

**Resolving  $B - CP$  puzzles in QCD factorization**

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Within the framework of QCD factorization (QCDF), power corrections due to penguin annihilation can account for the observed rates of penguin-dominated two-body decays of  $B$  mesons and direct  $CP$  asymmetries  $A_{CP}(K^- \pi^+)$ ,  $A_{CP}(K^{*-} \pi^+)$ ,  $A_{CP}(K^- \rho^0)$  and  $A_{CP}(\pi^+ \pi^-)$ . However, the predicted direct  $CP$ -violating effects in QCDF for  $B^- \rightarrow K^- \pi^0$ ,  $K^- \eta$ ,  $\pi^- \eta$  and  $\bar{B}^0 \rightarrow \pi^0 \pi^0$  are wrong in signs when confronted with experiment. We show that subleading  $1/m_b$  power corrections to the color-suppressed tree amplitude due to spectator scattering or final-state interactions will yield correct signs for aforementioned  $CP$  asymmetries and accommodate the observed  $\pi^0 \pi^0$  and  $\rho^0 \pi^0$  rates simultaneously. Implications are discussed.

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I. In the heavy quark limit, hadronic matrix elements can be expressed in terms of certain nonperturbative input quantities such as light-cone distribution amplitudes and transition form factors. Consequently, the decay amplitudes of charmless two-body decays of  $B$  mesons can be described in terms of decay constants and form factors. However, the predicted rates for penguin-dominated  $B \rightarrow PP$ ,  $VP$ ,  $VV$  decays ( $P$  and  $V$  denoting pseudoscalar and vector mesons, respectively) are systematically below the measurements (see the second column of Table I; for a review, see [5]).<sup>1</sup> Moreover, the calculated direct  $CP$  asymmetries for  $\bar{B}^0 \rightarrow K^- \pi^+$ ,  $K^{*-} \pi^+$ ,  $B^- \rightarrow K^- \rho^0$  and  $\bar{B}^0 \rightarrow \pi^+ \pi^-$  are wrong in signs when confronted with experiment as shown in the same Table. This implies the necessity of taking into account  $1/m_b$  power correction effects. In the QCD factorization (QCDF) approach [6], power corrections often involve endpoint divergences. For example, the hard spectator-scattering diagram at twist-3 order is power suppressed and possesses soft and collinear divergences arising from the soft spectator quark and the  $1/m_b$  annihilation amplitude has endpoint divergences even at twist-2 level. Since the treatment of endpoint divergences is model dependent, subleading power corrections generally can be studied only in a phenomenological way. While the endpoint divergence is regulated in the pQCD approach by introducing the parton's transverse momentum [7], it is parametrized in QCD factorization as

$$X_A \equiv \int_0^1 \frac{dy}{y} = \ln \frac{m_B}{\Lambda_h} (1 + \rho_A e^{i\phi_A}), \quad (1)$$

for penguin annihilation contributions with  $\Lambda_h$  being a typical scale of order 500 MeV.

<sup>1</sup>We have included chirally enhanced but power suppressed penguin contributions. Numerically, they are of order  $1/m_b^0$ .

In the so-called ‘‘S4’’ scenario of QCDF [8] with some appropriate choice of the parameters  $\rho_A$  and  $\phi_A$ , the above-mentioned discrepancies are resolved in the presence of power corrections due to the penguin annihilation topology. However, a scrutiny of the QCDF predictions reveals more puzzles in the regard of direct  $CP$  violation. When power corrections due to penguin annihilation are turned on, the signs of  $A_{CP}$  in  $B^- \rightarrow K^- \pi^0$ ,  $K^- \eta$ ,  $\pi^- \eta$  and  $\bar{B}^0 \rightarrow \pi^0 \pi^0$  will also get flipped in such a way that they disagree with experiment (see the third column of Table I). The so-called  $K\pi$   $CP$ -puzzle is related to the difference of  $CP$  asymmetries of  $B^- \rightarrow K^- \pi^0$  and  $\bar{B}^0 \rightarrow K^- \pi^+$ . This can be illustrated by considering the decay amplitudes of  $\bar{B} \rightarrow \bar{K}\pi$  in terms of topological diagrams

$$\begin{aligned} A(\bar{B}^0 \rightarrow K^- \pi^+) &= P' + T' + \frac{2}{3} P'_{EW}{}^c + P'_A, \\ A(\bar{B}^0 \rightarrow \bar{K}^0 \pi^0) &= -\frac{1}{\sqrt{2}} \left( P' - C' - P'_{EW} - \frac{1}{3} P'_{EW}{}^c + P'_A \right), \\ A(B^- \rightarrow \bar{K}^0 \pi^-) &= P' - \frac{1}{3} P'_{EW}{}^c + A' + P'_A, \\ A(B^- \rightarrow K^- \pi^0) &= \frac{1}{\sqrt{2}} \left( P' + T' + C' + P'_{EW} + \frac{2}{3} P'_{EW}{}^c \right. \\ &\quad \left. + A' + P'_A \right), \end{aligned} \quad (2)$$

where  $T'$ ,  $C'$ ,  $E'$ ,  $A'$ ,  $P'_{EW}$ , and  $P'_{EW}{}^c$  are color-allowed tree, color-suppressed tree,  $W$ -exchange,  $W$ -annihilation, color-allowed and color-suppressed electroweak penguin amplitudes, respectively, and  $P'_A$  is the penguin-induced weak annihilation amplitude. We use unprimed and primed symbols to denote  $\Delta S = 0$  and  $|\Delta S| = 1$  transitions. We notice that if  $C'$ ,  $P'_{EW}$  and  $A'$  are negligible compared with  $T'$ , it is clear from Eq. (2) that the decay amplitudes of  $K^- \pi^0$  and  $K^- \pi^+$  will be the same apart from a trivial factor of  $1/\sqrt{2}$ .

TABLE I.  $CP$ -averaged branching fractions (in units of  $10^{-6}$ ) and direct  $CP$  asymmetries (in %) of some selective  $B \rightarrow PP$  decays obtained in QCD factorization for three distinct cases: (i) without any power corrections, (ii) with power corrections from penguin annihilation, and (iii) with power corrections to both penguin annihilation and color-suppressed tree amplitudes. The parameters  $\rho_A$  and  $\phi_A$  are taken from Eq. (4),  $\rho_C = 1.3$  and  $\phi_C = -70^\circ$ . Sources of theoretical uncertainties are discussed in the text.

Modes	W/o $\rho_{A,C}, \phi_{A,C}$	With $\rho_A, \phi_A$	With $\rho_{A,C}, \phi_{A,C}$	Expt. [1]
$\mathcal{B}(\bar{B}^0 \rightarrow K^- \pi^+)$	$13.1^{+5.8+0.7}_{-3.5-0.7}$	$19.3^{+7.9+8.2}_{-4.8-6.2}$	$19.3^{+7.9+8.2}_{-4.8-6.2}$	$19.4 \pm 0.6$
$\mathcal{B}(\bar{B}^0 \rightarrow \bar{K}^0 \pi^0)$	$5.5^{+2.8+0.3}_{-1.7-0.3}$	$8.4^{+3.8+3.8}_{-2.3-2.9}$	$8.6^{+3.8+3.8}_{-2.2-2.9}$	$9.5 \pm 0.5$
$\mathcal{B}(B^- \rightarrow \bar{K}^0 \pi^-)$	$14.9^{+6.9+0.9}_{-4.5-1.0}$	$21.7^{+9.2+9.0}_{-6.0-6.9}$	$21.7^{+9.2+9.0}_{-6.0-6.9}$	$23.1 \pm 1.0$
$\mathcal{B}(B^- \rightarrow K^- \pi^0)$	$9.1^{+3.6+0.5}_{-2.3+0.5}$	$12.6^{+4.7+4.8}_{-3.0-3.7}$	$12.5^{+4.7+4.9}_{-3.0-3.8}$	$12.9 \pm 0.6$
$\mathcal{B}(B^- \rightarrow K^- \eta)$	$1.6^{+1.1+0.3}_{-0.7-0.4}$	$2.4^{+1.8+1.3}_{-1.1-1.0}$	$2.4^{+1.8+1.3}_{-1.1-1.0}$	$2.3 \pm 0.3^a$
$\mathcal{B}(\bar{B}^0 \rightarrow \pi^+ \pi^-)$	$6.2^{+0.4+0.2}_{-0.6-0.4}$	$7.0^{+0.4+0.7}_{-0.7-0.7}$	$7.0^{+0.4+0.7}_{-0.7-0.7}$	$5.16 \pm 0.22$
$\mathcal{B}(\bar{B}^0 \rightarrow \pi^0 \pi^0)$	$0.42^{+0.29+0.18}_{-0.11+0.08}$	$0.52^{+0.26+0.21}_{-0.10-0.10}$	$1.1^{+1.0+0.7}_{-0.4-0.3}$	$1.55 \pm 0.19^b$
$\mathcal{B}(B^- \rightarrow \pi^- \pi^0)$	$4.9^{+0.9+0.6}_{-0.5-0.3}$	$4.9^{+0.9+0.6}_{-0.5-0.3}$	$5.9^{+2.2+1.4}_{-1.1-1.1}$	$5.59^{+0.41}_{-0.40}$
$\mathcal{B}(B^- \rightarrow \pi^- \eta)$	$4.4^{+0.6+0.4}_{-0.3-0.2}$	$4.5^{+0.6+0.5}_{-0.3-0.3}$	$5.0^{+1.2+0.9}_{-0.6-0.7}$	$4.1 \pm 0.3^a$
$A_{CP}(\bar{B}^0 \rightarrow K^- \pi^+)$	$4.0^{+0.6+1.1}_{-0.7-1.1}$	$-7.4^{+1.7+4.3}_{-1.5-4.8}$	$-7.4^{+1.7+4.3}_{-1.5-4.8}$	$-9.8^{+1.2}_{-1.1}$
$A_{CP}(\bar{B}^0 \rightarrow \bar{K}^0 \pi^0)$	$-4.0^{+1.2+3.5}_{-1.8-3.0}$	$0.75^{+1.88+2.56}_{-0.94-3.32}$	$-10.6^{+2.7+5.6}_{-3.8-4.3}$	$-1 \pm 10$
$A_{CP}(B^- \rightarrow \bar{K}^0 \pi^-)$	$0.72^{+0.06+0.05}_{-0.05-0.05}$	$0.28^{+0.03+0.09}_{-0.03-0.10}$	$0.28^{+0.03+0.09}_{-0.03-0.10}$	$0.9 \pm 2.5$
$A_{CP}(B^- \rightarrow K^- \pi^0)$	$7.3^{+1.6+2.3}_{-1.2-2.7}$	$-5.5^{+1.3+4.9}_{-1.8-4.6}$	$4.9^{+3.9+4.4}_{-2.1-5.4}$	$5.0 \pm 2.5$
$A_{CP}(B^- \rightarrow K^- \eta)$	$-22.1^{+7.7+14.0}_{-16.7-7.3}$	$12.7^{+7.7+13.4}_{-5.0-15.0}$	$-11.0^{+8.4+14.9}_{-21.6-10.1}$	$-37 \pm 9^a$
$A_{CP}(\bar{B}^0 \rightarrow \pi^+ \pi^-)$	$-6.2^{+0.4+2.0}_{-0.5-1.8}$	$17.0^{+1.3+4.3}_{-1.2-8.7}$	$17.0^{+1.3+4.3}_{-1.2-8.7}$	$38 \pm 6$
$A_{CP}(\bar{B}^0 \rightarrow \pi^0 \pi^0)$	$33.4^{+6.8+34.8}_{-10.6-37.7}$	$-26.9^{+8.4+48.5}_{-6.0-37.5}$	$57.2^{+14.8+30.3}_{-20.8-34.6}$	$43^{+25}_{-24}$
$A_{CP}(B^- \rightarrow \pi^- \pi^0)$	$-0.06^{+0.01+0.01}_{-0.01-0.02}$	$-0.06^{+0.01+0.01}_{-0.01-0.02}$	$-0.11^{+0.01+0.06}_{-0.01-0.03}$	$6 \pm 5$
$A_{CP}(B^- \rightarrow \pi^- \eta)$	$-11.4^{+1.1+2.3}_{-1.0-2.7}$	$11.4^{+0.9+4.5}_{-0.9-9.1}$	$-5.0^{+2.4+8.4}_{-3.4-10.3}$	$-13 \pm 7^a$

<sup>a</sup>We have taken into account the new measurement of  $B^- \rightarrow (K^-, \pi^-)\eta$  [2] to update the average.

<sup>b</sup>This is the average of  $1.83 \pm 0.21 \pm 0.13$  by *BABAR* [3] and  $1.1 \pm 0.3 \pm 0.1$  by *Belle* [4]. If an  $S$  factor is included, the average will become  $1.55 \pm 0.35$ .

Hence, one will expect that  $A_{CP}(K^- \pi^0) \approx A_{CP}(K^- \pi^+)$ , while they differ by  $5.3\sigma$  experimentally,  $\Delta A_{K\pi} \equiv A_{CP}(K^- \pi^0) - A_{CP}(K^- \pi^+) = 0.148 \pm 0.028$  [1]. We also notice that the decay  $B^- \rightarrow K^- \eta$  has a world average  $-0.37 \pm 0.09$  for  $A_{CP}(K^- \eta)$  [1,2,9] different from zero by 4.1 standard deviations.

Since in the heavy quark limit,  $CP$  asymmetries of the  $K^- \pi^0, K^- \eta, \pi^- \eta, \pi^0 \pi^0$  modes have the correct signs when compared with experiment, the  $B$ - $CP$  puzzles mentioned here are relevant to QCDF and may not occur in other approaches such as pQCD. In this work, we shall show that soft power corrections to the color-suppressed tree amplitude will bring the signs of  $A_{CP}$  back to the right track. As a bonus, the rates of  $\bar{B}^0 \rightarrow \pi^0 \pi^0, \rho^0 \pi^0$  can be accommodated.

2. The aforementioned direct  $CP$  puzzles indicate that it is necessary to consider subleading power corrections other than penguin annihilation. For example, the large power corrections due to  $P'_A$  cannot explain the  $\Delta A_{K\pi}$  puzzle as they contribute equally to both  $B^- \rightarrow K^- \pi^0$  and  $\bar{B}^0 \rightarrow K^- \pi^+$ . The additional power correction should have little impact on the decay rates of penguin-dominated decays but will manifest in the measurement of direct  $CP$  asymmetries. Note that all the ‘‘problematic’’ modes receive a contribution from  $c^{(\prime)} = C^{(\prime)} + P'_{EW}$ . Since  $A(B^- \rightarrow K^- \pi^0) \propto t' + c' + p'$  and  $A(\bar{B}^0 \rightarrow K^- \pi^+) \propto t' + p'$  with

$t' = T' + P'_{EW}$  and  $p' = P' - \frac{1}{3}P'_{EW} + P'_A$ , we can consider this puzzle resolved, provided that  $c'/t'$  is of order  $1.3 \sim 1.4$  with a large negative phase (naively  $|c'/t'| \sim 0.9$ ). There are several possibilities for a large  $c'$ : either a large color-suppressed  $C'$  or a large electroweak penguin  $P'_{EW}$  or a combination of them. Various scenarios for accommodating large  $C'$  [10–17] or  $P'_{EW}$  [18,19] have been proposed. To get a large  $C'$ , one can appeal to spectator scattering or final-state rescattering (see discussions below). However, the general consensus for a large  $P'_{EW}$  is that one needs new physics beyond the standard model (SM). In principle, one cannot tell the difference of these two possibilities in penguin-dominated decays as it is always the combination  $c' = C' + P'_{EW}$  that enters into the decay amplitude except for the decays involving  $\eta$  and/or  $\eta'$  in the final state where both  $c'$  and  $P'_{EW}$  present in the amplitudes [20]. Nevertheless, the two scenarios can lead to very distinct predictions for tree-dominated decays where  $P_{EW} \ll C$  as the electroweak penguin amplitude here does not get a CKM enhancement. The decay rates of  $\bar{B}^0 \rightarrow \pi^0 \pi^0, \rho^0 \pi^0$  will be substantially enhanced for a large  $C$  but remain intact for a large  $P_{EW}$ . Since  $P_{EW} \ll C$  in tree-dominated channels,  $CP$  puzzles with  $\pi^- \eta$  and  $\pi^0 \pi^0$  cannot be resolved with a large  $P_{EW}$ . Therefore, it is most likely that the color-suppressed tree amplitude is large and complex. Motivated by the above observation, in

this work we shall consider the possibility of a large complex  $a_2$ , the parameter for describing the color-suppressed tree topology, and parametrize power corrections to  $a_2$  as <sup>2</sup>

$$a_2 \rightarrow a_2^{\text{NLO}}(1 + \rho_C e^{i\phi_C}), \quad (3)$$

with the unknown parameters  $\rho_C$  and  $\phi_C$  to be inferred from experiment.

The reader is referred to [24] for details. We shall first consider soft corrections to weak annihilation dictated by the parameters  $\rho_A$  and  $\phi_A$ . A fit to the data of two-body hadronic decays of  $B^0$  and  $B^-$  mesons within QCDF yields the values

$$\begin{aligned} \rho_A^0 &\approx 1.10, & 1.07, & 0.87, \\ \phi_A^0 &\approx -50^\circ, & -70^\circ, & -30^\circ, \end{aligned} \quad (4)$$

for  $B \rightarrow PP$ ,  $PV$ ,  $VP$  respectively, where the superscript ‘‘0’’ of  $\rho_A$  and  $\phi_A$  indicates that they are the default values we shall use in this work. Basically, this is very similar to the ‘‘scenario S4’’ presented in [8]. For the annihilation diagram we use the convention that  $M_1$  ( $M_2$ ) contains an antiquark (a quark) from the weak vertex. Since the penguin annihilation effects are different for  $M_1 = P$  and  $M_1 = V$ , the parameters  $\rho_A$  and  $\phi_A$  are thus different for  $B \rightarrow PV$  and  $B \rightarrow VP$ .

Branching fractions and direct  $CP$  asymmetries for some selective  $B \rightarrow PP$  decays are shown in Table I. The theoretical errors correspond to the uncertainties due to variation of (i) the Gegenbauer moments, the decay constants, (ii) the heavy-to-light form factors and the strange quark mass, and (iii) the wave function of the  $B$  meson characterized by the parameter  $\lambda_B$ , the power corrections due to weak annihilation and hard spectator interactions described by the parameters  $\rho_{A,H}$ ,  $\phi_{A,H}$ , respectively. To obtain the errors shown in Table I, we first scan randomly the points in the allowed ranges of the above nine parameters (specifically, the ranges  $\rho_A^0 - 0.1 \leq \rho_A \leq \rho_A^0 + 0.1$ ,  $\phi_A^0 - 20^\circ \leq \phi_A \leq \phi_A^0 + 20^\circ$ ,  $0 \leq \rho_H \leq 1$  and  $0 \leq \phi_H \leq 2\pi$  are used in this work) and then add errors in quadrature. More specifically, the second error in the table is referred to the uncertainties caused by the variation of  $\rho_{A,H}$  and  $\phi_{A,H}$ , where all other uncertainties are lumped into the first error. Power corrections beyond the heavy quark limit generally give the major theoretical uncertainties.

<sup>2</sup>We use NLO results for  $a_2$  in Eq. (3) as a benchmark to define power corrections. The NNLO calculations of spectator-scattering tree amplitudes and vertex corrections at order  $\alpha_s^2$  have been carried out in [21,22], respectively. While NNLO corrections can in principle push the magnitude of  $a_2(\pi\pi)$  up to the order of 0.50 by lowering the value of the  $B$ -meson parameter  $\lambda_B$ , the strong phase of  $a_2$  relative to  $a_1$  cannot be larger than  $15^\circ$  [23].

For  $\rho_C \approx 1.3$  and  $\phi_C \approx -70^\circ$ , we find that all the  $CP$  puzzles in  $B \rightarrow PP$  decays are resolved as shown in fourth column of Table I. The corresponding  $a_2$ 's are

$$a_2(\pi\pi) \approx 0.60e^{-i55^\circ}, \quad a_2(K\pi) \approx 0.51e^{-i58^\circ}. \quad (5)$$

They are consistent with the phenomenological determination of  $C^{(i)}/T^{(i)} \sim a_2/a_1$  from a global fit to the available data [20]. Because of the interference between the penguin and the enhanced color-suppressed amplitudes with a sizable strong phase, it is clear from Table I that theoretical predictions for  $A_{CP}$  now agree with experiment in sign even for those modes with the measured  $A_{CP}$  less than  $3\sigma$  in significance. As first emphasized by Lunghi and Soni [25], in the QCDF analysis of the quantity  $\Delta A_{K\pi}$ , although the theoretical uncertainties due to power corrections from penguin annihilation are large for individual asymmetries  $A_{CP}(K^-\pi^0)$  and  $A_{CP}(K^-\pi^+)$ , they essentially cancel out in their difference, rendering the theoretical prediction more reliable. We find  $\Delta A_{K\pi} = (12.3_{-0.9-4.7}^{+2.2+2.1})\%$ , while it is only  $(1.9_{-0.4-1.0}^{+0.5+1.6})\%$  in the absence of power corrections to the topological amplitude ‘‘C’’ or  $a_2$ .

For the direct  $CP$  asymmetry of  $\bar{B}^0 \rightarrow \bar{K}^0 \pi^0$ , we predict  $A_{CP}(\bar{K}^0 \pi^0) = (-10.6_{-3.7-4.3}^{+2.7+5.5})\%$ . Experimentally, the current world average  $-0.01 \pm 0.10$  is consistent with no  $CP$  violation because the *BABAR* and *Belle* measurements,  $-0.13 \pm 0.13 \pm 0.03$  [26] and  $0.14 \pm 0.13 \pm 0.06$  [27] respectively, are opposite in sign. Nevertheless, there exist several model-independent determinations of this asymmetry: one is the SU(3) relation  $\Delta\Gamma(\pi^0 \pi^0) = -\Delta\Gamma(\bar{K}^0 \pi^0)$  [28], and the other is the approximate sum rule for  $CP$  rate asymmetries [29]

$$\Delta\Gamma(K^-\pi^+) + \Delta\Gamma(\bar{K}^0 \pi^-) \approx 2[\Delta\Gamma(K^-\pi^0) + \Delta\Gamma(\bar{K}^0 \pi^0)], \quad (6)$$

based on isospin symmetry, where  $\Delta\Gamma(K\pi) \equiv \Gamma(\bar{B} \rightarrow \bar{K}\pi) - \Gamma(B \rightarrow K\pi)$ . This sum rule allows us to extract  $A_{CP}(\bar{K}^0 \pi^0)$  in terms of the other three asymmetries in  $K^-\pi^+$ ,  $K^-\pi^0$ ,  $\bar{K}^0 \pi^-$  modes that have been measured. From the current data of branching fractions and  $CP$  asymmetries, the above SU(3) relation and  $CP$ -asymmetry sum rule lead to  $A_{CP}(\bar{K}^0 \pi^0) = -0.073_{-0.041}^{+0.042}$  and  $A_{CP}(\bar{K}^0 \pi^0) = -0.15 \pm 0.04$ , respectively. An analysis based on the topological quark diagrams also yields a similar result  $-0.08 \sim -0.12$  [30]. All these indicate that the direct  $CP$  violation  $A_{CP}(\bar{K}^0 \pi^0)$  should be negative and has a magnitude of order 0.10. As for the mixing-induced asymmetry  $S_{\pi^0 K_S}$ , it is found to be enhanced from 0.76 to  $0.79_{-0.04-0.04}^{+0.06+0.04}$  when  $\rho_C$  and  $\phi_C$  are turned on, while experimentally it is  $0.57 \pm 0.17$  [1]. The discrepancy between theory and experiment for  $S_{\pi^0 K_S}$  is one of possible hints of new physics [31]. Our result for  $S_{\pi^0 K_S}$  is consistent with [13–15] where soft corrections to  $a_2$  were considered, but not with [16] where  $S_{\pi^0 K_S} \sim 0.63$  was obtained. A correlation between  $S_{\pi^0 K_S}$  and  $A_{CP}(\pi^0 K_S)$

has been investigated recently in [32]. For the mixing-induced asymmetry in  $B \rightarrow \pi^+ \pi^-$ , we find  $S_{\pi^+ \pi^-} = -0.69_{-0.10-0.09}^{+0.08+0.19}$ , in accordance with the world average of  $-0.65 \pm 0.07$  [1].

From Table I we see that power corrections to the color-suppressed tree amplitude have almost no impact on the decay rates of penguin-dominated decays, but will enhance the color-suppressed tree-dominated decay  $B \rightarrow \pi^0 \pi^0$  substantially owing to the enhancement of  $|a_2| \sim \mathcal{O}(0.6)$ . Notice that the central values of the branching fractions of this mode measured by *BABAR* [3] and *Belle* [4] are somewhat different as noticed in Table I. It is generally believed that direct  $CP$  violation of  $B^- \rightarrow \pi^- \pi^0$  is very small. This is because the isospin of the  $\pi^- \pi^0$  state is  $I = 2$  and hence it does not receive QCD penguin contributions and receives only the loop contributions from electroweak penguins. Since this decay is tree dominated, the SM predicts an almost null  $CP$  asymmetry, of order  $10^{-3} \sim 10^{-4}$ . What will happen if  $a_2$  has a large magnitude and strong phase? We find that soft corrections to the color-suppressed tree amplitude will enhance  $A_{CP}(\pi^- \pi^0)$  substantially to the level of 2%. Similar conclusions were also obtained by the analysis based on the diagrammatic approach [20].

$$\begin{aligned} \sqrt{2}A(B^- \rightarrow K^- \eta) &= A_{\bar{K}\eta_q} \left[ \delta_{pu} \alpha_2 + 2\alpha_3^p + \frac{1}{2} \alpha_{3,EW}^p \right] \\ &+ \sqrt{2}A_{\bar{K}\eta_s} \left[ \delta_{pu} \beta_2 + \alpha_3^p + \alpha_4^p - \frac{1}{2} \alpha_{3,EW}^p - \frac{1}{2} \alpha_{4,EW}^p + \beta_3^p + \beta_{3,EW}^p \right] + \sqrt{2}A_{\bar{K}\eta_c} [\delta_{pc} \alpha_2 + \alpha_3^p] \\ &+ A_{\eta_q \bar{K}} [\delta_{pu} (\alpha_1 + \beta_2) + \alpha_4^p + \beta_3^p + \beta_{3,EW}^p], \end{aligned} \quad (7)$$

where the flavor states of the  $\eta$  meson,  $q\bar{q} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$ ,  $s\bar{s}$  and  $c\bar{c}$  are labeled by the  $\eta_q$ ,  $\eta_s$  and  $\eta_c^0$ , respectively. The reader is referred to [8] for other notations. The physical states  $\eta$ ,  $\eta'$ , and  $\eta_c$  can be expressed in terms of flavor states  $\eta_q$ ,  $\eta_s$ , and  $\eta_c^0$ . Since the two penguin processes  $b \rightarrow ss\bar{s}$  and  $b \rightarrow sq\bar{q}$  contribute destructively to  $B \rightarrow K\eta$ , the penguin amplitude is comparable in magnitude to the tree amplitude induced from  $b \rightarrow us\bar{u}$ , contrary to the decay  $B \rightarrow K\eta'$  which is dominated by large penguin amplitudes. Consequently, a sizable direct  $CP$  asymmetry is expected in  $B^- \rightarrow K^- \eta$  but not in  $K^- \eta'$  [34].

Quantities relevant to the calculation are the decay constants  $f_\eta^q$ ,  $f_\eta^s$  and  $f_\eta^c$  defined by  $\langle 0 | \bar{q} \gamma_\mu \gamma_5 q | \eta \rangle = i f_\eta^q / \sqrt{2} q_\mu$ ,  $\langle 0 | \bar{s} \gamma_\mu \gamma_5 s | \eta \rangle = i f_\eta^s q_\mu$ , and  $\langle 0 | \bar{c} \gamma_\mu \gamma_5 c | \eta \rangle = i f_\eta^c q_\mu$ , respectively. A straightforward perturbative calculation gives [35]

$$f_\eta^c = -\frac{m_\eta^2}{12m_c^2} \frac{f_\eta^q}{\sqrt{2}}. \quad (8)$$

For the decay constants  $f_\eta^q$  and  $f_\eta^s$ , we shall use the values  $f_\eta^q = 107$  MeV and  $f_\eta^s = -112$  MeV obtained in [36] with the convention of  $f_\pi = 132$  MeV. Although the decay constant  $f_\eta^c \approx -2$  MeV is much smaller than  $f_\eta^{q,s}$ , its

effect is CKM enhanced by  $V_{cb}V_{cs}^*/(V_{ub}V_{us}^*)$ . In the absence of power corrections to  $a_2$ ,  $A_{CP}(K^- \eta)$  is found to be 0.127 (see Table I). When  $\rho_C$  and  $\phi_C$  are turned on,  $A_{CP}(K^- \eta)$  will be reduced to 0.004 if there is no intrinsic charm content in the  $\eta$ . When the effect of  $f_\eta^c$  is taken into account,  $A_{CP}(K^- \eta)$  finally reaches at the level of  $-11\%$  and has a sign in agreement with experiment. Hence,  $CP$  violation in  $B^- \rightarrow K^- \eta$  is the place where the charm content of the  $\eta$  plays a role.

We add a remark here that the pQCD prediction for  $A_{CP}(K^- \eta)$  is very sensitive to  $m_{qq}$ , the mass of the  $\eta_q$ , which is generally taken to be of order  $m_\pi$ . It was found in [37] that for  $m_{qq} = 0.14, 0.18$  and  $0.22$  GeV,  $A_{CP}(K^- \eta)$  becomes 0.0562, 0.0588 and  $-0.3064$ , respectively. There are two issues here: (i) Is it natural to have a large value of  $m_{qq}$ ? and (ii) The fact that  $A_{CP}(K^- \eta)$  is so sensitive to  $m_{qq}$  implies that the pQCD prediction is not stable. Within the framework of pQCD, the authors of [38] rely on the NLO corrections to get a negative  $CP$  asymmetry and to avoid the aforementioned issues. At the lowest order, pQCD predicts  $A_{CP}(K^- \eta) \approx 9.3\%$ . Then NLO corrections will change the sign and give rise to  $A_{CP}(K^- \eta) = (-11.7_{-11.4}^{+8.4})\%$  [38].

As for the decay  $B^- \rightarrow \pi^- \eta$ , it is interesting to see that penguin annihilation will flip the sign of  $A_{CP}(\pi^- \eta)$  into a wrong one without affecting its magnitude (see Table II). Again, soft corrections to  $a_2$  will bring the  $CP$  asymmetry back to the right track. Contrary to the previous case, the charm content of the  $\eta$  here does not play a role as it does not get a CKM enhancement relative to the noncharm content of the  $\eta$ . Our result of  $A_{CP}(\pi^- \eta) = -0.05^{+0.09}_{-0.11}$  is consistent with the measurement of  $-0.13 \pm 0.07$ . For comparison, the pQCD approach predicts  $-0.37^{+0.09}_{-0.07}$  [41] and SCET gives two solutions [42],  $0.05 \pm 0.29$  and  $0.37 \pm 0.29$  with signs opposite to the data.

3. What is the origin of power corrections to  $a_2$ ? There are two possible sources: spectator scattering and final-state interactions. The flavor operators  $a_i^p$  are basically the Wilson coefficients in conjunction with short-distance non-factorizable corrections such as vertex corrections, penguin contractions and hard spectator interactions. In

general, they have the expression [6,8]

$$a_i^p(M_1 M_2) = \left( c_i + \frac{c_{i\pm 1}}{N_c} \right) N_i(M_2) + \frac{c_{i\pm 1}}{N_c} \frac{C_F \alpha_s}{4\pi} \times \left[ V_i(M_2) + \frac{4\pi^2}{N_c} H_i(M_1 M_2) \right] + P_i^p(M_2), \quad (9)$$

where  $i = 1, \dots, 10$ , the upper (lower) signs apply when  $i$  is odd (even),  $c_i$  are the Wilson coefficients,  $C_F = (N_c^2 - 1)/(2N_c)$  with  $N_c = 3$ ,  $N_i(M_2) = 0$  for  $i = 6, 8$  and equals to 1 otherwise,  $M_2$  is the emitted meson and  $M_1$  shares the same spectator quark with the  $B$  meson. The quantities  $V_i(M_2)$  account for vertex corrections,  $H_i(M_1 M_2)$  for hard spectator interactions with a hard gluon exchange between the emitted meson and the spectator quark of the  $B$  meson and  $P_i(M_2)$  for penguin contractions. A typical hard spectator term  $H_i(M_1 M_2)$  has the expressions [6,8]:

TABLE II. Same as Table I except for some selective  $B \rightarrow VP$  decays with  $\rho_C = 0.8$  and  $\phi_C = -80^\circ$ .

Modes	W/o $\rho_{A,C}, \phi_{A,C}$	With $\rho_A, \phi_A$	With $\rho_{A,C}, \phi_{A,C}$	Expt. [1]
$\mathcal{B}(\bar{B}^0 \rightarrow K^- \rho^+)$	$6.5^{+5.4+0.4}_{-2.6-0.4}$	$8.6^{+5.7+7.4}_{-2.8-4.5}$	$8.6^{+5.7+7.4}_{-2.8-4.5}$	$8.6^{+0.9}_{-1.1}$
$\mathcal{B}(\bar{B}^0 \rightarrow \bar{K}^0 \rho^0)$	$4.7^{+3.3+0.3}_{-1.7+0.3}$	$5.5^{+3.5+4.3}_{-1.8-2.8}$	$5.4^{+3.3+4.3}_{-1.7-2.8}$	$5.4^{+0.9}_{-1.0}$
$\mathcal{B}(B^- \rightarrow \bar{K}^0 \rho^-)$	$5.5^{+6.1+0.7}_{-2.8+0.5}$	$7.8^{+6.3+7.3}_{-2.9-4.4}$	$7.8^{+6.3+7.3}_{-2.9-4.4}$	$8.0^{+1.5}_{-1.4}$
$\mathcal{B}(B^- \rightarrow K^- \rho^0)$	$1.0^{+2.5+0.3}_{-1.0-0.2}$	$3.3^{+2.6+2.9}_{-1.1-1.7}$	$3.5^{+2.9+2.9}_{-1.2-1.8}$	$3.81^{+0.48}_{-0.46}$
$\mathcal{B}(\bar{B}^0 \rightarrow K^{*-} \pi^+)$	$3.7^{+0.5+0.4}_{-0.5-0.4}$	$9.2^{+1.0+3.7}_{-1.0-3.3}$	$9.2^{+1.0+3.7}_{-1.0-3.3}$	$10.3 \pm 1.1$
$\mathcal{B}(\bar{B}^0 \rightarrow \bar{K}^{*0} \pi^0)$	$1.1^{+0.2+0.2}_{-0.2-0.2}$	$3.5^{+0.4+1.7}_{-0.5-1.5}$	$3.5^{+0.4+1.6}_{-0.4-1.4}$	$2.4 \pm 0.7$
$\mathcal{B}(B^- \rightarrow \bar{K}^{*0} \pi^-)$	$4.0^{+0.7+0.6}_{-0.9-0.6}$	$10.4^{+1.3+4.3}_{-1.5-3.9}$	$10.4^{+1.3+4.3}_{-1.5-3.9}$	$9.9^{+0.8}_{-0.9}$
$\mathcal{B}(B^- \rightarrow K^{*-} \pi^0)$	$3.2^{+0.4+0.3}_{-0.4-0.3}$	$6.8^{+0.7+2.3}_{-0.7-2.2}$	$6.7^{+0.7+2.4}_{-0.7-2.2}$	$6.9 \pm 2.3$
$\mathcal{B}(\bar{B}^0 \rightarrow \bar{K}^{*0} \eta)$	$11.0^{+6.9+1.7}_{-3.5-1.0}$	$15.4^{+7.7+9.4}_{-4.0-7.1}$	$15.6^{+7.9+9.4}_{-4.1-7.1}$	$15.9 \pm 1.0$
$\mathcal{B}(\bar{B}^0 \rightarrow \rho^0 \pi^0)$	$0.76^{+0.96+0.66}_{-0.37+0.31}$	$0.58^{+0.88+0.60}_{-0.32-0.22}$	$1.3^{+1.7+1.2}_{-0.6-0.6}$	$2.0 \pm 0.5^a$
$\mathcal{B}(B^- \rightarrow \rho^- \pi^0)$	$11.6^{+1.2+0.9}_{-0.9+0.5}$	$11.8^{+1.3+1.0}_{-0.9-0.6}$	$11.8^{+1.8+1.4}_{-1.1-1.4}$	$10.9^{+1.4}_{-1.5}$
$\mathcal{B}(B^- \rightarrow \rho^0 \pi^-)$	$8.2^{+1.8+1.2}_{-0.9-0.6}$	$8.5^{+1.8+1.2}_{-0.9-0.6}$	$8.7^{+2.7+1.7}_{-1.3-1.4}$	$8.3^{+1.2}_{-1.3}$
$\mathcal{B}(\bar{B}^0 \rightarrow \rho^- \pi^+)$	$15.3^{+1.0+0.5}_{-1.5-0.9}$	$15.9^{+1.1+0.9}_{-1.5-1.1}$	$15.9^{+1.1+0.9}_{-1.5-1.1}$	$15.7 \pm 1.8$
$\mathcal{B}(\bar{B}^0 \rightarrow \rho^+ \pi^-)$	$8.4^{+0.4+0.3}_{-0.7-0.5}$	$9.2^{+0.4+0.5}_{-0.7-0.7}$	$9.2^{+0.4+0.5}_{-0.7-0.7}$	$7.3 \pm 1.2$
$A_{CP}(\bar{B}^0 \rightarrow K^- \rho^+)$	$-1.3^{+0.7+3.8}_{-0.3-3.8}$	$31.9^{+11.5+19.6}_{-11.0-12.7}$	$31.9^{+11.5+19.6}_{-11.0-12.7}$	$15 \pm 6$
$A_{CP}(\bar{B}^0 \rightarrow \bar{K}^0 \rho^0)$	$6.8^{+1.1+4.9}_{-1.2-4.9}$	$-5.0^{+3.2+6.0}_{-6.4-4.5}$	$8.7^{+1.2+8.7}_{-1.2-6.8}$	$6 \pm 12$
$A_{CP}(B^- \rightarrow \bar{K}^0 \rho^-)$	$0.24^{+0.12+0.08}_{-0.15-0.07}$	$0.27^{+0.19+0.46}_{-0.27-0.17}$	$0.27^{+0.19+0.46}_{-0.27-0.17}$	$-12 \pm 17$
$A_{CP}(B^- \rightarrow K^- \rho^0)$	$-8.3^{+3.5+7.0}_{-0.9-7.0}$	$56.5^{+16.1+30.0}_{-18.2-22.8}$	$45.4^{+17.8+31.4}_{-19.4-23.2}$	$44^{+12}_{-17}$
$A_{CP}(\bar{B}^0 \rightarrow K^{*-} \pi^+)$	$15.6^{+0.9+4.5}_{-0.7-4.7}$	$-12.1^{+0.5+12.6}_{-0.5-16.0}$	$-12.1^{+0.5+12.6}_{-0.5-16.0}$	$-18 \pm 8$
$A_{CP}(\bar{B}^0 \rightarrow \bar{K}^{*0} \pi^0)$	$-12.0^{+2.4+11.3}_{-4.6-7.6}$	$-0.87^{+1.71+6.04}_{-0.89-6.79}$	$-10.7^{+1.8+9.1}_{-2.8-6.3}$	$-15 \pm 12$
$A_{CP}(B^- \rightarrow \bar{K}^{*0} \pi^-)$	$0.97^{+0.11+0.12}_{-0.07-0.11}$	$0.39^{+0.04+0.10}_{-0.03-0.12}$	$0.39^{+0.04+0.10}_{-0.03-0.12}$	$3.2 \pm 5.4$
$A_{CP}(B^- \rightarrow K^{*-} \pi^0)$	$17.5^{+2.0+6.3}_{-1.3-8.0}$	$-6.7^{+0.7+11.8}_{-1.1-14.0}$	$1.6^{+3.1+11.1}_{-1.7-14.3}$	$4 \pm 29$
$A_{CP}(\bar{B}^0 \rightarrow \bar{K}^{*0} \eta)$	$3.0^{+0.4+1.9}_{-0.4-1.8}$	$0.20^{+0.51+2.00}_{-1.00-1.21}$	$3.5^{+0.4+2.7}_{-0.5-2.4}$	$19 \pm 5$
$A_{CP}(\bar{B}^0 \rightarrow \rho^0 \pi^0)$	$-2.3^{+2.4+9.9}_{-3.7-9.2}$	$31.5^{+13.3+21.5}_{-12.5+30.9}$	$11.0^{+5.0+23.5}_{-5.7-28.8}$	$-30 \pm 38^b$
$A_{CP}(B^- \rightarrow \rho^- \pi^0)$	$-5.4^{+0.4+2.0}_{-0.3-2.1}$	$16.3^{+1.1+7.1}_{-1.2+10.5}$	$9.7^{+2.1+8.0}_{-3.1-10.3}$	$2 \pm 11$
$A_{CP}(B^- \rightarrow \rho^0 \pi^-)$	$6.7^{+0.5+3.5}_{-0.8-3.1}$	$-19.8^{+1.7+12.6}_{-1.2-8.8}$	$-9.8^{+3.4+11.4}_{-2.6-10.2}$	$18^{+9}_{-17}$
$A_{CP}(\bar{B}^0 \rightarrow \rho^- \pi^+)$	$-3.5^{+0.2+1.0}_{-0.2-0.9}$	$4.4^{+0.3+5.8}_{-0.3-6.8}$	$4.4^{+0.3+5.8}_{-0.3-6.8}$	$11 \pm 6$
$A_{CP}(\bar{B}^0 \rightarrow \rho^+ \pi^-)$	$0.6^{+0.1+2.2}_{-0.1-2.2}$	$-22.7^{+0.9+8.2}_{-1.1-4.4}$	$-22.7^{+0.9+8.2}_{-1.1-4.4}$	$-18 \pm 12$

<sup>a</sup>If an  $S$  factor is included, the average will become  $2.0 \pm 0.8$ .

<sup>b</sup>This is the average of  $10 \pm 40 \pm 53$  by *BABAR* [39] and  $-49 \pm 36 \pm 28$  by *Belle* [40].

$$\begin{aligned}
 H_i(M_1 M_2) &= \frac{i f_B f_{M_1} f_{M_2} m_B}{X^{(\bar{B} M_1 M_2)} \lambda_B} \int_0^1 dx dy \\
 &\times \left( \frac{\Phi_{M_1}(x) \Phi_{M_2}(y)}{\bar{x} \bar{y}} + r_\chi^{M_1} \frac{\Phi_{m_1}(x) \Phi_{M_2}(y)}{\bar{x} \bar{y}} \right),
 \end{aligned} \tag{10}$$

for  $i = 1 - 4, 9, 10$ , where  $X^{(\bar{B} M_1 M_2)}$  is the factorizable amplitude for  $\bar{B} \rightarrow M_1 M_2$ ,  $\bar{x} = 1 - x$ ,  $\lambda_B$  is the first inverse moment of the  $B$  meson light-cone wave function and

$$\begin{aligned}
 r_\chi^P(\mu) &= \frac{2m_P^2}{m_b(\mu)(m_2 + m_1)(\mu)}, \\
 r_\chi^V(\mu) &= \frac{2m_V}{m_b(\mu)} \frac{f_V^\perp(\mu)}{f_V}.
 \end{aligned} \tag{11}$$

Power corrections from the twist-3 amplitude  $\Phi_m$  are divergent and can be parametrized as

$$X_H \equiv \int_0^1 \frac{dy}{y} = \ln \frac{m_B}{\Lambda_h} (1 + \rho_H e^{i\phi_H}), \tag{12}$$

Since  $c_1 \sim \mathcal{O}(1)$  and  $c_9 \sim \mathcal{O}(-1.3)$  in units of  $\alpha_{em}$ , it turns out that spectator-scattering contributions to  $a_i$  are usually small except for  $a_2$  and  $a_{10}$  which are essentially governed by hard spectator interactions [43]. The value  $a_2(K\pi) \approx 0.51e^{-i58^\circ}$  corresponds to  $\rho_H \approx 4.9$  and  $\phi_H \approx -77^\circ$ .<sup>3</sup> Therefore, there is no reason to restrict  $\rho_H$  to the range  $0 \leq \rho_H \leq 1$ .

A sizable color-suppressed tree amplitude also can be induced via color-allowed decay  $B^- \rightarrow K^- \eta'$  followed by the rescattering of  $K^- \eta'$  into  $K^- \pi^0$  as depicted in Fig. 1. Recall that among the 2-body  $B$  decays,  $B \rightarrow K \eta'$  has the largest branching fraction, of order  $70 \times 10^{-6}$ . This final-state rescattering has the same topology as the color-suppressed tree diagram [33]. One of us (CKC) has studied the FSI effects through residual rescattering among  $PP$  states and resolved the  $B$ - $CP$  puzzles [13].

4. Power corrections to  $a_2$  for  $B \rightarrow VP$  and  $B \rightarrow VV$  are not the same as that for  $B \rightarrow PP$  as described by Eq. (5). From Table II we see that an enhancement of  $a_2$  is needed to improve the rates of  $B \rightarrow \rho^0 \pi^0$  and the direct  $CP$  asymmetry of  $\bar{B}^0 \rightarrow \bar{K}^{*0} \eta$ . However, it is constrained by the measured rates of  $\rho^0 \pi^-$  and  $\rho^- \pi^0$  modes. This means that  $\rho_C(VP)$  is preferred to be smaller than  $\rho_C(PP) = 1.3$ . In Table II we show the branching fractions and  $CP$  asymmetries in  $B \rightarrow VP$  decays for  $\rho_C(VP) = 0.8$  and  $\phi_C(VA) = -80^\circ$ . The corresponding values of  $a_2(VP)$  are

<sup>3</sup>As pointed out in [23,44], a smaller value of  $\lambda_B$  of order 200 MeV can enhance the hard spectator interaction [see Eq. (10)] and hence  $a_2$  substantially. However, the recent BABAR data on  $B \rightarrow \gamma \ell \bar{\nu}$  [45] seems to imply a larger  $\lambda_B$  ( $> 300$  MeV at the 90% CL). In this work we reply on  $\rho_C$  and  $\phi_C$  to get a large complex  $a_2$ .

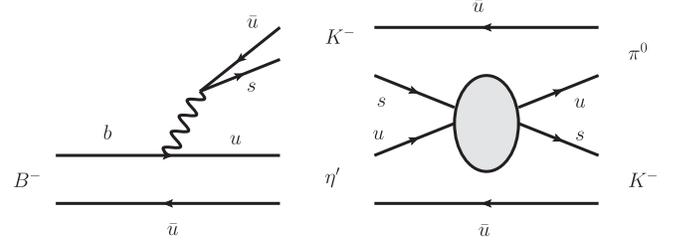


FIG. 1. Contribution to the color-suppressed tree amplitude of  $B^- \rightarrow K^- \pi^0$  from the weak decay  $B^- \rightarrow K^- \eta'$  followed by the final-state rescattering of  $K^- \eta'$  into  $K^- \pi^0$ . This has the same topology as the color-suppressed tree diagram.

$$\begin{aligned}
 a_2(\pi\rho) &\approx 0.40e^{-i51^\circ}, & a_2(\rho\pi) &\approx 0.38e^{-i52^\circ}, \\
 a_2(\rho\bar{K}) &\approx 0.36e^{-i52^\circ}, & a_2(\pi\bar{K}^*) &\approx 0.39e^{-i51^\circ}.
 \end{aligned} \tag{13}$$

It is clear from Table II that in the heavy quark limit, the predicted rates for  $\bar{B} \rightarrow \bar{K}^* \pi$  are too small by a factor of  $2 \sim 3$ , while  $\mathcal{B}(\bar{B} \rightarrow \bar{K} \rho)$  are too small by (15 ~ 50)% compared with experiment. The rate deficit for penguin-dominated decays can be accounted by the subleading power corrections from penguin annihilation. Soft corrections to  $a_2$  will enhance  $\mathcal{B}(B \rightarrow \rho^0 \pi^0)$  to the order of  $1.3 \times 10^{-6}$ , while the BABAR and Belle results,  $(1.4 \pm 0.6 \pm 0.3) \times 10^{-6}$  [46] and  $(3.0 \pm 0.5 \pm 0.7) \times 10^{-6}$  [47] respectively, differ in their central values by a factor of 2. Improved measurements are certainly needed for this decay mode. As for direct  $CP$  asymmetries, we see that penguin annihilation will flip the sign of  $A_{CP}(K^- \rho^0)$  into the right direction. Power corrections to the color-suppressed tree amplitude are needed to improve the prediction for  $A_{CP}(\bar{K}^{*0} \eta)$ . Our prediction is of order 0.035 to be compared with the experimental value of  $0.19 \pm 0.05$ . The pQCD prediction of  $A_{CP}(\bar{K}^{*0} \eta) \sim 0.0057$  [48] is too small, while the SECT result of  $\sim -0.01$  [49] has a wrong sign. For  $A_{CP}(\bar{K}^0 \rho^0)$ , it gets a sign flip after including soft effects on  $a_2$ . Our prediction is  $(8.7^{+8.8}_{-6.9})\%$ , while it is  $0.06 \pm 0.20$  experimentally. Defining  $\Delta A_{K^* \pi} \equiv A_{CP}(K^{*-} \pi^0) - A_{CP}(K^{*-} \pi^+)$  in analog to  $\Delta A_{K\pi}$ , we predict that  $\Delta A_{K^* \pi} = (13.7^{+2.9+3.6}_{-1.4-6.9})\%$ , while it is naively expected that  $K^{*-} \pi^0$  and  $K^{*-} \pi^+$  have similar  $CP$ -violating effects. It is of importance to measure  $CP$  asymmetries of these two modes to test our prediction. For mixing-induced  $CP$  violation, we obtain  $\Delta S_{\phi_{K_S}} = 0.022^{+0.004}_{-0.002}$ ,  $\Delta S_{\omega_{K_S}} = 0.17^{+0.06}_{-0.08}$  and  $\Delta S_{\rho^0_{K_S}} = -0.17^{+0.09}_{-0.18}$  [24], where  $\Delta S_f \equiv -\eta_f S_f - \sin 2\beta$ . It turns out that soft corrections to  $a_2$  have significant effects on the last two quantities.

As for  $B \rightarrow VV$  decays, we notice that the calculated  $B^0 \rightarrow \rho^0 \rho^0$  rate in QCDF is  $\mathcal{B}(B^0 \rightarrow \rho^0 \rho^0) = (0.88^{+1.46+1.06}_{-0.41-0.20}) \times 10^{-6}$  for  $\rho_C = 0$  [50], while the world average is  $(0.73^{+0.27}_{-0.28}) \times 10^{-6}$  [1]. Therefore, soft power correction to  $a_2$  or  $\rho_C(VV)$  should be small for  $B^0 \rightarrow \rho^0 \rho^0$ . Consequently, a pattern follows: Effects of power correc-

tions on  $a_2$  are large for  $PP$  modes, moderate for  $VP$  ones and very small for  $VV$  cases.<sup>4</sup> This is consistent with the observation made in [15] that soft power correction dominance is much larger for  $PP$  than  $VP$  and  $VV$  final states. It has been argued that this has to do with the special nature of the pion which is a  $q\bar{q}$  bound state on the one hand and a nearly massless Nambu-Goldstone boson on the other hand [15]. The two seemingly distinct pictures of the pion can be reconciled by considering a soft cloud of higher Fock states surrounding the bound valence quarks. From the FSI point of view, since  $B \rightarrow \rho^+ \rho^-$  has a rate much larger than  $B \rightarrow \pi^+ \pi^-$ , it is natural to expect that  $B \rightarrow \pi^0 \pi^0$  receives a large enhancement from the weak decay  $B \rightarrow \rho^+ \rho^-$  followed by the rescattering of  $\rho^+ \rho^-$  to  $\pi^0 \pi^0$  through the exchange of the  $\rho$  particle. Likewise, it is anticipated that  $B \rightarrow \rho^0 \rho^0$  will receive a large enhancement via isospin final-state interactions from  $B \rightarrow \rho^+ \rho^-$ . The fact that the branching fraction of this mode is rather small and is consistent with the theory prediction implies that the isospin phase difference of  $\delta_0^\rho$  and  $\delta_2^\rho$  and the final-state interaction must be negligible [51].

5.  $B$ - $CP$  puzzles arise in the framework of QCD factorization because power corrections due to penguin annihilation, that account for the observed rates of penguin-dominated two-body decays of  $B$  mesons and direct  $CP$

asymmetries  $A_{CP}(K^- \pi^+)$ ,  $A_{CP}(K^{*-} \pi^+)$ ,  $A_{CP}(K^- \rho^0)$  and  $A_{CP}(\pi^+ \pi^-)$ , will flip the signs of direct  $CP$ -violating effects in  $B^- \rightarrow K^- \pi^0$ ,  $B^- \rightarrow K^- \eta$ ,  $B^- \rightarrow \pi^- \eta$  and  $\bar{B}^0 \rightarrow \pi^0 \pi^0$  to wrong ones when confronted with experiment. We have shown that power corrections to the color-suppressed tree amplitude due to hard spectator interactions and/or final-state interactions will yield correct signs again for aforementioned  $CP$  asymmetries and accommodate the observed  $\pi^0 \pi^0$  and  $\rho^0 \pi^0$  rates simultaneously.  $CP$ -violating asymmetries of  $B^- \rightarrow K^- \eta$  can be understood as a consequence of soft corrections to  $a_2$ .  $A_{CP}(\bar{K}^0 \pi^0)$  is predicted to be of order  $-0.10$ , in agreement with that inferred from the  $CP$ -asymmetry sum rule, or SU(3) relation or the diagrammatical approach. For direct  $CP$  violation in  $B^- \rightarrow K^{*-} \eta$ ,  $\pi^- \eta$ , our predictions are in better agreement with experiment than pQCD and SCET. For  $\bar{B}^0 \rightarrow \bar{K}^0 \rho^0$ , we obtained  $A_{CP}(\bar{K}^0 \rho^0) = 0.087_{-0.069}^{+0.088}$ . We argued that the smallness of  $CP$  asymmetry of  $B^- \rightarrow \pi^- \pi^0$  is not affected by the soft corrections under consideration. For the  $CP$  asymmetry difference in  $K^* \pi$  modes defined by  $\Delta A_{K^* \pi} \equiv A_{CP}(K^{*-} \pi^0) - A_{CP}(K^{*-} \pi^+)$ , we predict that  $\Delta A_{K^* \pi} \sim 14\%$ , while these two modes are naively expected to have similar direct  $CP$ -violating effects. For mixing-induced  $CP$  violation, we found  $\Delta S_{\pi^0 K_S} = 0.12_{-0.06}^{+0.07}$ ,  $\Delta S_{\phi K_S} = 0.022_{-0.002}^{+0.004}$ ,  $\Delta S_{\omega K_S} = 0.17_{-0.08}^{+0.06}$  and  $\Delta S_{\rho^0 K_S} = -0.17_{-0.18}^{+0.09}$ .

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<sup>4</sup>Since the chiral factor  $r_\chi^V$  for the vector meson is substantially smaller than  $r_\chi^P$  for the pseudoscalar meson (typically,  $r_\chi^P = \mathcal{O}(0.8)$  and  $r_\chi^V = \mathcal{O}(0.2)$  at the hard collinear scale  $\mu = \sqrt{\Lambda m_b}$ ), one may argue that Eq. (10) naturally explains why the power corrections to  $a_2$  is smaller when  $M_1$  is a vector meson, provided that soft corrections arise from spectator rescattering. Unfortunately, this is not the case. Numerically, we found that, for example,  $H(K^* \pi)$  is comparable to  $H(K \pi)$ . This is due to the fact that  $\int_0^1 dx r_\chi^M \Phi_m(x)/\bar{x}$  is equal to  $X_H r_\chi^P$  for  $M = P$  and approximated to  $3(X_H - 2)r_\chi^V$  for  $M = V$ .

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