chi}_i}(q)\frac{\not{q} + m_{\tilde{\chi}_i}}{q^2 - m_{\tilde{\chi}_i}^2} = \bar{u}_{\chi_i}(q). \quad (4.32)$$

(ii) No mixing between the physical fields when these are on mass shell

$$\widetilde{\mathrm{Re}}\hat{\Sigma}_{\tilde{\chi}_i\tilde{\chi}_j}(q)u_{\chi_j}(q) = 0 \quad \text{for } q^2 = m_{\tilde{\chi}_j}^2, \qquad (i \neq j). \quad (4.33)$$

With these conditions, we do not have to consider any loop correction on the external legs. Note that as usual [14], $\widetilde{\mathrm{Re}}$ signifies that the imaginary absorptive part of the loop function is discarded so as to maintain Hermiticity at one loop. Equation (4.33) gives the diagonal element of the wave function renormalization constants

$$\begin{aligned}\delta Z_{ii}^L = {} & -\widetilde{\mathrm{Re}}\Sigma_{\tilde{\chi}_i\tilde{\chi}_i}^{LV}(m_{\tilde{\chi}_i}^2) - m_{\tilde{\chi}_i}^2(\widetilde{\mathrm{Re}}\Sigma_{\tilde{\chi}_i\tilde{\chi}_i}^{LV'}(m_{\tilde{\chi}_i}^2) \\ & + \widetilde{\mathrm{Re}}\Sigma_{\tilde{\chi}_i\tilde{\chi}_i}^{RV'}(m_{\tilde{\chi}_i}^2)) - 2m_{\tilde{\chi}_i}\widetilde{\mathrm{Re}}\Sigma_{\tilde{\chi}_i\tilde{\chi}_i}^{LS'}(m_{\tilde{\chi}_i}^2), \\ \delta Z_{ii}^R = {} & -\widetilde{\mathrm{Re}}\Sigma_{\tilde{\chi}_i\tilde{\chi}_i}^{RV}(m_{\tilde{\chi}_i}^2) - m_{\tilde{\chi}_i}^2(\widetilde{\mathrm{Re}}\Sigma_{\tilde{\chi}_i\tilde{\chi}_i}^{LV'}(m_{\tilde{\chi}_i}^2) \\ & + \widetilde{\mathrm{Re}}\Sigma_{\tilde{\chi}_i\tilde{\chi}_i}^{RV'}(m_{\tilde{\chi}_i}^2)) - 2m_{\tilde{\chi}_i}\widetilde{\mathrm{Re}}\Sigma_{\tilde{\chi}_i\tilde{\chi}_i}^{RS'}(m_{\tilde{\chi}_i}^2), \end{aligned} \quad (4.34)$$

where we have used the fact that in the case of CP conservation $\Sigma^{LS}_{\tilde{\chi}_i\tilde{\chi}_i}(m^2_{\tilde{\chi}_i}) = \Sigma^{RS}_{\tilde{\chi}_i\tilde{\chi}_i}(m^2_{\tilde{\chi}_i})$. The prime on a function such as $\Sigma^{RV'}_{\tilde{\chi}_i\tilde{\chi}_i}(m^2_{\tilde{\chi}_i})$ stands for the derivative $\partial\Sigma^{RV}_{\tilde{\chi}_i\tilde{\chi}_i}(q^2)/\partial q^2|_{q^2=m^2_{\tilde{\chi}_i}}$.

The nondiagonal elements ($i \neq j$) of $\delta Z^{L,R}$ are derived from the constraints of Eq. (4.33)

$$\begin{aligned}\delta Z^L_{ij} = \frac{2}{m^2_{\tilde{\chi}_i} - m^2_{\tilde{\chi}_j}}&(m_{\tilde{\chi}_i}\widetilde{\mathrm{Re}}\,\Sigma^{LS}_{\tilde{\chi}_i\tilde{\chi}_j}(m^2_{\tilde{\chi}_j})\\ &+ m_{\tilde{\chi}_j}\widetilde{\mathrm{Re}}\Sigma^{RS}_{\tilde{\chi}_i\tilde{\chi}_j}(m^2_{\tilde{\chi}_j}) + m_{\tilde{\chi}_i}m_{\tilde{\chi}_j}\widetilde{\mathrm{Re}}\Sigma^{RV}_{\tilde{\chi}_i\tilde{\chi}_j}(m^2_{\tilde{\chi}_j})\\ &+ m^2_{\tilde{\chi}_j}\widetilde{\mathrm{Re}}\Sigma^{LV}_{\tilde{\chi}_i\tilde{\chi}_j}(m^2_{\tilde{\chi}_j}) - m_{\tilde{\chi}_i}\delta\tilde{M}_{ij} - m_{\tilde{\chi}_j}\delta\tilde{M}^*_{ji}),\\ \delta Z^{R*}_{ij} = \frac{2}{m^2_{\tilde{\chi}_i} - m^2_{\tilde{\chi}_j}}&(m_{\tilde{\chi}_j}\widetilde{\mathrm{Re}}\Sigma^{LS}_{\tilde{\chi}_i\tilde{\chi}_j}(m^2_{\tilde{\chi}_j})\\ &+ m_{\tilde{\chi}_i}\widetilde{\mathrm{Re}}\Sigma^{RS}_{\tilde{\chi}_i\tilde{\chi}_j}(m^2_{\tilde{\chi}_j}) + m^2_{\tilde{\chi}_j}\widetilde{\mathrm{Re}}\Sigma^{RV}_{\tilde{\chi}_i\tilde{\chi}_j}(m^2_{\tilde{\chi}_j})\\ &+ m_{\tilde{\chi}_i}m_{\tilde{\chi}_j}\widetilde{\mathrm{Re}}\Sigma^{LV}_{\tilde{\chi}_i\tilde{\chi}_j}(m^2_{\tilde{\chi}_j}) - m_{\tilde{\chi}_i}\delta\tilde{M}^*_{ji} - m_{\tilde{\chi}_j}\delta\tilde{M}_{ij}).\end{aligned} \tag{4.35}$$

Specializing to the case of the charginos, we will have to take $D_R = U$ and $D_L = V$, see Eq. (4.7) and $M = X$, see Eq. (4.8), where both U and V are real matrices as is the mass matrix X in our case with CP conservation. In this case, $\delta Z^{L,R}$ can be taken real. For the neutralinos $D_L = D_R = N$, see Eq. (4.17) and $M = Y$ is a symmetric real matrix, see Eq. (4.16). In this case, as expected, we have $\delta Z^L = \delta Z^R = \delta Z^0$ which is a result of the symmetry of Y and the Majorana constraints of Eq. (4.31). In fact, Eq. (4.35) can be recast into

$$\begin{aligned}\delta Z^0_{ij} = \frac{1}{m_{\tilde{\chi}^0_i} - m_{\tilde{\chi}^0_j}}&(m_{\tilde{\chi}^0_j}(\widetilde{\mathrm{Re}}\Sigma^{LV}_{\tilde{\chi}^0_i\tilde{\chi}^0_j}(m^2_{\tilde{\chi}^0_j}) + \widetilde{\mathrm{Re}}\Sigma^{LV*}_{\tilde{\chi}^0_i\tilde{\chi}^0_j}(m^2_{\tilde{\chi}^0_j}))\\ &+ (\widetilde{\mathrm{Re}}\Sigma^{LS}_{\tilde{\chi}^0_i\tilde{\chi}^0_j}(m^2_{\tilde{\chi}^0_j}) + \widetilde{\mathrm{Re}}\Sigma^{LS*}_{\tilde{\chi}^0_i\tilde{\chi}^0_j}(m^2_{\tilde{\chi}^0_j})) - (\delta\tilde{Y}_{ij} + \delta\tilde{Y}^*_{ij}))\\ &+ \frac{1}{m_{\tilde{\chi}^0_i} + m_{\tilde{\chi}^0_j}}(-m_{\tilde{\chi}^0_j}(\widetilde{\mathrm{Re}}\Sigma^{LV}_{\tilde{\chi}^0_i\tilde{\chi}^0_j}(m^2_{\tilde{\chi}^0_j}) - \widetilde{\mathrm{Re}}\Sigma^{LV*}_{\tilde{\chi}^0_i\tilde{\chi}^0_j}(m^2_{\tilde{\chi}^0_j}))\\ &+ (\widetilde{\mathrm{Re}}\Sigma^{LS}_{\tilde{\chi}^0_i\tilde{\chi}^0_j}(m^2_{\tilde{\chi}^0_j}) - \widetilde{\mathrm{Re}}\Sigma^{LS*}_{\tilde{\chi}^0_i\tilde{\chi}^0_j}(m^2_{\tilde{\chi}^0_j})) - (\delta\tilde{Y}_{ij} - \delta\tilde{Y}^*_{ij})).\end{aligned} \tag{4.36}$$

Note that while Z^0_{ii} is real, Z^0_{ij} ($i \neq j$) is either purely real [given by the first two lines in Eq. (4.36)] or purely imaginary [given by the last two lines in Eq. (4.36)], when using $N = J\hat{N}$ in order to have positive masses. Using $N = \hat{N}$, we can have Z^0_{ij} as real but we have to keep track of the sign of the masses in Eq. (4.36). For example, with $N = \hat{N}$ (real), when both $m_{\tilde{\chi}_{i,j}} > 0$ are obtained, we get

$$\begin{aligned}\delta Z^0_{ij} = \frac{2}{m_{\tilde{\chi}^0_i} - m_{\tilde{\chi}^0_j}}&(\widetilde{\mathrm{Re}}\Sigma^{LS}_{\tilde{\chi}^0_i\tilde{\chi}^0_j}(m^2_{\tilde{\chi}^0_j})\\ &+ m_{\tilde{\chi}^0_j}\widetilde{\mathrm{Re}}\Sigma^{LV}_{\tilde{\chi}^0_i\tilde{\chi}^0_j}(m^2_{\tilde{\chi}^0_j}) - \delta\tilde{Y}_{ij}).\end{aligned} \tag{4.37}$$

It is important to note a common feature of our approach that we already encountered in the case of the mixing in the Higgs sector and the sfermion sector. The nondiagonal wave function renormalization constants in Eqs. (4.35), (4.36), and (4.37) are fully determined only once the mass counterterms $\delta\tilde{M}$ are fixed. In our case, this requires fixing δM_1, δM_2, and $\delta\mu$ to which we turn in the next section.

For completeness, let us give the corresponding counterterm matrices δX and δY. We have

$$\begin{aligned}\delta X &= \begin{bmatrix} \delta M_2 & \delta X_{12} \\ \delta X_{21} & \delta\mu \end{bmatrix},\\ \delta Y &= \begin{bmatrix} \delta M_1 & 0 & \delta Y_{13} & \delta Y_{14} \\ 0 & \delta M_2 & \delta Y_{23} & \delta Y_{24} \\ \delta Y_{13} & \delta Y_{23} & 0 & -\delta\mu \\ \delta Y_{14} & \delta Y_{24} & -\delta\mu & 0 \end{bmatrix},\end{aligned} \tag{4.38}$$

with

$$\begin{aligned}\delta X_{12} &= +\sqrt{2}M_W s_\beta\left(\frac{1}{2}\frac{\delta M^2_W}{M^2_W} + c^2_\beta\frac{\delta t_\beta}{t_\beta}\right),\\ \delta X_{21} &= +\sqrt{2}M_W c_\beta\left(\frac{1}{2}\frac{\delta M^2_W}{M^2_W} - s^2_\beta\frac{\delta t_\beta}{t_\beta}\right),\\ \delta Y_{13} &= -s_W M_Z c_\beta\left(\frac{1}{2}\frac{\delta M^2_Z}{M^2_Z} + \frac{1}{2}\frac{\delta s^2_W}{s^2_W} - s^2_\beta\frac{\delta t_\beta}{t_\beta}\right),\\ \delta Y_{14} &= +s_W M_Z s_\beta\left(\frac{1}{2}\frac{\delta M^2_Z}{M^2_Z} + \frac{1}{2}\frac{\delta s^2_W}{s^2_W} + c^2_\beta\frac{\delta t_\beta}{t_\beta}\right),\\ \delta Y_{23} &= +c_W M_Z c_\beta\left(\frac{1}{2}\frac{\delta M^2_Z}{M^2_Z} + \frac{1}{2}\frac{\delta c^2_W}{c^2_W} - s^2_\beta\frac{\delta t_\beta}{t_\beta}\right),\\ \delta Y_{24} &= -c_W M_Z s_\beta\left(\frac{1}{2}\frac{\delta M^2_Z}{M^2_Z} + \frac{1}{2}\frac{\delta c^2_W}{c^2_W} + c^2_\beta\frac{\delta t_\beta}{t_\beta}\right).\end{aligned} \tag{4.39}$$

C. Fixing δM_1, δM_2, $\delta\mu$

δM_1, δM_2, $\delta\mu$ can be fixed through the diagonal self-energies of the chargino-neutralino system which we have not fully exploited yet and which constrain the physical masses of the charginos and neutralinos. The most straightforward and simple choice is based on the fact that the chargino system is a 2×2 system which is easier to handle that the 4×4 system of the neutralinos. In SLOOPS, the default scheme is to choose the two chargino masses $m_{\tilde{\chi}^\pm_1}$ and $m_{\tilde{\chi}^\pm_2}$ as inputs to define the two parameters M_2 and μ and one neutralino mass to define M_1. The lightest neutralino mass $m_{\tilde{\chi}^0_1}$ is used by default to fix M_1. The three other neutralino masses $m_{\tilde{\chi}^0_{2,3,4}}$ are derived and receive one-loop quantum corrections. At one loop, these three input parameters translate into the usual definition of the pole masses in the on-shell scheme through the renormalized self-energies of the charginos and the lightest neutralino

$$\widetilde{\mathrm{Re}}\hat{\Sigma}_{\tilde{\chi}_i\tilde{\chi}_i}(q)u_{\chi_i}(q) = 0 \quad \text{for } q^2 = m^2_{\chi_i}, \quad \text{for } \chi_i \to \chi_1^{\pm}, \chi_2^{\pm}, \chi_1^0. \tag{4.40}$$

This translates into

$$\begin{aligned}\delta\tilde{X}_{11} &= \delta m_{\tilde{\chi}_1^{\pm}} \\ &= \widetilde{\mathrm{Re}}\Sigma^{LS}_{\tilde{\chi}_1^{\pm}\tilde{\chi}_1^{\pm}}(m^2_{\tilde{\chi}_1^{\pm}}) + \frac{1}{2}m_{\tilde{\chi}_1^{\pm}}(\widetilde{\mathrm{Re}}\Sigma^{LV}_{\tilde{\chi}_1^{\pm}\tilde{\chi}_1^{\pm}}(m^2_{\tilde{\chi}_1^{\pm}}) \\ &\quad + \widetilde{\mathrm{Re}}\Sigma^{RV}_{\tilde{\chi}_1^{\pm}\tilde{\chi}_1^{\pm}}(m^2_{\tilde{\chi}_1^{\pm}})), \\ \delta\tilde{X}_{22} &= \delta m_{\tilde{\chi}_2^{\pm}} \\ &= \widetilde{\mathrm{Re}}\Sigma^{LS}_{\tilde{\chi}_2^{\pm}\tilde{\chi}_2^{\pm}}(m^2_{\tilde{\chi}_2^{\pm}}) + \frac{1}{2}m_{\tilde{\chi}_2^{\pm}}(\widetilde{\mathrm{Re}}\Sigma^{LV}_{\tilde{\chi}_2^{\pm}\tilde{\chi}_2^{\pm}}(m^2_{\tilde{\chi}_2^{\pm}}) \\ &\quad + \widetilde{\mathrm{Re}}\Sigma^{RV}_{\tilde{\chi}_2^{\pm}\tilde{\chi}_2^{\pm}}(m^2_{\tilde{\chi}_2^{\pm}})), \\ \delta\tilde{Y}_{11} &= \delta m_{\tilde{\chi}_1^0} = \widetilde{\mathrm{Re}}\Sigma^{LS}_{\tilde{\chi}_1^0\tilde{\chi}_1^0}(m^2_{\tilde{\chi}_1^0}) + m_{\tilde{\chi}_1^0}\widetilde{\mathrm{Re}}\Sigma^{LV}_{\tilde{\chi}_1^0\tilde{\chi}_1^0}(m^2_{\tilde{\chi}_1^0}).\end{aligned} \tag{4.41}$$

These three counterterms can be inverted to derive the counterterms parameters δM_1, δM_2, $\delta\mu$ through $\delta\tilde{Y} = N^*\delta Y N^\dagger$ and $\delta\tilde{X} = U^*\delta X V^\dagger$, see Eq. (4.27). In fact, δM_2, $\delta\mu$ can be derived more directly without going through the mixing matrices from Eq. (4.11). We get

$$\begin{aligned}\delta M_2 &= \frac{1}{M_2^2 - \mu^2}\bigg((M_2 m^2_{\tilde{\chi}_1^+} - \mu\,\mathrm{det}X)\frac{\delta m_{\tilde{\chi}_1^+}}{m_{\tilde{\chi}_1^+}} \\ &\quad + (M_2 m^2_{\tilde{\chi}_2^+} - \mu\,\mathrm{det}X)\frac{\delta m_{\tilde{\chi}_2^+}}{m_{\tilde{\chi}_2^+}} - M_W^2(M_2 + \mu s_{2\beta}) \\ &\quad \times \frac{\delta M_W^2}{M_W^2} - \mu M_W^2 s_{2\beta}c_{2\beta}\frac{\delta t_\beta}{t_\beta}\bigg), \\ \delta\mu &= \frac{1}{\mu^2 - M_2^2}\bigg((\mu m^2_{\tilde{\chi}_1^+} - M_2\,\mathrm{det}X)\frac{\delta m_{\tilde{\chi}_1^+}}{m_{\tilde{\chi}_1^+}} \\ &\quad + (\mu m^2_{\tilde{\chi}_2^+} - M_2\,\mathrm{det}X)\frac{\delta m_{\tilde{\chi}_2^+}}{m_{\tilde{\chi}_2^+}} - M_W^2(\mu + M_2 s_{2\beta}) \\ &\quad \times \frac{\delta M_W^2}{M_W^2} - M_2 M_W^2 s_{2\beta}c_{2\beta}\frac{\delta t_\beta}{t_\beta}\bigg),\end{aligned} \tag{4.42}$$

$$\begin{aligned}\delta M_1 &= \frac{1}{N_{11}^{*2}}(\delta m_{\chi_1^0} - N_{12}^{*2}\delta M_2 + 2N_{13}^*N_{14}^*\delta\mu \\ &\quad - 2N_{11}^*N_{13}^*\delta Y_{13} - 2N_{12}^*N_{13}^*\delta Y_{23} - 2N_{11}^*N_{14}^*\delta Y_{14} \\ &\quad - 2N_{12}^*N_{14}^*\delta Y_{24}).\end{aligned} \tag{4.43}$$

The physical masses of the other three neutralinos ($i = 2$, 3, 4) receive a correction at one loop given by

$$\begin{aligned}m^{\mathrm{phys}}_{\tilde{\chi}_i^0} &= m_{\tilde{\chi}_i^0} + \delta\tilde{Y}_{ii} - \widetilde{\mathrm{Re}}\Sigma^{LS}_{\tilde{\chi}_i^0\tilde{\chi}_i^0}(m^2_{\tilde{\chi}_i^0}) \\ &\quad - m_{\tilde{\chi}_i^0}\widetilde{\mathrm{Re}}\Sigma^{LV}_{\tilde{\chi}_i^0\tilde{\chi}_i^0}(m^2_{\tilde{\chi}_i^0}) \quad \text{with} \\ \delta\tilde{Y}_{ii} &= N_{i1}^{*2}\delta M_1 + N_{i2}^{*2}\delta M_2 - 2N_{i3}^*N_{i4}^*\delta\mu + 2N_{i1}^*N_{i3}^*\delta Y_{13} \\ &\quad + 2N_{i1}^*N_{i4}^*\delta Y_{14} + 2N_{i2}^*N_{i3}^*\delta Y_{23} + 2N_{i3}^*N_{i4}^*\delta Y_{24}.\end{aligned} \tag{4.44}$$

Checking the cancellation of the ultraviolet divergences in Eq. (4.44) is an important nontrivial test on the validity and correctness of the procedure and its implementation. Other schemes in the neutralino/chargino sector can be implemented in SLOOPS as will be shown in a forthcoming publication. A deviation from the commonly used scheme adopted here was taken in Ref. [27], where the input parameters are the masses of $\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, and $\tilde{\chi}_2^{\pm}$.

There are a few important remarks to make about Eqs. (4.42) and (4.43). The choice of $m_{\tilde{\chi}_1^0}$ as an input parameter is appropriate only if the lightest neutralino is mostly bino or if the binolike neutralino is not too heavy compared to the other neutralinos. Otherwise, the extraction of M_1 would be subject to uncertainties. This shows in Eq. (4.43) since N_{11} would be too small which would, in turn, induce large radiative corrections. Another difficulty arises with the special configuration $M_2 \sim \pm\mu$. Equation (4.42) shows that an apparent singularity might be present. We had already pointed out in [5] that this configuration can induce a large t_β-scheme dependence in the counterterms $\delta M_{1,2}$ and $\delta\mu$ and therefore to the annihilation of the lightest supersymmetric particle into W's for a mixed lightest supersymmetric particle, see also [28]. Let us look at this configuration again. We can rewrite Eq. (4.42) as

$$\begin{aligned}\delta M_2 &= \frac{1}{M_2^2 - \mu^2}(\epsilon_\mu\mu\delta E_\chi + (M_2 - \epsilon_\mu\mu)\delta F_\chi) = \frac{1}{M_2^2 - \mu^2}(|\mu|\delta E_\chi + (M_2 - |\mu|)\delta F_\chi), \\ \delta\mu &= \frac{1}{\mu^2 - M_2^2}(\epsilon_\mu M_2\delta E_\chi + (\mu - \epsilon_\mu M_2)\delta F_\chi) = \frac{\epsilon_\mu}{\mu^2 - M_2^2}(M_2\delta E_\chi + (|\mu| - M_2)\delta F_\chi), \\ \delta E_\chi &= \frac{1}{2}\delta(m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_2^+})^2 - M_W^2\bigg(\frac{\delta M_W^2}{M_W^2}(1 + \epsilon_\mu s_{2\beta}) + \epsilon_\mu s_{2\beta}c_{2\beta}\frac{\delta t_\beta}{t_\beta}\bigg), \\ \delta F_\chi &= \frac{1}{2}(\delta m^2_{\tilde{\chi}_1^+} + \delta m^2_{\tilde{\chi}_2^+}) - \delta M_W^2.\end{aligned} \tag{4.45}$$

It is important to note that the contributions proportional to δF_χ are regular in the limit $M_2 \rightarrow |\mu|$, moreover δF_χ does not introduce any t_β dependence. Only terms in δE_χ may cause trouble. The problem is confined to the finite part (in the ultraviolet sense) of δE_χ. Indeed, we have checked explicitly that in the limit $M_2 \rightarrow |\mu|$, δE_χ is finite. This is a strong check on the validity of the code. Therefore any nonregular term comes from the finite part (in the ultraviolet sense) of δE_χ and calls for a good choice of the renormalization scheme in order not to induce too large corrections or ill-defined constants.

D. Input parameters and parameter reconstruction

In practice, in the on-shell scheme that is generally used for the chargino/neutralino sector and that we adopt here, we need to reconstruct from experiments the value of μ, M_2, and M_1 from three physical masses. If we invert the mass relations of the chargino sector, we would, in general, get four solutions (M_2, μ) for one set of chargino masses $(m_{\tilde{\chi}_1^+}, m_{\tilde{\chi}_2^+})$

$$\mu^2 = \frac{m^2_{\tilde{\chi}_1^+} + m^2_{\tilde{\chi}_2^+} - 2M_W^2}{2} - \frac{\epsilon_\chi}{2}[(m^2_{\tilde{\chi}_1^+} + m^2_{\tilde{\chi}_2^+} - 2M_W^2)^2 - 4(M_W^2 s_{2\beta} + \epsilon_\mu m_{\tilde{\chi}_1^+} m_{\tilde{\chi}_2^+})^2]^{1/2},$$
$$M_2 = [m^2_{\tilde{\chi}_1^+} + m^2_{\tilde{\chi}_2^+} - 2M_W^2 - \mu^2]^{1/2}, \tag{4.46}$$

where $\epsilon_{\mu,\chi}$ can take the value ± 1 and summarize the ambiguities in the reconstruction [29]. ϵ_μ represents the sign of μ so that $\mu = \epsilon_\mu \sqrt{\mu^2}$. ϵ_χ represents the $M_2 \leftrightarrow \mu$ symmetry in the reconstruction so that $\mathrm{Sgn}\epsilon_\chi = \mathrm{Sgn}(\mu^2 - M_2^2)$. In the numerical computations of the one-loop correction to the neutralino masses in Sec. V B, we have taken the set corresponding to $\epsilon_\chi = \epsilon_\mu = 1$. Once M_2 and μ are known, the remaining parameter M_1 can be extracted from the knowledge of one of the masses of the neutralinos. For example, in the case where the neutralino is mostly binolike and corresponds to the lightest neutralino with mass $m_{\tilde{\chi}_1^0}$ as what occurs with the models with gaugino mass unification at the grand unified theories (GUT) scale, we have[3]

$$M_1 = \frac{m^4_{\tilde{\chi}_1^0} - M_2 m^3_{\tilde{\chi}_1^0} - (\mu^2 + M_Z^2) m^2_{\tilde{\chi}_1^0} - (s_{2\beta} M_Z^2 \mu - (\mu^2 + s_W^2 M_Z^2) M_2) m_{\tilde{\chi}_1^0} + s_{2\beta} s_W^2 M_Z^2 \mu M_2}{m^3_{\tilde{\chi}_1^0} - M_2 m^2_{\tilde{\chi}_1^0} - (\mu^2 + c_W^2 M_Z^2) m_{\tilde{\chi}_1^0} - s_{2\beta} c_W^2 M_Z^2 \mu + \mu^2 M_2}. \tag{4.47}$$

Having M_1, M_2, μ, a consistency check can be made to make sure that M_1 is indeed given through Eq. (4.47) with $m_{\tilde{\chi}_1^0}$ as input and not some other neutralino. This somehow shows the ambiguity, already encountered in extracting M_2, μ from the 2 chargino masses, in reconstructing the Lagrangian parameters from the knowledge of three masses only. This said, considering that, with the present limits on the chargino masses, the effect of mass splitting is small like, as we will see, the effect of the radiative corrections on the neutralino masses, discovery of both charginos almost certainly guarantees the discovery of the two Higgsino and the winolike neutralinos with masses of the same order as the corresponding charginos therefore allowing to select the correct (M_2, μ) from the chargino reconstruction. If the binolike is not too heavy it will then be easy to single out and hence measure M_1. Another exploration about the correct extraction of M_2, μ, M_1, can also be done through the measurements of some couplings of the charginos (see, for example, [30] for a tree-level analysis) and the neutralinos (see, for example, [31]). We will see below how one can extract these parameters in decays involving the neutralinos combined with the measurements of the chargino masses. Although the situation here is quite different from the mixing in the sfermions, exploiting decays as inputs, to fix the underlying parameters less unambiguously when mixing takes place, is promising. We will get back to this issue in a forthcoming publication. Meanwhile, let us give an example about the reconstruction. As an example the measured masses that we take as input are the two chargino masses with $m_{\tilde{\chi}_1^+} = 232$ GeV and $m_{\tilde{\chi}_2^+} = 426$ GeV and the lightest neutralino mass $m_{\tilde{\chi}_1^0} = 98$ GeV. From the chargino masses, we obtain four solutions for (M_2, μ) according to Eq. (4.46). For each one of these solutions, we first reconstruct the corresponding M_1 by imposing the mass of the $\tilde{\chi}_1^0$ taken as input. In the example we have taken, the four solutions for (M_2, μ, M_1) are (all given in GeV)

$$\begin{aligned} &(250.39\ 399.78\ 100.38) && \text{for } \epsilon_\chi = 1, \epsilon_\mu = 1, \\ &(240.39, -405.86\ 98.22) && \text{for } \epsilon_\chi = 1, \epsilon_\mu = -1, \\ &(399.78\ 250.39\ 103.68) && \text{for } \epsilon_\chi = -1, \epsilon_\mu = 1, \\ &(405.86, -240.39\ 100.05) && \text{for } \epsilon_\chi = -1, \epsilon_\mu = -1. \end{aligned} \tag{4.48}$$

Each solution will lead to different predictions on the observables in the chargino and neutralino sector as well as the sfermion/Higgs sector. Comparing the theoretical predictions to the measurements of a minimal set of these observables lifts the four-fold ambiguity. Theoretically, with each set of solutions in Eq. (4.48), we can give one-loop predictions given some other model parameters that indirectly enter in the one-loop calculation. For simplicity, and to avoid having to deal with QED corrections, we consider the prediction on the 3 other neutralino masses and the decays $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0(\gamma, Z^0)$. The former is a pure one-

[3]Equation (4.47) was derived in [26]; however, there is a typo. s_W^2 in the last term in the numerator of Eq. (4.47) is missing in [26].

loop effect. We take the pseudoscalar mass $M_{A^0} = 300$ GeV, a common soft-SUSY sfermion mass $M_{\tilde{f}} = 500$ GeV, a common $A_f = 0$, the $SU(3)$ gaugino mass is set at $M_3 = 1000$ GeV and $t_\beta = 10$. For t_β, the results we present below are within the MH scheme. $\delta\Gamma(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0)$ is the one-loop correction to the rate $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0$.

The results in Table I show that disentangling between the possible solutions is, in principle, possible even if not all neutralino masses are measured. For example, the rate $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0$ is a clear cut indicator for the sign of ϵ_χ since this rate is an order of magnitude larger if the Higgsinolike neutralino is lighter than the winolike neutralino. If a precision measurement below the 10% level can be achieved on this observable, it can, by itself, also disentangle between all four solutions. Considering the smallness of the rate $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0$, this observable is perhaps of academic interest. Note, however, that it can, in principle, be used to lift the degeneracy between all four solutions.

TABLE I. Disentangling between the four solutions for M_2, μ, and M_1 from the input with $m_{\tilde{\chi}_1^+} = 232$ GeV, $m_{\tilde{\chi}_1^+} = 426$ GeV, and $m_{\tilde{\chi}_1^0} = 98$ GeV. All masses and decay widths are in GeV units. $\delta\Gamma$ is the one-loop correction in the MH scheme.

$\epsilon_\chi = 1$, $\epsilon_\mu = 1$

$m^{\text{tree-level}}_{\tilde{\chi}_2^0} = 232.34$, $m^{\text{phys}}_{\tilde{\chi}_2^0} = 232.19$,

$m^{\text{tree-level}}_{\tilde{\chi}_3^0} = 405.26$, $m^{\text{phys}}_{\tilde{\chi}_3^0} = 407.41$,

$m^{\text{tree-level}}_{\tilde{\chi}_4^0} = 425.69$, $m^{\text{phys}}_{\tilde{\chi}_4^0} = 425.77$,

$\Gamma(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \gamma) = 0.308 \times 10^{-8}$,

$\Gamma(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0)^{\text{tree-level}} = 0.223 \times 10^{-2}$,

$\delta\Gamma(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0) = 0.533 \times 10^{-4}$.

$\epsilon_\chi = 1$, $\epsilon_\mu = -1$

$m^{\text{tree-level}}_{\tilde{\chi}_2^0} = 231.83$, $m^{\text{phys}}_{\tilde{\chi}_2^0} = 231.74$,

$m^{\text{tree-level}}_{\tilde{\chi}_3^0} = 414.02$, $m^{\text{phys}}_{\tilde{\chi}_3^0} = 414.19$,

$m^{\text{tree-level}}_{\tilde{\chi}_4^0} = 422.79$, $m^{\text{phys}}_{\tilde{\chi}_4^0} = 423.46$,

$\Gamma(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \gamma) = 0.182 \times 10^{-7}$,

$\Gamma(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0)^{\text{tree-level}} = 0.202 \times 10^{-2}$,

$\delta\Gamma(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0) = 0.780 \times 10^{-4}$.

$\epsilon_\chi = -1$, $\epsilon_\mu = -1$

$m^{\text{tree-level}}_{\tilde{\chi}_2^0} = 236.17$, $m^{\text{phys}}_{\tilde{\chi}_2^0} = 236.17$,

$m^{\text{tree-level}}_{\tilde{\chi}_3^0} = 256.54$, $m^{\text{phys}}_{\tilde{\chi}_3^0} = 254.71$,

$m^{\text{tree-level}}_{\tilde{\chi}_4^0} = 425.00$, $m^{\text{phys}}_{\tilde{\chi}_4^0} = 425.81$,

$\Gamma(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \gamma) = 0.142 \times 10^{-7}$,

$\Gamma(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0)^{\text{tree-level}} = 0.197 \times 10^{-1}$,

$\delta\Gamma(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0) = 0.271 \times 10^{-2}$.

$\epsilon_\chi = -1$, $\epsilon_\mu = -1$

$m^{\text{tree-level}}_{\tilde{\chi}_2^0} = 231.76$, $m^{\text{phys}}_{\tilde{\chi}_2^0} = 232.59$,

$m^{\text{tree-level}}_{\tilde{\chi}_3^0} = 249.64$, $m^{\text{phys}}_{\tilde{\chi}_3^0} = 249.53$,

$m^{\text{tree-level}}_{\tilde{\chi}_4^0} = 425.80$, $m^{\text{phys}}_{\tilde{\chi}_4^0} = 425.65$,

$\Gamma(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \gamma) = 0.368 \times 10^{-10}$,

$\Gamma(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0)^{\text{tree-level}} = 0.277 \times 10^{-1}$,

$\delta\Gamma(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0) = 0.157 \times 10^{-2}$.

Combining measurements like this with measurements of some of the other neutralino masses or measuring all the neutralino masses is certainly a good way to lift the ambiguity.

V. APPLICATIONS AND EXAMPLES AT ONE LOOP

Our code has been checked extensively. We have written a script that automatically calculates cross sections for all $2 \to 2$ process in the MSSM at one loop. We check ultraviolet finiteness as well as the independence in each of the nonlinear gauge parameters. Results of these extensive checks can be found in [32].

Moreover, we have compared the results of the code and the renormalization procedure with quite a few observables that have appeared in the literature. Apart from these comparisons, which we will report here, the flexibility of the code allows us to study the scheme dependence of the result. We show, here, a few examples, taken from studies by different groups, of comparisons ranging from mass corrections, two-body decays as well as $2 \to 2$ processes paying a particular attention to the important t_β scheme dependence. For the latter, we consider the schemes introduced in [9] and summarized in Sec. II. The examples we will review here cover the sectors we studied in this paper, leaving aside the Higgs sector that we studied at length in [9].

Before embarking on showing our results for some observables at one loop, let us briefly describe how we treat infrared divergences. The one-loop corrections can still contain infrared divergences due to photon virtual exchanges. These are regulated by a small photon mass. The photon mass regulator contribution contained in the virtual correction should cancel exactly against the one present in the photon final state radiation. The photonic contribution is, in fact, split into a soft part, where the photon energy is less than some small cutoff k_c, $\mathcal{M}_\gamma^{\text{soft}}(E_\gamma < k_c)$ and a hard part with $\mathcal{M}_\gamma^{\text{hard}}(E_\gamma > k_c)$. The former requires a photon mass regulator. We use the usual universal factorized form with a simple rescaling for the case of the gluon correction in all processes we have studied where the non-Abelian coupling of the gluon is not at play. The test on the infrared finiteness is performed by including both the loop and the soft bremsstrahlung contributions and checking that there is no dependence on the fictitious photon mass λ_γ or gluon mass λ_g. For the bremsstrahlung part, we use VEGAS adaptive Monte Carlo integration package provided in the FFL bundle and verify the result of the cross section against COMPHEP [33]. We choose k_c small enough and check the stability and independence of the result with respect to k_c.

A. Corrections to the sbottom and stau masses

We compare our results with those of Ref. [34], where an approach similar to ours in this sector is taken. For t_β, the authors of [34] take a DCPR scheme and compare with $\overline{\text{DR}}$. The mixing parameter in [34] is, however, defined through the naive scheme of Eq. (3.38).

In order to conduct this comparison, we first need to implement the same set of input parameters as in [34]. We therefore slightly change our scheme to predict the heaviest sbottom mass $m_{\tilde{b}_1}$ at one loop instead of the heaviest stop mass $m_{\tilde{t}_1}$ which is therefore taken as input. Because our code is quite flexible, this change can be made very easily. The set of parameters corresponds to the (tree level) choice $\mu = 100$ GeV, $M_1 = 95$ GeV, $M_2 = 200$ GeV, $M_3 = 719$ GeV, $M_{A^0} = 150$ GeV, and $M_{\tilde{f}_R} = M_{\tilde{f}_L} = A_f = 300$ GeV. This assumes implicitly that these Lagrangian parameters have been reconstructed from the physical inputs. Let us discuss our results first, taking the same scheme for the sfermion mixing parameter as in Ref. [34] before commenting on the impact of taking the SLOOPS default scheme for this parameter. As Fig. 1 shows, the corrections are almost insensitive to the t_β scheme in the case of the correction to the sbottom mass, which is very welcome. Indeed, the $A_{\tau\tau}$, $\overline{\text{DR}}$, and DCPR are within 0.03% and thus indistinguishable, they are shown as one prediction in Fig. 1. The MH scheme deviates very slightly from the other schemes, especially for small t_β, this difference is at most of order 0.3%. However, in this case, the uncertainty introduced by the MH scheme is an order of magnitude smaller compared to the total correction which is of order 3–4%. For the sbottom, the corrections are due essentially to the QCD/SQCD (supersymmetric quantum chromodynamics) corrections increasing with t_β from 3% to about 4%. This correction is by itself small. The correction in the case of the stau mass is even smaller by an order of magnitude at least. However, here the MH uncertainty at small t_β is noticeable at small t_β of order 0.1–0.2% from the other three t_β schemes which agree with each other to better than 0.01%. The reason for the (almost) scheme independence is that the t_β-scheme dependence of the sbottom mass as well as of the stau mass is proportional

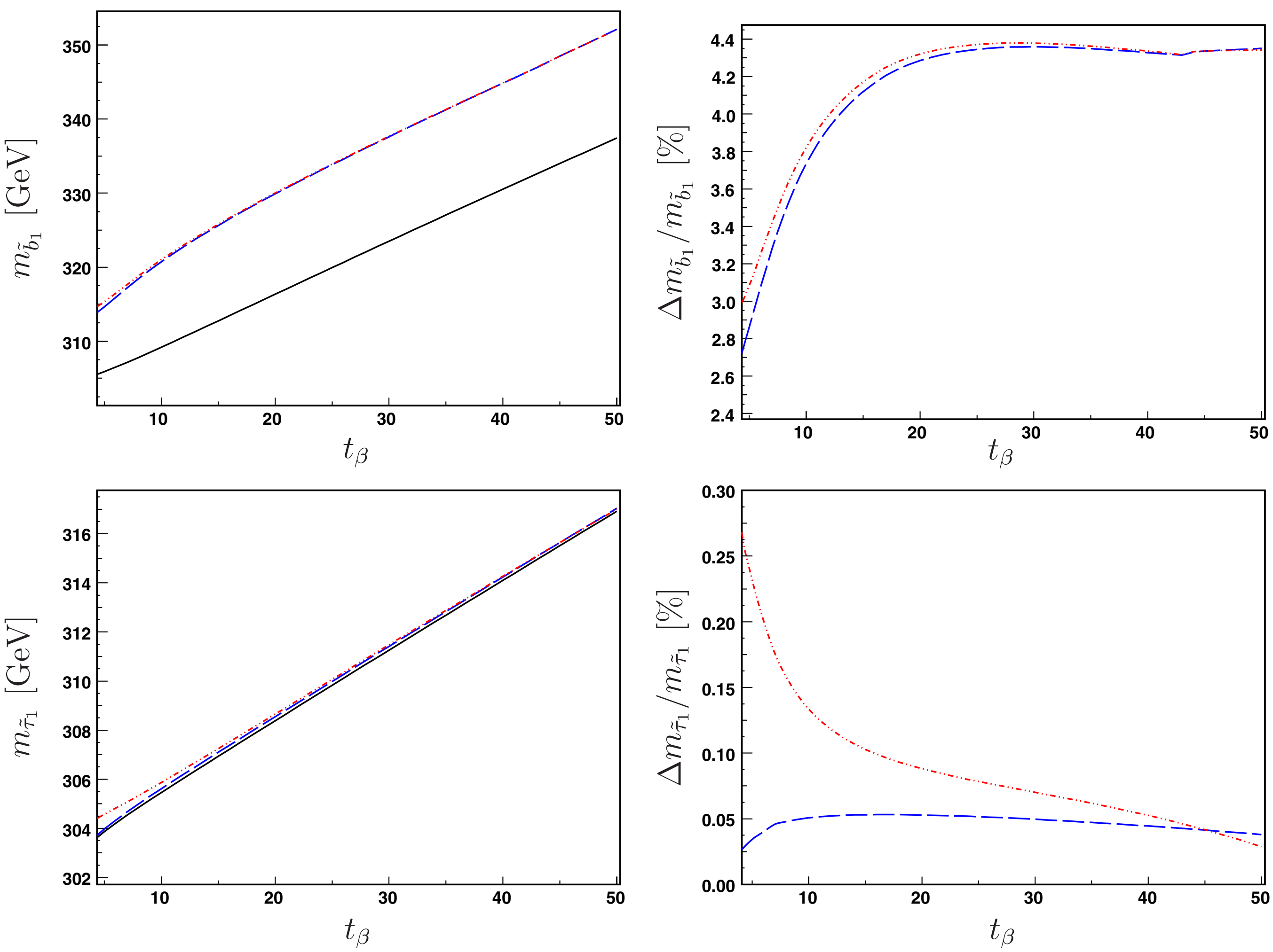

FIG. 1 (color online). Heaviest sbottom mass $m_{\tilde{b}_1}$ and heaviest stau mass $m_{\tilde{\tau}_1}$ at tree level (solid line) and at one loop for the $A_{\tau\tau}$ (and also, $\overline{\text{DR}}$ and DCPR) scheme (dashed line) and for the MH scheme (dash-dot-dotted line) as a function of t_β. The percentage correction is also given.

to $s_{2\beta}^2 \simeq 4/t_\beta^2$ which is strongly suppressed for large t_β. Our results for the DCPR and $\overline{\rm DR}$ schemes are in excellent agreement with those of Ref. [34]. Concerning the choice of the mixing parameter $\delta m_{f_{12}}^2$, we observe a small difference between the default choice in SLOOPS given by Eq. (3.37) and the one given by Eq. (3.38). To give an idea, the difference is about 0.2% in the sbottom mass correction for both $t_\beta = 10$ and $t_\beta = 50$.

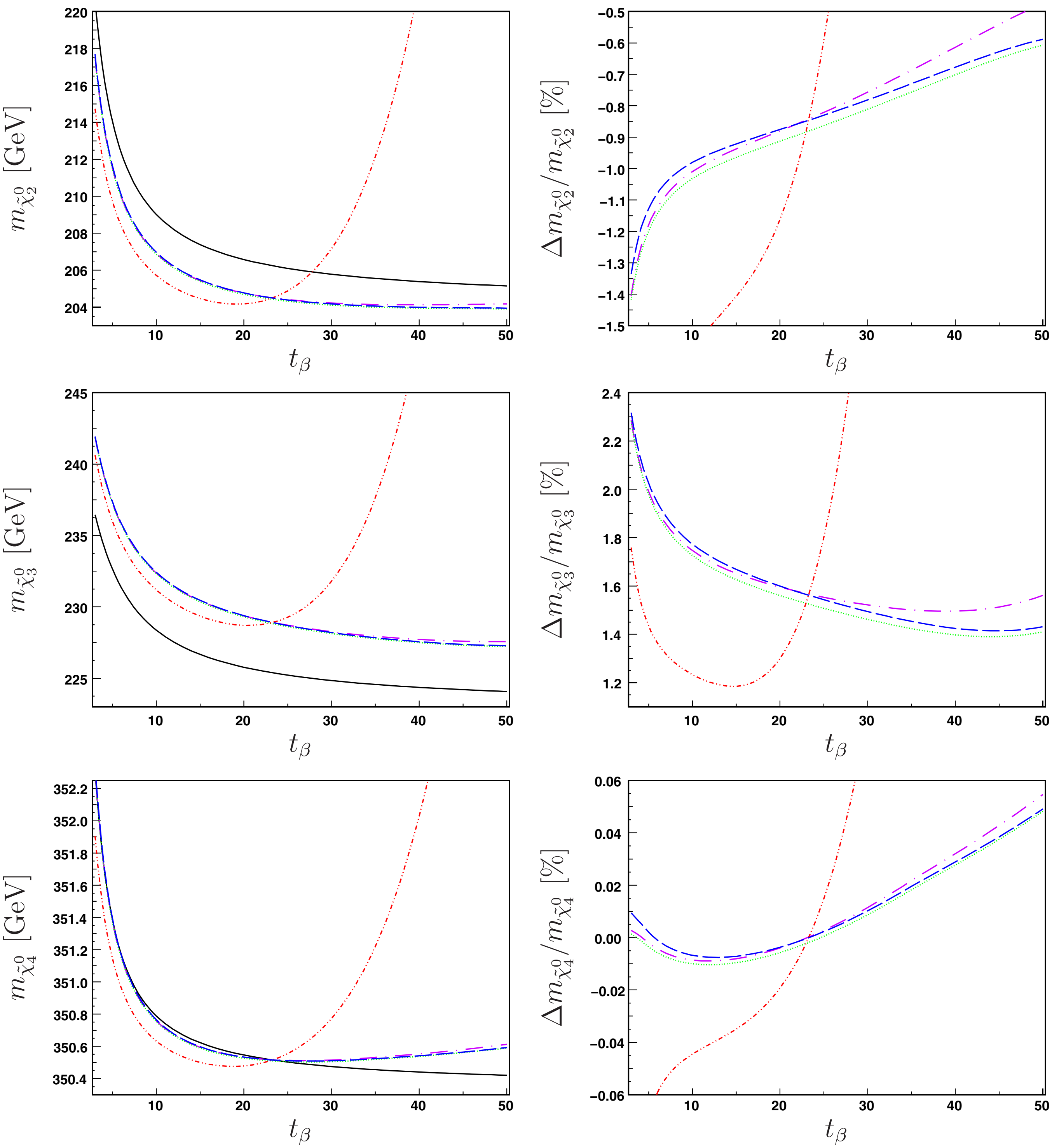

FIG. 2 (color online). Neutralino masses at tree level (solid/black line) and at one loop by using the $A_{\tau\tau}$ scheme (dashed/blue line), the $\overline{\rm DR}$ scheme (dotted/light green line), the DCPR scheme (dash-dotted/ purple line), and the MH scheme (dash-dot-dotted/red line) as a function of t_β.

B. Corrections to the masses of the heaviest neutralinos, $m_{\chi^0_{2,3,4}}$

We calculated the quantum corrections to the masses of the three neutralinos for the different schemes of t_β implemented in our code and compared our results with Ref. [26], which works within the DCPR scheme but otherwise takes the same input parameters, namely, the chargino masses and the lightest supersymmetric particle (LSP) mass. The input chargino/neutralino parameters are $m_{\chi_1^+} = 180$ GeV, $m_{\chi_2^+} = 350$ GeV, $m_{\chi_1^0} = 160$ GeV, we choose, as in [26], the model corresponding to $\epsilon_\mu = \epsilon_\chi = 1$ in Eqs. (4.46) and (4.47) in order to reconstruct the three fundamental parameters M_1, M_2, and μ. The other input parameters given in Ref. [26] which enter indirectly in the loop calculation are $M_{A^0} = 150$ GeV, $M_3 = 600$ GeV and for the sfermion sector, $M_{\tilde{L}_L} = M_{\tilde{e}_R} = M_{\tilde{Q}_L} = M_{\tilde{q}_R} = 300$ GeV, $A_e = A_d = 900$ GeV, and $A_u = 100$ GeV. Figure 2 shows our findings. The results obtained by using the DCPR scheme is in complete agreement with Fig. 2 of the Ref. [26]. The corrections within the $A_{\tau\tau}$ scheme, $\overline{\rm DR}$ scheme, and DCPR scheme are very modest. They are largest for $m_{\chi_3^0}$, where they reach a maximum of 5 GeV, which corresponds to a mere 2.5% relative correction. The results between the $A_{\tau\tau}$ scheme and the $\overline{\rm DR}$ scheme are almost indistinguishable for all values of t_β and all three masses. The DCPR scheme is also very close to the latter schemes, a slight deviation occurs for values of t_β in excess of 30. The largest corrections are found with the MH scheme which deviates considerably from all other schemes when t_β is in excess of 30. Therefore, once again, this scheme does not look very suitable.

C. Some decays of the two charginos

We compute the full electroweak corrections to a few decays of the charginos that were considered in Ref. [35] with the help of the code GRACE-SUSY at one loop. One of the main differences between our approach and the one adopted in [35] is the definition of t_β. In [35], t_β is closely related to our MH definition. [35] works with renormalized mixing matrices apart from the case of sfermions where a shift in the angle defining the diagonalizing matrix is performed. To conduct the comparison, we take set (A) of [35] given in Table II; moreover, we have $m_{\chi_1^+} = 184.2$ GeV, $m_{\chi_2^+} = 421.2$ GeV, $m_{\chi_1^0} = 97.75$ GeV. We study also the t_β scheme dependence of the result. As we find an excellent agreement at tree level, Table III shows the tree-level result for both codes in one column. For one-loop results, the agreement is generally good when we switch to the MH scheme apart from the corrections to the $\chi^0_{3,4}$ masses where a difference is noticeable.[4] The correction to the χ_2^0 mass is quite good. The corrections to the masses are negligible especially in the $\overline{\rm DR}$ scheme and $A_{\tau\tau}$ scheme. In the one-loop corrections to the decays, this additional negligible mass correction is not taken into account in a decay such as $\tilde{\chi}_1^+ \to W^+\chi_2^0$, for example, especially because of the large mass difference between $\tilde{\chi}_2^+$ and χ_2^0. For the decays, the largest discrepancy is for $\tilde{\chi}_1^+ \to W^+\chi_1^0$ and $\tilde{\chi}_2^+ \to Z\chi_1^+$. However, we note that when this discrepancy is largest, the correction within our MH scheme deviates drastically from the prediction within the $A_{\tau\tau}$ and $\overline{\rm DR}$ schemes. The MH scheme leads, in some decays, to too large corrections. For example, for $\tilde{\chi}_1^+ \to W^+\chi_1^0$ the MH scheme gives 23% correction whereas the correction in $\overline{\rm DR}$ is only 5%. A similar observation can be made for $\tilde{\chi}_2^+ \to \tilde{\tau}_2^+\nu_\tau$, where in the $A_{\tau\tau}$ scheme the correction is ~0% whereas it reaches 24% within our MH scheme. These examples also show that for *all* decays considered in Table III the predictions of the $A_{\tau\tau}$ and $\overline{\rm DR}$ are within 2% and very often even much better. Once more, these examples show that the MH scheme is not to be recommended, we suspect strongly that the differences we find between GRACE-SUSY and SLOOPS are essentially due to the peculiar choice of the scheme based on the heavy neutral CP-even Higgs that greatly amplifies the corrections and the differences.

TABLE II. Set of supersymmetric parameters defined as set (A) in [35]. All mass parameters are in GeV.

t_β	M_{A^0}	μ		$\tilde{e}$	$\tilde{\mu}$	$\tilde{\tau}$
10.00	424.90	399.31	$M_{\tilde{L}_L}$	184.12	184.11	182.19
M_1	M_2	M_3	$M_{\tilde{l}_R}$	118.01	117.99	111.29
100.12	197.52	610.00	A_l	−398.93	−452.58	−444.84

	$\tilde{u}$	$\tilde{d}$	$\tilde{c}$	$\tilde{s}$	$\tilde{t}$	$\tilde{b}$
$M_{\tilde{Q}_L}$	565.97		565.91		453.05	
$M_{\tilde{q}_R}$	546.78	544.95	546.84	544.97	460.52	538.13
A_q	−775.58	−979.08	−784.72	−1025.74	−535.40	−938.50

D. $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$

We now turn to the full $\mathcal{O}(\alpha)$ correction to chargino production at a linear collider. We consider the same process as the one computed in [35] within GRACE-SUSY, namely, $e^+e^- \to \chi_1^+\chi_1^-(\gamma)$. We use the same set of parameters, Set (A), defined in Table II and study the energy dependence of the total cross section. The same cross section has been studied in [36,37]. The QED radiation in view of an event generator has been studied in [38]. Figure 3 shows the cross section of this process computed at tree level and also at one loop for different t_β schemes. We find excellent agreement with the results of Ref. [35], when specializing to the MH scheme. The $A_{\tau\tau}$, $\overline{\rm DR}$, and DCPR give corrections within the per-mil level and one can hardly distinguish between the three schemes. For this

[4] Note that we have found perfect agreement with Ref. [26] as concerns corrections to all $\tilde{\chi}_i^0$ ($i = 2, 3, 4$) masses in the $\overline{\rm DR}$ and DCPR scheme, see Sec. V B.

TABLE III. Some $\tilde{\chi}^+_{1,2}$ decays at tree level and at one loop with three different t_β schemes in SLOOPS compared to GRACE-SUSY for set (A) defined in Table II. Corrections to the masses of $\chi^0_{2,3,4}$ are also given.

Decays (GeV)	Tree Level	GRACE	SLOOPS MH	SLOOPS $\overline{\rm DR}$	SLOOPS $A_{\tau\tau}$
$\tilde{\chi}^+_1 \to \nu_\tau \tilde{\tau}^+_1$	3.91×10^{-2}	$3.78 \times 10^{-2}(-3\%)$	$3.79 \times 10^{-2}(-3\%)$	$4.18 \times 10^{-2}(+7\%)$	$4.15 \times 10^{-2}(+6\%)$
$\tilde{\chi}^+_1 \to \tau^+ \tilde{\nu}_\tau$	1.47×10^{-2}	$1.48 \times 10^{-2}(0\%)$	$1.47 \times 10^{-2}(0\%)$	$1.44 \times 10^{-2}(-2\%)$	$1.49 \times 10^{-2}(+1\%)$
$\tilde{\chi}^+_1 \to W^+ \tilde{\chi}^0_1$	9.65×10^{-4}	$1.28 \times 10^{-3}(+33\%)$	$1.19 \times 10^{-3}(+23\%)$	$1.01 \times 10^{-3}(+5\%)$	$1.03 \times 10^{-2}(+7\%)$
$\tilde{\chi}^+_2 \to \nu_\tau \tilde{\tau}^+_2$	1.54×10^{-1}	$1.48 \times 10^{-1}(-4\%)$	$1.40 \times 10^{-1}(-9\%)$	$1.52 \times 10^{-1}(-1\%)$	$1.51 \times 10^{-1}(-2\%)$
$\tilde{\chi}^+_2 \to \tau^+ \tilde{\nu}_\tau$	6.89×10^{-2}	$5.70 \times 10^{-2}(-17\%)$	$5.27 \times 10^{-2}(-24\%)$	$6.75 \times 10^{-2}(-2\%)$	$6.88 \times 10^{-2}(0\%)$
$\tilde{\chi}^+_2 \to W^+ \tilde{\chi}^0_1$	1.93×10^{-1}	$2.07 \times 10^{-1}(+7\%)$	$2.02 \times 10^{-1}(+5\%)$	$2.08 \times 10^{-1}(+7\%)$	$2.08 \times 10^{-1}(+7\%)$
$\tilde{\chi}^+_2 \to W^+ \tilde{\chi}^0_2$	8.67×10^{-1}	$9.93 \times 10^{-1}(+15\%)$	$9.75 \times 10^{-1}(+12\%)$	$8.75 \times 10^{-1}(+1\%)$	$8.80 \times 10^{-1}(+1\%)$
$\tilde{\chi}^+_2 \to Z \tilde{\chi}^+_1$	7.53×10^{-1}	$8.56 \times 10^{-1}(+14\%)$	$8.06 \times 10^{-1}(+7\%)$	$7.64 \times 10^{-1}(+1\%)$	$7.68 \times 10^{-1}(+2\%)$
Neutralino masses (GeV)					
χ^0_2	184.55	184.62	184.60	184.44	184.46
χ^0_3	405.14	398.30	405.93	407.51	407.38
χ^0_4	420.49	413.39	420.23	419.54	419.60

process and with Set (A), the MH scheme gives systematically about -1 to -1.5% difference from the other schemes which is very small compared to the discrepancies we have noted for some decays of the charginos with the same set of parameters. This suggests that the t_β scheme dependence is quite small and explains why our results for this process agree very well with those of GRACE-SUSY. In any case, over the whole range of energies, the full $\mathcal{O}(\alpha)$ corrections in the $\overline{\rm DR}$ scheme amounts to about -9% for a center of mass energy $\sqrt{s} = 500$ GeV reaching a maximum of about -7% at $\sqrt{s} = 700$ GeV and dropping to about -11% at $\sqrt{s} = 1300$ GeV.

E. $e^+e^- \to \tilde{\tau}_i \bar{\tilde{\tau}}_j$

$e^+e^- \to \tilde{\tau}_1\bar{\tilde{\tau}}_1$, $\tilde{\tau}_2\bar{\tilde{\tau}}_2$, $\tilde{\tau}_1\bar{\tilde{\tau}}_2$ have been calculated in Refs. [39–41]. In Refs. [39,40], only the electroweak non-QED corrections are computed, the QED corrections are dismissed on a diagrammatic level by leaving out one-loop Feynman diagrams with virtual photon exchange. In Ref. [41], the full $\mathcal{O}(\alpha)$ is performed with a resummation of the leading log QED corrections within a structure function approach for the universal initial state radiation. We perform here a complete $\mathcal{O}(\alpha)$ calculation of these processes and compare our results to those of [40] as it concerns the electroweak non-QED corrections. We therefore take scenario 1 of [40] with the following set of parameters: $t_\beta = 20$, $\mu = 1000$ GeV, $M_1 = 94.92$ GeV, $M_2 = 200$ GeV, $M_3 = 669.18$ GeV, $M_{A^0} = 300$ GeV, $M_{\tilde{L}_L} = M_{\tilde{e}_R} = M_{\tilde{Q}_L} = M_{\tilde{u}_R,\tilde{d}_R} = 400$ GeV, $A_f = -500$ GeV, $M_{\tilde{t}_R} = 360$ GeV, and $M_{\tilde{b}_R} = 440$ GeV. In [40], the electromagnetic coupling is not taken in the Thomson limit but is fixed from $\alpha^{\overline{\rm MS}}(M_Z^2)$ with $\alpha^{\overline{\rm MS}}(M_Z^2) = 1/127.934$. This

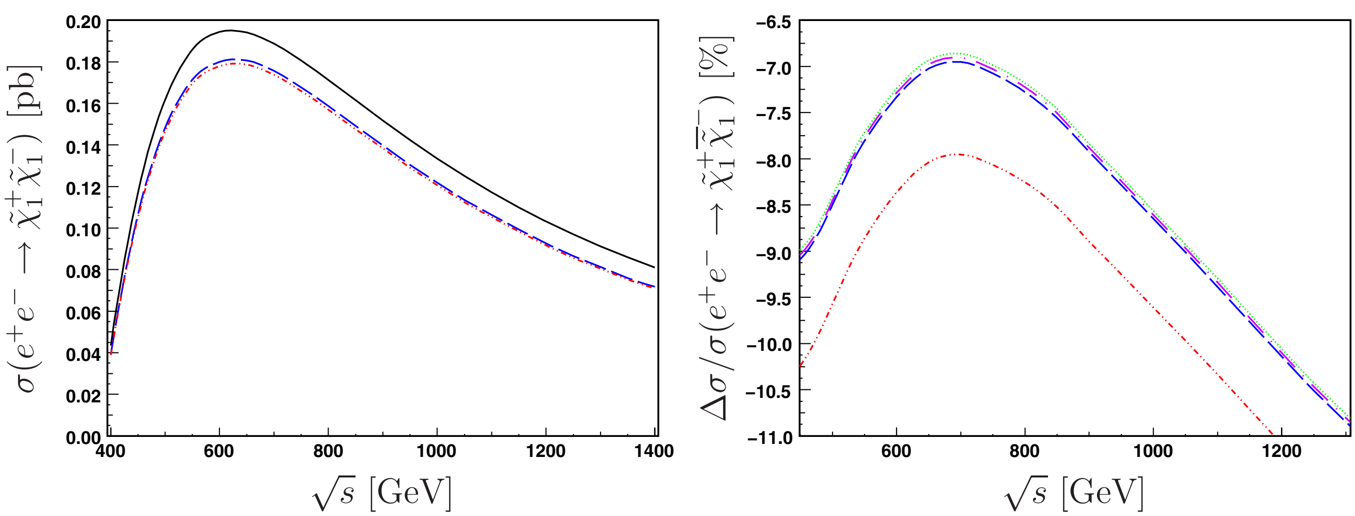

FIG. 3 (color online). Total cross section of $e^+e^- \to \chi^+_1\chi^-_1(\gamma)$ as a function of $\sqrt{s}$ at tree level (solid/black line) and at one loop (full order $\mathcal{O}(\alpha)$) in the $A_{\tau\tau}$ scheme (dashed/blue line), the $\overline{\rm DR}$ scheme (dotted/light green line), the DCPR scheme (dash-dotted/purple line), and the MH scheme (dash-dot-dotted/red line). The right panel gives the percentage correction. In the left panel considering the $A_{\tau\tau}$ scheme, the $\overline{\rm DR}$ and the DCPR scheme are not distinguishable and we therefore only show the result of the $A_{\tau\tau}$ scheme beside the tree level and the MH scheme.

absorbs large logarithms compared to our on shell scheme based on $\alpha(0) = 1/137.036$. In [40], the mixing parameter in the stau sector is parametrized through the mixing angle which is renormalized according to Eq. (3.38). For the sake of comparison, we will here also switch to this scheme for the sfermion mixing.

In addition to the tree level cross section calculated with $\alpha(0) = 1/137.036$ and the complete $\mathcal{O}(\alpha)$ one-loop correction, we compute the improved tree-level cross section with $\alpha^{\overline{\rm MS}}(M_Z^2) = 1/127.934$. Our evaluation of the weak non-QED correction is different from the one in [40]. In our case, the weak correction is obtained by subtracting the leading QED corrections. The initial state radiation factor, including the virtual photon correction and the soft bremsstrahlung photon below the cutoff energy k_c, is universal and known, see, for example, [42],

$$\delta_{V+S}^{\rm QED} = \frac{2\alpha}{\pi}\left((L_e - 1)\ln\frac{k_c}{E_b} + \frac{3}{4}L_e + \frac{\pi^2}{6} - 1\right), \quad L_e = \ln(s/m_e^2), \tag{5.1}$$

where m_e is the electron mass and E_b the beam energy $s = 4E_b^2$. To subtract not only the initial but also the final state radiation and the final-initial interference QED effect, we take the result of the virtual one-loop correction and the soft radiation factor obtained by the code and subtract the following:

$$\sigma^{\rm weak}(\sqrt{s}) = \sigma^{\rm virtual+soft}(\sqrt{s}, k_c) - \frac{\alpha}{\pi}A(\sqrt{s})\ln\left(\frac{2k_c}{\sqrt{s}}\right) - \frac{3\alpha}{2\pi}\sigma^{\rm tree}(\sqrt{s})\ln\left(\frac{s}{m_e^2}\right). \tag{5.2}$$

The last term in Eq. (5.2) stems from the collinear singularity due to initial state radiation and we neglect nonlog terms, the latter that arise from initial radiation are negligible of order 0.3% relative correction. The term $A(\sqrt{s})$ is extracted numerically based on the fact that the weak non-QED correction is independent of the cutoff k_c. We take two small enough cutoff k_{c_1}, k_{c_2} to extract $A(\sqrt{s})$,

$$\frac{\alpha}{\pi}A(\sqrt{s}) = \frac{\sigma^{\rm virtual+soft}(\sqrt{s}, k_{c_1}) - \sigma^{\rm virtual+soft}(\sqrt{s}, k_{c_2})}{\ln(\frac{k_{c_2}}{k_{c_1}})}. \tag{5.3}$$

We have checked that $\sigma^{\rm weak}(\sqrt{s})$ defined this way is independent of the cutoff k_c by taking other values of k_c. Such a definition of the weak correction has been introduced in [43].

Our tree-level results for the improved tree level with $\alpha = \alpha^{\overline{\rm MS}}(M_Z^2)$ reproduces the corresponding cross section in Ref. [40] perfectly.

To help compare our results with those Ref. [40], the right panel of Fig. 4 shows also the relative weak non-QED correction with $\alpha^{\overline{\rm MS}}(M_Z^2)$ as input rather than $\alpha(0)$, hence subtracting large logs from the running of α. Our predictions for the weak correction defined this way are within 1% of those in [40] within the DCPR scheme used in [40].

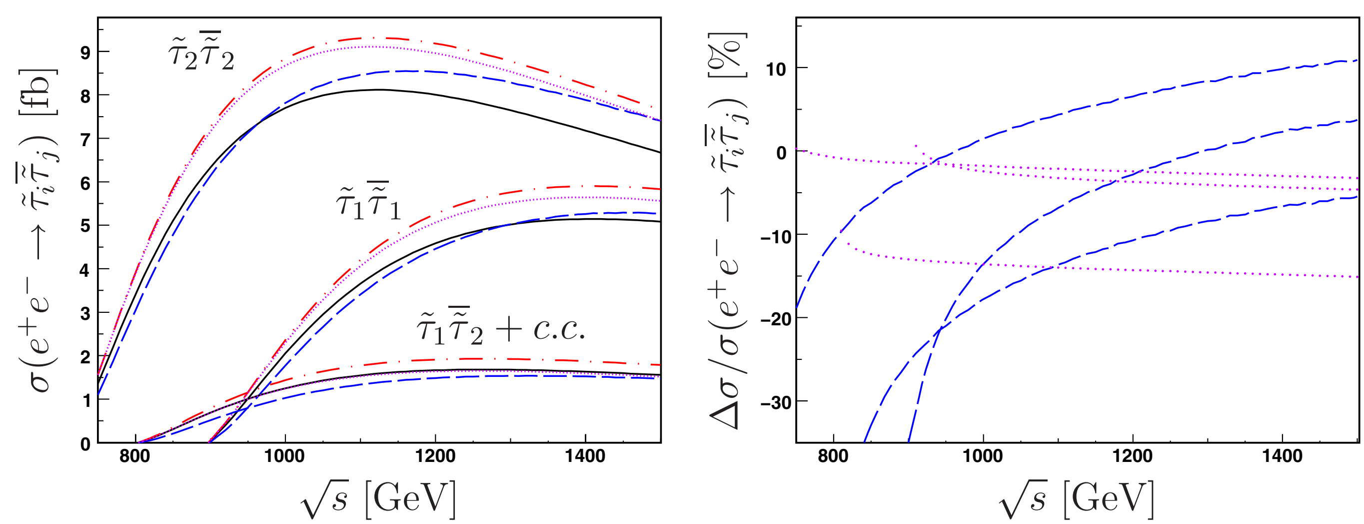

FIG. 4 (color online). Total cross section of $e^+e^- \to \tilde{\tau}_i\bar{\tilde{\tau}}_j(\gamma)$ as a function of $\sqrt{s}$ at tree level (solid/black line) and at full one loop in the DCPR scheme (dashed/blue line). We also show the tree level improved cross section with $\alpha^{\overline{\rm MS}}$ (M_Z^2) (dash-dotted/red line) and the pure weak correction in the on-shell scheme as defined in the text (dotted/purple line). The full $\mathcal{O}(\alpha)$ relative corrections for the three channels with respect to the tree-level cross sections with $\alpha(0)$ is shown in the panel on the right (dashed/blue line). We also show the weak non-QED relative correction (dotted/purple line) where the improved tree-level cross section with $\alpha^{\overline{\rm MS}}$ (M_Z^2) is used to absorb large logs from the running of α. This correction should be contrasted to the one obtained in Ref. [40]. In order not to crowd the figure, the channels are not labeled. They can be easily identified as they have different thresholds.

We have traced this small difference to the different ways the weak correction is defined from the subtraction of the QED corrections. The energy dependence of the weak corrections matches perfectly.

We can now comment on the t_β scheme dependence and the sfermion mixing renormalization scheme. The corrections induced by the different t_β schemes are very small. Even the MH scheme departs by no more than 0.3% from the $\overline{\rm DR}$. The other schemes, DCPR and $A_{\tau\tau}$, agree within better than 0.01% with $\overline{\rm DR}$. The difference in the choice of the sfermion mixing parameter $\delta m^2_{\tilde{f}_{12}}$ is even more negligible here. For example, for a center of mass energy $\sqrt{s} = 1000$ GeV, the one-loop correction to the process $e^+e^- \to \tilde{\tau}_1\bar{\tilde{\tau}}_2$ differs only about 0.003% when we switch from the default definition in SLOOPS Eq. (3.37) to the one that has been usually used Eq. (3.38).

These calculations show that not only it is important to take into account the QED corrections but also that the pure electroweak corrections are certainly not negligible, for example, even after absorbing the effect due to the running of α, the weak corrections for $\tilde{\tau}_1\bar{\tilde{\tau}}_2$ production is about -15%.

VI. CONCLUSIONS

We have presented in detail a complete renormalization of the sfermion sector as well as of the chargino/neutralino sector of the MSSM in the case of *CP* conservation. We critically analyzed the renormalization of the mixing parameter in the sfermion sector and discussed different ways to define it in a consistent manner. This paper is a sequel to our study in Ref. [9] and completes the presentation of all the ingredients that are built into our automatized code for one-loop calculations in the MSSM, SLOOPS. Although other approaches to renormalizing the MSSM have been worked out, we believe that our approach treats all the sectors consistently within the same general on-shell framework, in particular, about the treatment of mixing and how one deals with the rotation and diagonalizing matrices. Moreover, our code permits powerful gauge checks with the help of the nonlinear gauge-fixing condition and allows us to easily switch between different renormalization schemes. Some very powerful and extensive tests have been conducted on the code as concerns ultraviolet finiteness and gauge parameter independence on an almost exhaustive list of $2 \to 2$ processes, see [32]. In the present paper, we choose to concentrate on a few key observables in the sfermion and chargino/neutralino sector and compared our results with some that are found in the literature while at the same time studying the impact of different renormalization schemes. We have calculated one-loop corrections to sfermion masses and also neutralino masses. We have also derived some chargino decay widths and presented a calculation of the production of charginos and sleptons at e^+e^- colliders. We find the genuine electroweak corrections in these cross sections to be rather important and should therefore be taken into account. Having at our disposal a code that allows the one-loop calculation for any process in the MSSM, it is now possible to envisage revisiting analyses for the extraction of the fundamental supersymmetric parameters from precision measurements at the colliders and use them in turn for a precision calculation of the relic density, for example. Finally, let us mention that other renormalization schemes, with different choices of the input parameters from the one described in this paper, for the chargino/neutralino sector are already implemented in the code and would be part of a forthcoming study. Although in the many examples we have shown here the QCD corrections are calculated, a complete treatment of the gluon/gluino sector within an automated code such as SLOOPS and, in particular, how to easily implement within the code a regulator for the infrared singularity is work in progress.

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