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# Supersymmetry and $CP$ violation in $B_s^0$-$\bar{B}_s^0$ mixing and $B_s^0 \to J/\psi\phi$ decay

Alakabha Datta

*Department of Physics and Astronomy, 108 Lewis Hall, University of Mississippi, Oxford, Mississippi 38677-1848, USA*

Shaaban Khalil

*Center for Theoretical Physics at the British University in Egypt, Sherouk City, Cairo 11837, Egypt and Department of Mathematics, Ain Shams University, Faculty of Science, Cairo, 11566, Egypt*

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Supersymmetric contributions to time independent asymmetry in $B_s^0 \to J/\psi\phi$ process are analyzed in view of recent Tevatron experimental measurements. We show that the experimental limits of the mass difference $\Delta M_{B_s}$ and the mercury electric dipole moment significantly constrain the supersymmetric (SUSY) contribution to $B_s^0$-$\bar{B}_s^0$ mixing, so that $\sin 2\beta_s \lesssim 0.1$. We also point out that the one loop SUSY contribution to $B_s^0 \to J/\psi\phi$ decay can be important and can lead to large indirect $CP$ asymmetries which are different for different polarization states. These new physics effects in the decay amplitude can be consistent with $CP$ measurements in the $B_d$ system.

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## I. INTRODUCTION

Recently, the CDF and $D0$ collaborations have announced the observation of $CP$ violation in $B_s^0$-$\bar{B}_s^0$ mixing. The following results, for the $B_s^0$ mixing $CP$ violating phase, have been reported [1,2]:

$$2\beta_s = 0.57^{+0.30}_{-0.24}(\text{stat})^{+0.02}_{-0.07}(\text{syst}) \qquad (\text{D0}), \tag{1}$$

$$2\beta_s \in [0.32, 2.82](68\%) \qquad (\text{CDF}). \tag{2}$$

These results indicate that the phase $\beta_s$ deviates more that $3\sigma$ from the standard odel (SM) prediction [3]. Therefore, the experimental observation of $CP$ violation in $B_s^0$ mixing, along with the Belle and *BABAR* measurement for direct and indirect $CP$ asymmetries of $B_d$ decays, opens the possibility of the probing new physics effect at low energy.

It is a common feature for any physics beyond the SM to possess additional sources of $CP$ violation besides the SM phase in the quark mixing matrix. In supersymmetric extension of the SM, the soft supersymmetric (SUSY) breaking terms are in general complex and can give new contributions to $CP$ violating processes. The SUSY $CP$ violating phases can be classified as flavor independent phases, like the phases of the gaugino masses and $\mu$ term, and flavor-dependent phases, like the phases of the off diagonal $A$-terms. The flavor independent phases are stringently constrained by the experimental limits on the electric dipole moment (EDM) of the electron and the neutron. However, the flavor-dependent phases are much less constrained. This may imply that SUSY $CP$ violation has a flavor off diagonal character just as in the standard model. In this case, the origin of $CP$ violation is closely related to the origin of the flavor structures rather than to the origin of SUSY breaking [4].

The SUSY flavor-dependent phases can induce sizeable contributions to direct and indirect $CP$ asymmetries of $B_d$ decays [5–7], as in $B_d \to \phi K_S$, $B_d \to \eta' K_S$, and $B_d \to K\pi$, which show some discrepancy with the SM expectations. In this paper, we revisit the supersymmetric contributions to $B_s^0$-$\bar{B}_s^0$ mixing. We investigate the possibility that SUSY may be responsible for the large observed value of the $B_s$ mixing phase without enhancing the mass difference $\Delta M_s$ over the measured value. In addition, we analyze the one loop SUSY contribution to $B_s^0 \to J/\psi\phi$ decay, which turns out to be important and can lead to large indirect $CP$ asymmetries.

The paper is organized as follows. In Sec. II, we analyze the possible new physics contributions to $B_s^0$-$\bar{B}_s^0$ mixing and the indirect $CP$ asymmetries of $B_s^0 \to J/\psi\phi$, taking into account the constraints imposed by the experimental measurements of the mass difference $\Delta M_{B_s}$ and the mercury EDM. In Sec. III, we discuss the supersymmetric contributions to the effective Hamiltonian for $\Delta B = 2$ and $\Delta B = 1$ transitions. In Sec. IV, we show that the mercury EDM imposes stringent constraints on the supersymmetric contribution to the phase $\beta_s$, such that the $B_s^0$ mixing phase can not exceed 0.1. In Sec. V, we analyze the supersymmetric contribution to the $B_s^0 \to J/\psi\phi$ decay. We emphasize that the one loop SUSY contribution to $B_s^0 \to J/\psi\phi$ can be important and can lead to large indirect $CP$ asymmetries which are in general different for different polarization states. Finally, we give our conclusions in Sec. VI.

## II. $B_s^0$-$\bar{B}_s^0$ MIXING AND $CP$ ASYMMETRY IN $B_s^0 \to J/\psi\phi$

In the $B_s^0$ and $\bar{B}_s^0$ system, the flavor eigenstates are given by $B_s^0 = (\bar{b}s)$ and $\bar{B}_s^0 = (b\bar{s})$. The corresponding mass eigenstates are defined as $B_L = pB_s^0 - q\bar{B}_s^0$ and $B_H = pB_s^0 + q\bar{B}_s^0$, where $L$ and $H$ refer to the light and the heavy mass eigenstates, respectively. The mixing angles $q$ and $p$

 

are defined in terms of the transition matrix element $\mathcal{M}_{12} = \langle B_s^0 | H_{\rm eff}^{\Delta B=2} | \bar{B}_s^0 \rangle$, where $H_{\rm eff}^{\Delta B=2}$ is the effective Hamiltonian responsible for $\Delta B = 2$ transitions:

$$\frac{q}{p} = \sqrt{\frac{\mathcal{M}_{12}^*}{\mathcal{M}_{12}}}, \tag{3}$$

where we have assumed that $\Delta\Gamma_{B_s} \ll \Delta M_{B_s}$ and $\Delta\Gamma_{B_s} \ll \Gamma_{B_s}^{\rm total}$. The strength of $B_s^0$-$\bar{B}_s^0$ mixing is described by the mass deference,

$$\Delta M_{B_s} = M_{B_H} - M_{B_L} = 2\,{\rm Re}\left[\frac{q}{p}\mathcal{M}_{12}\right] = 2|\mathcal{M}_{12}(B_s)|. \tag{4}$$

The decay $B_s^0 \to J/\psi\phi$ involves vector-vector final states with three polarization amplitudes. Therefore, an angular distribution is necessary to separate out the three polarizations for a measurement of indirect $CP$ violation without dilution. The amplitudes for the decay of $B_s^0 \to f$ and $\bar{B}_s^0 \to f$ are given by $A^\lambda(f) = \langle f | H_{\rm eff}^{\Delta B=1} | B_s^0 \rangle$ and $\bar{A}^\lambda(f) = \langle f | H_{\rm eff}^{\Delta B=1} | \bar{B}_s^0 \rangle$ with

$$\bar{\rho}^{\,\lambda}(f) = \frac{\bar{A}^\lambda(f)}{A^\lambda(f)} = \frac{1}{\rho^\lambda(f)}. \tag{5}$$

Here, $\lambda$ is the polarization index. Therefore, the source of $CP$ violation in decays to $CP$ eigenstates with oscillation are: oscillation if $q/p \neq 1$, decay if $\bar{\rho}^\lambda(f) \neq 1$, both oscillation and decay if $\{q/p, \bar{\rho}^\lambda(f)\} \neq 1$. The time-dependent $CP$ asymmetry of $B_s^0 \to J/\psi\phi$, for each polarization state $\lambda$, is given by

$$\begin{aligned} A^\lambda_{J/\psi\phi}(t) &= \frac{\Gamma^\lambda(\bar{B}_s^0(t) \to J/\psi\phi) - \Gamma^\lambda(B_s^0(t) \to J/\psi\phi)}{\Gamma^\lambda(\bar{B}_s^0(t) \to J/\psi\phi) + \Gamma^\lambda(B_s^0(t) \to J/\psi\phi)}, \\ &= C^\lambda_{J/\psi\phi}\cos\Delta M_{B_s}t + S^\lambda_{J/\psi\phi}\sin\Delta M_{B_s}t, \end{aligned} \tag{6}$$

where $C^\lambda_{J/\psi\phi}$ and $S^\lambda_{J/\psi\phi}$ represent the direct and the mixing $CP$ asymmetry, respectively, and they are given by

$$\begin{aligned} C^\lambda_{J/\psi\phi} &= \frac{|\bar{\rho}^\lambda(J/\psi\phi)|^2 - 1}{|\bar{\rho}^\lambda(J/\psi\phi)|^2 + 1}, \\ S^\lambda_{J/\psi\phi} &= \eta^\lambda \frac{2\,{\rm Im}[\frac{q}{p}\bar{\rho}^\lambda(J/\psi\phi)]}{|\bar{\rho}^\lambda(J/\psi\phi)|^2 + 1}, \end{aligned} \tag{7}$$

where $\eta^\lambda$ is $\pm$ depending on the polarization states. In the SM, the mixing $CP$ asymmetry in the $B_s^0 \to J/\psi\phi$ process is the same for all polarization, to a very good approximation, up to a sign. Hence we will omit the polarization index when discussing the SM results. We have in the SM

$$\sin 2\beta_s = S_{J/\psi\phi}. \tag{8}$$

If $\rho(J/\psi\phi) = 1$, which is the case in SM, then $\beta_s$ is defined as $2\beta_s = \arg[\mathcal{M}_{12}(B_s)]$.

In the SM, the mass difference is given by

$$\Delta M_{B_s}^{\rm SM} = \frac{G_F^2}{6\pi^2}\eta_B m_B(\hat{B}_{B_s}F_{B_s}^2)M_W^2|V_{ts}|^2 S_0(x_t). \tag{9}$$

One may estimate the SM contribution to $\Delta M_{B_s}$ through the ratio $\Delta M_{B_s}^{\rm SM}/\Delta M_{B_d}^{\rm SM}$, where the uncertainties due to the short-distance effect cancel. More importantly, theoretical uncertainties from nonperturbative dynamics are also expected to cancel in the ratio. Hence, one has

$$\frac{\Delta M_{B_s}^{\rm SM}}{\Delta M_{B_d}^{\rm SM}} = \frac{M_{B_s}}{M_{B_d}}\frac{B_{B_s}f_{B_s}^2}{B_{B_d}f_{B_d}^2}\frac{|V_{ts}|^2}{|V_{td}|^2}. \tag{10}$$

We assume that $\Delta M_{B_d}^{\rm SM} = \Delta M_{B_d}^{\rm exp} \simeq 0.507~{\rm ps}^{-1}$. Thus, for the quark mixing angle $\gamma \simeq 67°$, one finds $\Delta M_{B_s}^{\rm SM} \simeq 15~{\rm ps}^{-1}$, which is consistent with the recent results reported by CDF and $D0$ [8,9]:

$$\Delta M_{B_s} = 17.77 \pm 0.10({\rm stat}) \pm 0.07({\rm syst}) \qquad ({\rm CDF}), \tag{11}$$

$$\Delta M_{B_s} = 18.53 \pm 0.93({\rm stat}) \pm 0.30({\rm syst}) \qquad ({\rm D0}). \tag{12}$$

On the other hand, the SM contribution ($\rho(J/\psi\phi) = 1$) to the $CP$ asymmetry $S_{J/\psi\phi}$ is given by

$$\begin{aligned} S_{J/\psi\phi} &= \sin 2\beta_s^{\rm SM}, \quad \text{with} \\ \beta_s^{\rm SM} &= \arg\left(\frac{-V_{cs}V_{cb}^*}{V_{ts}V_{tb}^*}\right) \simeq \mathcal{O}(0.01), \end{aligned} \tag{13}$$

where $V_{ij}$ are the elements of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. This result clearly conflicts with the experimental measurements reported in Eqs. (1) and (2). Therefore, a confirmation of these measurements would be a clear signal for new physics beyond the SM.

In a model independent way, the effect of new physics (NP), with $\rho(J/\psi\phi) = 1$, can be described by the dimensionless parameter $r_s^2$ and a phase $2\theta_s$ defined as follows:

$$r_s^2 e^{2i\theta_s} = \frac{\mathcal{M}_{12}(B_s)}{\mathcal{M}_{12}^{\rm SM}(B_s)} = 1 + \frac{\mathcal{M}_{12}^{\rm NP}(B_s)}{\mathcal{M}_{12}^{\rm SM}(B_s)}. \tag{14}$$

Therefore, $\Delta M_{B_s} = 2|\mathcal{M}_{12}^{\rm SM}(B_s)|r_s^2 = \Delta M_{B_s}^{\rm SM} r_s^2$. In this respect, $r_s^2$ is bounded by $r_s^2 = \Delta M_{B_s}^{\rm exp}/\Delta M_{B_s}^{\rm SM} \lesssim 1.2$. This constrains the ratio between the NP and SM amplitudes defined as $R = |A_{\rm NP}/A_{\rm SM}|$ as follows:

$$|1 + Re^{i\theta_{\rm NP}}| \lesssim 1.2. \tag{15}$$

Note that for vanishing NP phase, i.e., $\theta_{\rm NP} = 0$, one finds that $R \lesssim 0.2$. However, for $\theta_{\rm NP} \neq 0$, the constrain on $R$ is relaxed as shown in Fig. 1. It is clear that $R$ can be of order one if the NP phase is tuned to be within the range $\pi/2 < \theta_{\rm NP} < \pi$.

In the presence of NP contribution, the $CP$ asymmetry in $B_s^0 \to J/\psi\phi$ is modified and now we have

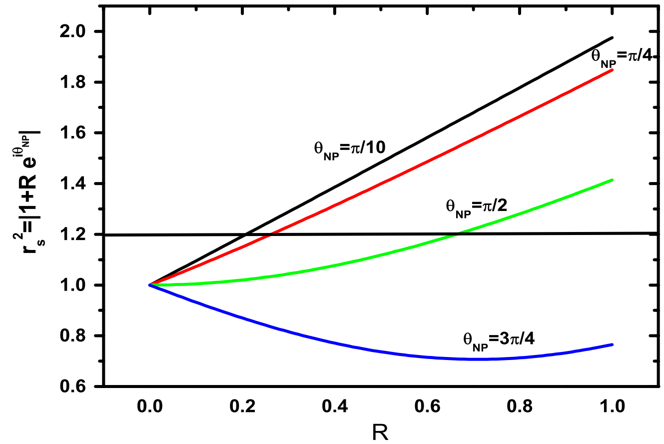


FIG. 1 (color online). The constraint on $R = |A_{NP}/A_{SM}|$ in case of $\theta = \pi/10$, $\pi/4$, $\pi/2$, and $3\pi/4$.

$$S_{J/\psi\phi} = \sin 2\beta_{\text{eff}} = \sin(2\beta_s^{\text{SM}} + 2\theta_s), \tag{16}$$

where

$$2\theta_s = \arg(1 + Re^{i\theta_{\text{NP}}}). \tag{17}$$

Therefore, in order to enhance the NP effects, large values of $R$ are required. Now we consider the effect of NP that leads to $\rho(J/\psi\phi) \neq 1$. Let us write the amplitude as

$$\bar{A}^{\lambda}(J/\psi\phi) = \bar{A}^{\lambda}_{\text{SM}}(J/\psi\phi) + \bar{A}^{\lambda}_{\text{NP}}(J/\psi\phi), \tag{18}$$

and define

$$\frac{A^{\lambda}(J/\psi\phi)}{A^{\lambda}_{\text{SM}}(J/\psi\phi)} = S^{\lambda}_A e^{i\theta^{\lambda}_A}, \tag{19}$$

where $\theta^{\lambda}_A$ is a weak phase, $\lambda$ is the polarization index, and we have assumed that the strong phases in the amplitude ratio cancel. One can now write $\bar{\rho}(J/\psi\phi)$ as

$$\bar{\rho}(J/\psi\phi) = e^{-2i\theta^{\lambda}_A}. \tag{20}$$

Thus, one obtains

$$\frac{q}{p}\bar{\rho}(J/\psi\phi) = e^{-2i(\beta_{\text{SM}} + \theta_s + \theta^{\lambda}_A)}. \tag{21}$$

In this case, the *CP* asymmetry $B_s^0 \to J/\psi\phi$ is modified and now we have

$$S^{\lambda}_{J/\psi\phi} = \pm\sin(2\beta_s^{\text{SM}} + 2\theta_s + 2\theta^{\lambda}_A). \tag{22}$$

However, as pointed out in Ref. [5], this parametrization is true only when the strong phase of the full amplitude is assumed to be the same as the SM amplitude. In fact, as discussed in Ref. [10], the NP strong phases can be different and is generally smaller than the SM strong phase thus invalidating the assumption about strong phases made in Eq. (19). In general, the SM and NP amplitudes can be parametrized as

$$A^{\lambda}_{\text{SM}} = |A^{\lambda}_{\text{SM}}|e^{i\delta^{\lambda}_{\text{SM}}}, \qquad A^{\lambda}_{\text{NP}} = \sum_i |A^{\lambda}_{i\text{NP}}|e^{i\theta^{\lambda}_{i\text{NP}}}e^{i\delta^{\lambda}_{i\text{NP}}}, \tag{23}$$

where $\delta^{\lambda}_{i\text{NP}}$ are the strong phases and $\theta^{\lambda}_{i\text{NP}}$ are the *CP* violating phases. If there is one dominant NP amplitude then we can parametrize the NP amplitude as

$$A^{\lambda}_{\text{NP}} = |A^{\lambda}_{\text{NP}}|e^{i\theta^{\lambda}_{\text{NP}}}e^{i\delta^{\lambda}_{\text{NP}}}. \tag{24}$$

Thus, the *CP* asymmetry $S_{J/\psi\phi}$ can be approximately written as

$$S^{\lambda}_{J/\psi\phi} = \sin(2\beta_s^{\text{SM}} + 2\theta_s) + 2r^{\lambda}_A\cos(2\beta_s^{\text{SM}} + 2\theta_s) \times \sin\theta^{\lambda}_{\text{NP}}\cos\delta^{\lambda}, \tag{25}$$

where $r^{\lambda}_A = |A^{\lambda}_{\text{NP}}/A^{\lambda}_{\text{SM}}|$ and $\delta^{\lambda} = \delta^{\lambda}_{\text{SM}} - \delta^{\lambda}_{\text{NP}}$. Here $\lambda$ represents the various polarization states of the vector-vector final state.

In the SUSY case considered in this paper, there will be two dominant operators. In this case we can write the new physics amplitude as

$$A^{\lambda}_{\text{NP}} = |A^{\lambda}_{1\text{NP}}|e^{i\theta^{\lambda}_{1\text{NP}}}e^{i\delta^{\lambda}_{1\text{NP}}} + |A^{\lambda}_{2\text{NP}}|e^{i\theta^{\lambda}_{2\text{NP}}}e^{i\delta^{\lambda}_{2\text{NP}}}. \tag{26}$$

Now using the result in Ref. [10], we will neglect the NP strong phases and hence the new physics amplitude can be rewritten as an effective single NP amplitude

$$\begin{aligned} A^{\lambda}_{\text{NP}} &= |A^{\lambda}_{\text{NP}}|e^{i\theta^{\lambda}_{\text{NP}}}, \\ \tan\theta^{\lambda}_{\text{NP}} &= \frac{|A^{\lambda}_{1\text{NP}}|\sin\theta^{\lambda}_{1\text{NP}} + |A^{\lambda}_{2\text{NP}}|\sin\theta^{\lambda}_{2\text{NP}}}{|A^{\lambda}_{1\text{NP}}|\cos\theta^{\lambda}_{1\text{NP}} + |A^{\lambda}_{2\text{NP}}|\cos\theta^{\lambda}_{2\text{NP}}}, \\ |A^{\lambda}_{\text{NP}}| &= \sqrt{(|A^{\lambda}_{1\text{NP}}|\sin\theta^{\lambda}_{1\text{NP}} + |A^{\lambda}_{2\text{NP}}|\sin\theta^{\lambda}_{2\text{NP}})^2 + (|A^{\lambda}_{1\text{NP}}|\cos\theta^{\lambda}_{1\text{NP}} + |A^{\lambda}_{2\text{NP}}|\cos\theta^{\lambda}_{2\text{NP}})^2}. \end{aligned} \tag{27}$$

Hence, the expression in Eq. (25) can still be used provided we set the NP strong phases to zero.

## III. SUPERSYMMETRIC CONTRIBUTIONS TO $\Delta B = 2$ AND $\Delta B = 1$ TRANSITIONS

In this section, we analyze the SUSY contribution to the $B_s^0$-$\bar{B}_s^0$ mixing and $B_s^0 \to J/\psi\phi$ decay. As pointed out in Ref. [11], gluino exchanges through $\Delta B = 2$ box diagrams give the dominant contribution to $B_s^0$-$\bar{B}_s^0$ mixing, while the chargino exchanges are subdominant and can be neglected. The general $H_{\text{eff}}^{\Delta B=2}$ induced by gluino exchanges can be expressed as

$$H_{\rm eff}^{\Delta B=2} = \sum_{i=1}^{5} C_i(\mu) Q_i(\mu) + \sum_{i=1}^{3} \tilde{C}_i(\mu) \tilde{Q}_i(\mu) + \text{H.c.}, \tag{28}$$

where $C_i(\mu)$, $\tilde{C}_i(\mu)$, $Q_i(\mu)$, and $\tilde{Q}_i(\mu)$ are the Wilson coefficients and operators, respectively, normalized at the scale $\mu$, with

$$Q_1 = \bar{s}_L^\alpha \gamma_\mu b_L^\alpha \bar{s}_L^\beta \gamma_\mu b_L^\beta, \tag{29}$$

$$Q_2 = \bar{s}_R^\alpha b_L^\alpha \bar{s}_R^\beta b_L^\beta, \tag{30}$$

$$Q_3 = \bar{s}_R^\alpha b_L^\beta \bar{s}_R^\beta b_L^\alpha, \tag{31}$$

$$Q_4 = \bar{s}_R^\alpha b_L^\alpha \bar{s}_L^\beta b_R^\beta, \tag{32}$$

$$Q_5 = \bar{s}_R^\alpha b_L^\beta \bar{s}_L^\beta b_R^\alpha. \tag{33}$$

In addition, the operators $\tilde{Q}_{1,2,3}$ are obtained from $Q_{1,2,3}$ by exchanging $L \leftrightarrow R$. The results for the gluino contributions to the above Wilson coefficients at the SUSY scale, in the framework of the mass insertion approximation, are give by [12]

$$C_1^{\tilde{g}} = -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2}[24 x f_6(x) + 66 \tilde{f}_6(x)](\delta_{23}^d)_{LL}^2, \tag{34}$$

$$C_2^{\tilde{g}} = -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2} 204 x f_6(x)(\delta_{23}^d)_{RL}^2, \tag{35}$$

$$C_3^{\tilde{g}} = -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2} 36 x f_6(x)(\delta_{23}^d)_{RL}^2, \tag{36}$$

$$C_4^{\tilde{g}} = -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\{[504 x f_6(x) - 72 \tilde{f}_6(x)](\delta_{23}^d)_{LL}(\delta_{23}^d)_{RR} - 132 \tilde{f}_6(x)(\delta_{23}^d)_{LR}(\delta_{23}^d)_{RL}\}, \tag{37}$$

$$C_5^{\tilde{g}} = -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\{[24 x f_6(x) + 120 \tilde{f}_6(x)](\delta_{23}^d)_{LL}(\delta_{23}^d)_{RR} - 180 \tilde{f}_6(x)(\delta_{23}^d)_{LR}(\delta_{23}^d)_{RL}\}, \tag{38}$$

where $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$ with $m_{\tilde{g}}$ and $m_{\tilde{q}}$ being the gluino mass and the average squark mass, respectively. The expressions for the functions $f_6(x)$ and $\tilde{f}_6(x)$ can be found in Ref. [12]. The Wilson coefficients $\tilde{C}_{1,2,3}$ are obtained by interchanging the $L \leftrightarrow R$ in the mass insertions appearing in $C_{1,2,3}$.

Note that the mass insertions $(\delta_{23}^d)_{LL}(\delta_{23}^d)_{RR}$ may give the dominant contribution to the transition matrix element, due to its large coefficient in $C_4^{\tilde{g}}$. In order to connect $C_i(M_S)$ at the SUSY scale $M_S$ with the corresponding low energy ones, $C_i(\mu)$ with $\mu \sim \mathcal{O}(m_b)$, one has to solve the renormalization group equations for the Wilson coefficients. The matrix elements of the operators $Q_i$ can be found in Ref. [13].

Now, we turn to the supersymmetric contribution to the amplitude for $B_s \to J/\psi \phi$. It turns out that the gluino exchanges through the $\Delta B = 1$ penguin diagrams give the dominant contributions to this process. The effective Hamiltonian for the $\Delta B = 1$ transitions through the penguin process can, in general, be expressed as

$$\mathcal{H}_{\rm eff}^{\Delta B=1} = \sum_{i=3}^{6} C_i O_i + C_g O_g + \sum_{i=3}^{6} \tilde{C}_i \tilde{O}_i + \tilde{C}_g \tilde{O}_g, \tag{39}$$

where

$$O_3 = \bar{s}_L^\alpha \gamma^\mu b_L^\alpha \bar{c}_L^\beta \gamma_\mu c_L^\beta, \tag{40}$$

$$O_4 = \bar{s}_L^\alpha \gamma^\mu b_L^\beta \bar{c}_L^\beta \gamma_\mu c_L^\alpha, \tag{41}$$

$$O_5 = \bar{s}_L^\alpha \gamma^\mu b_L^\alpha \bar{c}_R^\beta \gamma_\mu c_R^\beta, \tag{42}$$

$$O_6 = \bar{s}_L^\alpha \gamma^\mu b_L^\beta \bar{c}_R^\beta \gamma_\mu c_R^\alpha, \tag{43}$$

$$O_g = \frac{g_s}{8\pi^2} m_b \bar{s}_L^\alpha \sigma^{\mu\nu} \frac{\lambda_{\alpha\beta}^A}{2} b_R^\beta G_{\mu\nu}^A. \tag{44}$$

At the first order in the mass insertion approximation, the gluino contributions to the Wilson coefficients $C_{i,g}$ at the SUSY scale $M_S$ are given by [12]

$$\begin{aligned}
C_3(M_S) &= \frac{\alpha_s^2}{m_{\tilde{q}}^2}(\delta_{LL}^d)_{23}\left[\frac{1}{9}B_1(x) + \frac{5}{9}B_2(x) + \frac{1}{18}P_1(x) + \frac{1}{2}P_2(x)\right],\\
C_4(M_S) &= \frac{\alpha_s^2}{m_{\tilde{q}}^2}(\delta_{LL}^d)_{23}\left[\frac{7}{3}B_1(x) - \frac{1}{3}B_2(x) - \frac{1}{6}P_1(x) - \frac{3}{2}P_2(x)\right],\\
C_5(M_S) &= \frac{\alpha_s^2}{m_{\tilde{q}}^2}(\delta_{LL}^d)_{23}\left[-\frac{10}{9}B_1(x) - \frac{1}{18}B_2(x) + \frac{1}{18}P_1(x) + \frac{1}{2}P_2(x)\right],\\
C_6(M_S) &= \frac{\alpha_s^2}{m_{\tilde{q}}^2}(\delta_{LL}^d)_{23}\left[\frac{2}{3}B_1(x) - \frac{7}{6}B_2(x) - \frac{1}{6}P_1(x) - \frac{3}{2}P_2(x)\right],\\
C_g(M_S) &= \frac{\alpha_s \pi}{m_{\tilde{q}}^2}\left[(\delta_{LL}^d)_{23}\left(\frac{1}{3}M_3(x) + 3M_4(x)\right) + (\delta_{LR}^d)_{23}\frac{m_{\tilde{g}}}{m_b}\left(\frac{1}{3}M_1(x) + 3M_2(x)\right)\right].
\end{aligned} \tag{45}$$

The absolute values of the mass insertions $(\delta^d_{AB})_{23}$, with $A, B = (L, R)$ are constrained by the experimental results for the branching ratio of the $B \to X_s \gamma$ decay. These constraints are very weak on the $LL$ and $RR$ mass insertions and the only limits we have come from their definition, $|(\delta^d_{LL,RR})_{23}| < 1$. The $LR$ and $RL$ mass insertions are more constrained and, for instance with $m_{\tilde{g}} \simeq m_{\tilde{q}} \simeq 500$ GeV, one obtains $|(\delta^d_{LR,RL})_{23}| \lesssim 1.6 \times 10^{-2}$ [7,12]. Note that although the $LR(RL)$ mass insertion are constrained severely, their effects to the decay are enhanced by a large factor $m_{\tilde{g}}/m_b$, as can be seen from the above expression for $C_g(M_S)$.

In light of the discussion above, the phases of $(\delta^d_{LR})_{23}$, $(\delta^d_{LL})_{23}$, and $(\delta^d_{RR})_{23}$ are the relevant *CP* violating phases for our process. In the next section, we discuss possible constraints imposed on these phases by the mercury EDM.

## IV. MERCURY EDM VERSUS LARGE $B_s^0$-$\bar{B}_s^0$ MIXING PHASE

It has been pointed out [14,15] that large values of $(\delta^d_{23})_{RR}$ may enhance the chromoelectric dipole moment of the strange quark, which is constrained by the experimental bound on the EDM of mercury atom $H_g$. In this section, we show that the $H_g$ EDM imposes a constraint on $\mathrm{Im}[(\delta^d_{LL})_{23}(\delta^d_{RR})_{23}]$, which may limit the supersymmetric contribution to the $B_s^0$-$\bar{B}_s^0$ mixing.

Using the $T$-odd nucleon-nucleon interaction, the mercury EDM is given by [14],

$$d_{Hg} = -e(d_d^C - d_u^C - 0.012 d_s^C) \times 3.2 \times 10^{-2}. \quad (46)$$

The chromoelectric EDM of the strange quark $d_s^C$ is given by

$$d_s^C = \frac{g_s \alpha_s}{4\pi} \frac{m_{\tilde{g}}}{m_{\tilde{d}}^2} \mathrm{Im}(\delta^d_{22})_{LR} M_2(x), \quad (47)$$

where $x = m_{\tilde{g}}^2/m_{\tilde{d}}^2$, $g_s$ is the $SU(3)_C$ gauge coupling, and the function $M_2(x)$ can be found in Ref. [12]. For $m_{\tilde{d}} = 500$ GeV and $x = 1$, the experimental limit on $H_g$ EDM leads to the following constraint on $(\delta^d_{23})_{LR}$:

$$\mathrm{Im}\,(\delta^d_{22})_{LR} < 5.6 \times 10^{-6}. \quad (48)$$

The mass insertion $(\delta^d_{22})_{LR}$ may be generated effectively through three mass insertions as follows:

$$(\delta^d_{22})_{LR} \simeq (\delta^d_{23})_{LL}(\delta^d_{33})_{LR}(\delta^d_{32})_{RR}, \quad (49)$$

where $(\delta^d_{33})_{LR} \simeq \frac{m_b(A_b - \mu \tan\beta)}{m_{\tilde{d}}^2} \simeq \mathcal{O}(10^{-2})$. Therefore, the $H_g$ EDM imposes the following constraint on the $LL$ and $RR$ mixing between the second and the third generations:

$$\mathrm{Im}\,[(\delta^d_{23})_{LL}(\delta^d_{23})^\dagger_{RR}] \lesssim 5.6 \times 10^{-4}. \quad (50)$$

If one assumes that $(\delta^d_{23})_{LL} \sim \lambda^2$ with negligible weak phase, then one gets the following bound on the $(\delta^d_{23})_{RR}$ mass insertion:

$$|(\delta^d_{23})_{RR}| \sin(\arg[(\delta^d_{23})_{RR}]) \lesssim 10^{-2}. \quad (51)$$

Therefore, in case $|(\delta^d_{23})_{RR}| \sim \mathcal{O}(0.01)$, the associated weak phase is essentially unconstrained. However, if $|(\delta^d_{23})_{RR}| \sim \mathcal{O}(0.1)$, the weak phase is constrained to be of order 0.1. In both cases, this will limit the SUSY contributions to the $B_s^0$-$\bar{B}_s^0$ mixing phase. It is worth noting that in the above estimation we consider the $\mu$ parameter to be of the order of the average down squark mass due to the implementation of the radiative electroweak symmetry breaking conditions. If one assumes that $\mu \tan\beta \gg m_{\tilde{d}}$, then $(\delta^d_{33})_{LR}$ is of order $\mathcal{O}(0.1)$ and hence a stronger constrain is imposed on $|(\delta^d_{23})_{RR}|$ [16].

We start our analysis for SUSY contribution to $\sin 2\beta_s$ by assuming that the $B_s^0$-$\bar{B}_s^0$ mixing may receive a significant SUSY contribution, while the decay of $B_s^0 \to J/\psi\phi$ is dominated by the SM. Therefore, we have $\mathrm{Im}[\rho(J/\psi\phi)] = 0$ and the induced *CP* asymmetry is given by $S_{J/\psi\phi} = \sin(2\beta_s^{\mathrm{SM}} + 2\theta_s)$. As an example for the SUSY contribution, we consider $m_{\tilde{q}} = 500$ GeV and $x = 1$, which leads to the following expression for $R = |\mathcal{M}_{12}^{\mathrm{SUSY}}/\mathcal{M}_{12}^{\mathrm{SM}}|$ [11]:

$$R = |1.44[(\delta^d_{23})^2_{LL} + (\delta^d_{23})^2_{RR}] + 27.57[(\delta^d_{23})^2_{LR} + (\delta^d_{23})^2_{RL}] - 44.76[(\delta^d_{23})_{LR}(\delta^d_{23})_{RL}] - 175.79[(\delta^d_{23})_{LL}(\delta^d_{23})_{RR}]|. \quad (52)$$

From this equation, it is noticeable that the dominant contribution to the $B_s^0$-$\bar{B}_s^0$ mixing is due to the mass insertions $(\delta^d_{23})_{LL}(\delta^d_{23})_{RR}$.

If one assumes that $(\delta^d_{23})_{LL}$ is induced by the running from the high scale, where left-handed squark masses are universal, down to the electroweak scale, then one finds $(\delta^d_{23})_{LL} \sim \lambda^2 \sim 0.04$. With a small source of nonuniversality in the right-handed squark sector, one can easily get $(\delta^d_{23})_{RR}$ of order $\mathcal{O}(0.1)$. Therefore, one gets $R \sim 0.7$. However in this case, the $H_g$ EDM implies that $\arg[(\delta^d_{RR})_{23}] \lesssim 0.1$, which limits significantly the SUSY effect for enhancing $\sin 2\beta_s$.

In Fig. 2, we present our results for the $B_s^0$-$\bar{B}_s^0$ mixing phase $2\beta_s$ as a function of $\arg[(\delta^d_{23})_{RR}]$ for $|(\delta^d_{23})_{RR}| = 0.025$, 0.05, and 0.1. At these values, the ratio $R$ is of order $\lesssim 0.17$, 0.35, and 0.7, respectively. As can be seen from this figure, the values of the $B_s^0$ mixing phase, which are consistent with the $H_g$ EDM constraints, are typically of order $\lesssim 0.1$. Therefore, one concludes that the SUSY contribution to the $B_s^0$-$\bar{B}_s^0$ mixing implies limited enhancement for $\sin 2\beta_s$ and thus cannot account for the new experimental results reported in Eqs. (1) and (2). Moreover, a salient feature of this scenario with large *RR* mixing is that it predicts a reachable mercury EDM in the future experiments.

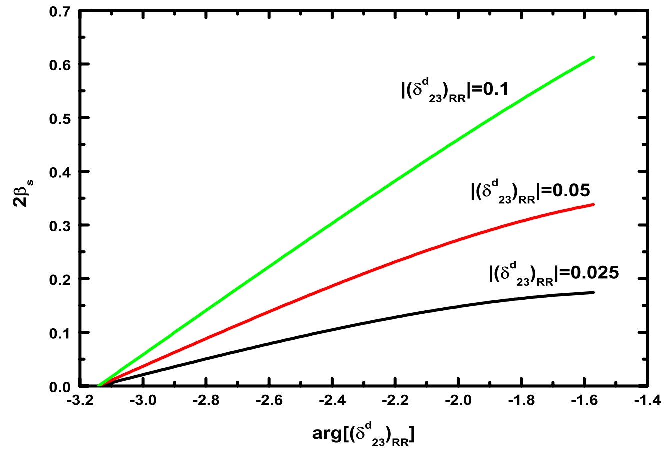


FIG. 2 (color online). The $B_s^0$-$\bar{B}_s^0$ mixing phase as a function of the $\arg[(\delta_{23}^d)_{RR}]$ (in radians) for $|(\delta_{23}^d)_{RR}| = 0.025$, 0.05, and 0.1.

## V. SUSY CONTRIBUTION TO $\bar{B}_s^0 \to J/\psi\phi$ DECAY

In this section, we will consider SUSY contribution to the decay $\bar{B}_s^0 \to J/\psi\phi$. However, let us discuss the complexities in analyzing new physics effects in the decay amplitude for vector-vector final state [17].

Consider a $B \to V_1 V_2$ decay which is dominated by a single weak decay amplitude within the SM. This holds for processes which are described by the quark-level decays $\bar{b} \to \bar{c} c \bar{s}$, which is the underlying quark transition in $\bar{B}_s^0 \to J/\psi\phi$. In this case, the weak phase of the SM amplitude is zero in the standard parametrization [18]. Suppose now that there is new physics in the decay amplitude, with different weak phases. The decay amplitude for each of the three possible helicity states may be written, following Eq. (27), as

$$\begin{aligned} A_\lambda &\equiv \mathrm{Amp}(B \to V_1 V_2)_\lambda = a_\lambda e^{i\delta_\lambda^a} + b_\lambda e^{i\phi_\lambda}, \\ \bar{A}_\lambda &\equiv \mathrm{Amp}(\bar{B} \to V_1 V_2)_\lambda = a_\lambda e^{i\delta_\lambda^a} + b_\lambda e^{-i\phi_\lambda}, \end{aligned} \tag{53}$$

where $a_\lambda$ and $b_\lambda$ represent the SM and NP amplitudes, respectively, $\phi_\lambda$ are the new physics weak phases, the $\delta_\lambda^a$ are the strong phases, and the helicity index $\lambda$ takes the values $\{0, \|, \perp\}$. Using $CPT$ invariance, the full decay amplitudes can be written as

$$\begin{aligned} \mathcal{A} &= \mathrm{Amp}(B \to V_1 V_2) = A_0 g_0 + A_\| g_\| + i A_\perp g_\perp, \\ \bar{\mathcal{A}} &= \mathrm{Amp}(\bar{B} \to V_1 V_2) = \bar{A}_0 g_0 + \bar{A}_\| g_\| - i \bar{A}_\perp g_\perp, \end{aligned} \tag{54}$$

where the $g_\lambda$ are the coefficients of the helicity amplitudes written in the linear polarization basis. The $g_\lambda$ depend only on the angles describing the kinematics [19]. Equations (53) and (54) above enable us to write the time-dependent decay rates as [17],

$$\begin{aligned} \Gamma(\bar{B}_s^0(t) \to V_1 V_2) = e^{-\Gamma t} \sum_{\lambda \le \sigma} (\Lambda_{\lambda\sigma} \pm \Sigma_{\lambda\sigma} \cos(\Delta M t) \\ \mp \rho_{\lambda\sigma} \sin(\Delta M t)) g_\lambda g_\sigma. \end{aligned} \tag{55}$$

Thus, by performing a time-dependent angular analysis of the decay $B(t) \to V_1 V_2$, one can measure 18 observables. These are

$$\begin{aligned} &\Lambda_{\lambda\lambda} = \frac{1}{2}(|A_\lambda|^2 + |\bar{A}_\lambda|^2), \quad \Sigma_{\lambda\lambda} = \frac{1}{2}(|A_\lambda|^2 - |\bar{A}_\lambda|^2), \\ &\Lambda_{\perp i} = -\mathrm{Im}(A_\perp A_i^* - \bar{A}_\perp \bar{A}_i^*), \quad \Lambda_{\|0} = \mathrm{Re}(A_\| A_0^* + \bar{A}_\| \bar{A}_0^*), \\ &\Sigma_{\perp i} = -\mathrm{Im}(A_\perp A_i^* + \bar{A}_\perp \bar{A}_i^*), \quad \Sigma_{\|0} = \mathrm{Re}(A_\| A_0^* - \bar{A}_\| \bar{A}_0^*), \\ &\rho_{\perp i} = \mathrm{Re}\left(\frac{q}{p}[A_\perp^* \bar{A}_i + A_i^* \bar{A}_\perp]\right), \quad \rho_{\perp\perp} = \mathrm{Im}\left(\frac{q}{p} A_\perp^* \bar{A}_\perp\right), \\ &\rho_{\|0} = -\mathrm{Im}\left(\frac{q}{p}[A_\|^* \bar{A}_0 + A_0^* \bar{A}_\|]\right), \quad \rho_{ii} = -\mathrm{Im}\left(\frac{q}{p} A_i^* \bar{A}_i\right), \end{aligned} \tag{56}$$

where $i = \{0, \|\}$. In the above, $q/p$ is the weak phase factor associated with $B_s^0$-$\bar{B}_s^0$ mixing. For $B_s^0$ meson, $q/p = \exp(-2i\beta_s)$. Note that $\beta_s$ may include NP effects in $B_s^0$-$\bar{B}_s^0$ mixing. Note also that the signs of the various $\rho_{\lambda\lambda}$ terms depend on the $CP$ parity of the various helicity states. We have chosen the sign of $\rho_{00}$ and $\rho_{\|\|}$ to be $-1$, which corresponds to the final state $J/\psi\phi$.

Not all of the 18 observables are independent. There are a total of six amplitudes describing $B \to V_1 V_2$ and $\bar{B} \to V_1 V_2$ decays [Eq. (53)]. Thus, at best one can measure the magnitudes and relative phases of these six amplitudes, giving 11 independent measurements.

The 18 observables given above can be written in terms of the 13 theoretical parameters: three $a_\lambda$'s, three $b_\lambda$'s, $\beta_s$, $\phi_\lambda$, and the strong phases $\delta_\lambda^a$. In the presence of new physics, one cannot extract the phase $\beta_s$. There are 11 independent observables, but 13 theoretical parameters. Since the number of measurements is fewer than the number of parameters, one cannot express any of the theoretical unknowns purely in terms of observables. In particular, it is impossible to extract $\beta_s$ cleanly.

In the absence of NP, the $b_\lambda$ are zero in Eq. (53). The number of parameters is then reduced from 13 to 6: three $a_\lambda$'s, two strong phase differences, and $\beta_s$. It is straightforward to show that all six parameters can be determined cleanly in terms of the observables. This is exactly what is done in the experimental measurements to measure $\beta_s$, the value of which appears to be inconsistent with the SM. This might indicate a new non-SM phase in $B_s$ mixing or NP in the decay amplitude in which case the general angular analysis in Eq. (55) should be used. In the presence of NP, the indirect $CP$ asymmetries for the various polarization states may no longer be the same as it is in the SM (up to a sign).

In this section, we will consider the scenario where SUSY gives significant contributions to both $B_s^0$-$\bar{B}_s^0$ mixing and the decay of $B_s^0 \to J/\psi\phi$. In this case, the induced $CP$ asymmetry is given by Eq. (25). As shown in Fig. 3, in the SM the decay of $B_s^0 \to J/\psi\phi$ takes place at tree level through the $b \to c$ transition. While the dominant SUSY

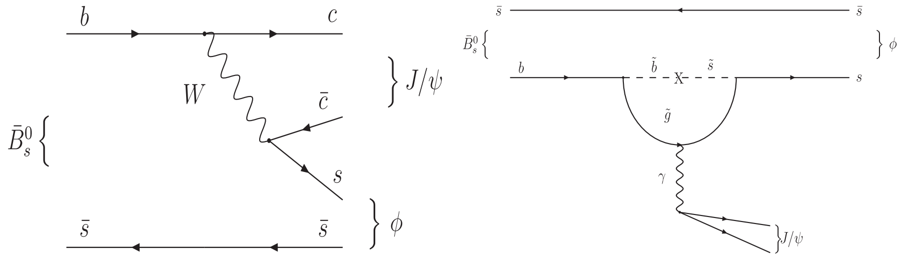


FIG. 3. SM tree level (left) and SUSY one loop (right) contributions to $\bar{B}^0_s \to J/\psi\phi$ decay.

contribution to this decay is given by the one loop level gluino exchange for $b \to s$ transition. It is interesting to note that the SM amplitude is proportional to $G_F \times V_{bc}V_{cs} \sim 10^{-7}$, while the SUSY amplitude is given in terms of $\alpha_s^2/m_{\tilde{q}}^2((\delta^d_{LR})_{23} \times m_{\tilde{g}}/m_b)$. Therefore, although the SUSY contribution is at the loop level, it can be important relative to the SM one. Hence, it is important to consider the impact of this contribution on the induced *CP* asymmetry $S^\lambda_{J/\psi\phi}$, as the phase of the mass insertion $(\delta^d_{LR})_{23}$ is not constrained by the EDM measurements.

Let us now write down the SM and SUSY contribution to $B^0_s(p) \to J/\psi(k_1, \epsilon_1)\phi(k_2, \epsilon_2)$, where we have labeled the momentum and polarization of the final-state particles. To proceed with our calculation, we will first specify the momentum and the polarization vectors of the final-state particles. We will work in the rest frame of the $B^0_s$ meson. We define the momentum and polarization of the vector $\phi$ meson as [20]

$$k_2^\mu = (E_\phi, 0, 0, -k) \qquad \varepsilon_2^\mu(0) = \frac{1}{m_\phi}(-k, 0, 0, E_\phi)$$
$$\varepsilon_2^\mu(\mp) = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0). \tag{57}$$

The momentum and polarization vectors for $J/\psi$ are defined as

$$k_1^\mu = (E_{J/\psi}, 0, 0, k) \qquad \varepsilon_1^\mu(0) = \frac{1}{m_{J/\psi}}(k, 0, 0, E_{J/\psi}),$$
$$\varepsilon_1^\mu(\pm) = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0). \tag{58}$$

The general amplitude for $\bar{B}^0_s(p) \to J/\psi(k_1, \varepsilon_1)\phi(k_2, \varepsilon_2)$ can be expressed as [21]

$$\bar{A} = \bar{a}\varepsilon_1^* \cdot \varepsilon_2^* + \frac{\bar{b}}{m^2_{B_s}}(p \cdot \varepsilon_1^*)(p \cdot \varepsilon_2^*) + i\frac{\bar{c}}{m^2_{B_s}}\epsilon_{\mu\nu\rho\sigma}p^\mu q^\nu \varepsilon_1^{*\rho}\varepsilon_2^{*\sigma}, \tag{59}$$

where $q = k_1 - k_2$. For angular analysis it is useful to use the linear polarization basis. In this basis, one decomposes the decay amplitude into components in which the polarizations of the final-state vector mesons are either longitudinal ($A_0$) or transverse to their directions of motion and parallel ($A_\parallel$) or perpendicular ($A_\perp$) to one another. One writes [22,23]

$$\bar{A} = \bar{A}_0\varepsilon_1^{*L} \cdot \varepsilon_2^{*L} - \frac{1}{\sqrt{2}}\bar{A}_\parallel \vec{\varepsilon}_1^{*T} \cdot \vec{\varepsilon}_2^{*T} - \frac{i}{\sqrt{2}}\bar{A}_\perp \vec{\varepsilon}_1^{*T} \times \vec{\varepsilon}_2^{*T} \cdot \hat{p}, \tag{60}$$

where $\hat{p}$ is the unit vector along the direction of motion of $V_2$ in the rest frame of $V_1$, $\varepsilon_i^{*L} = \vec{\varepsilon}_i^* \cdot \hat{p}$, and $\vec{\varepsilon}_i^{*T} = \vec{\varepsilon}_i^* - \varepsilon_i^{*L}\hat{p}$. $\bar{A}_0$, $\bar{A}_\parallel$, $\bar{A}_\perp$ are related to $\bar{a}$, $\bar{b}$, and $\bar{c}$ of Eq. (59) via

$$\bar{A}_\parallel = \sqrt{2}\bar{a}, \qquad \bar{A}_0 = -\bar{a}x - \frac{m_1 m_2}{m_B^2}\bar{b}(x^2 - 1),$$
$$\bar{A}_\perp = 2\sqrt{2}\frac{m_1 m_2}{m_B^2}\bar{c}\sqrt{x^2 - 1}, \tag{61}$$

where $x = k_1 \cdot k_2/(m_1 m_2)$. [A popular alternative basis is to express the decay amplitude in terms of helicity amplitudes $A_\lambda$, where $\lambda = 1, 0, -1$ [22,24]. The helicity amplitudes can be written in terms of the linear polarization amplitudes via $A_{\pm 1} = (A_\parallel \pm A_\perp)/\sqrt{2}$, with $A_0$ the same in both bases.]

We will now proceed to calculate the polarization dependent *CP* asymmetry given in Eq. (25). We will use factorization to calculate the ratio $r_A^\lambda = |A^\lambda_{\rm NP}/A^\lambda_{\rm SM}|$. In factorization there are no strong phases and we will keep them as a free unknown parameter in the expression for $S^\lambda_{J/\psi\phi}$ in Eq. (25). The amplitude for the process $\bar{B}_s(p) \to J/\psi(k_1, \varepsilon_1)\phi(k_2, \varepsilon_2)$ in the SM is given by

$$\bar{A}[\bar{B}_s \to J/\psi\phi] = \frac{G_F}{\sqrt{2}}XL_{J/\psi}, \tag{62}$$

with

$$X = V_{cb}V_{cs}^* a_2 - \sum_{q=u,c,t} V_{qb}V_{qs}^*(a_3^q + a_5^q + a_7^q + a_9^q),$$
$$L_{J/\psi} = m_{J/\psi}g_{J/\psi}\varepsilon_1^{*\mu}\langle\phi|\bar{s}\gamma_\mu(1 - \gamma_5)b|\bar{B}_s\rangle, \tag{63}$$

where $a_2 = c_2 + \frac{c_1}{N_c}$ and for $i > 2$, $a_i = c_i + \frac{c_{i+i}}{N_c}$, with $c_i$ being the Wilson's coefficient. Here $g_{J/\psi}$ is the $J/\psi$ decay constant defined in the usual manner.

We can simplify $X$ using several facts. First $a_2$ is much larger than $a_i^t$ with $i = 3, 5, 7, 9$ [25]. Second, in the penguin contributions in Eq. (63), we have included the rescattering contributions from the tree operators. However these are small and the contributions $a_3^{u,c}$ and $a_5^{u,c}$ due to perturbative QCD rescattering vanish because of the following relations:

$$c_{3,5}^{u,c} = -c_{4,6}^{u,c}/N_c = P_s^{u,c}/N_c, \quad (64)$$

where $N_c$ is the number of color. The leading contributions to $P_s^i$ are given by $P_s^i = (\frac{\alpha_s}{8\pi})c_1(\frac{10}{9} + G(m_i, \mu, q^2))$ with $i = u, c$. The function $G(m, \mu, q^2)$ is given by

$$G(m, \mu, q^2) = 4\int_0^1 x(1-x)\ln\frac{m^2 - x(1-x)q^2}{\mu^2}dx. \quad (65)$$

The rescattering via electroweak interactions are given by [26],

$$c_{7,9}^{u,c} = P_e^{u,c}, \qquad c_{8,10}^{u,c} = 0 \quad (66)$$

with $P_e^i = (\frac{\alpha_{\rm em}}{9\pi})(N_c c_2 + c_1)(\frac{10}{9} + G(m_i, \mu, q^2))$. These contributions are again much smaller than the dominant tree contributions and can be neglected.

In light of the above facts, we can conclude that the dominant contributions in $X$ in Eq. (63) come from the tree level terms where $c_1 = 1.081$ and $c_2 = -0.190$ are the relevant Wilson coefficients [25]. This leads to

$$X \approx V_{cb}V_{cs}^* a_2 = 0.17 V_{cb}V_{cs}^*. \quad (67)$$

At this point, we will discuss the validity of the factorization approximation in $B_s^0(p) \to J/\psi(k_1, \epsilon_1)\phi(k_2, \epsilon_2)$. One can compare this decay with $B^0(p) \to J/\psi(k_1, \epsilon_1)K^*(k_2, \epsilon_2)$. Both decays are related to one another in the $SU(3)$ limit. The decay $B^0(p) \to J/\psi(k_1, \epsilon_1)K^*(k_2, \epsilon_2)$ was studied in QCD factorization in Ref [27]. Naive factorization is unable to explain the branching ratio and the various polarization fractions in this decay. Using various models of form factors, one can extract $a_2$ from the experiment [27], which is found to be helicity dependent. It should be remembered that by the addition of new physics contribution to the decay amplitude the extracted values of $a_2$ in Ref [27] will be affected. Nonetheless, the extracted value of $a_2$ in Ref [27] for the different helicity amplitudes are not greatly different from the value in Eq. (67). We do not expect the situation to change much by including the new physics contribution to the decay amplitude. For our purpose, the use of $a_2$ in Eq. (67) is justified because the parameters in our new physics models are not precisely known. Hence our calculation should be understood as an estimate rather than a precise calculation.

The matrix elements in Eq. (63) above can be expressed in terms of form factors. This can be done as follows [28]:

$$\begin{aligned}
\langle V_2(k_2)|\bar{q}'\gamma_\mu b|\bar{B}_s(p)\rangle &= i\frac{2V^{(2)}(r^2)}{(m_B + m_2)}\epsilon_{\mu\nu\rho\sigma}p^\nu k_2^\rho \varepsilon_2^{*\sigma}, \\
\langle V_2(k_2)|\bar{q}'\gamma_\mu\gamma_5 b|B(p)\rangle &= (m_B + m_2)A_1^{(2)}(r^2) \\
&\quad\times\left[\varepsilon_{2\mu}^* - \frac{\varepsilon_2^*\cdot r}{r^2}r_\mu\right] - A_2^{(2)}(r^2) \\
&\quad\times\frac{\varepsilon_2^*\cdot r}{m_B + m_2}\left[(p_\mu + k_{2\mu})\right. \\
&\quad\left.-\frac{m_B^2 - m_2^2}{r^2}r_\mu\right] + 2im_2\frac{\varepsilon_2^*\cdot r}{r^2}r_\mu \\
&\quad\times A_0^{(2)}(r^2),
\end{aligned} \quad (68)$$

where $r = p - k_2$, and $V^{(2)}$, $A_1^{(2)}$, $A_2^{(2)}$, and $A_0^{(2)}$ are form factors.

Using Eq. (68) in Eq. (63) one obtains

$$\begin{aligned}
\bar{a}_{\rm SM} &= -\frac{G_F}{\sqrt{2}}m_{J/\psi}g_{J/\psi}x(m_{B_s} + m_\phi)A_1^{(2)}(m_{J/\psi}^2)X, \\
\bar{b}_{\rm SM} &= \frac{G_F}{\sqrt{2}}2m_{J/\psi}g_{J/\psi}\frac{m_{B_s}}{(m_{B_s} + m_\phi)}m_{B_s}A_2^{(2)}(m_{J/\psi}^2)X, \\
\bar{c}_{\rm SM} &= -\frac{G_F}{\sqrt{2}}m_{J/\psi}g_{J/\psi}\frac{m_{B_s}}{(m_{B_s} + m_\phi)}m_{B_s}V^{(2)}(m_{J/\psi}^2)X.
\end{aligned} \quad (69)$$

Let us turn now to the SUSY contribution. We will consider only the dominant chromomagnetic operators. The gluon in these operators can split into a charm-anticharm ($c\bar{c}$) quark pair, thereby contributing to $b \to s\bar{c}c$. We begin with a discussion on the matrix elements of the chromomagnetic operators $O_g$ and $\tilde{O}_g$. These are given by

$$\begin{aligned}
\langle J/\psi\phi|O_g|\bar{B}_s\rangle &= \langle O_g\rangle \\
&= -\frac{\alpha_s m_b}{\pi q^2}\langle J/\psi\phi|\left(\bar{s}_\alpha\gamma_\mu \not{q}(1+\gamma_5)\frac{\lambda_{\alpha\beta}^A}{2}b_\beta\right) \\
&\quad\times\left(\bar{c}_\rho\gamma^\mu\frac{\lambda_{\rho\sigma}^A}{2}c_\sigma\right)|\bar{B}_s\rangle, \\
\langle J/\psi\phi|\tilde{O}_g|\bar{B}_s\rangle &= \langle\tilde{O}_g\rangle \\
&= -\frac{\alpha_s m_b}{\pi q^2}\langle J/\psi\phi|\left(\bar{s}_\alpha\gamma_\mu \not{q}(1-\gamma_5)\frac{\lambda_{\alpha\beta}^A}{2}b_\beta\right) \\
&\quad\times\left(\bar{c}_\rho\gamma^\mu\frac{\lambda_{\rho\sigma}^A}{2}c_\sigma\right)|\bar{B}_s\rangle,
\end{aligned} \quad (70)$$

where $q^\mu$ is the momentum carried by the gluon in the penguin diagram. In our case $q^\mu$ coincides with the four momentum of the $J/\psi$.

After a color Fierz identity, we can write the operator $O_g$ as

$$O_g = Y_g\Big[-\frac{2}{N_c}\Big(\bar{s}_\alpha\gamma_\mu\frac{\not{q}}{m_b}(1+\gamma_5)b_\alpha\Big)(\bar{c}_\beta\gamma^\mu c_\beta)+\ldots\Big],$$
$$\tilde{O}_g = Y_g\Big[-\frac{2}{N_c}\Big(\bar{s}_\alpha\gamma_\mu\frac{\not{q}}{m_b}(1-\gamma_5)b_\alpha\Big)(\bar{c}_\beta\gamma^\mu c_\beta)+\ldots\Big],$$
$$Y_g = -\frac{\alpha_s m_b^2}{4\pi m_{J/\psi}^2}. \quad (71)$$

In the above we have only retained terms that contribute to the decay $\bar{B}_s(p)\to J/\psi(k_1,\varepsilon_1)\phi(k_2,\varepsilon_2)$. In factorization, after using equation of motion, we can write the matrix element of $O_g$ as

$$\langle O_g\rangle = T_1+T_2+T_3,$$
$$T_1 = C_gY_g\Big[-\frac{2}{N_c}L_{J/\psi}\Big],$$
$$L_{J/\psi} = m_{J/\psi}g_{J/\psi}\varepsilon_1^{*\mu}\langle\phi|\bar{s}\gamma_\mu(1-\gamma_5)b|\bar{B}_s\rangle,$$
$$T_2 = C_gY_g\frac{m_s}{m_b}\Big[-\frac{2}{N_c}R_{J/\psi}\Big],$$
$$R_{J/\psi} = m_{J/\psi}g_{J/\psi}\varepsilon_1^{*\mu}\langle\phi|\bar{s}\gamma_\mu(1+\gamma_5)b|\bar{B}_s\rangle,$$
$$T_3 = C_gY_g\frac{2\varepsilon_1^*\cdot k_2}{m_b}\Big[\frac{2}{N_c}S_{J/\psi}\Big],$$
$$S_{J/\psi} = m_{J/\psi}g_{J/\psi}\langle\phi|\bar{s}(1+\gamma_5)b|\bar{B}_s\rangle. \quad (72)$$

In the above equation, $m_{s,b}$ are the strange and the bottom quark masses.

In the above equation, it is clear that $T_2$ is suppressed relative to $T_1$ by $\frac{m_s}{m_b}$ and we will neglect it. From the structure of the polarization vectors in Eq. (57), it is also clear that the $\pm$ polarizations do not contribute to $T_3$. Hence for the $\pm$ polarizations, we can obtain a clear prediction for $r_A^\lambda$ defined below Eq. (25), as the form factors and other hadronic quantities cancel in the ratio.

For the matrix element of the operator $\tilde{O}_g$, focussing only on the transverse amplitudes we can write

$$\langle\tilde{O}_g\rangle = Y_g\Big[-\frac{2}{N_c}R_{J/\psi}\Big],$$
$$R_{J/\psi} = m_{J/\psi}g_{J/\psi}\varepsilon_1^{*\mu}\langle\phi|\bar{s}\gamma_\mu(1+\gamma_5)b|\bar{B}_s\rangle. \quad (73)$$

Hence, again focussing only on the transverse amplitudes we can write, using Eq. (68) in Eqs. (72) and (73)

$$\bar{a}_{\rm SUSY} = -\frac{G_F}{\sqrt{2}}m_{J/\psi}g_{J/\psi}(m_{B_s}+m_\phi)A_1^{(2)}(m_{J/\psi}^2)(Y-\tilde{Y}),$$
$$\bar{c}_{\rm SUSY} = -\frac{G_F}{\sqrt{2}}m_{J/\psi}g_{J/\psi}\frac{m_{B_s}}{(m_{B_s}+m_\phi)}m_{B_s}V^{(2)}(m_{J/\psi}^2)(Y+\tilde{Y}),$$
$$Y = \frac{\sqrt{2}C_g}{G_F}Y_g\Big[-\frac{2}{N_c}\Big],\quad \tilde{Y} = \frac{\sqrt{2}\tilde{C}_g}{G_F}Y_g\Big[-\frac{2}{N_c}\Big],$$
$$Y_g = -\frac{\alpha_s m_b^2}{4\pi m_{J/\psi}^2}. \quad (74)$$

Combining the SM and SUSY contributions we can now compute,

$$r_A^{\parallel} = |A_{\rm NP}^{\parallel}/A_{\rm SM}^{\parallel}| = \Big|\frac{(Y-\tilde{Y})}{X}\Big|,$$
$$r_A^{\perp} = |A_{\rm NP}^{\perp}/A_{\rm SM}^{\perp}| = \Big|\frac{(Y+\tilde{Y})}{X}\Big|. \quad (75)$$

Using the values of $V_{cb}$ and $V_{cs}$ from Ref. [18], we obtain $X\approx 0.0069$. Furthermore, with $m_{\tilde{g}} = m_{\tilde{q}} = 500$ GeV, $m_b(m_b) = 4.5$ GeV, we obtain

$$Y \approx= 2.1315(\delta_{LR}^d)_{23}\Big[\frac{-2}{N_c}Y_g\Big] = 0.0477(\delta_{LR}^d)_{23},$$
$$\tilde{Y} \approx= 2.1315(\delta_{RL}^d)_{23}\Big[\frac{-2}{N_c}Y_g\Big] = 0.0477(\delta_{RL}^d)_{23}. \quad (76)$$

We can then write, using Eq. (75),

$$r_A^{\parallel} \approx 0.07\frac{\sqrt{(|(\delta_{LR}^d)_{23}|)^2+(|(\delta_{RL}^d)_{23}|)^2-2|(\delta_{LR}^d)_{23}||(\delta_{RL}^d)_{23}|\cos(\theta_{LR}-\theta_{RL})}}{0.01},$$
$$r_A^{\perp} \approx 0.07\frac{\sqrt{(|(\delta_{LR}^d)_{23}|)^2+(|(\delta_{RL}^d)_{23}|)^2+2|(\delta_{LR}^d)_{23}||(\delta_{RL}^d)_{23}|\cos(\theta_{LR}-\theta_{RL})}}{0.01}, \quad (77)$$

where $\theta_{LR}$ and $\theta_{RL}$ are the phases of $(\delta_{LR}^d)_{23}$ and $(\delta_{RL}^d)_{23}$. We will set $|(\delta_{LR}^d)_{23}| = |(\delta_{RL}^d)_{23}| = 0.01$ and we can then now consider the following cases:
Case a: $(\delta_{LR}^d)_{23} = (\delta_{RL}^d)_{23}$. In this case we obtain

$$S_{J/\psi\phi}^{\parallel} = \sin(2\beta_s^{\rm SM}+2\theta_s),$$
$$S_{J/\psi\phi}^{\perp} = \sin(2\beta_s^{\rm SM}+2\theta_s)+0.28\cos(2\beta_s^{\rm SM}+2\theta_s)\times\sin\theta_{\rm NP}^{\perp}\cos\delta^{\perp}. \quad (78)$$

If we neglect the contribution from mixing then $S_{J/\psi\phi}^{\perp}$ can reach a value of up to $\pm 0.3$ for $\sin\theta_{\rm NP}^{\perp}\sim\pm 1$ and $\cos\delta^{\perp}\sim 1$.Case b: $(\delta_{LR}^d)_{23} = -(\delta_{RL}^d)_{23}$. In this case we obtain

$$S_{J/\psi\phi}^{\parallel} = \sin(2\beta_s^{\rm SM}+2\theta_s)+0.28\cos(2\beta_s^{\rm SM}+2\theta_s)\times\sin\theta_{\rm NP}^{\parallel}\cos\delta^{\parallel},$$
$$S_{J/\psi\phi}^{\perp} = \sin(2\beta_s^{\rm SM}+2\theta_s). \quad (79)$$

Again, if we neglect the contribution from mixing, then $S^{\parallel}_{J/\psi\phi}$ can reach a value of up to $\pm 0.3$ for $\sin\theta^{\parallel}_{\rm NP} \sim \pm 1$ and $\cos\delta^{\parallel} \sim 1$. Finally, we can consider the case where either $(\delta^d_{LR})_{23}$ or $(\delta^d_{RL})_{23}$ is zero. For the case $(\delta^d_{LR})_{23} \neq 0$, $(\delta^d_{RL})_{23} = 0$, we obtain

$$\begin{aligned} S^{\parallel}_{J/\psi\phi} &= \sin(2\beta_s^{\rm SM} + 2\theta_s) + 0.14\cos(2\beta_s^{\rm SM} + 2\theta_s) \\ &\quad \times \sin\theta^{\parallel}_{\rm NP}\cos\delta^{\parallel}, \\ S^{\perp}_{J/\psi\phi} &= \sin(2\beta_s^{\rm SM} + 2\theta_s) + 0.14\cos(2\beta_s^{\rm SM} + 2\theta_s) \\ &\quad \times \sin\theta^{\perp}_{\rm NP}\cos\delta^{\perp}. \end{aligned} \tag{80}$$

For the case $(\delta^d_{LR})_{23} = 0$, $(\delta^d_{RL})_{23} \neq 0$, we obtain

$$\begin{aligned} S^{\parallel}_{J/\psi\phi} &= \sin(2\beta_s^{\rm SM} + 2\theta_s) - 0.14\cos(2\beta_s^{\rm SM} + 2\theta_s) \\ &\quad \times \sin\theta^{\parallel}_{\rm NP}\cos\delta^{\parallel}, \\ S^{\perp}_{J/\psi\phi} &= \sin(2\beta_s^{\rm SM} + 2\theta_s) + 0.14\cos(2\beta_s^{\rm SM} + 2\theta_s) \\ &\quad \times \sin\theta^{\perp}_{\rm NP}\cos\delta^{\perp}. \end{aligned} \tag{81}$$

Now one may wonder how NP in $b \to s\bar{c}c$ transitions affect $CP$ measurements in the $B_d$ system. Let us first consider the indirect $CP$ asymmetry in the golden mode $B_d \to J/\psi K_s$. Note, this is a vector-pseudoscalar decay and so the strong phases involved here can be quite different from the ones involved in vector-vector decays. In other words, NP effects in different final states can be very different. More interestingly, it can be easily checked that for case b in Eq. (79) the contribution to the indirect asymmetry in $B_d \to J/\psi K_s$ cancels. However, the indirect $CP$ asymmetry in the vector-vector mode does not cancel for all polarization states. In other words, the range of NP effects obtained in the decay $B_s \to J/\psi\phi$ are consistent with $\sin 2\beta$ measurements in $B_d \to J/\psi K_s$ [29–31] for the various reasons discussed above.

The decay $B_d \to J/\psi K^*$ is related to $B_s^0 \to J/\psi\phi$ by $SU(3)$ flavor symmetry. Hence we should potentially see NP effects in $B_d \to J/\psi K^*$, up to $SU(3)$ breaking effects. The $CP$ measurements in this decay are not yet precise [29] and hence this decay also is an ideal place to look for new physics effects in the decay amplitude.

## VI. SUMMARY

In summary, we have analyzed the SUSY contribution to $B_s^0$-$\bar{B}_s^0$ mixing in light of the recent experimental measurement of the mixing phase. We showed that the experimental limits of the mass difference $\Delta M_{B_s}$ and the mercury EDM constrain significantly the SUSY contribution to $B_s^0$-$\bar{B}_s^0$ mixing, so that $\sin 2\beta_s \lesssim 0.1$. We then studied the one loop SUSY contribution to the $B_s^0 \to J/\psi\phi$ decay and found that new physics contribution to the decay amplitude can lead to significant indirect $CP$ asymmetries which are in general different for different polarization states.

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