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Dynamical generation and dynamical reconstruction

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A definition of "dynamical generation," a hotly debated topic at present, is proposed and its implications are discussed. This definition, in turn, leads to a method allowing one to distinguish, in principle, tetraquark and molecular states. The different concept of "dynamical reconstruction" is also introduced and applies to the generation of preexisting mesons (quark-antiquark, glueballs, ...) via unitarization methods applied to low-energy effective Lagrangians. Large-N_c arguments play an important role in all these investigations. A simple toy model with two scalar fields is introduced to elucidate these concepts. The large-N_c behavior of the parameters is chosen in order that the two scalar fields behave as quark-antiquark mesons. When the heavier field is integrated out, one is left with an effective Lagrangian with the lighter field only. A unitarization method applied to the latter allows one to "reconstruct" the heavier "quarkoniumlike" field, which was previously integrated out. It is shown that a Bethe-Salpeter analysis is capable of reproducing the preformed quark-antiquark state, and that the corresponding large-N_c behavior can be brought in agreement with the expected large-N_c limit; this is a subtle and interesting issue on its own. However, when only the lowest term of the effective Lagrangian is retained, the large-N_c limit of the reconstructed state is not reproduced: Instead of the correct large-N_c quarkonium limit, it fades out as a molecular state would do. Implications of these results are presented: It is proposed that axial-vector, tensor, and (some) scalar mesons just above 1 GeV, obtained via the Bethe-Salpeter approach from the corresponding low-energy, effective Lagrangian in which only the lowest term is kept, are quarkonia states, in agreement with the constituent quark model, although they might fade away as molecular states in the large-N_c limit.

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I. INTRODUCTION

A central topic of past and modern hadron physics is the determination of the wave function of resonances in terms of quark and gluon degrees of freedom, both in the baryon and meson sectors and both for light and heavy quarks (for reviews see Refs. [1,2]). In the mesonic sector, beyond conventional quark-antiquark ($\bar{q}q$) mesons, one has glueball states, multiquark states such as tetraquarks, and "dynamically generated resonances," most notably molecular states. Indeed, different authors used the term "dynamical generation" in rather different contexts. In the works of Refs. [3–6] a dynamically generated state is regarded as a resonance obtained via unitarization methods from a low-energy Lagrangian; in Refs. [7–9] it is considered as a state which does not follow the quark-antiquark pattern in large-N_c expansion. In Refs. [10–13] the concept of "additional, companion poles" as dynamically generated states is introduced, while in Refs. [14–16] a dynamically generated resonance is regarded as a loosely bound molecular state. These various definitions are not mutually exclusive and describe different points of view of the problem.

In this article (Secs. II and III) a definition for a dynamically generated state is proposed and its implications, also in connection with the aforementioned works, are presented. This definition, in turn, leads to a method allowing us to distinguish, in principle, tetraquark and molecular states, although they are both four-quark states. The concept of "dynamical reconstruction" is then introduced and discussed: It applies to resonances which are obtained from low-energy effective Lagrangians via unitarization methods, but still correspond to "fundamental" (not dynamically generated) $\bar{q}q$, glueball, or multiquark states. In this context, the study of the large-N_c behavior of these resonance constitutes a useful tool to discuss their nature. At the end of Sec. III some general thoughts about the form of an effective theory of hadrons valid up to 2 GeV are also presented.

In Sec. IV the attention is focused on a simple toy model, in which only two scalar fields are considered. The parameters of the toy model are chosen in such a way that both fields behave as quarkoniumlike states in the large-N_c limit. The heavier state is first integrated out in order to obtain an effective low-energy Lagrangian in the toy world and then is reobtained via a Bethe-Salpeter (BS) study applied to the low-energy Lagrangian. It is shown that this is (at least in some cases) possible and that the corresponding large-N_c behavior can be brought in agreement with the expected large-N_c limit; this is a subtle and interesting issue on its own. However, when only the lowest term of the effective, low-energy Lagrangian is retained, the large-N_c limit of the reconstructed state is not reproduced: Instead of the correct large-N_c quarkonium limit (which must hold by construction), it fades out as a molecular state would do. Implications of these results are presented: It is proposed that axial-vector, tensor, and (some) scalar mesons just above 1 GeV obtained via the BS approach from the corresponding low-energy, effective Lagrangian, in which only the lowest term is kept, are (reconstructed) quark-antiquark fields, in agreement with the constituent quark model, although they might fade away as molecular states in the large-N_c limit. Finally, in Sec. V the conclusions are presented.

II. DYNAMICAL GENERATION

Consider a physical system which is correctly and completely described in the energy range $0 \leq E \lesssim E_{\max}$ by a quantum field theory in which N fields $\phi_1, \phi_2, \ldots \phi_N$, their masses, and interactions are encoded in a Lagrangian $\mathcal{L} = \mathcal{L}(\phi_i, E_{\max})$. Each Lagrangian has such an $E_{\max}$ beyond which it cannot be trusted. In particular, we shall refer to $E_{\max}$ in the following sense: All the masses M_i of the states ϕ_i and the energy transfer in a two-body scattering $\phi_i + \phi_j \rightarrow \phi_i + \phi_j$ should be smaller than $E_{\max}$ ($M_i < E_{\max}$ and, in the s channel, $\sqrt{s} \lesssim E_{\max}$).

Moreover, we also assume that (i) the theory is not confining, and thus to each field ϕ_i there is a corresponding, measurable resonance (at least one with zero width); (ii) if not renormalizable, an appropriate regularization shall be specified.

A resonance R, emerging in the system described by $\mathcal{L}$, is said to be dynamically generated if it does *not* correspond to any of the original fields $\phi_1, \phi_2, \ldots \phi_N \subset \mathcal{L}$ in the Lagrangian and if its mass M_R lies below $E_{\max}$ ($M_R \lesssim E_{\max}$).

The last requirement $M_R \lesssim E_{\max}$ is natural because the state R can be regarded as an additional, dynamically generated resonance only if it belongs to the energy range in which the theory is valid. This simple consideration plays an important role in the following discussion. Clearly, the dynamically generated state R emerges via interactions of the original resonances ϕ_i. When switching them off, R must disappear. For this reason a dynamically generated mesonic resonance in QCD fades out in the large-N_c limit, which corresponds to a decreasing interaction strength of mesons; see below for more details.

Some examples and comments are in order:

(a) $\mathcal{L} = \mathcal{L}_{\rm QED}$, in which the electron and the photon fields are the basic fields. This theory is valid up to a very large $E_{\max}$ (grand unified theory scale). Positronium states are molecular, electron-positron bound states. They appear as poles close to the real axis just below the threshold $2m_e$, but slightly shifted due to their nonzero decay widths into photons. Clearly, positronium states are dynamically generated according to the given definition and should not be included in the original QED Lagrangian; otherwise they would be double counted. Note that the number of positronium states is infinite.

(b) In Lagrangians describing nucleon-nucleon interaction via meson exchange (ω, ρ, π, and σ), a bound state close to threshold, called the deuteron, emerges via Yukawa interactions; see, for instance, Ref. [17] and references therein. The deuteron is a dynamically generated molecular state. In this case, when lowering the interaction strength below a critical value [by reducing the coupling and/or increasing the mass of the exchanged particle(s)], the bound state disappears. In fact, the number of molecular states which can be obtained via a Yukawa potential is finite [18], and eventually zero if the attraction is too weak. In such models the deuteron should not be included in the original Lagrangian in order to avoid double counting [19].

(c) $\mathcal{L} = \mathcal{L}_{\rm F}$, the Fermi theory of the weak interaction, in which the neutrino and electron fields interact via a local, quartic interaction. This theory is valid up to $E_{\max} \ll M_W$, where W is the boson mediator of the weak force. As already mentioned in Ref. [2], the linear rise of the $\bar{\nu}_e e^-$ cross section—as calculated from $\mathcal{L}_{\rm F}$—shows a loss of unitarity at high energy. Unitarization applied to $\mathcal{L}_{\rm F}$ implies that a resonance well above $E_{\max}$ exists, and this resonance is exactly the W meson. However, with $M_W > E_{\max}$ one cannot state in the framework of the Fermi theory if the W meson is dynamically generated or not. A straightforward way to answer this question is with the knowledge of the corresponding theory, valid up to an energy $E_{\max} > M_W$. Of course, this theory is known: It is the electroweak theory described by the Lagrangian $\mathcal{L}_{\rm EW}$, which is part of the standard model [20] and is valid up to a very high energy (grand unified theory scale). In the framework of $\mathcal{L}_{\rm EW}$, the neutrino, the electron, and the W meson are all elementary fields. One can then conclude that the W meson is not a dynamically generated state.
Indeed, $\mathcal{L}_{\rm F}$ can be seen as the result of integrating out the W field from the electroweak Lagrangian $\mathcal{L}_{\rm EW}$. Unitarization arguments applied to the Fermi Lagrangian $\mathcal{L}_{\rm F}$ allow us, in a sense, to dynamically reconstruct the W, which is already present as a fundamental field in $\mathcal{L}_{\rm EW}$.

(d) It is important to discuss in more depth and to formalize the issue raised in the previous example. To this end let us consider the Lagrangian $\mathcal{L} = \mathcal{L}(\phi_i, E_{\max})$ as the low-energy limit of a Lagrangian $\mathcal{L}' = \mathcal{L}'(\phi_i, \varphi_k, E'_{\max})$ valid up to an energy $E'_{\max} > E_{\max}$. Beyond the fields ϕ_i, $\mathcal{L}'$ depends also on the fields φ_k, which are heavier than $E_{\max}$. Formally, when integrating out the fields φ_k from $\mathcal{L}'$, one obtains $\mathcal{L}$.

In general, a unitarization scheme uses the information encoded in a low-energy effective Lagrangian and the principle of unitarity in quantum field theories, in order to deduce the existence and some properties of resonances beyond the limit of validity of the theory itself. When applying a unitarization scheme to the Lagrangian $\mathcal{L}$, an energy window between $E_{\max}$ and a new energy scale $E_U > E_{\max}$—which depends on the details of the unitarization—becomes (partially) accessible. For our purposes, we assume that $E_U \lesssim E'_{\max}$.
Let R be a resonance with mass $E_{\max} < M_R < E_U$ obtained from $\mathcal{L}$ via a unitarization approach. Is this resonance R dynamically generated or not? A straightforward way to answer this question would be with the knowledge of $\mathcal{L}'$. If R corresponds to one of the fields, φ_k is not dynamically generated and vice versa. However, if $\mathcal{L}'$ is not known, it is not possible to answer this question at the level of the unitarized version of $\mathcal{L}$ only.
In conclusion, although the unitarization approach opens a window between $E_{\max}$ and E_U and the existence of resonances in this range can be inferred from the unitarized Lagrangian $\mathcal{L}$ only, the knowledge of the latter is still not complete [21]. If $\mathcal{L}'$ is unknown, some other kind of additional information is required to deduce the nature of R. In the framework of low-energy QCD, this additional information can be provided by large-N_c arguments; see Sec. III C.

(e) Let us consider a scalar field $\varphi = \varphi(t, x)$ in a $1 + 1$-dimensional world (t, x) subject to the potential $V(\varphi) = \frac{\lambda}{4!}(\varphi^2 - F^2)^2$. We assume that this theory is valid up to high energies. When expanding around one of the two minima $\varphi = \pm F$, the mass of φ is found to be $m = \lambda F^2/3$. In addition, this theory also admits a soliton with mass $M = \frac{2m^3}{\lambda}$, which is large if λ is small [22]. In this example the solitonic state with mass M can be regarded as a dynamically generated state.

(f) Mixing can take place among two "fundamental fields," ϕ_i and ϕ_k: Two physical resonances arise as an admixture of these two fields. One is predominantly ϕ_i and the other predominantly ϕ_k. Also, mixing can take place among a dynamically generated resonance R and one (or more) of the ϕ_i. It decreases when the interaction is lowered (large-N_c limit in the mesonic world): One state reduces to the original, preexisting resonance and the other disappears. In conclusion, mixing surely represents a source of technical complication which renders the identification of states (much) more difficult, but it does *not* change the number of states and the meaning of the previous discussion.

III. APPLICATION TO MESONS

A. Effective hadron theory up to 2 GeV

Let us turn to the hadronic world below 2 GeV. The basic ingredients of each low-energy hadronic Lagrangian are quark-antiquark mesons and three-quark baryons. In the framework of our formalism, we shall consider each quark-antiquark (3-quark) state as a fundamental state, which is described by a corresponding field in the hadronic Lagrangian (as long as its mass is below an upper energy $E_{\max}$).

Let us formalize this point in the mesonic sector as follows. Consider the correct, effective theory describing mesons up to $E_{\max} \simeq 2$ GeV given by

$$\mathcal{L}_{\rm eff}^{\rm had}(E_{\max}, N_c), \tag{1}$$

where N_c is the number of colors. Its precise form is, unfortunately, unknown. In fact, because confinement has not yet been analytically solved, it is not possible to derive $\mathcal{L}_{\rm eff}^{\rm had}(E_{\max}, N_c)$ from the QCD Lagrangian. In the limit $N_c \to \infty$ the effective Lagrangian $\mathcal{L}_{\rm eff}^{\rm had}(E_{\max}, N_c)$ is expected to be more simple. Although even in this limit a mathematical derivation is not possible, it is known that it must primarily consists of noninteracting quark-antiquark states. In fact, their masses scale as N_c^0 and their decay widths as N_c^{-1}, respectively [23–26]. Considering that the $\bar{q}q$ mass scales as N_c^0, these states shall also be clearly present when going from the large-N_c limit to the physical world $N_c = 3$. The next-expected states which are present in the large-N_c limit are glueballs, i.e. bound states of pure gluonic nature. Their masses also scale as N_c^0 and the decay widths as N_c^{-2}. They thus are also expected to be present in the real world for $N_c = 3$. In particular, the lightest glueball is a scalar field which is strongly related to the trace anomaly, i.e. the breaking of the classical dilatation invariance of the QCD Lagrangian (see also Sec. III D). In addition to quark-antiquark and glueball states, hybrid states also survive in the large-N_c limit [26]. They constitute an interesting subject of meson spectroscopy (see Ref. [27] and references therein), but will not be considered in the following discussion. All these states are therefore "preexisting" and not dynamically generated states of the mesonic Lagrangian under consideration.

An intermediate comment is devoted to baryon states: They have a linearly increasing mass with N_c ($M \sim N_c$), which exceeds $E_{\max}$ for a large enough N_c, and are therefore not present in the large-N_c limit of $\mathcal{L}_{\rm eff}^{\rm had}(E_{\max}, N_c)$. Thus, although they appear in the $N_c = 3$ world, they are not present in the effective Lagrangian because of the way in which the limit is constructed. If, instead, we construct the large-N_c limit as $\mathcal{L}_{\rm eff}^{\rm had}(\frac{N_c}{3}E_{\max}, N_c \to \infty)$ [in such a way that at $N_c = 3$ it coincides with Eq. (1)], baryons are well-defined states as proven originally in Ref. [24]. For simplicity, baryons will not be discussed in this paper but, together with hybrid states, should be included in a more complete treatment.

The reason why the value $E_{max} \simeq 2$ GeV is chosen is that all the resonances under study in this work are lighter than 2 GeV. Thus, they either correspond to a field in $\mathcal{L}_{eff}^{had}(E_{max}, N_c = 3)$ or arise as additional resonances (i.e. dynamically generated) via interaction of preexisting states of $\mathcal{L}_{eff}^{had}(E_{max}, N_c = 3)$. The full knowledge of the Lagrangian $\mathcal{L}_{eff}^{had}(E_{max} \simeq 2\ \text{GeV}, N_c = 3)$ would allow one to see if a resonance lighter than 2 GeV is dynamically generated or not by simply looking at it. Clearly, if one were interested in a resonance whose mass is heavier than 2 GeV, then E_{max} should be increased. Moreover, one expects to find below 2 GeV all the relevant ground-state mesons in the channel $J^{PC} = 0^{-+}, 0^{++}, 1^{-+}, 1^{++}, 2^{++}$. Thus, $\mathcal{L}_{eff}^{had}(2\ \text{GeV}, 3)$ may be described by an effective Lagrangian which exhibits linear realization of chiral symmetry and its spontaneous breakdown; see Sec. III D for a closer discussion.

Most of the mesonic resonances listed in the Particle Data Group paper [28] can be immediately associated to a corresponding, underlying quark-antiquark state. Yet, the question of whether some resonances of Ref. [28] are not $\bar{q}q$ is interesting and at the basis of many studies. In the mesonic sector, two alternative possibilities are well known:

(i) Molecular states: These are bound states of two distinct quark-antiquark mesons. They correspond to the example of the positronium [example (a) in Sec. II]. Just as the positronium states are not included in the QED Lagrangian, hadronic molecular states should *not* be included directly in $\mathcal{L}_{eff}^{had}(E_{max}, N_c)$. They arise upon meson-meson interactions in the $N_c = 3$ physical world; see the general discussion above. However, they inevitably fade out in the large-N_c limit because the interaction of an n-leg meson vertex decreases as $N_c^{-(n-2)/2}$. This is therefore a clear example of dynamically generated states within a mesonic system (see also the next subsection for a closer description of physical candidates below 1 GeV).

(ii) Tetraquark states: These consist of two distinct, colored "bumps," in contrast to a molecular state, which is made of two colorless, quark-antiquark bumps [29]. Loops of $\bar{q}q$ mesons, corresponding to the interaction of two colorless states, cannot generate the color distribution of a tetraquark. If present at $N_c = 3$, they shall be included directly in the effective Lagrangian $\mathcal{L}_{eff}^{had}(E_{max}, N_c = 3)$ and, in view of the given definition, should not be regarded as dynamically generated states.

A second, slightly different way to see it is the following: Let us imagine constructing the Lagrangian $\mathcal{L}_{eff}^{had}(E_{max}, N_c)$. We first put in quark-antiquark and glueball states, that is, those configurations which surely survive in the large-N_c limit and correspond to non-dynamically generates states: $\mathcal{L}_{eff}^{had}(E_{max}, N_c) = \mathcal{L}_{eff}^{\bar{q}q+\text{glueballs}}(E_{max}, N_c)$. Then the question is the following: Does this Lagrangian describe the physical world for $N_c = 3$? (Note that loops shall be taken into account and dynamically generated states can eventually emerge out of this Lagrangian.) If the answer is positive, no multiquark states are needed. If the answer is negative, the basic Lagrangian shall be extended to include, from the very beginning, multiquark states, most notably tetraquark states: $\mathcal{L}_{eff}^{had}(E_{max}, 3) = \mathcal{L}_{eff}^{\bar{q}q+\text{glueballs}}(E_{max}, 3) + \mathcal{L}_{eff}^{\text{multiquark}}(E_{max}, 3)$.

A third approach to the problem is via large-N_c arguments. In Refs. [24,25] it has been shown that a tetraquark state also vanishes in the large-N_c limit. However, for $N_c = 3$ the most prominent and potentially relevant for spectroscopy is the "good" diquark, which is antisymmetric in color space: $d_a = \varepsilon_{abc} q^b q^c$ (with $a, b, c = 1, 2, 3$) [30]. The tetraquark is the composition of a good diquark and a good antidiquark: $d_a^\dagger d_a$. The extension to N_c of a good diquark is the antisymmetric configuration $d_{a_1} = \varepsilon_{a_1 a_2 a_3 \ldots a_{N_c}} q^{a_2} q^{a_3} \ldots q^{a_{N_c}}$ with $a_1, \ldots, a_{N_c} = 1, \ldots N_c$, which constitutes $(N_c - 1)$ quarks. Thus, the generalization of the tetraquark to the N_c world is not a diquark-antidiquark object, but rather the state $\chi = \sum_{a_1=1}^{N_c} d_{a_1}^\dagger d_{a_1}$ which is made of $(N_c - 1)$ quarks and $(N_c - 1)$ antiquarks; see also the discussion in Ref. [31]. It is the dibaryonium already described in Ref. [24] which has a well-defined large-N_c limit: Its mass scales as $M_\chi \propto 2(N_c - 1)$ and decays into a baryon and an antibaryon. The state χ, while not present in $\mathcal{L}_{eff}^{had}(E_{max}, N_c \to \infty)$ because its mass overshoots E_{max}, appears in $\mathcal{L}_{eff}^{had}(N_c E_{max}, N_c \to \infty)$ in which the baryons also survive: This is contrary to a dynamically generated state, which also disappears in this case.

As a result of our discussion, tetraquark states and molecular states, although both formally four-quark states, are crucially different: The former are "elementary" and should be directly included in the effective Lagrangian $\mathcal{L}_{eff}^{had}(E_{max}, N_c = 3)$; the latter can emerge as dynamically generated resonances. We now turn to the particular case of the light scalar mesons, where all these concepts play an important role.

B. Light scalar mesons

One of the fundamental questions of low-energy QCD concerns the nature of the lightest scalar states $\sigma \equiv f_0(600)$, $k \equiv k(800)$, $f_0 \equiv f_0(980)$, and $a_0 \equiv a_0(980)$. Shall these states be included from the very beginning in $\mathcal{L}_{eff}^{had}(E_{max}, N_c)$? If yes, they correspond to quark-antiquark or tetraquark nonets (one of them can also be related to a light scalar glueball). If not, they shall be regarded as dynamically generated states. The main point of the following subsection is to discuss previous works about light scalar mesons in connection with the proposed definition of dynamical generation. In fact, it is easy to classify previous works into two classes (not dynamically generated and dynamically generated), thus allowing us to order different works of the last three decades in a clear way. We first review works in which scalar states are not dynamically generated and then works in which they are dynamically generated.

First we discuss the light scalar states that are not dynamically generated and should be directly included in $\mathcal{L}_{\rm eff}^{\rm had}(E_{\rm max}, N_c)$.

(i) In the quark-antiquark picture these light scalar states form the nonet of chiral partners of pseudoscalar mesons. Their flavor wave functions read $\sigma \simeq \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$, $f_0 \simeq \bar{s}s$, $a_0^+ \equiv u\bar{d}$, $k^+ \equiv u\bar{s}$. At the microscopic level this is the prediction of the NJL model [32], where a σ mass of about $2m^*$ is obtained and where $m^* \sim 300$ MeV is the constituent quark mass. This is usually the picture adopted in linear sigma models at zero [33] and at nonzero density and temperature [34]. However, this assignment encounters a series of problems: It can hardly explain the mass degeneracy of a_0 and f_0, the strong coupling of a_0 to kaon-kaons, and the large mass difference with the other p-wave nonets of tensor and axial-vector mesons [35]; it is at odds with large-N_c studies (see below) and with recent lattice works [36].

(ii) In the tetraquark picture, first proposed by Jaffe [37] and revisited in Refs. [38–40], $\sigma \simeq \frac{1}{2}[\bar{u}, \bar{d}][u, d]$, $f_0 \simeq \frac{1}{2\sqrt{2}}([\bar{u}, \bar{s}][u, s] + [\bar{d}, \bar{s}][d, s])$, $a_0^+ \equiv \frac{1}{2}[\bar{d}, \bar{s}] \times [u, d]$, $k^+ \equiv \frac{1}{2}[\bar{d}, \bar{s}][u, d]$, where $[.,.]$ stands for an antisymmetric configuration in flavor space (which, together with the already-mentioned antisymmetric configuration in color space, also implies an s-wave and spinless structure of the diquarks and of the tetraquarks). Degeneracy of a_0 and f_0 is a natural consequence. A good phenomenology of decays can be obtained if the next-to-leading order contribution in the large-N_c expansion is also taken into account [40] and/or if instanton induced terms are included [41]. Linear sigma models with an additional nonet of scalar states can be constructed [42–44]. The quark-antiquark states lie above 1 GeV [45] and mix with the scalar glueball whose mass is placed at ~1.7 GeV by lattice QCD calculations [46]. This reversed scenario directly affects the physics of chiral restoration at nonzero temperature [47].

(iii) Different assignments, in which the glueball state also shows up below 1 GeV, have been proposed; see Refs. [2,48,49] and references therein.

(iv) In all these assignments the very existence of the scalar mesons is due to some preformed compact bare fields entering in $\mathcal{L}_{\rm eff}^{\rm had}(E_{\rm max}, N_c = 3)$. By removing the corresponding bare resonances from $\mathcal{L}_{\rm eff}^{\rm had}(E_{\rm max}, N_c = 3)$, they disappear. Dressing via meson-meson loops, such as $\pi\pi$ for σ, $K\pi$ for k, and KK for a_0 and f_0, surely takes place. In particular, due to the intensity in these channels and the closeness to thresholds, they can cause a strong distortion and affect the properties of the scalar states [50]. However, the important point is that in all these scenarios mesonic loops represent a further complication of light scalars, but are not the reason for their existence.

(v) As discussed in Sec. III A, nondynamically generated scalar states survive in the large-N_c limit, although in a different way according to quarkonium, glueball, or tetraquark interpretations.

Next we discuss the light scalar states that are dynamically generated and should not be included in $\mathcal{L}_{\rm eff}^{\rm had}(E_{\rm max}, N_c)$.

(i) In Ref. [14] the σ pole arises as a broad enhancement due to the inclusion of ρ mesons in the t-channel isoscalar $\pi\pi$ scattering. In this case the σ is dynamically generated and arises because of a Yukawa-like interaction due to ρ meson exchange (pretty much as the deuteron described above, but above threshold). When reducing the $\rho\pi\pi$ coupling $g_{\rho\pi\pi}$ (which, in the large-N_c limit, scales as $1/\sqrt{N_c}$), the σ fades out. Alternatively, the limit $M_\rho \to \infty$ also implies a disappearance of the σ enhancement.

(ii) Similar conclusions for the $f_0(980)$ and $a_0(980)$ mesons, described as molecular $\bar{K}K$ bound states just below threshold, have been obtained in Refs. [15]. In particular, in Ref. [16] the origin of these states is directly related to a one-meson-exchange potential. Within all these approaches the $a_0(980)$ and the $f_0(980)$ are dynamically generated.

(iii) In the model of Ref. [10] the a_0 state also arises as an additional, dynamically generated state, but in a different way. Scalar and pseudoscalar quark-antiquark mesons are the original states. A bare scalar state with a mass of 1.6 GeV is the original, quark-antiquark "seed." When loops of pseudoscalar mesons are switched on, the mass is slightly lowered and the state is identified with $a_0(1450)$. In addition, a second state, arising in this model as a further zero of the real part of the denominator of the propagator, is identified with the $a_0(980)$ meson: It is dynamically generated and disappears in the large-N_c limit, where only the original quark-antiquark seed survives. More generally, we refer to [11–13] for the emergence of additional, companion poles not originally present as preexisting states in the starting Lagrangian. In particular, in Ref. [13] the conventional scalar quark-antiquark states, calculated within a harmonic oscillator confining potential, lie above 1 GeV. When meson loops are switched on, a *complete* second nonet of dynamically generated states below 1 GeV emerges. Note, in all these studies the validity of the employed theories lies well above 1 GeV, so that the definition of dynamical generation given in Sec. II holds for the light scalars.

(iv) Note, in (i) and (ii) the emergence of states is due to t-channel forces. This is not the case in (iii). However, a common point is that the light scalar states disappear in the large-N_c limit.

It is clear that the situation concerning light scalars is by far not understood. We wish, however, to stress once more that there is a crucial difference among the two outlined options in relation to the Lagrangian $\mathcal{L}_{\rm eff}^{\rm had}(E_{\rm max}, N_c)$. Note also that in this subsection we only discussed works for which it is possible to immediately conclude if the scalar mesons are dynamically generated or not according to the definition given in Sec. II. Unitarization methods were not discussed here; in fact, when the latter are applied, care is needed. This is the subject of the next subsection.

C. Low-energy Lagrangians, unitarization, and dynamical reconstruction

The Lagrangian $\mathcal{L}_{\rm eff}^{\rm had}(E_{\rm max}, N_c)$ with $E_{\rm max} \simeq 2$ GeV induces the breakdown of chiral symmetry $SU_A(N_f)$, where N_f is the number of light flavors. There are therefore $N_f^2 - 1$ Goldstone bosons: the pion triplet for $N_f = 2$, in addition to four kaonic states and the η meson for $N_f = 3$.

If we integrate out all the fields in $\mathcal{L}_{\rm eff}^{\rm had}(E_{\rm max}, N_c)$ besides the three light pions, we obtain the Lagrangian of chiral perturbation theory (see Ref. [51] and references therein) for $N_f = 2$:

$$\mathcal{L}_{\rm eff}^{\rm had}(E_{\rm max}, N_c) \rightarrow \mathcal{L}_{\chi\rm PT}(E_{\chi\rm PT}, N_c), \tag{2}$$

where $E_{\chi\rm PT}$ should be smaller than the mass of the first resonance heavier than the pions (~ 400 MeV). $\mathcal{L}_{\chi\rm PT}(E_{\chi\rm PT}, N_c)$ is recast in an expansion of the pion momentum $O(p^{2n})$, and for each n there is a certain number of low-energy coupling constants, which, in principle, could be calculated from $\mathcal{L}_{\rm eff}^{\rm had}(E_{\rm max}, N_c)$, if it were known. Since this is not the case, they are directly determined by experimental data. [Similar properties hold when the kaons and the η are retained in $\mathcal{L}_{\chi\rm PT}(E_{\chi\rm PT}, N_c)$].

For instance, the vector isotriplet ρ meson is predicted by a large variety of approaches (such as quark models) to be a preexisting 1^{--} quark-antiquark field. In this sense it is a fundamental field appearing in $\mathcal{L}_{\rm eff}^{\rm had}(2\text{ GeV}, 3)$, which is integrated out (together with other fields) to obtain $\mathcal{L}_{\chi\rm PT}(E_{\chi\rm PT}, 3)$. However, the ρ meson spectral function cannot be obtained from chiral perturbation theory unless a unitarization scheme is employed [3,4,7,8]. As an example, via the inverse amplitude method (IAM) unitarization scheme applied to $\mathcal{L}_{\chi\rm PT}(E_{\chi\rm PT}, N_c = 3)$ [3], a window between the original energy $E_{\chi\rm PT}$ and $4\pi f_\pi \sim 1$ GeV is opened: Resonances with masses in this window, such as the ρ meson, can be described within unitarized χPT. As discussed in point (d) of Sec. II, the very last question of whether the ρ meson is dynamically generated or not *cannot* be answered at the level of unitarized χPT. One still does not know if ρ corresponds to a basic, preexisting field entering in $\mathcal{L}_{\rm eff}^{\rm had}(2\text{ GeV}, N_c)$ or not.

Some additional information is needed. In the interesting and important case of large-N_c studies of unitarized χPT, the required additional knowledge is provided by the large-N_c scaling of the low-energy constants: It has been shown in Ref. [7] that the ρ mass scales as N_c^0 and the width as N_c^{-1}, and thus the ρ meson should be considered as a fundamental (not dynamically generated) quark-antiquark field, which shall be directly included in $\mathcal{L}_{\rm eff}^{\rm had}(E_{\rm max}, N_c = 3)$. We also refer to the analytic results of [8], where the large-N_c limit is evident.

Let us turn to the lightest scalar-isoscalar resonance $\sigma \equiv f_0(600)$ as obtained from (unitarized) χPT. In Refs. [52,53] precise determinations of the σ pole are obtained, but—as stated in Ref. [52]—it is difficult to understand its properties in terms of quarks and gluons. In Ref. [7] a study of the σ pole within the IAM scheme in the large-N_c limit has been performed: A result which is at odds with a predominantly quarkonium, or glueball, interpretation of the σ meson has been obtained. The mass is not constant and the width does not decrease. However, even at this stage one still cannot say if the σ is dynamically generated or reconstructed in relation to $\mathcal{L}_{\rm eff}^{\rm had}(E_{\rm max}, N_c = 3)$ because it is hard to distinguish the molecular and the tetraquark assignments in the large-N_c limit (see discussion above).

Recently, the Bethe-Salpeter unitarization approach has been used to generate various axial-vector [5,9], tensor, and scalar mesons above 1 GeV [6]. The starting points are low-energy Lagrangians for the vector-pseudoscalar (such as $\rho\pi$) and vector-vector (such as $\rho\rho$) interactions. These Lagrangians are also, in principle, derivable by integrating out heavier fields from the complete $\mathcal{L}_{\rm eff}^{\rm had}(E_{\rm max}, N_c = 3)$. For instance, in the $\rho\pi$ axial-vector channel the $a_1(1260)$ meson is obtained, and in the $\rho\rho$ tensor and scalar channels the states $f_2(1270)$ and $f_0(1370)$ are found. Are these dynamically generated states of molecular type? The answer is, not necessarily. In fact, the masses of the obtained states lie *above* the energy limit of the low-energy effective theories out of which they are derived. Even for these states the possibility of dynamical reconstruction—just as for the ρ meson described above—is not excluded: In this scenario, these resonances above 1 GeV are intrinsic, preexisting quark-antiquark or glueball (multiquark states are improbable here) fields of $\mathcal{L}_{\rm eff}^{\rm had}(E_{\rm max}, N_c = 3)$. While first integrated out to obtain the low-energy Lagrangians, unitarization methods applied to the latter allow us to reconstruct them. In the next section a toy model is presented, in which this mechanism is explicitly shown: Although a state obtained via the BS equation looks like a molecule, it still can represent a fundamental, preexisting quark-antiquark (or glueball) state.

As discussed in the summary of the PDG compilation [54] and in Refs. [1,2] (and references therein), the tensor resonances $f_2(1275)$, $f_2(1525)$, $a_2(1320)$, and $K_2(1430)$ represent a nonet of quark-antiquark states. The ideal mixing, the very well measured strong and electromagnetic decay rates [55], the masses, and the mass splitting are all in excellent agreement with the quark-antiquark assignment. In this case they are fundamental (intrinsic) fields of $\mathcal{L}_{\rm eff}^{\rm had}(E_{\max}, N_c = 3)$, which can be dynamically reconstructed (rather than generated) via unitarization scheme (s) applied to low-energy Lagrangians.

Although experimentally and theoretically more involved, the same can hold in the axial-vector channel: The resonances $f_1(1285)$, $f_1(1510)$, $a_1(1260)$, and $K_1(1270)$ are in good agreement with the low-lying 1^{++} quark-antiquark assignment. Even more complicated is the situation in the scalar channel: The low-lying quark-antiquark states mix with the scalar glueball [45]. Also in this case, however, the possibility of dynamical reconstruction rather than generation is upheld.

If dynamical reconstruction takes place, there is no conflict between the quark model assignment of Ref. [54] and the above-mentioned recent studies. Note, also, that dynamical reconstruction is in agreement with the discussion of Ref. [56].

D. A simplification of the Lagrangian $\mathcal{L}_{\rm eff}^{\rm had}(E_{\max}, N_c)$

The Lagrangian $\mathcal{L}_{\rm eff}^{\rm had}(E_{\max}, N_c)$ with $E_{\max} \sim 2$ GeV has been a key ingredient throughout the present discussion, but it has not been made explicit because it is unknown. An improved knowledge of (at least parts of) $\mathcal{L}_{\rm eff}^{\rm had}(E_{\max}, N_c)$ would surely represent progress in understanding the low-energy hadron system. As a last step we discuss which properties it might have. Clearly, the Lagrangian must reflect the symmetries of QCD, most notably spontaneous breakdown of chiral symmetry. The (pseudo)scalar meson matrix Φ, the (axial-)vector and tensor mesons, and the scalar glueball are its basic building blocks. Moreover, if additional nondynamically generated scalar states such as tetraquarks exist, they shall also be included. We thus have a complicated, general σ-model Lagrangian with many terms, in which operators of all orders can enter, because renormalization is not a property that an effective hadronic Lagrangian should necessarily have. The question is if it is possible to obtain a (relatively) simple form from this complicated picture; see also [42–44,57].

A possibility to substantially simplify the situation is via dilation invariance; let us consider the (pseudo)scalar meson matrix Φ, which transforms as $\Phi \rightarrow R\Phi L^\dagger$ $[R,\ L \subset SU(3)]$ under chiral transformation and the dilaton field G, subject to the potential [58] $V_G = \propto G^4(\log G/\Lambda_G + 1/4)$, where Λ_G is a dimensional parameter of the order of $\Lambda_{\rm QCD}$ (the glueball emerges upon shifting G around the minimum of its potential $G_0 \sim \Lambda_G$). Consider the Lagrangian $\mathcal{L}_{\rm eff}^{\rm had}(E_{\max}, N_c) = T - V$, where T is the dynamical part and $V = V[G, \Phi, \ldots]$ is the potential describing masses and interactions of the fields [dots refer to other degrees of freedom, such as (axial-)vector ones]. We assume that (i) in the chiral limit the only term in V which breaks dilation invariance—and thus mimics the trace anomaly of QCD—is encoded in V_G (via the dimensional parameter Λ_G), and that (ii) the potential V is finite for any finite value of the fields. As a consequence of (i), only operators of order (exactly) 4 can be included. They have the form $G^2\,{\rm Tr}[\Phi^\dagger\Phi]$, ${\rm Tr}[\Phi^\dagger\Phi\Phi^\dagger\Phi]$, ${\rm Tr}[\Phi^\dagger\Phi]^2, \ldots$. As a consequence of (ii), a huge set of operators are not admitted. In fact, an operator of the kind $G^{-2}\,{\rm Tr}[\partial_\mu\Phi^\dagger\partial^\mu\Phi]^2$ is excluded because, although of dimension 4, it blows up for $G \rightarrow 0$. In this way we are left with a sizably smaller number of terms, even smaller than what renormalizability alone would impose [59]. Work along this direction, including (axial-)vector degrees of freedom, is ongoing [60] and can constitute an important source of information for spectroscopy and for future developments at nonzero temperature and densities, where in the chirally restored phase a degeneration of chiral partners is manifest.

In conclusion, a way to implement these ideas and use the definition of dynamical generation can be sketched as follows: After writing a general chirally symmetric Lagrangian up to fourth order including the glueball and the quark-antiquark fields as basic states, one should attempt, without further inclusion of any other state, to describe physical processes up to $\sim$2 GeV, as pion-pion scattering, decay widths, etc. In doing this one should of course include loops. If, for instance, we start with a basic scalar-isoscalar quark-antiquark field above 1 GeV, do we correctly reproduce the resonance $f_0(600)$ when solving the Bethe-Salpeter channel in the $\pi\pi$ sector below 1 GeV? If the answer is positive, the latter resonance is dynamically generated and there is no need for any other additional state. If, albeit including loops, the attraction among pions turns out to be too weak to generate the resonance $f_0(600)$, we conclude that it is necessary to enlarge our model by explicitly introducing a field which describes it. As argued previously, this field can be identified as a tetraquark state. Whether or not this ambitious program will lead to a successful result is a matter of future research.

IV. A TOY MODEL FOR DYNAMICAL RECONSTRUCTION

A. Definitions and general discussion

In this section we start from a toy Lagrangian, in which two mesons, φ and S, interact. A large-N_c dependence is introduced in such a way that both fields behave as quarkonium states. Then, the field S, which is taken to be heavier than φ, is integrated out and a low-energy Lagrangian with the field φ only is obtained. A Bethe-Salpeter study is applied to the latter Lagrangian: The question is if the original state S, which was previously integrated out, can be reobtained in this way. The answer is generally positive; however, care is needed concerning the large-N_c limit. In a straight BS approach the quarkoniumlike large-N_c limit of the state S cannot be reproduced. However, as it shall be shown, within a modified BS approach the large-N_c limit can be correctly obtained.

The toy Lagrangian [61,62] consisting of the two fields φ (with mass m) and S (with bare mass $M_0 > 2m$) reads

$$\mathcal{L}_{\text{toy}}(E_{\max}, N_c) = -\tfrac{1}{2}\varphi(\Box + m^2)\varphi - \tfrac{1}{2}S(\Box + M_0^2)S + gS\varphi^2, \tag{3}$$

which we assume to be valid up to $E_{\max} >> M_0$. The N_c dependence of the effective Lagrangian is encoded in g only: $g = g(N_c) = g_0\sqrt{3/N_c}$. In this way both masses behave like N_c^0 and the decay amplitude for $S \to 2\varphi$ scales as $1/\sqrt{N_c}$, just as if φ and S were quarkonia states. $\mathcal{L}_{\text{toy}}(E_{\max}, N_c = 3)$ is the analogue of $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\max}, N_c = 3)$ in a simplified toy world. For definiteness we refer to values in GeV: $m = 0.3$, $M_0 = 1$, g_0 will be varied between 1.5 and 5.

The propagator of the field S is modified via φ-meson loops and takes the form (at the resummed one-loop level; see Fig. 1)

$$\Delta = i[p^2 - M_0^2 + (\sqrt{2}g)^2\Sigma_\Lambda(p^2)]^{-1} \tag{4}$$

where $\Sigma_\Lambda(p^2)$ is the one-loop contribution, which is regularized via a 3D sharp cutoff Λ [63]. The dressed mass can be defined via the zero of the real part of Δ^{-1}, i.e.

$$M^2 - M_0^2 + (\sqrt{2}g)^2\,\text{Re}\Sigma_\Lambda(M^2) = 0. \tag{5}$$

In the large-N_c limit $M \to M_0$. This is true whichever definition of the mass of the resonance is chosen. For finite N_c one has, in general, $M < M_0$ due to the loop corrections (see Ref. [61] for details). The T matrix for $\varphi\varphi$ scattering in the s channel upon one-loop resummation is depicted in Fig. 1 and reads [64]

$$T(p^2) = i(\sqrt{2}g)^2\Delta = \frac{1}{-K^{-1} + \Sigma_\Lambda(p^2)}, \qquad K = \frac{(\sqrt{2}g)^2}{M_0^2 - p^2}. \tag{6}$$

Note, the present focus is on the one-particle pole of the S resonance and its corresponding enhancement in the T matrix. Thus, for simplicity, we limit the study of two-body scattering to the exchange of one (dressed) meson S. Other diagrams, such as the exchange of two (or more) S mesons, are not considered here (see, for instance, Ref. [65]). A more refined and complete approach should also include the dressing of the φ propagator and of the $S\varphi^2$ vertex. All these complications, while important in a realistic treatment, can be neglected at this illustrative level.

We now turn to the development of a low-energy Lagrangian which involves only the light meson field φ. We assume that g_0 is not too large so that for $N_c = 3$ the mass M lies above the threshold $2m$. In this way one can integrate out the field S from $\mathcal{L}_{\text{toy}}$ and obtain a low-energy (le) effective Lagrangian for the $\varphi\varphi$ interaction valid up to $E_{\text{le}} \lesssim 2m < M_0$ [66]:

$$\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c) = -\frac{1}{2}\varphi(\Box + m^2)\varphi + V, \qquad V = \sum_{k=0}^{\infty} V^{(k)}, \tag{7}$$

$$V^{(k)} = L^{(k)}\varphi^2(-\Box)^k\varphi^2, \qquad L^{(k)} = \frac{g^2}{2M_0^{2+2k}}. \tag{8}$$

The Lagrangian $\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$ contains only quartic terms of the kind φ^4, $\varphi^2\Box\varphi^2, \ldots$. $\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$ is the analogue of chiral perturbation theory or, more generally, of a low-energy Lagrangian in this simplified system. The fact that we know explicitly the form of $\mathcal{L}_{\text{toy}}(E_{\max}, N_c)$ allows us to calculate the "low-energy constants" $L^{(k)}$ of Eq. (8). If the precise expression of $\mathcal{L}_{\text{toy}}(E_{\max}, N_c)$ were unknown, then $L^{(k)}$ would also be unknown. Note, each $L^{(k)}$ scales as N_c^{-1}.

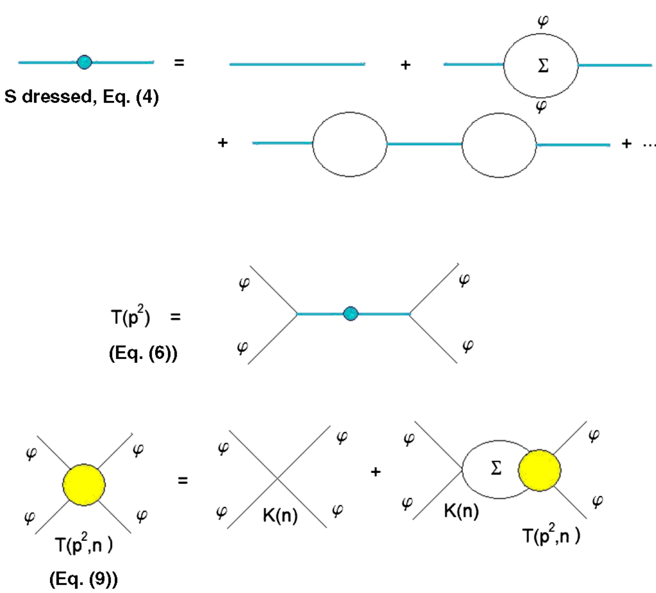

FIG. 1 (color online). Equations (4), (6), and (9) are depicted: In Eq. (4) the S resonance is dressed via loops of φ mesons. In Eq. (6) the T matrix is represented by an exchange of a dressed S meson. Finally, Eq. (9) represents a BS equation applied to the quartic terms of the Lagrangian $\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$, in which the T matrix appears both on the left- and the right-hand sides of the equation.

BS-inspired unitarization, way 1.—As a first exercise let us consider the low-energy Lagrangian $\mathcal{L}_{\text{le}}$ up to a certain order n by approximating the potential to $V(n) = \sum_{k=0}^{n} V^{(k)}$. By performing a Bethe-Salpeter study with this approximate potential (see Fig. 1), we obtain the following T matrix:

$$T(p^2, n) = -K(n) + K(n)\Sigma_\Lambda(p^2)T(p^2, n), \tag{9}$$

$$T(p^2, n) = \frac{1}{-K(n)^{-1} + \Sigma_\Lambda(p^2)}, \qquad K(n) = \frac{(\sqrt{2}g)^2}{M_0^2}\sum_{k=0}^{n}\left(\frac{p^2}{M_0^2}\right)^k, \tag{10}$$

where $K(n)$ is the bare tree-level amplitude corresponding to the sum of all the quartic terms up to order n.

Clearly, $T(p^2, n)$ is an approximate form of $T(p^2)$ of Eq. (6). The larger n, the better the approximation. Formally one has $\lim_{n\to\infty} T(p^2, n) = T(p^2)$. What we are doing is a dynamical reconstruction of the state S via a Bethe-Salpeter analysis applied to the low-energy Lagrangian $\mathcal{L}_{\rm le}$: We reobtain the state S which has been previously integrated out.

Let us keep n fixed and perform a large-N_c study of $T(p^2, n)$. Do we obtain the correct result, that is, $M = M_0$? The answer is no. In fact, in the large-N_c limit $K(n)$ scales as $1/N_c$ due to the dependence encoded in g, while $\Sigma_\Lambda(p^2)$ scales as N_c^0 (we assume that the cutoff does not scale with N_c [67]). In the large-N_c limit we obtain $T(p^2, n) \simeq -K(n)$. But $K(n)$ is a polynomial in p^2 and, for any finite n, does not admit poles for finite p^2, but only for $p^2 \to \infty$. Thus, we find the incorrect result that in the large-N_c limit the mass of the dynamically reconstructed state is infinity. This is shown in Fig. 2 for a particular numerical choice.

Although our analysis has been applied to a simple toy model, the form of Eq. (9) is general. One has a polynomial form for $K(n)$ as function of p^2 and a mesonic loop $\Sigma_\Lambda(p^2)$ which is independent of N_c. Complications due to different quantum numbers do not alter the conclusion. We also note

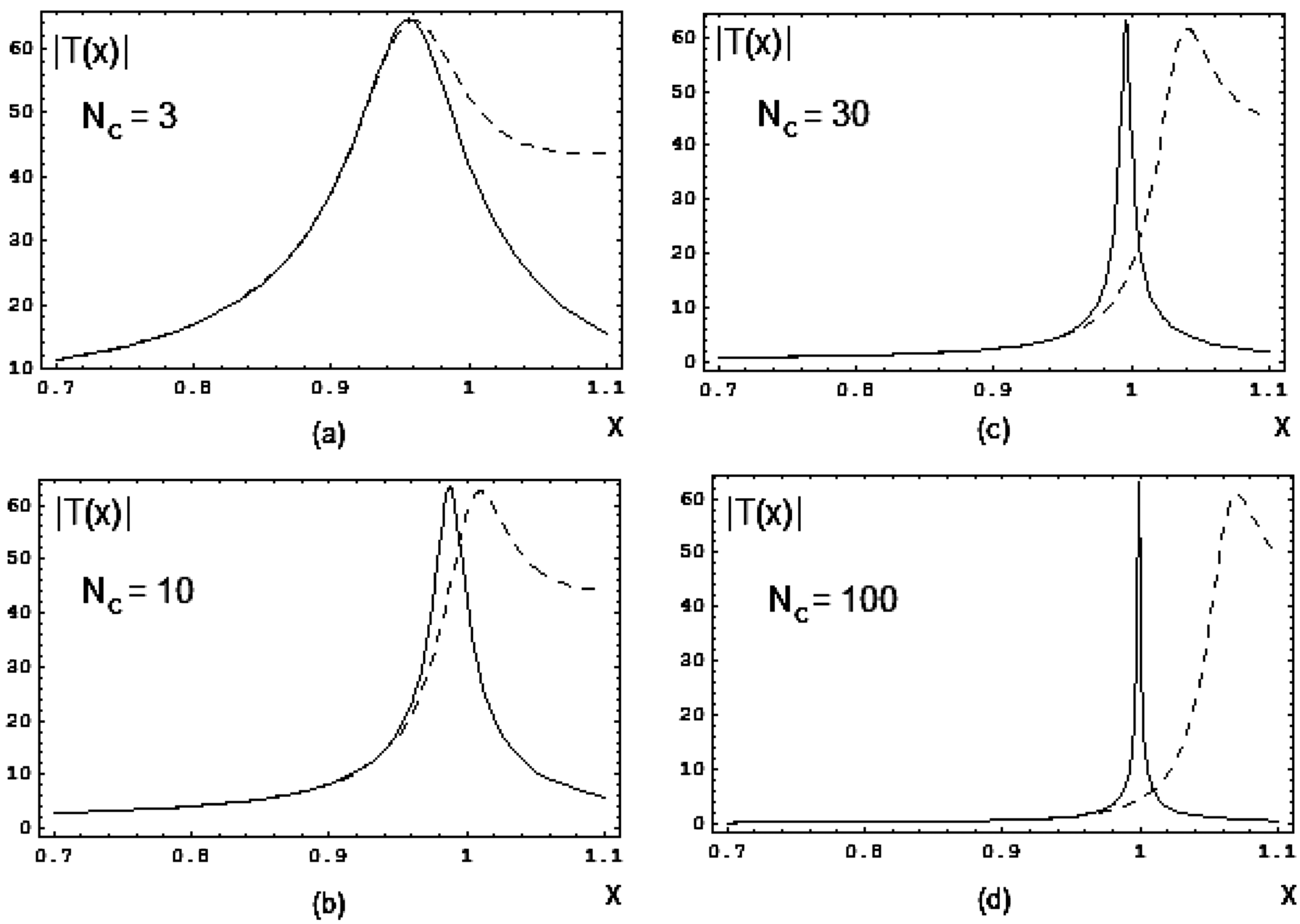

FIG. 2. Solid line: Absolute value of the full solution of the T matrix $|T(x)|$ with $x = \sqrt{p^2}$, Eq. (6). Dashed line: the approximate solution for $n = 10$, $|T(x, 10)|$, Eq. (10). The values (in GeV) $g_0 = \Lambda = 1.5$, $m = 0.3$, and $M_0 = 1$ are used. The dressed mass reads $M = 0.96$. The agreement is very good up to 1 GeV for $N_c = 3$ [panel (a)]. When increasing N_c the full solution is centered on M_0 and becomes narrower, as it should. On the contrary, the peak of the approximate solution increases and the width is only slightly affected by it. The approximate solution does not have the correct large-N_c expected behavior.

that these results are in agreement with the discussion of Ref. [68], where the scalar σ meson is first integrated out and then reconstructed in the framework of the linear sigma model.

IAM-inspired unitarization.—If we, instead, apply the IAM unitarization scheme to the $n = 1$ approximate form, we would obtain the correct result in the large-N_c expansion. In fact, in this case one schematically has (neglecting t and u channels)

$$T_{\rm IAM} \simeq T_2(T_2 - T_4 - iT_2\sigma T_2)^{-1}T_2, \tag{11}$$

where $\sigma = \sqrt{\frac{p^2}{4} - m^2}$ in our notation. Since $T_2 = \frac{(\sqrt{2}g)^2}{M_0^2}$ and $T_4 = -\frac{(\sqrt{2}g)^2}{M_0^4}p^2$, one finds

$$T_{\rm IAM} \simeq (\sqrt{2}g)^2(M_0^2 - p^2 - i(\sqrt{2}g)^2\sigma)^{-1}, \tag{12}$$

which represents a valid approximation of the full T matrix if g is not too large ($M \simeq M_0$). It is straightforward to see that the IAM approximation delivers the correct large-N_c result, namely, $M \to M_0$ and a width decreasing as $1/N_c$. Clearly one could repeat this study for increasing n, finding a better and better approximation of T.

BS-inspired unitarization, way 2.—Contrary to the BS-inspired unitarization described above (way 1), it is possible to follow a different BS-inspired approach which is in agreement with the large-N_c limit. For simplicity we discuss it in the explicit case $n = 1$ [69]. One has

$$K(1) = \frac{(\sqrt{2}g)^2}{M_0^2}\left(1 + \frac{p^2}{M_0^2}\right). \tag{13}$$

Now, instead of plugging $K(1)^{-1}$ directly into Eq. (10), we first invert it, obtaining the approximate form $K(1)^{-1}_{\rm way\,2}$ valid up to order $O(p^4/M_0^4)$:

$$K(1)^{-1}_{\rm way\,2} = \frac{M_0^2}{(\sqrt{2}g)^2}\left(1 - \frac{p^2}{M_0^2} + \ldots\right). \tag{14}$$

The next step is to write the T matrix in terms of $K(1)^{-1}_{\rm way\,2}$:

$$T(p^2, 1)_{\rm way\,2} = \frac{1}{-K(1)^{-1}_{\rm way\,2} + \Sigma_\Lambda(p^2)} \tag{15}$$

$$\simeq \frac{(\sqrt{2}g)^2}{p^2 - M_0^2 + (\sqrt{2}g)^2\Sigma_\Lambda(p^2)}. \tag{16}$$

Thus, this new approximate form derived from the BS equation is now in agreement with the large-N_c limit and is equivalent to the IAM-inspired unitarization approach described above. This shows an important fact in this discussion: It is not the BS method which fails in BS-way 1, but rather the adopted perturbative expansion. We could, as well, develop a second IAM-inspired unitarization which fails to reproduce the correct large-N_c results and that is equivalent to BS-way 1. From this perspective we can rearrange the unitarizations as "large-N_c correct" (BS-way 2 and IAM) and "large-N_c violating" (BS-way 1 and IAM-way 2). The reason why we associate the names BS or IAM to the different unitarizations is simply due to the way the equations settle down in the different cases. It offers a simple mnemonic to their development.

By studying the large-N_c limit one can see more closely the relations between the two described BS unitarizations: In the case $n = 1$ and in the large-N_c limit, the T matrix in the first BS form reads $T_{\rm way\,1} \simeq -K(1) = -\frac{(\sqrt{2}g)^2}{M_0^2} \times (1 + \frac{p^2}{M_0^2})$, which obviously has no pole. In the second BS unitarization one has in the large-N_c limit $T_{\rm way\,2} \simeq -K(1)_{\rm way\,2} \simeq -\frac{(\sqrt{2}g)^2}{M_0^2}(1 - \frac{p^2}{M_0^2})^{-1}$, and the correct pole $p^2 = M_0^2$ is recovered.

It is, however, important to notice that—just as in the IAM case—at least two terms in the expansion of the amplitude K are necessary to perform this second BS unitarization. This is the reason why it cannot be applied in the case studied in the next subsection (Sec. IV B), where only the lowest term of the amplitude is kept.

B. BS equation with the lowest term only

In most studies employing the BS analysis, only the lowest term of the effective low-energy Lagrangian is kept. Within the present toy model it is not possible to reconstruct a resonance with mass $M > 2m$ with only the lowest term ($n = 0$) [70]. However, a simple modification of the model which allows for such a study is possible:

$$\mathcal{L}^{\rm new}_{\rm toy}(E_{\max}, N_c) = \mathcal{L}_{\rm toy}(E_{\max}, N_c) + \frac{g^2}{2M_0^2}\varphi^4. \tag{17}$$

In this way an extra repulsion (whose quartic form is assumed to be valid up to $E_{\max}$) has been introduced. The T matrix takes the form

$$T(p^2) = \frac{1}{-K^{-1} + \Sigma_\Lambda(p^2)}, \qquad K = \frac{(\sqrt{2}g)^2}{M_0^2 - p^2} - \frac{(\sqrt{2}g)^2}{M_0^2}. \tag{18}$$

When deriving the low-energy Lagrangian, everything goes as before, but the $k = 0$ term is now absent:

$$V(n) = \sum_{k=1}^{n} V^{(k)}, \qquad V^{(k)} = L^{(k)}\varphi^2(-\Box)^k\varphi^2.$$

Note, in this case the $\varphi\varphi$ scattering vanishes at low momenta and in the chiral limit $m \to 0$ (just as the $\pi\pi$ scattering does in reality).

A study of the case $n = 1$ (corresponding to the first term only in the expansion) is now possible. We consider the following situation: Let the original Lagrangian $\mathcal{L}^{\rm new}_{\rm toy}$ of Eq. (17) be unknown. The low-energy potential at the lowest order reads $V \simeq V^{(1)} = L^{(1)}\varphi^2(-\Box)\varphi^2$, but the low-energy coefficient $L^{(1)}$ is also unknown. Moreover, from low-energy information only, one does not know the value of the cutoff Λ to be employed in mesonic loops: A new cutoff $\tilde{\Lambda}$, not necessarily equal to the original Λ, is also introduced as a free parameter. From the perspective of low-energy phenomenology, one writes down the following approximate form for the T matrix, which depends on two "free parameters," $L^{(1)}$ and $\tilde{\Lambda}$:

$$\tilde{T}(p^2) = T(p^2, 1) = \frac{1}{-\tilde{K}^{-1} + \Sigma_{\tilde{\Lambda}}(p^2)}, \quad \tilde{K} = K(1) = 4L^{(1)}p^2. \tag{19}$$

The question is if it is possible to vary $L^{(1)}$ and $\tilde{\Lambda}$ in such a way that the approximate T matrix $\tilde{T}(p^2)$ reproduces the "full" result $T(p^2)$ of Eq. (6) between, say, $2m = 0.6$ GeV and 1.3 GeV for $N_c = 3$.

The answer is that this is generally possible, but the results for $L^{(1)}$ and $\tilde{\Lambda}$ vary drastically with the coupling constant g_0 in the original Lagrangian. In particular, if g_0 is small, a good fit implies a very large and unnatural value of $\tilde{\Lambda}$ [Fig. 3(a)]. For instance, for $g_0 = g(N_c = 3) = 1.5$ GeV the mass $M = 0.95$ GeV is only slightly shifted from the bare mass $M_0 = 1$ GeV. In this case the approxi-

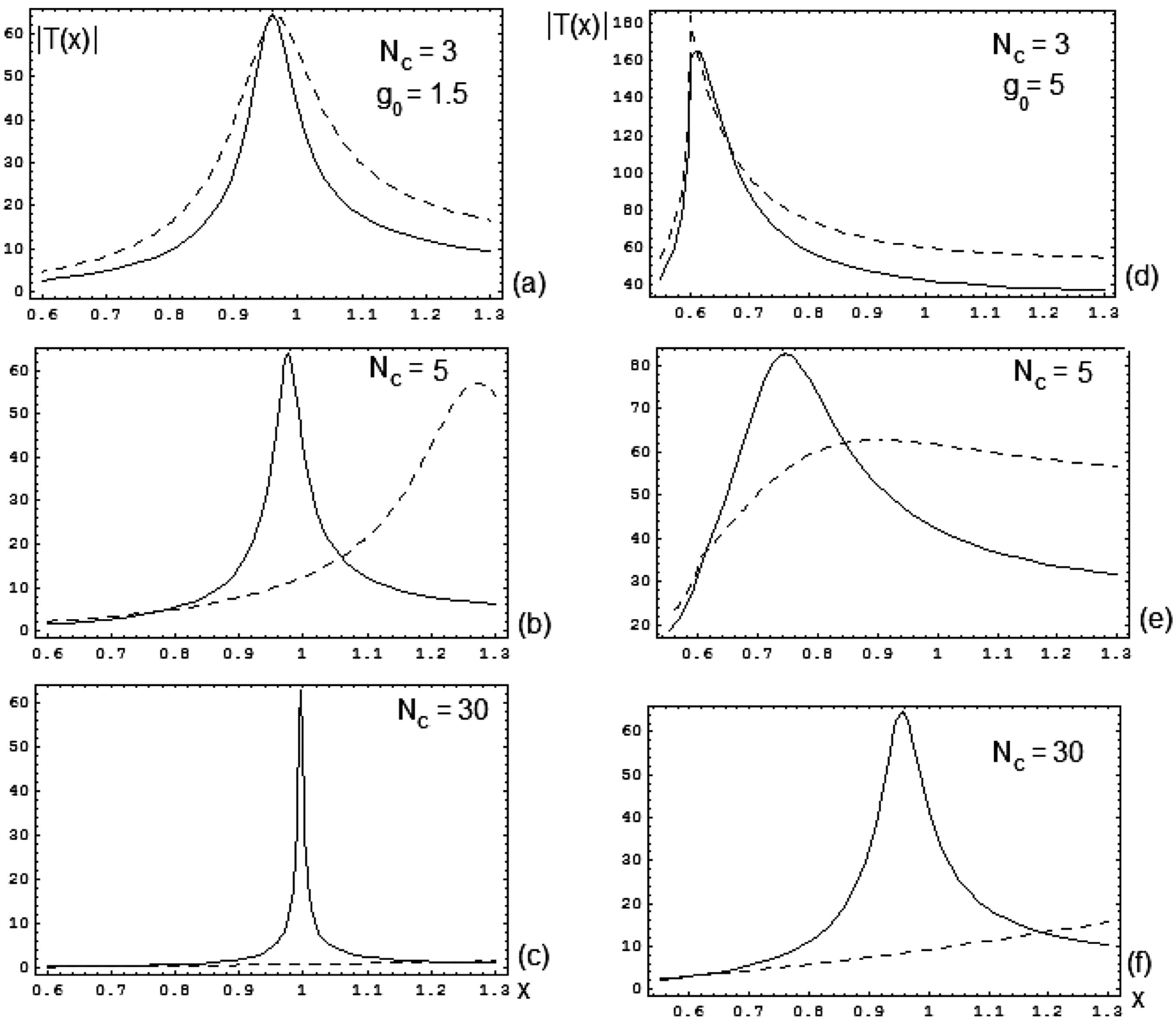

FIG. 3. Full $|T(x)|$ [solid line, Eq. (18) with $\Lambda = 1.5$ GeV] and approximate $|\tilde{T}(x)|$ [dashed line, Eq. (19)] in the cases $g_0 = 1.5$ GeV (left column) and $g_0 = 5$ GeV (right column) for different values of N_c. The values of $L^{(1)}$ and $\tilde{\Lambda}$, which determine the approximate dashed curve, are determined by fitting the approximate form to the full one in the $N_c = 3$ cases. One has $\tilde{\Lambda} \simeq 15\,000\Lambda$ in the left column, and $\tilde{\Lambda} = \Lambda$ in the right column. As soon as N_c is increased, the approximate solution quickly fades out, while the real solution approaches M_0 where it becomes more and more peaked.

mate form $|\tilde{T}(p^2)|$ reproduces $|T(p^2)|$ only if $\tilde{\Lambda} \sim 10^4 \Lambda$ (astronomically high and seemingly unnatural from the perspective of the low-energy theory).

The situation changes completely if g_0 is large: It is possible to find a satisfactory description in which $\tilde{\Lambda} \sim \Lambda$. For instance, for $g_0 = 5$ GeV one has $M = 0.65$ GeV and the approximate $|\tilde{T}(p^2)|$ reproduces well $|T(p^2)|$ for $\tilde{\Lambda} = \Lambda$ [Fig. 3(d)].

In the first case the failure of the dynamical reconstruction with a meaningful value of the cutoff $\tilde{\Lambda}$ is due to the quantitative inappropriate behavior of the Bethe-Salpeter approach when only the first term is kept. In the second case a rather satisfactory description is possible for a meaningful value of the cutoff. In light of the results of the low-energy Lagrangian only, one could also propose the interpretation that the obtained state S is dynamically generated, and shall be regarded as a $\varphi\varphi$ molecular state. This is *not*, however, the correct interpretation in the present example. We know, in fact, that this state corresponds—by construction—to the original, preexisting, quarkoniumlike state S.

In both cases, as soon as we increase the number of colors, the approximate T matrix $|\tilde{T}(p^2)|$ and the full $|T(p^2)|$ show a completely different behavior [Figs. 3(b), 3(c), 3(e), and 3(f)]: While the peak of $|T(p^2)|$ approaches $M_0 = 1$ GeV and becomes narrower according to the correct large-N_c limit of the S meson, the dynamically reconstructed state fades out, because of the *incorrect* behavior of BS unitarization with large N_c. This is clearly visible from the interaction term $V^{(1)} = L^{(1)}\varphi^2(-\Box)\varphi^2$, because $L^{(1)}$ scales as N_c^{-1}. However, although the interaction term disappears with large N_c, the state S is still the original quark-antiquark state. This example shows that the reconstruction of the state S is not possible in the large-N_c limit, but this does not mean that S is a dynamically generated state of molecular type. Note, this is just a subcase of the previous general discussion on large-N_c dependence: The fact that only one term is kept generates a much faster "fading out" of the reconstructed state; compare Figs. 2 and 3.

In the previous subsection it was shown that—while a straight application of the BS equation is at odds with the large-N_c limit—a second BS unitarization allows for a correct description of the large-N_c limit. The second BS unitarization is not, however, applicable in the present case. In fact, *at least* two terms in the expansion of $K(n)$ are needed to follow it. If only the lowest term is kept, as done here with the term $n = 1$ in Eq. (19), this is no longer feasible. This is similar to the fact that the IAM method also needs at least two terms in the expansion of the amplitude K in order to be applicable [71].

C. Analogy with the real world

The original toy Lagrangian $\mathcal{L}_{\text{toy}}(E_{\max}, N_c)$ of Eq. (3) is assumed to be valid up to an energy $E_{\max} \gg M_0$. The corresponding low-energy Lagrangian $\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$ of Eq. (7)—obtained by integrating out the S field—is valid up to an energy $E_{\text{le}} \ll M_0$. When unitarizing $\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$, one can enlarge the validity of the low-energy theory up to M_0 and then infer the existence of the resonance S with mass $M < M_0$. However, if no other input is known, the nature of the state S cannot be further studied; see the general discussion of point (d) in Sec. II.

This situation is similar to example (c) in Sec. II: The Fermi Lagrangian $\mathcal{L}_{\text{F}}$ alone does not allow one to deduce the nature of the W meson, even if the existence of the latter is inferred by unitarization arguments applied to $\mathcal{L}_{\text{F}}$. It is also similar to the cases studied in Sec. III C: When a resonance is obtained by unitarizing a low-energy mesonic Lagrangian, (at first) no statement about its nature can be made.

Further information is needed: In the case of the W meson, the full electroweak Lagrangian $\mathcal{L}_{\text{EW}}$ is known and leads to the straightforward conclusion that the W meson is not a dynamically generated state, but a fundamental field of the standard model. In the framework of the toy model, this corresponds to the knowledge of the "full Lagrangian" $\mathcal{L}_{\text{toy}}(E_{\max}, N_c)$ of Eq. (3). The "quarkonium" nature of S can then be easily deduced.

In the case of low-energy mesonic theories discussed in Sec. III C, the full hadronic Lagrangian is not known. The only additional knowledge is the large-N_c scaling of the low-energy constants of the low-energy Lagrangian(s). In the framework of the toy model, this corresponds to the knowledge of the low-energy Lagrangian $\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$ of Eq. (7) (up to a certain n) together with the scaling of the quantities $L^{(k)}$ in Eq. (8). The latter additional knowledge can lead to the correct conclusions about the nature of the S meson, although—as discussed in Sec. IV A—care is needed when the BS method is chosen.

Moreover, as further studied in Sec. IV B, when only *the lowest term* of the low-energy Lagrangian is kept, it is not possible to reproduce the correct large-N_c behavior of the resonance S. Although the "dynamical reconstruction" of the state S is possible, the state S "looks like" a molecular state which fades out in the large-N_c limit. This, however, is not true: In fact, we know from the very beginning that the state S corresponds "by construction" to a quark-antiquark state.

Although this discussion is based on toy models and the real world is much more complicate than this, the same qualitative picture can hold in low-energy QCD. In fact, the use of the BS equation in the literature is often limited to the lowest term of a low-energy Lagrangians for $\pi\pi$, $\rho\rho$, ... interactions. In our view, such low-energy Lagrangians emerge upon integrating out all the heavier fields in $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\max}, N_c = 3)$, in which tensor, axial-vector, and scalar quark-antiquark fields must exist below 2 GeV. Then, the use of the BS equation, similarly to the dynamical reconstruction of S in this simple example, leads to the dynamical reconstruction of the axial-vector, tensor, and scalar mesons above 1 GeV: They are preexisting, quark-antiquark states, which are reobtained from low-energy Lagrangians via unitarization methods. Future unitarization studies, involving the leading and the next-to-leading terms in the effective Lagrangians, may shed light on this point.

V. CONCLUSIONS AND OUTLOOK

In this work we studied the issue of dynamical generation, both in a general context and in low-energy QCD. A dynamically generated resonance has been defined as a state which does not correspond to any of the fields of the original Lagrangian describing the system up to a certain maximal energy $E_{\max}$, provided that its mass lies below this maximal energy. This discussion also offered us the possibility to distinguish, in principle, tetraquark from mesonic molecular states in low-energy QCD: While the former are fundamental and shall be included as bare fields in the (yet-unknown) hadronic Lagrangian $\mathcal{L}_{\rm eff}^{\rm had}(E_{\max}, N_c = 3)$, this is not the case for the latter. Note, $\mathcal{L}_{\rm eff}^{\rm had}(E_{\max}, N_c = 3)$ represents the complete hadron theory valid up to $E_{\max} \simeq 2$ GeV.

In the application to the hadronic world, we also discussed dynamical reconstruction of resonances: These are resonances which are obtained via unitarization methods from low-energy effective Lagrangians, but still represent fundamental fields (such as quark-antiquark states) in $\mathcal{L}_{\rm eff}^{\rm had}(E_{\max}, N_c = 3)$. Note, the low-energy effective Lagrangians can be seen as a result of integrating out heavier (quarkonia, glueballs, etc.) fields representing intrinsic, fundamental states in $\mathcal{L}_{\rm eff}^{\rm had}(E_{\max}, N_c = 3)$. In the scenario of dynamical reconstruction, one reconstructs these heavier resonances by unitarizing the appropriate low-energy Lagrangian.

Within a simple toy model, these issues have been examined. This model consists of two fields, φ and S, with the latter being heavier and with a nonzero decay width into $\varphi\varphi$. We introduced a large-N_c dependence which mimics that of quarkonium states in QCD. The field S was first integrated out and the emerging low-energy interaction Lagrangian involving only the field φ was derived. Out of this, the state S was dynamically reconstructed.

In order to do this, we have used a unitarization inspired by the Bethe-Salpeter equation, and we have shown that the original, quarkoniumlike large-N_c behavior of S cannot be reproduced if only the lowest term in the effective Lagrangian is kept. (Note, when more terms are kept this problem can be easily solved and the large-N_c result is correct also within the BS approach. The problem is not the latter but the adopted perturbative expansion; see Sec. IVA). We then proposed that a similar, although more complicated, dynamical reconstruction mechanism takes place for tensor, axial-vector, and scalar mesons above 1 GeV: These resonances, studied in recent works, can be interpreted as fundamental quark-antiquark states, which are reobtained when unitarizing low-energy effective Lagrangians. In this scenario there is no conflict between the ''old'' quark model assignments and recent developments, because they would both represent a dual description of the same, preexisting quark-antiquark resonances. This interpretation, although not yet conclusive, represents a possibility which deserves further study.

Dynamical reconstruction can also hold for light scalar mesons below 1 GeV, if they form a quarkonium (quite improbable) or a tetraquark nonet. The situation in this case is, as discussed in the text, still unclear. In this work we limited the study to the light mesonic sector, but the present discussion about dynamical generation/reconstruction can also hold, with due changes, in the baryon and the heavy quark sectors.

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[1] C. Amsler and N. A. Tornqvist, Phys. Rep. **389**, 61 (2004).
[2] E. Klempt and A. Zaitsev, Phys. Rep. **454**, 1 (2007).
[3] J. A. Oller, E. Oset, and J. R. Pelaez, Phys. Rev. Lett. **80**, 3452 (1998); Phys. Rev. D **59**, 074001 (1999); **60**, 099903 (E) (1999); **75**, 099903 (2007); A. Gomez Nicola and J. R. Pelaez, Phys. Rev. D **65**, 054009 (2002); A. Dobado and J. R. Pelaez, Phys. Rev. D **56**, 3057 (1997).
[4] E. Oset *et al.*, Int. J. Mod. Phys. E **18**, 1389 (2009); J. A. Oller and E. Oset, Nucl. Phys. **A620**, 438 (1997); **A652**, 407(E) (1999); Phys. Rev. D **60**, 074023 (1999).
[5] M. F. M. Lutz and E. E. Kolomeitsev, Nucl. Phys. **A730**, 392 (2004); M. Wagner and S. Leupold, Phys. Lett. B **670**, 22 (2008); Phys. Rev. D **78**, 053001 (2008); S. Leupold and M. Wagner, Int. J. Mod. Phys. A **24**, 229 (2009).
[6] L. S. Geng and E. Oset, Phys. Rev. D **79**, 074009 (2009); R. Molina, D. Nicmorus, and E. Oset, Phys. Rev. D **78**, 114018 (2008).
[7] J. R. Pelaez, Phys. Rev. Lett. **92**, 102001 (2004); Mod. Phys. Lett. A **19**, 2879 (2004); M. Uehara, arXiv:hep-ph/0401037.
[8] S. Leupold, Nucl. Phys. **A743**, 283 (2004).
[9] L. S. Geng, E. Oset, J. R. Pelaez, and L. Roca, Eur. Phys. J.

A **39**, 81 (2009).
[10] M. Boglione and M.R. Pennington, Phys. Rev. D **65**, 114010 (2002).
[11] D. Morgan and M.R. Pennington, Phys. Rev. D **48**, 1185 (1993).
[12] N.A. Tornqvist, Z. Phys. C **68**, 647 (1995); N.A. Tornqvist and M. Roos, Phys. Rev. Lett. **76**, 1575 (1996).
[13] E. van Beveren, T.A. Rijken, K. Metzger, C. Dullemond, G. Rupp, and J.E. Ribeiro, Z. Phys. C **30**, 615 (1986); E. van Beveren and G. Rupp, Eur. Phys. J. C **10**, 469 (1999); Int. J. Theor. Phys. Group Theory Nonlinear Opt. **11**, 179 (2006).
[14] O. Krehl and J. Speth, Nucl. Phys. **A623**, 162 (1997); C.L. Li, B.S. Zou, and G.l. Li, Phys. Rev. D **63**, 074003 (2001); N. Isgur and J. Speth, Phys. Rev. Lett. **77**, 2332 (1996); see, however, also M. Harada, F. Sannino, and J. Schechter, Phys. Rev. Lett. **78**, 1603 (1997).
[15] M.B. Voloshin and L.B. Okun, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 369 (1976); [JETP Lett. **23**, 333 (1976)]; K. Maltman and N. Isgur, Phys. Rev. Lett. **50**, 1827 (1983); Phys. Rev. D **29**, 952 (1984); V. Baru, J. Haidenbauer, C. Hanhart, Yu. Kalashnikova, and A.E. Kudryavtsev, Phys. Lett. B **586**, 53 (2004); T. Branz, T. Gutsche, and V.E. Lyubovitskij, arXiv:0712.0354.
[16] S. Krewald, R.H. Lemmer, and F.P. Sassen, Phys. Rev. D **69**, 016003 (2004).
[17] L. Hulthen and K.V. Laurikainen, Rev. Mod. Phys. **23**, 1 (1951); Y.B. Ding *et al.*, J. Phys. G **30**, 841 (2004); E. Golowich, arXiv:0803.3329.
[18] X.Q. Luo, Y.Y. Li, and H. Kroger, Sci. China G **35**, 631 (2005).
[19] In both cases (a) and (b) one can write a (nonlocal) Lagrangian in which the lightest positronium (deuteron) field P (D) directly couples to its constituents, the electron and the positron (the nucleon and the proton). In this case, the compositeness condition should be introduced; see S. Weinberg, Phys. Rev. **130**, 776 (1963); A. Salam, Nuovo Cimento **25**, 224 (1962); K. Hayashi, M. Hirayama, T. Muta, N. Seto, and T. Shirafuji, Fortschr. Phys. **15**, 625 (1967). It ensures that the field P (D) is a composite object and thus not on the same footing as its elementary constituents. For a recent application for the deuteron, we also refer to Y.b. Dong, A. Faessler, T. Gutsche, and V.E. Lyubovitskij, Phys. Rev. C **78**, 035205 (2008); and for the positronium, to F. Giacosa, Ph.D. thesis, Tuebingen (online at http://w210.ub.uni-tuebingen.de/dbt/volltexte/2005/1705/). For instance, in the positronium case the interaction Lagrangian (neglecting nonlocality) has the form $\mathcal{L} = gP\bar{\psi}i\gamma^5\psi$. Surely, this Lagrangian is useful to study properties of the positronium state, but can describe the e^--e^+ interaction only in a very small energy range close to $2m_e$, where the positronium resonance dominates. Moreover, as soon as the photon is added to the Lagrangian (a obviously necessary step), one shall not include P from the very beginning; otherwise it is double counted. Indeed, the compositeness condition can be directly related to the Bethe-Salpeter approach applied to $\mathcal{L}_{\text{QED}}$, where the composite nature of the positronium is manifest. Similar arguments hold for the deuteron.
[20] A. Djouadi, Phys. Rep. **457**, 1 (2008).
[21] The definition in Sec. II does not allow us to conclude that the resonance R obtained in the framework of unitarized $\mathcal{L}$ is dynamically generated because $M_R > E_{\text{max}}$. This is in agreement with the fact that at the level of the unitarized version of $\mathcal{L}$, one cannot deduce if the field R appears explicitly or not in the more complete Lagrangian $\mathcal{L}'$ (i.e. one cannot state if R is dynamically generated or not). Note, even in the case $E_U \sim E'_{\text{max}}$, the unitarized version of $\mathcal{L}$ is not equivalent to $\mathcal{L}'$ (therefore one should not use the definition in Sec. II with the energy scale E_U instead of E_{max}). Only some of the properties of $\mathcal{L}'$ can be inferred by the extrapolation to higher energy of the information contained in the Lagrangian $\mathcal{L}$.
[22] G. 't Hooft, arXiv:hep-th/0010225.
[23] G. 't Hooft, Nucl. Phys. **B72**, 461 (1974).
[24] E. Witten, Nucl. Phys. **B160**, 57 (1979).
[25] S.R. Coleman, Proceedings of the 1979 International School of Subnuclear Physics, Pointlike Structures Inside and Outside Hadrons, Erice, Italy, 1979 (Erice Subnuclear 1979:0011).
[26] R.F. Lebed, Czech. J. Phys. **49**, 1273 (1999).
[27] I.J. General, S.R. Cotanch, and F.J. Llanes-Estrada, Eur. Phys. J. C **51**, 347 (2007).
[28] C. Amsler *et al.*, Phys. Lett. B **667**, 1 (2008).
[29] While a *local* tetraquark current can be transformed into molecular and quarkonium currents, this does not hold when the tetraquark is regarded as a nonlocal object, where the diquarks are separated in space. This idea can be extended: Each hadron with an irreducible, nonlocal color substructure cannot be dynamically generated. Most of these hypothetical hadrons do not find a counterpart in the PDG because they do not bind or are too massive and broad to be seen. The tetraquark sector below 1 GeV can be an exception, due to the strong binding of the diquark in the color and flavor antitriplet channel and to the s wave, both in the diquark and in the tetraquark objects.
[30] R.L. Jaffe, Phys. Rep. **409**, 1 (2005); Nucl. Phys. B, Proc. Suppl. **142**, 343 (2005).
[31] C. Liu, Eur. Phys. J. C **53**, 413 (2008).
[32] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961). For reviews see T. Hatsuda and T. Kunihiro, Phys. Rep. **247**, 221 (1994); S.P. Klevansky, Rev. Mod. Phys. **64**, 649 (1992). See also the recent developments in M.X. Su, L.Y. Xiao, and H.Q. Zheng, Nucl. Phys. **A792**, 288 (2007); M.K. Volkov, M. Nagy, and V.L. Yudichev, Nuovo Cimento Soc. Ital. Fis. A **112**, 225 (1999) and references therein.
[33] M.D. Scadron, G. Rupp, F. Kleefeld, and E. van Beveren, Phys. Rev. D **69**, 014010 (2004); **69**, 059901(E) (2004).
[34] J.T. Lenaghan, D.H. Rischke, and J. Schaffner-Bielich, Phys. Rev. D **62**, 085008 (2000).
[35] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).
[36] N. Mathur *et al.*, Phys. Rev. D **76**, 114505 (2007).
[37] R.L. Jaffe, Phys. Rev. D **15**, 267 (1977); **15**, 281 (1977).
[38] L. Maiani, F. Piccinini, A.D. Polosa, and V. Riquer, Phys. Rev. Lett. **93**, 212002 (2004).
[39] D.V. Bugg, Eur. Phys. J. C **47**, 57 (2006); arXiv:hep-ex/0510014.
[40] F. Giacosa, Phys. Rev. D **74**, 014028 (2006).
[41] G. 't Hooft, G. Isidori, L. Maiani, A.D. Polosa, and V. Riquer, Phys. Lett. B **662**, 424 (2008).

[42] A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D **72**, 034001 (2005); A. H. Fariborz, Int. J. Mod. Phys. A **19**, 2095 (2004).

[43] M. Napsuciale and S. Rodriguez, Phys. Rev. D **70**, 094043 (2004).

[44] F. Giacosa, Phys. Rev. D **75**, 054007 (2007).

[45] C. Amsler and F. E. Close, Phys. Rev. D **53**, 295 (1996); W. J. Lee and D. Weingarten, Phys. Rev. D **61**, 014015 (1999); F. E. Close and A. Kirk, Eur. Phys. J. C **21**, 531 (2001); F. Giacosa, T. Gutsche, V. E. Lyubovitskij, and A. Faessler, Phys. Rev. D **72**, 094006 (2005); Phys. Lett. B **622**, 277 (2005); F. Giacosa, T. Gutsche, and A. Faessler, Phys. Rev. C **71**, 025202 (2005); H. Y. Cheng, C. K. Chua, and K. F. Liu, Phys. Rev. D **74**, 094005 (2006).

[46] Y. Chen *et al.*, Phys. Rev. D **73**, 014516 (2006).

[47] A. Heinz, S. Struber, F. Giacosa, and D. H. Rischke, Phys. Rev. D **79**, 037502 (2009).

[48] P. Minkowski and W. Ochs, Eur. Phys. J. C **9**, 283 (1999).

[49] V. Mathieu, N. Kochelev, and V. Vento, Int. J. Mod. Phys. E **18**, 1 (2009).

[50] C. Hanhart, Eur. Phys. J. A **31**, 543 (2007).

[51] S. Scherer, Adv. Nucl. Phys. **27**, 277 (2003).

[52] I. Caprini, G. Colangelo, and H. Leutwyler, Phys. Rev. Lett. **96**, 132001 (2006).

[53] F. J. Yndurain, R. Garcia-Martin, and J. R. Pelaez, Phys. Rev. D **76**, 074034 (2007).

[54] See the note on the quark model in the work of the Particle Data Group [28]. Online at http://pdg.lbl.gov/2008/reviews/rpp2008-rev-quark-model.pdf.

[55] F. Giacosa, T. Gutsche, V. E. Lyubovitskij, and A. Faessler, Phys. Rev. D **72**, 114021 (2005).

[56] G. Ecker, J. Gasser, A. Pich, and E. de Rafael, Nucl. Phys. **B321**, 311 (1989).

[57] A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D **79**, 074014 (2009).

[58] A. Salomone, J. Schechter, and T. Tudron, Phys. Rev. D **23**, 1143 (1981).

[59] The theory is nonrenormalizable because of the logarithmic term in V_G. Moreover, as soon as (axial-)vector fields are included, further nonrenormalizable terms emerge.

[60] D. Parganlija, F. Giacosa, and D. H. Rischke, AIP Conf. Proc. **1030**, 160 (2008); Proc. Sci. Confinement 8 (2008) 070 [arXiv:0812.2183]. See also M. Urban, M. Buballa, and J. Wambach, Nucl. Phys. **A697**, 338 (2002); S. Gallas, F. Giacosa, and D. H. Rischke, arXiv:0907.5084.

[61] F. Giacosa and G. Pagliara, Phys. Rev. C **76**, 065204 (2007).

[62] T. Bhattacharya and S. Willenbrock, Phys. Rev. D **47**, 4022 (1993).

[63] The details of the regularization are not important, as long as momenta larger than Λ are suppressed in the meson loop. Note that, although the toy model is formally renormalizable, we still work with a finite cutoff which directly affects the results. This is, namely, also the situation in the true world: A cutoff of the order of 1 GeV is an essential ingredient in many effective models of low-energy QCD.

[64] In the employed notation the full amplitude reads $A(s, t, u) = 2(T(s) + T(t) + T(u))$.

[65] J. I. Basdevant and B. W. Lee, Phys. Rev. D **2**, 1680 (1970).

[66] See also the lectures of Stefan Leupold, http://theorie.physik.uni-giessen.de/~leupold/, pp. 70–73.

[67] The assumption that Λ does not depend on N_c can be justified by assuming that $\Lambda \sim \Lambda_{\mathrm{QCD}}$, the latter being N_c independent. An alternative behavior of Λ as a function of N_c has been discussed in Ref. [9], where the case $\Lambda \sim 4\pi f_\pi$ leads to an increase of Λ with $N_c^{1/2}$. Here we note that in the context of the toy model the mass of the reconstructed state still diverges when admitting a power-like dependence $\Lambda \sim N_c^\alpha$ with $\alpha \geq 0$, although the quantitative way that the limit is reached is strongly affected by the value of α.

[68] Z. H. Guo, L. Y. Xiao, and H. Q. Zheng, Int. J. Mod. Phys. A **22**, 4825 (2007).

[69] The author thanks the referee for having pointed out this second possibility.

[70] This is due to the fact that the lowest term has no p^2 dependence and due to the particular form of $\mathrm{Re}\,\Sigma(p^2)$; see Ref. [61]. The reconstructed mass is actually infinite. The first term only, although inadequate in this case, could still work if the loops were so strong that $M < 2m$ (even if $M_0 > 2m$), but we do not discuss this case here.

[71] If, instead of $n = 1$, the case $n = 2$ is considered, then again the second BS unitarization is possible. In fact, with $K(2) = \frac{(\sqrt{2}g)^2 p^2}{M_0^4}(1 + \frac{p^2}{M_0^2})$ one has $K(2)^{-1}_{\mathrm{way}\ 2} = \frac{M_0^4}{(\sqrt{2}g)^2 p^2}(1 - \frac{p^2}{M_0^2} + \ldots)$ and $T_{\mathrm{way}\ 2} \simeq \frac{(\sqrt{2}g)^2(p^2/M_0^2)}{p^2 - M_0^2 - (\sqrt{2}g)^2(p^2/M_0^2)\Sigma_\Lambda}$, which is in agreement with the large-N_c expectation.