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Composite Higgs-mediated flavor-changing neutral current

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We discuss how, in the presence of higher-dimensional operators, the standard model fermion masses can be misaligned in flavor space with the Yukawa couplings to the Higgs boson, even with only one Higgs doublet. Such misalignment results in flavor-violating couplings to the Higgs and hence flavor-changing neutral current processes from tree-level Higgs exchange. We perform a model-independent analysis of such an effect. Specializing to the framework of a composite Higgs with partially composite standard model gauge and fermion fields, we show that the constraints on the compositeness scale implied by ϵ_K can be generically as strong as those from the exchange of heavy spin-1 resonances if the Higgs is light and strongly coupled to the new states. In the special and well-motivated case of a composite pseudo-Goldstone Higgs, we find that the shift symmetry acting on the Higgs forces an alignment of the fermion mass terms with their Yukawa couplings at leading order in the fermions' degree of compositeness, thus implying much milder bounds. As a consequence of the flavor-violating Higgs couplings, we estimate $\mathrm{BR}(t \to ch) \sim 10^{-4}$ and $\mathrm{BR}(h \to tc) \sim 5 \times 10^{-3}$ both for a pseudo-Goldstone (if t_R is fully composite) and for a generic composite Higgs. By virtue of the AdS/CFT correspondence, our results directly apply to 5-dimensional Randall-Sundrum compactifications.

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I. INTRODUCTION

According to the minimal description of the electroweak symmetry breaking (EWSB) in the standard model (SM), the Yukawa couplings of the up and down quarks to the Higgs boson are exactly aligned in flavor space with their mass terms, so that no flavor violation can arise mediated by the Higgs. There are however serious reasons to think of the standard model as an effective theory with a ~TeV cutoff, and this simple picture could be dramatically modified by the new physics. The leading flavor-violating contribution in the Higgs couplings to fermions can be parametrized by dimension-6 operators in the effective Lagrangian with more powers of the Higgs doublet than at the renormalizable level.

At first sight the Higgs contribution to $\Delta F = 2$ neutral currents is subdominant compared to that originating from generic dimension-6 four-fermion operators, as, for example, those arising from the exchange of new heavy vectors, since it requires two flavor-violating Higgs vertices, and as such it naively corresponds to a dimension-8 effect. However, as we will show in this paper, if the Higgs boson is light and strongly coupled to the new dynamics then its contribution can be comparable to dimension-6 effects and imply strong bounds on the scale of the new states. In this limit it will also parametrically dominate over $\Delta F = 2$ contributions generated through the exchange of the Z boson, which is light yet weakly coupled to the new physics. The Higgs contribution to $\Delta F = 1$ transitions will instead be largely negligible compared to the Z exchange, as further suppressed by a Yukawa coupling factor at the flavor-preserving vertex.

A strongly coupled light Higgs can naturally arise as the composite pseudo-Goldstone boson (PGB) of the new dynamics responsible for the electroweak symmetry breaking [1]. This possibility has recently attracted much attention as it resolves the Planck-weak hierarchy problem of the standard model while still being compatible with the precision tests performed at LEP (see, for example, [2]). In addition, if the SM quarks couple linearly to the new dynamics through composite fermionic operators [3], then the hierarchies in their masses and mixing angles can also be elegantly explained as the effect of a renormalization group evolution using only mild differences in the scaling dimensions of the operators. Remarkably, this scenario is explicitly realized in the 4D duals of 5D warped compactifications [4–7] as per the AdS/CFT correspondence [8,9]. Numerous theoretical studies on flavor-violating processes in such 5D realizations of composite Higgs models (and their 4D deconstructions) have been performed, mainly focusing on $\Delta F = 2$ transitions mediated by the tree-level exchange of heavy vectors or on $\Delta F = 1$ rare decays arising at tree- and one-loop levels; see [10–14] and references therein.

In this paper we show that the tree-level exchange of the Higgs boson can lead to quite strong constraints in $\Delta F = 2$ processes. In the following sections we first present a general model-independent operator analysis, then we specialize to the case of a composite Higgs (assuming linear couplings of the SM quarks to the strong dynamics), with a dedicated analysis of the pseudo-Goldstone limit.

Flavor-violating Higgs couplings were independently investigated in Refs. [13–15] in the context of specific

5D Randall-Sundrum models with a (non-PGB) Higgs doublet localized on or at the vicinity of the infrared brane. In both constructions, as a consequence of the Higgs localization, the effects were found to be small. In either case a full, general analysis of Higgs-mediated $\Delta F = 2$ neutral currents was not carried through. Finally, similar flavor-violating effects from the Higgs sector have been considered in [16,17], although in a different context.

II. MODEL-INDEPENDENT ANALYSIS OF HIGHER-DIMENSIONAL OPERATORS

In this section we present a model-independent analysis of the flavor-violating effects induced by the tree-level Higgs exchange, and derive the corresponding experimental constraints.[1] We focus on the down-type quark sector since it gives the strongest constraints, and leave to the reader the straightforward generalization to other sectors. At the level of dimension 6, there are four operators in the effective Lagrangian for a light Higgs doublet H which can lead to flavor-violating couplings to the SM down-type quarks. The first is a nonderivative operator

$$O_y = \frac{\tilde{y}^d_{ij}}{\Lambda^2}\bar{q}_{Li} H d_{Rj}(H^\dagger H) + \text{H.c.}, \tag{1}$$

while the other three are Higgs-dependent modifications of the quark kinetic terms:

$$\begin{aligned} O_q &= \frac{\kappa^q_{ij}}{\Lambda^2}\bar{q}_{Li} i\not{D} q_{Lj}(H^\dagger H) + \text{H.c.}, \\ O'_q &= \frac{\kappa'^q_{ij}}{\Lambda^2}\bar{q}_{Li} H H^\dagger i\not{D} q_{Lj} + \text{H.c.}, \\ O_d &= \frac{\kappa^d_{ij}}{\Lambda^2}\bar{d}_{Ri} i\not{D} d_{Rj}(H^\dagger H) + \text{H.c.} \end{aligned} \tag{2}$$

Here i, j denote generation indices, $\tilde{y}_{ij}$ and κ_{ij} are generic complex coefficients, and Λ stands for the mass scale of the new physics. Notice that the additional independent operators

$$\begin{aligned} &\bar{q}_{Li}\gamma^\mu q_{Lj}(H^\dagger i\overleftrightarrow{D}_\mu H) + \text{H.c.}, \\ &\bar{q}_{Li}\gamma^\mu T^a q_{Lj}(H^\dagger i\overleftrightarrow{D}_\mu T^a H) + \text{H.c.}, \\ &\bar{d}_{Ri}\gamma^\mu d_{Rj}(H^\dagger i\overleftrightarrow{D}_\mu H) + \text{H.c.}, \end{aligned} \tag{3}$$

with $H^\dagger \overleftrightarrow{D}_\mu H \equiv H^\dagger D_\mu H - (D_\mu H)^\dagger H$ do not modify the couplings of the Higgs boson (though they do modify the couplings of the Z), and are thus not relevant here.

As noticed in [20], the operators of Eq. (2) can be rewritten in terms of O_y by means of a field redefinition,[2] so that

$$\tilde{y}^{\,d}_{ij} \to \tilde{y}^d_{ij} + (\kappa^q \cdot y^d)_{ij} + (\kappa'^q \cdot y^d)_{ij} + (y^d \cdot \kappa^{d\dagger})_{ij}, \tag{4}$$

where y^d is the quark down Yukawa matrix. It is however convenient to distinguish between O_y and the derivative operators in Eq. (2), as different spurionic transformation rules can be assigned to their coefficients

$$\begin{aligned} &y^d \to V_L y^d V_R^\dagger, \qquad \tilde{y}^d \to V_L \tilde{y}^d V_R^\dagger, \qquad \kappa^q \to V_L \kappa^q V_L^\dagger, \\ &\qquad \kappa'^q \to V_L \kappa'^q V_L^\dagger, \qquad \kappa^d \to V_R \kappa^d V_R^\dagger, \end{aligned} \tag{5}$$

under flavor $SU(3)_{L,R}$ rotations $q_{Li} \to (V_L)_{ij} q_{Lj}$, $d_{Ri} \to (V_R)_{ij} d_{Rj}$. In particular, we will see in Sec. IV that in theories where the Higgs is a pseudo-Goldstone boson, $\tilde{y}^d$ can be naturally aligned with y^d [so that the only flavor spurion transforming as a (3, 3) under $SU(3)_L \times SU(3)_R$ is the Yukawa coupling]. In this case flavor-violating effects arise only from the derivative operators.

By use of Eq. (4), after expanding around the EWSB vacuum [$H^0 = v + h(x)$, with $v = 174$ GeV], and keeping at most terms linear in the Higgs field $h(x)$, the effective Lagrangian involving down-type quarks reads

$$\begin{aligned} \mathcal{L} = {} & \bar{d}_L i\not{\partial} d_L + \bar{d}_R i\not{\partial} d_R - v\bar{d}_{Li} d_{Rj} \\ & \times\left[y^d_{ij} - (\tilde{y}^d + \kappa^q y^d + \kappa'^q y^d + y^d \kappa^{d\dagger})_{ij}\frac{v^2}{\Lambda^2}\right] \\ & - h\bar{d}_{Li} d_{Rj}\left[y^d_{ij} - 3(\tilde{y}^d + \kappa^q y^d + \kappa'^q y^d + y^d\kappa^{d\dagger})_{ij}\frac{v^2}{\Lambda^2}\right] \\ & + \text{H.c.} \end{aligned} \tag{6}$$

The down quark mass matrix,

$$m^d_{ij} = y^d_{ij} v - (\tilde{y}^d + \kappa^q y^d + \kappa'^q y^d + y^d \kappa^{d\dagger})_{ij}\frac{v^3}{\Lambda^2}, \tag{7}$$

can be diagonalized as usual through a bi-unitary transformation, $q_{Li} \to (D_L)_{ij} q_{Lj}$ and $d_{Ri} \to (D_R)_{ij} d_{Rj}$, so that $m^d_i \delta_{ij} = (D_L^\dagger m^d D_R)_{ij}$. In this mass-eigenstate basis the couplings to the physical Higgs boson h are not flavor diagonal:

$$\begin{aligned} \mathcal{L} = {} & \bar{d}_L i\not{\partial} d_L + \bar{d}_R i\not{\partial} d_R - \bar{d}_{Li} m^d_i d_{Ri} \\ & - h\bar{d}_{Li} d_{Rj}\left(\frac{m^d_i}{v}\delta_{ij} - \hat{y}^d_{ij}\frac{v^2}{\Lambda^2}\right) + \text{H.c.}, \end{aligned} \tag{8}$$

where we have defined

$$\begin{aligned} \hat{y}^d_{ij} \equiv {} & -2[D_L^\dagger \cdot \tilde{y}^d \cdot D_R]_{ij} - 2[D_L^\dagger \cdot (\kappa^q + \kappa'^q)\cdot D_L]_{ij}\frac{m^d_j}{v} \\ & - 2\frac{m^d_i}{v}[D_R^\dagger \cdot \kappa^{d\dagger} \cdot D_R]_{ij} \end{aligned} \tag{9}$$

and neglected higher-order terms in (v/Λ). As a consequence, the tree-level exchange of the Higgs boson will generate $\Delta F = 2$ transitions at low energy.

[1] Similar operator analyses can be found in [18–20], though no experimental bound was there derived.

[2] Such field redefinition is equivalent to using the classical equations of motion upon the higher-dimensional operators [21]. We thank J. A. Aguilar-Saavedra for a clarifying discussion on this point.

To illustrate the importance of the effect we consider, for example, the contribution to $K\bar{K}$ mixing: by integrating out the Higgs boson one obtains the low-energy Lagrangian (α denotes a color index)

$$\mathcal{L}_{\Delta S=2} = \mathcal{O}_2 C_2 + \tilde{\mathcal{O}}_2 \tilde{C}_2 + \mathcal{O}_4 C_4 + \text{H.c.}, \tag{10}$$

$$\mathcal{O}_4 \equiv (\bar{s}^\alpha_L d_{R\alpha})(\bar{s}^\alpha_R d_{L\alpha}), \qquad \mathcal{O}_2 \equiv (\bar{s}^\alpha_R d_{L\alpha})^2, \qquad \tilde{\mathcal{O}}_2 \equiv (\bar{s}^\alpha_L d_{R\alpha})^2, \tag{11}$$

with

$$(C_4, C_2, \tilde{C}_2) = \frac{1}{m_h^2}\left(\frac{v^2}{\Lambda^2}\right)^2 \left(\hat{y}^d_{12}\hat{y}^{d*}_{21}, \frac{1}{2}(\hat{y}^{d*}_{12})^2, \frac{1}{2}(\hat{y}^d_{21})^2\right). \tag{12}$$

Since these 4-fermion operators are generated through the exchange of the Higgs, we must then apply the experimental constraint on the Wilson coefficients C_i renormalized at the Higgs mass scale. Using the RGE equations from Ref. [22] to evolve the experimental constraints reported by the UTFit Collaboration [23], and choosing a reference Higgs mass $m_h = 200$ GeV,[3] we find

$$\text{Im}(C_4, C_2, \tilde{C}_2)(\mu = m_h = 200\ \text{GeV}) \lesssim \frac{1}{\{(1.4, 0.72, 0.72) \times 10^5\ \text{TeV}\}^2}. \tag{13}$$

By turning on one operator at a time (which gives a rough account of the global constraint if these contributions are uncorrelated), the above bound implies

$$\Lambda \gtrsim (145, 88, 88)\ \text{TeV}\sqrt{\frac{200\ \text{GeV}}{m_h}} \cdot [\text{Im}(\hat{y}^d_{12}\hat{y}^{d*}_{21}, (\hat{y}^{d*}_{12})^2, (\hat{y}^d_{21})^2)]^{1/4}. \tag{14}$$

Following the same steps as above, it is straightforward to derive the analogous bounds on Λ from the $B_d\bar{B}_d$ and $B_s\bar{B}_s$ systems. We leave all the intermediate formulas to the Appendix and quote here only the final results: we find

$$\Lambda \gtrsim (15, 10, 10)\ \text{TeV}\sqrt{\frac{200\ \text{GeV}}{m_h}} \cdot [\text{Im}(\hat{y}^d_{13}\hat{y}^{d*}_{31}, (\hat{y}^{d*}_{13})^2, (\hat{y}^d_{31})^2)]^{1/4} \tag{15}$$

from $B_d\bar{B}_d$ mixing, and

$$\Lambda \gtrsim (5.2, 3.4, 3.4)\ \text{TeV}\sqrt{\frac{200\ \text{GeV}}{m_h}} \cdot [\text{Im}(\hat{y}^d_{23}\hat{y}^{d*}_{32}, (\hat{y}^{d*}_{23})^2, (\hat{y}^d_{32})^2)]^{1/4} \tag{16}$$

from $B_s\bar{B}_s$ mixing.

[3]The bounds are only logarithmically sensitive to the renormalization scale.

III. FLAVOR VIOLATION FROM A COMPOSITE HIGGS

The constraint on the new physics scale Λ derived above can be made more explicit by indicating the origin of the higher-dimensional operators in Eqs. (1) and (2). Here we consider the motivated scenario in which the Higgs doublet arises as a bound state of a new strongly interacting dynamics. We assume that the SM fermions are linearly coupled to the strong sector through composite operators $O_{L,R}$ [3]

$$\lambda_L \bar{\psi}_L O_R + \lambda_R \bar{\psi}_R O_L + \text{H.c.}, \tag{17}$$

like in the 4D duals of 5D warped compactifications. The SM Yukawa term and the higher-order operators of Eqs. (1) and (2) then arise at low energy by expanding the two-point Green functions of $O_{L,R}$ in powers of the Higgs field.

The coefficients $\tilde{y}^d$, κ and the SM Yukawa coupling y^d can be estimated by means of naive dimensional analysis (NDA) as follows:

$$y^d \sim y_* \frac{\lambda_L \lambda_R}{16\pi^2}, \qquad \tilde{y}^d \sim y_*^3 \frac{\lambda_L \lambda_R}{16\pi^2}, \qquad \kappa^q, \kappa'^q \sim y_*^2 \frac{\lambda_L^2}{16\pi^2}, \qquad \kappa^d \sim y_*^2 \frac{\lambda_R^2}{16\pi^2}, \tag{18}$$

where the coupling of the composite Higgs to the other strong states, y_*, can be as large as 4π, this case corresponding to a maximally strongly coupled dynamics.

We further assume that the hierarchy in the SM Yukawa couplings y^d_{ij} entirely originates from the hierarchy in the couplings $\lambda_{L,R}$, as, for example, the effect of their RG evolution, whereas the strong sector is substantially flavor anarchic. This has been dubbed in the literature as the "anarchic scenario," and has been studied extensively in the 5D warped framework; see [10–14] and references therein. Then, making the flavor structure explicit, the matrix $\tilde{y}^d$ will have the same hierarchical structure of y^d in flavor space, but it will not be exactly aligned with it in general:

$$\tilde{y}^d_{ij} = y_*^2 a_{ij} \times y^d_{ij} \qquad (\text{no sum over } i, j), \tag{19}$$

where a_{ij} is an anarchic matrix with $O(1)$ entries. By applying the above estimates to Eq. (9) one finds, in the mass-eigenstate basis

$$\hat{y}^d_{ij} \sim \frac{2y_*^2}{16\pi^2}\Big[y_*(D_L^\dagger)_{il}(\lambda_L)_l(\lambda_R)_m(D_R)_{mj} + (D_L^\dagger)_{il}(\lambda_L)_l(\lambda_L)_m(D_L)_{mj}\frac{m^d_j}{v} + \frac{m^d_i}{v}(D_R^\dagger)_{il}(\lambda_R)_l(\lambda_R)_m(D_R)_{mj}\Big], \tag{20}$$

where we have omitted $O(1)$ factors. As it will be explicitly shown in the next section, the second and third terms, which have their origin in the derivative operators, are always subdominant or at best of the same order of the first term, which arises from the operator O_y.[4]

We thus concentrate on the dominant effect from O_y and drop the last two terms of Eq. (20). As a final simplification, we assume that the left rotation matrix has entries of the same order as those of the Cabibbo-Kobayashi-Maskawa (CKM) matrix:

$$(D_L)_{ij} \sim (V_{\rm CKM})_{ij}. \tag{21}$$

This in turn, combined with the anarchy assumption and the estimate of the Yukawa matrix in Eq. (18), fixes the form of the right couplings λ_R and rotation matrix D_R:

$$(\lambda_R)_i \sim \frac{m_i}{y_* v (\lambda_L)_i}, \qquad (D_R)_{ij} \sim \left(\frac{m_i}{m_j}\right)\frac{1}{(D_L)_{ij}} \quad \text{for } i < j. \tag{22}$$

Using the above estimates and specializing to $K\bar{K}$ mixing, we find that ($\lambda_C = 0.22$ denotes the Cabibbo angle)

$$\hat{y}^{\,d}_{12} \sim 2y_*^2 \frac{m_s}{v}\lambda_C, \tag{23}$$

which arises as the result of several equally important terms in the sum over l, m of Eq. (20), with flavor violation arising either from the vertex $\tilde{y}^d$, or from the rotation to the mass eigenstates basis, or from both. A similar estimate can be derived for $\hat{y}^d_{21}$. Using $m_{d,s} = 3$, 65 MeV, i.e. the value of the quark masses renormalized at the Higgs mass scale $m_h = 200$ GeV, and assuming $O(1)$ CP-violating phases, one has

$$\Lambda \gtrsim (1.9, 1.1, 1.1)\ \text{TeV} \times y_* \times \sqrt{\frac{200\ \text{GeV}}{m_h}}. \tag{24}$$

Considering that Λ should be identified with the mass scale of the fermionic resonances of the strong sector, and that y_* can in principle be as large as 4π, the above constraint is rather strong.

It is interesting to compare with the constraints on $\Delta F = 2$ flavor-changing neutral currents (FCNCs) that arise from the tree-level exchange of heavy colored vectors (such as KK or composite gluons) and from the Z boson. In the latter case the leading new physics effects are encoded by the dimension-6 operators in Eq. (3). After matching to the four-fermion low-energy effective Lagrangian, a rough NDA estimate of the size of a generic Wilson coefficient (evaluated at the matching scale and before rotating to the mass-eigenstate basis) gives

$$C_i(m_h) \sim \frac{y_*^2}{m_h^2}\left(\frac{\lambda^2}{16\pi^2}\right)^2\left(\frac{y_*^2 v^2}{\Lambda^2}\right)^2 \quad \text{from Higgs exchange,}$$
$$C_i(M_Z) \sim \frac{g^2}{M_Z^2}\left(\frac{\lambda^2}{16\pi^2}\right)^2\left(\frac{g_*^2 v^2}{\Lambda^2}\right)^2 \quad \text{from } Z \text{ exchange,} \tag{25}$$
$$C_i(M_*) \sim \frac{g_*^2}{\Lambda^2}\left(\frac{\lambda^2}{16\pi^2}\right)^2 \quad \text{from heavy vector exchange.}$$

Here g_* ($\sim y_*$) denotes the typical coupling of the heavy resonances of the strong sector, while λ stands for λ_L or λ_R. The first estimate, in particular, agrees with the more refined one in Eq. (20). This shows that the Z exchange is always suppressed by a factor $(g_*^2 v^2/\Lambda^2)$ compared to the heavy vector exchange, and this is the reason why it has been usually neglected in the literature. The Higgs exchange, on the other hand, while suffering from the same suppression, has a further enhancement factor $(y_*^2 v^2/m_h^2)$ which is large if the Higgs is light and strongly coupled to the new states. In this way it can become comparable to the genuine dimension-6 effects.

In the case of $\Delta S = 2$ transitions the dominant contribution from heavy vectors to ϵ_K is from C_4, and it leads to the following bound on the heavy vector mass [assuming an $O(1)$ CP-violating phase] [11,12,25]:

$$\Lambda \gtrsim 10\ \text{TeV} \times \left(\frac{g_*}{y_*}\right)^2. \tag{26}$$

This is comparable with the bound on the new physics scale of Eq. (24) from the Higgs exchange for $y_* \sim g_* \sim 5$ and $m_h = 200$ GeV.[5] Furthermore, the above constraint becomes weaker by making the coupling of the heavy vector g_* smaller, or y_* larger.[6] The bound of Eq. (23), instead, becomes stronger for larger y_*, and in this sense the two are complementary.

The constraints that follow from $\Delta B = 2$ processes due to the Higgs exchange are less severe than those obtained above from CP violation in $K\bar{K}$ mixing. Under the same assumptions which led to Eq. (23) and using $m_b = 3$ GeV (renormalized at the Higgs mass scale $m_h = 200$ GeV), one finds

$$\hat{y}^{\,d}_{13} \sim 2y_*^2\frac{m_b}{v}\lambda_C^3, \qquad \hat{y}^d_{31} \sim 2y_*^2\frac{m_b}{v}\left(\frac{m_d}{m_b\lambda_C^3}\right), \tag{27}$$

$$\hat{y}^{\,d}_{23} \sim 2y_*^2\frac{m_b}{v}\lambda_C^2, \qquad \hat{y}^d_{32} \sim 2y_*^2\frac{m_b}{v}\left(\frac{m_s}{m_b\lambda_C^2}\right). \tag{28}$$

[4]The occurrence of nonuniversal shifts in the Higgs couplings as a consequence of corrections to the SM fermion kinetic terms was already noticed in Ref. [2] in the general context of a composite Higgs. Formulas for the modified Higgs couplings analogous to the last two terms and the first term of Eq. (20) were reported, respectively, in Refs. [13,15] for the case of specific 5D Randall-Sundrum models with the Higgs localized on or at the vicinity of the infrared brane. See Ref. [24] for a further discussion on the effect of the first term in both bulk and brane Higgs 5D scenarios.

[5]Notice however, that in principle the two new physics scales entering Eqs. (24) and (26) might be different, as they naively correspond to the mass of, respectively, the heavy vectorial and fermionic resonances.

[6]For fixed SM Yukawa couplings this latter limit is equivalent to making $(\lambda_L\lambda_R)$ smaller.

After substituting these expressions in Eqs. (15) and (16) and assuming $O(1)$ phases, one obtains the following constraints:

$$\Lambda \gtrsim (480, 190, 570)\ \mathrm{GeV} \times y_* \times \sqrt{\frac{200\ \mathrm{GeV}}{m_h}} \tag{29}$$

and

$$\Lambda \gtrsim (360, 135, 420)\ \mathrm{GeV} \times y_* \times \sqrt{\frac{200\ \mathrm{GeV}}{m_h}}, \tag{30}$$

respectively, from $B_d\bar{B}_d$ and $B_s\bar{B}_s$ mixings.

IV. PGB COMPOSITE HIGGS AND FLAVOR ALIGNMENT

There is an important class of composite Higgs models where the strong constraint on Λ derived in the previous section does not hold in general: these are theories in which the Higgs, rather than being an ordinary bound state, is a pseudo-Goldstone boson associated to the spontaneous breaking of a global symmetry G of the strong sector [1]. In that case the shift symmetry acting on the Higgs dictates the structure of the higher-order Higgs-dependent operators.

The simplest possibility is that the operators O_L, O_R in Eq. (17) have definite quantum numbers under the global symmetry G, and transform as representations r_L, r_R (for all three generations). It is possible then to "uplift" the SM fermions to complete representations r_L, r_R of G, $q_L \to \psi_L$, $d_R \to \psi_R$, the extra components being nonphysical spurionic fields. In this way the Higgs dependence of all nonderivative operators must resum to a polynomial P of the sigma model field $\Sigma = e^{ih/f}$ (where f is the analog of the pion decay constant):

$$\bar{q}^{\,i}_L H\left(y^d_{ij} + \tilde{y}^d_{ij}\frac{H^\dagger H}{\Lambda^2} + \cdots\right)d^j_R \to \bar{\psi}^i_L P_{ij}(\Sigma)\psi^j_R, \tag{31}$$

where i, j are flavor indices. The polynomial P transforms as a $r_L \times r_R$. If its projection over the physical fields q_L, d_R contains only one term [transforming as $2_{1/2}$ under $SU(2)_L \times U(1)_Y$], then the flavor dependence in the right-hand side of the above equation factorizes and all higher-order terms in the Higgs field expansion are aligned: $\bar{\psi}^i_L P_{ij}(\Sigma)d^j_R = y^d_{ij}\bar{\psi}^i_L P(\Sigma)\psi^j_R$. In particular, $\tilde{y}^d_{ij}$ is aligned with y^d_{ij} and the constraints of Eqs. (24), (29), and (30) do not hold. On the other hand, if the projection of P over q_L, d_R contains more than one term, as, for example, if any of the SM fermion is coupled (with similar strength) to more than one composite operator, then the alignment in flavor space is broken and the same bounds as for a non-PGB Higgs apply.

An explicit example will best illustrate this general result[7]: Consider a strong sector with $G = SO(5) \times U(1)_X$ spontaneously broken to $SO(4) \times U(1)_X$, where $SO(4) \sim SU(2)_L \times SU(2)_R$ and $Y = T^3_R + X$. Massless excitations around the SO(4) vacuum are parametrized by the Goldstone field $\Sigma = e^{ih/f}$, which transforms as a 5 of SO(5). Both operators O_L, O_R are $5_{-1/3}$ of $SO(5) \times U(1)_X$, where $5 = (1, 1) + (2, 2)$ under $SU(2)_L \times SU(2)_R$. Accordingly, each q_L and d_R can be uplifted to a full $5_{-1/3}$ of $SO(5) \times U(1)_X$, respectively, denoted as ψ_L and ψ_R, so that d_R is the (1, 1) inside ψ_R and q_L is the $T^3_R = -1/2$ component of the (2, 2) inside ψ_L. Then, the polynomial $P(\Sigma)$ transforms as a 5×5 and its projection over the physical fields d_R, q_L contains only one term, hence only one flavor structure:

$$\begin{aligned}\bar{\psi}^{\,i}_L P_{ij}(\Sigma)\psi^j_R &= y^d_{ij}\bar{\psi}^i_L \Sigma\Sigma^T\psi^j_R\\ &= y^d_{ij}\sin(h/f)\cos(h/f)\bar{q}^i_L\hat{H}d^j_R,\end{aligned} \tag{32}$$

where we have defined

$$\hat{H} = \frac{1}{h}\begin{bmatrix} h^1 + ih^2 \\ h^3 + ih^4\end{bmatrix}, \qquad h = \sqrt{(h^a)^2}. \tag{33}$$

This shows how the shift symmetry acting on the Higgs forces all the nonderivative operators with higher powers of the Higgs field to be aligned in flavor space with the SM Yukawa term. On the other hand, there is still a flavor-universal shift in the couplings of the Higgs boson of order v^2/f^2.

In spite of the flavor alignment in the nonderivative operators, flavor violation in the Higgs couplings will still occur due to the derivative operators of Eq. (2). Starting from Eq. (20) and concentrating on the last two terms, it is straightforward to derive the estimate for $\hat{y}^d_{12}$ and $\hat{y}^d_{21}$ relevant for $K\bar{K}$ mixing assuming Eqs. (21) and (22). We find

$$\begin{aligned}\hat{y}^d_{12} &\sim 2y^2_*\left(\frac{m_s}{v}\frac{(\lambda_L)_1(\lambda_L)_2}{16\pi^2} + \frac{m_d}{v}\frac{(\lambda_R)_1(\lambda_R)_2}{16\pi^2}\right),\\ \hat{y}^d_{21} &\sim 2y^2_*\left(\frac{m_d}{v}\frac{(\lambda_L)_1(\lambda_L)_2}{16\pi^2} + \frac{m_s}{v}\frac{(\lambda_R)_1(\lambda_R)_2}{16\pi^2}\right).\end{aligned} \tag{34}$$

The first term in each of the above formulas is maximized in the limit of b_L fully composite [i.e. for $(\lambda_L)_3 \to 4\pi$], while the second term is maximized for b_R fully composite [$(\lambda_R)_3 \to 4\pi$]. For b_L fully composite the strongest constraint on Λ comes from $C_2 \propto (\hat{y}^d_{12})^2$:

[7] See, for example, Ref. [26] for an explicit realization in the context of a 5D warped model.

$$\hat{y}^d_{12} \sim 2y_*^2 \frac{m_s}{v} \lambda_C \left[\frac{(\lambda_L)_2}{4\pi}\right]^2 \sim 2y_*^2 \frac{m_s}{v} \lambda_C^5 \Rightarrow \Lambda \gtrsim 55 \text{ GeV} \times y_* \times \sqrt{\frac{200 \text{ GeV}}{m_h}}. \tag{35}$$

Compared to the estimate of $\hat{y}^d_{12}$ from the nonderivative operator in Eq. (23), it is clear that the effect of the derivative operators is at best suppressed by a factor $\zeta^2_{s_L} = ((\lambda_L)_2/4\pi)^2$, where ζ_{s_L} corresponds to the degree of compositeness of s_L. Similarly, for b_R fully composite the strongest constraint on Λ comes from $\tilde{C}_2 \propto (\hat{y}^d_{21})^2$:

$$\hat{y}^d_{21} \sim 2y_*^2 \frac{m_s}{v} \lambda_C \left[\frac{(\lambda_R)_2}{4\pi}\right]^2 \sim 2y_*^2 \frac{m_s}{v} \lambda_C \left(\frac{m_s}{m_b \lambda_C^2}\right)^2 \Rightarrow \Lambda \gtrsim 510 \text{ GeV} \times y_* \times \sqrt{\frac{200 \text{ GeV}}{m_h}}. \tag{36}$$

Again, compared to Eq. (23) the effect of the derivative operators is at best suppressed by a factor $\zeta^2_{s_R} = ((\lambda_R)_2/4\pi)^2$, with ζ_{s_R} equal to the degree of compositeness of s_R.

The suppression from the degree of compositeness of the SM quarks is instead absent in $\Delta B = 2$ processes in the limit of either b_L or b_R being fully composite, in which case the constraint from derivative operators becomes as important as that from O_y. In the case of $B_d\bar{B}_d$ mixing one gets

$$\begin{aligned} \hat{y}^d_{13} &\sim 2y_*^2 \left(\frac{m_b}{v} \frac{(\lambda_L)_1(\lambda_L)_3}{16\pi^2} + \frac{m_d}{v} \frac{(\lambda_R)_1(\lambda_R)_3}{16\pi^2}\right), \\ \hat{y}^d_{31} &\sim 2y_*^2 \left(\frac{m_d}{v} \frac{(\lambda_L)_1(\lambda_L)_3}{16\pi^2} + \frac{m_b}{v} \frac{(\lambda_R)_1(\lambda_R)_3}{16\pi^2}\right), \end{aligned} \tag{37}$$

so that the strongest constraint for b_L and b_R fully composite, respectively, comes from $C_2 \propto (\hat{y}^d_{13})^2$ and $\tilde{C}_2 \propto (\hat{y}^d_{31})^2$:

$$\hat{y}^d_{13} \sim 2y_*^2 \frac{m_b}{v} \lambda_C^3 \Rightarrow \Lambda \gtrsim 190 \text{ GeV} \times y_* \times \sqrt{\frac{200 \text{ GeV}}{m_h}}, \tag{38}$$

$$\hat{y}^d_{31} \sim 2y_*^2 \frac{m_b}{v} \left(\frac{m_d}{m_b \lambda_C^3}\right) \Rightarrow \Lambda \gtrsim 570 \text{ GeV} \times y_* \times \sqrt{\frac{200 \text{ GeV}}{m_h}}. \tag{39}$$

Similarly, for $B_s\bar{B}_s$ mixing we find

$$\begin{aligned} \hat{y}^d_{23} &\sim 2y_*^2 \left(\frac{m_b}{v} \frac{(\lambda_L)_2(\lambda_L)_3}{16\pi^2} + \frac{m_s}{v} \frac{(\lambda_R)_2(\lambda_R)_3}{16\pi^2}\right), \\ \hat{y}^d_{32} &\sim 2y_*^2 \left(\frac{m_s}{v} \frac{(\lambda_L)_2(\lambda_L)_3}{16\pi^2} + \frac{m_b}{v} \frac{(\lambda_R)_2(\lambda_R)_3}{16\pi^2}\right), \end{aligned} \tag{40}$$

and the strongest constraint for b_L and b_R fully composite, respectively, comes from $C_2 \propto (\hat{y}^d_{23})^2$ and $\tilde{C}_2 \propto (\hat{y}^d_{32})^2$:

$$\hat{y}^d_{23} \sim 2y_*^2 \frac{m_b}{v} \lambda_C^2 \Rightarrow \Lambda \gtrsim 135 \text{ GeV} \times y_* \times \sqrt{\frac{200 \text{ GeV}}{m_h}}, \tag{41}$$

$$\hat{y}^d_{32} \sim 2y_*^2 \frac{m_b}{v} \left(\frac{m_s}{m_b \lambda_C^2}\right) \Rightarrow \Lambda \gtrsim 420 \text{ GeV} \times y_* \times \sqrt{\frac{200 \text{ GeV}}{m_h}}. \tag{42}$$

V. DISCUSSION AND CONCLUSIONS

Our analysis has shown that $\Delta F = 2$ neutral currents generated by the tree-level exchange of a composite Higgs lead to rather strong constraints on the scale of new physics if the Higgs is light and strongly coupled. We have focused on scenarios where the SM fermions couple linearly to operators of the new strong sector that gives the Higgs as a bound state. We have further assumed that the hierarchy in the SM Yukawa couplings entirely originates from the RG running of such couplings, while the strong sector is flavor anarchic.

In the case of CP violation in $K\bar{K}$ mixing the bounds that we have derived are comparable to those from the exchange of heavy vectors, despite the fact that the Higgs exchange requires flavor violation on both vertices and thus naively counts as a dimension-8 effect. We showed that the lightness of the Higgs and its strong coupling to the EWSB dynamics, y_*, compensate for the naive suppression. Moreover, while the Higgs exchange is enhanced for larger values of y_*, the vector exchange is suppressed, and in that sense the two constraints are complementary. Milder bounds follow instead from $\Delta B = 2$ transitions.

The above picture is however substantially modified in the special and well-motivated case of a composite pseudo-Goldstone Higgs. In the simplest situations the shift symmetry acting on the Higgs forces a flavor alignment between the SM Yukawa term and the higher-order nonderivative operators with larger powers of the Higgs field. Flavor violation then occurs only through the derivative operators, implying an additional suppression of the flavor-violating Higgs vertex by the degree of compositeness of the SM fermions involved. As a result, the constraints from ϵ_K are negligible and the most stringent bounds come in this case from $\Delta B = 2$ transitions. Moreover, the latter bounds can become as strong as the corresponding ones for a non-PGB Higgs only if b_L or b_R is fully composite. In that limit however, the constraints from the heavy vector exchange are quite stringent and dominate [2,27]. Hence, we conclude that PGB Higgs models are only very mildly constrained by the Higgs contribution to $\Delta F = 2$ processes.

It is worth stressing that our results rely on assuming that the strong sector is flavor anarchic and that the hierarchical structure of the Yukawa couplings has its origin in the running of the linear couplings of the SM fermions. If any of these assumptions is relaxed, then the estimate of the higher-order operators must be reconsidered. As an interesting example, consider the case in which the strong sector has an approximate global $SU(3)^5$ flavor symmetry, broken only by quasimarginal operators O_u, O_d, and O_e with the quantum numbers of the SM Yukawa couplings [28,29]. In the scenario of Ref. [28], all the SM fermions are fully composite, and the coefficients of the marginal operators are small and reproduce the hierarchy of the SM Yukawa couplings. At low energy the theory satisfies the criterion of minimal flavor violation, forcing $\tilde{y}^d \propto y^d + (y^u y^{u\dagger}) y^d + (y^d y^{d\dagger}) y^d + \cdots$, $\kappa^q \sim \kappa^{q\prime} \propto 1 + y^u y^{u\dagger} + y^d y^{d\dagger} + \cdots$, and $\kappa^d \sim 1 + y^{d\dagger} y^d + \cdots$, where numerical coefficients multiplying all the terms have been understood and the dots stand for terms with more Yukawa insertions. This implies that the flavor-violating effects from O_y and from the derivative operators of Eq. (2) are of the same order and small. In the models of Ref. [29] instead, the SM fermions are partially composite and the coefficients of the operators O_u, O_d, and O_e are assumed to be sizable and essentially anarchic.[8] The flavor violation induced by O_u, O_d, and O_e feeds into the fermionic sector by splitting the anomalous dimensions of the fermionic operators, and it is amplified by their RG evolution down to low energy leading to hierarchical SM Yukawa matrices. For these models our estimate of the higher-order operators goes through essentially unchanged,[9] both for a PGB and a generic Higgs. Hence, despite the constrained pattern of flavor violation in terms of spurions with the quantum numbers of the SM Yukawa couplings, the low-energy theory is not minimally flavor violating, and the bounds are strong.[10] This shows how the initial assumptions on the structure of the theory are crucial for determining the strength of the Higgs-mediated FCNCs.

The effect of the Higgs exchange in $\Delta F = 1$ transitions is by far negligible compared to that of the Z exchange, due to the Yukawa coupling suppression at the flavor-preserving vertex. Remarkable exceptions to this rule are decay processes in which the Higgs is in the initial or final state. If the Higgs is light, a quite promising decay mode is $t \to Hc$, as first pointed out in Ref. [30]. Starting from the analog of Eq. (20) applied to the up-quark sector, a simple estimate shows that even in the case of a PGB Higgs the tch vertex can be sizable as long as t_R is maximally composite. In such a limit the largest contribution comes from $\hat{y}^u_{32}$,

$$\left(\frac{v}{\Lambda}\right)^2 \bar{t}[\hat{y}^{u*}_{23} P_L + \hat{y}^u_{32} P_R] c + \text{H.c.},$$
$$y^u_{23} \ll y^u_{32} \sim 2 y_*^2 \frac{m_t}{v}\left(\frac{m_c}{m_t \lambda_C^2}\right), \tag{43}$$

so that the charm quark is mainly right handed. For $\Lambda/y_* = 1$ TeV we estimate $\text{BR}(t \to hc) \sim 1 \times 10^{-4}$, which should be within the reach of the LHC [31]. Interestingly, the above estimate is similar to that for a non-PGB Higgs, except in that case, due to the contribution of the nonderivative operator O_y, both $\hat{y}^u_{23}$ and $\hat{y}^u_{32}$ are comparable in size.

If the Higgs is heavier, the same tch vertex implies a flavor-violating Higgs decay $h \to tc$. If all the remaining decay widths are as in the SM, the above estimate predicts $\text{BR}(h \to tc) \sim 5 \times 10^{-3}$, but larger values can be obtained if the rate to gauge bosons turns out to be suppressed due to modified couplings of the composite Higgs. See Ref. [24] for prospects of observing such a signal at the LHC.

Similar or even larger rates for $t \to hc$ and $h \to ct$ are also predicted in the different scenarios of Refs. [16,17], where Yukawa couplings of the light fermions involve higher powers of the Higgs field. In that case however, only very small shifts are expected in the flavor-preserving $t\bar{t}h$ coupling. On the contrary, shifts as large as $v^2/f^2 \sim$ 10%–20% in the $t\bar{t}h$ coupling are a generic prediction of composite Higgs theories, both for the PGB and the non-PGB cases, independently on whether the top quark is fully composite or not.

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APPENDIX

We collect here the formulas relative to the $B_d \bar{B}_d$ and $B_s \bar{B}_s$ mixing. Integrating out the Higgs boson generates the $\Delta B = 2$, $\Delta S = 0$ low-energy effective Lagrangian

$$\mathcal{L}^{\Delta B=2}_{\Delta S=0} = \mathcal{O}_2 C_2 + \tilde{\mathcal{O}}_2 \tilde{C}_2 + \mathcal{O}_4 C_4 + \text{H.c.}, \tag{A1}$$

[8] As proposed by Ref. [29], the approximate flavor symmetry of the strong sector can be smaller than $SU(3)^5$ and not all three marginal operators might be actually needed. This does not change our conclusions however.

[9] In fact, there will be an extra numerical suppression of Higgs-mediated FCNCs due to the fact that the relative misalignment between the operator O_y and the down Yukawa matrix arises only at higher order in the number of flavor spurions.

[10] Notice on the other hand that in this case, similarly to the minimally flavor violating theory of Ref. [28], flavor violation in the down sector requires the interplay of both spurions acting in the up and down sectors. This is to be contrasted with the anarchic scenario considered in the present paper, where flavor violation can arise even from the down sector in isolation.

$$\mathcal{O}_4 \equiv (\bar{b}_L^\alpha d_{R\alpha})(\bar{b}_R^\alpha d_{L\alpha}), \qquad \mathcal{O}_2 \equiv (\bar{b}_R^\alpha d_{L\alpha})^2, \qquad \tilde{\mathcal{O}}_2 \equiv (\bar{b}_L^\alpha d_{R\alpha})^2, \tag{A2}$$

with

$$(C_4, C_2, \tilde{C}_2) = \frac{1}{m_h^2}\left(\frac{v^2}{\Lambda^2}\right)^2\left(\hat{y}_{13}^d \hat{y}_{31}^{d*}, \frac{1}{2}(\hat{y}_{13}^{d*})^2, \frac{1}{2}(\hat{y}_{31}^d)^2\right). \tag{A3}$$

The corresponding bound on the Wilson coefficients at the scale $m_h = 200$ GeV which follows from Ref. [23] is

$$\mathrm{Im}(C_4, C_2, \tilde{C}_2)(\mu = m_h = 200\ \mathrm{GeV}) \lesssim \frac{1}{\{(1.44, 0.94, 0.94)\times 10^3\ \mathrm{TeV}\}^2}. \tag{A4}$$

The analogous formulas in the case of the $\Delta B = 2$, $\Delta S = 2$ effective Lagrangian read

$$L_{\Delta S=2}^{\Delta B=2} = \mathcal{O}_2 C_2 + \tilde{\mathcal{O}}_2 \tilde{C}_2 + \mathcal{O}_4 C_4 + \mathrm{H.c.}, \tag{A5}$$

$$\mathcal{O}_4 \equiv (\bar{b}_L^\alpha s_{R\alpha})(\bar{b}_R^\alpha s_{L\alpha}), \qquad \mathcal{O}_2 \equiv (\bar{b}_R^\alpha s_{L\alpha})^2, \qquad \tilde{\mathcal{O}}_2 \equiv (\bar{b}_L^\alpha s_{R\alpha})^2, \tag{A6}$$

$$(C_4, C_2, \tilde{C}_2) = \frac{1}{m_h^2}\left(\frac{v^2}{\Lambda^2}\right)^2\left(\hat{y}_{23}^d \hat{y}_{32}^{d*}, \frac{1}{2}(\hat{y}_{23}^{d*})^2, \frac{1}{2}(\hat{y}_{32}^d)^2\right), \tag{A7}$$

$$\mathrm{Im}(C_4, C_2, \tilde{C}_2)(\mu = m_h = 200\ \mathrm{GeV}) \lesssim \frac{1}{\{(176, 107, 107)\ \mathrm{TeV}\}^2}. \tag{A8}$$

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