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Topologically modified teleparallelism, passing through the Nieh-Yan functional

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Recently, generalizations of the Ashtekar constraints are derived via the Nieh-Yan topological term. To be fair, such canonical transformations have been applied before in a gauge framework of gravity. Moreover, in the case of the teleparallelism equivalent of Einstein's theory, one can go further and show that the Chern-Simons solutions of the Gauss-type constraints wind around *torsional instantons*, thus establishing an analogue to the θ vacua of Yang-Mills theory.

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I. INTRODUCTION: PARITY-VIOLATING TOPOLOGICAL INVARIANTS IN GRAVITY

In the one-dimensional harmonic oscillator model [1], a *canonical transformation* can be induced by a boundary term derived from the *Chern-Simons* (CS) type term $\mathcal{C} = q^2/2$ as a *generating function*. After quantization, the corresponding operator $\mathcal{S} = \exp(-C) = \exp(-q^2/2)$ induces a well-known renormalization of the Schrödinger wave function. On the other hand, for diffeomorphism invariant topological field theories, Horowitz [2] has shown that $\Psi = N\exp(i\theta \int \underline{C})$ is, up to an overall factor, the unique solution of the Hamiltonian constraints.

Let us investigate, whether this carries over to gravity when the constraint of flat gauge connections is replaced by teleparallelism: In general, there exist two *parity-violating* [3] boundary terms which are exact forms built from three-forms:

$$C_{\rm TT} := \frac{1}{2\ell^2}\vartheta^\alpha \wedge T_\alpha = -\frac{(-1)^{\rm sig}}{2\ell^2}{}^*\mathcal{A}, \tag{1.1}$$

$$C_{\rm RR} := -\tfrac{1}{2}(\Gamma_\alpha{}^\beta \wedge R_\beta{}^\alpha + \tfrac{1}{3}\Gamma_\alpha{}^\beta \wedge \Gamma_\beta{}^\gamma \wedge \Gamma_\gamma{}^\alpha) \tag{1.2}$$

are *translational* and Lorentz-rotational CS terms, where $\mathcal{A} := {}^*(\vartheta_\alpha \wedge T^\alpha) = \mathcal{A}_i dx^i$ is the *axial torsion* one-form.

The starting point of Ashtekar's formulation of gravity with complex variables is the parity-violating boundary [4] four-form

$$dC_{\rm TT} = \frac{1}{2\ell^2}(T^\alpha \wedge T_\alpha + R_{\alpha\beta} \wedge \vartheta^\alpha \wedge \vartheta^\beta) \tag{1.3}$$

of Nieh and Yan (NY).[1] A fundamental length ℓ necessarily enters in order to keep all topological invariants [15] dimensionless, a point ignored in Ref. [16].

Whereas, the Pontrjagin term

$$dC_{\rm RR} = -\tfrac{1}{2}R_\alpha{}^\beta \wedge R_\beta{}^\alpha \tag{1.4}$$

is a topological Lagrangian whose variation returns the second Bianchi identity

$$DR_{\alpha\beta} \equiv 0, \tag{1.5}$$

the less known torsion identity (1.3) is based on the first Bianchi identity

$$DT^\alpha \equiv R_\beta{}^\alpha \wedge \vartheta^\beta \tag{1.6}$$

in Riemann-Cartan (RC) geometry.

Both are intimately interrelated within a gauge theory with the linear group $SL(5, R)$ as the structure group, containing the de Sitter groups $SO(1, 4)$ or $SO(2, 3)$ as subgroups: Already in 1995, we realized the Chern-Simons decomposition

$$\hat{C} = C_{\rm RR} - 2C_{\rm TT} \tag{1.7}$$

into linear and translational terms, after applying a Wigner-Inönü type contraction; see footnote 31 of Ref. [17], cf. [7,18]. In contrast to the metric-free Pontrjagin form (1.4), in the NY term (1.3) a metric $g_{\alpha\beta}$ is needed to rise and lower the indices, for instance in $T_\alpha = g_{\alpha\beta}T^\beta$.

II. TOPOLOGICALLY MODIFIED GRAVITY WITH TORSION

Let consider first the case of the usual Einstein-Cartan (EC) Lagrangian

$$L_{\rm EC} = -\frac{1}{2\kappa}R^{\alpha\beta} \wedge \eta_{\alpha\beta} = -\frac{1}{2\kappa}R_{\alpha\beta} \wedge {}^*(\vartheta^\alpha \wedge \vartheta^\beta) \tag{2.1}$$

amended by topological terms. Here $\kappa = 8\pi G$ denotes the gravitational coupling constant. Let us generalize (2.1) by including, besides the Pointrjagin term, also a dynamical coupling to the NY term and liberating possible θ angles to scalar fields.

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[1]As in many branches of physics, the historical progress in understanding is not always well monitored: The self-dual formulation of gravity was anticipated already in 1977 by Plebanski [5], whereas Hojman *et al.*, as well as Nelson [6] discussed the pseudoscalar curvature as a *parity-violating* Lagrangian for gravity and noted already in 1980 its relation to a complete divergence, before Nieh and Yan [4,7]. Then Dolan [8] as well as the author [9] employed the so-called NY term as a generating functional in gauge gravity. Several decades later, the role of pseudoscalar curvature was "rediscovered" by Holst [10] without references to earlier work and is persisting [11,12], although the NY term had already been instrumental in inducing chiral supergravity [13]. Gradually, a full turn to its topological origin can be testified; cf. [14].

Then, in our rather concise exterior form notation [17], we will consider the topologically modified gravitational Lagrangian

$$L := L_{\rm EC} + L_\theta + L_{\rm Dirac} \tag{2.2}$$

where the θ-type boundary term

$$L_\theta = \theta_{\rm T} dC_{\rm TT} + \theta_{\rm L} dC_{\rm RR} \tag{2.3}$$

is a linear superposition[2] of the topological Nieh-Yan term (1.3) and the Pontrjagin (1.4) four-forms. In order to recover parity or CP invariance [3], the θ angles need to be axionlike pseudoscalars [21].

In the translational field momentum

$$H_\alpha := -\frac{\partial L}{\partial T^\alpha} = -\frac{\theta_{\rm T}}{\ell^2} T_\alpha, \tag{2.4}$$

there is only one torsion term, whereas the rotational field momenta

$$H_{\alpha\beta} := -\frac{\partial L}{\partial R^{\alpha\beta}} = \frac{1}{2\kappa}\eta_{\alpha\beta} - \frac{\theta_{\rm T}}{2\ell^2}\vartheta_\alpha \wedge \vartheta_\beta - \theta_{\rm L} R_{\alpha\beta} \tag{2.5}$$

of the EC theory gets amended by two contributions from θ terms.

In gauge gravity, the two *nonlinear* gauge field equations [17] are

$$DH_\alpha - E_\alpha = \Sigma_\alpha, \tag{2.6}$$

$$DH_{\alpha\beta} + \vartheta_{[\alpha} \wedge H_{\beta]} = \tau_{\alpha\beta}, \tag{2.7}$$

where the three-forms of the *energy momentum* $E_\alpha = e_\alpha \rfloor L + (e_\alpha \rfloor T^\beta) \wedge H_\beta + (e_\alpha \rfloor R^{\beta\gamma}) \wedge H_{\beta\gamma}$ and of the *angular momentum current* $\vartheta_{[\beta} \wedge H_{\alpha]}$ arise due to the universality of gravitational interactions.

In the topologically modified gravity, the first gauge field equation (2.6) reduces to

$$D\theta_{\rm T} \wedge T_\alpha + \theta_{\rm T} DT_\alpha + \ell^2 E_\alpha = -\ell^2 \Sigma_\alpha. \tag{2.8}$$

Using the second Bianchi identity (1.5) for the curvature, the second gauge field equation (2.7) reduces to

$$\frac{1}{2\kappa} T^\gamma \wedge \eta_{\alpha\beta\gamma} - \frac{D\theta_{\rm T}}{2\ell^2} \wedge \vartheta_\alpha \wedge \vartheta_\beta + D\theta_{\rm L} \wedge R_{\alpha\beta} = \vartheta_{[\alpha} \wedge \mu_{\beta]} = \frac{1}{4}\vartheta_\alpha \wedge \vartheta_\beta \wedge {}^* j_5. \tag{2.9}$$

This a generalization of the Cartan equation

$$C_{\rm TT} \cong \frac{\kappa}{4\ell^2} j_5 \tag{2.10}$$

for constant θ's.

[2] The translational angle $\theta_{\rm T} = 2/\gamma$ is at times identified [19] with the inverse Barbero-Immirzi parameter γ. Such θ terms and the canonical transformation induced by the translational Chern-Simons term $dC_{\rm TT}$ have been considered earlier [20].

Note that the angular momentum part $\vartheta_{[\beta} \wedge H_{\alpha]}$ of the gauge fields induced by the NY term cancels identically against one part of $DH_{\alpha\beta}$. In the case of Dirac fields [22], the *spin-energy potential* μ_α, a two-form, is related to the axial current[3] three-form $j_5 = \bar\psi^* \gamma\gamma_5 \psi$ via

$$\mu_\alpha = \tfrac{1}{4}\vartheta_\alpha \wedge {}^* j_5. \tag{2.11}$$

At first sight, it appears that Eq. (2.9) for $D\theta_{\rm L} \neq 0$ provides torsion with a dynamical coupling to RC curvature, as stated in Ref. [23]. However, in view of the first Bianchi identity (1.6), this is not quite true: By contracting Eq. (2.9) with the coframe ϑ^α, it converts into

$$T^\gamma \wedge \eta_{\gamma\beta} + \kappa D\theta_{\rm L} \wedge DT_\beta = 0. \tag{2.12}$$

This is a first order equation only for torsion, even in the presence of Dirac fields, since the antisymmetric piece of its spin-energy potential vanishes, i.e., $\mu_\alpha \wedge \vartheta^\alpha = 0$, in view of (2.11).

Equivalently, it can be rewritten as

$$\kappa D\theta_{\rm L} \wedge DT_\beta = -T \wedge \eta_\beta = D\eta_\beta, \tag{2.13}$$

where the vector torsion one-form $T = e_\alpha \rfloor T^\alpha$ enters as an intermediate source. Because of the Poincaré lemma $DD\theta = dd\theta \equiv 0$ for a (pseudo-) scalar field, Eq. (2.13) has the exact torsion solution

$$\kappa d\theta_{\rm L} \wedge T_\beta = -\eta_\beta \tag{2.14}$$

as a first integral. After a contraction with ϑ^β, the dual of the axial torsion one-form $\mathcal{A} := {}^*(\vartheta^\alpha \wedge T_\alpha)$, i.e., the translational CS term, turns out to be related to the volume four-form η via

$$\kappa\ell^2 d\theta_{\rm L} \wedge C_{\rm TT} = 2\eta. \tag{2.15}$$

This topological result[4] is independent of the RC curvature. There occurs, however, a coupling to a kinetic term arising from the axion-type field $\theta_{\rm L}$ rescaling the Pontrjagin term (1.4).

III. GENERAL RELATIVITY AND ITS TELEPARALLELISM EQUIVALENT

Besides the familiar Hilbert-Einstein (HE) Lagrangian

$$L_{\rm HE} = -\frac{1}{2\kappa} R^{\{\}}_{\alpha\beta} \wedge {}^*(\vartheta^\alpha \wedge \vartheta^\beta) \tag{3.1}$$

[3] The one-form $\gamma := \gamma_\alpha \vartheta^\alpha$ is Clifford algebra valued.

[4] Information on other irreducible torsion components can be obtained from Eq. (2.8) or from Eq. (2.9) after covariant differentiation with the result that $DT^\gamma \wedge \eta_{\alpha\beta\gamma} + \kappa D\theta_{\rm T} \wedge T_{[\alpha} \wedge \vartheta_{\beta]}/\ell^2 = 2\kappa(T_{[\alpha} \wedge \mu_{\beta]} - \vartheta_{[\alpha} \wedge D\mu_{\beta]})$. Second order derivatives of the axionlike field drop out due to the Poincaré lemma $DD\theta = dd\theta \equiv 0$ and a quadratic torsion term vanishes identically, i.e., $T^{[\gamma} \wedge T^{\mu]}\eta_{\alpha\beta\gamma\mu} = 0$. Again we end up with a first order equation for torsion, where, however, the spin-energy potential μ_α remains as a source.

of general relativity (GR), where $R^{\{\}}_{\alpha\beta}$ denotes the Riemannian curvature with respect to the Levi-Civita connection $\Gamma^{\{\}}_{\alpha\beta}$, the NY term, after a duality rotation, suggests another option for a viable gravitational Lagrangian: The torsion-square Lagrangian

$$L_{\|} := -\frac{1}{2\kappa} T^\alpha \wedge {}^*\left({}^{(1)}T_\alpha - 2{}^{(2)}T_\alpha - \frac{1}{2}{}^{(3)}T_\alpha\right), \tag{3.2}$$

involving a specific combination of irreducible torsion components. Here $H^{\|}_\alpha := -\partial L_{\|}/\partial T^\alpha = (1/\kappa)\eta_{\alpha\beta\gamma}K^{\beta\gamma}$ is dual to the contortion one-form $K_{\alpha\beta}$ which features in the decomposition $\Gamma_{\alpha\beta} = -\Gamma_{\beta\alpha} = \Gamma^{\{\}}_{\alpha\beta} - K_{\alpha\beta} = \Gamma^{\{\}}_{\alpha\beta} + e_\alpha \rfloor T_\beta + (e_\alpha \rfloor e_\beta \rfloor T_\gamma) \wedge \vartheta^\gamma$ of the RC connection.

Because of the geometric identity

$$L_{\|} \equiv L_{\rm HE} + \frac{1}{2\kappa} R_{\alpha\beta} \wedge {}^*(\vartheta^\alpha \wedge \vartheta^\beta) + \frac{2\ell^2}{\kappa} dC_{\rm TT^*}, \tag{3.3}$$

where $C_{\rm TT^*} := \vartheta^\alpha \wedge {}^*T_\alpha/2\ell^2$ is a dual CS term, proper *teleparallelism* ($\mathrm{GR}_{\|}$) specified by (3.2) is classically *equivalent* to GR up to a boundary term, when constrained by the vanishing of RC curvature, i.e.,

$$R_{\alpha\beta} = 0. \tag{3.4}$$

The NY term is again instrumental [20] for converting the teleparallel version (3.2) of Einstein's GR for the choice $\theta_{\rm T} = \pm i$ into a *chiral gauge theory of translations*:

$$L^{(\pm)}_{\|} := L_{\|} \pm i\frac{2\ell^2}{\kappa} dC_{\rm TT} = L_{\rm HE} - L_{\rm EC} \pm i\frac{2\ell^2}{\kappa} dC^{(\mp)}_{\rm TT}. \tag{3.5}$$

The deviation of chiral $\mathrm{GR}_{\|}$ from the Hilbert-Einstein action turns out to be a boundary term derived from the *chiral* CS term

$$C^{(\pm)}_{\rm TT} := \frac{1}{2\ell^2}\left(\vartheta^\alpha \wedge T^{(\pm)}{}_\alpha\right), \tag{3.6}$$

where $T^{(\pm)\alpha} := \frac{1}{2}(T^\alpha \pm i^*T^\alpha)$ denotes the self- or anti-self-dual torsion. The resulting complex field momenta $\Pi^{(\pm)}_\alpha = -\partial L^{(\pm)}/\partial T^\alpha = H^{\|}_\alpha \mp (i/\kappa)T_\alpha$ satisfy an algebraic identity [24] such that the complexified Lagrangian (3.5) becomes *quadratic* in these new field momenta

$$L^{(\pm)}_{\|} = \mp\frac{i}{4}\kappa\Pi^{(\pm)\alpha} \wedge \Pi^{(\pm)}_\alpha. \tag{3.7}$$

Then the gravitational field equations

$$D^{(\pm)}\Pi^{(\pm)}_\alpha = 0 \tag{3.8}$$

are formally those of Yang-Mills for the chiral *translational* gauge field momenta $\Pi^{(\pm)}_\alpha$ "living" on a *nondynamical RC background* fixed by the teleparallelism constraint (3.4).

IV. CHERN-SIMONS SOLUTIONS OF THE CHIRAL TELEPARALLELISM CONSTRAINTS

In the canonical analysis of forms [20], the tangential part of the basis one-forms, i.e., the *"triad densities"* $\underline{{}^*\vartheta}_\alpha$ and the tangential part of the self- or antiself dual connection $\underline{\mathcal{A}}^{(\pm)}_\alpha := \underline{{}^*\Pi}^{(\pm)}_\alpha$, the three dual[5] of the chiral momenta $\Pi^{(\pm)}_\alpha$, become the generalized coordinates q and momenta $p^\pm$ of the bosonic sector, similar to Eq. (15a) of Ref. [16].

In the transition to quantum gravity, in contrast to GR [25], for $\mathrm{GR}_{\|}$ the *Schrödinger representation*

$$q\colon \underline{{}^*\vartheta}^\beta \Psi_{\|}(\vartheta) = \underline{{}^*\vartheta}^\beta \Psi_{\|}(\vartheta), \tag{4.1}$$

$$p^\pm\colon \underline{{}^*\Pi}^{(\pm)}_\alpha \Psi_{\|}(\vartheta) = -i\ell^2 \frac{\delta}{\delta \underline{{}^*\vartheta}_\alpha}\Psi_{\|}(\vartheta), \tag{4.2}$$

convert [24] the complex field "momenta" $\underline{{}^*\Pi}^{(\pm)}_\alpha$ into differential operators, whereas the triad densities $\underline{{}^*\vartheta}^\beta$ remain generalized coordinates q, as in Ref. [16].

In the time gauge $\underline{\vartheta}^{\hat{0}} = 0$, most of the canonical constraints are satisfied automatically due to the teleparallel condition (3.4) of vanishing RC curvature. The remaining operator form of the Gauss constraint, i.e., the tangential version of (3.8), can be solved [24] by the state vector

$$\Psi_{\|}(\vartheta) = \exp\left(-\int \underline{C}^{(\pm)}_{\rm TT}\right) = \exp\left(\frac{-1}{2\ell^2}\int\left[\underline{{}^*\vartheta}_B \wedge \underline{{}^*T}^{(\pm)B}\right]\right), \tag{4.3}$$

where the integration is understood over a spacelike hypersurface. When the tangential complexified *translational* Chern-Simons term (3.6) is rewritten in terms of the triad densities $\underline{{}^*\vartheta}^B$ with $B = 1, 2, 3$ and the tangential part of the self- or anti-self-dual torsion, the chiral version of Eq. (16) of Ref. [16] for "large" gauge transformations is anticipated [24]. Then the momentum operator (4.2) returns the chiral torsion $\underline{T}^{(\pm)B}$ as a factor, which is then annihilated by the Bianchi identity (1.6) truncated to $\underline{D}^{(\pm)}\underline{T}^{(\pm)B} = 0$ in teleparallel space. Similarly, as in topological field theory [2] with a flat connection, we suspect that (4.3) is the unique solution in teleparallelism. Consequently, Wilson type solutions (4.3) of the corresponding quantum Gauss constraint are dominated by *self-dual* torsion solutions satisfying $\underline{T}^{(\pm)} = 0$.

Exact *torsion instantons* "live" on a conformally compactified Euclidean space $R^4 \cup \infty = S^4$ with the spherically symmetric metric $ds^2 = h^2dr^2 + f^2[d\psi^2 + \sin^2\psi(d\theta^2 + \sin^2\theta d\phi^2)]$, where f carries physical dimension (length). This space is *parallelizable* such that the RC curvature is vanishing but is endowed [26] with the nontrivial spacelike torsion

[5] In our differential form notation [17], the spatial Hodge dual is involutive, i.e., $= +\underline{*}\underline{*}1$.

$$T^A = \frac{1}{f}(df \wedge \vartheta^A - 2\eta^{0A\beta\gamma}\vartheta_\beta \wedge \vartheta_\gamma) = \pm {}^*T^A, \qquad T^{\hat{0}} = 0, \tag{4.4}$$

which is self- or anti-self-dual provided that $df = \pm 2hdr$. Working in the zero-connection gauge $\underline{\Gamma}_{\|}^{\alpha\beta} =^* 0$, the *translational* CS term reduces to

$$\ell^2 \underline{C}_{\rm TT} \stackrel{*}{=} \underline{\vartheta}^\alpha \wedge d\underline{\vartheta}_\alpha = \underline{\vartheta}^A \wedge d\underline{\vartheta}_A = 3!\underline{\vartheta}^{\hat{1}} \wedge \underline{\vartheta}^{\hat{2}} \wedge \underline{\vartheta}^{\hat{3}}. \tag{4.5}$$

Applying Stokes's theorem and integrating over the boundary three-sphere at radial infinity $r \to \infty$ yields

$$n_{\rm NY} := \int_{R^4} dC_{\rm TT} = \int_{S^3_\infty} \underline{C}_{\rm TT} = 3{\rm Vol}(S^3)k = 6\pi^2 k. \tag{4.6}$$

One can deduce [15] that k is winding or instanton number of Pontrjagin, in compliance with the CS decomposition (1.7). If torsion is self- or anti-self-dual, i.e., $\underline{T}^{(\mp)B} = 0$, the integration over the chiral CS term $\underline{C}_{\rm TT}^{(\pm)}$ yields the same value or zero, respectively. Interestingly, in the gauge $\underline{\vartheta}^{\hat{0}} = 3\kappa d\theta_{\rm L} = hdr = \pm df/2$, such instantons are solutions to the topological Eq. (2.15), due to $T^{\hat{0}} = 0$.

V. CONCLUSIONS

After reviewing the group-theoretical decent of the two parity-violating topological terms of Pontrjagin and NY, the modifications of the gravitational gauge equations by such θ terms are analyzed. Then the topological amendment (2.3) provides an intriguing relation (2.15) for axial torsion $\mathcal{A}$, independent of RC curvature. This result has repercussions on teleparallelism constrained by (3.4), where the path-integral type CS solution (4.3) of the quantum constraints are dominated by torsion instantons.

In classical EC theory, the net axial current production dj_5 seems [11,16,27] to establish a link to the NY term (1.3) via the Cartan relation (2.10). However, a careful analysis of the *axial and trace anomaly* [28,29] in gravity does not support this, but rather provides a relation to the *scale-invariant* Pontrjagin term, including a $U(1)$ type four-form $d\mathcal{A} \wedge d\mathcal{A}$ involving the axial torsion. Since torsion instantons are characterized via (4.6) by the instanton number k, ultimately, they would induce a periodic θ vacuum of quantum gravity, similarly as in Yang-Mills theory; cf. Ref. [21].

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