PHYSICAL REVIEW D **80,** 065007 (2009)

Inverse seesaw mechanism in noncommutative geometry

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(Received 5 May 2009; published 8 September 2009)

In this paper we will implement the inverse seesaw mechanism into the noncommutative framework on the basis of the AC extension of the standard model. The main difference from the classical AC model is the chiral nature of the AC fermions with respect to a $U(1)_X$ extension of the standard model gauge group. It is this extension which allows us to couple the right-handed neutrinos via a gauge invariant mass term to left-handed A particles. The natural scale of these gauge invariant masses is of the order of 10^{17} GeV while the Dirac masses of the neutrino and the AC particles are generated dynamically and are therefore much smaller (~ 1 to $\sim 10^6$ GeV). From this configuration, a working inverse seesaw mechanism for the neutrinos is obtained.

DOI: 10.1103/PhysRevD.80.065007 PACS numbers: 14.60.Pq, 11.10.Nx

I. INTRODUCTION

We present an extension of the standard model in its noncommutative formulation [1], which implements the inverse seesaw mechanism [2]. As with previous extensions of the standard model within the noncommutative framework [3–6], this model is based on the classification of finite spectral triples [7–12]. It is a variant of the AC model found in [3], which contains the standard model fermions as well as a two new species of particles, the A and the C particles. In this paper we work with a spectral triple where the KO dimension of the internal part is taken to be six [13,14]. In the original AC model [3], the new particles were electrically charged with twice the electron charge and turned out to be viable candidates for dark matter [15]. Here, we will assume them to posses chiral charges of a new $U(1)_X$ gauge group.

An open problem in noncommutative geometry is the realization of the mass mechanism for neutrinos. The spectral action implies not only the standard model action and the Einstein-Hilbert action but also a set of conditions imposed on the couplings of the bosonic and fermionic fields. One finds [14,16] that the condition imposed on the Yukawa couplings demands that the top quark coupling is accompanied by at least a second Yukawa coupling of the order of 1. This fact strongly suggests a seesawlike mechanism where the additional large Yukawa coupling is taken to be one of the neutrino couplings.

While in KO-dimension zero, the neutrino masses are of a Dirac type [17–19]; KO-dimension six also allows for Majorana masses [13,14] and the seesaw mechanism. The price to be paid for the Majorana mass is a violation of one of the axioms of noncommutative geometry, namely, the axiom of orientability [20]. This problem can be overcome by introducing a second layer for the internal algebra in the finite part of the spectral triple [1,14], or by a modification of the spectral action [21].

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Nevertheless, a numerical analysis of the standard model with seesaw mechanism [14,22,23] shows that at least one of the gauge invariant seesaw masses is of the order of $\sim 10^{14}$ GeV. One would expect the elements of the Majorana mass matrix to be of the order of $\sim 10^{17}$ GeV, the cutoff scale of the spectral action. The inverse seesaw shifts this discrepancy to an intermediate energy scale associated with the vacuum expectation value of a new scalar field. Furthermore, the present version of the AC model is completely compatible with the axioms of noncommutative geometry [1] and can, in principle, be employed in models based on spectral triples with KO-dimension six or zero (although a specific model for KO-dimension zero does not yet exist).

The model presented here has as a gauge group $G = U(1)_Y \times SU(2) \times SU(3) \times U(1)_X$, where the standard model subgroup $G_{\mathrm{SM}} = U(1)_Y \times SU(2) \times SU(3)$ is broken by the usual Higgs mechanism to $U(1)_{\mathrm{em}} \times SU(3)$ and the new subgroup $U(1)_X$ is broken to $\mathbb{Z}_2$ by a decoupled Higgs mechanism associated with a new scalar field. It is therefore the second extension of the standard model within the noncommutative framework after [6] which has an enlarged scalar sector. This new scalar field generates the masses in the AC sector. Previous attempts to extend the standard model within the framework of noncommutative geometry proved to be extremely difficult. Most of the early attempts, unfortunately, failed to produce physically interesting models [24].

It would, of course, also be desirable to gain a deeper understanding of the origin of the internal space, i.e. the source of the matrix algebra. There are hints that a connection to loop quantum gravity exists [25]. Also double Fell bundles seem to be a plausible structure in noncommutative geometry [26]. They could provide a deep connection to category theory and give better insights into the mathematical structure of almost-commutative geometries such as the standard model.

This paper is organized as follows: In Sec. II we give the construction of the internal space based on a minimal

Krajewski diagram. We calculate the lift of the gauge group and the fluctuated Dirac operator. This fluctuation leads to the standard model Higgs and a new scalar field. We then calculate the relevant parts of the spectral action, which provides the potential for the Higgs and the scalar field, the new parts in the fermionic Lagrangian as well as constraints on the quadratic couplings and the quartic couplings of the scalar fields, the Yukawa couplings of the fermions, and the gauge couplings of the non-Abelian subgroup of the gauge group.

In Sec. III we analyze the mass matrix for the neutrino sector coupled to the AC sector and calculate the mass eigenvalues for the light mass eigenstates and the heavy mass eigenstates. We then estimate the scale of the vacuum expectation value of the new scalar field and also give a few estimations of the range of masses in the AC sector.

II. THE MODEL

The model proposed in this paper is a variant of the AC model [3] but with a different charge assignment for the $U(1)$ subgroups of the gauge group. The internal Hilbert space, which encodes the multiplet structure of the gauge group, consists of the usual chiral standard model multiplets [i.e. six quarks as $SU(2)$ doublets/singlets and $SU(3)$ triplets and six leptons as $SU(2)$ doublets/singlets] plus A and C particles being $SU(2)$ and $SU(3)$ singlets.

Internal spaces of almost-commutative geometries are conveniently encoded in Krajewski diagrams [27]. The Krajewski diagram for this model is depicted in Fig. 1.

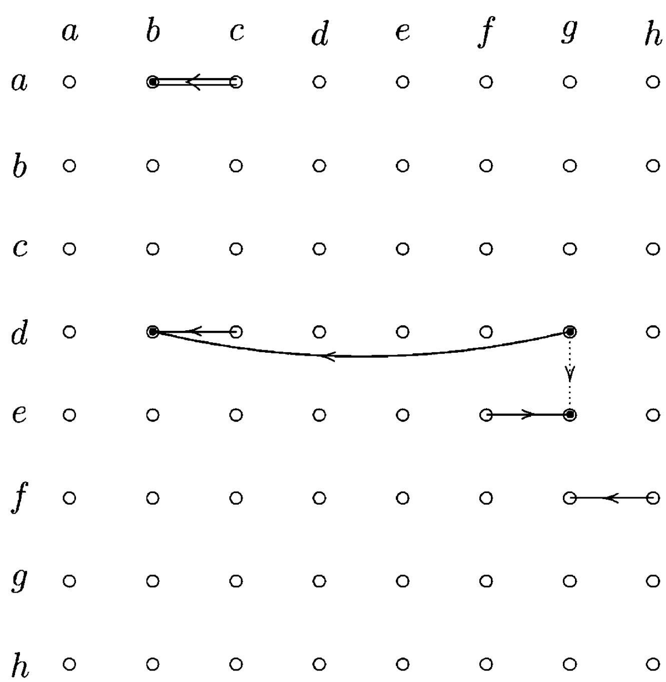

FIG. 1. Krajewski diagram of the extended standard model. The dotted line indicates the gauge invariant mass term connecting the right-handed neutrino to the left-handed A particle.

Note that the allowed mass term connecting right-handed C particles to right-handed A antiparticles does not appear explicitly since we have left out the antiparticles to keep the diagram simple. This Krajewski diagram is based on a minimal diagram that can be obtained by deleting the arrow for the right-handed neutrino.

As matrix algebra for the internal space, we choose $\mathcal{A} = \mathbb{C} \oplus M_2(\mathbb{C}) \oplus M_3(\mathbb{C}) \oplus \mathbb{C} \oplus \mathbb{C} \oplus \mathbb{C} \oplus \mathbb{C} \oplus \mathbb{C}$. This is exactly the AC-model algebra [3] in KO-dimension six. From the Krajewski diagram, we read off the representation for $\mathcal{A} \ni (a, b, c, d, e, f, g, h)$:

$$\rho_L = \begin{pmatrix} b \otimes 1_3 & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & g & 0 \\ 0 & 0 & 0 & \bar{g} \end{pmatrix},$$
$$\rho_R = \begin{pmatrix} c \otimes 1_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{c} \otimes 1_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{c} & 0 & 0 & 0 \\ 0 & 0 & 0 & g & 0 & 0 \\ 0 & 0 & 0 & 0 & e & 0 \\ 0 & 0 & 0 & 0 & 0 & h \end{pmatrix},$$
$$\rho_L^c = \begin{pmatrix} 1_2 \otimes a & 0 & 0 & 0 \\ 0 & d 1_2 & 0 & 0 \\ 0 & 0 & e & 0 \\ 0 & 0 & 0 & f \end{pmatrix}, \tag{1}$$
$$\rho_R^c = \begin{pmatrix} a & 0 & 0 & 0 & 0 & 0 \\ 0 & a & 0 & 0 & 0 & 0 \\ 0 & 0 & d & 0 & 0 & 0 \\ 0 & 0 & 0 & d & 0 & 0 \\ 0 & 0 & 0 & 0 & e & 0 \\ & 0 & 0 & 0 & 0 & f \end{pmatrix}.$$

The internal part $\mathcal{D}$ of the Dirac operator can be decomposed as follows:

$$\mathcal{D} = \begin{pmatrix} \Delta & T \\ \bar{T} & \bar{\Delta} \end{pmatrix}, \quad \text{with } \Delta = \begin{pmatrix} 0 & \mathcal{M} \\ \mathcal{M}^* & 0 \end{pmatrix}, \tag{2}$$

where the submatrix $\mathcal{M}$ is given by

$$\mathcal{M} = \begin{pmatrix} (M_u, M_d) \otimes 1_3 & 0 & 0 & 0 \\ 0 & (M_e, M_\nu) & 0 & 0 \\ 0 & (0, M_{\nu A}) & M_A & 0 \\ 0 & 0 & 0 & M_C \end{pmatrix}. \tag{3}$$

Here, (M_u, M_d) is the mass matrix of the quarks, (M_e, M_ν) is the mass matrix of the leptons, and M_A (M_C) is the mass matrix of the A particles (C particles). The gauge invariant mass matrix connecting the right-handed neutrinos to the left-handed A particles is $(0, M_{\nu A})$. The submatrix T is

$$T = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{AC} \\ 0 & 0 & M_{AC}^t & 0 \end{pmatrix}, \tag{4}$$

with M_{AC} a Majorana-type mass matrix connecting right-handed C particles to right-handed A antiparticles. It is not a gauge invariant mass term and will be associated with the new scalar field. We assume that the A and C particles come, as with the standard model particles, in three generations, so $M_{\nu A}, M_A, M_C, M_{AC} \in M_3(\mathbb{C})$.

The non-Abelian subgroup of unitarities of the matrix algebra $\mathcal{A}$ is $\mathcal{U}^{\mathrm{nc}} = U(2) \times U(3)$. It contains two $U(1)$ subgroups via the determinant that may be lifted by central extensions to the fermionic Hilbert space [28]. We will call these two subgroups $\det(U(2)) = U(1)_Y$ and $\det(U(3)) = U(1)_X$. The first one is nothing but the standard model hypercharge subgroup, and the second one is associated with the AC particles. The AC particles are neutral with respect to the standard model gauge group; i.e. the AC particles are $SU(2) \times SU(3)$ singlets and have zero hypercharge. On the other hand, the standard model particles are neutral with respect to the $U(1)_X$. It follows that the gauge group of our model is $G = U(1)_Y \times SU(2) \times SU(3) \times U(1)_X$.

An anomaly-free lift of $\mathcal{U}^{\mathrm{nc}}$ to the Hilbert space is achieved by the following central charge assignment, normalized to unity of A and C for the $U(1)_X$ subgroup:

	A_L	A_R	C_L	C_R
Q_X	0	1	1	0

It is remarkable that the representation (1) allows for a charge assignment which produces a new particle sector sterile to the standard model gauge group yet chiral under the new $U(1)_X$ subgroup while at the same time allowing for the gauge invariant mass term $M_{\nu A}$. The anomaly-free lift L decomposes into the usual standard model lift L_{SM}, which can be found in [28], and the lift L_X, acting on the AC particles. This can be written as

$$L(\det(u), \det(v), u, v) = L_{\mathrm{SM}}(\det(v), u, v) \oplus L_X(\det(u)), \tag{5}$$

where $u \in U(2)$, $v \in U(3)$. For the new part of the lift L_X, we find

$$L_X(\det(u)) = \mathrm{diag}(1, \det(u)^1, \det(u)^1, 1; 1, \det(u)^{-1}, \det(u)^{-1}, 1). \tag{6}$$

The semicolon divides the particles from the antiparticles, and the $U(1)_X$ charges of A and C have been used.

Next, we need to fluctuate the Dirac operator [1] to obtain the gauge bosons as well as the Higgs field ϕ and the new scalar field φ. We define the fluctuated Dirac operator ${}^f\mathcal{D}$ according to [7]

$${}^f\mathcal{D} = \sum_i r_i L(\det(u_i), \det(v_i), u_i, v_i)\mathcal{D}L(\det(u_i), \det(v_i), u_i, v_i)^{-1}, \qquad r_i \in \mathbb{R}. \tag{7}$$

One obtains the standard Higgs doublet ϕ, embedded into a quaternion and a new complex scalar field φ, because the lift does not commute with the Dirac operator $\mathcal{D}$. The only part of the mass matrix $\mathcal{M}$ of $\mathcal{D}$ commuting with the lift is $M_{\nu A}$, which is therefore a gauge invariant mass. We find for the fluctuated mass matrices

$${}^f\mathcal{M} = \begin{pmatrix} \phi(M_u, M_d) \otimes 1_3 & 0 & 0 & 0 \\ 0 & \phi(M_e, M_\nu) & 0 & 0 \\ 0 & (0, M_{\nu A}) & \varphi M_A & 0 \\ 0 & 0 & 0 & \varphi M_C \end{pmatrix} \tag{8}$$

and

$${}^fT = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{\varphi} M_{AC} \\ 0 & 0 & \bar{\varphi} M_{AC}^t & 0 \end{pmatrix}, \tag{9}$$

with $\varphi = \sum_i r_i \det(u_i)^{-1}$. The new scalar field φ is also neutral with respect to the standard model gauge group and has $U(1)_X$-charge $Q_X = -1$. From these mass matrices, we can calculate the spectral action which will give us the kinetic term of the scalars as well as the potential for the Higgs field and the new scalar field.

According to [1], the spectral action S_{CC} is given by the number of eigenvalues of the Dirac operator D up to a cutoff energy Λ. $D = \not{\partial} \otimes 1_{\dim \mathcal{H}_f} + \gamma^5 \otimes \mathcal{D}$ is the Dirac operator of the full almost-commutative geometry. The spectral action can be written approximately with help of a positive cutoff function f and then be calculated asymptotically via a heat-kernel expansion:

$$S_{CC} = \mathrm{tr}\left(f\left(\frac{D^2}{\Lambda^2}\right)\right) = \frac{1}{16\pi^2}\int dV(a_4 f_4 \Lambda^4 + a_2 f_2 \Lambda^2 + a_0 f_0 + o(\Lambda^{-2})). \tag{10}$$

Here, f_i are the first moments of the cutoff function f. They enter as free parameters into the model. The heat-kernel coefficients a_i are well known [29], and for the present calculation only a_2 and a_0 will be of concern. Note that we use the numerating convention of [14], where the number of the coefficient a_i corresponds to the power of Λ.

The coefficient a_2 will give us the mass terms of the potential for the scalar fields while a_0 will provide for the kinetic terms for the scalar fields, the quartic couplings of the potential, and, also, mass terms. All the following relations hold at the cutoff energy Λ and are not stable under the renormalization group flow.

To calculate the relevant parts of a_2 and a_0 we need the traces of ${}^f\mathcal{D}^2$ and ${}^f\mathcal{D}^4$. These calculations are very similar to those presented in detail in [6]. Therefore we will only present the final results.

One observes that the scalar fields so far have mass dimension zero. We have to normalize the scalar fields $\phi(M_u, M_d) \to \tilde{\phi}(\mathcal{Y}, \mathcal{Y})$ and $\varphi M \to \tilde{\varphi}\mathcal{Y}$ to obtain the standard kinetic terms of the Lagrangian. Here, $(\mathcal{Y}, \mathcal{Y})$ and $\mathcal{Y}$

are the Yukawa coupling matrices of the quarks/leptons and the AC particles. It is very convenient to immediately drop the $\tilde{\phi}/\tilde{\varphi}$ notation for the normalized scalar fields and return to ϕ/φ since only the normalized fields appear from now on.

For the real scalar fields ϕ_i and φ_j, we use the standard normalization

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \quad \text{and} \quad \varphi = \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2). \quad (11)$$

All the standard model fields acquire their well-known standard model Lagrangian; for details, see [14]. For example, the Higgs field Lagrangian is

$$\mathcal{L}_{\mathrm{Higgs}} = (D_\mu \phi)^*(D^\mu \phi) + \mu_1^2 |\phi|^2 - \frac{\lambda_1}{6}|\phi|^4. \quad (12)$$

For the new scalar φ, we find the following standard Lagrangian:

$$\mathcal{L}_{\mathrm{scalar}} = (D_\mu \varphi)^*(D^\mu \varphi) + \mu_2^2 |\varphi|^2 - \frac{\lambda_2}{6}|\varphi|^4. \quad (13)$$

The potential for φ will have a nontrivial minimum, just as the Higgs potential. It follows that the $U(1)_X$ subgroup is broken dynamically. Notice that there is no term proportional to $|\phi|^2|\varphi|^2$ mixing the standard model Higgs ϕ with the new scalar φ as it was the case in the model presented in [6]. We also get the standard Lagrangian $\mathcal{L}_X = -1/4 F_{X,\mu\nu}F_X^{\mu\nu}$ for the new subgroup $U(1)_X$ with coupling g_4 as well a conformal coupling of the new scalar φ to the curvature scalar R. These terms will not concern us in the following. The symmetry breaking pattern of the gauge group G is

$$U(1)_Y \times SU(2) \times SU(3) \times U(1)_X \to U(1)_{\mathrm{ew}} \times SU(3). \quad (14)$$

The fermionic action $(\psi, {}^f\mathcal{D}\psi)$, where $(\cdot,\cdot)$ denotes the scalar product on the Hilbert space, provides for the mass terms of the model. Apart from the Yukawa terms of the standard model, we have the following terms in the Lagrangian:

$$\mathcal{L}_{\mathrm{Yukawa}} = (\nu_R, M_{\nu A} A_L) + (A_L, \varphi \mathcal{Y}_A A_R) + (C_L, \varphi \mathcal{Y}_C C_R) + (\bar{A}_R, \bar{\varphi}\mathcal{Y}_{AC} C_R) + \mathrm{c.c.} \quad (15)$$

It will be the gauge invariant mass term $(\nu_R, M_{\nu A} A_L)$, connecting the right-handed neutrinos to the left-handed A particles, which is responsible for the inverse seesaw mechanism. The vacuum expectation value of the scalar field φ is a free parameter of the model. Later, we will estimate the vacuum expectation value of φ by using the requirement that the entries of $M_{\nu A}$ should be of the order of the cutoff Λ in the spectral action.

For all the parameters in the Lagrangian, the spectral action provides a set of constraints [14,16]. Let us first regard the constraints on the dimensionful and dimensionless couplings of the Higgs and the new scalar field. For the quadratic couplings, we find

$$\mu_1^2 = 2\frac{f_2}{f_0}\Lambda^2 - 2\frac{\mathrm{tr}(\mathcal{Y}_\nu^* \mathcal{Y}_\nu M_{\nu A}^* M_{\nu A})}{Y_2} \quad \text{and}$$
$$\mu_2^2 = 2\frac{f_2}{f_0}\Lambda^2 - \frac{\mathrm{tr}(\mathcal{Y}_A^* \mathcal{Y}_A M_{\nu A}^* M_{\nu A})}{\tilde{Y}_2}, \quad (16)$$

where as usual $Y_2 = \mathrm{tr}(\mathcal{Y}_u^*\mathcal{Y}_u + \mathcal{Y}_d^*\mathcal{Y}_d + \mathcal{Y}_e^*\mathcal{Y}_e + \mathcal{Y}_\nu^*\mathcal{Y}_\nu)$ is the trace of the standard model Yukawa matrices squared and $\tilde{Y}_2 = \mathrm{tr}(\mathcal{Y}_A^*\mathcal{Y}_A + \mathcal{Y}_C^*\mathcal{Y}_C + \mathcal{Y}_{AC}^*\mathcal{Y}_{AC})$ is its analogue for the AC sector.

Since $M_{\nu A}$ is gauge invariant, the entries of the matrix should be of the order of the cutoff scale, so $(M_{\nu A})_{ij} \sim \Lambda$. We observe that $\mathrm{tr}(\mathcal{Y}_\nu^*\mathcal{Y}_\nu M_{\nu A}^* M_{\nu A}) \sim \mathrm{tr}(\mathcal{Y}_A^*\mathcal{Y}_A M_{\nu A}^* M_{\nu A}) \sim \Lambda^2$ allows us to decouple the vacuum expectation values of ϕ and φ from the cutoff scale Λ. This means especially that the W mass is decoupled from the cutoff scale.

For the dimensionless quartic couplings, we find

$$\lambda_1 = 24\frac{\pi^2}{f_0}\frac{H}{Y_2^2}, \qquad \lambda_2 = 24\frac{\pi^2}{f_0}\frac{\tilde{H}}{\tilde{Y}_2^2}, \quad (17)$$

with $H = \mathrm{tr}((\mathcal{Y}_u^*\mathcal{Y}_u)^2 + (\mathcal{Y}_d^*\mathcal{Y}_d)^2 + (\mathcal{Y}_e^*\mathcal{Y}_e)^2 + (\mathcal{Y}_\nu^*\mathcal{Y}_\nu)^2)$ and $\tilde{H} = \mathrm{tr}((\mathcal{Y}_A^*\mathcal{Y}_A)^2 + (\mathcal{Y}_C^*\mathcal{Y}_C)^2 + (\mathcal{Y}_{AC}^*\mathcal{Y}_{AC})^2)$.

The constraints for the Yukawa couplings are $Y_2 = \tilde{Y}_2 = 4\pi^2/f_0$, which, together with $g_3^2 = g_2^2 = \pi^2/f_0$, gives the final set of constraints for the dimensionless couplings at the cutoff Λ:

$$g_2^2 = g_3^2 = \frac{\lambda_1}{24}\frac{Y_2^2}{H} = \frac{\lambda_2}{24}\frac{\tilde{Y}_2^2}{\tilde{H}} = \frac{Y_2}{4} = \frac{\tilde{Y}_2}{4}, \quad (18)$$

where g_2 is the $SU(2)$ coupling and g_3 is the $SU(3)$ coupling.

From these relations, it is now possible to deduce the cutoff scale Λ using renormalization group techniques [14]. From the constraint $g_2 = g_3$ follows $\Lambda = 1.1 \times 10^{17}$ GeV. With $\lambda_1 = 24 g_2^2 H/Y_2^2$ at Λ, a Higgs mass of the order of 170 GeV follows [14]. The latest data from the Tevatron [30] strongly suggest that this value of the standard model Higgs mass is excluded. But to obtain the above value only, one-loop renormalization group equations have been used, and considering the enormous energy range from $\Lambda \sim 10^{17}$ to $m_Z \sim 100$ GeV, more refined methods from perturbation theory may change the value of the Higgs mass sufficiently. Furthermore, the effects of the new particles of the model have not been taken into account, although they will only change the results through contributions in higher loop orders. The value of the Higgs mass may also change considerably for different models beyond the standard model [4–6]. With the same procedure we could also calculate the low energy value of λ_2. But since only the overall scale of the φ vacuum expectation values matters here, we will postpone these investigations to a later publication.

III. THE INVERSE SEESAW MECHANISM

The relevant mass terms for the inverse seesaw mechanism are the Dirac masses m_ν of the neutrinos, the Dirac masses m_A of the A particles, and the gauge invariant mass term $M_{\nu A}$ connecting the right-handed neutrinos to the left-handed A particles. To simplify the model a bit, we will assume that the Dirac masses of the C particles, as well as the mass terms connecting A particles and C antiparticles, are much smaller than the A particle masses.

It follows from the constraints (18) that the neutrino Yukawa couplings should be of the order of 1, so the Dirac masses are $m_\nu \sim 100$ GeV. We assume that the same holds for the Yukawa couplings of the A particles; therefore their masses should be of the order of the vacuum expectation value of the scalar field φ, which is a free parameter.

From the Lagrangian $\mathcal{L}_{\text{Yukawa}}$ (15), we can deduce the following mass matrix for the neutrinos and the A particles:

$$M = \begin{pmatrix} 0 & m_\nu & 0 & 0 \\ m_\nu & 0 & M_{\nu A} & 0 \\ 0 & M_{\nu A} & 0 & m_A \\ 0 & 0 & m_A & 0 \end{pmatrix}. \tag{19}$$

This type of matrix is well known [2]. To transparently calculate the eigenvalues of M, we will take m_ν, $M_{\nu A}$, and m_A to be the Dirac masses of the first standard model + AC family.

As was shown in [2], the eigenvalues of M are given by

$$m_{1/2}^2 = m_\nu^2 \frac{m_A^2}{M_{\nu A}^2} \quad \text{and} \quad m_{3/4}^2 = M_{\nu A}^2. \tag{20}$$

One sees immediately that one obtains two light mass eigenstates and two heavy mass eigenstates $M_{\nu A} \sim \Lambda = 1.1 \times 10^{17}$ GeV.

Assuming that the light mass eigenstates correspond to the light neutrino, which we assume to be of the order of 1 eV, we can deduce that the approximate mass scale for the vacuum expectation value of the new scalar field φ should be of the order of 10^6 GeV.

Speculating further, we may assume that the variation of the masses in the AC sector is similar to the standard model, i.e.

$$\frac{m_C}{m_A} \sim \frac{m_e}{m_{\text{top}}} \sim 10^{-6}, \tag{21}$$

where m_C is a generic C-particle mass, m_e is the electron mass, and m_{top} is the top quark mass. This would imply that the mass of the lightest C particle could be as low as $m_C \sim m_A \times 10^{-6} \sim 1$ GeV. These particles can easily escape detection since they do not couple to the standard model on a tree level. Only couplings in higher loops to the right-handed neutrino are possible, which can be seen from the Lagrangian (15). And these couplings are mediated by the $(\nu_R, M_{\nu A} A_L)$ part of the Lagrangian which is strongly suppressed by the assumptions that the eigenvalues of $M_{\nu A}$ are of the order of Λ. An interesting question for further investigation regards the stability of the lightest new particle, i.e. one of the C particles, and its ability to play the role of dark matter. The mass of the X boson would be $m_X \sim \sqrt{g_4} 10^6$ GeV, where the $U(1)_X$ gauge coupling g_4 is essentially a free parameter.

IV. CONCLUSION AND OUTLOOK

We have shown in this publication that the noncommutative framework [1] allows us to successfully implement the inverse seesaw mechanism [2] on the basis of the AC extension of the standard model [3]. The main difference to the classical AC model is the chiral nature of the AC fermions with respect to a $U(1)_X$ extension of the standard model gauge group. It is this extension which allows us to couple the right-handed neutrinos via a gauge invariant mass term to left-handed A particles. Since the natural scale of these gauge invariant masses is of the order of the cutoff scale of the spectral action ($\sim 10^{17}$ GeV), while the Dirac masses of the neutrino and the AC particles are generated dynamically and therefore much smaller (~ 1 to $\sim 10^6$ GeV), a working inverse seesaw mechanism is obtained. In the usual realization of the seesaw mechanism, at least one of the Majorana mass terms has to be several orders of magnitude smaller than the cutoff scale ($M_M \sim 10^{14}$ GeV, [22,23]).

Compared to the usual seesaw mechanism [1,13], there are several differences, some of which could turn out to be advantageous:

(i) The spectral triple on which the model is based fulfils all axioms of noncommutative geometry [1].
(ii) There is no principle obstacle to realize this or a similar mechanism in KO-dimension zero.
(iii) The AC particles could produce stable or sufficiently long-lived particles that could serve as dark matter.
(iv) Since the A particle mixes with the standard model neutrino, its Dirac mass matrix as well as the mass matrix connecting the two particles introduce the new CP violating phases. This may be interesting for leptogenesis.

An inconvenience of the model is the Higgs mass of the order of 170 GeV. This mass range is now practically excluded by Tevatron [30]. It will be interesting to see if the inverse seesaw mechanism can be implemented into other models beyond the standard model within the framework of noncommutative geometry. These models usually provide stronger constraints on the coupling constants [3–6] and may therefore be very predictive.

ACKNOWLEDGMENTS

The author gratefully acknowledges the funding of his work by the Deutsche Forschungsgemeinschaft.

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