Unitarity violation in sequential neutrino mixing in a model of extra dimensions

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We give the first demonstration of unitarity violation in the sequential neutrino mixing matrix in a scenario with extra compact spacelike dimensions. Gauge singlet neutrinos are assumed to propagate in one extra dimension, giving rise to an infinite tower of states in the effective four-dimensional theory. It is shown that this leads to small lepton-number violating entries in the neutrino mass matrix, which can violate unitarity on the order of 1%.

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I. INTRODUCTION

The ever-consolidating evidence in favor of neutrino masses and mixing has spawned a large volume of speculations on new physics possibilities that could be at their origin. Considering the three light sequential neutrinos, many proposed scenarios, including seesaw models of type I [1], II [2], or III [3], ensure unitarity to a high degree of precision in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix describing mixing in the lepton sector [4]. A measured departure from such unitarity, evinced from precision data in the neutrino sector, may thus point towards some novel mechanism for the generation of neutrino masses. One such possibility arises when at least one small gauge singlet Majorana mass term enters into an extended neutrino mass matrix. Under certain conditions, this situation passes off as an ‘‘inverse seesaw mechanism’’ [5–7]. It has been demonstrated in a number of recent works that this can lead to a violation of unitarity at the level of about 1% or more in the 3×3 light sequential neutrino mass matrix, due to mixing with additional sterile states [5–8]. A pertinent question to ask is, does such a situation fit into some of the popular scenarios of new physics at the TeV scale?

The experimental constraints on the loss of unitarity as well as its testability in neutrino oscillation experiments have been investigated recently [8,9]. As for theoretical models, a grand unified theory-inspired scenario, based on $SO(10)$ with a breaking chain involving an extra $U(1)$ gauge symmetry surviving at a low scale, has been considered recently for this purpose [6]. This scenario has been shown to lead to two-loop generation of some small Majorana masses and consequently to the inverse seesaw mechanism. It has also been suggested that a supersymmetric model including two types of gauge singlet neutrino superfields may produce effects of this kind [10]. In a number of other model-building ventures, too, the effect mentioned above emerges as a consequence [11].

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Phenomenological implications of unitarity violation in the PMNS matrix, including its signatures in phenomena driven by neutrino oscillation, have been recently investigated [9]. In this paper, we point out that a loss of unitarity in the PMNS matrix can also arise if one has extra flat spacelike dimensions, with gauge singlet neutrinos propagating in *one* extra dimension, and lending small diagonal elements to an extended neutrino mass matrix.

Mechanisms of neutrino mass generation have been frequently suggested in models of compact extra spacelike dimensions, both flat [12–18] and warped [19]. However, the very important issue of unitarity loss in sequential neutrino mixing has not been addressed in any earlier work. Here we demonstrate this feature, by considering a minimal higher-dimensional framework where the standard model (SM) fields all lie on a 3-brane, while one or more gauge singlet neutrinos propagate along *one flat extra dimension* [12,13]. However, there can, in principle, be several extra spacelike dimensions where gravity propagates, thereby evading the already established lower limits on the number of such dimensions [20]. An orbifold symmetry is further imposed along the compact direction containing the neutrino(s), so that one obtains only one (right-handed) chirality for the $n = 0$ Kaluza-Klein (KK) mode. It has been demonstrated earlier that this scenario can naturally suppress neutrino masses via a type I [1] seesaw mechanism.

The gauge singlet neutrinos can have Majorana masses in five dimensions to start with. We are especially interested in the situation where a strong cancellation between this mass and the KK tower mass leads to a very small entry in the effective neutrino mass matrix in four dimensions. We show that the resulting mass matrix has additional ‘‘sterile’’ states mixing appreciably with the sequential neutrinos. It is found that one can consequently expect the violation of unitarity in the 3×3 (PMNS) matrix in certain regions of the parameter space of such a model.

In Sec. II, we outline some scenarios that lead to loss of unitarity of the PMNS matrix, including the inverse seesaw mechanism. The extra-dimensional model under investigation is briefly reviewed in Sec. III. The viability of a substantial loss of PMNS unitarity is numerically demonstrated in Sec. IV. We summarize and conclude in Sec. V.

II. LOSS OF UNITARITY IN THE PMNS MATRIX

In general, the well-known type I seesaw mechanism involving a light and a heavy neutrino also involves a departure from unitarity in the PMNS matrix. However, this departure is immeasurably tiny, since the seesaw mass scale is invariably much higher than the light neutrino masses [1]. An exception to this may occur if a very small Majorana mass is introduced. However, this choice is inhibited by (a) the need of justifying such a small $\Delta L = 2$ mass in terms of new physics, and (b) the need of introducing excessively suppressed Dirac masses for generating light neutrinos, which essentially destroys the motivation of the seesaw mechanism.

The situation can be different when one has more than one two-component sterile neutrino. It has been shown in a number of recent works [5–8] that this allows one to insert small lepton-number violating mass terms in diagonal positions of the neutrino mass matrix, and the off-diagonal entries need not all be much smaller. Its most noticeable consequence is a loss of unitarity at the level of 1% or more in the 3×3 (PMNS) part of the neutrino mass matrix.

Several kinds of scenarios that meet this description are found in the literature [5,7,8]. Here we mention two classes only among these for illustration, considering one sequential and two sterile species in each case. The first [5] is one of the form

$$\mathcal{M} = \begin{pmatrix} 0 & m_D & 0 \\ m_D & m_R & m_N \\ 0 & m_N & m_L \end{pmatrix}, \tag{1}$$

in the basis $(\bar{\nu}_L, N_R, \bar{N}_L)$, where the last two are gauge singlets. The masses $m_{L,R}$ arise from $\Delta L = 2$ terms. For $m_{L,R} \ll m_D$, m_N, this not only yields an active neutrino mass eigenstate in the right order, but also leads to active-sterile mixing at the level of 1% for appropriate choices of the mass parameters (say, for example, $m_D \sim 10$ GeV, $m_N \sim 1$ TeV, and $m_L \sim 10$ keV). A corresponding situation with three sequential neutrinos will show unitarity violation at the same level in the PMNS matrix, but with an additional light sterile neutrino. Since the light (sequential) neutrino mass vanishes in the limit $m_L \to 0$, it is often called an inverse seesaw scenario.

Another situation that one can consider has the same choice of neutrino basis states, but a mass matrix of the form [8]

$$\mathcal{M} = \begin{pmatrix} 0 & m_D & m_N \\ m_D & m_R & 0 \\ m_N & 0 & m_L \end{pmatrix}, \tag{2}$$

with $m_D \ll m_R$ and $m_N^2/m_R \ll m_L \ll m_N \ll m_R$. It has been found that this situation, too, leads to light sequential neutrino(s) and unitarity violation at the same level ($\sim 1\%$), for appropriate choices of parameters (say, for example, m_D, $m_N \sim 1$ MeV, $m_R \sim 1$ TeV, and $m_L \sim 100$ eV). The difference with the previous situation is that (a) one obtains a light sterile neutrino even with one sequential family, and (b) the sequential neutrino mass does not vanish in the limit $m_L \to 0$. This makes it deviate from an inverse seesaw scenario in the strict sense, although it is equally interesting from the viewpoint of unitarity loss of the PMNS matrix. Since such unitarity loss is a very interesting consequence that is experimentally testable, it is worth exploring if it occurs in some otherwise well-motivated theories beyond the SM. In the next two sections we outline one such scenario, and go on to examine its potential for generating unitarity loss.

III. A MODEL WITH EXTRA DIMENSIONS

In this section we describe the model adopted for illustrating our point. It assumes extra flat compact spacelike dimensions where gravity can propagate. The SM fields are confined to a 3-brane which constitutes a "slice" in the higher-dimensional space. So far it is very similar to the Arkani-Hamed–Dimopoulos-Dvali (ADD) scenario [21], excepting that it includes an effort to account for neutrino masses, through the introduction of one gauge singlet neutrino propagating in *one extra dimension only* [12–17]. Thus, while all the phenomenology involving gravitons remains similar to that in the ADD framework with several extra dimensions, one can consider just the five-dimensional subspace for studying neutrino physics. We make our analysis simple by adhering to one generation of SM neutrinos. The fifth flat dimension (y), along which propagates the right-handed neutrino [$N(x, y)$], is compactified over an S^1/Z_2 orbifold where R is the radius of compactification. The preservation of the Z_2 invariance necessitates the existence of at least two symmetrically placed branes, and the SM fields lie on either of them. Thus the complete leptonic field content of the model is

$$L(x) = \begin{pmatrix} \nu_\ell(x) \\ \ell_L(x) \end{pmatrix}, \qquad \ell_R(x), \qquad N(x, y) = \begin{pmatrix} \xi(x, y) \\ \bar{\eta}(x, y) \end{pmatrix}, \tag{3}$$

where ν_ℓ, ℓ_L, ℓ_R are Weyl spinors in four dimensions, and ξ, η are two-component spinors in five dimensions. Under S^1/Z_2, the latter may be associated with opposite parities:

$$\xi(x, y) = \xi(x, -y), \qquad \eta(x, y) = -\eta(x, -y). \tag{4}$$

The brane where the SM is localized can be assumed to be at $y = a$ just for generality, instead of at the orbifold fixed point $y = 0$. We shall see later that this adds to the freedom of the model. The generic effective four-dimensional Lagrangian of this model is given by

$$\mathcal{L}_{\rm eff} = \int_0^{2\pi R} dy \bigg\{ \bar{N}(i\gamma^\mu \partial_\mu + \gamma_5 \partial_y) N - \frac{1}{2}(M N^T C^{(5)-1} \gamma_5 N + \text{H.c.}) + \delta(y-a)\bigg[\frac{h_1}{(M_F)^{1/2}} L\tilde{\Phi}^* \xi + \frac{h_2}{(M_F)^{1/2}} L\tilde{\Phi}^* \eta + \text{H.c.}\bigg] + \delta(y-a)\mathcal{L}_{\rm SM}\bigg\}, \quad (5)$$

where $\tilde{\Phi} = i\sigma_2 \Phi^*$, $\mathcal{L}_{\rm SM}$ is the SM Lagrangian, M is the Majorana mass for N (we do not specify its scale for the moment), $C^{(5)}$ is the five-dimensional charge conjugation operator, and M_F is the fundamental gravity scale. The Yukawa couplings in five dimensions, $h_{1,2}$, are assumed to be $\mathcal{O}(1)$. For gravity propagating in a d-dimensional bulk,

$$M_P = (2\pi M_F R)^{d/2} M_F, \quad (6)$$

for the simple case where all the compactification radii are of equal size R, M_P being the four-dimensional Planck scale. A Dirac mass term $m_D \bar{N} N$ is not allowed in Eq. (5) because of the Z_2 symmetry.

Following Eq. (4), the two-component spinors ξ and η can be expanded as

$$\xi(x,y) = \frac{1}{\sqrt{2\pi R}} \xi_0(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \xi_n(x) \cos\bigg(\frac{ny}{R}\bigg), \quad (7)$$

$$\eta(x,y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \eta_n(x) \sin\bigg(\frac{ny}{R}\bigg), \quad (8)$$

where the chiral spinors $\xi_n(x)$ and $\eta_n(x)$ form an infinite tower of KK fields. Using these expansions and integrating out the y coordinate, the effective Lagrangian reduces to

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \bar{\xi}_0(i\bar{\sigma}^\mu \partial_\mu)\xi_0 + \bigg(\bar{h}_1^{(0)} L\tilde{\Phi}^* \xi_0 - \frac{1}{2} M \xi_0 \xi_0 + \text{H.c.}\bigg) + \sum_{n=1}^{\infty} \bigg[\bar{\xi}_n(i\bar{\sigma}^\mu \partial_\mu)\xi_n + \bar{\eta}_n(i\bar{\sigma}^\mu \partial_\mu)\eta_n + \frac{n}{R}(\xi_n \eta_n + \bar{\xi}_n \bar{\eta}_n) - \frac{1}{2} M(\xi_n \xi_n + \bar{\eta}_n \bar{\eta}_n + \text{H.c.}) + \sqrt{2}(\bar{h}_1^{(n)} L\tilde{\Phi}^* \xi_n + \bar{h}_2^{(n)} L\tilde{\Phi}^* \eta_n + \text{H.c.})\bigg] \quad (9)$$

in a basis in which M is positive, and with

$$\bar{h}_1^{(n)} = \frac{h_1}{(2\pi M_F R)^{1/2}} \cos\bigg(\frac{na}{R}\bigg) = \bigg(\frac{M_F}{M_{\rm P}}\bigg)^{1/d} h_1 \cos\bigg(\frac{na}{R}\bigg) = \bar{h}_1 \cos\bigg(\frac{na}{R}\bigg), \quad (10)$$

$$\bar{h}_2^{(n)} = \frac{h_2}{(2\pi M_F R)^{1/2}} \sin\bigg(\frac{na}{R}\bigg) = \bigg(\frac{M_F}{M_{\rm P}}\bigg)^{1/d} h_2 \sin\bigg(\frac{na}{R}\bigg) = \bar{h}_2 \sin\bigg(\frac{na}{R}\bigg). \quad (11)$$

For deriving the last two equalities on the right-hand sides of Eqs. (10) and (11), we have made use of Eq. (6).

Equations (10) and (11) imply that the induced four-dimensional Yukawa couplings $\bar{h}_{1,2}^{(n)}$ can get suppressed by many orders depending on the hierarchy between M_P and M_F; for example, if gravity and the bulk neutrino feel the same number of extra dimensions, say $d = 1$, then these couplings are suppressed by a factor $M_F/M_{\rm P} \sim 10^{-15}$, for $M_F \approx 10$ TeV (see also [12,13]).

It is clear from Eq. (3) that ξ and $\bar{\eta}$ have the same lepton number. Thus, the simultaneous presence of the two operators $L\tilde{\Phi}^* \xi$ and $L\tilde{\Phi}^* \eta$ in Eq. (9) leads to lepton-number violation. Such coexistence of the two operators is possible only if we allow the brane to be shifted by an amount $a(\neq 0)$ from the orbifold fixed points ($y = 0, \pi R$). Such a shifting of the brane, respecting the Z_2 invariance of the original higher-dimensional Lagrangian, has been shown to be possible under certain restrictions in type I string theories [22]. As indicated in [12,18], the Z_2 invariance can be taken care of by allowing the replacements

$$\begin{aligned} \xi\delta(y-a) &\to \tfrac{1}{2}\xi[\delta(y-a) + \delta(y+a-2\pi R)], \\ \eta\delta(y-a) &\to \tfrac{1}{2}\eta[\delta(y-a) - \delta(y+a-2\pi R)], \end{aligned} \quad (12)$$

with $0 \le a < \pi R$ and $0 \le y \le 2\pi R$. Here we reiterate that a Z_2-invariant implementation of brane-shifted couplings requires the existence of at least two branes placed at $y = a$ and $y = 2\pi R - a$.

A remarkable feature of the brane-shifted framework was pointed out in [18], where it has been shown that in such a framework it is possible to completely decouple the effective Majorana-neutrino mass $\langle m \rangle$ and the scale of light neutrino masses, so as to have $\langle m \rangle$ within an observable range. Therefore, the Lagrangian (9) contains two types of Majorana-neutrino mass terms (involving, respectively, the parameters M and $\bar{h}_2^{(n)}$), both of which lead to a breaking of L. Such L breaking is a necessary ingredient of leptogenesis.

Following the notations of Ref. [12], we now introduce the weak basis for the KK Weyl spinors, by defining

$$\chi_{\pm n} = \frac{1}{\sqrt{2}}(\xi_n \pm \eta_n), \quad (13)$$

followed by a rearrangement of the states ξ_0 and $\chi_n^\pm$, such that, for a given value of n (say, $n = k_0$), the smallest diagonal entry of the neutrino mass matrix is

$$\varepsilon = \min\bigg(\bigg|M - \frac{k_0}{R}\bigg|\bigg) \le 1/(2R). \quad (14)$$

After reordering, we can define the multiplet Ψ_ν consisting of the Majorana spinors

$$\Psi_\nu^T = \left[\begin{pmatrix}\chi_{\nu_\ell}\\ \bar{\chi}_{\nu_\ell}\end{pmatrix}, \begin{pmatrix}\chi_{k_0}\\ \bar{\chi}_{k_0}\end{pmatrix}, \begin{pmatrix}\chi_{k_0-1}\\ \bar{\chi}_{k_0-1}\end{pmatrix}, \begin{pmatrix}\chi_{k_0+1}\\ \bar{\chi}_{k_0+1}\end{pmatrix}, \ldots, \begin{pmatrix}\chi_{k_0-n}\\ \bar{\chi}_{k_0-n}\end{pmatrix}, \begin{pmatrix}\chi_{k_0+n}\\ \bar{\chi}_{k_0+n}\end{pmatrix}, \ldots\right], \tag{15}$$

while the effective Lagrangian for right-handed neutrinos reduces to

$$\mathcal{L}_{\mathrm{kin}} = \tfrac{1}{2}\bar{\Psi}_\nu(i\not{\partial} - \mathcal{M}_\nu^{\mathrm{KK}})\Psi_\nu, \tag{16}$$

where $\mathcal{M}_\nu^{\mathrm{KK}}$ is the corresponding neutrino mass matrix given by

$$\mathcal{M}_\nu^{\mathrm{KK}} = \begin{pmatrix} 0 & m^{(0)} & m^{(-1)} & m^{(1)} & m^{(-2)} & m^{(2)} & \cdots \\ m^{(0)} & \varepsilon & 0 & 0 & 0 & 0 & \cdots \\ m^{(-1)} & 0 & \varepsilon - \frac{1}{R} & 0 & 0 & 0 & \cdots \\ m^{(1)} & 0 & 0 & \varepsilon + \frac{1}{R} & 0 & 0 & \cdots \\ m^{(-2)} & 0 & 0 & 0 & \varepsilon - \frac{2}{R} & 0 & \cdots \\ m^{(2)} & 0 & 0 & 0 & 0 & \varepsilon + \frac{2}{R} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{17}$$

The most important consequence of such a rearrangement is that the mass scale M, which we did not specify earlier but which could be arbitrarily large, is now replaced by the light mass scale ε. The entries in the first row and the first column of $\mathcal{M}_\nu^{\mathrm{KK}}$ are given by the relation

$$m^{(n)} = \frac{v}{\sqrt{2}}\left[\bar{h}_1 \cos\left(\frac{(n-k_0)a}{R}\right) + \bar{h}_2 \sin\left(\frac{(n-k_0)a}{R}\right)\right] = m\cos\left(\frac{na}{R} - \phi_h\right), \tag{18}$$

with

$$m = \frac{v}{2}\sqrt{\frac{h_1^2 + h_2^2}{\pi M_F R}} = \frac{m_{\mathrm{max}}}{\sqrt{M_F R}}, \tag{19}$$

$$\phi_h = \tan^{-1}\left(\frac{h_2}{h_1}\right) + k_0\frac{a}{R}, \tag{20}$$

where v is the vacuum expectation value of the SM Higgs boson.

IV. UNITARITY LOSS WITH EXTRA DIMENSIONS: SOME NUMERICAL ILLUSTRATIONS

Here we show that a substantial loss of unitarity of the PMNS matrix can occur in different allowed regions of the parameter space of the model described in the previous section. The first issue is, of course, insuring at least one small entry in diagonal positions of the neutrino mass matrix $\mathcal{M}_\nu^{\mathrm{KK}}$. Equation (14) tells us that $\varepsilon \le 1/2R$. There is no other theoretical or phenomenological constraint on ε. Thus ε qualifies to be the small diagonal element which can be potentially responsible for a departure from unitarity.

M_F, the Planck mass in five dimensions, is expected to be $\gtrsim$ TeV, since gravitational effects will otherwise become important in low-energy physics. At the same time, in order to ensure that physics along the compact dimension(s) is not plagued with trans-Planckian effects, one should have $1/R \le M_F$. Thus, in the expression for m in Eq. (19), $M_F R$ is at least of order unity. Given the fact that the five-dimensional Yukawa couplings $h_{1,2}$, too, are *prima facie* of the order of unity, this implies that m can at most be around v, the electroweak symmetry breaking scale.

We indicated two scenarios of unitarity loss in Sec. II: (i) where the off-diagonal elements in the extended neutrino mass matrix are all larger than the diagonal ones, and (ii) where diagonal elements excepting the smallest one are larger than the off-diagonal ones. Let us first examine whether the extra-dimensional model under scrutiny answers to both of these scenarios.

The first possibility demands

$$m > \frac{1}{R} \tag{21}$$

since, with $a \neq 0$, $m^{(n)}$ can approach m for some value of n along the tower. Using Eq. (19) in (21), one obtains

$$\frac{m_{\mathrm{max}}^2}{M_F} > \frac{1}{R}. \tag{22}$$

The inequality should hold for the maximum value of the right-hand side for a given M_F, which is M_F itself. Thus we have

$$M_F < m_{\mathrm{max}}. \tag{23}$$

Therefore, demanding $m > 1/R$ implies that the five-dimensional Planck scale has to be brought down below m_{max} which is just about the electroweak symmetry breaking scale, and hence is inadmissible.

The first scenario is thus disfavored in this model. On the other hand, since the diagonal elements $\mathcal{M}_\nu^{\mathrm{KK}}(i, i)$, $i \ge 3$ are always greater in magnitude than $m^{(n)}$ for all values of n, one can say that for sufficiently small ε, this model provides an opportunity for unitarity loss in the PMNS matrix in the sense of the second scenario mentioned in Sec. II.

The value of $1/R$, on the other hand, is not subject to any general constraint stronger than that arising from the validity of Newton's law of gravitation down to about 10^{-2} mm, which essentially allows $1/R$ to have any value $\gtrsim 10^{-8}$ MeV [20]. Precision electroweak constraints do not tighten the constraint, since the tower resulting from the compactification of the extra dimension corresponds to gauge singlet neutrinos only. Thus, in order to have loss of unitarity in the PMNS matrix, we are faced with two possibilities, namely, (a) $\varepsilon \ll 1/R$ and (b) $\varepsilon \simeq 1/R$. We show below that both of these situations are possible.

Our principal aim is to check if it is possible to have the sequential neutrino masses in the right order ($\sim 10^{-2}$ eV), and at the same time have substantial violation of unitarity. The latter requires that the squares of elements of some particular column in the full mixing matrix beyond the PMNS block add up to $\mathcal{O}(10^{-4})$. This sum is defined as δ^2 here.[1]

We carry out this investigation in the simplified situation, with one sequential neutrino flavor and just one gauge singlet neutrino in the bulk. We shall comment later on the generalizations necessary to generate the actual pattern of masses and mixing. As far as the violation of PMNS unitarity is concerned, however, the conclusions we reach below remain valid even when such generalizations are made.

Case (a): This implies a fine cancellation between the bulk mass M and some integral multiple of $1/R$. While a dynamical explanation of this is difficult, it is not entirely unlikely, as both M and $1/R$ can rather naturally be around the TeV scale, and there is a distinct possibility of the two of them having near-coincident values.

A few illustrative points in the parameter space for this case are shown in Table I. We have confined ourselves to 1 MeV $\leq 1/R \leq$ 10 TeV. It is found that, to get a substantial unitarity violation ($\delta \geq 0.5\%$) and neutrino mass in the right order, the largest possible value of ε that we can take is $\simeq 10^{-6}$ GeV. In this case, the 2×2 block in the upper left corner of the neutrino mass matrix effectively determines the masses of the sequential and the lightest sterile neutrino, given as $(m^{(0)})^2/\varepsilon$ and ε, respectively. One further has

$$\delta \sim \left[\frac{(m^{(0)})^2}{\varepsilon}\right]\frac{1}{m^{(0)}}. \quad (24)$$

Therefore if ε is increased, the concomitant enhancement in $m^{(0)}$, required to keep the sequential neutrino mass unaffected, ends up suppressing δ. The values of $m^{(0)}$ required point towards $\phi_h \simeq \pi/2$. On the other hand, the

[1]In Refs. [8,9], the violation of unitarity has been defined in terms of a parameter η. It is easy to check that $2\eta = \delta^2$.

TABLE I. Different sample points in the parameter space of the model where substantial unitarity violation takes place, for $\varepsilon \ll 1/R$. The corresponding sequential neutrino masses are also presented. All mass parameters are in GeV.

$1/R$	ε	$m^{(0)}$	$m^{(-1)} = m^{(+1)}$	δ (%)	m_ν
10 000	10^{-7}	10^{-9}	100	1.7	1.0×10^{-11}
	10^{-9}	10^{-10}	0.001	0.99	9.9×10^{-12}
	10^{-9}	10^{-7}	0.001	1.0	9.9×10^{-12}
	10^{-8}	3×10^{-10}	0.001	3.3	9.1×10^{-12}
	10^{-7}	10^{-9}	0.001	1.0	1.0×10^{-11}
	10^{-7}	10^{-9}	0.005	1.0	9.9×10^{-12}
1000	10^{-7}	10^{-9}	0.01	1.0	9.9×10^{-12}
	10^{-7}	10^{-9}	0.1	1.0	1.0×10^{-11}
	10^{-7}	10^{-9}	1.0	1.0	9.8×10^{-12}
	10^{-7}	10^{-9}	10.0	1.7	9.9×10^{-12}
	10^{-7}	10^{-9}	20.0	3.0	7.0×10^{-11}
	10^{-7}	10^{-9}	25.0	3.6	1.1×10^{-10}
1000	3×10^{-8}	3×10^{-10}	3×10^{-10}	1.0	3.1×10^{-12}
10	10^{-7}	10^{-9}	0.001	1.0	1.0×10^{-11}
	10^{-7}	10^{-9}	0.001	1.0	9.8×10^{-12}
1	10^{-6}	3×10^{-9}	0.005	0.8	4.1×10^{-11}
	10^{-6}	6×10^{-9}	0.005	0.9	1.4×10^{-11}
0.01	10^{-7}	10^{-9}	10^5	1.2	9.8×10^{-12}
0.001	10^{-7}	10^{-9}	10^5	1.7	1.0×10^{-11}

fact that $m^{(1)}$ can vary over a wide range implies that the brane-shift parameter a can vary from zero to $0.1R$ approximately. It should be noted that Table I includes one sample corresponding to $m^{(0)} = m^{(1)}$, which means $a = 0$. Thus, in this case, large unitarity violation is consistent with both the cases where the brane is located at the orbifold fixed point and where it is noticeably shifted.

Progressively smaller values of ε lead to correspondingly enhanced mixing between the sequential and lightest sterile neutrinos. In the limit $\varepsilon \to 0$, this leads to two degenerate states with maximal mixing. However, such a high degree of loss of unitarity is incompatible with experimental limits.

Case (b): In this case ε and $1/R$ can be relatively close to each other. Therefore, no drastic cancellation is required between them, and no allegation of fine-tuning can be leveled against such a scenario.

Some sample results for this case are presented in Table II. It should be noted that values of $1/R$ of comparable smallness as that of ε imply that the part of $\mathcal{M}_\nu^{\rm KK}$ beyond the upper left 2×2 block no longer tends to decouple. An immediate consequence is that $m^{(\pm1)}$ have to be as small as $m^{(0)}$ when $1/R$ is small. This in turn drives the value of the brane-shift parameter a to very small values. Therefore, unlike in the previous case, the brane is compelled to be close to the orbifold fixed points.

The figures included in Table II are self-explanatory; the provision for substantial unitarity violation is clearly there. However, there is a relative paucity of available points

TABLE II. Different sample points in the parameter space of the model where substantial unitarity violation takes place, for $\varepsilon \simeq 1/R$. The corresponding sequential neutrino masses are also presented. All mass parameters are in GeV.

$1/R$	ε	$m^{(0)}$	$m^{(-1)} = m^{(+1)}$	δ (%)	m_ν
	10^{-6}	10^{-8}	3.5×10^{-8}	1.0	7.5×10^{-11}
10^{-5}	2×10^{-6}	10^{-8}	3.5×10^{-8}	0.7	1.0×10^{-12}
	5×10^{-6}	10^{-8}	3.0×10^{-8}	0.7	1.0×10^{-10}
10^{-5}	5×10^{-6}	3×10^{-8}	3×10^{-8}	0.9	6.0×10^{-11}
10^{-6}	5×10^{-7}	3×10^{-9}	10^{-8}	2.2	1.2×10^{-10}
10^{-7}	5×10^{-8}	5×10^{-10}	3×10^{-10}	1.2	3.8×10^{-12}
10^{-7}	3×10^{-8}	5×10^{-10}	5×10^{-10}	1.9	6.7×10^{-12}

compared to the previous case. This is because the part of $\mathcal{M}_\nu^{\rm KK}$ beyond the 2×2 block is not ineffectual in determining the sequential neutrino mass and its mixing with light sterile states. In order to comply with all constraints there, one therefore requires a correlation between ε and $1/R$, in contrast to the situation with $\varepsilon \ll 1/R$, thereby restricting the allowed points in the parameter space.

We have shown values of $1/R$ as small as 10^{-7} GeV in Table II. While it is possible to have even smaller values of $1/R$ and go down to the limit quoted earlier, the value of δ becomes unacceptably large as $1/R$ approaches the mass eigenvalues of the sequential neutrinos.

The numerical results presented by us are obtained through the diagonalization of the 4×4 neutrino mass matrix, including a tower of states up to the first KK excitation only. We have, however, checked that the results do not change qualitatively upon the inclusion of additional towers and the resulting augmentation of the mass matrix. For example, for $1/R = 10$ TeV, $\varepsilon = 10^{-7}$ GeV, $m^{(0)} = 10^{-9}$ GeV, $|m^{(\pm 1)}| = 100$ GeV, we obtain practically the same results on extending the tower to include the second excitations, thus using a 6×6 mass matrix. The level of unitarity violation changes from 1.7% to 1.9%.

The range of unitarity loss, according to the tabulated numbers, can be 1% to more than 3.5% in this scenario. The models proposed in [6], too, predict unitarity loss to the tune of 1%, with the choice of parameters presented there. In the sample study on a supersymmetric model with R-parity breaking [10], the large Majorana mass has been set at 500 GeV, and the small one, at 100 eV. As far as the level of unitarity loss goes, this again falls at the same level. As for the experimental reach of unitarity loss, the first reference in [9], for example, brings the current capacities down to just around this value, based on oscillation data as well as rare decays. However, it has been claimed that future experiments, including those at neutrino factories, may be able to probe unitarity loss down to the level of 1 in 10^{-3} or thereabout. It can therefore be expected that the prediction of the model studied here will then come as much under the microscope as the other ones investigated so far. The distinctive features of the model will be revealed through other, supplementary studies, including those at high-energy colliders.

V. SUMMARY AND CONCLUSIONS

We have studied a popular model of flat extra compact spacelike dimensions. A gauge singlet neutrino is assumed to propagate in one extra dimension. The ‘‘bulk’’ mass possessed by this neutrino can undergo cancellation with the KK tower mass for some member of the tower, giving rise to at least one small diagonal entry (ε) in the infinite-dimensional neutrino mass matrix in four dimensions. We show that this can cause substantial mixing between the sequential and sterile neutrinos without violating any existing constraint. The consequence is a departure from unitarity of the PMNS matrix, both for $\varepsilon \ll 1/R$ and $\varepsilon \simeq 1/R$, R being the radius of the compact dimension housing the gauge singlet neutrino. In the former case, a small ϵ arises due to a strong cancellation between the bulk Majorana mass and some multiple of $1/R$, for which there may not be any deep theoretical reason. For small $1/R$, however, no strong cancellation is required, and a small ϵ is the only possibility, as demonstrated by Eq. (14).

For the sake of simplicity, we have presented our results for one sequential neutrino. It can be easily checked that the conclusions are valid with additional generations. In fact, the constraints on δ are easy to satisfy, since the strongest constraint on the PMNS matrix is on its (1,2)th element [8,9]. The relatively unconstrained mixing of, for example, ν_τ with a sterile neutrino can accommodate the values of δ obtained here. On the other hand, it may be difficult to accommodate the neutrino mixing data and mass hierarchies with one sterile bulk neutrino only [23]. At least two such neutrinos can, however, accommodate everything rather easily, thanks to the additional Yukawa couplings available, which are essentially free parameters. Our general conclusions are unaffected by such extensions.

In conclusion, the phenomenon of unitarity violation in the PMNS matrix can be motivated rather well in a model of extra dimensions. This brings to the fore the likely connection between subtleties of the neutrino sector and theories which advocate strikingly new physics around the TeV scale.

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