Seeking the loop quantum gravity Barbero-Immirzi parameter and field in 4D, $\mathcal{N} = 1$ supergravity

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We embed the loop quantum gravity Barbero-Immirzi parameter and field within an action describing 4D, $\mathcal{N} = 1$ supergravity and thus within a low-energy effective action of superstring/M theory. We use the fully gauge-covariant description of supergravity in (curved) superspace. The gravitational constant is replaced with the vacuum expectation value of a scalar field, which in local supersymmetry is promoted to a complex, covariantly chiral scalar superfield. The imaginary part of this superfield couples to a supersymmetric Holst term. The Holst term also serves as a starting point in the loop quantum gravity action. This suggest the possibility of a relation between loop quantum gravity and supersymmetric string theory, where the Barbero-Immirzi parameter and field of the former play the role of the supersymmetric axion in the latter. Adding matter fermions in loop quantum gravity may require the extension of the Holst action through the Nieh-Yan topological invariant, while in pure, matter-free supergravity their supersymmetric extensions are the same. We show that, when the Barbero-Immirzi parameter is promoted to a field in the context of 4D supergravity, it is equivalent to adding a dynamical complex chiral (dilaton-axion) superfield with a nontrivial kinetic term (or Kähler potential), coupled to supergravity.

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I. INTRODUCTION

Currently, there are mainly two mathematical constructs which are often presented in adversarial positions to one another for the role of providing a road to a completely consistent quantum theory of gravity. One of these, possessing a larger community of supporters among theoretical physicists, is superstring/M theory (SSMT), and the other, with a corresponding smaller community of supporters, is loop quantum gravity (LQG). Theoretical physics is not a democracy as nature possesses the final and only vote to serve as the arbiter on the value of any theoretical construct. From this standpoint, the number of supporters is irrelevant when deciding which model better describes nature. It is not the purpose of this work to debate whether one or the other of these proposals has been most successfully argued. Relevant to this work, there have appeared both arguments against the combination of these approaches [1,2] as well as attempts to carry out such a combination [3–6]. To our knowledge, however, there is no completely rigorous exclusion principle that forbids the ultimate theory of quantum gravity to be the result of a combination of these approaches (and perhaps others), which we believe makes it worthwhile to investigate possible regions of overlap.

LQG searches for the unification of general relativity (GR) and quantum mechanics by postulating that spacetime itself is discrete [7–9]. This model is formulated in terms of so-called *connection variables*, two versions of which currently exist: the Ashtekar connection [self-dual $SL(2, C)$] [10] and the Ashtekar-Barbero connection [real $SU(2)$] [11]. Both formalisms can be derived from the Holst action which modifies the Einstein-Hilbert action through the addition of a surface term that consists of the product of a parameter γ [known as the Barbero-Immirzi (BI) parameter] times the fully contracted dual Riemann curvature tensor [12].

The BI parameter is a constant related to the minimum eigenvalue of the discrete area and discrete volume operators [13] and whose value is determined by the entropy-area relation when studying black hole thermodynamics. Regardless of the numerical choice of this parameter, it does not affect the classical field equations since it appears solely through the addition of a surface term to the Einstein-Hilbert action.

Recently, however, the possibility of promoting this parameter to a scalar field (''scalarization'') was proposed [14]. Replacing the BI parameter by the BI field has the important consequence that the modification of the Einstein-Hilbert action is *no* longer by the addition of a pure surface term. Instead, the field equations are now modified at the classical level, since integration by parts of the Holst term leads to corrections to the field equations that depend on derivatives of the BI field. One can easily show that these corrections reduce to GR in the presence of a scalar field stress-energy tensor [14] upon field redefinition. Shortly after its introduction, the canonical formula-

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tion of the quantum theory with a BI field was studied [15], followed by its relation to the Peccei-Quinn mechanism [16] and a possible topological interpretation [17].

In the presence of spacetime torsion (e.g., as induced by fermions), however, the Holst extension must be further augmented by the addition of another term to avoid classical modifications to the field equations. Such an augmentation consists of adding a torsion squared term to the action, which thus transforms the Holst term to the so-called Nieh-Yan invariant [18]. When the BI parameter is promoted to a scalar field in the Nieh-Yan invariant, one finds generically that the Pontryagin topological density arises naturally in LQG [19]. The effective action of LQG with a BI field is then equivalent to dynamical Chern-Simons modified gravity (DCSMG) [20–22], which interestingly has already been connected to heterotic string theory [23].

Though the possibility of any direct link between SSMT and LQG may be deemed remote by some, we believe it is of interest to probe for any such interconnection possessing the property of supersymmetry. Supergravity appears to be the natural crossing because, on the one hand, supergravity is the least *ad hoc* way to include fermions into any field theory with gravity, while, on the other hand, supergravity arises as the low-energy effective action of superstrings. In this paper, we shall attempt to make the possibility of interconnections among LQG, DCSMG, and SSMT more clear and explicit, in the context of 4D, $\mathcal{N} = 1$ (pure) supergravity. Our choice of the four-dimensional (4D), $\mathcal{N} = 1$ supergravity theory is for the sake of simplicity.

Let us first recall the basic (standard) motivation for supergravity. Supergravity is the theory of local supersymmetry, and the latter is a symmetry that converts bosons into fermions and vice versa. Supersymmetry seems well motivated in particle physics for unification of bosons and fermions, as well as for the consistency of SSMT. From the viewpoint of quantum gravity, supersymmetry is the tool for its consistent coupling to matter as well as the improvement of its ultraviolet (UV) behavior (e.g., UV divergences). Supergravity is the only known consistent route to couple spin-3/2 particles (gravitinos) to gravity in field theory. In addition, supergravity (with matter) naturally arises as the low-energy effective action of SSMT. Such considerations therefore suggest that supergravity might also be unavoidable in LQG, if one desires to couple LQG to matter and fermions, in particular. As regards discretizing supersymmetry on a lattice, see, e.g., [24–26].

The above relations are schematically portrayed in Fig. 1. LQG and modified Holst or modified Nieh-Yan gravity are connected through the promotion of the BI parameter into a scalar field. In turn, we here show that modified Holst gravity also generically arises in SSMT, where the BI field is related to the string theory axion (for real values of the BI parameter). In the presence of fermions, modified Nieh-Yan gravity can also be mapped to DCSMG. In turn, the latter has already been shown to be generically unavoidable in string theory [23]. In this way, we reinforce the possibility of interconnection between the seemingly disparate LQG and SSMT.

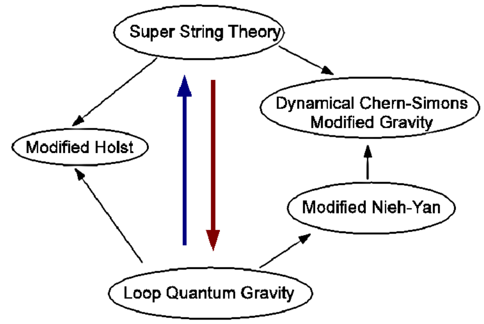

FIG. 1 (color online). Schematic diagram of the relations drawn in this paper between SSMT and LQG.

A deep comparison between LQG and SSMT requires not only the embedding of Holst or Holst-like actions in supergravity as discussed in this paper but also a mapping of the connection-triad variables at the classical level and the background independent, non-Fock quantization procedure at the quantum level to some SSMT equivalent. In all irreducible off-shell supergravity theories known, the connection variables do not appear in any superspace formulation. Thus, any attempt to reconcile LQG and SG theories begins with an intrinsic disagreement over the fundamental variables of the theory. Relinquishing any of the requirements of irreducibility, off-shell structure or a superspace formulation, however, will open the way to the introduction of connection variables. Such issues are relegated to future work.

The remainder of this paper is divided as follows: Sec. II discusses the Holst action in LQG and its coupling of fermions with the Nieh-Yan invariant; Sec. III describes the basics of 4D, $\mathcal{N} = 1$ supergravity in superspace; Sec. IV presents the relation between the BI parameter and 4D, $\mathcal{N} = 1$ supergravity; Sec. V shows how the BI field is related to superstring theory; Sec. VI discusses the Nieh-Yan invariant in supergravity; Sec. VII concludes and points to future research.

We shall employ the following conventions in this paper: lowercase *middle* Greek letters $\mu, \nu, \ldots = 0, 1, 2, 3$ are used for curved spacetime vector indices; the lowercase *middle* Latin letters $m, n, \ldots = 0, 1, 2, 3$ are used for flat (target) space vector indices; the lowercase *early* Greek letters $\alpha, \beta, \ldots = 1, 2$ are used for chiral spinor indices of one chirality, whereas the lowercase *early* Greek letters *with dots* $\dot{\alpha}, \dot{\beta}, \ldots = \dot{1}, \dot{2}$ are used for chiral spinor indices of the opposite chirality. Symmetrization and antisymmetrization on the indices is denoted via $A_{(ab)} := (A_{ab} + A_{ba})/2$ and $A_{[ab]} := (A_{ab} - A_{ba})/2$, respectively. We also use natural units where $\hbar = c = 1$, and the gravitational coupling constant is introduced via $\kappa^2 = 8\pi G_{\rm N}$.

II. LOOP QUANTUM GRAVITY

A. The Holst action

The Holst action with the BI field is defined as the sum of three terms: the Einstein-Hilbert term, a term involving the fully contracted dual Riemann tensor, and terms involving coupling to matter. Its explicit form can be written as follows:

$$S_{\rm Holst} = \frac{1}{4\kappa^2}\int \epsilon_{mnpq}e^m \wedge e^n \wedge F^{pq} + \frac{1}{2\kappa^2}\int \bar{\gamma}e^m \wedge e^n \wedge F_{mn} + S_{\rm mat}, \qquad (2.1)$$

where ϵ_{mnpq} is the Levi-Civita tensor, e is the determinant of the tetrad (vierbein) e^m and e_m is its inverse, with spacetime indices here suppressed, F^{pq} is the curvature tensor constructed with the Lorentz spin connection w^{mn}, while $\bar{\gamma} = 1/\gamma$ is the inverse of the BI field. We shall use exterior calculus in this section, and we refer the reader to Refs. [14,27] for details of its implementation in LQG. The quantity $S_{\rm mat}$ represents additional matter degrees of freedom.

The field equations can be derived by varying the action of Eq. (2.1) with respect to the tetrad and the connection. Variation with respect to the tetrad leads to the torsion condition

$$2T_{[m} \wedge e_{n]} = \frac{\partial_q \bar{\gamma}}{2\bar{\gamma}^2 + 2}[\epsilon_{rpmn}e^r \wedge e^p \wedge e^q - 2\bar{\gamma}e_m \wedge e_n \wedge e^q], \qquad (2.2)$$

which is solved to obtain

$$T^m = \frac{1}{2}\frac{1}{\bar{\gamma}^2 + 1}[\epsilon^m{}_{npq}\partial^q\bar{\gamma} + \bar{\gamma}\delta^m_{[n}\partial_{p]}\bar{\gamma}]e^n \wedge e^p. \qquad (2.3)$$

Clearly, in the limit as the BI field becomes constant, one recovers the standard torsion-free limit of GR.

Variation of the action with respect to the connection leads to the modified field equations. The on-shell field equations can be shown to reduce to the Einstein equations in the presence of a scalar field

$$G_{\mu\nu} = k^2[(\partial_\mu\phi)(\partial_\nu\phi) - \tfrac{1}{2}g_{\mu\nu}(\partial^\sigma\phi)(\partial_\sigma\phi)] \qquad (2.4)$$

upon the field redefinition

$$\phi = \sqrt{3}\sinh^{-1}\bar{\gamma}. \qquad (2.5)$$

Variation of the action with respect to this new field leads simply to the massless Klein-Gordon equation $\Box\phi = 0$. When the BI field is a constant, the field equations classically reduce to the Einstein equations.

The scalarization of the BI parameter has also been studied in the quantum theory [15]. In fact, at a quantum level, the closing of the algebra requires that the Ashtekar-Barbero connection be defined with a constant BI parameter [15]. This constant value can be thought of as the expectation value of the BI field on the states of the theory or as the asymptotic value of this field [28].

The introduction of torsion to the connection suggests that higher-order curvature corrections to the action will lead to noncanonical kinetic terms for the scalar field. If the scalar field ϕ is then identified with the inflaton, one could possibly arrive at a realization of K inflation from LQG [14,29]. Of course, these comments are speculative as LQG does not require as of now higher-order curvature corrections to the Holst action. Moreover, not just any modification to the kinetic energy of ϕ suffices to induce K inflation, as precise conditions would have to be satisfied [14].

B. The Nieh-Yan action

The Holst action has been shown to lead to certain problems when coupling the theory to fermions. Even when the BI parameter is treated as constant, fermion couplings have been shown to lead to torsion, which in turn leads to a classical effective action with fermion interaction terms that depend on the BI parameter at a classical level [30,31]. Since the BI parameter is thought to be related to the quantum effect of spacetime discretization, its appearance at the level of an on-shell effective action has been thought somewhat problematic [32,33]. For this reason, it was suggested that the Holst term should by replaced by the Nieh-Yan invariant in the presence of fermions [18,32,33]:

$$S_{\text{Nieh-Yan}} = S_{\rm mat} + \frac{1}{4k^2}\int \epsilon_{mnpq}e^m \wedge e^n \wedge F^{pq} + \frac{1}{2k^2}\int \bar{\gamma}(e^m \wedge e^n \wedge F_{mn} - T^m \wedge T_m). \qquad (2.6)$$

In the absence of fermions, this formulation is equivalent to the Holst one since torsion vanishes, thus also leading to the Ashtekar and Ashtekar-Barbero connections. In the presence of fermions, this formulation also leads to torsion, but the fermion interactions induced in the effective action are independent of the BI parameter [32,33].

Inspired by the scalarization of the BI parameter in the Holst action [14], there has been recently an effort to also study the scalarization of this parameter in the Nieh-Yan action. When doing so, it was found again that variation of the action with respect to the tetrad leads to a torsion condition similar to Eq. (2.2), whose solution is again nonvanishing torsion [16]:

$$T^m = -\tfrac{1}{2}\epsilon^m{}_{npq}(\partial^n\bar{\gamma})e^p \wedge e^q + \cdots, \qquad (2.7)$$

where the dots stand for possible fermion contributions. The inclusion of the $T^m \wedge T_m$ term in the action simplifies the torsion tensor, removing the prefactor in Eq. (2.3). Naturally then, the on-shell variation of the action with respect to the connection leads to the field equations of Eq. (2.4) with the identification of $\phi \to \bar{\gamma}$. Variation of the action with respect to the BI field reveals that the BI field in the Nieh-Yan formulation satisfies a source-free, massless Klein-Gordon equation.

One of the advantages of the Nieh-Yan action is the ability to consistently include fermions, which when combined with a BI field lead to BI field-fermion interactions that are proportional to $\star J_A \wedge d\bar{\gamma}$, where J_A is the axial fermion current. Upon integration by parts, one can relate such terms to $\bar{\gamma} R \wedge R$, where R is the (torsion-free) Riemann tensor, because the axial fermion current is anomalous due to quantum effects [16].

The anomalous Pontryagin density ($R \wedge R$) can be used to cancel other CP-violating terms that arise in gauge theories that are also proportional to this density. For example, such terms arise by the local gauge group due to the diagonalization of the quark mass matrices by a chiral rotation. Through the analog of the Peccei-Quinn mechanism of quantum chromodynamics (QCD) [16,34], the BI field could be used to remove such CP violation from the action. In fact, the analogy to the Peccei-Quinn mechanism is so strong that the BI field could be interpreted as the QCD axion to solve the strong CP problem through the chiral anomaly [16,34].

The interaction terms, arising due to the coupling of fermions with the BI field, also naturally lead to a topological interpretation. Indeed, in Yang-Mills theories, the requirement that states be invariant under large gauge transformations leads to CP-violating anomalous terms of the form $\theta R \wedge R$, where θ is the Yang-Mills angle. Such a term has been shown to also arise in LQG [35]. Since the BI field-fermion coupling leads to the same type of terms in the effective action, one can then identify the expectation value of the BI field with the θ angle [17].

The casting of the effective action of modified Nieh-Yan theory with a BI field in terms of the Pontryagin density leads to a interesting connection with DCSMG [19]. Indeed, the latter is precisely defined by the sum of the Einstein-Hilbert action and the Pontryagin density multiplied by a dynamical field [20–22]. This connection becomes yet more interesting when one realizes that DCSMG contains terms that generically arise in the low-energy limit of heterotic and type IIb string theories. In the latter, these terms arise due to a ten-dimensional generalization of the Abbott-Deser-Jackiw anomaly, required for the Ward identities to be preserved. This then suggests a deep connection between string theory and LQG, which is at the heart of this paper, but before such a connection can be established, we shall briefly review supergravity in superspace.

III. 4D, $\mathcal{N} = 1$ SUPERGRAVITY IN SUPERSPACE

A concise, manifestly supersymmetric and gauge-invariant description of supergravity is provided by *superspace* [36–38]. The 4D, $\mathcal{N} = 1$ superspace is an extension of spacetime, parametrized by (bosonic) spacetime coordinates x^m and by extra fermionic (Grassmann) anticommuting spinor coordinates θ_α and $\bar{\theta}_{\dot{\alpha}}$. General coordinate transformations of GR are extended in curved superspace to general supercoordinate transformations mixing x's and θ's. A superfield amounts to a finite set (supermultiplet) of the ordinary fields (called superfield components) appearing as the coefficients in the superfield expansion in powers of θ's. Superspace supergravity is the most natural way of unifying gravity and supersymmetry. We refer the reader to the available textbooks [36–39] for details about supergravity, superfields, and their field components. In this section we formulate some basic ideas and give a few equations needed for the purposes of this paper.

An off-shell solution to the superspace Bianchi identities and the constraints, defining the $\mathcal{N} = 1$ Poincaré-type minimal supergravity, gives rise to only three relevant tensor superfields: $\mathcal{R}$, $\mathcal{G}_m$, and $\mathcal{W}_{\alpha\beta\gamma}$ (as parts of the supertorsion field). These fields are subject to the relations [36–38]

$$\mathcal{G}_m = \bar{\mathcal{G}}_m, \qquad \mathcal{W}_{\alpha\beta\gamma} = \mathcal{W}_{(\alpha\beta\gamma)}, \\ \bar{\nabla}_{\dot{\alpha}} \mathcal{R} = \bar{\nabla}_{\dot{\alpha}} \mathcal{W}_{\alpha\beta\gamma} = 0, \tag{3.1}$$

and

$$\bar{\nabla}^{\dot{\alpha}} \mathcal{G}_{\alpha\dot{\alpha}} = \nabla_\alpha \mathcal{R}, \\ \nabla^\gamma \mathcal{W}_{\alpha\beta\gamma} = \frac{i}{2} \nabla_\alpha{}^{\dot{\alpha}} \mathcal{G}_{\beta\dot{\alpha}} + \frac{i}{2} \nabla_\beta{}^{\dot{\alpha}} \mathcal{G}_{\alpha\dot{\alpha}}, \tag{3.2}$$

where $(\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}.\nabla_{\alpha\dot{\alpha}})$ represent the curved superspace $\mathcal{N} = 1$ supercovariant derivatives and bars denote complex conjugation. For instance, the covariantly chiral complex scalar superfield $\mathcal{R}$ has the scalar curvature R as the coefficient of its θ^2 term. Similarly, the real vector superfield $\mathcal{G}_{\alpha\dot{\alpha}}$ has the traceless Ricci tensor $2R_{(mn)} - \frac{1}{2}\eta_{mn} R$ as the coefficient of its $\theta \sigma^m \bar{\theta}$ term.[1] The covariantly chiral, complex, totally symmetric, fermionic superfield $\mathcal{W}_{\alpha\beta\gamma}$ has the Weyl tensor $W_{\alpha\beta\gamma\delta}$ as the coefficient of its linear θ^δ-dependent term.

Gauge-fixed, off-shell, pure $\mathcal{N} = 1$ supergravity can be also formulated in terms of the more conventional field components of supergravity superfields in spacetime (of Minkowski signature). We shall here consider the minimal (Poincaré) $\mathcal{N} = 1$ supergravity in a *Wess-Zumino* (WZ) supersymmetric gauge [36–38]. The physical fields are a vierbein (or tetrad) $e^m_\mu(x)$ and a Majorana gravitino $\psi_\mu(x)$, while the auxiliary fields are a complex scalar $B(x)$ and a real vector $A_m(x)$.[2] The tetrad is dimensionless, while the gravitino field is of canonical (mass) dimension $3/2$, while

[1]Here $\sigma^m = (\mathbf{1}, i\vec{\sigma})$ stand for Pauli matrices.

[2]There are no physical degrees of freedom associated with auxiliary fields by definition, and, thus, these fields are non-propagating. These fields are needed in supersymmetry, however, for an off-shell closure of the supersymmetry algebra and to make supersymmetry manifest. The auxiliary fields can become propagating in the presence of generic higher derivative modifications to the supergravity action.

the auxiliary fields are of dimension 2. All of the auxiliary fields vanish on the pure supergravity equations of motion; i.e. on-shell we have $B = A_m = 0$.

The off-shell superspace constraints, defining a Poincaré supergravity, are necessary to reduce a curved superspace geometry to a geometry of supergravity. These constraints include the following: the *representation-preserving* constraints, which allow for the existence of covariantly chiral superfields; the *conventional* constraints, which fix the vector covariant derivative in terms of the spinor ones and the spinor superconnections in terms of the spinor supervielbeins; and the *Einstein* (or WZ) constraints, which allow the passage from conformal supergeometry to Poincaré supergeometry in curved superspace. In particular, the minimal Poincaré supergravity constraints imply that the spin connection $\omega_\mu{}^{mn}(x)$ has the following structure [36–39]:

$$\omega_{\mu mn} = \omega_{\mu mn}(e) + k_{\mu mn}(\psi) - \tfrac{2}{3}\varepsilon_{\mu mnp}A^p, \qquad (3.3)$$

where $\omega_{\mu mn}(e)$ is the torsion-free spin connection of GR, and the gravitino-induced contorsion tensor $k_{\mu mn}(\psi)$ is given by

$$k_{\mu mn} = \frac{\kappa^2}{4}(\bar{\psi}_m\gamma_\mu\psi_n + \bar{\psi}_\mu\gamma_m\psi_n - \bar{\psi}_\mu\gamma_n\psi_m). \quad (3.4)$$

In other words, we are in the 2nd-order formalism with a fixed spin connection $\omega_\mu{}^{mn}(e, \psi, A)$, where we are now explicitly including spacetime indices. The physical (on-shell) spacetime torsion in supergravity is given by

$$T^p_{\mu\nu} = \frac{1}{2}\nabla_{[\mu}(\omega)e^p{}_{\nu]} = \frac{\kappa^2}{4}\bar{\psi}_\mu\gamma^p\psi_\nu, \qquad (3.5)$$

which obeys the identity

$$\varepsilon^{\mu\nu\lambda\rho}T^p_{\mu\nu}T^p_{\lambda\rho} = 0, \quad \text{or} \quad \mathrm{tr}(T\wedge T) = 0, \qquad (3.6)$$

as the consequence of Fierz identities for the Majorana gravitino field. Hence, though the gravitino-induced torsion does not vanish in pure supergravity, its contribution to the Nieh-Yan invariant in the last term of Eq. (2.6) vanishes on-shell because of Eq. (3.6).

The gauge-invariant action of pure supergravity in superspace is given by [36–38]

$$S_{\rm sg} = -\frac{3}{\kappa^2}\int d^8zE^{-1} = -\frac{3}{2\kappa^2}\int d^6z\mathcal{E}\mathcal{R} + \text{H.c.} \qquad (3.7)$$

The form of the action on the first line of Eq. (3.7) is the well-known, standard one of the superspace supergravity action [37] in terms of the supervielbein density $E^{-1} = \mathrm{sDet}E^M_A$ in *full* curved superspace[3] $z^{(4+2+2)} = (x, \theta, \bar{\theta})$, where sDet is the superdeterminant. The second line in Eq. (3.7) is an equivalent representation of the same pure supergravity action in *chiral* superspace. The quantities $\mathcal{E}$ and $\mathcal{R}$ are supersymmetric generalizations of the volume element and the Lagrangian density in the chiral superspace.

The chiral superspace density (in WZ gauge) reads

$$\mathcal{E}(x, \theta) = e(x)[1 - \kappa 2i\theta\sigma_m\bar{\psi}^m(x) + \kappa\theta^2B(x)], \qquad (3.8)$$

where $e = \sqrt{-\det g_{\mu\nu}}$, $g_{\mu\nu} = \eta_{mn}e^m_\mu e^n_\nu$ is a spacetime metric of Minkowski signature, $\psi^m_\alpha = e^m_\mu\psi^\mu_\alpha$ is a chiral gravitino, and $B \equiv S - iP$ is the complex auxiliary field, with S and P defined as its real and imaginary parts, respectively.

The chiral density integration formula, relating a curved superspace action to the corresponding component action in spacetime, reads [36–38]

$$\int d^4xd^2\theta\mathcal{E}\mathcal{L} = \int d^4xe\{\mathcal{L}_{\rm last} + B\mathcal{L}_{\rm first}\} + \cdots, \qquad (3.9)$$

where the dots stand for the gravitino-dependent terms. Here we have introduced the field components of the covariantly chiral superfield Lagrangian $\mathcal{L}(x, \theta)$ and $\bar{\nabla}^{\dot{\alpha}}\mathcal{L} = 0$ (the vertical bars denote the leading component of a superfield) as

$$\mathcal{L}| = \mathcal{L}_{\rm first}(x), \qquad \nabla^2\mathcal{L}| = \mathcal{L}_{\rm last}(x). \qquad (3.10)$$

Noting that $\mathcal{L} \propto \mathcal{R}$ in Eq. (3.7), we must then compute $\mathcal{R}|$ and $\nabla^2\mathcal{R}|$, which reduce to [36–38]

$$\mathcal{R}| = \tfrac{1}{3}\bar{B} = \tfrac{1}{3}(S + iP), \qquad (3.11)$$

$$\nabla^2\mathcal{R}| = \frac{1}{3}\left(R - \frac{i}{2}\varepsilon^{mn}{}_{pq}R^{pq}{}_{mn}\right) + \frac{4}{9}\bar{B}B, \qquad (3.12)$$

where the spacetime curvature is given by

$$R^{pq}{}_{mn} = e^\mu_m e^\nu_n R^{pq}{}_{\mu\nu} = e^\mu_m e^\nu_n(\partial_\mu\omega^{pq}_\nu + \omega^{pr}_\mu\omega^{rq}_\nu - \mu\leftrightarrow\nu) \qquad (3.13)$$

and the scalar curvature is $R = e^\mu_m e^\nu_n R^{mn}{}_{\mu\nu}$.

Combining all of these ingredients, Eq. (3.7) gives rise to the standard supergravity action in components in WZ gauge [36–39]:

$$S_{\rm sg} = \int d^4xe\left\{-\frac{1}{2\kappa^2}R[e, \omega(e, \psi)] + \frac{i}{2}\varepsilon^{\mu\nu\lambda\rho}\bar{\psi}_\mu\gamma_5\gamma_\nu\nabla_\lambda\psi_\rho - \frac{1}{3}\bar{B}B + \frac{4}{3}A^nA_n\right\}. \qquad (3.14)$$

The auxiliary terms $B\bar{B}$ and A^nA_n arise here from the integration formula in Eq. (3.12) and from the decomposition of the spin connection in Eq. (3.3), respectively. The dual Riemann contribution in Eq. (3.12) cancels in the

[3] In the next sections we are going to use the supergravity action in *chiral* curved superspace $z^{(4+2)} = (x, \theta)$ only, with the chiral supervielbein density $\mathcal{E}$. The meaning of x and θ is *different* in full and chiral superspace [36–38].

action of Eq. (3.7) because of the identities for the torsion-free Riemann tensor.

IV. HOLST ACTION IN SUPERGRAVITY WITH A BI PARAMETER

The gauge-invariant supergravity action in Eq. (3.7) in chiral superspace can be slightly generalized by *complexifying* the gravitational coupling constant in front of the first term via

$$\frac{1}{\kappa^2} \rightarrow \frac{1}{\kappa^2}(1 + i\eta) \tag{4.1}$$

with the new dimensionless real parameter η. Such a complexification makes sense in supersymmetry since the second line of Eq. (3.7) is the sum of a term (which is in general complex) and its complex conjugate.

The complexification gives rise to a modified superfield supergravity action of the form

$$S_{\rm sgH} = -\frac{3}{2\kappa^2}\int d^6 z\mathcal{E}(1 + i\eta)\mathcal{R} + \text{H.c.}, \tag{4.2}$$

which induces new terms proportional to η. When using the component expansion of the WZ-gauge-fixed supergravity superfields (see, e.g., Sec. 5.8 of Ref. [38]), one can find these *extra* η-dependent terms:

$$S_{\rm sH} = -\frac{\eta}{2}\int d^4 x \varepsilon^{\mu\nu\lambda\rho}\left[\frac{1}{\kappa^2}R_{\lambda\rho\mu\nu}(e,\omega) + \bar{\psi}_\mu \gamma_\nu \nabla_\lambda \psi_\rho\right]. \tag{4.3}$$

The η-independent terms in Eq. (4.2) [not shown in Eq. (4.3)] are just the standard supergravity action.

The first gravitational term of Eq. (4.3) coincides with the second term of the Holst action in the second line of Eq. (2.1) [12] with a constant BI parameter. This identification allows us to relate the reciprocal of η with γ, namely, $\bar{\gamma} = \eta = \gamma^{-1}$ [11,40]. Therefore, the action

$$S_{\rm sgH} = -\frac{3i\eta}{2\kappa^2}\left(\int d^6 z\mathcal{E}\mathcal{R} - \text{H.c.}\right) \tag{4.4}$$

is to be identified with the gauge-invariant and manifestly supersymmetric extension of the second term of Eq. (2.1) in superspace. In WZ gauge, Eq. (4.3) just represents the relevant (i.e. depending upon physical fields only) part of that supersymmetric extension in components, in agreement with the first calculation of the component supersymmetric Holst action (without auxiliary fields) [41]. The auxiliary fields of the *pure* $\mathcal{N} = 1$ supergravity theory vanish on-shell (in the absence of supersymmetric matter). The off-shell superfield action in Eq. (4.2), which represents the full supersymmetric extension of the Holst action, depends, of course, on the auxiliary fields.

The original significance of the bosonic part of the action in Eq. (4.3) stems from dealing with the reality conditions required in the Ashtekar formulation of Euclidean quantum gravity [10]. Since we work with a Minkowski signature, our BI parameter η must be real, in accordance with the Ashtekar-Barbero formulation of LQG. In Euclidean space, however, η is purely imaginary, and the (anti)self-dual case $\eta = \pm i$ just leads to the Ashtekar formulation of LQG.

The purely gravitational Lagrangian $\varepsilon^{\mu\nu\lambda\rho}R_{\lambda\rho\mu\nu}(e,\omega(e))$ with the standard (no-torsion) connection $\omega_\mu{}^{mn}(e)$ vanishes because of Bianchi identities for the curvature. In supergravity, with the gravitino-induced torsion and $\omega_\mu{}^{mn}(e,\psi)$ of Eq. (3.3), the supersymmetric Holst-Tsuda Lagrangian in Eq. (4.3) in components amounts to a total derivative [41]

$$-\frac{1}{2}\eta\varepsilon^{\mu\nu\lambda\rho}\partial_\lambda(\bar{\psi}_\mu\gamma_\nu\psi_\rho) = \frac{2\eta}{\kappa^2}\varepsilon^{\mu\nu pq}\partial_\mu T_{\nu pq}, \tag{4.5}$$

where we have used the spacetime torsion tensor of Eq. (3.5). Therefore, adding the supersymmetric (Holst) term with a constant BI parameter to pure $\mathcal{N} = 1$ supergravity has no effect on its supergravitational equations of motion (though it is relevant in quantum supergravity), in agreement with Refs. [41,42].

The independence of the supergravitational equations of motion upon a constant BI parameter in the *extended* $\mathcal{N} = 2$ and $\mathcal{N} = 4$ pure supergravities without their coupling to supermatter (and for $\mathcal{N} = 1$ as well) was established by Kaul [42]. Our analysis in $\mathcal{N} = 1$ superspace confirms those conclusions in the case of pure $\mathcal{N} = 1$ supergravity without its coupling to $\mathcal{N} = 1$ matter superfields. In a matter-coupled $\mathcal{N} = 1$ supergravity, the supergravity auxiliary fields couple to matter fields, so that they do not vanish generically. Accordingly, the BI dependence (or independence) of the equations of motion in supergravity theories coupled to matter at the classical level should be checked separately and is beyond the scope of this paper.

V. HOLST ACTION IN SUPERGRAVITY WITH A BI FIELD

The gravitational constant can be thought of as the vacuum expectation value (VEV) of a real scalar field, which leads to the well-known *Brans-Dicke* (BD) gravitational theories. In $\mathcal{N} = 1$ supersymmetry, the BD real scalar field must be promoted to a complex chiral scalar superfield $\mathcal{Z}(x,\theta)$, for the same reasons that the gravitational constant was complexified in the previous section. The leading field component of this complex chiral scalar superfield is the complex scalar

$$\mathcal{Z}(x,\theta)| = \phi(x) + ia(x). \tag{5.1}$$

The real part $\phi(x)$ of the complex BD scalar couples to the scalar curvature, and, therefore, it should be identified with a *dilaton*. The imaginary part $a(x)$ of the BD complex scalar field couples to the 2nd term in the Holst action (the dual scalar curvature), and, therefore, it should be identified with an *axion*.

The independence of the Holst action on a (constant) BI parameter is crucial for claiming an associated *Peccei-Quinn* (PQ) symmetry. Such a symmetry of the Holst action with a BI field (cf. Ref. [16])

$$a(x) \to a(x) + \text{const} \tag{5.2}$$

is often taken as the defining property of the axion field. From this perspective, it is clear that the BI field should be identified with the supergravity axion.

More specifically, let us replace the coupling constants in the supergravity action of Eq. (4.2), inside the superspace integration, by a complex covariantly chiral BI (or BD) superfield $\mathcal{Z}(x, \theta)$ as

$$-\frac{3}{2k^2}(1 + i\eta) \to \mathcal{Z}(x, \theta). \tag{5.3}$$

Such a replacement gives rise to the modified $\mathcal{N} = 1$ supergravity action

$$S_Z = \int d^6z \mathcal{E} \mathcal{Z} \mathcal{R} + \text{H.c.} \tag{5.4}$$

in chiral $\mathcal{N} = 1$ superspace. The action in Eq. (5.4) appears to be the very special case of the modified supergravity actions introduced recently in Refs. [43,44]—see, e.g., Eq. (5.19) in Ref. [43]—with the superpotential $V = 0$. We shall thus employ the results of Refs. [43,44] when $V = 0$.

A super-Weyl transform of the superfield action in Eq. (5.4) can also be performed entirely in superspace, i.e. with manifest local $\mathcal{N} = 1$ supersymmetry. In terms of field components, the superfield Weyl transform amounts to a Weyl transform, a chiral rotation, and a (superconformal) S-supersymmetry transformation [45]. The chiral density superfield $\mathcal{E}$ is a chiral compensator of the super-Weyl transformation

$$\mathcal{E} \to e^{3\Phi} \mathcal{E}, \tag{5.5}$$

whose parameter Φ is an arbitrary covariantly chiral superfield $\bar{\nabla}_{\dot{\alpha}} \Phi = 0$. Under the transformation (5.5) the covariantly chiral superfield $\mathcal{R}$ transforms as

$$\mathcal{R} \to e^{-2\Phi}(\mathcal{R} - \tfrac{1}{4}\bar{\nabla}^2) e^{\bar{\Phi}}. \tag{5.6}$$

The super-Weyl chiral superfield parameter Φ can be traded for the chiral Lagrange multiplier $\mathcal{Z}$ by using a simple holomorphic gauge condition

$$\mathcal{Z} = \Phi. \tag{5.7}$$

With all of these ingredients at hand, the super-Weyl transform of the action in Eq. (5.4) results in the classically equivalent action

$$S_\Phi = \int d^4x d^4\theta E^{-1} e^{\Phi + \bar{\Phi}} [\Phi + \bar{\Phi}] \tag{5.8}$$

in full curved superspace of $\mathcal{N} = 1$ supergravity. Equation (5.8) has the standard form of the action of a chiral matter superfield coupled to supergravity [36–38]

$$S[\Phi, \bar{\Phi}] = \int d^4x d^4\theta E^{-1} \Omega(\Phi, \bar{\Phi}), \tag{5.9}$$

with the kinetic potential $\Omega(\Phi, \bar{\Phi})$, where we have defined

$$\Omega(\Phi, \bar{\Phi}) = e^{\Phi + \bar{\Phi}} [\Phi + \bar{\Phi}]. \tag{5.10}$$

The Kähler potential $K(\Phi, \bar{\Phi})$ is given by [37,38]

$$K = -3 \ln\left(-\frac{\Omega}{3}\right) \quad \text{or} \quad \Omega = -3e^{-K/3}. \tag{5.11}$$

The proposed action in Eq. (5.4) with the chiral BI superfield $\mathcal{Z}$ is classically equivalent to the standard action of a chiral matter superfield Φ coupled to the minimal $\mathcal{N} = 1$ supergravity and having a noncanonical kinetic term, i.e. a nontrivial Kähler potential. The equivalent action in Eq. (5.9) may be suitable as the starting point for generating K inflation in the BI-field-modified supergravity along the lines of Ref. [14].

VI. NIEH-YAN ACTION IN SUPERGRAVITY WITH A BI FIELD

In this section we comment on the embedding of the Nieh-Yan action with a BI field into supersymmetry. The Nieh-Yan invariant was defined by the integrand of the last term in Eq. (2.6), without the BI field. Because of the identity in Eq. (3.6) it is clear that the supersymmetric extension of the Nieh-Yan density should be *the same* as the supersymmetric extension of the Holst action; i.e. both are given by Eq. (4.4) in superspace, up to an overall normalization factor, as far as the *pure* supergravity with a *constant* BI term is concerned. Moreover, because of Eq. (4.5), the Nieh-Yan density (neglecting the Einstein-Hilbert term) can be rewritten as a divergence of the axial gravitino current $J^\mu = \frac{1}{2} \varepsilon^{\mu\nu\lambda\rho} \bar{\psi}_\nu \gamma_\rho \psi_\lambda$:

$$S_{\text{Nieh-Yan}} = -\frac{1}{2} \int d^4x \nabla_\mu J^\mu, \tag{6.1}$$

where ∇_μ is the covariant derivative. Equation (6.1) is again in agreement with Ref. [42]. In superspace supergravity, it appears to be the consequence of the on-shell identity

$$i(\nabla^2 \mathcal{R} - \text{H.c.}) \propto \nabla_\mu J^\mu, \tag{6.2}$$

where the H.c. stands for the Hermitian conjugate of the preceding term.

The invariance of an action in supersymmetry and supergravity is *usually* defined modulo a surface term or up to a total derivative in the Lagrangian. A supersymmetric completion of a topological term in supersymmetry is often ambiguous (i.e. defined up to a surface term) since it does not change the bosonic value of a topological action. The superspace approach can, nevertheless, be effectively used in selecting the most natural (or minimal) consistent supersymmetric topological action that should be used in quantum gravity. When a constant BI parameter is promoted to a (spacetime-dependent) BI *field*, there is no ambiguity in defining a (nontopological) supersymmetric action.

When supergravity is coupled to matter superfields, as is the case with a BI superfield, the supergravity auxiliary fields do not vanish, while their (algebraic) equations of motion determine them in terms of the physical (supergravity and matter) fields. In particular, the matter contribution to the spin connection enters via the vector auxiliary field in Eq. (3.3), giving rise to matter-generated contributions to spacetime torsion and rendering Eq. (3.6) no longer valid contrary to the pure supergravity case. As a result, the supersymmetric Holst action in components gets modified by matter-dependent terms. We believe that the modification should be precisely in the form of the Nieh-Yan-type extra term, though we did not verify it by an explicit calculation. The reason for this belief is the PQ symmetry [e.g. Eq. (5.2)] of the superspace formulation of Holst supergravity with the BI superfield: the action in Eq. (5.8) is invariant with respect to the transformations in Eq. (5.2) as the former merely depends on the sum $(\Phi + \bar{\Phi})$.

We have thus established a direct mathematical relation between the Nieh-Yan action with a BI field and the $\mathcal{N} = 1$ supergravity action in superspace. This mapping relates the BI field $\bar{\gamma}$ to the supergravity axion. In the nonsupersymmetric four-dimensional theory, when the axion is coupled to the Holst term only, its kinetic energy becomes noncanonical, while with the Nieh-Yan term one recovers the standard canonical kinetic term. In the supersymmetric case of a BI superfield coupled to supergravity, we get the genuine (supersymmetric) nonlinear sigma model representing the BI kinetic terms in the action of Eq. (5.8).

VII. CONCLUSIONS

We have shown that the possibility of migrating concepts and constructs across the LQG/SSMT boundary can yield interesting perspectives on issues that can arise. While there is currently no known requirement of the presence of supersymmetry in the LQG program, embedding LQG considerations within supergravity theory may offer hints on the possibility of a synthesis, daunting though this may seem. If supersymmetry were ever to be found as a requirement in the LQG program, then the stage would be set for a fascinating line of investigation on the issue of Bekenstein-Hawking black hole entropy within the two distinct approaches represented by LQG and SSMT. In the former all indications are that the axion must have a nonvanishing VEV, while in the latter no such requirement arises.

A connection between the BI field and the axion also leads to two possible and interesting interpretations: either the BI field and the axion field are one and the same, or the BI field is another massless degree of freedom that happens to possess similar couplings to curvature relative to the axion coupling. In the former case, one can then attempt to constrain the BI field by studying the effect of its potential on cosmological observables. Work along these lines is currently underway [46].

Should the identification of the BI field with the axion hold true generically, one may recall the possibility of dark matter composed at least in part of axions. In such a case, the BI field/particle might become a viable candidate for dark matter.

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