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Highly predictive ansatz for leptonic mixing and *CP* violation

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We suggest a simple highly predictive ansatz for charged lepton and light neutrino mass matrices, based on the assumption of universality of Yukawa couplings. Using as input the charged lepton masses and light neutrino masses, the six parameters characterizing the leptonic mixing matrix $V_{\rm PMNS}$ are predicted in terms of a single phase ϕ, which takes a value around $\phi = \frac{\pi}{2}$. Correlations among various physical quantities are obtained; in particular $V_{13}^{\rm PMNS}$ is predicted as a function of Δm_{21}^2, Δm_{31}^2, and $\sin^2(\theta_{\rm sol})$, and restricted to the range $0.167 < |V_{13}^{\rm PMNS}| < 0.179$.

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I. INTRODUCTION

Understanding the pattern of fermion masses and flavor mixing is still one of the open fundamental questions in particle physics. The discovery of large leptonic mixing, in contrast to small quark mixing, has rendered the flavor puzzle even more intriguing.

In the standard model and in most of its extensions, the arbitrariness of fermion masses and mixing stems from the fact that the gauge invariance does not constrain the flavor structure of the Yukawa couplings. The fact that, in the standard model, only Yukawa couplings can be complex, has motivated the hypothesis of universality of strength of Yukawa couplings (USY) [1], which would all have the same strength, with flavor-dependent phases. The consequences of USY have been analyzed in various works, both for the quark [2] and lepton sectors [3]. Such an USY structure for the Yukawa couplings could arise from higher-dimensional theories [4]. It is worth recalling that when applied to the quark sector, the USY hypothesis can accommodate the main features of the Cabibbo-Kobayashi-Maskawa matrix, but cannot account for the observed strength of *CP* violation in the quark sector, measured by the rephasing invariant $|{\rm Im}[V_{ub}V_{cb}V_{ub}^*V_{cs}^*]|$. However, sufficient *CP* violation can be obtained [5] in extensions of the standard model where an USY structure is assumed, but where extra down singlet quarks are introduced and mix with the standard quarks.

In this paper, we suggest a highly predictive USY ansatz which is able to accommodate our present experimental knowledge on lepton masses and mixing and makes definite predictions, which can be tested in the near future. More specifically, in this USY ansatz, once the charged lepton and neutrino masses are fixed, the three leptonic mixing angles, the Dirac phases, and the two Majorana phases are all predicted in terms of only one free parameter. This highly constrained system implies interesting correlations among various physical quantities.

The size of $V_{13}^{\rm PMNS}$ is predicted as a function of $\tan(\theta_{\rm sol})$ and the neutrino mass differences Δm_{21}^2, Δm_{31}^2. For central values of $\sin^2(\theta_{\rm sol})$ and Δm_{ij}^2, one obtains $|V_{13}^{\rm PMNS}| = 0.178$, clearly at the reach of the next round of experiments [6]. The ansatz also predicts the strength of Dirac-type *CP* violation, measured by the invariant quartet $I_{CP} \equiv |{\rm Im}[V_{12}V_{23}V_{22}^*V_{13}^*]|^{\rm PMNS}$. For central values of $\sin^2(\theta_{\rm atm})$, $\sin^2(\theta_{\rm sol})$, and Δm_{ij}^2, one obtains $I_{CP} = 0.009\,06$, which can be measured in neutrino oscillation experiments [6].

This paper is organized as follows. In the next section, we describe the Ansatz and its parameter space, both in the charged lepton and neutrino sectors. In Sec. III, we evaluate the lepton mixing and derive some predictions of the ansatz for various physical quantities, including $|V_{13}^{\rm PMNS}|$, double beta decay, and the strength of the Dirac-type *CP* violation. Section IV contains some numerical results and figures illustrating correlations among various physical quantities. Finally, our conclusions are contained in Sec. V.

II. THE ANSATZ AND ITS PARAMETER SPACE

A. The charged lepton sector

We propose the following USY structure for the charged lepton mass matrix

$$M_l = \frac{c_l}{\sqrt{3}} \cdot K_\phi^\dagger \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{ia_l} & 1 \\ 1 & 1 & e^{ib_l} \end{bmatrix}; \qquad K_\phi = {\rm diag}(1, 1, e^{i\phi}). \tag{1}$$

The phase ϕ does not affect the charged lepton mass spectrum but contributes to the leptonic mixing. Using the trace, determinant, and second invariant of $H_l \equiv M_l M_l^\dagger$, one can derive exact expressions for the phases

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a_l, b_l, and the parameter c_l in terms of the masses:

$$\begin{aligned} c_l &= \frac{1}{\sqrt{3}}\sqrt{m_\tau^2 + m_\mu^2 + m_e^2} \\ 3\sin^2\left(\frac{a_l}{2}\right) &+ 3\sin^2\left(\frac{b_l}{2}\right) + \sin^2\left(\frac{a_l + b_l}{2}\right) \\ &= \frac{81}{4}\frac{m_\mu^2 m_\tau^2 + m_e^2 m_\tau^2 + m_e^2 m_\mu^2}{(m_\tau^2 + m_\mu^2 + m_e^2)^2} \\ \left|\sin\left(\frac{a_l}{2}\right)\sin\left(\frac{b_l}{2}\right)\right| &= \frac{27}{4}\frac{m_e m_\mu m_\tau}{\sqrt{(m_\tau^2 + m_\mu^2 + m_e^2)^3}}. \end{aligned} \tag{2}$$

From the charged lepton hierarchy, one obtains to an excellent approximation

$$|a_l| \simeq 6\frac{m_e}{m_\tau}, \qquad |b_l| \simeq \frac{9}{2}\frac{m_\mu}{m_\tau}. \tag{3}$$

Obviously, in Eq. (2) a_l and b_l enter in a symmetric way. The choice of Eq. (3) is required in order to obtain the right eigenvalue ordering.

B. The effective neutrino mass matrix

We assume that lepton number is violated at a high energy scale, leading at low energies to the following effective neutrino mass matrix:

$$M_\nu = \frac{c_\nu}{\sqrt{3}}\begin{bmatrix} e^{ia_\nu} & 1 & 1 \\ 1 & e^{-ia_\nu} & 1 \\ 1 & 1 & e^{ib_\nu} \end{bmatrix}. \tag{4}$$

The three parameters, c_ν, b_ν, and a_ν of the neutrino mass matrix ansatz in Eq. (4) are entirely determined by the three neutrino masses. We find from the trace, second invariant, and determinant of $H_\nu \equiv M_\nu M_\nu^\dagger$

$$\begin{gathered} 3c_\nu^2 = m_3^2 + m_2^2 + m_1^2 \qquad \cos(a_\nu) = 1 - \frac{27}{2}d_\nu \\ \cos(b_\nu) = \frac{1 + \frac{27}{2}d_\nu - \frac{81}{8}\chi_\nu}{1 - \frac{27}{4}d_\nu} \end{gathered} \tag{5}$$

where

$$\begin{aligned} d_\nu &= \frac{m_1 m_2 m_3}{(m_3^2 + m_2^2 + m_1^2)^{3/2}}; \\ \chi_\nu &= \frac{m_1^2 m_2^2 + m_1^2 m_3^2 + m_2^2 m_3^2}{(m_3^2 + m_2^2 + m_1^2)^2}. \end{aligned} \tag{6}$$

At this stage, it is worth emphasizing the predictive power of the ansatz. From Eqs. (2) and (5), it is clear that once the parameters (c_l, b_l, a_l), and (c_ν, b_ν, a_ν) are fixed by the charged lepton and neutrino masses, the six parameters of the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix, $V_{\rm PMNS}$, are completely determined in terms of a single parameter, the phase ϕ.

III. EVALUATION OF LEPTON MIXING

A. Diagonalization and parametrization of the lepton mass matrices

The diagonalization of the Hermitian charged lepton mass matrix $H_l \equiv M_l M_l^\dagger$ is carried out through

$$V_l^\dagger \cdot H_l \cdot V_l = \mathrm{diag}(m_e^2, m_\mu^2, m_\tau^2)$$

with the unitary matrix V_l given by

$$V_l = K_\phi^\dagger \cdot F \cdot W_l$$

where F

$$F = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix} \tag{7}$$

and W_l is a unitary matrix close to the identity. Given the strong hierarchy of the charged lepton masses, to an excellent approximation, one obtains for W_l

$$W_l \simeq \begin{pmatrix} 1 & \frac{m_e}{\sqrt{3}m_\mu} & -i\sqrt{\frac{2}{3}}\frac{m_e}{m_\tau} \\ -\frac{m_e}{\sqrt{3}m_\mu} & 1 - \frac{1}{2}(\frac{m_\mu}{m_\tau})^2 & i\frac{m_\mu}{\sqrt{2}m_\tau} \\ -i\sqrt{\frac{3}{2}}\frac{m_e}{m_\tau} & i\frac{m_\mu}{\sqrt{2}m_\tau} & 1 - \frac{1}{2}(\frac{m_\mu}{m_\tau})^2 \end{pmatrix}. \tag{8}$$

The diagonalization of the neutrino mass matrix is achieved through

$$V_\nu^\dagger \cdot M_\nu \cdot V_\nu^* = D_\nu = \mathrm{diag}(m_1, m_2, m_3) \tag{9}$$

where m_i denote the neutrino masses. In order to understand the main features of V_ν in the framework of our ansatz, it is useful to introduce a convenient parametrization. Let us now introduce the dimensionless parameters ε, δ defined by

$$\varepsilon = \frac{m_2}{\sqrt{m_3^2 + m_2^2 + m_1^2}}; \qquad \delta = \frac{m_1}{m_2}. \tag{10}$$

The neutrino masses can then be written

$$\begin{gathered} m_1 = \sqrt{3}c_\nu\varepsilon\delta \qquad m_2 = \sqrt{3}c_\nu\varepsilon \\ m_3 = \sqrt{3}c_\nu\sqrt{1 - \varepsilon^2 - \delta^2\varepsilon^2}. \end{gathered} \tag{11}$$

By substituting m_i as functions of ε, δ in Eqs. (5) and (6), we obtain d_ν, χ_ν as well as a_ν, b_ν as functions of ε and δ

$$\begin{aligned} d_\nu &= \delta\varepsilon^2\sqrt{1 - \varepsilon^2(1 + \delta^2)}; \\ \chi_\nu &= \varepsilon^2[1 + \delta^2 - \varepsilon^2(1 + \delta^2 + \delta^4)]. \end{aligned} \tag{12}$$

The matrix V_ν is then entirely given as a function of these two parameters (ε, δ), which are fixed by neutrino mass ratios. Furthermore, for our ansatz, V_ν is exactly factorizable in the following way:

$$V_\nu = F \cdot K_\gamma \cdot O_\nu \cdot K_M \tag{13}$$

where F was given in Eq. (7) and K_γ, K_M are diagonal unitary matrices containing phases, which will contribute to the Dirac and Majorana-type phases of the lepton mixing, $K_\gamma = \mathrm{diag}(1, e^{i\gamma}, -i)$ and $K_M = \mathrm{diag}(e^{i\hat{\alpha}_M}, e^{i\hat{\beta}_M}, e^{i\hat{\gamma}_M})$. As mentioned, all these phases and the angles of orthogonal matrix O_ν can be expressed as functions of δ and ε.

So far all our results are exact. Our numerical results for V_{PMNS} will be obtained through exact numerical diagonalization of H_l and H_ν. However, in order to get an overview of the physical implications of this USY ansatz, it is useful to derive some analytical expressions which hold to a good approximation. Let us parametrize O_ν in the following way:

$$O_\nu = O_{23} \cdot O_{13} \cdot O_{12} \tag{14}$$

with

$$O_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\hat{\theta}_{23}) & \sin(\hat{\theta}_{23}) \\ 0 & -\sin(\hat{\theta}_{23}) & \cos(\hat{\theta}_{23}) \end{bmatrix};$$

$$O_{13} = \begin{bmatrix} \cos(\hat{\theta}_{13}) & 0 & \sin(\hat{\theta}_{13}) \\ 0 & 1 & 0 \\ -\sin(\hat{\theta}_{13}) & 0 & \cos(\hat{\theta}_{13}) \end{bmatrix}$$

$$O_{12} = \begin{bmatrix} \cos(\hat{\theta}_{12}) & \sin(\hat{\theta}_{12}) & 0 \\ -\sin(\hat{\theta}_{12}) & \cos(\hat{\theta}_{12}) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

It turns out that in the relevant region of parameter space, ε is relatively small, $\varepsilon \approx 0.2$. Therefore, we make an expansion in powers of ε which yield

$$\begin{aligned} \tan(\hat{\theta}_{12}) &= -\sqrt{\delta}\left(1 + \frac{8\delta + 8\delta^2 - 3\delta^3 - 3}{4(1-\delta)}\varepsilon^2 + O(\varepsilon^4)\right) \\ \tan(\hat{\theta}_{23}) &= \varepsilon\frac{(1-\delta)}{\sqrt{2}}\left(1 + \frac{37\delta - 2\delta^2 - 2}{8}\varepsilon^2 + O(\varepsilon^4)\right) \\ \tan(\hat{\theta}_{13}) &= \varepsilon\sqrt{2\delta}\left(1 + \frac{4 + \delta + 4\delta^2}{8}\varepsilon^2 + O(\varepsilon^4)\right) \\ \tan(\gamma) &= -\varepsilon\frac{(10\delta - 3\delta^2 - 3)}{4(1-\delta)} + O(\varepsilon^3). \end{aligned} \tag{15}$$

The leptonic mixing matrix is given by

$$V_{\mathrm{PMNS}} = V_l^\dagger \cdot V_\nu = (W_l^\dagger F^T K_\phi) \cdot (F K_\gamma O_\nu K_M). \tag{16}$$

This formula is exact and it will be used in the numerical computation of V_{PMNS}. However, it is useful to obtain analytical approximate expressions for V_{PMNS}. Using Eqs. (16) and (14), and neglecting the small contribution from W_l given by Eq. (8), one obtains

$$\begin{aligned} \left|\frac{V_{12}^{\mathrm{PMNS}}}{V_{11}^{\mathrm{PMNS}}}\right| &\equiv |\tan(\theta_{\mathrm{sol}})| = |\tan(\hat{\theta}_{12})| \\ |V_{13}^{\mathrm{PMNS}}| &= |\sin(\hat{\theta}_{13})| \end{aligned} \tag{17}$$

which identifies these two lepton mixing angles in terms of our parametrization. Up to second order in ε, from Eq. (15), one obtains $\tan^2(\theta_{\mathrm{sol}})$ and $|V_{13}^{\mathrm{PMNS}}|$ expressed in terms of the measured Δm_{31}^2, Δm_{21}^2 and the lightest neutrino mass, m_1

$$\begin{aligned} \tan^2(\theta_{\mathrm{sol}}) &= \frac{m_1}{\sqrt{\Delta m_{21}^2 + m_1^2}} \\ |V_{13}^{\mathrm{PMNS}}|^2 &= \frac{2m_1\sqrt{\Delta m_{21}^2 + m_1^2}}{\Delta m_{31}^2 + \Delta m_{21}^2 + 3m_1^2}. \end{aligned} \tag{18}$$

Eliminating m_1 from Eq. (18), one obtains the interesting sum rule expressing $|V_{13}^{\mathrm{PMNS}}|$ in terms of measured quantities

$$|V_{13}^{\mathrm{PMNS}}| = \sqrt{2}|\tan(\theta_{\mathrm{sol}})|\sqrt{\frac{\Delta m_{21}^2}{\Delta m_{31}^2}}\frac{1}{\sqrt{1 - \tan^4(\theta_{\mathrm{sol}}) + (1 + 2\tan^4(\theta_{\mathrm{sol}}))\frac{\Delta m_{21}^2}{\Delta m_{31}^2}}}. \tag{19}$$

For central values of $\sin^2(\theta_{\mathrm{sol}})$ and Δm_{ij}^2 one finds

$$|V_{13}^{\mathrm{PMNS}}| = 0.178. \tag{20}$$

For θ_{atm} one obtains

$$\sin^2(\theta_{\mathrm{atm}}) = \tfrac{4}{9}\left[1 - \cos(\phi) + \tfrac{3}{2}\varepsilon(1-\delta)\sin(\phi) + O(\varepsilon^2)\right]. \tag{21}$$

It is clear that θ_{atm} crucially depends on ϕ, the phase defined in Eq. (1). It is interesting to note that a good fit of θ_{atm} is obtained for $\phi = \frac{\pi}{2}$.

B. Double beta decay

We evaluate now M_{ee}, which controls the strength of double beta decay and is given by

$$M_{ee} \equiv |m_1(V_{11}^{\mathrm{PMNS}})^2 + m_2(V_{12}^{\mathrm{PMNS}})^2 + m_3(V_{13}^{PMNS})^2|. \tag{22}$$

We compute M_{ee} in two steps. First, we evaluate the contribution to Majorana phases from $K_M = \mathrm{diag}(e^{i\hat{\alpha}_M}, e^{i\hat{\beta}_M}, e^{i\hat{\gamma}_M})$. This can be done by focusing only on the diagonalization of the neutrino mass matrix: $V_\nu^\dagger \cdot M_\nu \cdot V_\nu^* = \mathrm{diag}(m_1, m_2, m_3)$. It is clear that these phases appear when diagonalizing M_ν only with $FK_\gamma O_\nu$, without K_M

$$(FK_\gamma O_\nu)^\dagger \cdot M_\nu \cdot (FK_\gamma O_\nu)^* = \mathrm{diag}(m_1 e^{2i\hat{\alpha}_M}, m_2 e^{2i\hat{\beta}_M}, m_3 e^{2i\hat{\gamma}_M}). \tag{23}$$

In leading order, we find[1]

$$\begin{aligned} 2\hat{\alpha}_M &= -\frac{\pi}{2} - \frac{9 - 12\delta - \delta^2}{4(1-\delta)}\varepsilon \\ 2\hat{\beta}_M &= \frac{\pi}{2} + \frac{1 + 12\delta - 9\delta^2}{4(1-\delta)}\varepsilon \\ 2\hat{\gamma}_M &= \pi + \frac{(1-\delta)}{2}\varepsilon. \end{aligned} \tag{24}$$

We can then write

$$M_{ee} = |m_1 e^{2i\hat{\alpha}_M}(V_{11})^2 + m_2 e^{2i\hat{\beta}_M}(V_{12})^2 + m_3 e^{2i\hat{\gamma}_M}(V_{13})^2| \tag{25}$$

where here V is the lepton mixing matrix V_{PMNS} but without the last K_M phases, i.e. $V = V_{\mathrm{PMNS}} \cdot K_M^* = W_l^\dagger F^T K_\phi F K_\gamma O_\nu$.

Since, the matrix $F^T K_\phi F$ in V only gives a contribution in the 2–3 plane, and W_{12}^l and W_{13}^l are all of the order of ε^5 or smaller, we may read the expressions for V_{11}, V_{12}, and V_{13} directly from the leading order expressions for

$$\begin{aligned} \tan(\hat{\theta}_{12}) &= -\sqrt{\delta}\left(1 + \frac{8\delta + 8\delta^2 - 3\delta^3 - 3}{4(1-\delta)}\varepsilon^2\right) \\ \tan(\hat{\theta}_{13}) &= \varepsilon\sqrt{2\delta}\left(1 + \frac{4 + \delta + 4\delta^2}{8}\varepsilon^2\right) \\ \tan(\gamma) &= -\varepsilon\frac{(10\delta - 3\delta^2 - 3)}{4(1-\delta)}. \end{aligned} \tag{26}$$

Using Eqs. (24)–(26) together with m_1, m_2, and m_3 expressed in terms of c_ν, δ, and ε [as in Eq. (11)], we find the following leading order expression:

$$M_{ee} = \frac{9\sqrt{3}}{2}\delta\varepsilon^2\left(1 - \frac{(1+\delta^2)}{2}\varepsilon^2 - \frac{(1+\delta^2)^2}{8}\varepsilon^4\right)c_\nu. \tag{27}$$

C. Dirac-type *CP* violation

The strength of the Dirac-type *CP* violation is given by the imaginary part of any rephasing invariant quartet of V_{PMNS}, e.g.

$$I_{CP} = |\mathrm{Im}[V_{12}V_{23}V_{22}^*V_{13}^*]^{\mathrm{PMNS}}|. \tag{28}$$

Using Eqs. (1), (7), (8), and (14), we can evaluate I_{CP} in terms of ε, δ, and ϕ, obtaining in second order of ε

$$I_{CP} = \frac{2\delta\varepsilon}{9(1+\delta)} \times \left[1 - \cos(\phi) - \varepsilon\frac{3(10\delta - 3\delta^2 - 3)}{4(1-\delta)}\sin(\phi)\right]. \tag{29}$$

From Eq. (21), it is clear that the phase ϕ is strongly correlated with $\sin(\theta_{\mathrm{atm}})$. Then, for the central value of $\sin^2(\theta_{\mathrm{atm}})$, which is obtained with $\phi = \frac{\pi}{2}$, and central values of $\sin^2(\theta_{\mathrm{sol}})$ and Δm_{ij}^2, one gets $|I_{CP}| = 0.0105$, a value obtained neglecting the charged lepton contribution, which is small. Further on, in Sec. IV, we shall give an exact numerical example.

D. Majorana-type *CP* violation

It is well known that in the case of Majorana neutrinos, the basic rephasing invariants, in the leptonic sector, are bilinears of the type $V_{jk}^{\mathrm{PMNS}}V_{jl}^{\mathrm{PMNS}*}$ with $k \neq l$. In fact, in the case of three leptonic flavors, it has recently been shown that there are six rephasing invariant independent "Majorana-type" phases from which one can reconstruct the full V_{PMNS} matrix using 3×3 unitarity [7]. One can choose as basic Majorana phases

$$\begin{aligned} \gamma_1 &= \mathrm{Arg}[V_{11}(V_{13})^*] \qquad & \beta_1 &= \mathrm{Arg}[V_{12}(V_{13})^*] \\ \gamma_2 &= \mathrm{Arg}[V_{21}(V_{23})^*] \qquad & \beta_2 &= \mathrm{Arg}[V_{22}(V_{23})^*] \\ \gamma_3 &= \mathrm{Arg}[V_{31}(V_{33})^*] \qquad & \beta_3 &= \mathrm{Arg}[V_{32}(V_{33})^*]. \end{aligned} \tag{30}$$

where we have dropped the PMNS superscript in the V_{ij}'s. The γ_i, β_i can be evaluated in the present USY ansatz and we obtain in leading order,

$$\begin{aligned} \gamma_1 &= -\frac{3\pi}{4} - \frac{11 - 16\delta + \delta^2}{8(1-\delta)}\varepsilon \\ \beta_1 &= -\frac{\pi}{4} + \frac{16\delta - 11\delta^2 - 1}{8(1-\delta)}\varepsilon \\ \gamma_2 &= -\frac{\pi}{4} - \arctan\left(\frac{3\sin(\phi)}{1-\cos(\phi)}\right) \\ \beta_2 &= \frac{5\pi}{4} - \arctan\left(\frac{3\sin(\phi)}{1-\cos(\phi)}\right) \\ \gamma_3 &= \frac{3\pi}{4} - \arctan\left(\frac{3\sin(\phi)}{1-\cos(\phi)}\right) \\ \beta_3 &= \frac{\pi}{4} - \arctan\left(\frac{3\sin(\phi)}{1-\cos(\phi)}\right). \end{aligned} \tag{31}$$

IV. NUMERICAL RESULTS

The predictive power of our ansatz is best shown with a set of figures. Figure 1 demonstrates the dependence of the

[1]Obviously, the phases $\hat{\alpha}_M$, $\hat{\beta}_M$, $\hat{\gamma}_M$ are defined modulo π.

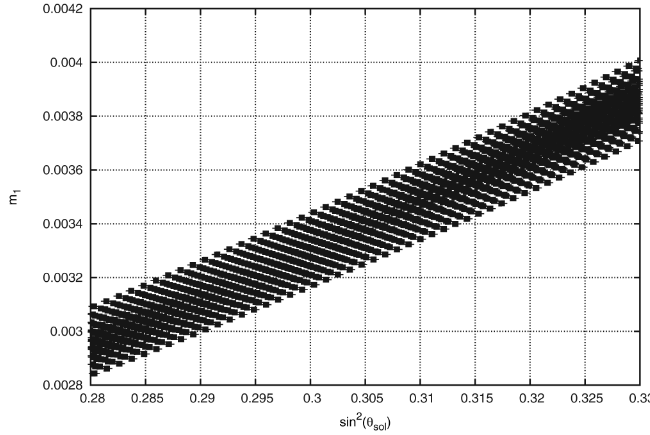

FIG. 1. m_1 as a function of $\sin^2\theta_{\mathrm{sol}}$, assuming 1σ uncertainties in neutrino mass differences.

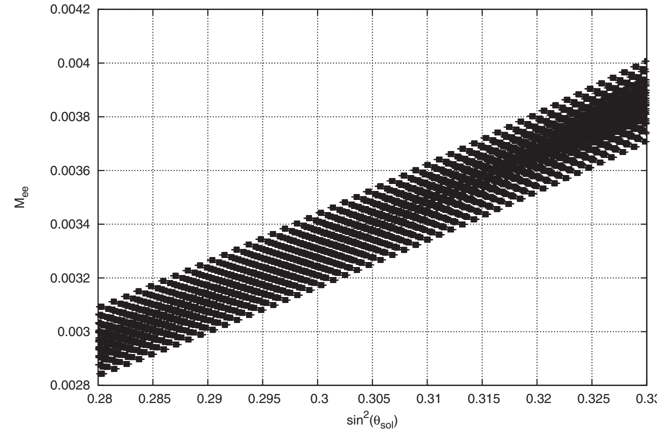

FIG. 3. M_{ee} as a function of $\sin^2\theta_{\mathrm{sol}}$, assuming 1σ uncertainties in neutrino mass differences.

solar mixing angle on the value of m_1. We allow for explicit experimental uncertainties of $\Delta m_{21}^2 = 7.65^{+23}_{-20} \times 10^{-5}\ \mathrm{eV}^2$ and $\Delta m_{31}^2 = 2.40^{+12}_{-11} \times 10^{-3}\ \mathrm{eV}^2$. It is clear, that a central value for $\sin^2(\theta_{\mathrm{sol}}) = 0.3$ implies a prediction for the value of m_1: $0.0316\ \mathrm{eV} < m_1 < 0.0345\ \mathrm{eV}$. From Fig. 2 and 3, it also follows that $0.167 < |V_{13}^{\mathrm{PMNS}}| < 0.179$ and that $0.003\,15 < M_{ee} < 0.003\,45$. Notice that, choosing the neutrino mass differences and $\sin^2(\theta_{\mathrm{sol}}) = 0.304^{+22}_{-16}$ within these 1σ experimental constraints, our model accommodates the upper limit for $|V_{13}^{\mathrm{PMNS}}|^2 < 0.004$. The mixing angle $\sin^2(\theta_{\mathrm{atm}})$ and the experimental observable measuring CP violation I_{CP} depend crucially on the angle ϕ and thus we may plot the two experimental observables against each other. From Fig. 4 we find, for a central value of $\sin^2(\theta_{\mathrm{sol}}) = 0.5$, that $0.0090 < I_{CP} < 0.0098$.

Next, we give an explicit numerical example, where six of the input parameters of the ansatz are fixed by the known charged lepton masses, two neutrino mass differences Δm_{21}^2, Δm_{31}^2, together with a chosen value for the lightest neutrino mass m_1. Then, the six parameters of V_{PMNS} are all predicted with a single free parameter, namely, the phase ϕ, which is taken to be $\phi = \frac{\pi}{2}$.

INPUT:

$$c_l = 1023.72\ \mathrm{eV} \qquad a_l = 1.729 \times 10^{-3} \qquad a_\nu = 0.66$$
$$b_l = 0.2677 \qquad b_\nu = 0.5077$$
$$c_\nu = 0.029\,035\,2\ \mathrm{eV} \qquad \phi = \frac{\pi}{2}$$

where, for this particular example we have $\delta = 0.4286$ and $\varepsilon = 0.1927$. We then find

OUTPUT

$$|V_{\mathrm{PMNS}}| = \begin{bmatrix} 0.815\,73 & 0.550\,15 & 0.178\,67 \\ 0.301\,73 & 0.662\,98 & 0.685\,14 \\ 0.493\,50 & 0.507\,73 & 0.706\,16 \end{bmatrix};$$

with

$$\sin^2(\theta_{\mathrm{sol}}) = 0.313; \qquad \sin^2(\theta_{\mathrm{atm}}) = 0.485;$$
$$|V_{13}^{\mathrm{PMNS}}|^2 = 0.0319$$

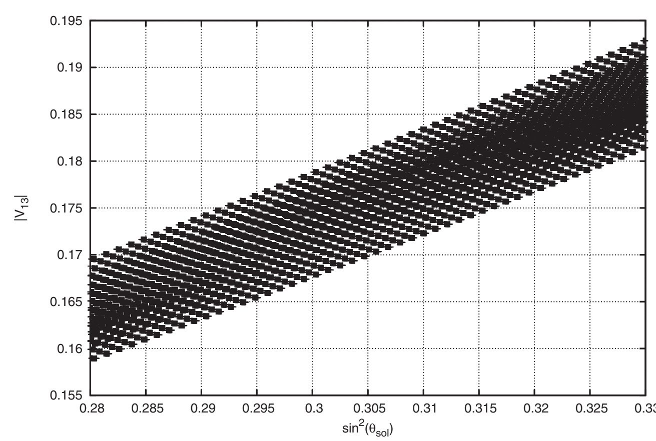

FIG. 2. $|V_{13}|$ as a function of $\sin^2\theta_{\mathrm{sol}}$, assuming 1σ uncertainties in neutrino mass differences.

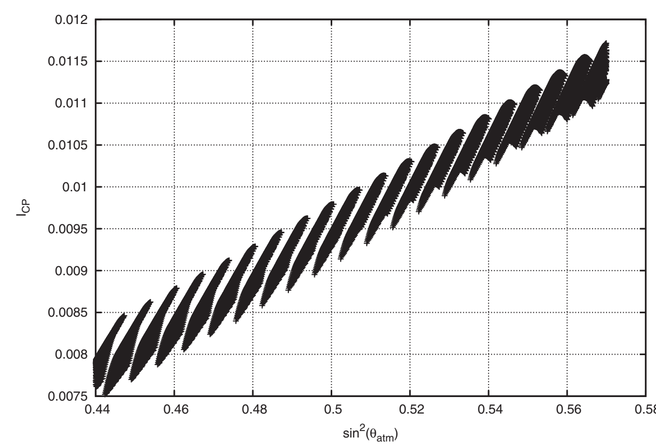

FIG. 4. I_{CP} as a function of $\sin^2\theta_{\mathrm{atm}}$, assuming 1σ uncertainties in neutrino mass differences.

and

$$\begin{aligned} m_e &= 0.51\ \text{MeV} & m_1 &= 4.15 \times 10^{-3}\ \text{eV} & \Delta m^2_{21} &= 7.664 \times 10^{-5}\ \text{eV}^2 \\ m_\mu &= 105.5\ \text{MeV} & m_2 &= 9.69 \times 10^{-3}\ \text{eV} & \Delta m^2_{31} &= 2.401 \times 10^{-3}\ \text{eV}^2 \\ m_\tau &= 1770\ \text{MeV} & m_3 &= 0.049\,17\ \text{eV}. \end{aligned}$$

We obtain for the Majorana observables

$$\text{Arg}\begin{pmatrix} V_{11}V^*_{13} & V_{12}V^*_{13} \\ V_{21}V^*_{23} & V_{22}V^*_{23} \\ V_{31}V^*_{33} & V_{32}V^*_{33} \end{pmatrix}^{\text{PMNS}} = \begin{pmatrix} -2.535 & -0.6145 \\ -2.230 & 2.731 \\ 0.7857 & -0.3545 \end{pmatrix}$$

and for the strength of the Dirac-type CP violation and double beta decay

$$M_{ee} = 3.53 \times 10^{-3}\ \text{eV}; \qquad I_{CP} = 0.009\,06.$$

V. CONCLUSIONS

We have pointed out that a simple ansatz, inspired by the hypothesis of universality of Yukawa couplings, leads to a highly predictive scheme for leptonic mixing. If one uses as input the charged lepton and neutrino masses, then the three mixing angles and the three CP violating phases entering in V_{PMNS} are all predicted in terms of a single phase which takes the value $\phi \approx \frac{\pi}{2}$. The ansatz predicts a relatively large value of $|V_{13}^{\text{PMNS}}|$ and of I_{CP}, clearly at the reach of the next round of experiments [6]. Furthermore, the ansatz predicts various testable correlations among physical quantities.

The USY ansatz has clearly a great appeal. A crucial open question is finding a symmetry principle, possibly implemented in a framework with extra dimensions [4], which can naturally lead to the universality of the strength of Yukawa couplings.

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