PHYSICAL REVIEW D **80,** 074009 (2009)

Topological dark matter in little Higgs models

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(Received 23 May 2009; published 9 October 2009)

We show that certain little Higgs models with symmetry breaking $SU(N) \to SO(N)$ for $N \geq 4$ admit topologically stable solitons that may contribute to cosmological dark matter. We have constructed a spherically symmetric soliton and estimated its mass in the case of $SU(5) \to SO(5)$. Its mass is estimated to be around 10.3 TeV. Whether this particle is a fermion or a boson depends on the value of an integer-valued parameter of the underlying theory, analogous to the number of colors of QCD. In either case, the particle is neutral. If it is a fermion, it is a Majorana particle, which could take part in a seesaw mechanism for neutrino masses.

DOI: 10.1103/PhysRevD.80.074009 PACS numbers: 95.35.+d, 12.39.Dc, 12.60.Rc, 14.80.Cp

I. INTRODUCTION

The hot big bang model of cosmology tells us that the Universe cooled down from a primordial hot and dense state to the present state of galaxies and other large-scale structures with a mean temperature of about 2.73 K [1]. Certain theories of grand unification predict that the Universe underwent a series of phase transitions as it cooled down, similar to what we observe in condensed matter systems. Phase transitions in the early Universe can give rise to certain stable configurations of matter known as topological defects. Different types of topological defects can arise depending on the symmetry breaking mechanism of the underlying field theory [2]. They can appear in the forms of magnetic monopoles, cosmic strings, domain walls, textures, and skyrmions [3].

A field theory described by a continuous symmetry group G, spontaneously breaks down to a subgroup $H \subset G$, and the space of all accessible vacua of the theory called the vacuum manifold is defined to be the space of cosets of H in G. The theory possesses a topological defect if some homotopy group of the coset space $\pi_d(\mathcal{M} \equiv G/H)$ is nontrivial. When $d = 0$, 1, and 2 the defects, respectively, are domain walls, strings (vortices), and magnetic monopoles or textures. The case $d = 3$, which plays a major role in this paper, gives rise to pointlike topological defects called skyrmions.

Recently there has been much interst in a class of field theoretic models called the little Higgs models [4–6] in the context of weak scale symmetry breaking. These models provide a new logical possibility for natural electroweak symmetry breaking and a new partial resolution of the hierarchy problem in elementary particle physics. Introduction of new symmetries at the TeV scale by these models provides the cancellation of all quadratically divergent contributions to the Higgs mass at the one-loop level and pushes up the hierarchy problem to an energy scale of around 10 TeV. Little Higgs models have generated a lot of interest since any potential candidate to solve the hierarchy problem deserves serious attention.

Among the many possible ways of implementing the little Higgs paradigm, the littlest Higgs model [5] is the simplest and most economical. This theory introduces a weakly coupled new physics at TeV energies, stabilizes the electroweak scale with a naturally light Higgs, and is the smallest extension of the standard model to date.

In this paper we address the interesting new possibility of bridging the natural electroweak symmetry breaking and cosmological dark matter—the nonbaryonic, nonrelativistic, and weakly interacting matter that constitutes about 22% of matter in the Universe. Since the Higgs particles appear as pseudo Nambu-Goldstone bosons in little Higgs models, skyrmion solutions that are stable and electrically neutral can also come out quite generically. We demonstrate the existence of a pointlike, electrically neutral, and topologically stable structure—a particle with a Z_2 charge, which could be a viable dark matter candidate. Its mass, estimated in the context of the littlest Higgs model with T parity [7], is found to be around 10.3 TeV which is well below the unitarity limit [8] of viable dark matter particles. (The existence of other topological defects in the little Higgs model was investigated in [9].)

We start with a class of nonrenormalizable effective field theories for pseudo Nambu-Goldstone bosons, in which a symmetry group $SU(N)$ is broken down to its real subgroup $SO(N)$ for $N \geq 4$. The case $N = 5$ is of most interest, as it appears in the little Higgs models.

At small energy scale compared to the symmetry breaking scale, the effective action has the form

$$S_1 = \frac{f^2}{8} \int d^4x \, \mathrm{tr} \partial_\mu \Phi \partial^\mu \Phi^\dagger + \cdots, \qquad (1)$$

where f is a parameter with dimension of energy and Φ is a scalar field given by a differentiable map from the Minkowsky space $R^{1,3}$ to a nonlinear target manifold $\mathcal{M}_N$,

$$\Phi\colon R^{1,3} \to \mathcal{M}_N. \tag{2}$$

The target manifold $\mathcal{M}_N$ is the subset of symmetric matrices in $SU(N)$

$$\mathcal{M}_N = \{\Phi | \Phi = \Phi^T, \Phi\Phi^\dagger = 1, \det\Phi = 1\}. \tag{3}$$

It has a global symmetry $\Phi \to g\Phi g^T$, $g \in SU(N)$. Any $\Phi \in \mathcal{M}_N$ can be reduced to the identity by this transformation [10]. That is, there is a $g \in SU(N)$ such that $\Phi = gg^T$. If we change $g \mapsto gh$, with $h \in SO(N)$, the product gg^T is unchanged. Thus we can identify $\mathcal{M}_N = SU(N)/SO(N)$. The canonical projection to the cosets is p: $SU(N) \to \mathcal{M}_N$, $p(g) = gg^T$.

At spatial infinity, the field $\Phi(x)$ must approach a constant; the choice of this constant among all matrices satisfying Eq. (3) will break the symmetry $SU(N)$ down to its real subgroup $SO(N)$. The parameter f sets the scale of the symmetry breaking; in the little Higgs models it is expected to be of the order of a TeV. The ellipsis in Eq. (1) indicates that we are ignoring higher derivative terms, which are expected to be unimportant in the limit of "low" energies, that is, energies of the order of f.

Among the higher derivative terms we can add a new term that does not change the hyperbolic nature of the field equations and is still second order in time. This is the "Skyrme term" [3] given by

$$S_2 = \frac{1}{32e^2}\int d^4x\, \mathrm{tr}[\partial^\mu\Phi, \partial^\nu\Phi^\dagger][\partial^\mu\Phi, \partial^\nu\Phi^\dagger]^\dagger. \tag{4}$$

The value of the dimensionless constant e depends on the details of the renormalizable theory of which Eq. (1) is the effective action. We will see that in the presence of this term, the effective action supports a topological soliton, whose mass is proportional to $M = fI/e$, with I given in Eq. (22).

The more familiar Skyrme model [11,12] is for the spontaneous breakdown of the symmetry $SU(2) \times SU(2)$ to $SU(2)$. The Nambu-Goldstone bosons are then the π mesons. The action of the Skyrme model is then

$$S = \frac{f_\pi^2}{2}\int d^4x\, \mathrm{tr}\partial_\mu g \partial^\mu g^\dagger + \frac{1}{32e^2}\int d^4x\, \mathrm{tr}[\partial^\mu g, \partial^\nu g^\dagger] \times [\partial^\mu g, \partial^\nu g^\dagger]^\dagger + \cdots. \tag{5}$$

Closer in spirit to these papers are the Hopf soliton [13] (the case of $\mathcal{M}_2$) and even more so, the model studied in [14], which is the case of $\mathcal{M}_3$.

II. TOPOLOGICAL CONSERVED CHARGE

A continuous function Φ, $R^3 \to \mathcal{M}_N$, that approaches a constant at infinity can also be thought of as a map Φ, $S^3 \to \mathcal{M}_N$, by identifying the points at infinity. The homotopy group $\pi_3(\mathcal{M}_N)$ has as elements the equivalence classes of such maps that can be deformed continuously into each other. It is well known that [15]

$$\pi_3(\mathcal{M}_2) = Z, \qquad \pi_3(\mathcal{M}_3) = Z_4, \qquad \pi_3(\mathcal{M}_N) = Z_2, \qquad N \geq 4. \tag{6}$$

The case $N = 3$ was studied in a different context some years ago [14]. We will focus here on the case $N \geq 4$, which includes the little Higgs models. There is just one nontrivial equivalence class of maps Φ: $R^3 \to \mathcal{M}_N$; we will need to determine which representative of this class has the least energy. The nontrivial element of $\pi_3(SO(N)/SO(N))$ is just the projection of the generator of $\pi_3(SU(N))$.

III. A SPHERICALLY SYMMETRIC ANSATZ

Recall Skyrme's spherically symmetric ("hedgehog") ansatz for a soliton of winding number 1:

$$g_2(x) = e^{i\boldsymbol{\sigma}\cdot\hat{\mathbf{x}}\omega(r)}, \qquad \omega(0) = -\pi, \qquad \omega(\infty) = 0. \tag{7}$$

The boundary conditions on ω ensure that the limits at $r = 0, \infty$ are direction independent:

$$g_2(\infty) = 1_2, \qquad g_2(0) = -1_2. \tag{8}$$

This ansatz is spherically symmetric in the sense that a rotation in space can be compensated by the adjoint action of $SU(2)$:

$$g_2(R(A)x) = Ag_2(x)A^\dagger, \tag{9}$$

where R, $SU(2) \to SO(3)$, is the usual homomorphism. The obvious topologically nontrivial map $g_2 g_2^T$ is not spherically symmetric: it is just cylindrically symmetric around the x_2 axis. This is because the representative A of the rotation matrix does not cancel (is not orthogonal) unless the rotation is around the x_2 axis. In fact, the energy minimizing configuration in $\mathcal{M}_2$ is only cylindrically symmetric [13]. If there is a spherically symmetric configuration, it is likely to have less energy.

There [14] is another spherically symmetric map g_3, $R^3 \to SU(3)$, which interpolates between the identity at infinity and a cube root of unity at the origin:

$$g_3(\infty) = 1_3, \qquad g_3(0) = e^{((2\pi i)/3)}1_3. \tag{10}$$

It is a generator or $\pi_3(SU(3))$. To construct it we start with the spherically symmetric ansatz

$$[g_3(x)]_{kl} = A(r)[\delta_{kl} - \hat{x}_k\hat{x}_l] + B(r)\epsilon_{kln}\hat{x}_n + C(r)\hat{x}_k\hat{x}_l, \qquad \hat{x}_i \equiv x_i/|x|, \tag{11}$$

with the constraints $|C| = 1$, $A^*B = B^*A$, $|A|^2 + |B|^2 = 1$ to be unitary and $C(A^2 + B^2) = 1$ to have determinant 1. Under the action $g_3(x) \to Rg_3(Rx)R^T$ this is spherically symmetric.

So we get

$$A(r) = e^{-(i/2)\chi(r)}\cos\alpha(r), \qquad C(r) = e^{i\chi(r)}, \qquad B(r) = e^{-(i/2)\chi(r)}\sin\alpha(r). \tag{12}$$

The boundary conditions

$$\chi(\infty) = 0, \qquad \chi(0) = 2\pi/3, \qquad \alpha(\infty) = 0, \qquad \alpha(0) = \pi \tag{13}$$

ensure that the winding number is 1. Computing $\Phi_3(x) = g_3(x)g_3^T(x)$,

$$[\Phi_3(x)]_{kl} = e^{-i\chi(r)}\delta_{kl} + [e^{2i\chi(r)} - e^{-i\chi(r)}]\hat{x}_k\hat{x}_l. \tag{14}$$

Finally, we can embed in $\mathcal{M}_N$ to get a spherically symmetric representative for the generator of $\pi_3(\mathcal{M}_N)$ for $N \geq 4$:

$$\Phi_N(x) = \begin{pmatrix} \Phi_3(x) & 0 \\ 0 & 1_{N-3} \end{pmatrix}. \tag{15}$$

For $\Phi_N(x) = g_N(x)g_N^T(x)$ with

$$g_N(x) = \begin{pmatrix} g_3(x) & 0 \\ 0 & 1_{N-3} \end{pmatrix} \tag{16}$$

and g_N: $R^3 \to SU(N)$ has winding number 1 by the above construction. The configuration is spherically symmetric under the action $\Phi(x) \mapsto R\Phi(Rx)R^T$.

IV. MINIMUM ENERGY SOLITON

The mass of the soliton in the theory with action $S_1 + S_2$ will be the minimum of the energy

$$H(\Phi) \equiv f^2 I_1(\Phi) + \frac{1}{e^2} I_2(\Phi), \tag{17}$$

where

$$I_1(\Phi) = \frac{1}{8}\int d^3x \, \mathrm{tr}\partial_i\Phi\partial_i\Phi^\dagger, \tag{18}$$

$$I_2(\Phi) = \frac{1}{32}\int d^3x \, \mathrm{tr}[\partial_i\Phi, \partial_j\Phi^\dagger][\partial_i\Phi, \partial_j\Phi^\dagger]^\dagger, \tag{19}$$

among all functions Φ: $R^3 \to \mathcal{M}_N$ equivalent to the nontrivial element of $\pi_3(\mathcal{M}_N)$. Since this topological charge is valued in Z_2, the topological soliton and its antiparticle are identical. In the absence of other interactions a single such soliton will be stable. Their number is not conserved—a pair of them can annihilate when they come in contact with each other.

As with skyrmions in QCD, it is clear that under a scaling $\Phi_\lambda(x) = \Phi(\lambda x)$, the two terms in the energy scale are opposite to each other:

$$I_1(\Phi_\lambda) = \lambda^{-1} I_1(\Phi), \qquad I_2(\Phi_\lambda) = \lambda I_2(\Phi). \tag{20}$$

Minimizing in the scale parameter, we see that the minimum energy will be proportional to f/e:

$$H_{\min} = (f/e)\sqrt{I_1(\Phi)I_2(\Phi)}. \tag{21}$$

We can make a variational estimate for the constant

$$I = \min_{\Phi}\sqrt{I_1(\Phi)I_2(\Phi)}. \tag{22}$$

The minimizing configuration should be invariant under the simultaneous rotation of the coordinate x and a rotation by some $SU(2)$ subgroup (analogous to isospin in the Skyrme model) of $SU(N)$.

It is not difficult to make an estimate for the soliton mass M. Substituting the spherically symmetric ansatz Eq. (15) in Eq. (17) and after some calculation we find

$$\begin{aligned} E(\chi) = {} & \pi\int f^2[3r^2\chi'^2 + 4(1-\cos 3\chi)]dr \\ & + \pi\int \frac{2}{e^2r^2}(1-\cos 3\chi)^2 dr \\ & + \pi\int \frac{2\chi'^2}{e^2}(3 - \cos 3\chi - 2\cos 6\chi)dr. \end{aligned} \tag{23}$$

We can find the minimum of energy in two ways: (i) by taking a variational ansatz for $\chi(r)$ or (ii) by solving $E(\chi)$ numerically. The variational ansatz gives an answer almost as good as the numerical solution.

We tried the following "stereographic" ansatz for $\chi(r)$

$$\chi(r) = \frac{4\pi}{3}\arctan\left(\frac{R^n}{r^n}\right) \tag{24}$$

for $n = 1, 2, 3$, and 4. They satisfy the boundary conditions given in Eq. (13). The lowest value for energy was obtained for $n = 2$ and $R = R_0 \approx \frac{1.13}{ef}$. The value of the minimum energy is $E(R_0) \equiv M = 105\frac{f}{e}$. The numerical solution of the differential equation for χ gives a slightly lower value of energy close to the variational ansatz:

$$E = 102.8\frac{f}{e}. \tag{25}$$

We plot the solution in Fig. 1 in units where $e = f = 1$; the dashed curve is the variational ansatz with $n = 2$ and the solid curve is the numerical solution.

We need an estimate for the value of the dimensionless parameter e as well as f to get a number for the mass of the

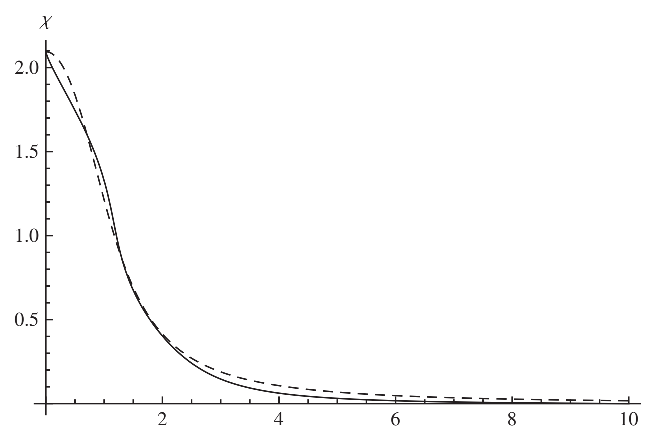

FIG. 1. The solution for $\chi(r)$ in units where $e = f = 1$.

soliton M. Precision electroweak constraints put a lower bound on the symmetry breaking scale f typically of about 500 GeV [16] for little Higgs models with T parity [7]. The Skyrme constant is in principle determined by the underlying renormalizable theory of which the little Higgs model is the effective theory. At the moment we do not know what this effective theory is; even if we knew it, we do not yet know how to compute such constants. But it is reasonable to expect that e will have the same order of magnitude as for QCD; this is the best we can do with our current knowledge. In QCD $e \approx 5$, as we can deduce from the value of the nucleon mass. Assuming that the size of the Skyrme term in the little Higgs model has the same size as that of QCD, we get an estimate for the mass of the soliton

$$M \gtrsim 10.3 \text{ TeV}. \tag{26}$$

Since the mass of this particle is below the unitarity bound ($\lesssim 340$ TeV) [8], it cannot be excluded from the list of viable dark matter candidates. Possible cosmological implications such as relic abundance, decay[1] and annihilation cross sections of these particles should be explored.

The coefficient N_c of the Wess-Zumino-Witten term (which is equal to the number of colors of QCD) determines whether the baryon is a boson or a fermion: for odd N_c it is a fermion and for even N_c it is a boson. We do not yet know if the analogous parameter in the little Higgs models is even or odd: both possibilities would give the same effective theory at the electroweak scale. When our skyrmion is a boson, it can be represented by a real scalar field S whose couplings have the discrete symmetry $S \rightarrow -S$. The conserved quantity associated with this symmetry is the topological charge of the little Higgs model. The phenomenological consequences of such scalars have been investigated in [17]. When our particle is a fermion, it is a Majorana particle. In this case it could be the fermion responsible for the neutrino masses in a seesaw mechanism [18]. To flesh out this idea, we need to understand the mixing matrix of the topological soliton with neutrinos, induced by the anomalous coupling of neutrinos of the bosons of the little Higgs models [19,20].

[1]These particles can be metastable with very long lifetimes since the global symmetry in the little Higgs models is approximate.

ACKNOWLEDGMENTS

A. J. thanks Jay Hubisz for useful discussions and the Department of Physics and Astronomy, University of Rochester, for their friendly hospitality. S. G. R. thanks Fred Cohen for discussions on homotopy groups, Mark Trodden on cosmology, and A. P. Balachandran on solitons. We also thank Marc Gillioz for pointing out the numerical error in Eq. (23). S. G. R.'s work is supported in part by a grant from the U.S. Department of Energy under Contract No. DE-FG02-91ER40685. A. J.'s work is supported in part by a U.S. Department of Energy grant under Contract No. DE-FG02-85ER40231.

Note added.—While this manuscript was in preparation a preprint with a significant overlap with the work presented here appeared in the arXiv [21].

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