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Stagflation: Bose-Einstein condensation in the early universe

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Our universe experienced the accelerated expansion at least twice; an extreme inflationary acceleration in the early universe and the recent mild acceleration. By introducing the Bose-Einstein condensation (BEC) phase of a boson field, we have been developing a unified model of dark energy (DE) and dark matter (DM) for the later mild acceleration. In this scenario, two phases of BEC (= DE) and normal gas (= DM) transform with each other through BEC phase transition. This unified model has successfully explained the mild acceleration as an attractor. We extend this BEC cosmology to the early universe without introducing new ingredients. In this scenario, the inflation is naturally initiated by the condensation of the bosons in the huge vacuum energy. This inflation and even the cosmic expansion eventually terminates exactly at zero energy density. We call this stage as stagflation. At this stagflation era, particle production and the decay of BEC take place. The former makes the universe turn into the standard hot big bang stage and the latter makes the cosmological constant vanishingly small after the inflation. Furthermore, we calculate the density fluctuations produced in this model, which turns out to be in the range allowed by the present observational data. We also show that the stagflation is quite robust and easily appears when one allows negative region of the potential. Further, we comment on the possibility that BEC generation/decay series might have continued all the time in the cosmic history from the inflation to present.

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I. INTRODUCTION

Accelerated expansion of the universe seems not to be exceptional. Theoretical and observational studies have revealed that the universe experienced the era of accelerated cosmic expansion at least twice; an extreme inflationary acceleration in the early universe (EA) [1] and a late mild acceleration (LA) [2,3]. We would like to figure out the basic physics behind these accelerations and reveal the inevitability, if any, of these accelerations. In the context of the Einstein equation, we need special matter which is endowed with strong negative pressure in order to guarantee the accelerated cosmic expansion.

However, it is often very difficult to find such matter in baryons, and therefore new fields such as the *classical scalar field* has been hypothesized though we have no fundamental understanding of it. Probably reflecting this uncertainty, this field has been named differently in various contexts in the literature such as Higgs field, inflaton in EA, and *K*-essence, tachyon, quintessence, Chaplygin gas, phantom field, ghosts, etc. in LA.

We do not want to be involved in these particular contexts but instead, we would like to reconsider how the degrees of freedom of such scalar field can arise within a firm physics even if we still cannot fully identify this field. To this aim, we reconfirm the fact that the basic physics which describes the universe from the fundamental level is no doubt the quantum field theory. Then how the classical dynamical degrees of freedom of scalar field can arise within this theory? The most natural mechanism we can consider would be the *Bose-Einstein condensation* (BEC) of some boson field [4]. This condensation process is a phase transition and a typical mechanism that a classical degrees of freedom emerges as an order parameter within the quantum field theory [5].

Along this line of thought, we have developed a cosmological scenario based on BEC in order to explain LA in the contemporary universe [6–8]. In this scenario, the dark energy (DE) is identified with the BEC of the bosons and the dark matter (DM) with the excited gas of the bosons. This cosmological situation is analogous to the liquid ^{4}He system in the laboratory in the sense that the two distinguished components, *super and normal, coexist and interchange with each other*. Our unified model of DE and DM has been successful to describe the contemporary universe so far.

In this paper, we extend this BEC cosmology to the early universe in order to examine how our BEC cosmology reveal another acceleration EA. Since we treat LA and EA on an equal footing, we should not introduce any new ingredients into the previously developed BEC cosmology except for the cosmological background. Thus the goal of this paper is to examine to what extent this BEC cosmology can be verified when applied to the early universe. We are especially interested in how the mechanisms of LA and EA are different with each other in BEC cosmology. We remember that LA, described in BEC cosmology, has been an attractor [7,8] which guarantees its autonomous realization. Can we also expect EA be an autonomous mechanism in BEC cosmology?

More precisely, when we analyze EA, we should clarify the *autonomous initiation and termination processes of the inflationary era*, which naturally turns into *the reheated* radiation dominated era. Actually, the BEC makes the order parameter develop from zero to some finite value, and therefore the inflationary dynamics with finite duration would initiate autonomously if sufficient vacuum energy is provided by quantum fluctuations or something. We would verify this mechanism first. That is, we derive BEC production rate from the microscopic Lagrangian by using the generalized effective action in the in-in formalism. As for the termination of the inflationary era, it may seem to be impossible to control the ever increasing order parameter since our BEC model assumes the negative quartic interaction (attractive force) without bound. However it is often seen in many physical phenomena that a violently unstable mode is taken over by an another stable mode via complex nonlinear processes. We will show that this is the case for BEC cosmology. Actually an extreme development of the order parameter completely prevents the violent cosmic expansion. We call this process as *stagflation*, where the order parameter ever inflates while the cosmic expansion becomes stagnant. Thus the inflationary era terminates autonomously within a finite time scale. Moreover, this extreme development of the ever accelerating order parameter easily gives rise to the particle production, which provides a natural process of the reheating. Furthermore, the instability analysis reveals that the homogeneous mode immediately decays into the inhomogeneous localized modes at this stagflation era, when the cosmic expansion ceases.

This stagflation era also provides a key mechanism to solve the *cosmological constant problem*. The issue is to explain the dynamics of the cosmological 'constant' Λ, which had once been a huge value $(10^{21-26}\ \mathrm{eV})^2$ that makes EA, turns into a tiny but nonzero value about $(10^{-31}\ \mathrm{eV})^2$ that makes LA. Though this initial huge value for Λ would be naturally provided from ubiquitous vacuum energy, there should be a certain mechanism for the very fine tuning of Λ to the vanishingly small but finite value at present. In the previous analysis of BEC cosmology applied to LA, we assumed that the zero-point of the potential energy is exactly zero, and shown that the vanishingly small but finite value of the effective Λ term appears as a fixed point, in which the condensation speed and the potential force balance with each other. In this paper, we propose a new mechanism which autonomously adjust the effective Λ term exactly zero. This would justify the above assumption for LA. The essence of this Λ-extinction mechanism is the fact that *the stagflation takes place exactly at the zero total energy*. As explained above, the uniform mode of BEC becomes unstable at this stagflation point and suddenly decays into localized objects. Thus after the inflation, the universe evolves into the stagflation point, where Λ becomes zero and then the universe becomes dominated by the reheated radiation. We would like to quantitatively verify this mechanism in this paper. It will turn out that the Λ-extinction is asymptotically realized after many BEC decays.

The above instability of the uniform mode is the same as that appears in BEC cosmology application in LA. Moreover, the condensation process of BEC commonly triggers all the essential dynamics of the cosmic history in both the cases of EA and LA. It is important to notice that these decay due to instability and the condensation of BEC are not a coherent time change described by any Lagrangian but are rather phase transitions and the incoherent evolution with dissipation and fluctuations. This point is further discussed in the following sections.

This paper is organized as follows. After applying the in-in formalism into BEC formation, we review the essence of BEC cosmology applied in LA in Sec. II. In Sec. III, we apply the BEC cosmology to the early universe in the order of initiation, termination of inflation, reheating, cosmological constant problem, and generation of density fluctuations. In the last Sec. IV, we summarize our study emphasizing the robust stagflation and clarifying unsolved problems at present.

II. BASICS OF THE BEC COSMOLOGY

A. BEC in the universe

In this section, we first review our model of BEC cosmology from the microscopic physics. Various observations and theoretical analysis have revealed that the DE and DM dominate the matter contents of the universe and DE/DM ratio is of order unity at present [3]. These facts may indicate some deep relation between DE and DM. Therefore it seems natural to construct a unified model of DE and DM based on a single physical principle. The simplest thought will be to consider that both DE and DM are actually the same dynamical matter but in different phases of existence. DE phase should be almost uniformly distributed and is supposed to provide accelerated cosmic expansion. On the other hand, DM phase should be inhomogeneously distributed and is supposed to trace the structure of galaxies and clusters.

According to the Einstein equation, the acceleration of the cosmic expansion requires isotropic negative pressure comparable to the energy density $p < -\rho/3$ for DE. This is because, in general relativity, pressure as well as energy density works as a source of gravity. Such strong negative pressure sounds strange if we consider fermionic fields which constitute ordinary baryonic matter. On the other hand for bosons, familiar static magnetic field $\vec{H}$, for example, shows such behavior $p = -\rho = -\vec{H}/(8\pi)$ in one spatial direction, reflecting that the magnetic force line has a positive tension. This negative pressure would be the origin of the variety of bursts and accelerations in solar physics such as flares and coronas. Unfortunately this is a vector field and shows positive pressure in the other two spatial directions. A natural possibility to obtain the isotropic negative pressure is to consider the classical scalar field. This would be the standard line of thought in the modern cosmology as explained in the introduction.

However, when we construct a unified model of DE and DM, we further have to specify the origin of such scalar field in order to clarify the connection between DE and DM. The problem is to find a phase which behaves as classical scalar field in the matter which is originally described by quantum physics. The most natural case would be the quantum condensation of bosons in low temperature, *i.e.* the Bose-Einstein condensation. Macroscopic number of boson particles share a single ground state in common and behaves as a coherent wave. This condensed phase is well described by a classical mean field. On the other hand the rest of the boson gas in excited states should macroscopically behaves as classical gas. Therefore we naturally arrive at the thought that the boson gas is composed of two phases, condensed phase and gas phase, each of which can be regarded as DE and DM, respectively. Thus the BEC cosmology arises. Since the basic structure of the universe in this model is quantum physical and cosmological, we have a unique mass scale $m = (H_0^2 h^3/(Gc^3))^{1/4} = 0.0092$ eV obtained by equating the present energy density $\rho = H_0^2/G$ to the energy density associated with mass m and the Compton wave length $l = h/(mc)$. This scale characterizes the cosmic BEC.

The above two-phase structure is quite common in physics in the sense that any special interactions among boson particles are not required for BEC and it inevitably takes place provided the temperature is lower than the critical one. A well known such structure would be the liquid ^{4}He system in the laboratory. The super phase and normal phase correspond, respectively, to DE and DM; they coexist and interchange with each other. Although the origin of the two-phase is quantum phase transition, the condensed phase is a classical mean field and should not be confused with the macroscopic quantum state, which is described by a wave function.

The BEC of boson mass m takes place provided the de Broglie wave length $\lambda_{dB} \equiv \sqrt{2\pi\hbar^2/(mkT)}$ exceeds the mean separation $n^{1/3}$

$$kT < kT_{\rm cr} \equiv \frac{2\pi\hbar^2 n^{2/3}}{(\zeta(3/2))^{2/3} m}. \tag{1}$$

This condition requires the boson gas to be 'cold', if we were to realize BEC for reasonable boson mass appropriate for DM. Actually in the universe, if the boson is the cold dark matter, irrespective of its interaction, its temperature (or quasitemperature) behaves as the same as above $T \propto n^{3/2}$ in the cosmic expansion. Therefore BEC can always take place and continue provided the boson temperature was less than the critical temperature at some moment of cosmic evolution. The corresponding critical boson mass for BEC can be arbitrary in general, but if we temporary assume that the boson was in equilibrium with the rest of the matter in the past, then the upper limit of the mass turns out to be about 19 eV [8].

In the universe the ratio of DE and DM cannot simply be given by the standard expression for the ratio of condensation $1 - (T/T_{\rm cr})^{3/2}$ in equilibrium with the temperature T. The universe is actually almost adiabatic and therefore the transition toward the ground state is forbidden in the first approximation. However a small nonadiabaticity allows the very slow condensation. We need a firm formulation which describes this condensation process based on microscopic physics. The most appropriate, but not necessarily complete, method for this purpose would be the generalized effective action formalism [9–11], on which we now briefly explain.

The partition function, which describes all the fluctuations, is given by

$$\tilde{Z}[\tilde{J}] \equiv {\rm tr}\left[\tilde{T}\left(\exp\left[i\int_c \tilde{J}\,\tilde{\phi}\right]\rho\right)\right] \equiv \exp[i\tilde{W}[\tilde{J}]], \tag{2}$$

where the integration is taken over the entire space and the *doubled time contour* which extends from $t = -\infty$ toward $t = +\infty$ and then back to $t = -\infty$ again. The symbol $\tilde{T}$ rearranges the operators on its right hand side in the order of this doubled time contour. All the quantities with overtilde represents that they are defined on this contour. $\tilde{J}$ is the external source field, ρ is the initial density matrix, and the quantum field ϕ is in the Heisenberg representation. This partition function can also be represented in the interaction picture as

$$\tilde{Z}[\tilde{J}] = {\rm tr}\left[\tilde{T}\left(\exp\left[i\int \tilde{J}\,\tilde{\phi} - i\int V(\phi)\right]\rho\right)\right] \tag{3}$$

$$= \exp\left[\tilde{T}\int V\left[\frac{\delta}{i\delta\tilde{J}}\right]\right]\exp\left[-\frac{i}{2}\iint \tilde{J}(x)\tilde{G}_0(x,y)\tilde{J}(y)\right] \times {\rm tr}\left[:\exp\left[i\int \tilde{J}\,\tilde{\phi}\right]:\rho\right] \tag{4}$$

where the Lagrangian is decomposed as the free part $L_0[\phi]$ and the interaction part $V[\phi]$, $L[\phi] = L_0[\phi] - V[\phi]$. The Wick theorem is used for the last equality. It is useful to introduce a vector and matrix representation for the functions on the generalized time contour; a field $\tilde{X}(x)$ can be written as $X_+(x) + X_-(x)$, where the quantity with suffix $+(-)$ has its support only on the forward (respectively, backward) time contour. Then $\tilde{X}(x)$ can be represented as a vector $(X_+(x), X_-(x))$. For example $\tilde{J}(x) = (J_+(x), J_-(x))$ and $\tilde{\phi}(x) = (\phi_+(x), \phi_-(x))$. We also use the notation $X_\Delta(x) \equiv X_+(x) - X_-(x)$ and $X_C(x) \equiv (X_+(x) + X_-(x))/2$ in the following. Then the notation $\int \tilde{J}\,\tilde{\phi}$ means

$\int \tilde{J}\tilde{\phi} = \int dx J_+(x)\phi_+(x) - \int dx J_-(x)\phi_-(x)$, where the subtraction on the right hand side represents the fact that the direction is opposite in the backward time contour. This can be conveniently expressed in the two-by-two matrix notation,

$$\int \tilde{J}\tilde{\phi} = \int dx (J_+(x), J_-(x))\sigma_3(\phi_+(x), \phi_-(x))^T, \quad (5)$$

where

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and T is the transpose.

The generalized two-point function $\tilde{G}_0(x, y) \equiv -i\langle \tilde{T}\phi(x)\phi(y)\rangle$ is also expressed in this matrix notation as

$$\tilde{G}_0(x, y) = \begin{pmatrix} G_F(x, y) & G_+(x, y) \\ G_-(x, y) & G_{\bar{F}}(x, y) \end{pmatrix} = \begin{pmatrix} -i\langle T\phi(x)\phi(y)\rangle & -i\langle \phi(y)\phi(x)\rangle \\ -i\langle \phi(x)\phi(y)\rangle & -i\langle \bar{T}\phi(x)\phi(y)\rangle \end{pmatrix}. \quad (6)$$

The fact that the only three of them are independent is most clearly observed in their momentum representations,

$$\hat{G}_F(k) = \frac{-D(k) + iB(k)}{D(k)^2 + A(k)^2}, \qquad \hat{G}_{\bar{F}}(k) = \frac{-D(k) - iB(k)}{D(k)^2 + A(k)^2},$$
$$\hat{G}_\pm(k) = i\frac{A(k) \mp B(k)}{D(k)^2 + A(k)^2}, \quad (7)$$

where $A(k)$, $B(k)$, and $D(k)$ are mutually independent functions.

If we were to define the classical field as $\tilde{\varphi}(x) \equiv \delta\tilde{W}/\delta\tilde{J}$ and define the Legendre transform $\tilde{\Gamma}[\tilde{\varphi}] \equiv W[\tilde{J}] - \int \tilde{J}\tilde{\varphi}$ as the effective action as usual, then by definition, the equality holds $\delta\tilde{\Gamma}/\delta\tilde{\varphi}(x) = -\tilde{J}(x)$, which may be regarded as the equation of motion for $\tilde{\varphi}(x)$ including all the quantum fluctuations. However, reflecting the fact that the effective action is not in general real $\tilde{\Gamma} = \mathrm{Re}\Gamma + i\,\mathrm{Im}\Gamma$, the above method makes the field $\tilde{\varphi}(x)$ unphysically complex. This ordinary procedure cannot describe the phase transition dynamics. Therefore we go beyond the standard method and decompose the effective action. We first notice that $\mathrm{Im}\Gamma$ is even in the variable $\varphi_\Delta(x) \equiv \varphi_+(x) - \varphi_-(x)$,

$$\mathrm{Im}\,\tilde{\Gamma} = \frac{1}{2}\iint \varphi_\Delta(x)B(x-y)\varphi_\Delta(y)dxdy + \ldots \quad (8)$$

and the kernel $B(x - y)$ is positive definite [9]. Therefore, introducing an auxiliary real field $\xi(x)$, we have the following path-integral expression.

$$e^{i\tilde{\Gamma}[\varphi]} = \int [d\xi]P[\xi]\exp\left[i\,\mathrm{Re}\Gamma + \int i\xi(x)\varphi_\Delta(x)dx\right] \quad (9)$$

where the weight function is defined as

$$P[\xi] = N\exp\left[-\frac{1}{2}\iint \xi(x)B^{-1}(x-y)\xi(y)dxdy\right], \quad (10)$$

where N is the normalization factor. Thus the field $\xi(x)$ can be regarded as Gaussian random field. Inclusion of higher order terms in the above makes us possible to treat the non-Gaussian properties as well. On the other hand, the real part of the action

$$\mathrm{Re}\,\Gamma + \int \xi\varphi_\Delta \quad (11)$$

should be regarded as the standard action which describes dynamics. Applying the variational principle for the variable $\varphi_\Delta(x)$

$$\left(\frac{\delta(\mathrm{Re}\Gamma + \int \xi\varphi_\Delta)}{\delta\varphi_\Delta(x)}\right)_{\varphi_\Delta(x)=0} = -J_C(x) \quad (12)$$

we obtain the evolution equation for $\varphi_C(x) \equiv (\varphi_+(x) + \varphi_-(x))/2$, which is simply denoted as $\varphi(x)$ hereafter,

$$(\partial\partial + m^2)\varphi + \int_{-\infty}^{t} dt' \int dx' A(x - x')\varphi(x') = \xi(x), \quad (13)$$

where the source term $J_C(x)$ is dropped. This is the Langevin equation for the classical order parameter $\varphi(x)$ with classical random field $\xi(x)$. Actually, if we define the statistical average by

$$\langle \ldots \rangle_\xi \equiv \int [d\xi]\ldots P[\xi], \quad (14)$$

then the correlation function for the auxiliary fields becomes

$$\langle \xi(x)\xi(y)\rangle_\xi = B(x - y). \quad (15)$$

Moreover, Eq. (13) is necessarily causal [9] since the kernel $A(x - x')$ has the property $A(x - x') = 0$ for $x^0 < x'^0$. Thus the full effective action is now rewritten as the statistical superposition of dynamical effective action. Note that the above dissipative nature arises as a result of an implicit and systematic coarse graining of the original full theory through the process of renormalization in the in-in formalism. This renormalization process is generally inevitable in quantum field theory to obtain finite physical quantities. The dissipative nature however cannot arise in the ordinary vacuum as in the standard quantum field theory. For dissipation to arise, the system must be unstable or permit particle production or be immersed in the dissipative environment.

The above Langevin equation can, in principle, be transformed into the equivalent Fokker-Plank equation for the distribution function. This process is actually tractable if we can use the Markovian property.

The most of the application of the above formalism to the BEC is given in [11]. The Langevin equation is transformed into the Fokker-Plank equation

$$i\hbar\frac{\partial}{\partial t}P[\phi;t] = -\sum_k \frac{\partial}{\partial\phi_k}((\varepsilon' - iR - \mu)\phi_k P[\phi;t]) - \frac{1}{2}\sum_k \frac{\partial^2}{\partial\phi_k^2}(\hbar\Sigma^K P[\phi;t]), \tag{16}$$

for the probability distribution function $P[\phi;t]$, where ε' and μ are, respectively, renormalized energy and chemical potential which are related to the function D, and R, Σ^K are the transport coefficient functions which are related to the function A and B. This Fokker-Plank equation is further transformed into the rate equation for the energy density of the condensation $\rho(k;t)$. In the limit of small condensation, i.e. the backreaction of the condensation to the gas is negligible, the rate equation reduces to

$$\frac{\partial}{\partial t}\rho(k;t) \approx \Gamma^{\rm in}\left(1 - \frac{\rho(k;t)}{\rho^{eq}(k;t)}\right) \approx \Gamma^{\rm in}, \tag{17}$$

where $\rho^{\rm eq}(k;t)$ is the equilibrium value of the condensation and $\Gamma^{\rm in} \equiv -R + i\Sigma^K/2$. The order estimate of the results in [11] becomes,

$$\Gamma^{\rm in} \approx |\lambda|^2 m^{-3}\rho_{\rm gas}^2, \tag{18}$$

if the dominant interaction is $\lambda(\phi\phi^*)^2$, and

$$\Gamma^{\rm in} \approx |\alpha|^2 m\rho_{\rm gas}, \tag{19}$$

if the dominant interaction is of type $\alpha\phi A\partial\phi^*$, where A is the radiation field. In either case, the induced process is neglected in the RHS of Eq. (17). It is most interesting that the detail of the form of $\Gamma^{\rm in}$ is not relevant for the BEC cosmology ([8]). Any choice of $\Gamma^{\rm in}$ (or Γ in the later argument) yields qualitatively the same cosmology. In this flexible possibilities, we adopt Eq. (18) in the following although Eq. (19) is equally applicable.

The above results determine the BEC condensation rate in the uniform boson gas. Note that the above condensation rates are transport coefficients which cannot be found directly in any Lagrangian. This fact yields essential difference between our approach and the presently popular interacting DE-DM scalar field models. For example [12,13] study DE-DM coupling as an interaction term in Lagrangian. The recent Ref. [14] study DE-DM coupling phenomenologically by setting $w \equiv p/\rho$ as a constant parameter. The quantity w actually evolves in time in our model according to the equation of motion. In these aspects, our BEC model should be distinguished from general DE-DM coupling models.

So far we have considered the generation of BEC. While this condensation proceeds, the uniform BEC eventually becomes unstable due to the Jeans instability and collapses into localized objects (inhomogeneous DM) leaving no uniform component. If the boson gas has attractive interaction, then this collapse would be further enhanced.

Thus in the universe, BEC is generated and collapses. These dynamics can be regarded as generalized phase transitions which cannot be described only by the coherent time change governed by a Lagrangian. We now examine the detail of the BEC phase transition in the following subsections.

B. Homogeneous mode: steady BEC

The gradual generation and subsequent decay of the spatially uniform BEC can be summarized by the following set of equations in a compact form

$$\begin{aligned} H^2 &= \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3c^2}(\rho_g + \rho_\phi + \rho_l),\\ \dot\rho_g &= -3H\rho_g - \Gamma\rho_g,\\ \dot\rho_\phi &= -6H(\rho_\phi - V) + \Gamma\rho_g - \Gamma'\rho_\phi,\\ \dot\rho_l &= -3H\rho_l + \Gamma'\rho_\phi, \end{aligned} \tag{20}$$

where we assume a simple form of potential $V \equiv V_0 + m^2\phi^*\phi + \frac{\lambda}{2}(\phi^*\phi)^2$ with attractive interaction $\lambda < 0$. This potential form will be fully fixed if we can identify the boson field in the future. The Einstein equation in the first line of Eq. (20) gives the cosmic expansion rate H in terms of the energy densities of gas ρ_g, BEC ρ_ϕ, and the localized objects ρ_l. The second line represents the evolution of the gas energy density ρ_g which reduces due to the cosmic expansion $-3H\rho_g$ and the consumption for BEC $-\Gamma\rho_g$. The third line is the relativistic form of Gross-Pitaevski equation describing BEC dynamics with the steady condensation rate $\Gamma\rho_g$, where $\Gamma = $ const [for Eq. (19)] or $\Gamma \propto \rho_g$ [for Eq. (18)]. This form of source and loss terms $\pm\Gamma\rho_g$ is the most characteristic to our BEC model although they are small. In the same line of Eq. (20), the term $-\Gamma'\rho_\phi$ symbolically represents a sudden collapse of BEC. The detail of this collapse will be explained in the next subsection. The last line represents the evolution of the localized energy density ρ_l which reduces due to the cosmic expansion $-3H\rho_l$ and increases due to the BEC collapse $+\Gamma'\rho_\phi$. We now explain the BEC instability and the collapse.

C. Inhomogeneous modes: BEC collapse

Uniform BEC is not an absolutely stable phase if the interaction is attractive or gravity is concerned. It collapses into localized object after the appearance of instability. Let us estimate the scale of BEC instability, which determines the time scale of the BEC collapse and the resultant mass scale of the localized objects. We start the space-time metric of the form

$$ds^2 = (1 + 2\Phi)dt^2 - a^2(1 - 2\Phi)d\mathbf{x}^2, \tag{21}$$

where $\Phi = \Phi(t,\vec x)$ represents the gravitational potential and $a = a(t)$ the cosmic scale factor. BEC is described by the equation of motion for the order parameter $\phi = \phi(t,\vec x)$, i.e. the third line of Eq. (20) neglecting Γ and Γ'

since the instability time scale is very short compared with Γ^{-1} and Γ'^{-1}. Up to the first order in Φ, this equation becomes

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{1}{a^2}\nabla^2\phi + m^2(1+2\Phi)\phi + \lambda(1+2\Phi)|\phi|^2\phi = 0. \tag{22}$$

Gravity is described by the Poisson equation, in the Einstein equation,

$$\nabla^2\Phi = 4\pi G a^2\left\{\dot{\phi}^\dagger\dot{\phi} + \frac{\nabla}{a}\phi\cdot\frac{\nabla}{a}\phi^\dagger + m^2|\phi|^2 + \frac{\lambda}{2}|\phi|^4 - \rho_0\right\}, \tag{23}$$

where ρ_0 is the uniform background energy density. We consider the linear perturbation on the uniform background

$$\phi(t,\vec{x}) = \phi_0(t) + \phi_1(t,\vec{x}),\ \Phi(t,\vec{x}) = 0 + \Phi_1(t,\vec{x}). \tag{24}$$

where small perturbations ϕ_1 and Φ_1 are decomposed into Fourier modes,

$$\phi_1 = \phi_1\exp(\Omega t + i\vec{k}\vec{r}), \qquad \Phi_1 = \Phi_1\exp(\Omega t + i\vec{k}\vec{r}). \tag{25}$$

The background variables satisfy, from Eqs. (22) and (23),

$$\ddot{\phi}_0 + 3\frac{\dot{a}}{a}\dot{\phi}_0 + (m^2 + \lambda|\phi_0|^2)\phi_0 = 0, \tag{26}$$

$$\dot{\phi}_0^\dagger\dot{\phi}_0 + \left(m^2 + \frac{\lambda}{4}|\phi_0|^2\right)|\phi_0|^2 - \rho_0 = 0. \tag{27}$$

In these Eqs. (26) and (27), the variable ϕ_0 can be assumed to be real without generality since all the coefficients are real. Then the existence of a nontrivial solution requires that the above set of linear equations are dependent with each other. Thus we have the condition

$$\left[\left(\Omega^2 + 3\frac{\dot{a}}{a}\Omega + \frac{\mathbf{k}^2}{a^2} + m^2 + 3\lambda\phi_0^2\right)\frac{\mathbf{k}^2}{4\pi G a^2} - 2(m^2+\lambda\phi_0^2)\phi_0(2\dot{\phi}_0\Omega + 2(m^2+\lambda\phi_0^2)\phi_0)\right] \times\left(\Omega^2 + 3\frac{\dot{a}}{a}\Omega + \frac{\mathbf{k}^2}{a^2} + m^2 + \lambda\phi_0^2\right) = 0, \tag{28}$$

which determines the instability strength Ω as a function of the wave number $\vec{k}$. The wave number $\vec{k}$ which corresponds to the maximum value of Ω (*i.e.* most unstable mode) is not necessarily the relevant scale since we also have to consider the completion time of the collapse $a/(\alpha k)$, where $\alpha(<1)$ is the decay speed of BEC. Thus the relevant scale will be the wave number k_* which should satisfy the condition that the collapse time is just the instability time i.e. $\alpha k_*/a = \Omega$. Choosing the value associated with the most unstable mode among four solutions of Ω, we have

$$\frac{k_*}{a} = \left(\frac{-m_{\rm eff}^2 + \sqrt{m_{\rm eff}^4 + 64\pi G(1+\alpha^2)(m^4\phi_0^2 - 2\kappa m^2\phi_0^4 + \kappa^2\phi_0^6)}}{2(1+\alpha^2)}\right)^{1/2}, \tag{29}$$

where $m_{\rm eff}^2 \equiv m^2 - 3\kappa\phi_0^2$, $\kappa = -\lambda(>0)$ and adiabatic approximation $H = 0$, $\dot{\phi}_0 = 0$ is utilized since the collapse time scale is much smaller than cosmic and condensation time scales in the late universe.

The actual scale critically depends on the boson mass [8]. If it is about 1 eV, then the typical mass of the structure will be $M_* \approx 1.6\times10^{11}M_\odot$ which is of order of a galaxy. If it is about 10^{-3} eV, then the typical scale in tiny $M_* \approx 10^{-1}g$ and therefore BEC collapse cannot form macroscopic cosmological structures. In any case, these collapsed clumps would work as the ordinary DM and the scenario of large scale structure formation exactly reduces to the standard model.

A typical numerical demonstration of uniform BEC mode is given in Fig. 1. Most of the characteristic features of BEC cosmology are shown in this figure. The gas density ρ_g (grey line) reduces monotonically while the BEC density ρ_ϕ (solid line) gradually increases toward some instability point where it suddenly decays into localized objects ρ_l (dashed line). Then the BEC density disappears and a new BEC begins. This cycle repeats several times and finally the phase of ρ_ϕ = const. appears. Thus

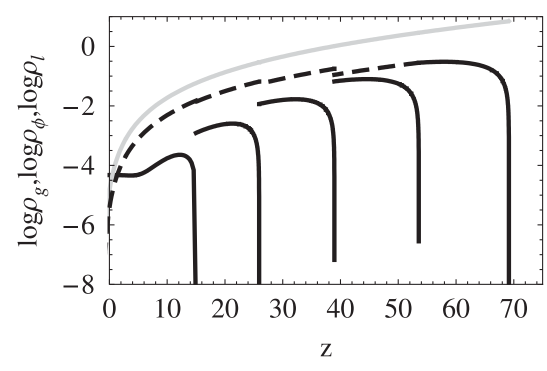

FIG. 1. A typical numerical demonstration of the BEC cosmology. This graph represents the evolution of energy densities of the boson gas ρ_g (grey line), BEC ρ_ϕ (solid line), and the localized objects ρ_l (dashed line) as a function of redshift z. We solved Eq. (20) with the parameters $\lambda = -0.51$, $\Gamma = 0.2601$, $m = 0.0028$ eV. The BEC Collapse time sequence is $z = 69$, 54, 39, 36, 15. After four collapses of BEC, the energy density of BEC ρ_ϕ becomes constant, which induces the accelerated expansion of the present universe.

the universe inevitably enters into the final accelerated expansion phase. This phase is guaranteed by the stable balance of the condensation flow and the potential force, and is an attractor of Eq. (20). Several other features and predictions of this model are found in [8].

III. BEC IN THE EARLY UNIVERSE

We now extend the BEC cosmology to the early universe. As was studied previously, for the relativistic case, the critical temperature $T_{\rm cr} \propto \rho^{1/2}$ increases much faster than the cosmic temperature $T \propto \rho^{1/4}$ in the course of increasing energy density ρ. Therefore we safely expect the universe is always under critical temperature provided so at present.

The acceleration in the early universe is quite different from that in late universe in the sense that a huge vacuum energy V_0 is present from the beginning in the former. However we do not want to introduce any new ingredients into the model used in the original analysis in LA. This is because we want to understand the cosmic accelerations from a unified point of view. Actually, the mass m of the boson is about eV and therefore is completely negligible in the early universe. The condensation rate Γ has been quite robust in the previous study in the contemporary universe and therefore we take the same form of Γ for BEC condensation as used previously.

The goal of this section is to examine the following individual steps to complete the whole BEC cosmological scenario and verify this model: 1. Natural initiation of the inflation from the fireball stage is expected from the steady BEC processes. 2. This inflation stage terminates autonomously due to the stagflation. 3. Reheating process inevitably takes place due to the nonlinear coupling in BEC. 4. The cosmological constant is autonomously adjusted to be zero due to the BEC instability at the stagflation point. 5. We calculate the generated density fluctuations and compare them with observations.

A. Natural initiation of the inflation

The original model of the BEC cosmology is described by Eq. (20). Now in the application to the early universe whose energy scale is extremely higher than eV, a typical mass scale of the boson, relativistic damping terms should be used:

$$\begin{aligned} H^2 &= \left(\frac{\dot a}{a}\right)^2 = \mu^2(\rho_g + \rho_\phi + \rho_l), \\ \dot\rho_g &= -4H\rho_g - \Gamma\rho_g, \\ \dot\rho_\phi &= -6H(\rho_\phi - V) + \Gamma\rho_g - \Gamma'\rho_\phi, \\ \dot\rho_l &= -4H\rho_l + \Gamma'\rho_\phi. \end{aligned} \tag{30}$$

where $\mu^2 = 8\pi G/(3c^2)$, the potential is $V \equiv V_0 + m^2\phi^*\phi + \frac{\lambda}{2}(\phi^*\phi)^2$ with $\lambda < 0$, and the energy density for BEC is $\rho_\phi = \dot\phi^*\dot\phi + V$.

In the early universe, tiny mass term m^2 is neglected. On the other hand, the condensation rate Γ, whose energy scale dependence is not clear, may still be relevant in the early universe. We should consider an initial vacuum energy $V_0 \approx m_{\rm pl}^4$, which is huge up to 10^{120} times the present value.

Initial boson gas $\rho_g(t) = \rho_{gi}(t_i/t)^2$ and the temperature will be reduced by the cosmic expansion $a(t) = a_0(t/t_i)^{1/2}$. On this background, BEC gradually proceeds since $T_{\rm cr} < T$ is guaranteed from the energy density dependence of them as argued above. Since the initial evolution velocity of the BEC $\dot\phi$ is small and the potential energy is negligible compared with the boson gas density, the BEC ϕ approximately obeys

$$3H\dot\phi^2 = \Gamma\rho_g, \tag{31}$$

whose solution, under the choice $\Gamma(t) = \Gamma_0\rho_g(t)$, is given by the fire ball solution,

$$\phi_{\rm fire}(t) = 2t_i\left(\frac{\Gamma_0\rho_{gi}^{3/2}}{3\mu}\right)^{1/2}\left(1 - \left(\frac{t_i}{t}\right)^{1/2}\right). \tag{32}$$

The situation of each stage of BEC will be given in Fig. 4. Then, if we identify the initiation time of inflation $t_{\rm iinf}$ by the condition

$$\rho_g(t_{\rm iinf}) = V_0, \tag{33}$$

then the initial value of the BEC when the inflation begins will be

$$\phi_{\rm fire}(t_{\rm iinf}) = 2t_i\left(\frac{\Gamma_0\rho_{gi}^{3/2}}{3\mu}\right)^{1/2}\left(1 - \left(\frac{V_0}{\rho_{gi}}\right)^{1/4}\right) \tag{34}$$

or

$$\delta \equiv \dot\phi_{\rm fire}(t_{\rm iinf}) = \left(\frac{\Gamma_0\rho_{gi}^{3/2}}{3\mu}\right)^{1/2}\left(\frac{V_0}{\rho_{gi}}\right)^{3/4}. \tag{35}$$

Although the boson gas will be diluted, this initial BEC triggers the further evolution of BEC, and BEC proceeds consuming the vacuum energy. During the inflation, BEC evolves as

$$\dot V = -3H\dot\phi^2, \qquad H^2 = \mu^2 V \tag{36}$$

which solves, under the initial condition Eq. (35), as

$$\phi_{\rm inf}(t) = \left(\frac{3H_{\rm inf}}{-2\lambda}\right)^{1/2}\left(t_{\rm iinf} - t + \frac{1}{2}\left(\frac{3H_{\rm inf}}{-\lambda\delta^2}\right)^{1/3}\right)^{-1/2}, \tag{37}$$

where we approximate $H^2 = H_{\rm inf}^2 \equiv \mu^2 V_0$. Since the potential is flat around the origin, this solution actually satisfies the slow-role condition

$$\left|\frac{\dot\phi_{\rm inf}^2}{2V}\right| \ll 1, \qquad \left|\frac{\ddot\phi_{\rm inf}}{3H_{\rm inf}\dot\phi_{\rm inf}}\right| \ll 1 \tag{38}$$

or equivalently under the condition Eq. (36),

$$|\varepsilon| \ll 1, \qquad |\eta| \ll 1, \tag{39}$$

where

$$\varepsilon \equiv \frac{m_{\rm pl}^2}{16\pi}\left(\frac{V'}{V}\right)^2, \qquad \eta \equiv \frac{m_{\rm pl}^2}{8\pi}\frac{V''}{V}. \tag{40}$$

These conditions hold until just before $\phi_{\rm inf}(t)$ hits the (virtual) singularity at $t = t_{\rm einf}$ (where $\phi_{\rm inf}$ diverges). Thus the duration of the inflation is

$$\Delta_{\rm inf} \equiv t_{\rm einf} - t_{\rm iinf} = \frac{1}{2}\left(\frac{3H_{\rm inf}}{-\lambda\delta^2}\right)^{1/3} + O(H_{\rm inf}^{-1}) \tag{41}$$

and the e-folding number N becomes $N = H_{\rm inf}\Delta_{\rm inf}$.

In Fig. 2, we have depicted a typical time evolution of the scale factor. We do not seek for the most favorable values of parameters in this paper since our aim is to clarify the basic mechanism and to show the robustness of the BEC cosmological model in the early universe.

This mechanism of inflation is the same as the standard models and especially the period of inflation itself is determined as usual. The BEC mode (the order parameter) behaves in quite similar ways to the ordinary scalar field. However, the initiation of the inflationary era is quite different from the standard model as shown above. Moreover, the termination of it is also different as we will see in the next subsection. In the inflationary phase, BEC develops by consuming the vacuum energy contrary to the case of BEC evolution in LA in which BEC develops by consuming the boson gas energy.

Though the order parameter is complex, the condensation is simply assumed to have a common phase and therefore the field behaves as a real scalar field. However actually there must be a finite coherent scale only on such domain this assumption is applicable. The phase boundary may have an extra energy density, which we simply neglected in the present paper. However these structures are much more complicated than the usual treatment of multiple uniform scalar field models which may give rise to isocurvature modes (Refs. [1,15]). This issue goes beyond the scope of the present paper and should be addressed in our future research.

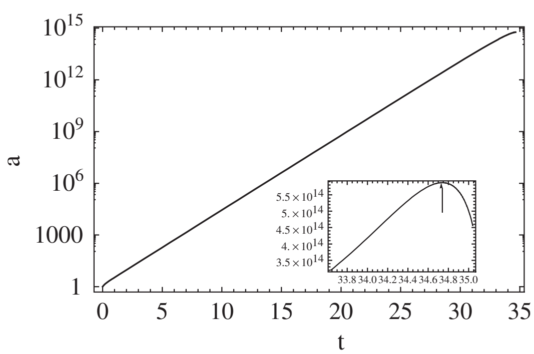

FIG. 2. The time evolution of the scale factor $a(t)$. The scale factor is normalized by itself at the beginning of the inflation. This is a solution of Eq. (30) with parameters: $\lambda = -0.5$, $V_0 = 1$, $\phi(0) = 0$, $\dot{\phi}(0) = 0.001$, $\rho_g(0) = 5$, $\Gamma_0 = 1$, in the units of $H_{\rm inf}$ for H, $H_{\rm inf}^{-1}$ for t, $H_{\rm inf}^2/\mu^2$ for ρ, where $H_{\rm inf}^2 \equiv \mu^2 V_0$. The inflationary phase, i.e. $a \propto e^{H_{\rm inf}t}$, naturally appears with sufficient e-folding number. The total energy density vanishes and the expansion ceases at the stagflation point $t = t_s \approx 34.8$, where the strong instability of the uniform BEC mode actually invalidates the evolution after t_s. The inset magnifies the local region around t_s, which is marked by an arrow.

B. Termination of inflation

As is shown in Fig. 2, this inflation sharply terminates within a finite time. This can be easily understood as a general feature of our BEC cosmology. In the late stage of inflation Γ, Γ', ρ_g are neglected, and Eq. (30) reduces to

$$H^2 = \frac{8\pi G}{3c^2}\rho_\phi, \qquad \dot{\rho}_\phi = -3H\dot{\phi}^2, \tag{42}$$

and, provided $H \neq 0$, further to

$$\dot{H} = -\frac{3}{2}\mu^2\dot{\phi}^2. \tag{43}$$

Since the right hand side of Eq. (43) is negative and $\dot{\phi}^2$ increases in time, the expansion rate H eventually crosses 0, at time $t = t_s$, after that the universe terns into the contraction phase. The dynamics of BEC in this process corresponds to the over-hill regime in the original BEC cosmology [7].

When the cosmic expansion ceases at $t = t_s$, Eq. (30) yields, neglecting ρ_g,

$$H = 0, \qquad \rho_\phi = 0, \qquad \dot{\rho}_\phi = 0, \tag{44}$$

which guarantee the energy density smoothly vanishes at $t = t_s$. However, we should notice that $\dot{H} \neq 0$ from Eq. (43) and the continuity. We call this phase as *stagflation* regime, i.e. the cosmic expansion is 'stagnant' while the BEC is steadily accelerating and 'inflating'. In the previous approximate solution Eq. (37), the termination time $t_{\rm einf}$ is almost t_s: $t_s \approx t_{\rm einf}$.

Figure 3 shows the energy densities of the boson gas ρ_g (grey line), and of BEC ρ_ϕ (solid line) in the same time evolution as in Fig. 2. The stagflation regime is magnified in the inset of this figure. The dashed line shows the Hubble parameter which smoothly changes its signature, expanding to contracting at t_s.

We would like to comment briefly on the virtual history of the universe after the stagflation regime though it actually does not exist, as will be explained in the following subsections. The stagflation regime Eq. (44) does not last forever; the kinetic term $\dot{\phi}^2$ eventually develops and surpasses $\lambda\phi^4$ term. In this kinetic term-dominant regime, Eq. (30) yields $d(\dot{\phi}^2/2)/dt = -3H\dot{\phi}^2$ and $H^2 = \mu^2\dot{\phi}^2/2$, and therefore

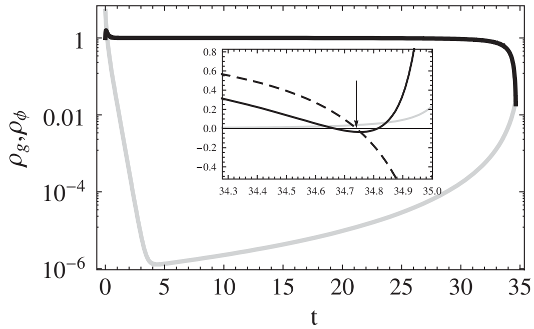

FIG. 3. This graph represents the evolution of energy densities of the boson gas ρ_g (grey line), and of BEC ρ_ϕ (solid line) as a function of time t. The energy density is normalized by $(H_{\rm inf}/\mu)^2$ and the time by $H_{\rm inf}^{-1}$. The inflationary phase eventually terminates and the universe turns into the contraction phase. It finally approaches the virtual singularity, if we do not consider the instability and the decay of BEC. Inset is the magnified graph at the last stage of the inflation. Dashed line is the Hubble parameter, which crosses zero and the total energy density vanishes at the stagflation point $t = t_s \approx 34.74$ (marked by an arrow). The strong instability of the uniform BEC mode actually invalidates the evolution after t_s.

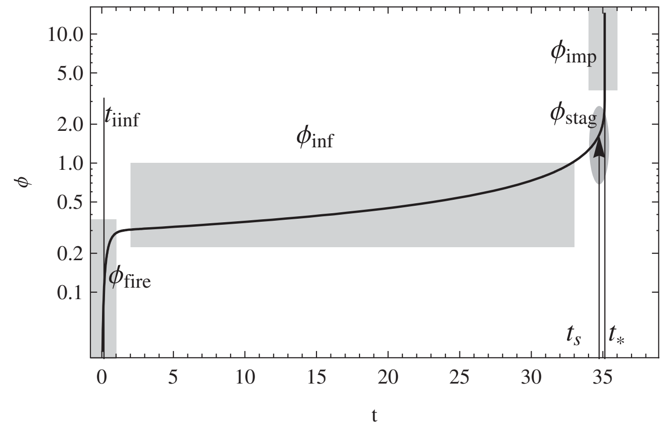

FIG. 4. This figure represents the evolution of BEC field $\phi(t)$, numerically obtained, and several analytical approximations, $\phi_{\rm fire}$, $\phi_{\rm inf}$, $\phi_{\rm stag}$, and $\phi_{\rm imp}$. The applicability range for each solution is shown by shaded region. The solutions $\phi_{\rm fire}$, $\phi_{\rm inf}$ are matched together at $t_{\rm iinf} \approx 0.16$. The stagflation time $t_s \approx 34.7$ is marked by an arrow. The BEC field ϕ diverges at the implosion time $t_* \approx 35.1$.

$$\ddot{\phi} = \frac{3}{\sqrt{2}}\mu\dot{\phi}^2, \tag{45}$$

where we must choose $H < 0$ and $\dot{\phi} > 0$ for consistent solution. These lead to the implosive solution

$$\phi_{\rm imp} = {\rm const} - \frac{\sqrt{2}}{3\mu}\log(t_* - t), \tag{46}$$

where the universe hits the singularity at time t_*. The cosmic contraction becomes

$$a = {\rm const}(t_* - t)^{1/3}, \tag{47}$$

which explains the virtual behavior in Fig. 2 for $t \longrightarrow t_*$. (See also Fig. 4.)

As a whole, the early universe suffers the BEC phase transition as depicted in Fig. 4. This figure represents the whole evolution of BEC field $\phi(t)$, numerically obtained, and several analytical approximations whose validity is limited within each local region. Analytic solutions are $\phi_{\rm fire}$ in Eq. (32), $\phi_{\rm inf}$ in Eq. (37), $\phi_{\rm stag}$ in Eq. (51) (which will be described shortly), and $\phi_{\rm imp}$ in Eq. (46). The solutions $\phi_{\rm fire}$ and $\phi_{\rm inf}$ are matched at $t_{\rm iinf}$, defined by Eq. (33). The stagflation time t_s, marked by an arrow, is defined by Eq. (44). The implosion time t_* is the time when BEC field diverges as Eq. (46).

In the real world however, the infinite amplitude $\phi \longrightarrow \infty$ and velocity $\dot{\phi} \longrightarrow \infty$ for BEC are not allowed. Actually, before the universe enters into this phase, there appear two new features in this scenario and the singularity, which we call *implosion*, is avoided. One new feature is the particle production due to the ever accelerating BEC $\dot{\phi} \longrightarrow \infty$. This is the origin of the reheating that some portion of the original vacuum energy V_0 is converted into thermalized gas of bosons. Another new feature is the instability of the uniform mode of BEC at $t \approx t_s$ and $\rho_\phi \approx 0$, and subsequent decay of the uniform BEC into the localized objects, without leaving any vacuum energy. This is the mechanism which did adjust the final vanishing vacuum energy, or vanishing cosmological 'constant', despite the huge vacuum energy set before inflation. Let us examine each of these features separately in the following.

C. Initiation of reheating

The rapid increase of BEC $\dot{\phi} \longrightarrow \infty$ at the late stage of the inflation yields plenty of particle production, *which will reheat the universe*, through the self-interaction. If this particle production is effective to turn sufficient amount of kinetic energy of BEC into thermal gas, the universe can go into the big bang phase. In order to demonstrate this scenario, we consider a simple model of particle production; an addition of the term $-\gamma\dot{\phi}^2$ ($+\gamma\dot{\phi}^2$) to the right hand side of $\dot{\rho}_\phi$ ($\dot{\rho}_g$, respectively) in Eq. (30). We can approximately evaluate the particle production during the inflationary era. By using Eq. (37), we obtain

$$\rho_g(t_0) = \gamma\int_{t_{\rm iinf}}^{t_0}\dot{\phi}_{\rm inf}^2 dt \le \frac{\gamma V_0}{3H_{\rm inf}} = \frac{\gamma H_{\rm inf}}{3\mu^2}. \tag{48}$$

If we set $\gamma \approx m_\phi \approx 1$ eV, then the reheating temperature, using the expression $\rho_g = (\frac{\pi^2 g_* k_B^4}{30\hbar^3 c^3})T^4$, would be

$$k_B T_{\rm rh} < \left(\frac{10}{\pi^2}\gamma m_{\rm pl}^3\right)^{1/4}(H_{\rm inf}/m_\phi)^{1/4} \approx 1.17 \times 10^{12}(H_{\rm inf}/m_{\rm pl})^{1/4}\ {\rm GeV}. \tag{49}$$

Note that the particle production in the BEC model is caused simply by the accelerated evolution of the background field ϕ.

D. Autonomous adjustment toward the vanishing cosmological constant

Let us discuss on another new feature which does not allow the implosive singularity. As in the previous arguments on the BEC cosmology in LA, the instability of BEC actually develops and the homogeneous BEC phase collapses into many localized objects. Thus dynamics of BEC is not closed within the homogeneous mode contrary to the recent arguments on the DE-DM coupling ([13]), for example.

In the middle of the inflationary phase, the BEC fluctuations cannot develop. However, they have a chance to grow after the inflationary phase. We now examine this around the stagflation time when the energy density vanishes at $t = t_s$.

The middle of Eq. (44) allows us to approximate the evolution equation, neglecting V_0, as

$$\dot{\phi} = \sqrt{\kappa}\phi^2, \tag{50}$$

which immediately solves

$$\phi_{\rm stag} = \kappa^{-1/2}(t_s - t)^{-1}, \tag{51}$$

where $\kappa = -\lambda(>0)$. In this regime $t \approx t_s$, the instability scale Eq. (29) becomes

$$\frac{k_*}{a} = (8\pi)^{1/4}G^{1/4}\kappa^{1/2}\phi_{\rm stag}^{3/2}$$
$$= (8\pi)^{1/4}G^{1/4}\kappa^{-1}(t_s - t)^{-3/2}, \tag{52}$$

which increases without bound for $t \to t_s$. On the other hand, the cosmic expansion is almost ceased in this regime $t \approx t_s$. Therefore, we expect a strong instability takes place and BEC rapidly collapses into localized objects, whose density is denoted as ρ_l. Moreover, there is sufficient time $(t_s - t)$ for the completion of the BEC collapse before the universe escapes from this regime $t \approx t_s$:

$$(t_s - t) > a/k_* \propto (t_s - t)^{3/2}. \tag{53}$$

This is similar to what happens in the over-hill regime in the late time BEC cosmology. Thus all the BEC component turns into localized objects, which behave as dust fluid, leaving vanishing vacuum energy or cosmological 'constant', despite the initial value of the vacuum energy V_0. The fate of these localized objects is not yet clear. If they form light black holes, they may eventually evaporate into radiation including the boson gas. If they form heavy black holes, they may remain until now and yield significant effects on the cosmic structure. We simply choose the former case in our numerical demonstration in this paper.

More precisely, we have numerically checked the above instability directly based on Eq. (28) without the approximation Eq. (29). It has turned out that the collapse of BEC, estimated from the linear instability analysis, takes place just before the stagflation point t_s. This means that a small amount of vacuum energy remains at this point since $\rho = 0$ realizes only exactly at $t = t_s$. However further evolution of the uniform BEC from that point leads to the next instability of this BEC, which yields much smaller remnant vacuum energy. These sequence of BEC collapse continues many times and the vacuum energy reduces at each time. Figures 5–7 show a typical example of the numerical demonstration of this sequence. Figure 5 shows the time evolution of the BEC order parameter ϕ. Each segment represents a history from the generation toward the decay of BEC. The average value of ϕ reduces in time and the duration of each history becomes longer. Figure 6 shows the time evolution of the scale factor $a(t)$. Initial inflationary era, which is a straight line on this graph, is followed by much milder expansion era. The latter is shown in the inset of this figure and is almost the same

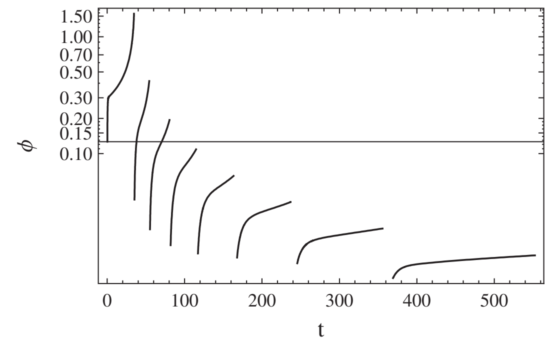

FIG. 5. Time evolution of the BEC order parameter ϕ is shown. It shows a sequence of BEC collapses. It may happen that this BEC reduction sequence leads to the era of LA in Ref. [1]. However, the numerical calculation extending over 100 digits is almost impossible and we could not reveal the full extension of the reduction sequence.

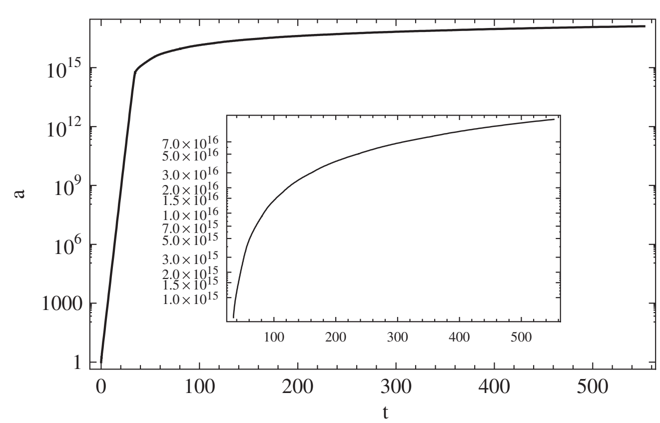

FIG. 6. Time evolution of the scale factor $a(t)$ is shown in logarithmic scale against the cosmic time t. Initial inflation era is followed by the almost radiation dominated era. The inset is the later evolution.

as radiation dominated cosmic expansion. Figure 7 shows the time evolution of BEC energy density ρ_ϕ (solid line), boson gas energy density ρ_g (gray line), and the Hubble parameter H (dashed line). The reduction sequence is apparent. In our demonstration Fig.(7), the vacuum energy at each segment era reduces as 1.0, 0.0063, 0.00072, 0.00016, 0.000046, 0.000014, 4.6×10^{-6}, $1.5 \times 10^{-6}\dots$ in the unit of the original vacuum energy V_0.

Thus there is no artificial fine tuning in simultaneously realizing the inflationary acceleration due to the huge vacuum energy and the reduction of the vacuum energy to zero in our model. The essences of the autonomous fine tuning are, (a) the decay of the uniform mode, which contributes to the huge vacuum energy, into the localized states, (b) this decay always takes place whenever the universe enters into the stagflation era, (c) this stagflation always exists whenever the kinetic term can reduce the cosmic expansion in Eq. (43) to zero, and (d) such a case is guaranteed by allowing the negative region of the potential, which will be further discussed in the summary section.

E. Generation of density fluctuations-comparison with observations

As we have examined in the previous two subsections, both the reheating due to the particle production and the decay of the vacuum energy due to the BEC instability are most effective at around the stagflation time $t = t_s$. Furthermore, we observed that this set of BEC generation and decay repeats many times and this sequence makes the effective cosmological constant reduce toward zero.

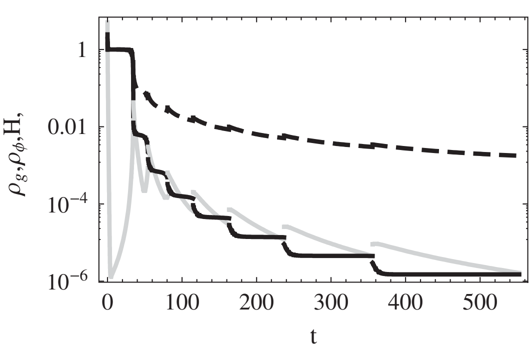

FIG. 7. Time evolution of BEC energy density ρ_ϕ(solid line), boson gas energy density ρ_g (gray line), and the Hubble parameter H (dashed line) in cosmic time t. The original gas density ρ_g reduces almost completely due to the exponential expansion during the inflationary era. In the later stage of the inflationary era, uniform BEC becomes unstable and collapses leaving some fraction of ρ_ϕ. At the same time, ρ_g gradually recovers due to the particle production process caused by the rapid change of the background field ϕ and eventually dominates ρ_ϕ. However ρ_g reduces faster than ρ_ϕ in the subsequent cosmic evolution. Eventually the second instability of BEC takes place. These sequence of BEC and its decay repeats and the effective cosmological constant reduces toward zero.

Although the detail depends on the BEC decay dynamics which is quite nonlinear and goes beyond the present paper, we may naturally assume that the universe approaches toward the hot FRW universe. Then its fate will be well described by the standard cosmology until it enters into LA regime in most recent era. Therefore the most characteristic feature of our model will be related to the phenomena around the stagflation point t_s. Then one of the remnant characteristic features to our BEC model will be the power spectrum of the produced density fluctuations reflecting ever increasing development of the BEC component toward the end of the inflationary era.

We now calculate the produced density fluctuations during the inflation. Since our BEC behaves almost like scalar field during the inflation and therefore we can safely apply the standard analysis on the density fluctuations. Based on the squeezed quantum state in de Sitter space with slow-rolling approximation, we can estimate the curvature fluctuations and the tensor fluctuations. The power index n_R for the former is given [1] by

$$n_R = 1 - 6\varepsilon + 2\eta \tag{54}$$

where ε and η are defined in Eq. (40). We can calculate them based on the approximate solution $\phi_{\rm inf}(t)$ during the inflation Eq. (37)

$$n_R = 1 - \frac{3(1 + \frac{9\mu^2 H_{\rm inf}^2}{16\kappa N^2})}{N(1 - \frac{9\mu^2 H_{\rm inf}^2}{16\kappa N^2})^2}, \tag{55}$$

which reduces, after use of Eq. (41), to

$$n_R = 1 - \frac{3}{N} + O\left(\frac{1}{N^3}\right), \tag{56}$$

where N is the e-folding number defined by Eq. (41) and the next line. The tensor to scalar ratio $r = P_T/P_R = 16\varepsilon$ becomes

$$r = \frac{9\mu^2 H_{\rm inf}^2}{\kappa N^3(1 - \frac{9\mu^2 H_{\rm inf}^2}{16\kappa N^2})^2} = \frac{9\mu^2 H_{\rm inf}^2}{\kappa N^3} + O\left(\frac{1}{N^4}\right). \tag{57}$$

In our model, they are estimated as

$$n_R \approx 0.95 \quad \text{for } N = 60, \tag{58}$$

$$r \approx 4.2 \times 10^{-12} \quad \text{for } N = 60, \qquad \mu H_{\rm inf} = 10^{-4}, \qquad \kappa = 0.1. \tag{59}$$

These values are well within the observationally allowed range so far [15]:

$$n_R = 0.960 \pm 0.013, \tag{60}$$

$$r < 0.22. \tag{61}$$

We will need to examine other observational tests in the next step in order to confirm the unified BEC cosmology.

We will save the sequel of this analysis for our future papers.

IV. SUMMARY AND DISCUSSIONS

In the previous papers, we developed the BEC cosmology to describe DE/DM in a unified manner in LA. In this paper, we have applied this model to the early universe and unified EA and LA without introducing new ingredients.

It turns out that the BEC cosmology well fits to describe inflationary dynamics. Being triggered by the condensation of bosons in the environment of huge vacuum energy, the inflation naturally initiates. This inflation autonomously terminates due to the stagflation stage which inevitably takes place exactly at zero energy density. At the stagflation point, particle production and the decay of BEC occur. The former makes the universe connect to the standard hot big bang stage and the latter guarantees the vanishingly small cosmological constant after the inflation. Further, we have calculated the density fluctuations produced in this model, which turns out to be in the range allowed by the present observational data.

There is no artificial fine tuning in realizing the inflationary acceleration and the reduction of the vacuum energy in our model. The key mechanism to reduce the huge energy scale of inflation exactly to zero is the stagflation, *i.e.* the moment of the cosmic expansion stops (Fig. 2). This stagflation is quite robust as will be discussed shortly bellow. In this stagflation era the uniform mode of BEC, *i.e.* the energy density of the negative pressure component, becomes unstable and decays into localized objects [Eq. (52)]. Even if this decay were not complete, due to the fact that the decay is completed just before the exact stagflation point, the universe inevitably points toward the second stagflation era and the uniform mode further reduces and so on until finally the uniform mode of BEC completely vanishes (Fig. 7). Thus the reduction of the initial huge energy to the vanishing energy is not due to the fine tuning by hand but is due to the instability of the stagflation which is a solution of the Einstein equation including all the energy density [Eqs. (44) and (51)].

The above mechanism in the early universe sets the vacuum energy to be exactly zero irrespective of the initial energy density V_0. The energy scale of the present acceleration is due to the tiny mass scale of the boson, which we have to assume about 1 eV. However this is not a fine tuning; it can be 10^{-3} eV or smaller (Ref. [8]). In EA we have huge cosmological constant V_0, and the boson mass is negligible. The initial vacuum energy V_0 is eaten by ρ_ϕ (Fig. 3) and dissipates into localized objects during the stagflation (Fig. 7). After the stagflation we have no cosmological constant.

Whereas in LA, we have no cosmological constant but the boson mass is not negligible, and therefore we have a finite hill in the potential. After possible several BEC collapses, the condensation flow finally balances with the potential force, $V' = \Gamma\rho_g\dot{\phi}$ (Fig. 1). The realization of this balance is also autonomous, *i.e.* the attractor (Ref. [8]).

It should be remarked that this stagflation regime is important to solve several problems in the early universe. Although this stagflation regime seems to be unique to the BEC model, it is not true. The stagflation is actually general and robust in various cosmological models provided that the potential of the condensation or the inflaton can become negative. It means that there is no necessity to adjust the potential minimum as the zero point of energy from the beginning but it is realized autonomously through the stagflation. The stagflation is a normal physical mechanism and does not violate any fundamental principles. The total energy density, for example, is guaranteed to be nonnegative despite the negative potential. Furthermore the potential unbounded from below for a uniform mode of the condensation does not mean the catastrophe of the whole system, but only yields BEC collapse into nonuniform modes.

In order to demonstrate that the stagflation is a general mechanism in the universe, we pick up several typical models of inflation and slightly generalize them by adding small negative constant term $-V_1$. The models are the chaotic inflation, the new inflation, and the inflation with exponential potential. The potentials of them are generalized to

$$V_{\rm chaotic}(\phi) = \frac{1}{2}m^2\phi^2 - V_1, \tag{62}$$

$$V_{\rm new}(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4 - V_1, \tag{63}$$

$$V_{\rm exponential}(\phi) = e^{-\phi} - V_1, \tag{64}$$

with $m^2 > 0$, $\lambda > 0$, $V_1 > 0$. We numerically calculate the fate of the inflaton field $\phi(t)$ in Fig. 8. In this figure, the columns show, from left to right, inflation models of chaotic, new and exponential potential. The rows show, from top to bottom, the potentials $V(\phi)$, time evolutions of BEC order parameter $\phi(t)$, time evolutions of scale factor $a(t)$, and time evolutions of energy densities (grey line for $\rho_g(t)$, solid line for $\rho_\phi(t)$) as well as the Hubble parameter $H(t)$(dashed line). All of the models show stagflation, market by arrows in the figure, without any fine tuning.

BEC cosmology seems to be able to describe the two accelerating stages of the universe in different ways, one is the early inflationary acceleration (EA) and the another is the contemporary mild acceleration (LA). The dynamics of the inflation is triggered by the BEC condensation and the instability of BEC around the stagflation point guarantees that the cosmological constant is vanishingly small irrespective of the initial vacuum energy. On the other hand, the contemporary mild acceleration inevitably takes place from the balance of the condensation speed and the potential force. Its attractor nature guarantees the existence of the tiny cosmological constant presently observed. EA

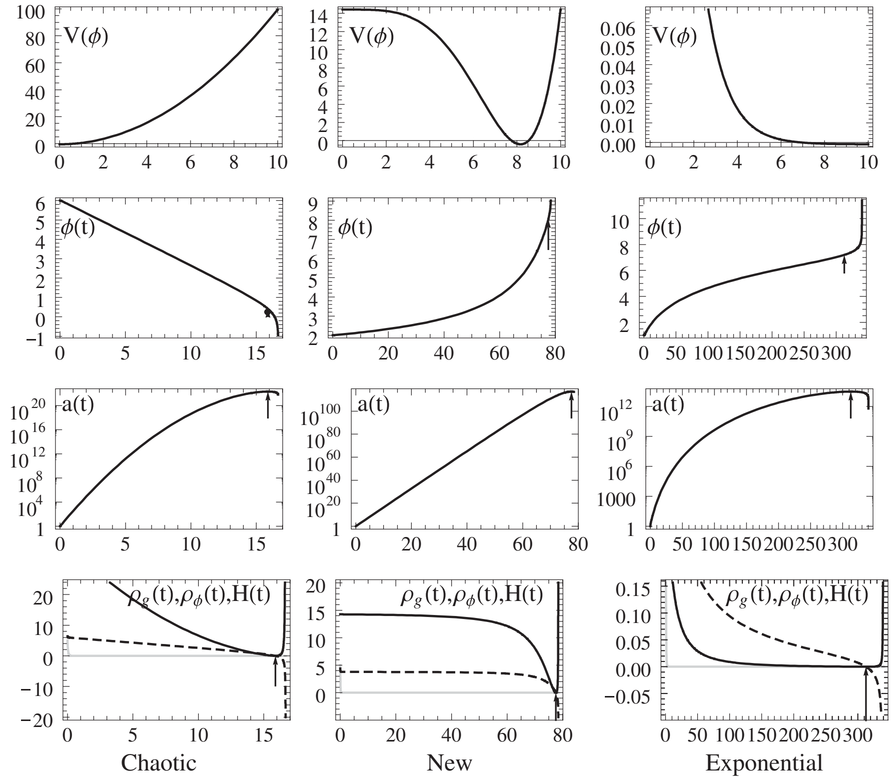

FIG. 8. Demonstration of the robustness of stagflation in various models of inflation. The columns show, from left to right, chaotic inflation, new inflation and the inflation with exponential potential. The rows show, from top to bottom, the potentials $V(\varphi)$, time evolutions of BEC order parameter $\varphi(t)$, those of scale factor $a(t)$, and those of energy densities (grey line for $\rho_g(t)$, solid line for $\rho_\phi(t)$) as well as the Hubble parameter $H(t)$(dashed line). All of them show stagflation, market by arrows, without any fine-tuning.

corresponds to the over-hill regime and LA the mild-inflation regime in the general BEC cosmology [7].

These two acceleration regimes may be connected with each other by multiple collapses of BEC, as suggested in Figs. 5–7. In this paper, we have demonstrated the post-sequence of BEC collapse after inflation and the presequence of it before the final acceleration. These two sequences may actually be connected to form a single sequence of BEC collapse. Unfortunately, the detailed analysis on this possibility and the mechanism of BEC collapse itself go beyond the present paper. We will figure out the dynamics of BEC probably by numerical technique and report them in the near future.

Novel properties of BEC cosmology in this paper is realized by the phase transition of BEC, which cannot directly described by a unitary Lagrangian dynamics. It will be important to notice that the universe may not be a mechanical machine whose time evolution is unitary, but be a whole sequence of phase transitions allowing the emergence of new structures in various stages. This paper is also a first proposal for the resolution of the cosmological constant problem based on the nonunitary phase transitions including nonuniform modes of condensation. We believe the most appropriate method to describe such incoherent evolution is the generalized effective action formalism [10] on which we would like to report comprehensively in our future paper including various applications and verifications of BEC cosmology.

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