Using final state pseudorapidities to improve *s*-channel resonance observables at the LHC

Ross Diener,[1] Stephen Godfrey,[1,2,*] and Travis A. W. Martin[1,†]

[1] *Ottawa-Carleton Institute for Physics, Department of Physics, Carleton University, Ottawa, Canada K1S 5B6*
[2] *TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3*

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We study the use of final state particle pseudorapidity for measurements of *s*-channel resonances at the LHC. Distinguishing the spin of an *s*-channel resonance can, in principle, be accomplished using angular distributions in the center-of-mass frame, possibly using a center-edge asymmetry measurement, $A_{\rm CE}$. In addition, forward-backward asymmetry measurements, $A_{\rm FB}$, can be used to distinguish between models of extra neutral gauge bosons. In this article we show how these measurements can be improved by using simple methods based on the pseudorapidity of the final state particles and present the expected results for $A_{\rm FB}$ and $A_{\rm CE}$ for several representative models.

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I. INTRODUCTION

The startup of the CERN Large Hadron Collider (LHC) will allow the exploration of the TeV energy regime and the testing of the multitude of proposed theories of physics beyond the standard model. Many of these theories predict the existence of massive, neutral *s*-channel resonances [1–10]. For some models of new neutral gauge bosons (Z'), the LHC is expected to have a discovery reach upwards of 5 TeV with 100 fb^{-1} of integrated luminosity [11]. This is a significant improvement over the current experimental limits for most models, which constrain Z' masses to values greater than $\sim$1 TeV [12–14].

If a TeV scale *s*-channel resonance were discovered, the immediate task would be to identify its origins. Many observables have been proposed to this end, primarily focused on the dilepton channel (e and μ), which would produce the cleanest and most easily measured signal for a nonleptophobic Z' [1–5] with the ATLAS [15,16] and CMS [17] detectors. The proposed measurements for the dilepton channel are the Z' width, total cross section, forward-backward asymmetry ($A_{\rm FB}$) [18], central-to-edge rapidity ratio [19], and a comprehensive analysis of all rapidity regions [20,21].

There are challenges associated with some of these measurements that we argue can be alleviated by using the pseudorapidity of the final state fermions. In particular, we focus on two measurements: determining (or at least constraining) the spin of an *s*-channel resonance, and determining the forward-backward asymmetry of a Z'. Distinguishing whether the new resonance is a scalar, such as an *R*-parity violating sneutrino [9,10], a spin-2 boson, such as a KK graviton [6–8], or a spin-1 Z' [1–5] will be challenging and is typically determined through the study of the angular distribution in the center-of-mass frame of the initial state quark and antiquark (c.m.) [8,22–25]. The forward-backward asymmetry measurement at the LHC has to deal with the ambiguity in defining the forward direction due to the inability to unambiguously determine the direction of the initial state quark in a symmetric proton-proton collision.

Presently, some solutions exist to deal with these challenges. To distinguish the spin of the resonance, a center-edge asymmetry, $A_{\rm CE}$, [23] can be defined that is sensitive to the angular distribution of the events. The center-edge asymmetry is a simple means of binning the events in the central and edge regions of $\cos\theta^*$, the c.m. scattering angle, which will be weighted differently depending on the angular distribution. This has the benefit of eliminating some of the systematic uncertainties of a fit to the angular distribution. However, the $A_{\rm CE}$ observable still relies on boosting the particle four-momentum from the lab frame to the c.m. frame.

The forward-backward ambiguity in a symmetric pp collision can be resolved by exploiting the fact that the valence quarks have, on average, larger momentum than the sea antiquarks. The quark direction can then be identified with the boost direction of the dilepton system [26]. Restricting the measurement to those events that have a large boost (i.e., $|Y_{Z'}| > 0.8$) reduces the misidentification of the initial state quarks and antiquarks, resulting in greater than 70% of dilepton events being correctly identified as being boosted by the quark [26]. Both of these methods have been explored in great detail and remain the standard approach used in the literature [20,21,24,25].

Both the $A_{\rm CE}$ and the $A_{\rm FB}$ measurements require analysis of the center-of-mass (c.m.) angular distribution of the dilepton events—directly for $A_{\rm CE}$, and when tagging forward or backward events in $A_{\rm FB}$. In this article we propose a simpler method of measuring these asymmetries without reconstructing the angular distributions. Specifically, we exploit the direct measurements of the lepton pseudorapidities to calculate the observables, rather than using derived quantities that may propagate uncertainties into the result. The proposed methods also take advantage of the fact that

*godfrey@physics.carleton.ca
†tmartin@physics.carleton.ca

differences in pseudorapidities are Lorentz invariant quantities, so that all calculations can be performed using quantities measured in the lab frame.

The point of these methods is not to provide new phenomenological insight into the models, but rather to demonstrate how the use of final state pseudorapidities provides a simpler and cleaner means of obtaining the $A_{\rm CE}$ and $A_{\rm FB}$ values. The dimuon signal is very clean and error propagation should not be a big issue. The real power of this approach will be seen when applied to heavy quark final states [27]. In the following sections, we give some calculational details which are followed by a description of our approaches to the center-edge asymmetry and the forward-backward asymmetry. We conclude with some final comments.

II. CALCULATIONAL DETAILS

The basic ingredients in our calculations are the cross sections, $\sigma(pp \to R \to \mu^+\mu^-)$, where $R = Z'$, $\tilde{\nu}$ or G. The cross section for $R = Z'$ is described by the Drell-Yan process with the addition of a Z' [11,18,28]. Analagous expressions for the spin-0 $\tilde{\nu}$ and spin-2 graviton are given in Refs. [7,9,10], respectively. We computed the cross sections using Monte-Carlo phase space integration with weighted events and imposed kinematic cuts to take into account detector acceptances, as described in the following sections.

In our numerical results we take $\alpha = 1/128.9$, $\sin^2\theta_w = 0.231$, $M_Z = 91.188$ GeV, and $\Gamma_Z = 2.495$ GeV [29]. We used the CTEQ6M parton distribution functions [30] and included a K-factor to account for next-to-leading-order (NLO) QCD corrections [31]. We neglected next-to-next-to-leading order (NNLO) corrections, which are not numerically important to our results [32,33], and final state QED radiation effects, which are potentially important [34] but require a detailed detector level simulation that is beyond the scope of the present analysis. The Z' widths only include decays to standard model fermions and include NLO QCD and electroweak radiative corrections [35]. For the $\tilde{\nu}$ width, we take $\Gamma_{\tilde{\nu}} = 1$ GeV following Ref. [10]. Expressions for the G width can be found in Refs. [22,24,36,37].

III. SPIN DISCRIMINATION USING CENTRE-EDGE ASYMMETRY, $A_{\rm CE}$

The parton-level angular distributions, $d\hat{\sigma}/d\cos\theta^*$, of the spin-0, -1, -2 bosons, shown in Fig. 1, are distinct enough that, in principle, such a measurement would uniquely identify the spin [8,22]. However, these distributions are not directly accessible due to the convolution with the parton distributions of the protons, the boosting of measured lab frame quantities to the center-of-mass frame, detector limitations and finite statistics, all of which will make the measurement challenging [22].

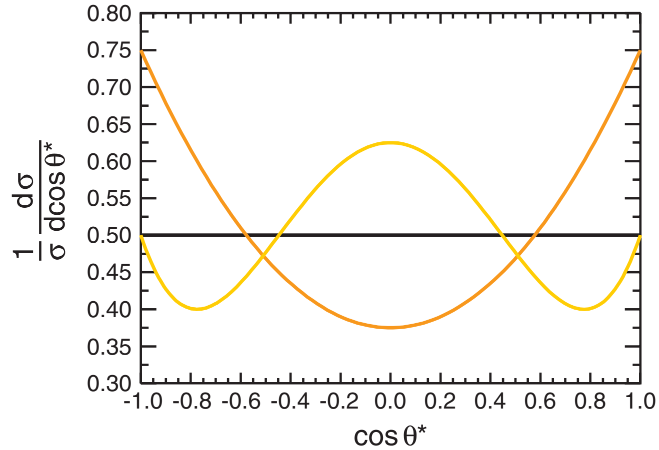

FIG. 1 (color online). Normalized parton-level angular distribution of spin-0 (black), spin-1 (dark grey/orange) and spin-2 (light grey/yellow) bosons decaying to fermions.

The center-edge asymmetry is almost entirely model-independent for spin-0 and spin-1 bosons. For example, assuming the narrow width approximation for a Z', we find that $A_{\rm CE} \approx 3/4\bar{z}(1 + 1/3\bar{z}^2) - 1/2$, for some value $\bar{z}$ that separates the center and edge regions of z^*, independent of the couplings to fermions. Spin-2 KK gravitons have contributions from gg and $q\bar{q}$ processes that have slightly different angular distributions, and the $A_{\rm CE}$ depends on the weighted contribution of each. The specific model will have an effect on the expected statistical uncertainties, but this should not be significant to the measurement due to the low backgrounds associated with leptonic final states. Thus, with limited statistics, an $A_{\rm CE}$ measurement could have an advantage over a fit to the angular distribution.

For a hadron collider, the center-of-mass angle of the outgoing fermion is not directly measurable on an event by event basis due to the unknown values of the parton momentum fractions. However, there exists a direct mapping between the c.m. angular distribution and the difference in pseudorapidity of the final state lepton and antilepton, $\Delta\eta$. Furthermore, it is straightforward to show that $\Delta\eta$ is a Lorentz invariant quantity, so that measuring this quantity in the lab frame is equivalent to measuring it in the center-of-mass frame:

$$\Delta\eta_{\rm lab} = \Delta\eta^* = \ln\left(\frac{1+\cos\theta^*}{1-\cos\theta^*}\right). \tag{1}$$

The normalized $\Delta\eta_{\rm lab}$ distributions for spin-0, -1, -2 resonances are shown in Fig. 2, where it is clear that they are distinct from one another. One can therefore construct a new center-edge asymmetry using the lab frame $\Delta\eta_{\rm lab}$ distribution in place of the c.m. frame angular distribution.

Using the mapping given by Eq. (1), we define the center-edge asymmetry:

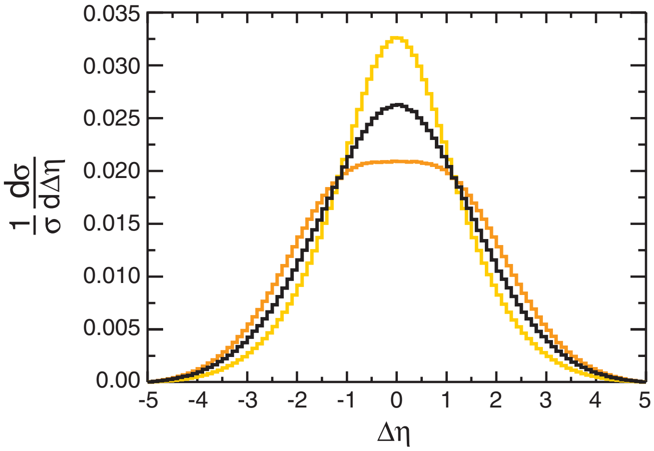

FIG. 2 (color online). Normalized $\Delta\eta$ distribution including detector acceptance cuts ($|\eta_l| < 2.5$, $p_{T_l} > 20$ GeV) and only including events from the resonance peak. These cuts reduce the number of measurable events with large values of $|\Delta\eta|$. $R = \tilde{\nu}$ (black), Z' (dark grey/orange), G (light grey/yellow), where only one spin-1 distribution is shown due to the model-independent nature of the spin-1 measurement.

$$\tilde{A}_{\rm CE} = \frac{(\int_{-\Pi}^{\Pi} - \int_{-\infty}^{-\Pi} - \int_{\Pi}^{\infty}) \frac{d\sigma}{d\Delta\eta} d\Delta\eta}{\int_{-\infty}^{\infty} \frac{d\sigma}{d\Delta\eta} d\Delta\eta}. \quad (2)$$

Following Osland, *et al.* [24], we take $\bar{z} = 0.5$, which they find to be the "optimal" value, and which translates to $\Pi = \Delta\eta = 1.099$. Experimentally, detector acceptance constrains the pseudorapidity of each fermion to $|\eta| < 2.5$, which limits $|\Delta\eta| < 5$ for the distribution.

As is common in the literature, we take an R-parity violating $\tilde{\nu}$ with $\lambda\lambda' = (0.05)^2$ as an example of a spin-0 resonance [9,10], and a Randall-Sundrum graviton with $c = 0.1$ for a spin-2 resonance [6]. Current experimental limits and studies on model parameters are given in Refs. [9,38]. For the Z' case we explored the E_6 models (ψ, χ and η) [1], the left-right symmetric model (LRSM, $g_R = g_L$) [39], both the littlest Higgs (LHM, $\tan\theta_H = 1.0$) [40] and simplest little Higgs (SLHM) [41] models, and the sequential standard model (SSM).

The spin-0 model, spin-2 and some Z' models we study predict narrow resonances, such that including events within several widths of the peak will be impossible in practice due to detector resolution effects smearing the Breit-Wigner distribution. Instead, we examine events within one dilepton invariant mass bin as defined in the ATLAS TDR [16], using $\Delta M = 42.9$ GeV for the 1.5 TeV resonance as in Ref. [24]. A more precise measurement could be obtained by including events from a wider invariant mass window, if a broader peak were to be observed.

In Table I we show the expected center-edge asymmetry for a spin-0, spin-1 and spin-2 resonance, analogous to the study performed by Dvergsnes, *et al.* [23], assuming muon final states with 96% detection efficiency [15]. From Table I one sees that if a Z' were observed, a G or $\tilde{\nu}$ could be ruled out. Likewise, an $A_{\rm CE}$ measurement would strongly discriminate against the Z' or $\tilde{\nu}$ hypothesis if a G were observed. However, the Z' and G hypothesis could only be ruled out at approximately 2.5σ if a $\tilde{\nu}$ signal was observed. The primary limitation in distinguishing between the different possibilities is the low statistics for $\tilde{\nu}$ production, as shown in the table, which is due to the tight constraints on the allowed values of its couplings. Other hypothetical spin-0 resonances may not be as tightly constrained and could therefore be distinguished from a Z' or G with higher statistical significance.

TABLE I. $\tilde{A}_{\rm CE}$ values with corresponding statistical uncertainties for 100 fb^{-1} integrated luminosity, $p_{T_l} > 20$ GeV, $|\eta_l| < 2.5$, within one bin $\Delta M_{l^+l^-} = 42.9$ GeV and $M_R = 1.5$ TeV. Also shown are the expected number of total events for each model assuming 100 fb^{-1} integrated luminosity.

Model	$\tilde{A}_{\rm CE}$	$\delta\tilde{A}_{\rm CE}$	N Events
$E_6\chi$	-0.106	± 0.017	3875
$E_6\psi$	-0.095	± 0.022	2223
$E_6\eta$	-0.092	± 0.021	2480
LR symmetric	-0.099	± 0.018	3350
Sequential SM	-0.097	± 0.016	4162
Littlest Higgs	-0.095	± 0.001	6217
Simplest little Higgs	-0.094	± 0.017	3542
RS graviton	$+0.228$	± 0.011	8208
R-parity violating $\tilde{\nu}$	$+0.055$	± 0.066	251

IV. FORWARD-BACKWARD ASYMMETRY

The forward-backward asymmetry is a well-established measurement for distinguishing between models of Z''s [18]. For $p\bar{p}$ collisions at the Tevatron, the proton direction provides an obvious choice to define the "forward" direction. The choice of forward direction at the LHC is more subtle, and is conventionally defined as the direction of the Z' rapidity, $Y_{Z'}$. The Z' rapidity is chosen because the parton distribution functions for the valence quarks peak at a higher momentum fraction than those of the antiquarks, so the system has a higher probability of being boosted in the quark direction. This observation is statistical in nature and is more likely to hold true for larger values of $|Y_{Z'}|$. For smaller values of $|Y_{Z'}|$, the momentum fractions of the quark and antiquark are generally closer in magnitude, so that using $|Y_{Z'}|$ in the low rapidity region is less likely to correctly identify the quark direction.

A simpler method of defining a "forward" or "backward" event uses pseudorapidity. As before, we define the quark direction to be that of the higher momentum parton, or equivalently the direction of the Z' rapidity. One can then show that a forward event is one in which $|\eta_f| > |\eta_{\bar{f}}|$ in the lab frame, and vice-versa for a backward event. Using these definitions for forward and backward, one can define the forward-backward asymmetry:

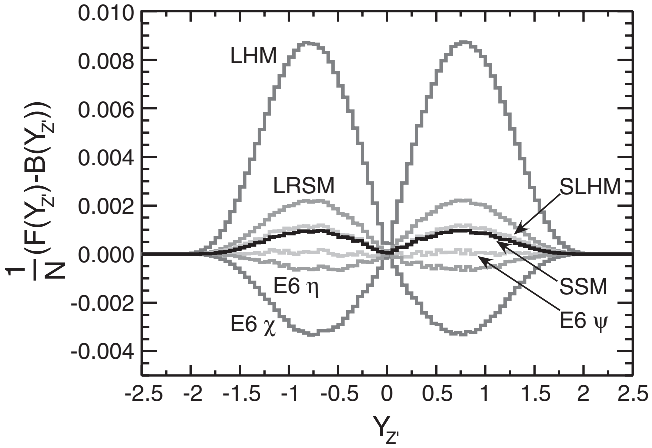

FIG. 3. A_{FB} as a function of the Z' rapidity following Eq. (3) except that these results are not integrated over rapidity. From top to bottom, the models are LHM, LRSM, SLHM, SSM, $E_6\psi$, $E_6\eta$, $E_6\chi$.

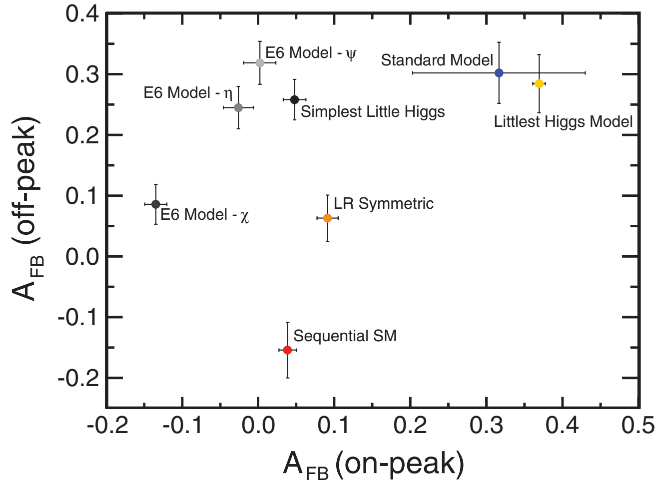

FIG. 4 (color online). A_{FB} off-peak versus on-peak for a variety of models, including detector acceptance limits and kinematic cuts as previously listed. Standard model measurement determined from the standard model Drell-Yan cross section, with on-peak events within $|M_{l^+l^-} - M_{Z'}| < 100$ GeV and off-peak events within $2/3M_{Z'} < M_{l^+l^-} < M_{Z'} - 300$ GeV to include large enough statistics.

$$A_{FB} = \frac{\int[F(y) - B(y)]dy}{\int[F(y) + B(y)]dy} \tag{3}$$

where $F(y)$ is the number of forward events and $B(y)$ is the number of backward events for a given y, the Z' rapidity (i.e., $Y_{Z'}$). The $F(y) - B(y)$ distribution under this definition is clearly shown in Fig. 3 to be symmetric in Z' rapidity. This method of finding A_{FB} has the advantage of being very straightforward and clean. It simply relies on counting events with $|\eta_f| > |\eta_{\bar{f}}|$ and those with $|\eta_f| < |\eta_{\bar{f}}|$. We note that a related technique is employed by the CDF Collaboration [42] for the Z^0 A_{FB} in $p\bar{p}$ collisions at the Tevatron. However, the natural choice of the quark direction in $p\bar{p}$ collisions at the Tevatron in contrast to pp collisions at the LHC results in important differences between the methods.

As in the conventional method for finding A_{FB}, for small values of $|Y_{Z'}|$, there is a higher probability to wrongly assume that the quark is the parton with the higher momentum fraction. This results in incorrectly assigning the forward or backward direction and gives a small "wrong" contribution to the A_{FB} measurement. For this reason, it has been suggested that the central region, $|Y_{Z'}| < Y_{min}$, be excluded in the measurement [26]. However, the coupling dependency can still be determined without this constraint on $|Y_{Z'}|$ [19].

Another consideration for excluding the central region is that the number of events that remain after subtracting $F - B$ is small, as shown in Fig. 3, while the total number of events in this region is large. Excluding the events in the central region would increase the magnitude of A_{FB}, potentially making models more distinguishable. However, we found that increasing Y_{min} resulted in an increase in the relative uncertainty. We therefore conclude that little is gained by excluding events with small $Y_{Z'}$, and suggest that the whole rapidity region be included to decrease uncertainty and further simplify the A_{FB} measurement.

Using this method, we calculate A_{FB} for the E_6 models (ψ, χ, η) [1], the left right symmetric model [39], the littlest Higgs model [40], the simplest little Higgs model [41], and the sequential standard model. The on-peak versus off-peak A_{FB} are shown in Fig. 4, where on-peak includes events which satisfy $|M_{l^+l^-} - M_{Z'}| < 3\Gamma_{Z'}$ and off-peak includes events which satisfy $2/3M_{Z'} < M_{l^+l^-} < M_{Z'} - 3\Gamma_{Z'}$, similar to the cuts used by Petriello and Quackenbush [20].

We conclude with the important observation that it might also be possible to include some events in the forward regions of the calorimeter (FCAL) using this technique [43]. While a muon signature appears as missing E_T in the FCAL, it may be possible to distinguish an electron from a jet in the FCAL due to the differences in the showering. The signal would require triggering off of a single, high p_T electron in the $|\eta| < 2.5$ region, with an electron-jet in the FCAL. Determining the charge sign of the single electron would distinguish whether this is a forward or backward tagged event. It is not clear what the signal efficiency of this method is, as reducible backgrounds include $W + j$ and others that might have low rejection rates. Extending the rapidity range has the potential of increasing the statistics and remains an interesting possibility for further study.

V. SUMMARY

In this paper we described an approach for discriminating between various spin hypotheses for a newly discovered s-channel resonance at the LHC using a center-edge symmetry, $\tilde{A}_{\mathrm{CE}}$, that is based on the difference of the rapidities of final state fermions. We also described a simple way to measure the forward-backward asymmetry, A_{FB}, using the properties of pseudorapidity. Both of these measurements have an advantage over previous approaches as they rely solely on the measurement of pseudorapidity, a fairly basic quantity. The new measurements require simple counting and should propagate fewer errors than previous approaches that rely on boosting the four-momentum into the center-of-mass frame in order to perform the analysis. Our approaches reproduce the results found in other analyses but via a more straightforward analysis.

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