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Environment-dependent dark sector

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We propose a new framework unifying cold dark matter (CDM) and modified Newtonian dynamics (MOND) to solve their respective problems on galactic scales and large scale structure formation. In our framework the dark matter clusters on large scales but not on galactic scales. This *environment dependence* of the dark matter behaviors is controlled by a vector field, which also produces the MOND effects in galaxies. We find that in this framework only a *single* mass scale needs to be introduced to produce the phenomena of CDM, MOND, and also dark energy.

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I. INTRODUCTION

The observed universe appears to not be made purely of standard model particles and Einstein gravity. We have yet to identify the physics of the missing constituent(s). Data on galactic and larger scales are often used to argue for the modified Newtonian dynamics (MOND) and the cold dark matter (CDM) frameworks, respectively. Linear growth of large scale structure in the early universe, such as the cosmic microwave background (CMB), favors the CDM idea, and nonlinear structures on the scale of galaxy clusters agree very well with numerical CDM simulations. On the other hand, on smaller scales, Milgrom's MOND formula [1] captures the tight correlation of the observed baryonic mass distribution within a spiral galaxy vs the observed gravitational acceleration at each radii of that galaxy. This applies to galaxies with a wide range of scales, formation histories, and environments, from dwarf to elliptical galaxies [2–6]. The amazing accuracy of this relation and the fact that it predicts a correct Tully-Fisher relation even for tidal dwarf galaxies [7] motivate the noncovariant MOND theory [8] and a class of covariant theories [9–15], to eliminate the need for the CDM particles.

However, both CDM and covariant MOND have their own problems. So far, the most challenging difficulty for covariant MOND theories is in producing early growths of large scale structure and fitting the CMB data [16–18] on which CDM works very well. Also, massive neutrinos seem to be indispensable even in covariant MOND to explain the lensing data in galaxy clusters [19,20]. In comparison, thanks to its simplicity, the CDM framework enjoys many tools for sophisticated numerical simulations, yet the properties of galaxies in these simulations are not in good agreement with observations. The overproduction of dark structures in small scales is well known as the substructure problem and the cusp problem. A common assumption to solve this problem is that the CDM particle is ballistic and will always follow the same geodesic equation as a collisionless star would.

Indeed, CDM and MOND are both complementary and mutually exclusive: if both exist in galaxies, then obviously new problems will arise. A natural way out of this dilemma is to have a "CDM" which is no longer cold in the environments where MOND dominates, e.g., letting it develop a nonzero pressure or have a much smaller mass (for particles) there. This environment dependence may be controlled by a scalar field, but in this case the different dark matter behaviors in different regimes (galactic, cluster, and cosmological) indicate that the coupling between the scalar field and the dark matter must be fine-tuned (if it is possible anyway), because of the dynamical nature of the scalar field. In order to see why, note that the MONDian behavior is only expected where the Newtonian acceleration $|\nabla\Phi|$ is smaller than the MOND parameter a_0 (on galactic scales), but not in the solar system (where $|\nabla\Phi| \gg a_0$), nor on cosmological scales in most of the cosmic history (where $cH \gg a_0$). These suggest that we should use *both* $|\nabla\Phi|/a_0$ *and* cH/a_0 as the criteria about the environment dependence of the dark matter behaviors. A scalar field dark matter faces not only the challenge to reproduce MOND when $|\nabla\Phi| \lesssim a_0$ [21], but also to follow *both* $\nabla\Phi$ *and* cH through a correct dynamical evolution, because there are no inherent characteristic quantities which mimic $\nabla\Phi$ or cH in normal scalar field models.

In most attempts to construct relativistic MOND, a vector field is used, which can easily overcome the second challenge faced by scalar fields. Furthermore, from previous studies of timelike unit-norm vector fields (the Æther field $Æ_a$ [22]), we know that there are four possible kinetic terms for $Æ_a$, which have different properties in different regimes. If we write these kinetic terms as $\mathcal{K} \equiv K^{ab}{}_{cd}\nabla_a Æ^c \nabla_b Æ^d$ with $K^{ab}{}_{cd} = c_1 g^{ab} g_{cd} + c_2 \delta^a_c \delta^b_d + c_3 \delta^a_d \delta^b_c + c_4 Æ^a Æ^b g_{cd}$, in which c_i's are dimensionless

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TABLE I. The behaviors of the c_i terms in different relevant regimes (+ means there is effect and − means no effect). The static limit of the terms actually depend on spatial configuration of the vector field, but this can be consistently regarded as of higher order.

Terms	c_1	c_2	c_3	c_4
Cosmological background	+	+	+	−
Cosmological perturbation	+	+	+	+
Static limit	+	−	−	+

constants, then Table I briefly summarizes the behaviors of these terms in different regimes. More specifically, in the static limit the c_1 and c_4 terms are $\propto (\nabla\Phi)^2$, while in the background the $c_{1,2,3}$ terms are $\propto (cH)^2$ (see [11] for an earlier discussion about this point).

These facts suggest that the Æther field $Æ_a$ should be a natural candidate to control the environment dependence of the dark matter's behavior, most probably through a coupling to the latter. In this work we shall give a simple example to illustrate this principle. The idea is very straightforward: let the behavior of the dark matter depend on $\mathcal{K}/a_0^2$, which could be very large on cosmological scales in most of the cosmic history ($\mathcal{K} \sim c^2H^2 \gg a_0^2$) and in the solar system ($\mathcal{K} \sim |\nabla\Phi|^2 \gg a_0^2$), but becomes of order unity or less on galactic scales $\mathcal{K} \sim |\nabla\Phi|^2 \lesssim a_0^2$. In the first case the dark matter has (nearly) zero pressure and zero sound speed, and thus is actually cold, while in the latter situations it acquires a nonzero pressure and nonzero sound speed, and thus no longer clusters; instead then, the $Æ_a$ field will produce the MOND effect.

II. OUR MODEL AND ITS BEHAVIORS

A distinction between our model and previous ones [9–12,18] is that we are not using $Æ_a$ to grow the large scale structure, which proves difficult. Instead, we introduce a dark matter φ which is *controlled by* $Æ_a$ through a coupling. For illustration purposes, we take φ as a k-essence field [23], though our principle can be applied much more generally (see a discussion below). We start from the following Lagrangian density:

$$\mathcal{L} = -g(\varphi, X, \mathcal{K}) + V(\varphi, \mathcal{K}) + \lambda(Æ^aÆ_a - 1), \quad (1)$$

in which g and V are arbitrary functions, φ is a dimensionless scalar field, $X \equiv \frac{1}{2}\nabla^a\varphi\nabla_a\varphi$, and $\mathcal{K}$ and $Æ_a$ are defined above; λ is a Lagrange multiplier ensuring that $Æ_a$ has unit norm. For $g(\varphi, X, \mathcal{K})$, we use a generic power law in X with the power dependent on $\mathcal{K}$:

$$g(\varphi, X, \mathcal{K}) = g(X, \mathcal{K}) = 2w\left(\frac{X}{a_0^2}\right)^{(w+1)/2w} a_0^2, \quad (2)$$

in which the normalization a_0^2 is always justified because φ (hence X) could be rescaled; $w = w(\frac{\mathcal{K}_3}{a_0^2})$, which has the meaning of the dark matter equation of state, is a free function introduced to produce the desired effects, and $\mathcal{K}_3 \equiv c_3\delta^a_d\delta^b_c\nabla_aÆ^c\nabla_bÆ^d$ only has the c_3 term. Here c_3 is a constant, and we adopt $\frac{1}{w} = 3 + (\frac{\mathcal{K}}{a_0^2})^n$, where $n = 1$ for illustration. We further choose $V(\varphi, \mathcal{K}) = V(\mathcal{K}_4)$, where $\mathcal{K}_4 \equiv c_4(w)Æ^aÆ^bg_{cd}\nabla_aÆ^c\nabla_bÆ^d$ has only the c_4 term; here c_4 can be a function of w, but for the simplicity of arguments, we shall first treat c_4 as a constant first. Note that our choices of the c_3 and c_4 terms above are designed so that we have a clean separation between the dark matter behaviors in cosmological background (g) and in the static limit (V). This is achieved because the c_3 term only has effects on the former while the c_4 term only affects the latter (cf. Table I).

A variation with respect to the metric tensor gives the following energy momentum tensor[1]:

$$\begin{aligned}8\pi GT_{ab} = {}& g_X\nabla_a\varphi\nabla_b\varphi - g_{ab}[g(X, \mathcal{K}) - V(\mathcal{K})] \\ & - 2\nabla_c[(V_{\mathcal{K}} - g_{\mathcal{K}})(Æ^cJ_{(ab)} + Æ_{(a}J^c{}_{b)} \\ & - Æ_{(a}J_{b)}{}^c)] + 2\nabla_c[(V_{\mathcal{K}} - g_{\mathcal{K}})J^{cd}]Æ_dÆ_aÆ_b \\ & - 2c_4V_{\mathcal{K}}(Æ^d\nabla_dÆ_c)(Æ^e\nabla_eÆ^c)Æ_aÆ_b \\ & + 2c_4V_{\mathcal{K}}(Æ^c\nabla_cÆ_a)(Æ^d\nabla_dÆ_b),\end{aligned} \quad (3)$$

where we have defined $J^a{}_c \equiv K^{ab}{}_{cd}\nabla_bÆ^d$ and

$$g_{\mathcal{K}} \equiv \frac{\partial g(X, \mathcal{K})}{\partial\mathcal{K}}, \qquad V_{\mathcal{K}} \equiv \frac{\partial V(\varphi, \mathcal{K})}{\partial\mathcal{K}}. \quad (4)$$

We could also derive the scalar and vector field equations of motion, but they are not needed here.

The energy momentum tensor for the k-essence field is defined as (see below for a discussion)

$$8\pi GT^{\varphi}_{ab} = g_X\nabla_a\varphi\nabla_b\varphi - g_{ab}g(X, \mathcal{K}), \quad (5)$$

which resembles the energy momentum tensor of a perfect fluid $T^{\varphi}_{ab} = (\rho_\varphi + p_\varphi)u_au_b - p_\varphi g_{ab}$ with $u_a = \frac{\nabla_a\varphi}{\sqrt{2X}}$, energy density ρ_φ, and pressure p_φ:

$$8\pi G\rho_\varphi = 2Xg_X - g(X, \mathcal{K}), \quad (6)$$

$$8\pi Gp_\varphi = g(X, \mathcal{K}). \quad (7)$$

Substituting Eq. (2) into Eqs. (6) and (7) it is easy to check

$$w = \frac{p_\varphi}{\rho_\varphi} = \frac{1}{3 + \frac{\mathcal{K}}{a_0^2}}. \quad (8)$$

So we see that when $\mathcal{K} \gg a_0^2$ this behaves as dust while when $\mathcal{K} \ll a_0^2$ it behaves as radiation. Furthermore, when it behaves as dust the sound speed c_s^2 satisfies

[1] Note that in Eq. (3) the $\mathcal{K}$ in $g(X, \mathcal{K})$ can be different from that in $V(\varphi, \mathcal{K})$ (e.g., having different c_i's). Consequently, if, for example, the $\mathcal{K}$ in $V(\varphi, \mathcal{K})$ has no c_4 term, then $c_4V_{\mathcal{K}}$ is zero, but this does not necessarily mean that $c_4g_{\mathcal{K}}$ is also zero and vice versa.

$$c_s^2 = \frac{g_X}{2Xg_{XX} + g_X} = \frac{1}{3 + \frac{\mathcal{K}}{a_0^2}} \rightarrow 0, \tag{9}$$

so that it has the desired clustering property of CDM.

Remember that we have chosen the $\mathcal{K}$ in $w(\frac{\mathcal{K}}{a_0^2})$ as to have only a c_3 term. It requires no fine-tuning to choose c_3 so that all through the cosmic history $\mathcal{K} \sim 3c_3H^2c^2 \gg a_0^2$, since $cH_0 \sim 6a_0$ and $H > H_0$ at earlier times, as the result of which a choice of $c_3 \sim \mathcal{O}(10^0\text{–}10^2)$ guarantees $\frac{1}{w} \sim \frac{\mathcal{K}}{a_0^2} \geq 108c_3 \gg 1$ (if we use $n > 1$ in w then c_3 could even be set to 1). This indicates that in our model the scalar field does behave like CDM in the background expansion and the large scale structure formation, where there is significant Hubble expansion.

At this stage one may be worried about the other terms in the energy momentum tensor in Eq. (3): are they large enough to spoil the good CDM behaviors we have obtained so far? We discuss how $V(\mathcal{K})$ and $V_{\mathcal{K}}$ are negligible separately below. For the $g_{\mathcal{K}}$ terms, from Eq. (2) we get

$$g_{\mathcal{K}} = \frac{8\pi G p_\varphi}{2a_0^2}\left[\log\frac{X}{a_0^2} - 2w\right]. \tag{10}$$

To see that $g_{\mathcal{K}} \ll 1$, note that $p_\varphi = w\rho_\varphi \sim \rho_\varphi a_0^2/\mathcal{K}$ when $\mathcal{K} = 3c_3(cH)^2 \gg a_0^2$ and $8\pi G\rho_\varphi/3(cH)^2 \sim O(1)$, so

$$\frac{8\pi G p_\varphi}{2a_0^2} \sim \frac{4\pi G\rho_\varphi}{\mathcal{K}} \sim \frac{4\pi G\rho_\varphi}{c^2H^2} \sim \mathcal{O}(1). \tag{11}$$

Meanwhile, the current fractional energy density of dark matter is 0.2, which means that $8\pi G\rho_{\varphi 0} \sim 0.6(cH_0)^2 \sim 20a_0^2$, so we have

$$8\pi G\rho_\varphi = 2\left(\frac{X}{a_0^2}\right)^{w+1/2w} a_0^2 \sim 20a_0^2(1+z)^3,$$

in which z is the redshift, or

$$\left(\frac{X}{a_0^2}\right)^{w+1/2w} = B(1+z)^3$$

with $B \sim 10$. As a result

$$\log\frac{X}{a_0^2} = \frac{2w}{w+1}[\log B + 3\log(1+z)] \sim \mathcal{O}(w) \ll 1$$

today; $\log(\frac{X}{a_0^2})/w$ increases with redshift logarithmically at high redshifts, e.g., $\log(\frac{X}{a_0^2})/w \sim \mathcal{O}(10^2)$ at $z \sim 10^{10}$. But $w \propto (1+z)^{-3}$ and $(1+z)^{-4}$ in the matter and radiation dominated eras so that indeed both w and $\log(\frac{X}{a_0^2})$ decrease quickly with redshift. This above analysis shows that $g_{\mathcal{K}} \sim \mathcal{O}(10^0\text{–}10^2)w \ll 1$ in all the cosmological epochs of interests, which is easy to understand because $g \sim a_0^2$ is very small while $\frac{\mathcal{K}}{a_0^2}$ is very large (this order-of-magnitude estimate holds for general n's). The smallness of $g_{\mathcal{K}}$ strongly suppresses the effects of the Æther terms in Eq. (3), making them negligible. In fact, the T^φ_{ab} in Eq. (5) is not conserved, but the smallness of $g_{\mathcal{K}}$ implies that the energy exchange between φ and $Æ_a$ is just negligible. Then, as $\frac{p}{\rho}$, $\frac{\dot{p}}{\dot{\rho}}$, and $\frac{\delta p}{\delta\rho} \sim \mathcal{O}(w) \ll 1$, Eqs. (35), (36) of [24] show that the perturbation growth also mimics that of CDM for reasonable parameters. Numerical results will be reported in a forthcoming paper.

We next consider the cluster scales, where the observations are not compatible with MOND alone but necessarily incur a certain amount of dark matter. An example is the bullet cluster, in which the offset between the gas and dark matter distributions is hard to explain by MOND. These scales generally have not decoupled from the background expansion, where according to our model the dark matter is still cold. Thus this model has the potential to explain the observations on cluster scales.

On galactic scales, the spacetime is more or less static, which means that $\frac{\mathcal{K}_3}{a_0^2}$ is small enough to make $w = c_s^2 \rightarrow \frac{1}{3}$, so that the pressure support is strong enough to prevent any further collapse of dark matter. This eliminates the CDM in galaxy systems as we expected, since otherwise CDM and MOND will coexist, spoiling MOND's good fit with data. Now, with the scalar field dark matter not clustering and the $g_{\mathcal{K}}$ (with only c_3 term) having no effect in the static weak field, it is the role of the $V(\mathcal{K})$ (only c_4 term) to produce the MOND effect. To do this, let us use the metric $ds^2 = (1 + 2\epsilon\Phi)dt^2 - (1 - 2\epsilon\Psi)dx^idx^j$ and write $Æ^a = \delta^a_0 + \epsilon æ^a$ in which $æ^a$ is the perturbation of $Æ^a$ and ϵ is a small positive quantity. Then up to first order in ϵ, it is easy to derive that $G_{00} = -2\Phi^{,i}_{,i} = 2\partial_i\partial_i\Phi$, where we have used the fact that $\Phi = \Psi$ thanks to the absence of anisotropic stresses. For the energy density of the fields [cf. the right-hand side of Eq. (3)], we already know that the first line as well as all $g_{\mathcal{K}}$ terms have negligible or zero effects; also it is easy to show that up to first order in ϵ the last three lines all vanish, while the second line reduces to $-2\nabla_i(c_4V_{\mathcal{K}}\Phi^{,i})$. Defining $\mu \equiv 1 - c_4V_{\mathcal{K}}$ the Poisson equation now reads [actually there is also a $V(\mathcal{K})$ on the right-hand side of Eq. (12), but this is like a cosmological constant and will not cluster]

$$2\partial_i[\mu(x)\partial_i\Phi] = 8\pi G\rho_b, \tag{12}$$

where ρ_b is the local baryon energy density and the argument of $\mu(x)$ is

$$x \equiv \left(\frac{\mathcal{K}}{a_0^2}\right)^{1/2} = \frac{|\nabla\Phi|}{(-c_4)^{-1/2}a_0},$$

where we have used $\mathcal{K} = -c_4|\nabla\Phi|^2$ up to $\mathcal{O}(\epsilon^2)$ in the static limit. We could choose the form of $V(\mathcal{K})$ or μ as in [11]

$$1 - \mu(x) = \left(1 + \frac{x}{3}\right)^{-3} = \frac{V(\mathcal{K})}{V(0)(1 + \frac{x}{3})(1 + \frac{2x}{3})}, \tag{13}$$

where $V(0) = (-c_4)^{-1}(3a_0)^2$. Clearly the MOND limit

$\mu(x) \to x$ is recovered when $x \to 0$ if we choose $c_4 = -1$. In general $V(\mathcal{K})$ serves as a nonuniform dark energy potential [11], whose local minimum can recover the MOND equation, and the background value behaves as a cosmological constant far away from galaxies $V(0) = (3a_0)^2$. Such a $V(0)$ with $c_4 = -1$ is however not enough to account for the dark energy with $8\pi G\rho_{\rm DE} \sim 81a_0^2$, and we will come back to this point later.

In the Solar System, again the k-essence field φ does not cluster, and the c_3 term in $g_{\mathcal{K}}$ has no effect. But here the MOND effect and cosmological constant effect are both suppressed because, from Eq. (13), $V(\mathcal{K}) \to 54x^{-1}a_0^2 \ll a_0^2$, and $\mu(x) \to 1 - (1 + x/3)^{-3} \to 1$ in the strong gravity regime in which $x \gtrsim \mathcal{O}(10^6) \gg 1$. In fact, the Newtonian gravity and the parametrized post-Newtonian limits are recovered [11].

III. DISCUSSION

We want to point out that the model described above is only a very simple one for the Lagrangian equation (1). One can also, for example, use the oscillation of a canonical scalar field around its potential minimum to provide the dark matter, with the steepness of the potential depending on $\mathcal{K}/a_0^2$. In a more phenomenological way, we could simply postulate a coupling between dark matter particles and the vector field (like the coupling with a scalar field) as a result of which the dark matter particle mass depends on $\frac{\mathcal{K}}{a_0^2}$. Furthermore, it is also interesting to see if the parameter a_0^2 is indeed determined dynamically. These possibilities will be considered in details in forthcoming papers.

The interesting fact $a_0 \sim cH_0$ suggests that there may be some fundamental relations between MOND and dark energy. In fact, there are many possible ways by which our model can be generalized to include dark energy as well. One way is that at late times when $H \sim H_0$ the dark matter decays into dark energy (e.g., its equation of state w becomes -1). The idea here is to use the quantity a_0 to determine both the transitions from CDM to MOND and from CDM to dark energy. A more straightforward method is to have a cosmological constant in $V(\mathcal{K})$: as is shown above, the MOND effect only depends on $V_{\mathcal{K}}$ but not $V(0)$, and we can use the dark energy density to fix $V(0)$ so that the combination of dark energy and MOND completely determines $V(\mathcal{K})$. Another interesting possibility is to note that in Eq. (13) the MOND effect requires $c_4 = -1$ while dark energy requires $c_4 \sim -\frac{1}{9}$. This can be easily achieved, again using our principle of environment dependence: let c_4 depend on $\mathcal{K}_3$, for example, with $c_4 = -(3\text{–}6w)^{-2}$. In this case $V(\mathcal{K})$ acts as an environment-dependent cosmological constant, which accounts for the cosmic acceleration in background cosmology ($w \to 0$) and approaches zero in the solar system ($w \to 1/3$). Note that dark energy and MOND are unified with a single $V(\mathcal{K})$ in the latter two possibilities, and we thus have a full Lagrangian as

$$\mathcal{L} = -2wa_0^2\left[\frac{X}{a_0^2}\right]^{1+w/2w} + V(\mathcal{K}), \tag{14}$$

in which $V(\mathcal{K}) = (9a_0)^2(1 - 2w)^2(1 + \frac{2x}{3})/(1 + \frac{x}{3})^2$ and $x = \sqrt{\mathcal{K}_4}/a_0$ for the third possibility. Interestingly, $a_0 \sim cH_0$ is a single mass scale introduced for this model to relate CDM, MOND, and dark energy together. All the other parameters (c_i's, n) are dimensionless and $\sim\mathcal{O}(1)$, and there are no fine-tunings of them: the huge difference between the dark matter density ρ_φ at earlier times and the scale $\frac{a_0^2}{8\pi G}$ comes as a generic result of the dynamical evolution of the vector field. In this sense the vector field acts as a leverage, making the tiny mass scale a_0 capable of characterizing the large energy density of dark matter. Meanwhile, this could also shed further light on the dark energy coincidence problem, since the dark energy dominance begins at the time when galaxies have formed (and we observers come into existence), both characterized by our fundamental mass scale a_0.

IV. SUMMARY

In this work we have tried to tackle the problem of how to unify CDM and MOND in a consistent way. The idea is to give the dark matter an environment dependence, making it behave like CDM on large scales, while reproducing the MOND (Newtonian dynamics) in the static and weak (strong) field limits, respectively. Although the idea of an environment dependence is not new, it is novel to use the vector ($Æ_a$) field as the switch. We show how the particular properties of the vector field make it very effective for this purpose. Our model provides a general framework which can potentially solve the problems of CDM on galactic scales and of MOND on larger scales. It could also be generalized to include dark energy in a way such that all the phenomena of CDM, MOND, and dark energy are related to one parameter a_0, which is the single mass scale introduced in our model. Both fields in Eq. (14), likely effective, should provide insights to people seeking the fundamental fields in particle physics theories.

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