Flavor violation in supersymmetric Q_6 model

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We investigate flavor violation mediated by Higgs bosons and supersymmetric particles in a predictive class of models based on the non-Abelian flavor symmetry Q_6. These models, which aim to reduce the number of parameters of the fermion sector and to solve the flavor changing problems of generic supersymmetry (SUSY) setup, assume three families of Higgs bosons and spontaneous or soft violation of CP symmetry. Tree-level contributions to meson-antimeson mixings mediated by Higgs bosons are shown to be within experimental limits for Higgs masses in the (1–5) TeV range. Calculable flavor violation induced by SUSY loops are analyzed for meson mixing and lepton decays and found to be consistent with data. Significant new SUSY contributions arise in B_s–$\bar{B}_s$ mixing, but nonstandard CP violation is suppressed. A simple solution to the SUSY CP problem is found, which requires light Higgsinos.

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I. INTRODUCTION

The gauge interactions of the standard model (SM) fermions are invariant under separate $U(3)_L \times U(3)_R$ transformations. This global symmetry is broken explicitly by the fermion Yukawa couplings. In the light fermion sector violation of this symmetry is small, being proportional to their masses. This feature has played a crucial role in the success of the SM in the flavor sector. In extensions of the SM this property is generally lost, often leading to excessive flavor changing neutral current (FCNC) processes.

A case in point is the supersymmetric (SUSY) standard model which is the subject of this paper. While the gauge interactions of the SUSY SM respect the $U(3)_L \times U(3)_R$ global symmetry, there are new sources of violation of this symmetry, in the soft SUSY breaking sector. Indeed, generic soft SUSY breaking scenarios lead to excessive FCNC in processes such as K^0–$\bar{K}^0$ mixing, B^0–$\bar{B}^0$ mixing, D^0–$\bar{D}^0$ mixing, and flavor changing leptonic decays such as $\mu \to e\gamma$ [1]. This problem is most severe in the K^0–$\bar{K}^0$ system. SUSY box diagrams involving gluino and squarks modify the successful SM prediction for ΔM_K and ϵ_K, leading to the following constraints for the real and imaginary parts of the amplitude [2]:

$$|(\mathrm{Re}, \mathrm{Im})(\delta^d_{LL})_{12}(\delta^d_{RR})_{12}|^{1/2} \leq (9.6 \times 10^{-4}, 1.3 \times 10^{-4}) \times \left(\frac{\tilde{m}}{500\ \mathrm{GeV}}\right). \quad (1.1)$$

Here $(\delta_{AB})_{ij} = (m^2_{AB})_{ij}/\tilde{m}^2$ is a flavor violating squark mass insertion parameter, for $(A, B) = (L, R)$, with $\tilde{m}$ being the average mass of the relevant squarks ($\tilde{d}$ and $\tilde{s}$ in this case). For this estimate the gluino mass was assumed to equal the average squark mass. Now, the natural magnitude of the mixing parameters $(\delta^d_{LL})_{12}$ and $(\delta^d_{RR})_{12}$, in the absence of additional symmetries, should be of order the Cabibbo angle, ~ 0.2. Since the parameters $(\delta_{AB})_{ij}$ split the masses of the squarks, one sees from Eq. (1.1) that a high degree of squark mass degeneracy is needed for consistency.

Analogous limits from B^0_d–$\bar{B}^0_d$ mixing are less severe, as given by [3]:

$$|(\mathrm{Re}, \mathrm{Im})(\delta^d_{LL})_{13}(\delta^d_{RR})_{13}|^{1/2} \leq (2.1 \times 10^{-2}, 9.0 \times 10^{-3}) \times \left(\frac{\tilde{m}}{500\ \mathrm{GeV}}\right). \quad (1.2)$$

Note that the natural value of this mixing parameter, in the absence of other symmetries, is $V_{ub} \sim 3 \times 10^{-3}$. The constraints from Eq. (1.2) are well within limits. B_s–$\bar{B}_s$ mixing provides even weaker constraints.

It can be argued that a natural explanation for solving this problem is to enhance the symmetry of the SUSY SM by assuming a non-Abelian symmetry G (a subgroup of the $U(3)_L \times U(3)_R$) that pairs the first two families into a doublet, with the third family transforming trivially [4].[1] Invariance under G will then lead to degeneracy of squarks, as needed for phenomenology. A variety of such models have been proposed in the literature [4–8]. In Ref. [4], $SU(2)$ family symmetry and its variants were proposed to solve the SUSY FCNC problem. If the symmetry is global, one has to deal with the Goldstone bosons associated with its spontaneous breaking. Global symmetries are susceptible to violations from quantum gravity. Local gauge symmetries are more natural, but in the SUSY context there would be new FCNC processes arising from the

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[1] Grouping all three families into an irreducible triplet representation of G is also possible. The large top quark mass however reduces the original $U(3)_L \times U(3)_R$ symmetry to $U(2)_L \times U(2)_R$, so we find it is easier to work with (2 + 1) assignment.

family $SU(2)$ D terms [9]. Exceptions to this generic problem are known to exist [5].

A more natural solution to the problem is perhaps to choose G to be a non-Abelian discrete symmetry group [6]. In this case there would be no D term problem, since there are no gauge bosons associated with G. Spontaneous breaking of such symmetries will not lead to Goldstone bosons. If the symmetry breaking occurs before the inflationary era, such models should also be safe from potential cosmological domain wall problems. Such non-Abelian discrete symmetries have found application in understanding the various puzzles associated with the quark and lepton masses and mixing angles with or without supersymmetry [10], more recently for understanding the tribimaximal neutrino mixing pattern [11]. It would be desirable to find a symmetry that sheds light on the fermion mass and mixing puzzle, and at the same time solves the SUSY FCNC problem.

The supersymmetric standard model has another problem. In the flavor conserving sector CP violation is generically too large. Neutron and electron electric dipole moments (EDM) receive new contributions from SUSY loops. Unless the new phases in the SUSY breaking sector are small or conspire to be small, experimental limits on the EDM of the neutron (d_n), electron (d_e), and atoms will be violated by 2 to 3 orders of magnitude (depending on the squark and slepton masses) [12,13]. The imaginary parts of the left-right squark mixing parameters must satisfy the constraints (from the experimental constraints $d_n < 6.3 \times 10^{-26}$ e-cm, $d_e < 4.3 \times 10^{-27}$ e-cm) [14]

$$\begin{aligned} \mathrm{Im}[(\delta^d_{LR})_{11}] &\le 1.9 \times 10^{-6}\left(\frac{\tilde{m}}{500\ \mathrm{GeV}}\right), \\ \mathrm{Im}[(\delta^e_{LR})_{11}] &\le 1.7 \times 10^{-7}\left(\frac{\tilde{m}}{100\ \mathrm{GeV}}\right), \end{aligned} \tag{1.3}$$

assuming that the gluino and Bino have the same mass as the squark and slepton. Now, since these mixing parameters are expected to be suppressed by fermion helicity factors (but enhanced by the minimal supersymmetric standard model (MSSM) parameter $\tan\beta$) the natural values for these mixing parameters are of order $(1 \times 10^{-4}, 3 \times 10^{-6})$, respectively (for $\tan\beta = 10$ and assuming order one phases). This implies that the CP violating phases arising from the soft SUSY breaking sector must satisfy $\theta_d \le 1/53$, $\theta_e \le 1/63$ [for gluino (Bino) mass of 500 GeV (100 GeV)]. Why this is so, while the Kobayashi-Maskawa phase takes order 1 value, is the SUSY CP puzzle. It would be desirable to resolve this puzzle based on a symmetry principle in the same context where the SUSY FCNC problem is solved.

The purpose of this paper is to study a recently proposed SUSY model based on the non-Abelian symmetry group Q_6 [7] which addresses these issues. Q_6 is a finite subgroup of $SU(2)$ with 12 elements. Apart from providing a solution to the SUSY flavor problem, this class of models can also constrain the quark masses and mixings. It was shown in Ref. [7] that with the assumption of spontaneous (or soft) CP violation, there is a nontrivial relation between quark masses and mixings in this model. This sum rule was found to be consistent with experimental data.

A crucial aspect of the Q_6 model relevant for the quark mixing sum rule is that CP violation occurs either spontaneously or softly. This can help ameliorate the SUSY CP problem mentioned above. CP invariance requires that the gaugino masses, the μ terms, and the trilinear A terms be all real. In the Q_6 model of Ref. [7] it was found that there is a phase alignment mechanism that makes the phases of the sfermion mixing terms arising from the A terms to align with the phases of the fermion masses. So SUSY CP violation is suppressed to a large extent. However, spontaneously induced complex vacuum expectation values (VEVs) do lead to nonzero contributions to EDM. Here we analyze these contributions. Since these complex VEVs are accompanied by the Higgsino μ terms, a simple solution to the problem is found by making the Higgsinos to be lighter than the squarks. Adequate suppression of EDM is obtained for $\mu \sim 100$ GeV, while squark masses are of order 500 GeV. This suggestion obviously has testable implications for physics that will be probed at the LHC.

The fermion mass matrices that allow for a nontrivial prediction and the phase alignment is a generalization of well studied models [15]. The mass matrices for up and down quarks and the charged leptons take the following form:

$$\mathbf{M} = \begin{pmatrix} 0 & C & 0 \\ \pm C & 0 & B \\ 0 & B' & A \end{pmatrix}. \tag{1.4}$$

The main feature of such mass matrices is that the phases can be factorized, i.e., $M = P \cdot M^0 \cdot Q$, with M^0 being real and P, Q being diagonal phase matrices. This feature, when combined with the Q_6 symmetry, has the interesting consequence that CP violation induced by SUSY loops are suppressed. This will be discussed in more detail in Sec. IV.

The form of Eq. (1.4) can be obtained in renormalizable theories based on Q_6 symmetry. This requires the introduction of three families of Higgs doublets, which fall into $2 + 1$ representations of the Q_6 group, very much like the quarks and leptons. With multiple Higgs fields coupling to fermions, invariably there will be tree-level FCNC mediated by the Higgs bosons. The flavor changing Higgs couplings are not arbitrary, but can be computed in terms of the fermion masses and mixings. We will show that these FCNC processes are within acceptable range, provided that the Higgs boson masses lie in the (1–5) TeV range [except of course for the standard model-like Higgs boson, which has a mass in the (100–130) GeV range]. While Higgsinos are naturally light in this scenario, in the bosonic sector only the lightest SM-like Higgs will be accessible to LHC experiments.

One of our major results is that nonstandard *CP* violation is highly suppressed in this class of models. The phase factorizability of the fermion mass matrices implies that much of the SUSY induced *CP* violation is small. The structure of the Yukawa couplings in the model implies that the amplitudes for tree-level FCNC induced by neutral Higgs bosons are nearly real (see discussions in Sec. V). While there can be significant new contributions to meson-antimeson mixings, there is very little *CP* violation beyond the standard model.

Our analysis is similar in spirit to that of Ref. [8]. Our approach is slightly different, with some differences in analytical results, fits, spectrum, and conclusions. In particular, we have presented complete analytical results for the Higgs boson spectrum, and we have a new proposal to solve the SUSY EDM problem, which requires light Higgsinos. We have also derived generalized constraints on SUSY FCNC parameters for the $B_{d,s}$–$\bar{B}_{d,s}$ system appropriate for a $(2+1)$ mass spectrum.

The plan of the paper is as follows. In Sec. II we describe the SUSY Q_6 model, lay out the parameter choice, and summarize the prediction for the quark sector. In Sec. III we analyze the Higgs potential involving the three pairs of Higgs doublets. We provide analytic expressions for the mass spectrum of Higgs bosons as well as numerical fits. Consistency of symmetry breaking and spontaneous *CP* violation will be established here. In Sec. IV we address tree-level FCNC processes mediated by the heavy Higgs bosons. Section V is devoted to analysis of the SUSY flavor violation and EDM within the model. In Sec. VI we conclude.

II. SUPERSYMMETRIC Q_6 MODEL

Q_6 is the binary dihedral group, a subgroup of $SU(2)$, of order 12. It has the presentation

$$\{A, B; A^6 = E, B^2 = A^3, B^{-1}AB = A^{-1}\}. \tag{2.1}$$

The 12 elements of Q_6 can be represented as

$$\{E, A, A^2, \ldots, A^5, B, BA, BA^2, \ldots, BA^5\}. \tag{2.2}$$

In the two dimensional representation the generators are given in a certain basis by

$$\mathbf{A} = \begin{pmatrix} \cos\frac{\pi}{3} & \sin\frac{\pi}{3} \\ -\sin\frac{\pi}{3} & \cos\frac{\pi}{3} \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}. \tag{2.3}$$

The irreducible representation of Q_6 fall into $2, 2', 1, 1', 1'', 1'''$, where the 2 is complex-valued but pseudoreal, while the $2'$ is real valued. (Q_6 is the simplest group with two distinct doublet representations, which is very useful for model building.) The 1 and $1'$ are real representations, while $1''$ and $1'''$ are complex conjugates to each other. The group multiplication rules are given as

$$\begin{aligned} &1' \times 1' = 1, \qquad 1'' \times 1'' = 1', \qquad 1''' \times 1''' = 1', \\ &1'' \times 1''' = 1, \qquad 1' \times 1''' = 1'', \qquad 1' \times 1'' = 1''' \end{aligned} \tag{2.4}$$

$$\begin{aligned} &2 \times 1' = 2, \qquad 2 \times 1'' = 2', \qquad 2 \times 1''' = 2', \\ &2' \times 1' = 2', \qquad 2' \times 1'' = 2, \qquad 2' \times 1''' = 2 \end{aligned} \tag{2.5}$$

$$\begin{aligned} 2 \times 2 = 1 + 1' + 2', \qquad 2' \times 2' = 1 + 1' + 2', \\ 2 \times 2' = 1'' + 1''' + 2. \end{aligned} \tag{2.6}$$

The Clebsch-Gordon coefficients for these multiplication can be found in Ref. [7].

The fermions of all sectors (up-quark, down-quark, charged leptons) are assigned to $2+1$ representations of Q_6. The model assumes three families of Higgs bosons, which are also assigned to $2+1$ under Q_6. Their transformation properties are given by

$$\begin{aligned} \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = 2, \qquad \psi^c = \begin{pmatrix} -\psi_1^c \\ \psi_2^c \end{pmatrix} = 2', \\ \psi_3 = 1' \qquad \psi_3^c = 1''', \end{aligned} \tag{2.7}$$

$$H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = 2', \qquad H_3 = 1'''. \tag{2.8}$$

Here ψ generically denotes the fermion fields, and H denotes the up-type and the down-type Higgs fields which are doublets of $SU(2)_L$. Because of the constraints of supersymmetry, H^u and H_3^u couple only to up quarks, while H^d and H_3^d couple to down-type quarks and leptons. The Yukawa couplings of the model in the down quark sector arise from the superpotential

$$\begin{aligned} W = {} & \alpha_d \psi_3 \psi_3^c H_3 + \beta_d \psi^T \tau_1 \psi_3^c H - \beta'_d \psi_3 \psi^{cT} i\tau_2 H \\ & + \delta_d \psi^T \tau_1 \psi^c H_3 + \text{H.c.} \end{aligned} \tag{2.9}$$

with similar results for up-type quarks and charged leptons. This leads to the mass matrix for the down quarks given by

$$\mathbf{M_d} = \begin{pmatrix} 0 & \delta_d v_{d3} & \beta_d v_{d2} \\ -\delta_d v_{d3} & 0 & \beta_d v_{d1} \\ \beta'_d v_{d2} & \beta'_d v_{d1} & \alpha_d v_{d3} \end{pmatrix}. \tag{2.10}$$

Here v_{d1}, v_{d2}, v_{d3} are the vacuum expectation values of $H^d_{1,2,3}$ fields, which break the Q_6 symmetry.

Now, the potential of the Q_6 model admits an unbroken S_2 symmetry which interchanges $H_1^{u,d} \leftrightarrow H_2^{u,d}$. This unbroken symmetry allows us to choose a VEV pattern

$$v_{u1} = v_{u2}, \qquad v_{d1} = v_{d2},. \tag{2.11}$$

Consequently, a 45° rotation of the matrix in Eq. (2.10) in the 1–2 plane can be done both in the up and the down quark sectors without inducing Cabibbo-Kobayashi-Maskawa (CKM) mixing. This will bring the mass matrices to the desired form of Eq. (1.4). By using the unbroken S_2 symmetry, we make a 45° rotation on the Higgs fields, $\hat{H}_{1,2} = (H_1 \pm H_2)/\sqrt{2}$, so that $\hat{H}_1$ acquires a VEV, while $\langle \hat{H}_2 \rangle = 0$. We shall drop the hat on these redefined fields, and simply denote the VEV of the redefined H_1 as v_1.

We assume that CP is a good symmetry of the Lagrangian, and that it is broken spontaneously by the VEVs of scalar fields. If the full theory contains SM singlet Higgs fields, spontaneous CP violation in the singlet sector will show up as soft CP violation in the Higgs doublet sector. Explicit examples of this sort have been given in Ref. [7]. For now we simply assume that the Yukawa couplings in Eq. (2.9) are real, and the CKM CP violation has a spontaneous origin, via complex VEVs of the Higgs doublet fields. We denote the phase of these (redefined) VEVs as

$$\begin{aligned} \Delta\theta_u &= \arg(v_{u3}) - \arg(v_{u1}), \\ \Delta\theta_d &= \arg(v_{d3}) - \arg(v_{d1}). \end{aligned} \tag{2.12}$$

We make an overall 45° rotation on the Q_6 doublets, Q, D^c, and U^c, and then a phase rotations on these fields:

$$U \to P_u U, \qquad U^c \to P_{u^c} U^c \tag{2.13}$$

and similarly for D and D^c fields, where

$$\begin{aligned} P_{u,d} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \exp(i2\Delta\theta_{u,d}) & 0 \\ 0 & 0 & \exp(i\Delta\theta_{u,d}) \end{pmatrix}, \\ P_{u^c,d^c} &= \begin{pmatrix} \exp(-i2\Delta\theta_{u,d}) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \exp(-i\Delta\theta_{u,d}) \end{pmatrix}. \end{aligned} \tag{2.14}$$

This will make the originally complex mass matrices of Eq. (1.4) real, which we parametrize as

$$M_{u,d} = m^0_{t,b} \begin{pmatrix} 0 & q_{u,d}/y_{u,d} & 0 \\ -q_{u,d}/y_{u,d} & 0 & b_{u,d} \\ 0 & b'_{u,d} & y^2_{u,d} \end{pmatrix}. \tag{2.15}$$

These real mass matrices can be diagonalized by the following orthogonal transformations:

$$\begin{aligned} O^T_{u,d} M_{u,d} M^T_{u,d} O_{u,d} &= \begin{pmatrix} m^2_{u,d} & 0 & 0 \\ 0 & m^2_{c,s} & 0 \\ 0 & 0 & m^2_{t,b} \end{pmatrix}, \\ O^T_{u^c,d^c} M^T_{u,d} M_{u^c,d^c} O_{u^c,d^c} &= \begin{pmatrix} m^2_{u,d} & 0 & 0 \\ 0 & m^2_{c,s} & 0 \\ 0 & 0 & m^2_{t,b} \end{pmatrix}. \end{aligned} \tag{2.16}$$

The CKM matrix $V_{\rm CKM}$ is then given by

$$V_{\rm CKM} = O^T_u P_q O_d, \tag{2.17}$$

where

$$P_q = P^\dagger_u P_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i2\theta_q} & 0 \\ 0 & 0 & e^{i\theta_q} \end{pmatrix} \tag{2.18}$$

with $\theta_q = \Delta\theta_d - \Delta\theta_u$.

Now it is clear how the Q_6 setup reduces the number of parameters in the quark sector. The total number of parameters in the quark sector is nine (four real parameters each in M_u and M_d, plus a single phase θ_q), which should fit ten observables. Spontaneous CP violation is crucial for this reduction of parameters. With explicit CP violation, there would have been one more phase parameter. The single prediction of this model was numerically studied in Ref. [7], and shown to be fully consistent with data. Here we present a numerical fit to all the quark sector observables, which deviates somewhat from the fit given in Ref. [7]. The difference arises since here we have attempted to be consistent with the recent lattice determination of light quark masses. An excellent fit to the quark masses and mixings, including CKM CP violation, is obtained with the following choice of parameters at a momentum scale of $\mu = 1$ TeV:

$$\begin{gathered} m^0_t = 150.7\ \text{GeV}, \qquad m^0_b = 2.5515\ \text{GeV}, \\ \theta_q = \Delta\theta_d - \Delta\theta_u = -1.40, \qquad q_u = 1.5142 \times 10^{-4}, \\ b_u = 0.0395, \qquad b'_u = 0.0770474, \qquad y_u = 0.99746, \\ q_d = 0.0043435, \qquad b_d = 0.02609, \\ b'_d = 0.69138, \qquad y_d = 0.8100. \end{gathered} \tag{2.19}$$

This choice yields at $\mu = 1$ TeV, the following masses and mixings for the quarks:

$$\begin{gathered} m_u = 1.13\ \text{MeV}, \qquad m_c = 0.461\ \text{GeV}, \\ m_t = 150.50\ \text{GeV}, \qquad m_d = 2.53\ \text{MeV}, \\ m_s = 50.99\ \text{MeV}, \qquad m_b = 2.43\ \text{GeV}, \\ |V_{\rm CKM}| = \begin{pmatrix} 0.9745 & 0.2244 & 0.0033 \\ 0.2242 & 0.9737 & 0.0408 \\ 0.0093 & 0.0399 & 0.9991 \end{pmatrix}, \\ \eta_W = 0.3465, \end{gathered} \tag{2.20}$$

where η_W is the CP violation parameter in the Wolfenstein parametrization. These values, when extrapolated to lower energy scales, give extremely good agreement with data [16].

We have computed the orthogonal matrices that diagonalize M_u and M_d. These rotation matrices will be relevant for our discussion of Higgs-induced flavor violation, as well as FCNC arising via SUSY loop diagrams. We find

$$O_d = \begin{pmatrix} 0.9840 & -0.1782 & 0.0041 \\ 0.1781 & 0.9838 & 0.0188 \\ -0.0074 & -0.0178 & 0.9998 \end{pmatrix},$$
$$O_{d^c} = \begin{pmatrix} 0.9645 & -0.2640 & -0.0001 \\ -0.1817 & 0.6642 & 0.7251 \\ 0.1915 & -0.6994 & 0.6886 \end{pmatrix}, \tag{2.21}$$
$$O_u = \begin{pmatrix} 0.9988 & -0.0495 & 1.17\times 10^{-5} \\ 0.0494 & 0.9980 & 0.0395 \\ -0.0020 & -0.0394 & 0.9992 \end{pmatrix},$$
$$O_{u^c} = \begin{pmatrix} 0.9988 & 0.0496 & -6.00\times 10^{-6} \\ -0.0494 & 0.9958 & 0.0771 \\ 0.0038 & -0.0770 & 0.9970 \end{pmatrix}.$$

In the case of charged leptons, there is some arbitrariness in the values of $(A, B, B', C)_\ell$ of Eq. (1.4), since we have three observables (charged lepton masses) and four parameters (without including the neutrino sector). We shall present a fit with a simplifying assumption $B'_\ell = B_\ell$. At $\mu = 1$ TeV, a consistent fit for all the lepton masses is found with the following input values:

$$A_\ell = 1.675\,36 \text{ GeV}, \qquad B_\ell = B'_\ell = 0.430\,588 \text{ GeV}, \qquad C_\ell = 0.007\,428\,77 \text{ GeV}. \tag{2.22}$$

These yield the following eigenvalues at $\mu = 1$ TeV:

$$m_e = 0.4963 \text{ MeV}, \qquad m_\mu = 104.686 \text{ MeV}, \qquad m_\tau = 1779.5 \text{ MeV}. \tag{2.23}$$

These values correspond to the central values of charged lepton masses when extrapolated down to their respective mass scales [16]. The orthogonal matrix that diagonalizes M_e is given by

$$O_e = \begin{pmatrix} 0.9976 & 0.0688 & 9.81\times 10^{-4} \\ 0.0664 & -0.9697 & 0.2352 \\ -0.0171 & 0.2346 & 0.9720 \end{pmatrix}, \tag{2.24}$$

with O_{e^c} obtained from the above by flipping the signs in the first row and column.

III. SYMMETRY BREAKING AND THE HIGGS BOSON SPECTRUM

We now turn to the discussion of symmetry breaking and the Higgs boson spectrum in the model. We shall confine here to the case of having three pairs of Higgs doublets, and no Higgs singlets in the low energy theory. It is however, assumed that singlet fields are present in the full theory, so that spontaneous Q_6 breaking in the singlet sector appears as soft breaking in the doublet sector. As shown in Ref. [7], it is possible to realize such a scenario while preserving the $1 \leftrightarrow 2$ interchange symmetry for members (1, 2) inside Q_6 doublets. We seek a consistent picture where CP violating phases are generated in the Higgs doublet VEVs. As it turns out, CP also has to be softly broken in the bilinear soft SUSY breaking terms, or else there would be no CP phases in the VEVs.

The superpotential that we consider is the most general one consistent with softly broken Q_6 symmetry, but preserving the S_2 interchange symmetry:

$$\begin{aligned} W_{\text{eff}} = {} & \mu_1(H_1^u H_1^d + H_2^u H_2^d) + \mu_3 H_3^u H_3^d \\ & + \mu_{13}(H_1^u + H_2^u)H_3^d + \mu_{31}H_3^u(H_1^d + H_2^d) \\ & + \mu_{12}(H_1^u H_2^d + H_1^d H_2^u). \end{aligned} \tag{3.1}$$

As mentioned earlier, we make a 45° rotation in H_1^d, H_2^d and H_1^u, H_2^u space, with $\hat{H}^u_{1,2} = \frac{H_1^u \pm H_2^u}{\sqrt{2}}$ and $\hat{H}^d_{1,2} = \frac{H_1^d \pm H_2^d}{\sqrt{2}}$, so that the superpotential becomes

$$\begin{aligned} W_{\text{eff}} = {} & (\mu_1 + \mu_{12})\hat{H}_1^u \hat{H}_1^d + (\mu_1 - \mu_{12})\hat{H}_2^u \hat{H}_2^d \\ & + \mu_3 \hat{H}_3^u \hat{H}_3^d + \sqrt{2}\mu_{13}\hat{H}_1^u \hat{H}_3^d + \sqrt{2}\mu_{31}\hat{H}_3^u \hat{H}_1^d. \end{aligned} \tag{3.2}$$

The redefined fields have $\langle \hat{H}_2^u \rangle = \langle \hat{H}_2^d \rangle = 0$. We work in the hatted basis from now on, and drop the hat on the new fields.

The soft SUSY breaking Lagrangian is given, in the rotated basis, as

$$\begin{aligned} V_{\text{soft}} = {} & (b_1 + b_{12})H_1^u \epsilon H_1^d + (b_1 - b_{12})H_2^u \epsilon H_2^d \\ & + b_3 H_3^u \epsilon H_3^d + \sqrt{2}b_{13}H_1^u \epsilon H_3^d + \sqrt{2}b_{31}H_3^u \epsilon H_1^d \\ & + \text{H.c.} + m_{d1}^2(|H_1^d|^2 + |H_2^d|^2) + m_{d3}^2|H_3^d|^2 \\ & + m_{u1}^2(|H_1^u|^2 + |H_2^u|^2) + m_{u3}^2|H_3^u|^2, \end{aligned} \tag{3.3}$$

where $\epsilon = i\sigma_2$.

The full scalar potential including the soft terms, the F terms, and the D terms has the form

$$\begin{aligned} V = {} & M_{d1}^2(|H_1^{d0}|^2 + |H_1^{d-}|^2) + M_{d3}^2(|H_3^{d0}|^2 + |H_3^{d-}|^2) + M_{u1}^2(|H_1^{u0}|^2 + |H_1^{u+}|^2) + M_{u3}^2(|H_3^{u0}|^2 + |H_3^{u+}|^2) \\ & + \{M_{13}^2(H_1^{d0*}H_3^{d0} + H_1^{d-*}H_3^{d-}) + M_{31}^2(H_3^{u0*}H_1^{u0} + H_3^{u+*}H_1^{u+}) + \text{H.c.}\} + M_{d2}^2(|H_2^{d0}|^2 + |H_2^{d-}|^2) \\ & + M_{u2}^2(|H_2^{u0}|^2 + |H_2^{u+}|^2) + \{b_1'(H_1^{u+}H_1^{d-} - H_1^{u0}H_1^{d0}) + b_3(H_3^{u+}H_3^{d-} - H_3^{u0}H_3^{d0}) + \sqrt{2}b_{13}(H_1^{u+}H_3^{d-} - H_1^{u0}H_3^{d0}) \\ & + \sqrt{2}b_{31}(H_3^{u+}H_1^{d-} - H_3^{u0}H_1^{d0}) + b_2'(H_2^{u+}H_2^{d-} - H_2^{u0}H_2^{d0}) + \text{H.c.}\} + \frac{1}{8}(g_1^2 + g_2^2)(|H_1^{u+}|^2 + |H_1^{u0}|^2 + |H_3^{u+}|^2 \\ & + |H_3^{u0}|^2 + |H_2^{u+}|^2 + |H_2^{u0}|^2 - |H_1^{d-}|^2 - |H_1^{d0}|^2 - |H_3^{d-}|^2 - |H_3^{d0}|^2 - |H_2^{d-}|^2 - |H_2^{d0}|^2)^2 \\ & + \frac{1}{2}g_2^2|H_1^{u+}H_1^{d0*} + H_1^{u0}H_1^{d-*} + H_3^{u+}H_3^{d0*} + H_3^{u0}H_3^{d-*} + H_2^{u+}H_2^{d0*} + H_2^{u0}H_2^{d-*}|^2. \end{aligned} \tag{3.4}$$

Here we have redefined new effective parameters for convenience as

$$\begin{aligned} M_{d1}^2 &= |\mu_1 + \mu_{12}|^2 + 2|\mu_{31}|^2 + m_{d1}^2, \\ M_{d3}^2 &= |\mu_3|^2 + 2|\mu_{13}|^2 + m_{d3}^2, \\ M_{u1}^2 &= |\mu_1 + \mu_{12}|^2 + 2|\mu_{13}|^2 + m_{u1}^2, \\ M_{u3}^2 &= |\mu_3|^2 + 2|\mu_{31}|^2 + m_{u3}^2, \\ M_{d2}^2 &= |\mu_1 - \mu_{12}|^2 + m_{d1}^2, \\ M_{u2}^2 &= |\mu_1 - \mu_{12}|^2 + m_{u1}^2, \\ M_{13}^2 &= \sqrt{2}(\mu_1 + \mu_{12})^*\mu_{13} + \sqrt{2}\mu_3\mu_{31}^*, \\ M_{31}^2 &= \sqrt{2}(\mu_1 + \mu_{12})\mu_{31}^* + \sqrt{2}\mu_3^*\mu_{13}, \\ b_1' &= b_1 + b_{12}, \qquad b_2' = b_1 - b_{12}. \end{aligned} \tag{3.5}$$

Before analyzing the spectrum, let us note that the potential should be bounded from below along all D-flat directions. The following conditions should be satisfied:

$$\begin{gathered} M_{d1}^2 + M_{u1}^2 - 2|b_1'| > 0, \qquad M_{d1}^2 + M_{u2}^2 > 0, \\ M_{d1}^2 + M_{u3}^2 - 2\sqrt{2}|b_{31}| > 0 \qquad M_{d2}^2 + M_{u1}^2 > 0, \\ M_{d2}^2 + M_{u2}^2 - 2|b_2'| > 0, \qquad M_{d2}^2 + M_{u3}^2 > 0, \\ M_{d3}^2 + M_{u1}^2 - 2\sqrt{2}|b_{13}| > 0, \qquad M_{d3}^2 + M_{u2}^2 > 0, \\ M_{d3}^2 + M_{u3}^2 - 2|b_3| > 0. \end{gathered} \tag{3.6}$$

In our numerical analysis, we shall verify that these conditions are indeed met.

We parametrize the VEVs of the four neutral Higgs fields as

$$v_{u1} = v\sin\beta\sin\gamma_u e^{i\theta_{u1}}, \qquad v_{u3} = v\sin\beta\cos\gamma_u e^{i\theta_{u3}}, \qquad v_{d1} = v\cos\beta\sin\gamma_d e^{i\theta_{d1}}, \qquad v_{d3} = v\cos\beta\cos\gamma_d e^{i\theta_{d3}}. \tag{3.7}$$

Thus we have $|v_{u1}|^2 + |v_{u3}|^2 + |v_{d1}|^2 + |v_{d3}|^2 = v^2 = (174\ \text{GeV})^2$. $\gamma_{u(d)}$ reflect the orientation of the VEVs in the $H_{u(d)1}$–$H_{u(d)3}$ space, while $\tan\beta$ is analogous to the up and down VEV ratio of MSSM.

We can rewrite the potential of the H_1–H_3 sector of the neutral Higgs fields which acquire VEVs in a compact form

$$\begin{aligned} V_N^{(1-3)} = {} & \begin{pmatrix} H_1^{u0*} & H_3^{u0*} \end{pmatrix} \begin{pmatrix} M_{u1}^2 & M_{31}^2 \\ M_{31}^{2*} & M_{u3}^2 \end{pmatrix} \begin{pmatrix} H_1^{u0} \\ H_3^{u0} \end{pmatrix} + \begin{pmatrix} H_1^{d0*} & H_3^{d0*} \end{pmatrix} \begin{pmatrix} M_{d1}^2 & M_{13}^2 \\ M_{13}^{2*} & M_{d3}^2 \end{pmatrix} \begin{pmatrix} H_1^{d0} \\ H_3^{d0} \end{pmatrix} \\ & + \left[\begin{pmatrix} H_1^{u0} & H_3^{u0} \end{pmatrix} \begin{pmatrix} -b_1' & -\sqrt{2}b_{13} \\ -\sqrt{2}b_{31} & -b_3 \end{pmatrix} \begin{pmatrix} H_1^{d0} \\ H_3^{d0} \end{pmatrix} + \text{H.c.} \right] \\ & + \frac{1}{8}(g_1^2 + g_2^2)\left[\begin{pmatrix} H_1^{u0*} & H_3^{u0*} \end{pmatrix} \begin{pmatrix} H_1^{u0} \\ H_3^{u0} \end{pmatrix} - \begin{pmatrix} H_1^{d0*} & H_3^{d0*} \end{pmatrix} \begin{pmatrix} H_1^{d0} \\ H_3^{d0} \end{pmatrix} \right]^2. \end{aligned} \tag{3.8}$$

This suggests a unitary transformation that would diagonalize the first two matrices in Eq. (3.8), while leaving the D term unaffected. With such a rotation we have

$$\begin{aligned} V_N^{(1-3)} = {} & \begin{pmatrix} h_1^* & h_2^* \end{pmatrix} \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} + \begin{pmatrix} h_3^* & h_4^* \end{pmatrix} \begin{pmatrix} m_3^2 & 0 \\ 0 & m_4^2 \end{pmatrix} \begin{pmatrix} h_3 \\ h_4 \end{pmatrix} + \left[\begin{pmatrix} h_1 & h_2 \end{pmatrix} \begin{pmatrix} m_{13}^2 & m_{14}^2 \\ m_{23}^2 & m_{24}^2 \end{pmatrix} \begin{pmatrix} h_3 \\ h_4 \end{pmatrix} + \text{H.c.} \right] \\ & + \frac{1}{8}(g_1^2 + g_2^2)\left[\begin{pmatrix} h_1^* & h_2^* \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} - \begin{pmatrix} h_3^* & h_4^* \end{pmatrix} \begin{pmatrix} h_3 \\ h_4 \end{pmatrix} \right]^2. \end{aligned} \tag{3.9}$$

The unitary transformations to go from Eq. (3.8) to Eq. (3.9) are defined as

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = U_U \begin{pmatrix} H_1^u \\ H_3^u \end{pmatrix} = Q_u \begin{pmatrix} \cos\omega_u & -\sin\omega_u \\ \sin\omega_u & \cos\omega_u \end{pmatrix} \begin{pmatrix} e^{i\phi_u} & 0 \\ 0 & e^{i(\phi_u+\theta_{M_{31}})} \end{pmatrix} \begin{pmatrix} H_1^u \\ H_3^u \end{pmatrix},$$
$$\begin{pmatrix} h_3 \\ h_4 \end{pmatrix} = U_D \begin{pmatrix} H_1^d \\ H_3^d \end{pmatrix} = Q_d \begin{pmatrix} \cos\omega_d & -\sin\omega_d \\ \sin\omega_d & \cos\omega_d \end{pmatrix} \begin{pmatrix} e^{i\phi_d} & 0 \\ 0 & e^{i(\phi_d+\theta_{M_{13}})} \end{pmatrix} \begin{pmatrix} H_1^d \\ H_3^d \end{pmatrix}, \tag{3.10}$$

with $\theta_{M_{31}} = \arg(M_{31}^2)$, $\theta_{M_{13}} = \arg(M_{13}^2)$, and

$$\omega_u = \frac{1}{2}\tan^{-1}\left(\frac{2|M_{31}^2|}{M_{u3}^2 - M_{u1}^2}\right),$$
$$\omega_d = \frac{1}{2}\tan^{-1}\left(\frac{2|M_{13}^2|}{M_{d3}^2 - M_{d1}^2}\right). \tag{3.11}$$

The two phases ϕ_u and ϕ_d here are arbitrary. $\phi_u - \phi_d$ does not appear in the potential[being proportional to $U(1)_Y$ charges]. $\phi_u + \phi_d$ can be used to remove one phase of the bilinear terms in the potential. $Q_{u,d}$ are arbitrary diagonal phase matrices. If desired, one can take advantage of these phases to remove all but one phase from the parameters of the potential. Since we are interested in going back to the original basis from this rotated basis, we find it convenient to set $Q_{u,d}$ to be identity.

The other parameters of this transformation are

$$m_{1,2}^2 = \frac{1}{2}[M_{u3}^2 + M_{u1}^2 \pm \sqrt{(M_{u3}^2 - M_{u1}^2)^2 + 4|M_{31}^2|^2}],$$
$$m_{3,4}^2 = \frac{1}{2}[M_{d3}^2 + M_{d1}^2 \pm \sqrt{(M_{d3}^2 - M_{d1}^2)^2 + 4|M_{13}^2|^2}]. \tag{3.12}$$

and

$$\begin{pmatrix} m_{13}^2 & m_{14}^2 \\ m_{23}^2 & m_{24}^2 \end{pmatrix} = U_U^* \begin{pmatrix} -b_1' & -\sqrt{2}b_{13} \\ -\sqrt{2}b_{31} & -b_3 \end{pmatrix} U_D^\dagger. \tag{3.13}$$

If we choose

$$\phi_u + \phi_d = \pi + \arg[b_1' \sin\omega_u \sin\omega_d + \sqrt{2}b_{31}\cos\omega_u \sin\omega_d e^{-i\theta_{M_{31}}} + \sqrt{2}b_{13}\sin\omega_u\cos\omega_d e^{-i\theta_{M_{13}}} + b_3 \cos\omega_u \cos\omega_d e^{-i(\theta_{M_{31}}+\theta_{M_{13}})}], \tag{3.14}$$

m_{24}^2 is real and positive (with $Q_{u,d}$ set to identity). We shall adopt this phase convention in our numerical study. However, we shall present analytical results that hold in an arbitrary phase convention.

The task at hand is somewhat simplified, since Eq. (3.9) is relatively simple to analyze. The eight real neutral Higgs bosons in $H_{1,3}^{u,d}$ can be conveniently parametrized as (see e.g. Ref. [17])

$$h_1 = e^{i\delta_1}\left[v_1 + \frac{1}{\sqrt{2}}\left(\phi_1 + ie\phi_5 + ia\phi_7 + i\frac{v_1}{v}G\right)\right],$$
$$h_2 = e^{i\delta_2}\left[v_2 + \frac{1}{\sqrt{2}}\left(\phi_2 + if\phi_6 + ib\phi_7 + i\frac{v_2}{v}G\right)\right],$$
$$h_3 = e^{i\delta_3}\left[v_3 + \frac{1}{\sqrt{2}}\left(\phi_3 + ig\phi_5 + ic\phi_7 - i\frac{v_3}{v}G\right)\right],$$
$$h_4 = v_4 + \frac{1}{\sqrt{2}}\left(\phi_4 + ih\phi_6 + id\phi_7 - i\frac{v_4}{v}G\right). \tag{3.15}$$

Here v_i ($i = 1, 2, 3, 4$) are the magnitudes of the VEVs of the redefined fields h_i, and δ_i are their phases. Without loss of generality we have taken v_4 to be real. G in Eq. (3.15) is the Goldstone field eaten up by the Z gauge boson. We shall work in the unitary gauge and set $G = 0$. We have checked explicitly that the G field does not mix with other scalar fields, and that its mass is exactly zero. The coefficients of various fields in Eq. (3.15) are functions of the v_i's:

$$a = \frac{v_1\sqrt{v_2^2+v_4^2}}{v\sqrt{v_1^2+v_3^2}}, \qquad b = -\frac{v_2\sqrt{v_1^2+v_3^2}}{v\sqrt{v_2^2+v_4^2}},$$
$$c = -\frac{v_3\sqrt{v_2^2+v_4^2}}{v\sqrt{v_1^2+v_3^2}}, \qquad d = \frac{v_4\sqrt{v_1^2+v_3^2}}{v\sqrt{v_2^2+v_4^2}},$$
$$e = \frac{v_3}{\sqrt{v_1^2+v_3^2}}, \qquad f = \frac{v_4}{\sqrt{v_2^2+v_4^2}},$$
$$g = \frac{v_1}{\sqrt{v_1^2+v_3^2}}, \qquad h = \frac{v_2}{\sqrt{v_2^2+v_4^2}}. \tag{3.16}$$

We shall allow for the soft SUSY breaking parameters (b_i) in the Higgs potential to be complex. Phase rotations cannot remove all phases from the potential; one phase is unremovable. Without this phase, the model cannot induce complex VEVs to the doublets, as shown in Ref. [18] by a geometric argument. For the case when all parameters in the Higgs potential are real, we have numerically verified that the CP violating extremum would generate two massless modes, signaling inconsistency with symmetry breaking [18].

We take the soft bilinear terms m_{13}^2, m_{14}^2, m_{23}^2, m_{24}^2 of Eq. (3.9) to be complex, and denote the phase of m_{ij}^2 as θ_{ij}. The minimization conditions then read as

$$
\begin{aligned}
m_1^2 v_1 + |m_{13}^2| v_3 \cos(\theta_{13} + \delta_1 + \delta_3) + |m_{14}^2| v_4 \cos(\theta_{14} + \delta_1) + \tfrac{1}{4}(g_1^2 + g_2^2) v_1 (v_1^2 + v_2^2 - v_3^2 - v_4^2) &= 0,\\
m_2^2 v_2 + |m_{23}^2| v_3 \cos(\theta_{23} + \delta_2 + \delta_3) + |m_{24}^2| v_4 \cos(\delta_2 + \theta_{24}) + \tfrac{1}{4}(g_1^2 + g_2^2) v_2 (v_1^2 + v_2^2 - v_3^2 - v_4^2) &= 0,\\
m_3^2 v_3 + |m_{13}^2| v_1 \cos(\theta_{13} + \delta_1 + \delta_3) + |m_{23}^2| v_2 \cos(\theta_{23} + \delta_2 + \delta_3) - \tfrac{1}{4}(g_1^2 + g_2^2) v_3 (v_1^2 + v_2^2 - v_3^2 - v_4^2) &= 0,\\
m_4^2 v_4 + |m_{14}^2| v_1 \cos(\theta_{14} + \delta_1) + |m_{24}^2| v_2 \cos(\delta_2 + \theta_{24}) - \tfrac{1}{4}(g_1^2 + g_2^2) v_4 (v_1^2 + v_2^2 - v_3^2 - v_4^2) &= 0,\\
|m_{13}^2|(v_1^2 + v_3^2) \sin(\theta_{13} + \delta_1 + \delta_3) + |m_{23}^2| v_1 v_2 \sin(\theta_{23} + \delta_2 + \delta_3) + |m_{14}^2| v_3 v_4 \sin(\theta_{14} + \delta_1) &= 0,\\
|m_{24}^2|(v_2^2 + v_4^2) \sin(\theta_{24} + \delta_2) + |m_{14}^2| v_1 v_2 \sin(\theta_{14} + \delta_1) + |m_{23}^2| v_3 v_4 \sin(\theta_{23} + \delta_2 + \delta_3) &= 0,\\
|m_{14}^2| v_1 v_4 \sin(\theta_{14} + \delta_1) - |m_{23}^2| v_2 v_3 \sin(\theta_{23} + \delta_2 + \delta_3) &= 0.
\end{aligned}
\tag{3.17}
$$

Denoting the squared matrix for ϕ_i, $i = 1, 2, \ldots 7$ from the H_1–H_3 sector as

$$
\mathcal{M}^2_{0,(1-3)} = \mathcal{M}^2_{ij}, \tag{3.18}
$$

we obtain

$$
\begin{aligned}
\mathcal{M}_{11}^2 &= \lambda v_1^2 + \kappa \frac{v_2 v_4}{v_1^2}[\cot(\theta_{14} + \delta_1) - \cot(\theta_{13} + \delta_1 + \delta_3)],\\
\mathcal{M}_{22}^2 &= \lambda v_2^2 + \kappa \frac{v_4}{v_2}[\cot(\theta_{23} + \delta_2 + \delta_3) - \cot(\theta_{24} + \delta_2)],\\
\mathcal{M}_{33}^2 &= \lambda v_3^2 + \kappa \frac{v_2 v_4}{v_3^2}[\cot(\theta_{23} + \delta_2 + \delta_3) - \cot(\theta_{13} + \delta_1 + \delta_3)],\\
\mathcal{M}_{44}^2 &= \lambda v_4^2 + \kappa \frac{v_2}{v_4}[\cot(\theta_{14} + \delta_1) - \cot(\theta_{24} + \delta_2)],\\
\mathcal{M}_{55}^2 &= \kappa \frac{v_2 v_4}{v_1^2 + v_3^2}\left[\frac{v_3^2}{v_1^2}\cot(\theta_{14} + \delta_1) + \frac{v_1^2}{v_3^2}\cot(\theta_{23} + \delta_2 + \delta_3) - \frac{(v_1^2 + v_3^2)^2}{v_1^2 v_3^2}\cot(\theta_{13} + \delta_1 + \delta_3)\right],\\
\mathcal{M}_{66}^2 &= \kappa \frac{1}{v_2 v_4 (v_2^2 + v_4^2)}[v_2^4 \cot(\theta_{14} + \delta_1) + v_4^4 \cot(\theta_{23} + \delta_2 + \delta_3) - (v_2^2 + v_4^2)^2 \cot(\theta_{24} + \delta_2)],\\
\mathcal{M}_{77}^2 &= \kappa \frac{v_2 v_4 (v_1^2 + v_2^2 + v_3^2 + v_4^2)}{(v_1^2 + v_3^2)(v_2^2 + v_4^2)}[\cot(\theta_{14} + \delta_1) + \cot(\theta_{23} + \delta_2 + \delta_3)],
\end{aligned}
$$

$$
\mathcal{M}_{12}^2 = \lambda v_1 v_2, \qquad \mathcal{M}_{13}^2 = -\lambda v_1 v_3 + \kappa \frac{v_2 v_4}{v_1 v_3}\cot(\theta_{13} + \delta_1 + \delta_3), \qquad \mathcal{M}_{14}^2 = -\lambda v_1 v_4 - \kappa \frac{v_2}{v_1}\cot(\theta_{14} + \delta_1),
$$

$$
\mathcal{M}_{15}^2 = -\kappa \frac{v_2 v_4}{v_3\sqrt{v_1^2 + v_3^2}}, \qquad \mathcal{M}_{16}^2 = \kappa \frac{v_2^2}{v_1\sqrt{v_2^2 + v_4^2}}, \mathcal{M}_{17}^2 = \kappa \frac{v_2 v_4}{v_1}\frac{\sqrt{v_1^2 + v_2^2 + v_3^2 + v_4^2}}{\sqrt{v_1^2 + v_3^2}\sqrt{v_2^2 + v_4^2}},
$$

$$
\mathcal{M}_{23}^2 = -\lambda v_2 v_3 - \kappa \frac{v_4}{v_3}\cot(\theta_{23} + \delta_2 + \delta_3), \qquad \mathcal{M}_{24}^2 = -\lambda v_2 v_4 + \kappa \cot(\theta_{24} + \delta_2), \qquad \mathcal{M}_{25}^2 = \kappa \frac{v_1 v_4}{v_3\sqrt{v_1^2 + v_3^2}},
$$

$$
\mathcal{M}_{26}^2 = -\kappa \frac{v_2}{\sqrt{v_2^2 + v_4^2}}, \qquad \mathcal{M}_{27}^2 = -\kappa \frac{v_4\sqrt{v_1^2 + v_2^2 + v_3^2 + v_4^2}}{\sqrt{v_1^2 + v_3^2}\sqrt{v_2^2 + v_4^2}}, \qquad \mathcal{M}_{34}^2 = \lambda v_3 v_4, \qquad \mathcal{M}_{35}^2 = -\kappa \frac{v_2 v_4}{v_1\sqrt{v_1^2 + v_3^2}},
$$

$$
\mathcal{M}_{36}^2 = \kappa \frac{v_4^2}{v_3\sqrt{v_2^2 + v_4^2}}, \qquad \mathcal{M}_{37}^2 = -\kappa \frac{v_2 v_4}{v_3}\frac{\sqrt{v_1^2 + v_2^2 + v_3^2 + v_4^2}}{\sqrt{v_1^2 + v_3^2}\sqrt{v_2^2 + v_4^2}}, \qquad \mathcal{M}_{45}^2 = \kappa \frac{v_2 v_3}{v_1\sqrt{v_1^2 + v_3^2}},
$$

$$
\mathcal{M}_{46}^2 = -\kappa \frac{v_4}{\sqrt{v_2^2 + v_4^2}}, \qquad \mathcal{M}_{47}^2 = \kappa \frac{v_2\sqrt{v_1^2 + v_2^2 + v_3^2 + v_4^2}}{\sqrt{v_1^2 + v_3^2}\sqrt{v_2^2 + v_4^2}},
$$

$$\mathcal{M}^2_{56} = \kappa \frac{1}{v_1 v_3 \sqrt{v_1^2 + v_3^2}\sqrt{v_2^2 + v_4^2}}[v_2^2 v_3^2 \cot(\theta_{14} + \delta_1) + v_1^2 v_4^2 \cot(\theta_{23} + \delta_2 + \delta_3)],$$

$$\mathcal{M}^2_{57} = \kappa \frac{v_2 v_4}{v_1 v_3} \frac{\sqrt{v_1^2 + v_2^2 + v_3^2 + v_4^2}}{(v_1^2 + v_3^2)\sqrt{v_2^2 + v_4^2}}[v_3^2 \cot(\theta_{14} + \delta_1) - v_1^2 \cot(\theta_{23} + \delta_2 + \delta_3)],$$

$$\mathcal{M}^2_{67} = \kappa \frac{\sqrt{v_1^2 + v_2^2 + v_3^2 + v_4^2}}{\sqrt{v_1^2 + v_3^2}(v_2^2 + v_4^2)}[v_2^2 \cot(\theta_{14} + \delta_1) - v_4^2 \cot(\theta_{23} + \delta_2 + \delta_3)]. \quad (3.19)$$

Here we have defined $\lambda = (g_1^2 + g_2^2)/2 = M_Z^2/v^2$ and $\kappa = m_{24}^2 \sin(\theta_{24} + \delta_2)$.

The potential of the $H_2^u - H_2^d$ fields which do not acquire VEVs is

$$V_N^{(2)} = M_{u2}^2|H_2^u|^2 + M_{d2}^2|H_2^d|^2 - \{b_2' H_2^u H_2^d + \text{H.c.}\} + \frac{g_1^2 + g_2^2}{8}(|H_2^u|^2 - |H_2^d|^2 + |v_{u1}|^2 + |v_{u3}|^2 - |v_{d1}|^2 - |v_{d3}|^2)^2. \quad (3.20)$$

The corresponding squared mass matrix for the scalars in the basis $(\text{Re}H_2^u, \text{Im}H_2^u, \text{Re}H_2^d, \text{Im}H_2^d)$ is

$$\mathcal{M}^2_{0(2)} = \begin{pmatrix} M_{u2}^2 - \frac{m_Z^2}{2}\cos 2\beta & 0 & \text{Re}b_2' & -\text{Im}b_2' \\ 0 & M_{u2}^2 - \frac{m_Z^2}{2}\cos 2\beta & -\text{Im}b_2' & -\text{Re}b_2' \\ \text{Re}b_2' & -\text{Im}b_2' & M_{d2}^2 + \frac{m_Z^2}{2}\cos 2\beta & 0 \\ -\text{Im}b_2' & -\text{Re}b_2' & 0 & M_{d2}^2 + \frac{m_Z^2}{2}\cos 2\beta \end{pmatrix}. \quad (3.21)$$

This matrix has two pairs of degenerate eigenstates, owing to an unbroken $U(1)$ symmetry.

The $H_1^{u,d}$–$H_3^{u,d}$ sector charged Higgs boson mass matrix is, in the basis $\{H_1^{u+}, H_3^{u+}, H_1^{d-*}, H_3^{d-*}\}$,

$$\mathcal{M}^2_{\pm(1-3)} = (\mathcal{M}^2)_{ij},$$

with

$$\begin{aligned}
\mathcal{M}^2_{11} &= M_{u1}^2 - \tfrac{1}{2}m_Z^2\cos 2\beta + \frac{1}{2}g_2^2|v_{d1}|^2,\\
\mathcal{M}^2_{22} &= M_{u3}^2 - \tfrac{1}{2}m_Z^2\cos 2\beta + \tfrac{1}{2}g_2^2|v_{d3}|^2,\\
\mathcal{M}^2_{33} &= M_{d1}^2 + \tfrac{1}{2}m_Z^2\cos 2\beta + \tfrac{1}{2}g_2^2|v_{u1}|^2,\\
\mathcal{M}^2_{44} &= M_{d3}^2 + \tfrac{1}{2}m_Z^2\cos 2\beta + \tfrac{1}{2}g_2^2|v_{u3}|^2,\\
\mathcal{M}^2_{12} &= \mathcal{M}^{2*}_{21} = M_{31}^2 + \tfrac{1}{2}g_2^2 v_{d1}^* v_{d3},\\
\mathcal{M}^2_{13} &= \mathcal{M}^{2*}_{31} = b_1' + \tfrac{1}{2}g_2^2 v_{u1}^* v_{d1}^*,\\
\mathcal{M}^2_{14} &= \mathcal{M}^{2*}_{41} = \sqrt{2}b_{13} + \tfrac{1}{2}g_2^2 v_{u3}^* v_{d1}^*,\\
\mathcal{M}^2_{23} &= \mathcal{M}^{2*}_{32} = \sqrt{2}b_{31} + \tfrac{1}{2}g_2^2 v_{u1}^* v_{d3}^*,\\
\mathcal{M}^2_{24} &= \mathcal{M}^{2*}_{42} = b_3 + \tfrac{1}{2}g_2^2 v_{u3}^* v_{d3}^*,\\
\mathcal{M}^2_{34} &= \mathcal{M}^{2*}_{43} = M_{13}^2 + \tfrac{1}{2}g_2^2 v_{u1} v_{u3}^*.
\end{aligned} \quad (3.22)$$

Finally, the H_2^u–H_2^d sector charged Higgs mass matrix is, in the basis $\{H_2^{u+}, H_2^{d-*}\}$,

$$\mathcal{M}^2_{\pm(2)} = \begin{pmatrix} M_{u2}^2 - \frac{1}{2}m_Z^2\cos 2\beta & b_2' \\ b_2'^* & M_{d2}^2 + \frac{1}{2}m_Z^2\cos 2\beta \end{pmatrix} \quad (3.23)$$

Now we present two sets of numerical fits [cases (1) and (2)] which show the consistency of symmetry breaking. We are interested in choosing the SUSY breaking parameters (including the μ terms) around the TeV scale, guided by arguments of naturalness. At the same time we wish the spectrum to be consistent with FCNC constraints arising from meson-antimeson mixings. We have explored parameter space of the Higgs potential where both these constraints are met. For the FCNC constraint, we allow the new Higgs exchange contribution to ΔM be not more than the experimentally measured values. We emphasize that these two numerical examples are case studies, but they are representative of the allowed parameter space of the model. When we deviate significantly from these solutions, the (heavy) Higgs boson masses exceed several TeV, which would be in conflict with naturalness arguments.

Case (1)

The parameters in the original Higgs potential of Eq. (3.8) are taken to have the following values:

$$M_{d1} = 3.754\ \text{TeV}, \quad M_{d3} = 3.586\ \text{TeV},$$
$$M_{u1} = 4.782\ \text{TeV}, \quad M_{u3} = 2.152\ \text{TeV},$$
$$M_{31} = 2.336e^{i0.792}\ \text{TeV}, \quad M_{13} = 1.346e^{-i1.205}\ \text{TeV},$$
$$b'_1 = 3.144e^{i2.963}\ \text{TeV}^2, \quad b_3 = 3.196e^{i2.064}\ \text{TeV}^2,$$
$$b_{31} = 4.052e^{i2.186}\ \text{TeV}^2, \quad b_{13} = 3.438e^{i3.109}\ \text{TeV}^2,$$
$$M_{u2} = 4.550\ \text{TeV}, \quad M_{d2} = 4.850\ \text{TeV}$$
$$b'_2 = 0.000\ \text{TeV}^2. \tag{3.24}$$

In the representation of Eq. (3.9) this choice corresponds to

$$m_1 = 4.937\ \text{TeV}, \quad m_2 = 1.767\ \text{TeV},$$
$$m_3 = 3.923\ \text{TeV}, \quad m_4 = 3.401\ \text{TeV},$$
$$m_{13} = 1.851e^{-i1.437}\ \text{TeV}, \quad m_{14} = 2.736e^{-i0.732}\ \text{TeV},$$
$$m_{23} = 2.442e^{i1.347}\ \text{TeV}, \quad m_{24} = 2.104\ \text{TeV}. \tag{3.25}$$

For completeness we also give values of other parameters, $\omega_u = 0.70$, $\omega_d = 0.622$, $\phi_u + \phi_d = 1.005$.

We obtain numerically the VEV parameters to be

$$\tan\beta = 2.00, \quad \Delta\theta_d = -0.03, \quad \Delta\theta_u = 1.37,$$
$$\tan\gamma_d = 2.50, \quad \tan\gamma_u = 0.33. \tag{3.26}$$

The mass eigenvalues of the Higgs bosons in the H_1–H_3 sector are found to be

$$M_{h0} = (99.4, 115.1)\ \text{GeV}, \quad M_1 = 3.299\ \text{TeV},$$
$$M_2 - M_1 = 0.226\ \text{GeV}, \quad M_3 = 4.161\ \text{TeV},$$
$$M_4 - M_3 = 0.411\ \text{GeV}, \quad M_5 = 5.124\ \text{TeV},$$
$$M_6 - M_5 = 0.040\ \text{GeV}. \tag{3.27}$$

Note the appearance of nearly degenerate states (M_1, M_2) etc., with their mass splitting being proportional to $m_Z^2/4$. The Higgs bosons from the H_2 sector have degenerate masses given by

$$M_7 = M_8 = 4.850\ \text{TeV} \qquad M_9 = M_{10} = 4.550\ \text{TeV}. \tag{3.28}$$

The charged Higgs bosons are nearly degenerate with its neutral partner, so we list the mass splittings:

$$M_{\pm 1} - M_1 = -0.532\ \text{GeV},$$
$$M_{\pm 2} - M_3 = -0.156\ \text{GeV}, \tag{3.29}$$
$$M_{\pm 3} - M_5 = 0.032\ \text{GeV}.$$

In the $(H_2^u$–$H_2^d)$ sector, the two charged Higgs bosons are degenerate with the neutral ones given in Eq. (3.28).

The mass eigenstates H_i are mixtures of h_i, $i = 1, 2, \ldots 7$ states in the (1–3) sector. The orthogonal transformation that diagonalizes the mass matrix of Eq. (3.18) is

$$H^k = \begin{pmatrix} 0.066 & 0.892 & 0.271 & 0.356 & 8.60\times10^{-7} & 1.23\times10^{-6} & 2.15\times10^{-6} \\ 0.031 & -0.002 & 0.043 & -0.032 & -0.400 & 0.880 & 0.248 \\ 0.332 & -0.262 & -0.327 & 0.843 & 0.020 & 0.029 & 0.051 \\ -0.036 & 0.003 & -0.048 & 0.037 & 0.151 & 0.335 & -0.927 \\ -0.064 & 0.365 & -0.901 & -0.216 & 0.023 & 0.033 & 0.058 \\ 0.043 & -0.003 & 0.058 & -0.044 & 0.904 & 0.331 & 0.261 \\ -0.937 & -0.055 & -0.029 & 0.335 & 0.028 & 0.040 & 0.070 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \end{pmatrix}, \tag{3.30}$$

with $k = 0, \cdots 6$. Since $b'_2 = 0$ in this case, the H_2^0 mass matrix is diagonal, and thus the mass eigenstates are the original state.

Case (2)

Here we take the input parameters corresponding to Eq. (3.8) to be

$$M_{d1} = 3.980\ \text{TeV}, \quad M_{d3} = 5.412\ \text{TeV},$$
$$M_{u1} = 2.765\ \text{TeV}, \quad M_{u3} = 3.692\ \text{TeV},$$
$$M_{31} = 2.825e^{i0.781}\ \text{TeV}, \quad M_{13} = 1.693e^{-i0.949}\ \text{TeV},$$
$$b'_1 = 3.698e^{i1.495}\ \text{TeV}^2, \quad b_3 = 3.097e^{i1.522}\ \text{TeV}^2,$$
$$b_{31} = 7.420e^{i2.428}\ \text{TeV}^2, \quad b_{13} = 1.840e^{-i2.772}\ \text{TeV}^2,$$
$$M_{u2} = 3.550\ \text{TeV}, \quad M_{d2} = 5.850\ \text{TeV},$$
$$b'_2 = 1.234e^{i1.56}\ \text{TeV}^2. \tag{3.31}$$

This choice corresponds to parameters in Eq. (3.9) to be

$$m_1 = 4.377\ \text{TeV}, \quad m_2 = 1.154\ \text{TeV},$$
$$m_3 = 5.466\ \text{TeV}, \quad m_4 = 3.906\ \text{TeV},$$
$$m_{13} = 3.281e^{i1.271}\ \text{TeV}, \quad m_{14} = 1.702e^{i0.974}\ \text{TeV},$$
$$m_{23} = 3.190e^{-i0.501}\ \text{TeV}, \quad m_{24} = 2.326\ \text{TeV}, \tag{3.32}$$

with $\omega_u = -0.501$, $\omega_d = -0.606$, $\phi_u + \phi_d = 4.786$.

The Higgs VEV parameters are found for this input to be

$$\tan\beta = 2.40, \quad \Delta\theta_d = -0.06, \quad \Delta\theta_u = 1.34,$$
$$\tan\gamma_d = 1.80, \quad \tan\gamma_u = 1.00. \tag{3.33}$$

The mass spectrum of Higgs bosons in the H_1–H_3 sector is

$$M_{h0} = (104.1, 119.2)\ \text{GeV}, \qquad M_1 = 2.869\ \text{TeV},$$
$$M_2 - M_1 = 0.325\ \text{GeV} \qquad M_3 = 5.114\ \text{TeV},$$
$$M_4 - M_3 = 0.132\ \text{GeV}, \qquad M_5 = 5.658\ \text{TeV},$$
$$M_6 - M_5 = 0.087\ \text{GeV}, \tag{3.34}$$

while the mass eigenvalues of Eq. (3.21) are

$$M_7 = M_8 = 5.856\ \text{TeV} \qquad M_9 = M_{10} = 3.541\ \text{TeV}. \tag{3.35}$$

The charged Higgs boson masses are given by

$$M_{\pm 1} - M_1 = 0.225\ \text{GeV}, \qquad M_{\pm 2} - M_3 = 0.182\ \text{GeV},$$
$$M_{\pm 3} - M_5 = -0.064\ \text{GeV}, \tag{3.36}$$

with the remaining two charged Higgs bosons being degenerate with the neutral ones given in Eq. (3.35).

The orthogonal matrix that diagonalizes Eq. (3.18) is

$$H^k = \begin{pmatrix} 0.392 & 0.836 & 0.262 & 0.282 & 1.02\times 10^{-4} & -5.99\times 10^{-5} & 7.35\times 10^{-5} \\ 0.476 & -0.223 & -0.190 & 0.176 & 0.669 & 0.407 & 0.207 \\ -0.523 & -0.038 & 0.455 & 0.417 & 0.422 & -0.239 & 0.329 \\ 0.507 & -0.238 & 0.101 & -0.094 & 0.090 & -0.811 & 0.054 \\ -0.089 & 0.283 & 0.049 & -0.760 & 0.197 & 0.013 & 0.542 \\ -0.063 & 0.029 & 0.380 & -0.353 & 0.451 & 0.044 & -0.721 \\ 0.278 & -0.336 & 0.729 & -0.067 & -0.351 & 0.342 & 0.181 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \end{pmatrix}, \tag{3.37}$$

The matrix diagonalizing Eq. (3.21) is

$$H^k = \begin{pmatrix} -0.057 & 0.000 & 0.998 & 0.000 \\ 0.000 & -0.057 & 0.000 & 0.998 \\ 0.000 & 0.998 & 0.000 & 0.057 \\ 0.998 & 0.000 & 0.057 & 0.000 \end{pmatrix} \begin{pmatrix} \mathrm{Re}(H_2^u) \\ \mathrm{Im}(H_2^u) \\ \mathrm{Re}(H_2^d) \\ \mathrm{Im}(H_2^d) \end{pmatrix}, \tag{3.38}$$

with $k = 7, \cdots 10$.

In these fits, M_{h_0} is the light standard model-like Higgs boson mass, for which radiative corrections are significant. In our computation we have included known two loop corrections. The two values listed for M_{h_0} correspond to zero and maximal left-right stop mixing ($X_t = 0$ or 6). We have taken $m_t = 174$ GeV, $M_{\text{SUSY}} = 1.5$ TeV, and $\alpha_s(m_t) = 0.108$ for these evaluations and used the analytic approximation given in Ref. [19]. We see that the lightest Higgs boson mass cannot exceed about 121 GeV with these numerical fits (allowing for uncertainties in m_t and α_s). Recall that the bound on m_{h0} in the MSSM is about 130 GeV. Here since $\tan\beta$ has been chosen to be small so that FCNC constraints are satisfied [$\tan\beta = 2.4$ in case (2)], the tree-level contribution to m_{h0} is suppressed.

An interesting feature of these two fits is that the diagonal entries of the quadratic mass matrix of the potential of Eq. (3.8) are all positive. This of course does not preclude some soft squared masses turning negative as in the MSSM via large top quark Yukawa coupling (since the diagonal entries also receive μ term contributions), however, this is not necessary for symmetry breaking to be triggered. Yet, one of the eigenvalues of this matrix is negative, which facilitates symmetry breaking. For the two cases we find these eigenvalues to be

$$\text{Case (1)}: \{(5.123\ \text{TeV})^2, (4.161\ \text{TeV})^2, (3.300\ \text{TeV})^2, -(38.682\ \text{GeV})^2\},$$
$$\text{Case (2)}: \{(5.658\ \text{TeV})^2, (5.115\ \text{TeV})^2, (2.869\ \text{TeV})^2, -(45.40\ \text{GeV})^2\}. \tag{3.39}$$

The conditions for boundedness of the potential listed in Eq. (3.6) are found to be satisfied for both cases.

Neutralino and Chargino masses

The symmetry breaking parameters do not fully determine the masses of the neutralinos and the charginos. Here we present analytical results for their mass matrices.

The mass matrix of $\tilde{H}_1$–$\tilde{H}_3$ sector neutralino in the basis of $\{\tilde{B}, \tilde{W}^0, \tilde{H}_1^{u0}, \tilde{H}_3^{u0}, \tilde{H}_1^{d0}, \tilde{H}_3^{d0}\}$ is

$$\mathcal{M}_{\chi^0(13)} = \begin{pmatrix} M_{\tilde{B}} & 0 & \frac{g_2 v_{u1}}{\sqrt{2}} & \frac{g_2 v_{u3}}{\sqrt{2}} & -\frac{g_2 v_{d1}}{\sqrt{2}} & -\frac{g_2 v_{d3}}{\sqrt{2}} \\ 0 & M_{\tilde{W}} & -\frac{g_1 v_{u1}}{\sqrt{2}} & -\frac{g_1 v_{u3}}{\sqrt{2}} & \frac{g_1 v_{d1}}{\sqrt{2}} & \frac{g_1 v_{d3}}{\sqrt{2}} \\ \frac{g_2 v_{u1}}{\sqrt{2}} & -\frac{g_1 v_{u1}}{\sqrt{2}} & 0 & 0 & -(\mu_1 + \mu_{12}) & -\sqrt{2}\mu_{13} \\ \frac{g_2 v_{u3}}{\sqrt{2}} & -\frac{g_1 v_{u3}}{\sqrt{2}} & 0 & 0 & -\sqrt{2}\mu_{31} & -\mu_3 \\ -\frac{g_2 v_{d1}}{\sqrt{2}} & \frac{g_1 v_{d1}}{\sqrt{2}} & -(\mu_1 + \mu_{12}) & -\sqrt{2}\mu_{31} & 0 & 0 \\ -\frac{g_2 v_{d3}}{\sqrt{2}} & \frac{g_1 v_{d3}}{\sqrt{2}} & -\sqrt{2}\mu_{13} & -\mu_3 & 0 & 0 \end{pmatrix}. \tag{3.40}$$

The mass matrix of the $\tilde{H}_2$ sector in the basis $\{\tilde{H}_2^{u0}, \tilde{H}_2^{d0}\}$ is

$$\mathcal{M}_{\chi^0(2)}\begin{pmatrix} 0 & -(\mu_1 - \mu_{12}) \\ -(\mu_1 - \mu_{12}) & 0 \end{pmatrix}. \quad (3.41)$$

The mass matrix of charginos of the $\tilde{H}_1$–$\tilde{H}_3$ sector in the basis $\{\tilde{W}^+, \tilde{H}_1^{u+}, \tilde{H}_3^{u+}, \tilde{W}^-, \tilde{H}_1^{d-}, \tilde{H}_3^{d-}\}$ has a block-diagonal form:

$$\mathcal{M}_{\chi^\pm(13)} = \begin{pmatrix} 0 & \mathbf{X}^{\mathbf{T}} \\ \mathbf{X} & 0 \end{pmatrix} \quad (3.42)$$

with

$$\mathbf{X} = \begin{pmatrix} M_{\tilde{W}} & g_1 v_{u1} & g_1 v_{u3} \\ g_1 v_{d1} & \mu_1 + \mu_{12} & \sqrt{2}\mu_{31} \\ g_1 v_{d3} & \sqrt{2}\mu_{13} & \mu_3 \end{pmatrix}. \quad (3.43)$$

The chargino mass matrix in the $\tilde{H}_2^u$–$\tilde{H}_2^d$ sector in the basis of $\{\tilde{H}_2^{u+}, \tilde{H}_2^{d-}\}$ is

$$\mathcal{M}_{\chi^\pm(2)} = \begin{pmatrix} 0 & \mu_1 - \mu_{12} \\ \mu_1 - \mu_{12} & 0 \end{pmatrix}. \quad (3.44)$$

IV. TREE-LEVEL HIGGS INDUCED FCNC PROCESSES

In this section we discuss various FCNC processes mediated by tree-level neutral Higgs boson exchange.

A. Neutral meson mixing via Higgs exchange

Accurate measurements exist [20] for neutral meson-antimeson mixings in the K^0–$\bar{K}^0$, B_d^0–$\bar{B}_d^0$, B_s^0–$\bar{B}_s^0$, and in D^0–$\bar{D}^0$ sectors. In the Q_6 model there are new contributions to these mixings arising through tree-level Higgs exchange. These new contributions will modify the SM predictions, which are all in good agreement with data. Here we compute these new contributions, following the analysis of Ref. [21], with updated QCD corrections and hadronic matrix elements.

The Yukawa coupling $\alpha_{u,d}$, $\beta_{u,d}$, $\beta'_{u,d}$, $\delta_{u,d}$ of Eq. (2.10) can be determined from the mass matrix Eq. (2.15):

$$\alpha_{u,d} = \frac{m_{t,b}^0 y_{u,d}^2}{|v_{u,d3}|}, \qquad \beta_{u,d} = \frac{m_{t,b}^0 b_{u,d}}{|v_{u,d1}|}$$
$$\beta'_{u,d} = \frac{m_{t,b}^0 b'_{u,d}}{|v_{u,d1}|}, \qquad \delta_{y,d} = \frac{m_{t,b}^0 q_{u,d}/y_{u,d}}{|v_{u,d3}|}. \quad (4.1)$$

Using the input values given in Eq. (2.19) we get the following for the two cases.

Case (1)

$$\alpha_d = 0.0409, \qquad \beta_d = 6.51 \times 10^{-4},$$
$$\beta'_d = 0.0173, \qquad \delta_d = 3.35 \times 10^{-4},$$
$$\alpha_u = 0.7195, \qquad \beta_u = 0.0858,$$
$$\beta'_u = 0.1672, \qquad \delta_u = 1.10 \times 10^{-4}.$$

Case (2)

$$\alpha_d = 0.0526, \qquad \beta_d = 7.46 \times 10^{-4},$$
$$\beta'_d = 0.0198, \qquad \delta_d = 4.30 \times 10^{-4},$$
$$\alpha_u = 0.9354, \qquad \beta_u = 0.0372,$$
$$\beta'_u = 0.0724, \qquad \delta_u = 1.43 \times 10^{-4}.$$

After 45° rotation in the Q_6 doublet space, the Yukawa coupling matrices in the down sector are

$$Y_{d1} = O_d^T P_d \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \beta_d \\ 0 & \beta'_d & 0 \end{pmatrix} P_{d^c} O_{d^c},$$
$$Y_{d2} = O_d^T P_d \begin{pmatrix} 0 & 0 & \beta_d \\ 0 & 0 & 0 \\ \beta'_d & 0 & 0 \end{pmatrix} P_{d^c} O_{d^c}, \quad (4.2)$$
$$Y_{d3} = O_d^T P_d \begin{pmatrix} 0 & \delta_d & 0 \\ -\delta_d & 0 & 0 \\ 0 & 0 & \alpha_d \end{pmatrix} P_{d^c} O_{d^c},$$

where P_d, P_{d^c} are defined in Eq. (2.14). The Yukawa couplings in the up-quark sector and the charged lepton sector are similar.

The new Higgs-mediated contributions to $\Delta F = 2$ Hamiltonian, responsible for the neutral meson-antimeson mixings has the form [21]

$$H_{\rm eff} = -\frac{1}{2M_k^2}\left(\bar{q}_i\left[Y_{ij}^k \frac{1+\gamma_5}{2} + Y_{ji}^{k*}\frac{1-\gamma_5}{2}\right]q_j\right)^2. \quad (4.3)$$

Here $q_{i,j}$ are the relevant quark fields contained in the meson. Y_{ij}^k are the Yukawa couplings of q_i, q_j with Higgs mass eigenstate H^k mediating FCNC interactions, $k = 1, 2, \ldots 10$ in our model, 6 from the (H_1–H_3) sector, and 4 from the H_2 sector. (The light standard model-like Higgs boson has practically no FCNC couplings.) Y_{ij}^k can be obtained via inverse transformations, Eq. (3.10), (3.15), and (3.30) or (3.37).

We obtain

$$M_{12}^\phi = \langle\phi|H_{\rm eff}|\bar{\phi}\rangle$$
$$= -\frac{f_\phi^2 m_\phi}{2M_k^2}\left[-\frac{5}{24}\frac{m_\phi^2}{(m_{q_i}+m_{q_j})^2}(Y_{ij}^{k2} + Y_{ji}^{k*2})\right.$$
$$\cdot B_2 \cdot \eta_2(\mu) + Y_{ij}^k Y_{ji}^{k*}\left(\frac{1}{12} + \frac{1}{2}\frac{m_\phi^2}{(m_{q_i}+m_{q_j})^2}\right)$$
$$\left.\cdot B_4 \cdot \eta_4(\mu)\right]. \quad (4.4)$$

Here ϕ is the neutral meson (K^0, B_d^0, B_s^0, D^0). For our numerical study we use the modified vacuum saturation and factorization approximation results for the matrix elements [2,3]

$$\langle\phi|\bar{f}_i(1\pm\gamma_5)f_j\bar{f}_i(1\mp\gamma_5)f_j|\bar{\phi}\rangle = f_\phi^2 m_\phi\left(\frac{1}{6}+\frac{m_\phi^2}{(m_{q_i}+m_{q_j})^2}\right)\cdot B_4,$$
$$\langle\phi|\bar{f}_i(1\pm\gamma_5)f_j\bar{f}_i(1\pm\gamma_5)f_j|\bar{\phi}\rangle = -\frac{5}{6}f_\phi^2 m_\phi\frac{m_\phi^2}{(m_{q_i}+m_{q_j})^2}\cdot B_2. \quad (4.5)$$

B_2 and B_4 are equal to one in the vacuum saturation approximation, but are found to be slightly different from one in lattice simulations. We use $(B_2, B_4) = (0.66, 1.03)$ for the K^0 system, (0.82, 1.16) for the B_d^0 and B_s^0 systems, and (0.82, 1.08) for the D^0 system [2]. In Eq. (4.4) $\eta_2(\mu)$, $\eta_4(\mu)$ are QCD correction factors of the Wilson coefficients C_2 and C_4 of the effective $\Delta F = 2$ Hamiltonian in going from the SUSY scale M_s to the hadronic scale μ. These factors are computed as follows. The $\Delta F = 2$ effective Hamiltonian has the general form

$$\mathcal{H}_{\text{eff}}^{\Delta F=2} = \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3}\tilde{C}_i\tilde{Q}, \quad (4.6)$$

where

$$Q_1 = \bar{q}_{iL}^\alpha\gamma_\mu q_{jL}^\alpha\bar{q}_{iL}^\beta\gamma^\nu q_{jL}^\beta, \qquad Q_2 = \bar{q}_{iR}^\alpha q_{jL}^\alpha\bar{q}_{iR}^\beta q_{jR}^\beta,$$
$$Q_3 = \bar{q}_{iR}^\alpha q_{jL}^\beta\bar{q}_{iR}^\beta q_{jL}^\alpha, \qquad Q_4 = \bar{q}_{iR}^\alpha q_{jL}^\alpha\bar{q}_{iL}^\beta q_{jR}^\beta,$$
$$Q_5 = \bar{q}_{iR}^\alpha q_{jL}^\beta\bar{q}_{iL}^\beta q_{jR}^\alpha, \quad (4.7)$$

with $\tilde{Q}_{1,2,3}$ obtained from $Q_{1,2,3}$ by the interchange $L \leftrightarrow R$.

For computing $\eta_{2,4}$ we take the SUSY scale M_s to be 1 TeV. All the supersymmetric particles and heavy Higgs bosons are integrated out at 1 TeV. The Wilson coefficients evolve from M_s down to the hadron scale μ according to the equations

$$C_r(\mu) = \sum_i\sum_s(b_i^{(r,s)} + \eta c_i^{(r,s)})\eta^{a_i}C_s(M_s). \quad (4.8)$$

Here η is defined as $\eta = \alpha_s(M_s)/\alpha_s(m_t)$. The magic numbers a_i, $b_i^{(r,s)}$, and $c_i^{(r,s)}$ can be found in Ref. [2] for the K system, in Ref. [3] for the $B_{d,s}$ system and in Ref. [22] for the D system. With $M_s = 1$ TeV and $\alpha_s(m_Z) = 0.118$, and $m_t(m_t) = 163.6$ GeV we find $\eta = \alpha_s(1\text{ TeV})/\alpha_s(m_t) = 0.0882/0.108 = 0.8167$.

At the SUSY scale, the neutral Higgs bosons in our model generate only operators Q_2 and Q_4. Consequently, at the hadron scale, for the K^0 system, we find

$$C_2(\mu) = C_2(M_s)\cdot(2.54),$$
$$C_4(\mu) = C_4(M_s)\cdot(4.81),$$
$$C_3(\mu) = C_2(M_s)\cdot(-1.8\times10^{-3}),$$
$$C_5(\mu) = C_4(M_s)\cdot(0.186), \quad (4.9)$$

leading to $\eta_2(\mu) = 2.54$, $\eta_4(\mu) = 4.81$. Although operator mixings induce nonzero C_3 and C_5 at the hadronic scale, their coefficients are found to be rather small.

For the $B_{d,s}^0$ system, following the same procedure, we find

$$C_2(\mu) = C_2(M_s)\cdot(2.00),$$
$$C_4(\mu) = C_4(M_s)\cdot(3.12),$$
$$C_3(\mu) = C_2(M_s)\cdot(-2.44\times10^{-2}),$$
$$C_5(\mu) = C_4(M_s)\cdot(0.0874). \quad (4.10)$$

And for the D^0 system we have

$$C_2(\mu) = C_2(M_s)\cdot(2.31),$$
$$C_4(\mu) = C_4(M_s)\cdot(3.99),$$
$$C_3(\mu) = C_2(M_s)\cdot(-1.30\times10^{-2}),$$
$$C_5(\mu) = C_4(M_s)\cdot(0.144). \quad (4.11)$$

In all cases we see that the induced operators C_3 and C_5 are negligible.

K^0–$\bar{K}^0$ mixing constraint:

In the K^0 system, tree-level neutral Higgs boson exchange contributes to $K_L - K_S$ mass difference, as well as to the indirect CP violation parameter, modifying the successful SM predictions. The mass difference is computed from $\Delta m_K = 2\,\text{Re}M_{12}^K$, while the CP violation parameter is $|\epsilon_K| \simeq \frac{\text{Im}M_{12}^K}{\sqrt{2}\Delta m_K}$. We seek consistency with the precisely measured experimental values $\Delta m_K/m_K \simeq (7.1\pm0.014)\times10^{-15}$ and $|\epsilon_K| \simeq 2.3\times10^{-3}$. In our calculation, we choose $m_K = 498$ MeV and $f_K = 160$ MeV. For the two numerical fits we find the new contributions to be

$$\text{Case (1): } (\Delta m_K/m_K)^{\text{new}} = 7.361\times10^{-15}, \quad \epsilon_K^{\text{new}} = 2.00\times10^{-4},$$
$$\text{Case (2): } (\Delta m_K/m_K)^{\text{new}} = 5.721\times10^{-15}, \quad \epsilon_K^{\text{new}} = 2.28\times10^{-5}. \quad (4.12)$$

The contributions from H_1^0–H_3^0 sector and H_2^0 sector to $\text{Re}(M_{12}^K)$ are, respectively, $(3.033\times10^{-15}, -1.200\times10^{-15})$ GeV for case (1) and $(2.512\times10^{-15}, -1.088\times10^{-15})$ GeV for case (2). We see that the new contributions to the mass difference is significant, but consistent with data. New contributions to CP violation is suppressed, which is a generic feature of Higgs exchange in this class of models. We elaborate on this issue later in this section.

B_d^0–$\bar{B}_d^0$ mixing constraint:

For the B_d^0–$\bar{B}_d^0$ system we use as input $m_{B_d} = 5.281$ GeV, $f_{B_d} = 240$ MeV, and seek consistency with the experimental value $\Delta m_{B_d} = 3.12\times10^{-13}$ GeV. We find for the Higgs induced contribution

$$\text{Case (1): } (\Delta m_{B_d})^{\text{new}} = 2.997 \times 10^{-13}\ \text{GeV}, \quad \text{Case (2): } (\Delta m_{B_d})^{\text{new}} = 2.728 \times 10^{-13}\ \text{GeV}. \tag{4.13}$$

The contributions from H_1^0–H_3^0 sector and H_2^0 sector to $M_{12}^{b_d}$ are $(2.298 \times 10^{-14}, 1.269 \times 10^{-13})$ GeV for case (1) and $(2.137 \times 10^{-14}, 1.150 \times 10^{-13})$ GeV. Again, we see consistency with experimental values. *CP* violation parameter is found to be extremely tiny, $\sim 10^{-5}$, from the Higgs boson exchange.

B_s^0–$\bar{B}_s^0$ mixing constraint:

For the B_s^0–$\bar{B}_s^0$ system, we use $m_{B_s} = 5.37$ GeV, $f_{B_s} = 295$ MeV and compare the new contributions with $\Delta m_{B_s} = 1.067 \times 10^{-11}$ GeV.

$$\text{Case (1): } (\Delta m_{B_s})^{\text{new}} = 1.688 \times 10^{-12}\ \text{GeV}, \quad \text{Case (2): } (\Delta m_{B_s})^{\text{new}} = 1.396 \times 10^{-12}\ \text{GeV}. \tag{4.14}$$

The H_1^0–H_3^0 sector and the H_2^0 sector contribute to $M_{12}^{B_s^0}$ given by $(8.532 \times 10^{-13}, -9.460 \times 10^{-15})$ GeV for case (1) and $(7.067 \times 10^{-13}, -3.835 \times 10^{-15})$ GeV for case (2). These new contributions are within the experimentally allowed range. The Higgs mediated *CP* violation is again found to be highly suppressed.

D^0–$\bar{D}^0$ mixing constraint:

For the D^0–$\bar{D}^0$ mixing we use $m_D = 1.864$ GeV, $f_D = 200$ MeV, and compare the new contribution with $\Delta m_D = 1.27 \times 10^{-12}$ GeV.

$$\text{Case (1): } (\Delta m_D)^{\text{new}} = 8.620 \times 10^{-13}\ \text{GeV}, \quad \text{Case (2): } (\Delta m_D)^{\text{new}} = 2.645 \times 10^{-13}\ \text{GeV}. \tag{4.15}$$

The H_1^0–H_3^0 sector contribution has different sign from that of the H_2^0 sector. We find for $M_{12}^{D^0}$ these contributions to be $(4.402 \times 10^{-15}, -4.354 \times 10^{-13})$ GeV for case (1) and $(2.568 \times 10^{-15}, -1.348 \times 10^{-13})$ GeV for case (2). Again these limits are within the experimental range.

We have found that the new sources of *CP* violation through tree-level Higgs is very small in meson-antimeson mixings with typical values $\text{Im}(M_{12}) \sim 10^{-4}\ \text{Re}(M_{12})$. This can be understood heuristically as follows. There are two types of contributions to the meson mixing as given in Eq. (4.4). The first term, proportional to B_2 respects a global $U(1)$ symmetry (strangeness in the K^0 system), which is only broken by the mass-splittings in the neutral Higgs boson spectrum between a pair of particles. However, this splitting is very small, of order m_Z^2 in the squared mass [see Eq. (3.27)]. The couplings of the nearly degenerate Higgs in each pair differ by a factor i, owing to the $U(1)$ symmetry, and the two contributions cancel, in the limit of exact degeneracy. For both the real and imaginary parts of M_{12} the contribution from the first term is suppressed by a factor $m_Z^2/(4M_k^2)$. Such a suppression is absent in the second term of Eq. (4.4), since the operator Q_4 explicitly breaks the $U(1)$ symmetry. Thus, although the first term has *CP* violation, in relation to the *CP*-conserving second term, it is suppressed by a factor $m_Z^2/(4M_k^2) \sim 10^{-4}$. Now, the second term, while it has no suppression factor, it is purely real. This can be seen from the following observation. In the mass basis of fermions in the original basis we have the relation (owing to the vanishing of off-diagonal mass terms in the mass eigenbasis)

$$(Y_{d3})_{ij}\langle H_3^d\rangle = -(Y_{d1})_{ij}\langle H_1^d\rangle \tag{4.16}$$

for $i \neq j$. The couplings of mass eigenstates of the Higgs boson to down-type quarks are simply linear combinations of H_1^d and H_3^d. Since we assume *CP* to be spontaneously broken, all components of $(Y^k)_{ij}$ with $i \neq j$ have the same phase. As a result the second term of Eq. (4.4) becomes real. The constraint imposed by SUSY, that H_u^* fields do not couple to down-type quarks, and the fact that only two of the down-type Higgs bosons acquire VEVs is very crucial for this result.

B. Neutron electric dipole moment from Higgs exchange

The Higgs boson exchange can generate nonzero electric dipole moments for the fermions. These diagrams are however suppressed by the light fermion Yukawa couplings. For the d quark EDM arising from neutral Higgs boson exchange at the one-loop level we find [21]

$$d_d = \frac{Q_d e}{16\pi^2}\,\text{Im}(Y_{dq}^k Y_{qd}^k)\frac{m_q}{M_k^2}\left[\frac{3}{2} - \ln\left(\frac{M_k^2}{m_q^2}\right)\right]\xi_d, \tag{4.17}$$

where $\xi_d = (\alpha_s(M_k)/\alpha_s(\mu))^{16/23} \approx 0.12$, and q is summed over d, s, and b. The neutron EDM is determined using the quark model via

$$D_n = 4d_d/3 - d_u/3. \tag{4.18}$$

We find

$$\text{Case (1): } D_n = 1.809 \times 10^{-31}\ \text{e-cm}, \quad \text{Case (2): } D_n = 6.091 \times 10^{-31}\ \text{e-cm}, \tag{4.19}$$

which are well within experimental limits. The EDM of the electron is similarly found to be extremely small from the Higgs boson exchange diagrams.

C. $\mu \to 3e$ and $\tau \to 3\mu$ decays

Tree-level Higgs boson exchange can lead to flavor violating leptonic decays such as $\tau \to 3\mu$ and $\mu \to 3e$. The effective weak interaction mediating such decays can be parametrized as

$$G_{\text{eff}} = \left|\sum_k (Y_e)_{11}^k (Y_e)_{12}^k \frac{1}{M_k^2}\right|. \tag{4.20}$$

The effective couplings are found for $\mu \to 3e$ for the two cases to be

$$\begin{aligned}\text{Case (1): } G_{\text{eff}} &= 4.432 \times 10^{-13} G_F,\\ \text{Case (2): } G_{\text{eff}} &= 4.191 \times 10^{-13} G_F.\end{aligned} \tag{4.21}$$

And the couplings for $\tau \to 3\mu$ decay are

$$\begin{aligned}\text{Case (1): } G_{\text{eff}} &= 45.721 \cdot 10^{-8} G_F,\\ \text{Case (2): } G_{\text{eff}} &= 6.977 \cdot 10^{-8} G_F.\end{aligned} \tag{4.22}$$

Such small effective couplings will lead to negligible contributions to the decay branching ratios. For example, the branching ratio for $\tau \to 3\mu$ is of order 10^{-15}, well below the experimental sensitivity. We conclude that Higgs-mediated FCNCs in the lepton sector are all safe.

V. FCNC MEDIATED BY SUSY PARTICLES

In this section we turn attention to the flavor changing processes mediated by the supersymmetric particles. The main motivation for the non-Abelian Q_6 model was to bring such processes under control by a symmetry reason. Here we analyze meson-antimeson mixings, flavor violating leptonic decays, and the EDM of the neutron and the electron. We present our proposal to suppress SUSY contributions to the EDM by making the Higgsinos of the model light, with masses of order 100 GeV.

Owing to the Q_6 symmetry, the first two family squarks (and similarly sleptons) are degenerate in mass, while the third family, which is a Q_6 singlet has a different mass. In the fermion sector Q_6 symmetry is broken, which means that there will be SUSY loop induced flavor violation in the model. Constraints on such flavor violation have been listed in Refs. [2,3,22] assuming all three families of squarks are degenerate. While these results are applicable for the K^0 and D^0 system in our model, they do not work well for the $B^0_{d,s}$ system. This is because the masses of the $\tilde{b}$ and $\widetilde{d,s}$ masses are not the same.

A. Generalized constraints for B_d system

We have generalized the results of Ref. [3] by allowing for $\tilde{b}$ mass to be different from the masses of $\widetilde{d,s}$. We define new parameters

$$y^d_{A,B} = \frac{(\tilde{m}^2_b)_{A,B}}{\tilde{m}^2_{dA,B}} \tag{5.1}$$

for A, $B = L$, R. We expect these y parameters to be of order one, but not very close to 1. Taking account of $y \neq 1$ we have generalized the constraints on the squark mixing parameters from B^0_d system as follows.

The effective $\Delta F = 2$ Hamiltonian for $B_{d,s}$ system can be written as

$$\begin{aligned}\mathcal{H}_{\text{eff}} &= \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3} \tilde{C}_i \tilde{Q}\\ &= -\frac{\alpha_s}{216 m^2_{\tilde{d}}}\{(\delta^d_{13})^2_{LL}(24Q_1 x f_6(x,y) + 66Q_1\tilde{f}_6(x,y)) + (\delta^d_{13})^2_{RR}(24\tilde{Q}_1 x f_6(x,y) + 66\tilde{Q}_1\tilde{f}_6(x,y))\\ &\quad + (\delta^d_{13})_{LL}(\delta^d_{13})_{RR}(504Q_4 x f_6(x,y) - 72Q_4\tilde{f}_6(x,y) + 24Q_5 x f_6(x,y) + 120Q_5\tilde{f}_6(x,y))\\ &\quad + (\delta^d_{13})^2_{RL}(204Q_2 x f_6(x,y) - 36Q_3 x f_6(x,y)) + (\delta^d_{13})^2_{LR}(204\tilde{Q}_2 x f_6(x,y) - 36\tilde{Q}_3 x f_6(x,y))\\ &\quad + (\delta^d_{13})_{LR}(\delta^d_{13})^2_{RL}(-132Q_4\tilde{f}(x,y) - 180Q_5\tilde{f}_6(x,y))\}.\end{aligned} \tag{5.2}$$

The functions $f_6(x,y)$ and $\tilde{f}_6(x,y)$ are

$$\begin{aligned}f_6(x,y) &= \frac{1}{(x-1)^3(y-1)^3(z-1)^3}[-\ln x(x+y+xy-3x^3)(y-1)^3 + \ln y(x+y+xy-3y^2)(x-1)^3\\ &\quad + 2(x-1)(y-1)(-x+y+x^2-y^2-x^3+y^3+2x^2y-2xy^2)]\\ \tilde{f}_6(x,y) &= \frac{1}{(x-1)^3(y-1)^3(z-1)^3}[2\ln x \cdot x(x^2-y)(y-1)^3 + 2\ln y \cdot y(x-y^2)(x-1)^3\\ &\quad + (x-1)(y-1)(x^2-y^2+x^3-y^3-7x^2y+7xy^2+x^3y-xy^3)].\end{aligned} \tag{5.3}$$

Generalizing the results of Ref. [3] we obtain the squark mixing coefficients $(\delta^d_{13})_{AB}$ with $A, B = (L, R)$ as shown in Table I. Here we have used the same input as in Ref. [3], so that for $y = 1$ our results coincide. We have used the next-to-leading order lattice calculation results for the matrix elements. For some of the mixing parameters we made a simplifying assumption that y^d_L and y^d_R are equal.

TABLE I. Maximum allowed values for $|\mathrm{Re}(\delta^d_{13})_{AB}|$ and $|\mathrm{Im}(\delta^d_{13})_{AB}|$, with $A, B = (L, R)$. A new parameter y is introduced, with $y = m^2_{\tilde{b}}/m^2_{\tilde{d}}$. The definition of other parameters and their values follow Ref. [3].

$y \backslash x$	0.25	1.0	4.0	0.25	1.0	4.0
		$\lvert\mathrm{Re}(\delta^d_{13})_{LL}\rvert$			$\lvert\mathrm{Im}(\delta^d_{13})_{LL}\rvert$	
0.25	3.4×10^{-2}	1.6×10^{-1}	2.5×10^{-1}	7.2×10^{-2}	3.4×10^{-1}	1.2×10^{-1}
1.0	6.2×10^{-2}	1.4×10^{-1}	7.0×10^{-1}	1.3×10^{-1}	3.0×10^{-1}	3.4×10^{-1}
4.0	1.6×10^{-1}	2.7×10^{-1}	-	3.3×10^{-1}	5.8×10^{-1}	-
		$\lvert\mathrm{Re}(\delta^d_{13})_{RR}\rvert = \lvert\mathrm{Re}(\delta^d_{13})_{LL}\rvert$			$\lvert\mathrm{Im}(\delta^d_{13})_{RR}\rvert = \lvert\mathrm{Im}(\delta^d_{13})_{LL}\rvert$	
0.25	1.4×10^{-3}	2.4×10^{-2}	1.0×10^{-2}	4.4×10^{-3}	1.0×10^{-2}	4.3×10^{-3}
1.0	1.9×10^{-2}	2.1×10^{-2}	2.8×10^{-2}	8.0×10^{-3}	9.0×10^{-1}	1.2×10^{-2}
4.0	4.8×10^{-2}	4.0×10^{-2}	1×10^{-1}	2×10^{-2}	1.7×10^{-2}	4.6×10^{-2}
		$\lvert\mathrm{Re}(\delta^d_{13})_{LR}\rvert$			$\lvert\mathrm{Im}(\delta^d_{13})_{LR}\rvert$	
0.25	1.7×10^{-2}	3.7×10^{-2}	1.6×10^{-2}	3.6×10^{-2}	8.4×10^{-2}	3.6×10^{-2}
1.0	3.0×10^{-2}	3.3×10^{-2}	4.5×10^{-2}	6.6×10^{-2}	7.4×10^{-2}	1.0×10^{-1}
4.0	7.5×10^{-2}	6.4×10^{-2}	1.7×10^{-1}	1.7×10^{-1}	1.4×10^{-1}	3.9×10^{-1}
		$\lvert\mathrm{Re}(\delta^d_{13})_{LR}\rvert = \lvert\mathrm{Re}(\delta^d_{13})_{RL}\rvert$			$\lvert\mathrm{Im}(\delta^d_{13})_{LR}\rvert = \lvert\mathrm{Im}(\delta^d_{13})_{RL}\rvert$	
0.25	1.4×10^{-2}	5.9×10^{-2}	-	2.3×10^{-2}	4.4×10^{-1}	-
1.0	2.6×10^{-2}	5.2×10^{-2}	-	9.0×10^{-3}	2.3×10^{-2}	-
4.0	6.5×10^{-2}	1.0×10^{-1}	-	2.3×10^{-2}	4.4×10^{-2}	-

B. SUSY flavor change in Q_6 model

In the Q_6 model the mass matrices of squarks in the flavor basis can be written as

$$(m_{\tilde{q}})^2_{AA} = m^2_{\tilde{q}A}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & y \end{pmatrix}, \tag{5.4}$$

q can be u or d, and A can be L or R. Making the same unitary transformation on the squark fields as the ones on the quarks which diagonalize the quark mass matrices, we find the mass matrices of squarks in the SUSY basis (where the gluino coupling matrix is identity in the flavor space) to be:

$$\begin{aligned}(\tilde{m}_{\tilde{d}})^2_{LL} &= O_d^T P_d^* (m_{\tilde{d}})^2_{LL} P_d O_d = m^2_{\tilde{d}L}\left[I + (y^d_l - 1) O_d^T P_d^* \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} P_d O_d \right] \\ &= m^2_{\tilde{d}L}\left[I + (y^d_L - 1)\begin{pmatrix} 5.43\times10^{-5} & 1.31\times10^{-4} & -0.0074 \\ 1.31\times10^{-4} & 3.17\times10^{-4} & -0.0178 \\ -0.0074 & -0.0178 & 0.9996 \end{pmatrix}\right]. \end{aligned} \tag{5.5}$$

Note that this matrix is real, a consequence of the phase factorization of the fermion mass matrix. Similarly,

$$(\tilde{m}_{\tilde{d}})^2_{RR} = m^2_{\tilde{d}R}\left[I + (y^d_R - 1)\begin{pmatrix} 0.0367 & -0.1339 & 0.1318 \\ -0.1339 & 0.4891 & -0.4816 \\ 0.1319 & -0.4816 & 0.4742 \end{pmatrix}\right], \tag{5.6}$$

$$(\tilde{m}_{\tilde{u}})^2_{LL} = m^2_{\tilde{u}L}\left[I + (y^u_L - 1)\begin{pmatrix} 3.85\times10^{-6} & 7.74\times10^{-5} & -0.0020 \\ 7.74\times10^{-5} & 0.0016 & -0.0394 \\ -0.0020 & -0.0394 & 0.9984 \end{pmatrix}\right], \tag{5.7}$$

$$(\tilde{m}_{\tilde{u}})^2_{RR} = m^2_{\tilde{u}R}\left[I + (y^u_R - 1)\begin{pmatrix} 1.46\times10^{-5} & 2.94\times10^{-4} & 0.0038 \\ 2.94\times10^{-4} & 0.0059 & -0.0768 \\ 0.0038 & -0.0768 & 0.9941 \end{pmatrix}\right]. \tag{5.8}$$

K^0–$\bar{K}^0$ mixing via squark-gluino loops have several contributions. The most stringent limit arises from the (LL)–(RR) mixing, which requires [2]

$$\frac{|(y^d - 1)|}{(0.51 + 0.49y^d)^{1/4}} < 0.23\left(\frac{\tilde{m}}{500\text{ GeV}}\right). \qquad (5.9)$$

Here we have assumed $y^d_L = y^d_R = y^d$, and took the gluino mass to be equal to the first two family squark mass. For the first two family squark mass of 500 GeV, this translates to the limit $0.77 \leq y^d \leq 1.24$. For 1 TeV squarks, this limit is relaxed to $0.58 \leq y^d \leq 1.48$. We see that for y^d order 1, the most stringent limit on squark mediated FCNC is satisfied.

The Q_6 model also generates significant $(RR)(RR)$ contributions to the K^0–$\bar{K}^0$ mixing. We find

$$0.68 \leq y^d \leq 1.37 \qquad (5.10)$$

for squark and gluino mass of 500 GeV. This constraint is also easily satisfied in the model.

In the B^0_d system, the analogous constraints are [from the $(LL)(RR)$ operator]

$$\frac{|(y^d - 1)|}{(0.53 + 0.47y^d)^{1/4}} < 0.69\left(\frac{\tilde{m}}{500\text{ GeV}}\right). \qquad (5.11)$$

This limit leads to $0.48 \leq y^d \leq 1.85$ for squark-gluino mass of 500 GeV. The $(RR)(RR)$ squark mixing gives no constraint from the B_d system. Similarly, there are no constraints arising from the D^0 system, nor from other type of operators in the model.

In the leptonic sector, we find the (LL) slepton mixing [which is the same for the (RR) slepton mixing] to be

$$(\tilde{m}_{\tilde{e}})^2_{LL} = m^2_{\tilde{e}L}\left[I + (y^e_L - 1)\begin{pmatrix} 2.93\times10^{-4} & -4.02\times10^{-3} & -0.0167 \\ -4.02\times10^{-3} & 0.0550 & 0.2280 \\ -0.0167 & 0.2280 & 0.9447 \end{pmatrix}\right]. \qquad (5.12)$$

There are stringent constraints on the mixing parameter $((\delta^e)_{LL})_{12}$ from the decay $\mu \to e\gamma$ [23]. On the face of it, the mixing presented above would appear to be in mild conflict with data by a factor of few. However, since such a constraint is very weak for the $((\delta^e)_{RR})_{12}$ mixing, we point out that the flexibility in the lepton sector mass matrix can be used to make the (LL) contribution small in exchange for larger (RR) contributions. That is, assume $B \ll B'$ in Eq. (1.4).

C. Left-Right squark mixing and a solution to the EDM problem

So far we have ignored SUSY flavor violation arising from the left-right squark mixings. It turns out that these operators do not give significant contributions to meson-antimeson mixings, since such mixings have fermion chirality suppression. However, these mixings can generate new contributions to the neutron (and electron) electric dipole moments. Here we analyze constraints from the EDM and suggest a simple solution to the SUSY EDM problem.

First, as shown in Ref. [7], the trilinear A term induced phases align with the phases of the fermion mass matrices, even without assuming proportionality of the A terms with the respective Yukawa couplings. This feature arises due to the phase factorization of the fermion mass matrix. Left-right squark mixings also receive contributions from the superpotential μ terms. We derive the mass matrix for the down squark sector to be:

$$(m_{\tilde{d}})^2_{LR} = F^{d*}_1\begin{pmatrix} 0 & \delta_d & 0 \\ -\delta_d & 0 & 0 \\ 0 & 0 & \alpha_d \end{pmatrix} + \sqrt{2}F^{d*}_2\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \beta_d \\ 0 & \beta'_d & 0 \end{pmatrix}, \qquad (5.13)$$

with

$$F^d_1 = \mu_3 v_{u3} + \mu_{13} v_{u1}, \qquad F^d_2 = \mu_{31} v_{u3} + \frac{\mu_1 + \mu_{12}}{2} v_{u1}. \qquad (5.14)$$

After the unitary transformations to the left and the right squarks, corresponding to case (1), we have the (LR) mixing matrix in the flavor basis as

$$(\tilde{m}_{\tilde{d}})^2_{LR} = O^T_d P_d (m_{\tilde{d}})^2_{LR} P_{d^c} O_{d^c}$$
$$= F^{d*}_1\begin{pmatrix} -1.75\times10^{-4} & 4.14\times10^{-4} & 3.09\times10^{-5} \\ -4.46\times10^{-4} & 3.84\times10^{-4} & -5.45\times10^{-4} \\ 0.0078 & -0.0286 & 0.0282 \end{pmatrix}$$
$$+ \sqrt{2}F^{d*}_2 e^{i\Delta\theta_d}\begin{pmatrix} 4.53\times10^{-5} & -1.66\times10^{-4} & -1.24\times10^{-5} \\ 1.79\times10^{-4} & -6.52\times10^{-4} & 2.18\times10^{-4} \\ -0.0031 & 0.0115 & 0.0125 \end{pmatrix}, \qquad (5.15)$$

$$(\tilde{m}_{\tilde{u}})^2_{LR} = O_u^T P_u (m_{\tilde{u}})^2_{LR} P_{u^c} O_{u^c} = F_1^{u*}\begin{pmatrix} -1.62\times 10^{-5} & 2.18\times 10^{-4} & -0.0014 \\ -2.18\times 10^{-4} & 0.0022 & -0.0283 \\ 0.0027 & -0.0554 & 0.7168 \end{pmatrix} + \sqrt{2}F_2^{u*}e^{i\Delta\theta_u}\begin{pmatrix} 3.24\times 10^{-5} & -6.53\times 10^{-4} & 0.0042 \\ 6.53\times 10^{-4} & -0.0131 & 0.0848 \\ -0.0082 & 0.1661 & 0.0162 \end{pmatrix}. \tag{5.16}$$

with

$$F_1^u = \mu_3 v_{d3} + \mu_{31} v_{d1}, \qquad F_2^u = \mu_{13} v_{d3} + \frac{\mu_1 + \mu_{12}}{2} v_{d1}. \tag{5.17}$$

$$(\tilde{m}_{\tilde{e}})^2_{LR} = O_e^T P_e (m_{\tilde{e}})^2_{LR} P_{e^c} O_{e^c} = F_1^{d*}\begin{pmatrix} -1.10\times 10^{-5} & 2.85\times 10^{-4} & 6.72\times 10^{-4} \\ -2.75\times 10^{-4} & 0.0039 & 0.0257 \\ -4.89\times 10^{-5} & 0.0047 & 0.0313 \end{pmatrix} + \sqrt{2}F_2^{d*}e^{i\Delta\theta_d}\begin{pmatrix} 2.93\times 10^{-6} & -1.14\times 10^{-4} & -2.69\times 10^{-4} \\ 1.10\times 10^{-4} & -0.0043 & -0.0103 \\ 1.96\times 10^{-5} & -0.0019 & 0.0093 \end{pmatrix}, \tag{5.18}$$

Corresponding to case (2) these matrices are

$$(\tilde{m}_{\tilde{d}})^2_{LR} = O_d^T P_d (m_{\tilde{d}})^2_{LR} P_{d^c} O_{d^c} = F_1^{d*}\begin{pmatrix} -2.25\times 10^{-4} & 5.32\times 10^{-4} & 3.97\times 10^{-5} \\ -5.73\times 10^{-4} & 4.93\times 10^{-4} & -7.00\times 10^{-4} \\ 0.0100 & -0.0368 & 0.0362 \end{pmatrix} + \sqrt{2}F_2^{d*}e^{i\Delta\theta_d}\begin{pmatrix} 5.20\times 10^{-5} & -1.90\times 10^{-4} & -1.42\times 10^{-5} \\ 2.05\times 10^{-4} & -7.48\times 10^{-4} & 2.50\times 10^{-4} \\ -0.0036 & 0.0131 & 0.0144 \end{pmatrix}, \tag{5.19}$$

$$(\tilde{m}_{\tilde{u}})^2_{LR} = O_u^T P_u (m_{\tilde{u}})^2_{LR} P_{u^c} O_{u^c} = F_1^{u*}\begin{pmatrix} -2.11\times 10^{-5} & 2.83\times 10^{-4} & -0.0018 \\ -2.83\times 10^{-4} & 0.0028 & -0.0368 \\ 0.0036 & -0.0720 & 0.9319 \end{pmatrix} + \sqrt{2}F_2^{u*}e^{i\Delta\theta_u}\begin{pmatrix} 1.41\times 10^{-5} & -2.83\times 10^{-4} & 0.0018 \\ 2.83\times 10^{-4} & -0.0057 & 0.368 \\ -0.0036 & 0.0720 & 0.0070 \end{pmatrix}. \tag{5.20}$$

$$(\tilde{m}_{\tilde{e}})^2_{LR} = O_e^T P_e (m_{\tilde{e}})^2_{LR} P_{e^c} O_{e^c} = F_1^{d*}\begin{pmatrix} -1.41\times 10^{-5} & 3.66\times 10^{-4} & 8.62\times 10^{-4} \\ -3.53\times 10^{-4} & 0.0050 & 0.0330 \\ -6.28\times 10^{-5} & 0.0061 & 0.0402 \end{pmatrix} + \sqrt{2}F_2^{d*}e^{i\Delta\theta_d}\begin{pmatrix} 3.36\times 10^{-6} & -1.31\times 10^{-4} & -3.08\times 10^{-4} \\ 1.26\times 10^{-4} & -0.0049 & -0.0118 \\ 2.24\times 10^{-5} & -0.0022 & 0.0106 \end{pmatrix}. \tag{5.21}$$

Note that these matrices are in general complex, since $F_i^{u,d}$ are complex because of the spontaneously induced phases of the VEVs. This means that these matrices will contribute to neutron and electron EDM. Since these complex coefficients are proportional to $\mu v/\tilde{m}^2$, we find a simple solution to the SUSY EDM problem: Let the μ terms be of order 100 GeV, in which case one finds a suppression factor of 10^{-2} for the effective phase that enters the EDM expression. With this suppression factor, from the (1, 1) elements of these (LR) mixing matrices, we see that neutron and electron EDM constraints can be satisfied, even with the spontaneously induced phases in the VEVs being of order 1.

The proposed solution to the SUSY EDM problem has direct experimental consequences for LHC. We predict that the Higgsinos should be light, and three such pairs of doublet Higgsinos should be observable at the LHC. Their scalar partners, however, are inaccessible, since their masses lie in the few TeV range.

VI. CONCLUSION

In conclusion, we have presented a detailed analysis of the Higgs potential involving three pairs of Higgs doublets in a Q_6 model of flavor. We have found consistent numerical solutions to the Higgs spectrum which satisfy all the FCNC constraints, with the SUSY breaking scale in the TeV range. This class of models are motivated on two grounds: They lead to reduced number of parameters in the fermionic sector, and they can be helpful in alleviating the flavor changing problems of generic SUSY models.

We have shown that tree-level Higgs boson induced FCNC are within experimental limits, even for the most stringent K^0–$\bar{K}^0$ mixing amplitude. The Higgs boson masses must lie in the TeV range. New sources of CP violation in meson mixing are highly suppressed. We have also shown the consistency of the model with SUSY flavor violation. A simple solution to the SUSY EDM problem is suggested, which requires light Higgsinos.

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