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Cosmological constraints on the matter equation of state

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We investigate the impact of a nonstandard time evolution of the dark matter component on current cosmological bounds from cosmic microwave background (CMB) anisotropies. We found that a less than 0.1% variation in the effective dark matter equation of state $w_{\rm dm}$ can drastically change current CMB bounds on the matter density, the Hubble parameter and the age of the Universe. A flat universe without dark energy could provide an excellent fit to current CMB data, providing that $w_{\rm dm} \sim -10^{-2}$.

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I. INTRODUCTION

The last years have seen an extraordinary improvement in the quality and quantity of cosmological data. In particular, the measurements of cosmic Microwave Background (CMB) anisotropies from satellite experiments as well as the Wilkinson Microwave Anisotropy Probe (WMAP) have fully confirmed the predictions of the standard model of structure formation, based on dark matter and inflation [1]. When combined with the assumption of a flat universe, current CMB data analysis clearly shows the need of a dark energy component at the confidence level of several standard deviations. For example, the recent analysis of [2] shows that, from just CMB observations and in the framework of a flat universe, the energy density in a cosmological constant respect to the total energy density is constrained to $\Omega_\Lambda = 0.73 \pm 0.04$ at 68% c.l. This constraint is in full agreement with the most recent measurements of luminosity distances of type-Ia supernovae [3], when combined with complementary information as from galaxy clustering data [4].

Dark energy, however, introduces several theoretical problems, especially if interpreted as a cosmological constant (for recent reviews see e.g. [5,6]). While future data will be able to accurately determine some of the dark energy properties and possibly identify a theoretical candidate, another possibility is that current local data is strongly affected by systematic. Some authors, for example, are proposing that the emergence of local perturbations in the late universe could react back on the local geometry, mimicking an accelerated expansion in SN-Ia data (see e.g. [7] and references therein). In a few words, a conservative approach would be to not consider all current measurements of luminosity distance that yield a value of the reduced Hubble parameter of $h = 0.72 \pm 0.07$ with the Hubble parameter defined as $H_0 = 100h$ Km/s/Mpc [8] and a deceleration parameter $q_0 \sim -0.55$ [9] and to propose as a cosmological model a flat universe composed of just dark matter and with a Hubble parameter $H_0 \sim 45$ Km/s/Mpc. However, even if this model fits age constraints and reasonably reproduces the galaxy clustering on a large scale, is ruled out from CMB anisotropies. As is well known, a good fit to current WMAP data with a matter-dominated universe could be obtained assuming a positive curvature with $\Omega_m \sim 1.2$ (see e.g. [2]). A positive curved universe is clearly unattractive since it violates the common inflationary prediction for a flat universe (even if curved models could still be produced, see e.g. [10]). Some authors have therefore studied other possibilities for making a flat matter-dominated universe compatible with CMB data. The analysis of Sarkar *et al.* [11], for example, has showed that breaking the scale invariance in the primordial perturbations' inflationary spectrum could reconcile CMB anisotropies with a flat, matter-dominated, universe. In a few words, the CMB evidence for a cosmological constant depends on the assumption of flatness and of a scale invariant spectrum of primordial perturbations, which are both strong predictions from inflation.

In this paper we investigate the dependence of the evidence for a cosmological constant on the assumed properties of the dark matter component. Even if predicted by a wide range of models and theories, very little is known about the dark matter component. While most direct detection experiments are at the moment inconclusive, and no clear experimental evidence for a dark matter candidate has been reported, experiments such as DAMA/LIBRA [12] or PAMELA [13] have shown hints that could be interpreted as evidence for an (exotic) dark matter particle. It is therefore not implausible that a modification to the assumed nature of dark matter, and not of inflation, could be the solution to the dark energy puzzle.

Given the large amount of models and given our poor theoretical knowledge of dark matter, the main way to test for its properties would be to parametrize it as a fluid with a

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varying equation of state $w_{\rm dm}$, sound speed c_s^2 and anisotropic stresses $\sigma_{\rm dm}$ as in [14]. However, since we expect dark matter to be massive, we can neglect, for a first analysis, anisotropic stresses. Moreover, we can assume a constant equation of state and fit the data with an effective equation of state, integrating possible redshift dependence from the appearance of dark matter at equivalence, up to today. This approach has been already investigated in [15,16], where a fit to the current CMB data has been done varying a constant equation of state for dark matter. In this paper we update the previous constraints presented in those papers on $w_{\rm dm}$ using the most recent CMB data and we focus on the impact that small variations in $w_{\rm dm}$ can have in constraining the remaining standard cosmological parameters. In particular we focus on the possibility of having a flat universe without dark energy compatible with CMB data, but we will also look at the degeneracies with other parameters.

II. THE DARK MATTER EQUATION OF STATE

The class of cosmological models we consider in this work are spatially flat, Friedmann-Robertson-Walker space-times filled with ordinary matter (baryonic) and radiation, a dark energy acting as a cosmological constant and a dark matter component. The latter is modeled as a fluid which interacts with the other components only gravitationally and which is characterized by a generalized, constant equation of state (EoS):

$$w_{\rm dm} = \frac{p_{\rm dm}}{\rho_{\rm dm}}, \tag{1}$$

where $p_{\rm dm}$ and $\rho_{\rm dm}$ represent pressure and density, respectively. This is the case for $w_{\rm dm}$ time variation, small in comparison to the Universe's expansion rate. To completely describe the dark matter phenomenology we need to define its energy-momentum tensor. Allowing for perturbations at the linear order, the tensor's components in the synchronous gauge[1] read [17]:

$$\begin{aligned} T^0{}_0 &= -(\rho_{\rm dm} + \delta\rho_{\rm dm}),\\ T^0{}_i &= (\rho_{\rm dm} + p_{\rm dm})v_i = -T^i{}_0,\\ T^i{}_j &= (p_{\rm dm} + \delta p_{\rm dm})\delta^i{}_j, \end{aligned} \tag{2}$$

where $\delta\rho_{\rm dm}$ and $\delta p_{\rm dm}$ are the density and pressure perturbations, whereas v_i is the fluid velocity that for a fluid with a small coordinate velocity can be treated as a perturbation of the same order as the other two. As expected from the background homogeneity and isotropy, the properties of this component at the lowest order depend only on its EoS Eq. (1). In particular, since a noninteractive species is covariantly conserved, the evolution of the unperturbed dark matter density is given by

$$\frac{d\ln\rho_{\rm dm}}{d\ln((1+z)^{-1})} = -3(1 + w_{\rm dm}). \tag{3}$$

It follows that the Friedmann equation is

$$\left(\frac{H}{H_0}\right)^2 = \Omega_b(1+z)^3 + \Omega_{\rm dm}(1+z)^{3(1+w_{\rm dm})} + \Omega_r(1+z)^4 + \Omega_\Lambda, \tag{4}$$

expressed in terms of the density parameters, $\Omega_i = 8\pi G\rho_i/3H_0^2$, of each component i. H_0 is the Hubble constant, i.e. the expansion rate at present time. We want to stress that the background evolution is completely determined by the dark matter EoS. Nevertheless dark matter participates to gravitational instability, and to probe its remaining properties we have to investigate the behavior of its perturbations. Making no special assumptions about the nature of this component, our treatment of perturbations relies only on the speed of sound, a fundamental quantity in determining the clustering properties, hence it sets the Jeans scale of the fluid. For a perfect fluid it purely arises from adiabatic perturbations, and one can define the adiabatic speed of sound as

$$c_a^2 \equiv \frac{\dot p_{\rm dm}}{\dot\rho_{\rm dm}} = w_{\rm dm} - \frac{\dot w_{\rm dm}}{3H(1+w_{\rm dm})} = w_{\rm dm}, \tag{5}$$

where dots represent derivatives with respect to conformal time and the last equality holds for our assumption of a constant $w_{\rm dm}$. In this case $w_{\rm dm}$ is tightly constrained by observations [15,16]. In particular, in the regime $w_{\rm dm} < 0$ a concern could be that the sound speed becomes imaginary and small wave length perturbations are hydrodynamically unstable, driving a runaway growth of perturbations on these scales. This trend is not observed either as an excess in the small scale matter power spectrum nor as an excess in the CMB lensing due to the growing potentials. It appears then interesting to consider an *imperfect* fluid, in which dissipative processes could induce intrinsic entropic fluctuations that recover a non-negative square effective sound speed for the fluid [14]:

$$c_s^2 = \frac{\delta p_{\rm dm}}{\delta\rho_{\rm dm}} = c_a^2 + \frac{w_{\rm dm}\Gamma_{\rm dm}}{\delta_{\rm dm}}, \tag{6}$$

where $\Gamma_{\rm dm}$ is the entropy production rate and $\delta_{\rm dm}$ is the fluid density contrast.

The adiabatic sound speed, c_a, and the entropy production rate, $\Gamma_{\rm dm}$, are gauge invariant quantities. In general this is not true for the effective sound speed, that shares this property only in the dark matter rest frame, the only frame in which the density contrast, $\delta_{\rm dm}$, is gauge invariant (see Eq. (6)). We use the expression given in [18] to relate the gauge invariant, rest frame dark matter density contrast, $\hat\delta_{\rm dm}$, to the density contrast, $\delta_{\rm dm}$, and velocity perturbations in k-space, $\theta_{\rm dm}$, for the synchronous gauge:

$$\hat\delta_{\rm dm} = \delta_{\rm dm} + 3H(1+w_{\rm dm})\frac{\theta_{\rm dm}}{k^2}. \tag{7}$$

[1]In the synchronous gauge the lines of constant space-coordinates are orthogonal to the constant time hypersurfaces.

Combining Eqs. (6) and (7) we can derive the expression of the pressure perturbation $\delta p_{\rm dm}$, as a function of the rest frame effective sound speed, $\hat{c}_s$:

$$\delta p_i = \hat{c}_s^2 \delta\rho_{\rm dm} + 3H(1 + w_{\rm dm})(\hat{c}_s^2 - c_a^2)\rho_{\rm dm}\frac{\theta_{\rm dm}}{k^2}. \quad (8)$$

The energy-momentum conservation yields the equations for the evolution of the density and velocity perturbations. In the synchronous gauge these have been derived in [17] and substituting Eq. (8) one gets [19]:

$$\dot{\delta}_{\rm dm} = -(1 + w_{\rm dm})\left\{[k^2 + 9H^2(\hat{c}_s^2 - c_a^2)]\frac{\theta_{\rm dm}}{k^2} + \frac{\dot{h}_L}{2}\right\} - 3H(\hat{c}_s^2 - w_{\rm dm})\delta_{\rm dm} \quad (9)$$

$$\frac{\dot{\theta}_{\rm dm}}{k^2} = -H(1 - 3\hat{c}_s^2)\frac{\theta_{\rm dm}}{k^2} + \frac{\hat{c}_s^2}{1 + w_{\rm dm}}\delta_{\rm dm}, \quad (10)$$

where h_L are the metric perturbation sources.

Equations (9) and (10), together with Eq. (4), can be implemented into any of the standard codes that solve the Einstein-Boltzmann equations. For this purpose we have modified the publicly available code CAMB[2] and derived constraints on the dark matter EoS by comparing theoretical predictions thus obtained with cosmological data sets.

III. DARK MATTER EQUATION OF STATE AND PARAMETER DEGENERACY

In practice, any cosmological observable that depends on the expansion rate of the Universe will be sensitive to the background properties of the dark matter, namely, to $\Omega_{\rm dm}$ and $w_{\rm dm}$. Unfortunately, no single kind of measurement can isolate the EoS on its own [20]. In particular, CMB data sets are effective in constraining the baryon and dark matter physical densities $\Omega_b h^2$, $\Omega_{\rm dm} h^2$ and the angular diameter distance to the last scattering surface $d_A(\Omega_b h^2, \Omega_{\rm dm} h^2, w_{\rm dm}, \Omega_\Lambda)$, through the morphology and the angular location of the acoustic peak pattern. Conversely, $\Omega_{\rm dm}$, $w_{\rm dm}$ and h cannot be determined individually with high precision due to a degeneracy: different combinations of the values of these parameters lead to the same angular power spectrum of the CMB anisotropy. An example of this degeneracy can be see in Fig. 1, where we plot degenerate angular power spectra for different values of $w_{\rm dm}$, h and $\Omega_{\rm dm}$ for a flat universe. The sound speed is fixed to $c_s^2 = 0$ in order to be consistent with large-scale structure data.

As we can see, decreasing the Hubble parameter and $w_{\rm dm}$ while increasing the matter density provides nearly identical power spectra. Since we are considering flat universes this means that the introduction of $w_{\rm dm}$ allows the possibility of having purely matter-dominated flat models compatible with CMB data.

[2]http://camb.info/.

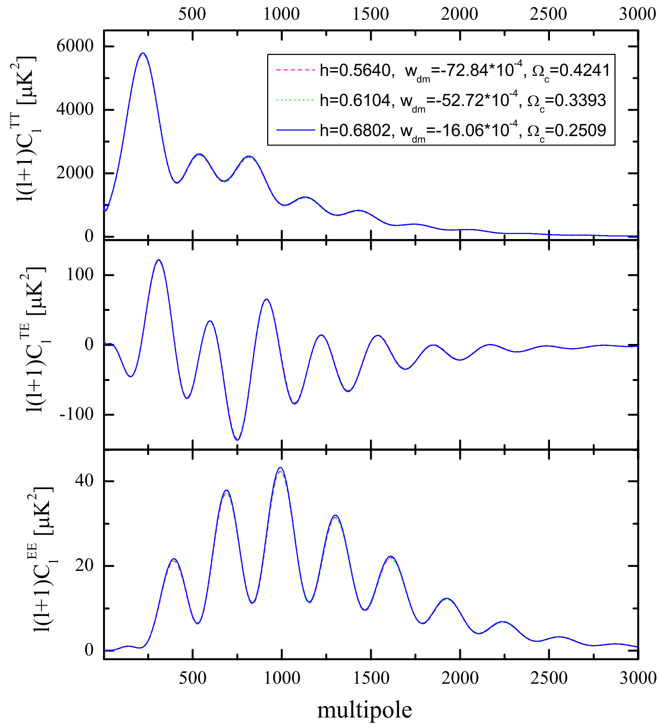

FIG. 1 (color online). Degenerate CMB spectra. In the case of a flat space, for different combinations of the values of the Hubble parameter, the dark matter equation of state parameter and the dark matter density parameter we obtain exactly the same TT, TE and EE CMB angular power spectra.

We indeed show this in Fig. 2 where we plot two degenerate spectra. As we can see a model with a negligible cosmological constant could reproduce the same spectra with a Hubble parameter $h \sim 0.4$.

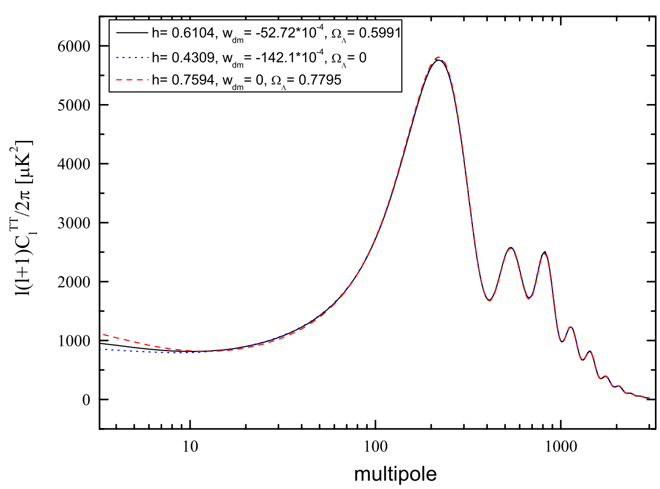

FIG. 2 (color online). Degenerate spectra. A model with a negligible cosmological constant can provide an equally good fit than Λ-CDM in a flat universe providing that $w_{\rm dm} < 0$. The only difference appears at large angular scales due to the ISW effect.

TABLE I. Limits on $w_{\rm dm}$ from WMAP data only marginalizing over a cosmological constant (first row) and imposing $\Omega_\Lambda = 0$ (second row), and from a larger sample of CMB experiments, plus SDSS and supernovae SNLS (third row). We report errors at 68% and 95% confidence level.

Experiment	Limits on $w_{\rm dm}$
WMAP	$-0.35^{+0.56+1.17}_{-0.58-0.98} \cdot 10^{-2}$
WMAP ($\Omega_\Lambda = 0$)	$-1.39^{+0.16+0.34}_{-0.54-0.95} \cdot 10^{-2}$
All CMB + SDSS + SNLS	$0.07^{+0.21+0.41}_{-0.21-0.42} \cdot 10^{-2}$

To alleviate these degeneracies one may include other cosmological data sets, e.g. large galaxy redshift surveys and supernovae measures.

IV. ANALYSIS METHOD

We constrain the dark matter parameter of state defined in the previous section by a COSMOMC analysis of a large set of cosmological data. The analysis method we adopt is based on the publicly available Markov Chain Monte Carlo package `cosmomc` [21] with a convergence diagnostics done through the Gelman and Rubin statistics. We sample the following nine-dimensional set of cosmological parameters, adopting flat priors on them: the baryon and cold dark matter densities ω_b and ω_c, the ratio of the sound horizon to the angular diameter distance at decoupling, θ_s, the scalar spectral index n_s, the overall normalization of the spectrum A_s at $k = 0.05$ Mpc^{-1}, the optical depth to reionization, τ, the age of the Universe Age/GYr, the Hubble parameter H_0 and, finally, the dark matter equation of state parameter $w_{\rm dm}$. Furthermore, we consider purely adiabatic initial conditions, we impose spatial flatness and we only consider models with $c_s^2 = 0$. In our analysis we always include a cosmological constant such that $\Omega_\Lambda = 1 - (\omega_c + \omega_b)/h^2$.

Our basic data set is the five-year WMAP data [1,2] (temperature and polarization) with the routine for computing the likelihood supplied by the WMAP team. We also consider a larger data set: this adds other CMB experiments like Acbar [22], Boomerang 2K2 [23] and CBI [24] but also the large-scale structure data in form of the Red Luminous Galaxies power spectrum [25], the supernovae measurements from SuperNova Legacy Survey (SNLS) [9], a prior on the Hubble's constant from the Hubble Key project [8] and, finally, a big bang nucleosynthesis prior of $\omega_b = 0.022 \pm 0.002$ at 68% c.l. to help break degeneracies.

V. CONSTRAINTS ON THE DARK MATTER EQUATION OF STATE PARAMETER AND IMPACT ON COSMOLOGICAL PARAMETERS

In Table I we report the constraints on the $w_{\rm dm}$ parameter obtained from different COSMOMC analyses. As we can see from the first and third row in the table, the most recent

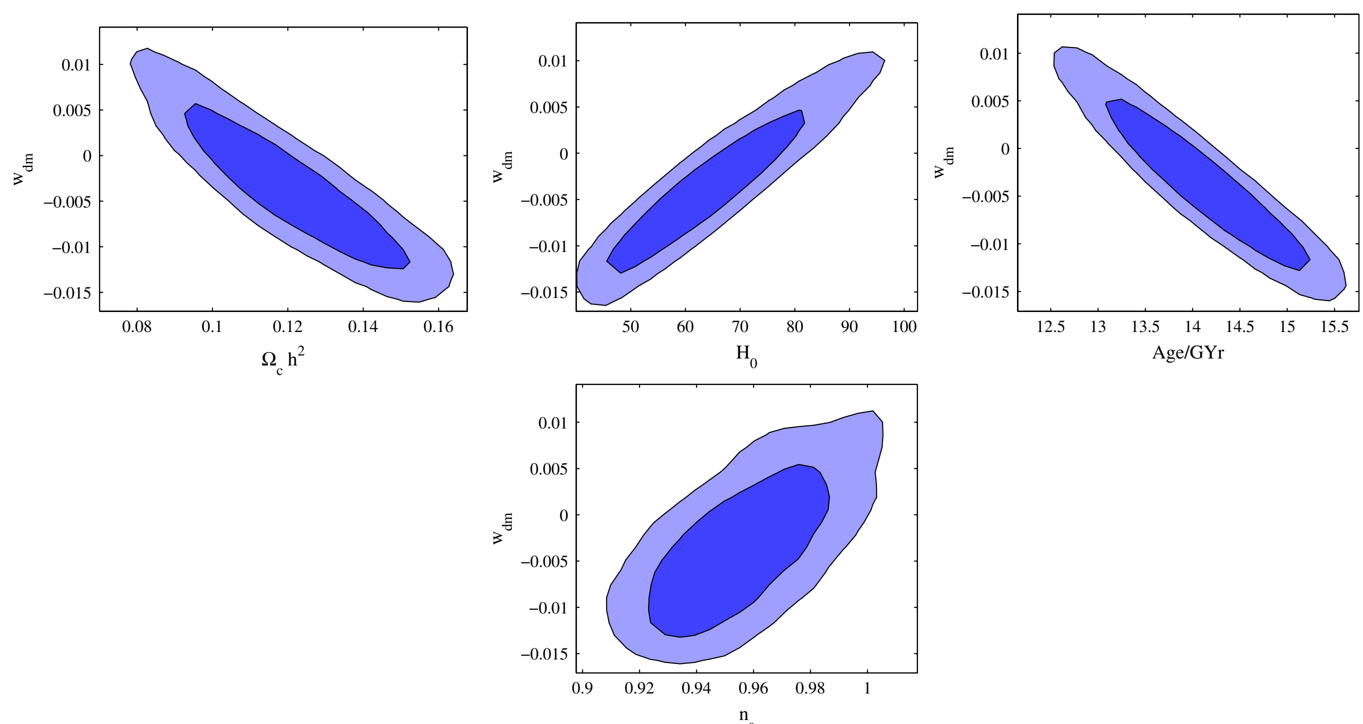

FIG. 3 (color online). 68% and 95% c.l. constraints on the $w_{\rm dm}$ vs (clockwise) the dark matter density, the Hubble parameter, the age of the Universe and the spectral index of primordial fluctuations from the WMAP data set. The results are obtained after marginalizing over a cosmological constant.

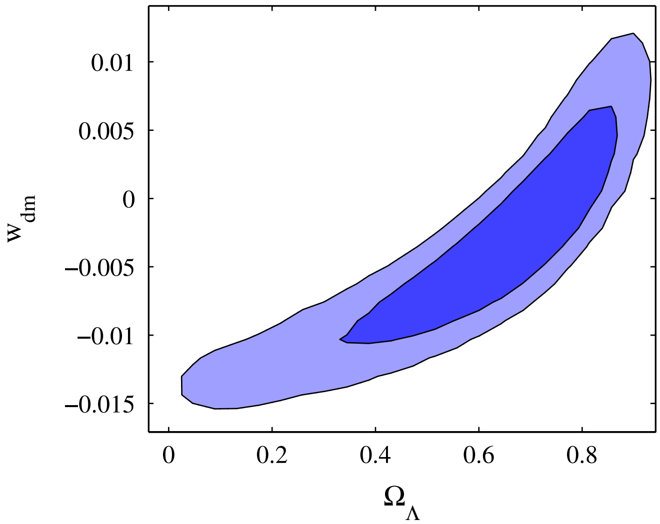

FIG. 4 (color online). Cosmological constraints on the Ω_Λ-w_{dm} plane from the WMAP. A degeneracy is clearly present.

data strongly constrains possible variations in w_{dm}. Also, reported in the table are the constraints on w_{dm} imposing a prior $\Omega_\Lambda = 0$. As we can see, in this case, w_{dm} must be different from 0 at more than 95% c.l. in agreement with the discussion presented in the previous section.

It is interesting to investigate the possible correlations between w_{dm} and the remaining, standard, cosmological parameters. We can notice the correlations in Fig. 3 where we plot the 2-D contours between w_{dm} and the dark matter density, the Hubble parameter, the age of the Universe, and the scalar spectral index, respectively, for WMAP data. The values are obtained after marginalizing over a cosmological constant. As we can see lower values of w_{dm} will make cosmological models with higher matter density and age of the Universe more compatible with the data. Vice versa, models with a higher Hubble parameter and scalar spectral index will be in agreement with CMB data if w_{dm} is lower than zero.

Since a model with larger matter density and hence a smaller cosmological constant can be put in agreement

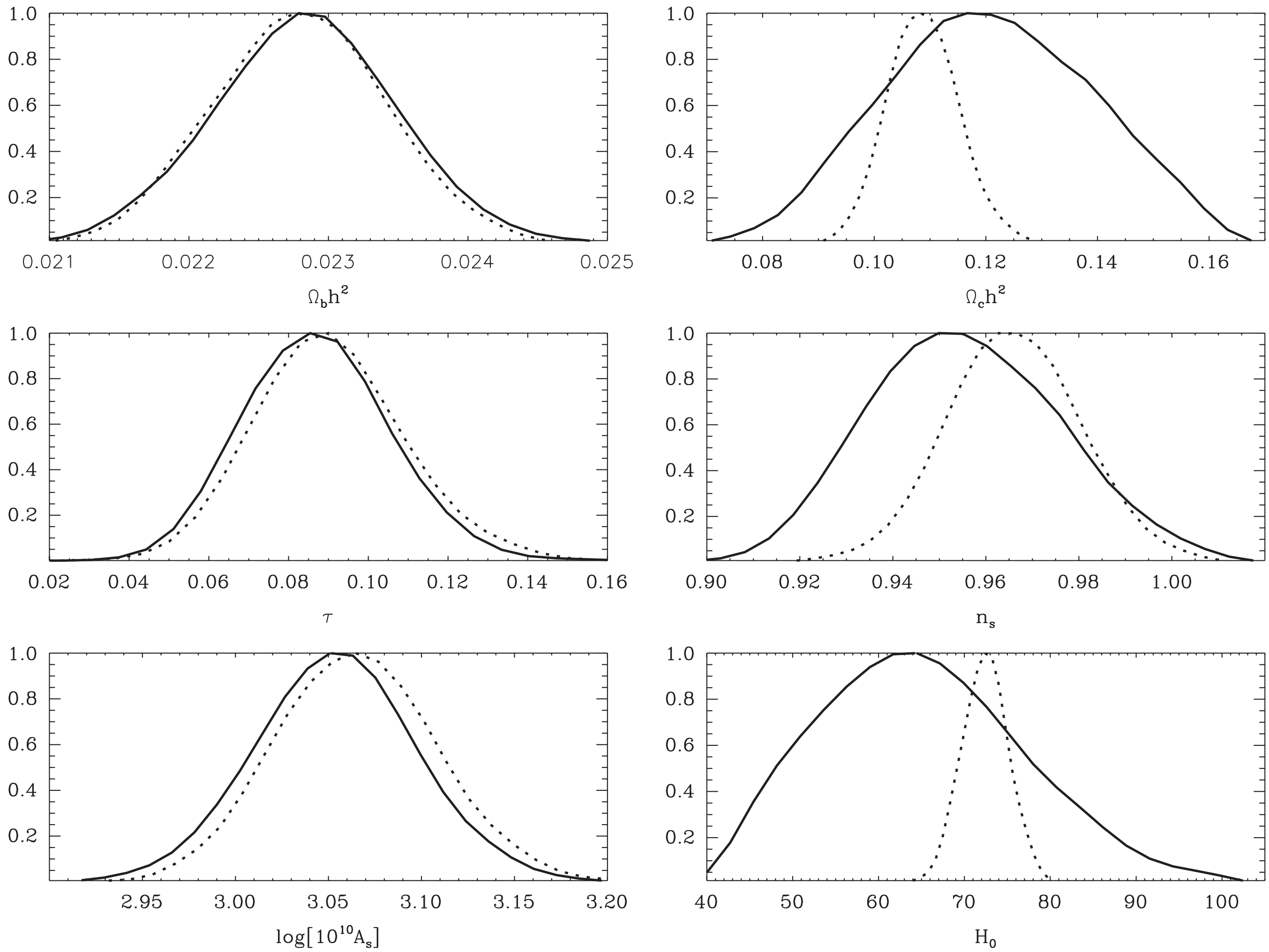

FIG. 5. Cosmological parameters marginalized 1-D likelihood obtained analyzing the WMAP data set once considering a standard model with $w_{dm} = 0$ (dashed line) and once considering a model with w_{dm} allowed to vary (solid line).

with observations it is interesting to consider the 2-D constraints on the $w_{\rm dm}$-Ω_Λ plane as we do in Fig. 4. As we can see, models without a cosmological constant can be in agreement with the data provided that $w_{\rm dm} \sim -0.015$. More specifically, we found that the best-fit flat cosmological model without a cosmological constant is only $\Delta\chi^2 \sim 3.8$ away from the overall best fit.

The impact of a variation in $w_{\rm dm}$ on constraining cosmological parameters can be clearly seen from Fig. 5, where we plot the 1-D likelihoods for several cosmological parameters in the case when $w_{\rm dm}$ is fixed to zero and when we let it vary. The major deviations are on the matter density, the Hubble parameter and the scalar spectral index, while constraints on the optical depth and the baryon density are left practically unchanged. In particular, we found the following constraints when variations in $w_{\rm dm}$ are permitted: $\Omega_c h^2 = 0.121 \pm 0.019$, $n_s = 0.956 \pm 0.02$ and $h = 0.65 \pm 0.12$ at 68% c.l. to be compared with the standard constraints ($w_{\rm dm}$) $\Omega_c h^2 = 0.1088 \pm 0.006$, $n_s = 0.966 \pm 0.014$ and $h = 0.724 \pm 0.025$.

Given the above degeneracies, it is clear that including extra data sets improves the constraints on $w_{\rm dm}$. Combining the CMB data with Sn-Ia, SDSS and Hubble Space Telescope (HST) constraints we improve the bound on $w_{\rm dm} = (0.07 \pm 0.4) \cdot 10^{-2}$ at 95% c.l. In this case the constraints on the remaining parameters are $\Omega_c h^2 = 0.104 \pm 0.010$, $n_s = 0.970 \pm 0.013$ and $h = 0.756 \pm 0.037$ at 68% c.l. As we can see, even if $w_{\rm dm}$ is better determined, the constraints obtained are still weaker than those obtained with $w_{\rm dm} = 0$ and using just WMAP data.

VI. CONCLUSIONS

In this brief paper we have investigated the constraints on the dark matter equation of state from current CMB observations. We have updated previous results and investigated in more detail the degeneracies present between $w_{\rm dm}$ and the remaining cosmological parameters. We have found that the latest data from the five-year survey of the WMAP satellite provide the constraint $w_{\rm dm} = -0.35^{+1.17}_{-0.98} \cdot 10^{-2}$ at 95% c.l. This constraint is comparable with the previous constraint obtained by [15] with the WMAP first-year data but combined with galaxy clustering and supernovae data. We have however shown the presence of degeneracies between $w_{\rm dm}$ and other parameters. We have indeed found that flat models with a smaller Hubble parameter and a higher matter density can exactly reproduce the same CMB angular spectra of the WMAP best-fit model when $w_{\rm dm}$ is lowered. In this respect we have found that the current CMB constraints on h and ω_m are strongly based on the assumption of $w_{\rm dm} = 0$. Moreover, we found that a flat model with zero cosmological constant is perfectly compatible with the CMB data if $w_{\rm dm} \sim -0.015$. The main question is of course if such an equation of state is physically acceptable. Models of dark matter particles with a varying equation of state have been proposed. For example, [26] showed that a gas of particles interacting with a condensate that spontaneously breaks Lorentz invariance has an equation of state that varies from $1/3$ to less than -1. Since what we need here is a $\sim$1% variation in the dark matter equation of state, it may be possible to think of an exotic dark matter component, additional to standard cold dark matter, with a varying equation of state around recombination, such that the final effect would be to bring the total, effective, dark matter equation of state to $w_{\rm eff} \sim -0.015$.

Finally, we have found that combining the CMB data with complementary observables such as SN-Ia and HST drastically improves the constraint on $w_{\rm dm}$. A global analysis of current cosmological data does not prefer a variation on $w_{\rm dm}$ over a cosmological constant. In particular, the expected amplitude of the Integrated Sachs-Wolfe Effect, recently detected by several analysis (see e.g. [27,28]), while nonzero in varying $w_{\rm dm}$ models, is definitely too small to match current observations. However, we have shown that also in the case of a global analysis, the impact of a varying $w_{\rm dm}$ is non-negligible and strongly affects the current determinations of most of the parameters.

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