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Zero point energy of renormalized Wilson loops

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The quark-antiquark potential, and its associated zero point energy, can be extracted from lattice measurements of the Wilson loop. We discuss a unique prescription to renormalize the Wilson loop, for which the perturbative contribution to the zero point energy vanishes identically. A zero point energy can arise nonperturbatively, which we illustrate by considering effective string models. The nonperturbative contribution to the zero point energy vanishes in the Nambu model, but is nonzero when terms for extrinsic curvature are included. At one loop order, the nonperturbative contribution to the zero point energy is negative, regardless of the sign of the extrinsic curvature term.

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I. INTRODUCTION

The Wilson loop has a privileged status in gauge theory. Although a nonlocal and composite operator, with dimensional regularization, smooth loops in $3 + 1$ dimensions are rendered finite by the usual renormalization constants for the gluon [1].

Lattice regularization, however, introduces an additional divergence. Considering the Wilson loop as the propagator for an infinitely heavy test quark, the new divergence, E_0, represents mass renormalization for the test quark. E_0 has dimensions of mass, so if a is the lattice spacing, $E_0 \sim 1/a$, and multiplies the length of the loop. Extracting the quark-antiquark potential from the Wilson loop in the usual manner, E_0 corresponds to the zero point energy of the potential at asymptotically large distances. Although the (bare) quark-antiquark potential has been measured with precision [2–5], E_0 is usually ignored.

It is thus of interest to know how to renormalize E_0 on the lattice. This is especially important for thermal Wilson loops, where in a pure gauge theory the Polyakov loop is the order parameter for deconfinement. While the bare Polyakov loop vanishes in the continuum limit, the renormalized loop does not [6–14]. However, as E_0 varies, the renormalized Polyakov loop ℓ changes as $\ell \to e^{-E_0/T}\ell$; see, e.g., Appendix C of Ref. [10]. This nonperturbative ambiguity cannot be fixed by appealing to the perturbative regime at high temperature.

In the first section of the paper, we follow previous analysis [6,7,9,10] and suggest that there is a natural way to renormalize the Wilson loop such that the perturbative contribution to the zero point energy vanishes identically, $E_0^{\mathrm{pert}} = 0$.

This does not exclude nonperturbative contributions to the zero point energy. We illustrate this by considering E_0^{non} in one possible model of confinement, that of string models. If σ is the string tension, then just on dimensional grounds, $E_0^{\mathrm{non}} \sim \sqrt{\sigma}$ is possible. Even so, in the Nambu model [15,16], or variants thereof [4,17,18], $E_0^{\mathrm{non}} = 0$.

A nonzero value of E_0^{non} is generated only by models with massive modes on the world sheet. This arises by adding terms for the extrinsic curvature of the world sheet. A simple computation shows that at one loop order, E_0^{non} is nonzero and negative, whether the coupling for the extrinsic curvature term is positive [19–22] or negative [23–26].

On the lattice, numerical simulations find that the quark-antiquark potential appears to agree well with the simplest Nambu model down to rather short distances [3]. This suggests that on the world sheet, any massive modes are heavy. In this case, the effective string theory is strongly coupled, so that the results of a one loop computation are only suggestive.

II. RENORMALIZED ZERO POINT ENERGY

Consider a rectangular Wilson loop of length t_{tot} and width R. When $t_{\mathrm{tot}} \gg R$, the vacuum expectation value of the bare Wilson loop can be used to define the quark-antiquark potential,

$$\langle \mathcal{W} \rangle = \left\langle \exp\left(ig \oint_{\mathcal{C}} A_\mu dx^\mu \right) \right\rangle = \exp(-V(R) t_{\mathrm{tot}}). \quad (1)$$

At large distances,

$$V(R \to \infty) \sim \sigma R + E_0 - \frac{\alpha}{R} + \cdots. \quad (2)$$

Here σ is the string tension, E_0 is the zero point energy, and α is a constant.

Assume that the theory has no dynamical quarks and confines, with the loop in the fundamental representation. Then $\sigma \neq 0$, and α is universal, $= \pi/12$ in four spacetime dimensions [2]. In quantum mechanics, the value of the zero point energy simply produces a phase which multiplies the wave function, and is of no physical consequence.

The potential above is a bare quantity, but only the zero point energy is ultraviolet divergent. The Wilson loop is related to the propagator for a massive test quark, with the zero point energy the additive shift in the mass. As the mass of the test field goes to infinity, the integral for this mass divergence is over three, instead of four, dimensions:

$$E_0^{\text{pert}} \sim -g^2 \int^{1/a} \frac{d^3k}{(2\pi)^3} \frac{1}{k^2} + \cdots$$
$$= -c_1 g^2 (1 + c_2 g^2 + \cdots) \frac{1}{a}. \quad (3)$$

The constants c_1, c_2, etc., depend upon the representation of the test particle, the details of lattice discretization, etc.

The possibility of a zero point energy was recognized when the renormalization of the Wilson loop was first considered [1]. In perturbative computations it is of little consequence, because then dimensional regularization is natural, and the integral in Eq. (3) automatically vanishes, as a purely powerlike divergence in an odd number of dimensions. This remains true with other gauge invariant regulators, such as higher derivatives or Pauli-Villars, since they also eliminate such power law divergences.

To define a renormalized loop on the lattice, we introduce the renormalization constant $\mathcal{Z}_{\mathcal{R}}$, which is a function of the (bare) coupling, g^2:

$$\mathcal{W}_{\mathcal{R}}^{\text{bare}} = \mathcal{Z}_{\mathcal{R}}(g^2)^{L/a} \mathcal{W}_{\mathcal{R}}^{\text{ren}}. \quad (4)$$

With L is the length of the loop, and L/a is the number of links for the path.

One way of extracting the renormalization constant $\mathcal{Z}_{\mathcal{R}}(g^2)$ is to compute for different values of the lattice spacing, a, holding the physical length, L, fixed [7,10]. This can be done directly from the numerical simulations, without using the perturbative expansion of Eq. (3).

Another way of computing $\mathcal{Z}_{\mathcal{R}}(g^2)$ is to compare the potential at short distances, $V(R)$ as $R \to 0$, to the result in perturbation theory [6,9,10]. In a pure $SU(3)$ gauge theory, the two methods agree with one another to the numerical accuracy tested [10].

Under this renormalization, it is possible to redefine

$$\mathcal{Z}_{\mathcal{R}}(g^2) \to \mathcal{Z}_{\mathcal{R}}(g^2)^{L/a} e^{-\tilde{E}_0^{\text{pert}} L} / \mathcal{Z}_0, \quad (5)$$

so that the renormalized Wilson loop becomes

$$\mathcal{W}_{\mathcal{R}}^{\text{ren}} \to e^{+\tilde{E}_0^{\text{pert}} L} \mathcal{Z}_0 \mathcal{W}_{\mathcal{R}}^{\text{ren}}. \quad (6)$$

In these expressions $\mathcal{Z}_0$ is a pure number, which just shifts the overall normalization of the loop. We can always choose this normalization by considering very small loops; for Polyakov loops, this corresponds to the limit of very high temperature. For small loops, perturbative corrections are computable as a power series in $\sim g^2(L) \sim 1/\log(L)$. We can thus eliminate $\mathcal{Z}_0$ by requiring that the small loops, suitably normalized, approach unity as $L \to 0$.

Less trivial is the change due to $\tilde{E}_0^{\text{pert}}$, which shifts the renormalized loop by $\exp(\tilde{E}_0^{\text{pert}} L)$. This cannot be eliminated considering small loops, $L \to 0$, as $\tilde{E}_0^{\text{pert}}$ represents a correction in a power of L, which in an asymptotically free theory is nonperturbative.

We suggest that under perturbative renormalization, the only consistent choice is to take $\tilde{E}_0^{\text{pert}} = 0$. We can renormalize the loop at zero temperature, where gluons are massless. For massless fields, the linearly divergent integral is uniformly proportional to the ultraviolet cutoff, which is $\sim 1/a$. The basic point is that there are no finite terms $\sim a^0$.

This depends crucially upon the fact that at zero temperature, gluons are massless order by order in perturbation theory. Consider how the integral in Eq. (3) changes if the gluons did have a mass, m:

$$E_0^{\text{pert}}(m) \sim -g^2 \int^{1/a} \frac{d^3k}{(2\pi)^3} \frac{1}{k^2 + m^2}$$
$$= -g^2 \left(c_1 \frac{1}{a} - e_2 m + e_3 m^2 a + \cdots \right), \quad (7)$$

for some constants e_2, e_3, etc. The ultraviolet divergent term, $\sim 1/a$, is independent of the mass m, and so the coefficient c_1 is the same as in Eq. (3). When $m \neq 0$, though, there is a term $\sim e_2 m$ which contributes a finite amount to the zero point energy. There are also terms at higher order which vanish as $a \to 0$, $\sim e_3 m^2 a$, etc.

Our point is simply that when $m = 0$, then $e_2 = 0$, and there is no contribution to the zero point energy. This can be made more general. Consider first a non-Abelian gauge theory without dynamical quarks, and let the renormalization mass scale of the theory be Λ_{ren}. To be invariant under the renormalization group, $\tilde{E}_0^{\text{pert}}$ can only depend upon Λ_{ren} as $\tilde{E}_0^{\text{pert}} \sim \Lambda_{\text{ren}} \exp(-\int dg/\beta(g^2))$, where $\beta(g^2)$ is the β function for the theory. Doing so, however, means that $\tilde{E}_0^{\text{pert}}$ is a dynamically generated mass scale. Such a nonperturbative mass scale is inconsistent with perturbative renormalization.

Strictly speaking, this argument fails in the presence of dynamical quarks, where one could have $\tilde{E}_0^{\text{pert}} \sim m_{\text{quark}}$, with m_{quark} a current quark mass. We suggest that $\tilde{E}_0^{\text{pert}}$ vanishes even in the presence of dynamical quarks. However quarks modify the gluon propagator, the perturbative contribution to the zero point energy arises from a linearly divergent integral over a massless field, as it does in the pure glue theory.

The bare zero point energy, Eq. (2), is then a sum of two contributions,

$$E_0 = E_0^{\text{pert}} + E_0^{\text{non}}. \quad (8)$$

The perturbative contribution is ultraviolet divergent, with $aE_0^{\text{pert}} \neq 0$ as $a \to 0$. This does not exclude nonperturbative contributions, E_0^{non}, for which $aE_0^{\text{non}} \to 0$ as $a \to 0$. We compute E_0^{non} in string models in the next section. Our point is simply that E_0^{pert} can be uniquely determined: there is no freedom to vary E_0^{pert} by a finite amount by introducing $\tilde{E}_0^{\text{pert}}$.

In particular, this implies that it is not possible to choose a $\tilde{E}_0^{\rm pert}$ so that the renormalized potential $V(R)$ vanishes at a given distance [8]. Of course the renormalized potential will vanish at some distance R_0, but this cannot be fixed *a priori*. Instead, it follows from the renormalized quark-antiquark potential.

There are several notable examples when the gluon develops a "mass," and a finite zero point energy arises perturbatively. In all of these cases, however, renormalization proceeds as for $m = 0$; there is no ambiguity in how to compute such terms.

The first is at nonzero temperature, where a Debye mass $m_{\rm Debye} \sim gT$ arises for A_0. Comparing to Eq. (7), this generates a zero point energy $\sim g m_{\rm Debye} \sim g^3 T$ [11]. The coefficient e_2 is positive, so at asymptotically high temperatures, the renormalized Polyakov loop approaches unity from above. A related phenomenon is familiar in Coulombic plasmas.

Another case where a gauge field develops a mass is when it is coupled to a scalar field, ϕ, which then undergoes spontaneous symmetry breaking. Then the gauge fields acquire a Higgs mass, $m_{\rm Higgs}$, and there is a zero point energy $\sim g^2 m_{\rm Higgs}$.

The last example where a perturbative zero point energy arises is for a gauge theory in three spacetime dimensions. Then the gauge coupling, g_{3d}^2, has dimensions of mass, and there can be a perturbative contribution to the zero point energy $\sim g_{3d}^2$. This cannot be distinguished from the nonperturbative contribution, since the square root of the string tension is also $\sim g_{3d}^2$. In this sense, it is much cleaner separating $E_0^{\rm non}$ from $E_0^{\rm pert}$ in four, instead of three, dimensions.

In fact, in three dimensions one cannot avoid a zero point energy proportional to g_{3d}^2 [1,7]. For a smooth Wilson loop in three dimensions, the only ultraviolet divergence is at one loop order; instead of Eq. (3), it is

$$E_0^{\rm pert,3d} \sim -g_{3d}^2 \int_{1/L}^{1/a} \frac{d^2k}{(2\pi)^2} \frac{1}{k^2} = -g_{3d}^2 c_2^{3d} \log\left(\frac{L}{a}\right). \quad (9)$$

This then implies that bare loops vanish in the continuum limit as

$$\mathcal{Z}_{\mathcal{R}}^{\rm div}(g_{3d}^2) \sim \exp(-\mathcal{C}_{\mathcal{R}} c_2^{3d} g_{3d}^2 L \log(L/a)) \sim \left(\frac{a}{L}\right)^{\mathcal{C}_{\mathcal{R}} c_2^{3d} g_{3d}^2 L}. \quad (10)$$

As in four dimensions, in three dimensions bare loops vanish in the continuum limit, $a \to 0$. The suppression in three dimensions is only by a power of a, though, and is much weaker than the exponential suppression seen in four dimensions. This is seen in numerical simulations, where the bare Polyakov loop approaches unity at temperatures as low as several times the critical temperature [27].

III. STRING MODELS

To epitomize how a nonperturbative zero point energy can arise, in this section we consider a string model of the flux sheet. Of course there are many other models of confinement; presumably generic models also generate $E_0^{\rm non} \neq 0$.

We consider a string action

$$\mathcal{S} = \int d^2z \sqrt{g} g^{ab} (\mathcal{D}_a x_\mu) \left(\sigma + \frac{1}{\kappa} \mathcal{D}^2 + \frac{1}{\lambda} \mathcal{D}^4 \right) (\mathcal{D}_b x^\mu). \quad (11)$$

Here $\mathcal{D}_a$ are covariant derivatives with respect to the induced metric $g_{ab} = \partial_a x^\mu \partial_b x_\mu$ on the surface $x^\mu(z)$; $\mathcal{D}^2 x^\mu = 1/\sqrt{g}\, \partial_a (\sqrt{g} g^{ab} \partial_b x^\mu)$.

In Eq. (11), σ is the string tension, with dimensions of mass squared. The coupling for the extrinsic curvature term, κ, is dimensionless. Lastly, the coupling λ also has dimensions of mass squared. This action can be considered as the first terms in a power series of covariant derivatives on the world sheet. It will be clear later that our qualitative conclusions will not be altered by the presence of higher terms.

A. Nambu model

The simplest model is that for which $\kappa = \lambda = \infty$, so the action only involves the area of the world sheet. In four dimensions, the exact solution for the potential is [16]

$$V(R) = \sigma \sqrt{R^2 - \frac{\pi}{6\sigma}} \approx \sigma R - \frac{\pi}{12R} - \frac{\pi^2}{288\sigma R^3} + \cdots. \quad (12)$$

The term $\sim 1/R$ is universal [2,4,20]. The potential is imaginary when $R < \sqrt{\pi/(6\sigma)}$, which reflects the inconsistency of the pure Nambu model in four dimensions.

For our purposes, all we need to recognize is that at large distances, the potential is σR times a power series in $1/(\sigma R^2)$. Thus there is no zero point energy in the Nambu model, $E_0^{\rm non} = 0$.

We shall see in the next section that this arises because there are only massless modes in the Nambu model. Thus $E_0^{\rm non}$ remains zero for the models of Polchinski and Strominger [17] and of Lüscher and Weisz [4]. While these models are rather different, in both higher derivative terms, like the extrinsic curvature term of Eq. (11), are treated as perturbations to the Nambu model. Thus there are only massless modes in such models, and $E_0^{\rm non}$ remains zero.

B. Rigid strings

We next consider the model with positive sign for the coupling to the extrinsic curvature, $\kappa > 0$, and neglect the coupling for a higher derivative term, $\lambda = \infty$.

We compute for small fluctuations about a flat sheet and follow Alvarez [15]. We let the number of spacetime dimensions, d, be arbitrary. At one loop order the $d-2$ transverse fluctuations contribute to give an effective action,

$$S_{\text{eff}} = \left(\frac{d-2}{2}\right) \text{tr} \log(-\partial^2)(-\partial^2 + m^2); \qquad -\partial^2 = -\partial_t^2 - \partial_r^2, \qquad m^2 = \kappa\sigma. \tag{13}$$

The time is continuous, so the corresponding momenta ω is continuous. In the spatial direction, the transverse directions vanish at the end of the flux sheet, $x^\mu_{\text{tr}}(0) = x^\mu_{\text{tr}}(R) = 0$. Thus the associated momenta are discrete, $p = n\pi/R$, for $n = 1, 2, \ldots$.

The effective Lagrangian involves a product of a massless mode, from the Nambu model, and a massive mode. The integral over the massless mode generates the term $\sim 1/R$. To do the integral over the massive mode, one can use analytic regularization [15], or consider the derivative with respect to m^2:

$$\frac{\partial S_{\text{eff}}}{\partial m^2} = \left(\frac{d-2}{2}\right) \text{tr} \frac{1}{-\partial^2 + m^2} = t_{\text{tot}}\left(\frac{d-2}{2}\right) \sum_{n=1}^{\infty} \int \frac{d\omega}{2\pi} \frac{1}{\omega^2 + (n\pi/R)^2 + m^2}. \tag{14}$$

The ω integral can be done either directly, or by contour integration:

$$\frac{\partial S_{\text{eff}}}{\partial m^2} = t_{\text{tot}}\left(\frac{d-2}{4}\right) \sum_{n=1}^{\infty} \frac{1}{(\omega^2 + (n\pi/R)^2 + m^2)^{1/2}}. \tag{15}$$

Integrating with respect to m^2, we obtain

$$S_{\text{eff}}(m^2) = t_{\text{tot}}\left(\frac{d-2}{2}\right) \sum_{n=1}^{\infty} \left(\left(\frac{n\pi}{R}\right)^2 + m^2\right)^{1/2}. \tag{16}$$

The sum over n is highly divergent, but can be done by using ζ-function regularization.

For $m = 0$, there is a single term, $\sim 1/R$. The sum over n gives $\zeta(-1)$, and generates the Lüscher term.

When $m \neq 0$, we can expand the power series in powers of $n\pi/R$. The first term is proportional to m times $\zeta(0) = -1/2$, so that the nonperturbative zero point energy is

$$E_0^{\text{non}} = -\left(\frac{d-2}{4}\right)\sqrt{\kappa\sigma}. \tag{17}$$

This result was obtained previously [21,22], by computing in a large d expansion in weak coupling, $\kappa \ll 1$. Braaten, Pisarski, and Tse computed to leading order, and obtained Eq. (17) [Eq. (16) of [21]]. Braaten and Tse then computed at next to leading order in κ, and so determined the corrections $\sim\kappa \log(m)$ to Eq. (17) [Eq. (4.24) of [22]].

Expanding S_{eff} in powers of the spatial momenta, $(n\pi/R)^2$, in principle we would expect corrections to the potential proportional to $\sim 1/R^2$, $\sim 1/R^4$, and so on. However, at one loop order a term $\sim 1/(R^2)^\ell$ is proportional to $\zeta(-2\ell)$; this vanishes when ℓ is an integer, as a "trivial" zero of the ζ function. Consequently, at one loop order there is *only* a contribution to the zero point energy, as *all* other corrections to the potential vanish. It is amusing to contrast this to the massless case, where simply on dimensional grounds the only contribution to Eq. (16) is $\sim 1/R$.

That corrections $\sim 1/(R^2)^\ell$ vanish is special to one loop order. This can be seen by the computations at next to leading order in κ at large d, where from Eq. (4.23) of [22], at large distances the potential is

$$V(R \to \infty) \approx \sigma R - \frac{d}{4} m - \frac{\pi d}{24}\frac{1}{R} - \frac{\pi^2 d^2}{1152}\frac{1}{\sigma R^3} + \frac{\pi^2 d^2}{384}\frac{1}{m\sigma R^4} + \cdots. \tag{18}$$

The result for the Nambu model is obtained by taking $d \to d-2$; then for $d = 4$, the terms $\sim 1/R$ and $\sim 1/R^3$ agree with Eq. (12). It is noteworthy that even at next to leading order in κ, there is no correction $\sim 1/R^2$. We do not know if this is peculiar to next to leading order, or persists at higher order.

There is a nonzero contribution $\sim 1/R^4$, which can be understood as follows. At leading order, Eq. (16), there is a term $\sim 1/(m^3 R^4)$ times $\zeta(-4)$, which vanishes. At next to leading order there is a contribution κ times $\sim 1/(m^3 R^4)$, or $\sim 1/(m\sigma R^4)$, which is nonzero.

C. Confining strings

We turn next to the case of confining strings [24–26]. We take this to mean a theory with negative coupling constant for the extrinsic curvature, $\kappa < 0$, and $\lambda > 0$. The effective Lagrangian is

$$S_{\text{eff}} = \left(\frac{d-2}{2}\right) \text{tr} \log(-\partial^2)((-\partial^2)^2 - 2M_1^2(-\partial^2) + M_2^4), \tag{19}$$

where

$$2M_1^2 = \frac{\lambda}{\kappa}, \qquad M_2^4 = \lambda\sigma. \tag{20}$$

As usual, the massless mode generates the usual term $\sim 1/R$.

The poles of the massive propagator are at

$$p^2 = M_1^2 \pm i\sqrt{M_2^4 - M_1^4} = M_2^2 \mathrm{e}^{\pm 2i\theta}, \qquad \tan(2\theta) = \sqrt{\left(\frac{M_2}{M_1}\right)^4 - 1}. \tag{21}$$

For the model to be physical, it is necessary that there are no poles on the real axis [25]. This gives the constraint,

$$M_2^4 > M_1^4, \qquad \kappa^2 > \frac{\lambda}{4\sigma}. \tag{22}$$

As in the previous section we compute the derivative of the effective Lagrangian with respect to M_2^4,

$$\frac{\partial S_{\rm eff}}{\partial M_2^4} = \left(\frac{d-2}{2}\right) {\rm tr} \frac{1}{(-\partial^2)^2 - 2M_1^2(-\partial^2) + M_2^4}. \tag{23}$$

In taking this derivative we can assume that M_1 is independent of M_2.

We first compute the zero point energy, neglecting the dependence upon the spatial momentum. This implies that the sum over the discrete spatial momenta generates $\zeta(0) = -1/2$.

It is then necessary to perform the integral over energies, ω. For rigid strings, the propagator has two poles, at $\pm im$. In the present case there are four poles. Two are in the upper half plane, $M_2 e^{i\theta}$ and $M_2 e^{i(\pi-\theta)}$, and two in the lower half plane, $M_2 e^{-i\theta}$ and $M_2 e^{-i(\pi+\theta)}$. Including the contribution just of the two poles in the upper half plane, the pole at $M_2 e^{i\theta}$ gives a residue $\sim e^{-i\theta}/(M_2^3 \sin(2\theta))$, while that at $-M_2 e^{-i\theta}$ gives a residue $\sim e^{+i\theta}/(M_2^3 \sin(2\theta))$. Hence,

$$\frac{\partial S_{\rm eff}}{\partial M_2^4} = -t_{\rm tot}\left(\frac{d-2}{16}\right)\frac{1}{\sin(\theta)M_2^3}. \tag{24}$$

Using $\sin(\theta) = \sqrt{(1 - M_1^2/M_2^2)/2}$, we find the nonperturbative contribution to the zero point energy to be

$$E_0^{\rm non} = -\left(\frac{d-2}{2\sqrt{2}}\right)\sqrt{M_2^2 - M_1^2}. \tag{25}$$

This vanishes when $M_2 = M_1$, but at this point the propagator has poles for real, Euclidean momenta, and the theory is not well defined [25].

Having obtained this result, one can also go through the algebra to determine corrections to the potential involving higher powers of $1/R^2$. When $p = n\pi/R$ is included, the poles in the propagator are shifted. Even so, the only way that the spatial momentum p enters is as p^2. Thus if one expands the result in powers of p, one will find that corrections are integral powers in p^2. At one loop order, as for the rigid string this only involves the trivial zeros of the ζ function, so that once again, the zero point energy is the *only* contribution to one loop order. As for the rigid string, we do not expect that this persists to higher loop order.

Clearly one could consider higher derivative terms for an effective string model. By a similar analysis one expects that at one loop order, the only correction to the Nambu potential is $E_0^{\rm non} < 0$.

IV. CONCLUSIONS

We argued in Sec. II that a renormalized quark-antiquark potential can be obtained from numerical simulations on the lattice, and that perturbative contributions to the associated zero point energy vanish. Nonperturbative contributions were computed in effective string models in Sec. III. They vanish in the Nambu model, but arise for either rigid or confining strings, with $E_0^{\rm non} < 0$ at one loop order.

Numerical simulations appear to find that at large distances, corrections to the Nambu model are small [3]. This suggests that on the world sheet, any massive modes are heavy on the scale of $\sqrt{\sigma}$. This is a regime of strong coupling in effective string models, and so the one loop result for $E_0^{\rm non}$ is not definitive. It will be interesting to see what numerical simulations find for both the sign and magnitude of $E_0^{\rm non}$.

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