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Generating primordial black holes via hilltop-type inflation models

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It has been shown that black holes would have formed in the early Universe if, on any given scale, the spectral amplitude of the cosmic microwave background exceeds $\mathcal{P}_\zeta \sim 10^{-4}$. This value is within the bounds allowed by astrophysical phenomena for the small scale spectrum of the cosmic microwave background, corresponding to scales which exit the horizon at the end of slow-roll inflation. Previous work by Kohri *et al.* (2007) showed that for black holes to form from a single field model of inflation, the slope of the potential at the end of inflation must be flatter than it was at horizon exit. In this work we show that a phenomenological hilltop model of inflation, satisfying the Kohri *et al.* criteria, could lead to the production of black holes, if the power of the inflaton self-interaction is less than or equal to 3, with a reasonable number or e-folds. We extend our analysis to the running mass model, and confirm that this model results in the production of black holes, and by using the latest WMAP year 5 bounds on the running of the spectral index, and the black hole constraint we update the results of Leach *et al.* (2000) excluding more of parameter space.

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I. INTRODUCTION

The temperature anisotropies of the cosmic microwave background (CMB) have been measured by WMAP on angular scales down to $\theta \sim 0.3°$, whereas they have yet to be measured to an effectual degree of accuracy on scales $\theta < 0.3°$ [1]. In fact, CMB data naively allows for a very large spectrum on these scales, i.e. a spectrum a few orders of magnitude above $\mathcal{P}_{\zeta_*} \simeq 10^{-9}$, evaluated at horizon exit. One can, however, place an upper limit on the smaller scale spectrum by taking into account astrophysical and cosmological constraints on black holes [2–11]. If this spectrum is in fact close to this upper limit, then the situation allows for "large" fluctuations; large enough to collapse into black holes, known as primordial black holes (PBHs) (cf. [12–18]). Since the uncertainties in the primordial spectrum at such small scales are dominated by instrumental noise [1], as opposed to cosmic variance which is the dominant source of uncertainties at larger angular scales, it may be that these uncertainties can be reduced in future surveys. Therefore, the question of whether the spectrum of perturbations on small scales is large enough to form PBHs is one that can, in theory, be answered.

The $\theta \gtrsim 0.3°$ spectrum has been used extensively in discriminating between models of inflation (cf. [19–25]). These analyses are based on the assumption that the anisotropies in the CMB, and hence the origin of large scale structure, are sourced by quantum fluctuations in a scalar field during or straight after inflation. As such, we assume that the fluctuations which sourced the PBHs are also generated during inflation, specifically towards the end of inflation, as it is during this epoch that the small scale fluctuations exit the horizon. Measuring the $\theta < 0.3°$ spectrum will therefore not only probe generic signatures of inflation [2,26] but also act as an indicator for the shape of the inflationary potential [18,27]. In this paper we aim to exploit the latter purpose of measuring the small scale spectrum, and investigate whether single field models of inflation can lead to the formation of PBHs.[1]

To be specific, the primordial black hole (PBH) condition [6,14,15,40,41] can be expressed as

$$\mathcal{P}^{1/2}_{\zeta_e} \simeq 0.03 = 10^3 \mathcal{P}^{1/2}_{\zeta_*}, \tag{1.1}$$

where the subscripts e and $*$ refer to the end of inflation and horizon exit, respectively. Assuming a constant spectral index then

$$n_s - 1 \simeq \frac{d \ln \mathcal{P}_\zeta}{dN} \simeq \frac{\Delta \ln \mathcal{P}_\zeta}{\Delta N} \simeq \frac{14}{\Delta N}, \tag{1.2}$$

where we used (1.1) in the last semiequality. $\Delta N \sim N$ refers to the number of e-folds that elapse from the time when scales of cosmological interest leave the horizon till the end of inflation.

Taking a standard value of $N \simeq 60$, it then follows from (1.1) that $n_s \simeq 1.3$. This is beyond the upper limit of 0.993 allowed by the recent WMAP data, at 95% confidence limit with no tensor modes or running. Therefore we consider the variation in the spectrum up to second order, and assume that the spectral index depends on scale, i.e. $\mathcal{P} \propto$

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[1]For multifield or multistage inflation models to produce PBHs, see Refs. [28–39] and references therein.

$k^{n_s(k)-1}$, then

$$\frac{d\ln\mathcal{P}}{d\ln k} = (n_s - 1) + n_s' \ln k$$
$$\ln\left[\frac{\mathcal{P}_{\zeta_e}}{\mathcal{P}_\zeta(N)}\right] = N(n_s - 1) + \frac{1}{2} n_s' N^2 \tag{1.3}$$

$$= 14, \tag{1.4}$$

where we used $\ln k = \ln(aH) = N$ in the second step. Taking $n_s \simeq 0.95$ and $N = 60$ then requires $n_s' \simeq 0.0061$ to satisfy the WMAP bounds and produce primordial black holes.

We now wish to rewrite the primordial black hole condition (1.1) in terms of the slow-roll parameters. Recalling that the spectrum can be written in terms of the potential V and the slow-roll parameter $\epsilon = m_{\rm Pl}^2 (V'/V)^2/2$, then [42]

$$\mathcal{P}_\zeta = \frac{1}{24\pi^2 m_{\rm Pl}^4} \frac{V}{\epsilon}. \tag{1.5}$$

Defining a new parameter $\mathcal{B} = \epsilon_e/\epsilon_*$, and combining Eqs. (1.1) and (1.5) we find that the condition for PBH formation, without violating the aforementioned astrophysical and cosmological bounds is

$$\mathcal{B} \simeq 10^{-6}. \tag{1.6}$$

An "absolute" upper bound on the spectrum is given by [10] as $\mathcal{P}_{\zeta_e}^{1/2} \sim 10^{-1}$, which translates to a lower bound $\mathcal{B} > 10^{-8}$.

Equation (1.6) tells us that for an inflationary potential to lead to the production of PBHs, its slope must flatten towards the end of inflation. This shape is satisfied by a phenomenological model akin to the one analyzed in [43], and also the running mass model, first introduced in [44]. In these types of models, we require that the inflaton initially be sitting at around the top of the hill, that is near a local maxima. This condition can be considered natural [43] from the viewpoint of eternal inflation [45], and can be understood as follows: via some mechanism, be it quantum tunneling or an inhomogeneous preinflationary universe, the inflaton will somewhere, at some time find itself sitting at the top of the potential, at which point the universe will start to inflate. As long as the inflaton is undisturbed, the universe will inflate indefinitely, and can end up volumetrically dominating the universe. Since this process can lead to an indefinitely large volume, then even if there was the smallest probability that inflation were to start, it would [46,47]. Within this patch, quantum fluctuations in some subpatches displace the inflaton from its vestige causing it to roll either to the left or the right, and ending inflation in those regions, while overall, the patch continues to inflate [47,48]. We introduce these models in Secs. II and III, respectively. We present our results in Sec. IV and discuss them in Sec. V.

A. The number of *e*-folds

In this paper we use the duration of inflation, otherwise known as the number of e-folds N, as a discriminator. It is defined as the ratio of the scale factor a at the end of inflation to a at the "beginning" of inflation:

$$N = \ln\left[\frac{a_e}{a_*}\right] \simeq m_{\rm Pl}^{-1} \int_{\phi_e}^{\phi_*} \frac{d\phi}{\sqrt{2\epsilon}}, \tag{1.7}$$

where the final semiequality comes from the slow-roll approximation.

To get a proper handle on how long inflation lasted from the time of horizon exit, one needs a complete history of the Universe. At present though, we do not have an agreed upon mechanism of reheating. Therefore, one assumes an instant transition from inflation to a radiation dominated universe, and gets the bounds [49]

$$10 \lesssim N \lesssim 110. \tag{1.8}$$

The lower bound comes from the assumption that nucleosynthesis is well bounded, and the upper bound assumes that the universe underwent a few bouts of "fast" roll inflation. We do note that these are extreme bounds, and that the limits $N = 54 \pm 7$ are more widely acceptable (cf. [20,42,49]).

B. Slow-roll and cosmological parameters

The lowest order slow order parameters are given by

$$\epsilon = \frac{m_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2, \tag{1.9}$$

$$\eta = m_{\rm Pl}^2 \frac{V''}{V}, \tag{1.10}$$

$$\xi^2 = m_{\rm Pl}^4 \frac{V'V'''}{V^2}, \tag{1.11}$$

which we then use to compute the spectral index and the running

$$n_s = 1 + 2\eta - 6\epsilon, \tag{1.12}$$

$$n_s' = \frac{dn}{d\ln k} = 16\epsilon\eta - 24\epsilon^2 - 2\xi^2. \tag{1.13}$$

We use the bounds on cosmological parameters given by the WMAP year 5, baryon acoustic oscillations and supernovae data sets [50]:

$$0.939 < n_s < 1.109 \tag{1.14}$$

$$-0.0728 < n_s' < 0.0087, \tag{1.15}$$

where a zero tensor mode was assumed in the prior. This prior is reasonable since we are considering small field models, characterized by a field variation that is smaller than the Planck mass $\Delta\phi < m_{\rm Pl}$. In these models the gravitational wave signature will be small [51], and by small we mean well below the sensitivity of WMAP5 parameter estimation, so we do not calculate the associated parameter.

II. THE TREE-LEVEL POTENTIAL

We consider the potential of the form

$$V = V_0\left(1 + \eta_p\left(\frac{\phi}{m_{\rm Pl}}\right)^p - \eta_q\left(\frac{\phi}{m_{\rm Pl}}\right)^q\right), \tag{2.1}$$

where $p \geq 2$ and $q > p$, plotted in Fig. 1. The case $p = 2$ and $q \geq 4$ can be generated from a flat direction in the minimal supersymmetric model (MSSM) [52–57]. In this case q is the nonrenormalizable operator that depends on the flat directions. One can also motivate the parameter range $p = 2$ and $q \geq 3$ in [57–59], in this scenario the inflaton higher order terms are not Planck suppressed, and one gets a lower energy scale inflation, on the order of the TeV—grand unified theory (GUT) scale. We also maintain $\phi < m_{\rm Pl}$, a realistic bound from an effective particle physics perspective, which demands that one not consider mass scales larger than the largest naturally occurring scale, in this case the Planck mass. Then due to the Lyth bound, the gravitational wave contribution of this model is negligible, regardless how long inflation lasts.

In this setup we require that the potential at the end of inflation be flatter than it was at the time of horizon exit, so the inflaton must roll towards the origin. We denote the inflaton value at the maximum of the potential as ϕ_m, at horizon exit as ϕ_* and at the end of inflation as ϕ_e. We impose the conditions

$$\phi_* < \phi_m \tag{2.2}$$

$$0 < \phi_e < \phi_o, \tag{2.3}$$

where ϕ_o is the inflection point. ϕ_m and ϕ_o are then given by (2.1)

$$\frac{\phi_m}{m_{\rm Pl}} = \left(\frac{p\eta_p}{q\eta_q}\right)^{1/(q-p)} \tag{2.4}$$

$$\phi_o = \phi_m\left(\frac{p-1}{q-1}\right)^{1/(q-p)}. \tag{2.5}$$

III. THE RUNNING MASS MODEL

With the exception of $p = 2$ and integral values $3 \leq q \leq 9$, the previous model is a phenomenological one. However the shape of potential does appear in a more theoretically motivated setup, the running mass model [44,60–67] which has the potential

$$V = V_0\left[1 - \frac{1}{2}\mu^2\frac{\phi^2}{m_{\rm Pl}^2}\right], \tag{3.1}$$

where the mass of the inflaton is scale dependent and can be expressed as

$$\mu^2(\phi) = \mu_0^2 + A_0\left[1 - \frac{1}{(1 + \alpha\ln(\phi/m_{\rm Pl}))^2}\right], \tag{3.2}$$

where μ_0^2 is the mass of the inflaton squared, A_0 is the gaugino mass squared in units of $m_{\rm Pl}$, and α is related to the gauge coupling.

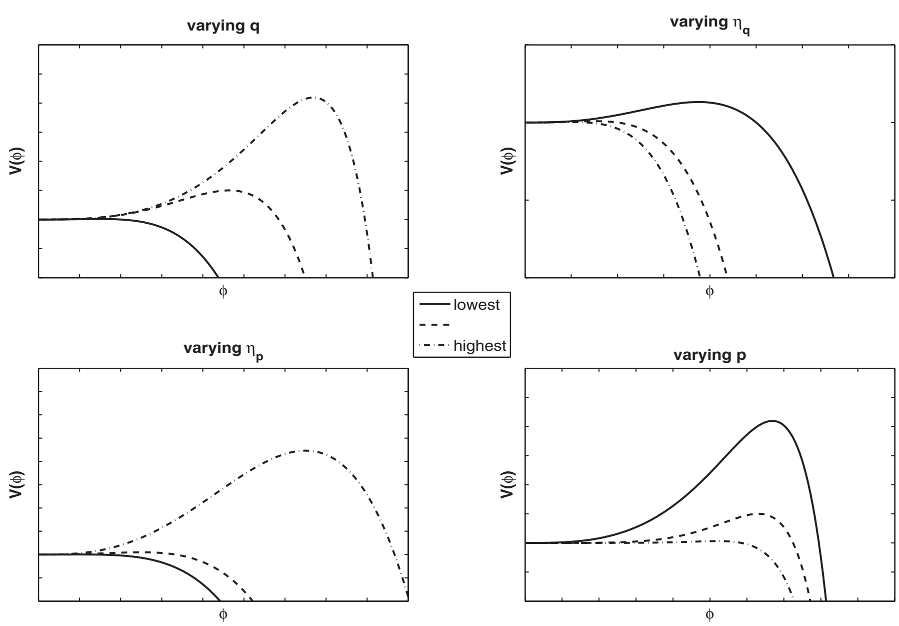

FIG. 1. For the figures on the left, increasing the variant decreases N, and the opposite is true for the variants defining the figures on the right.

The potential can then be written as

$$V = V_0\left[1 - \frac{1}{2}B_0^2\left(\frac{\phi}{m_{\rm Pl}}\right)^2 + \frac{A_0}{2(1+\alpha\ln(\phi/m_{\rm Pl}))^2}\left(\frac{\phi}{m_{\rm Pl}}\right)^2\right] \tag{3.3}$$

and $B_0^2 = \mu_0^2 + A_0$, with inflation occurring in the regime $\phi \ll m_{\rm Pl}$. This potential has the shape in Fig. 2, and the parameters have theoretically motivated constraints [61]

$$1 \lesssim \mu_0^2 \lesssim \mathcal{O}(10), \quad 0 \lesssim A_0 \lesssim \mathcal{O}(10), \quad 10^{-3} \lesssim \alpha \lesssim 10^{-1}. \tag{3.4}$$

Linear approximation

The linear approximation of the running mass potential is given by [27,62]

$$\frac{V}{V_0} = 1 - \frac{\phi^2}{2}(\mu_*^2 + c\ln(\phi/\phi_m)), \tag{3.5}$$

which is basically Eq. (3.3) expanded about the maximum of the potential. The terms c and μ_*^2 are related to the theoretical parameters A_0, μ_0^2 and α by

$$\mu_*^2 = \mu_0^2 + A_0\left[1 - \frac{1}{(1+\alpha\ln(\phi_m))^2}\right], \qquad c = \frac{2\alpha A_0}{(1+\alpha\ln(\phi_m))^3}. \tag{3.6}$$

Since ϕ_m defines the maximum of the potential then $\mu_*^2 = -c/2$, this allows us to write

$$N = -\frac{1}{c}\ln\left[\frac{\ln(\phi_m/\phi_*)}{\ln(\phi_m/\phi_e)}\right], \tag{3.7}$$

$$\frac{n-1}{2} = -c\left[\ln\left(\frac{\phi}{\phi_m}\right) + 1\right] \tag{3.8}$$

$$= \sigma e^{-cN} - c, \tag{3.9}$$

where $\sigma = -c\ln(\phi_e/\phi_m)$.

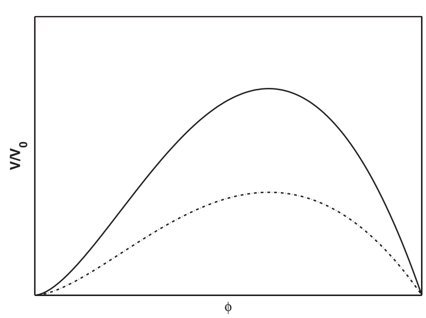

FIG. 2. Plot of the running mass model. The solid line represents a larger α than the dashed-dotted line.

This approximation is only valid near the maximum. We neither expect nor get reasonable values of ϕ_e using this estimate. However given ϕ_* and ϕ_e, we found that the linear approximation for N in (3.7) appears consistent with the numerical calculation.

IV. RESULTS

A. Hilltop

For the tree level potential, Fig. 3 depicts the dependence of N on p and q, we note that N increases sharply as p increases, as expected. We then searched for the range of p and q parameters that satisfy the bounds $10 < N < 110$ and found that the condition for PBH formation within this range is $2 \leq p < 3$ and $p < q < 4$. Larger values of p lead to $N \gg 110$ while larger values of q/p do not satisfy the WMAP bounds, since they steepen the potential and result in an increased n'. We also found that $p \simeq 2$ and $q < 3$ (note $q \neq 3$) places N in the range $N = 54 \pm 7$.

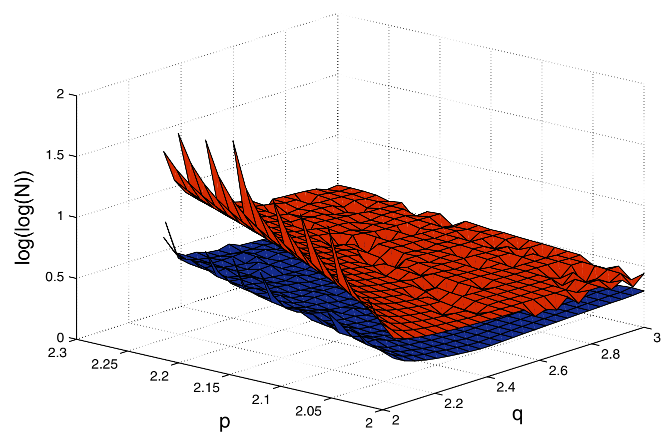

FIG. 3 (color online). A plot of the maximum and minimum values of $\log(\log(N))$ versus p and q, from a range values of η_p and η_q.

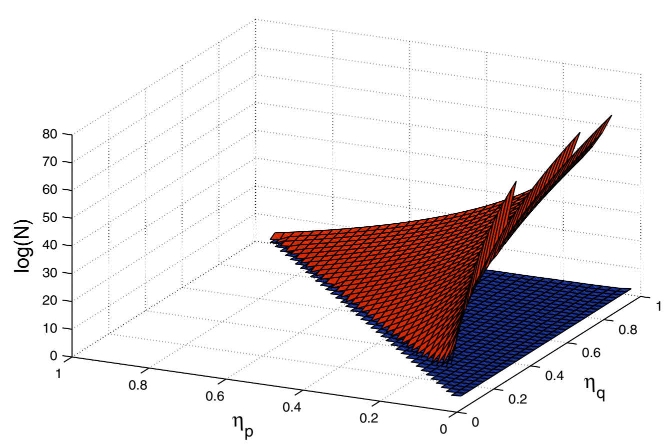

FIG. 4 (color online). A plot of the minimum values of $\log(N)$ versus η_p and η_q, from a range values of p and q.

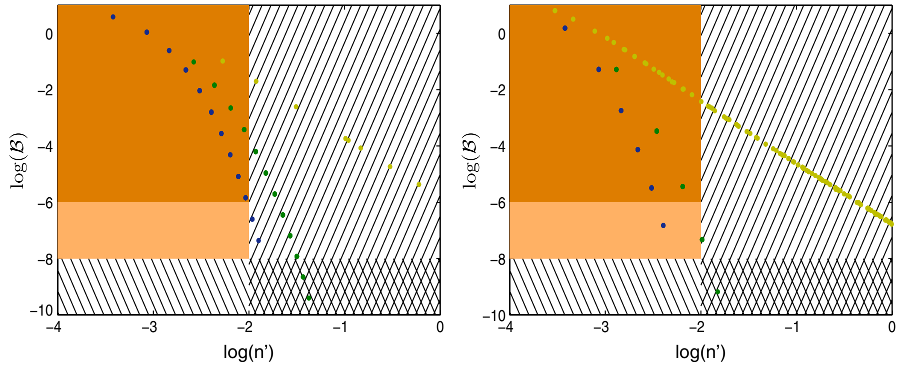

FIG. 5 (color online). Plot of $\log(\mathcal{B})$ versus $\log(n')$ for the hilltop model with $N = 60$ (figure on left), $N = 100$ (figure on right) and $n_s = 0.95$. The hatched region is excluded, representing $\log(n') > -2$ and $\log(\mathcal{B}) < -8$. The region $\log(\mathcal{B}) > -6$ does not lead to the formation of PBHs, and is represented by the tan color in the figure. PBHs can form in the region $-8 \leq \log(\mathcal{B}) \leq -6$ without violating astrophysical or cosmological bounds, and is represented by the light orange region. The yellow dots correspond to $\{p, q\} = \{3, 4\}$, the green dots to $\{p, q\} = \{2, 3\}$ and the blue dots to $\{p, q\} = \{2, 2.5\}$.

We then plot the dependence of N on η_p and η_q in Fig. 4. As expected N increases for decreasing η_p and vice versa for η_q. Once we have filtered out the reasonable values of N, we find that within the range $\{\eta_p, \eta_q\} = \{0, 1\}$ PBH formation will occur. It seems that the stronger constraints for PBH formation with $10 < N < 110$ come from p and q.

Next we consider the case of defining ϕ_* by the condition that $n(\phi_*) = 0.95$ and ϕ_e by $N = 60$ and $N = 100$. Figure 5 shows that for both $N = 60$ and $N = 100$, the parameter ranges satisfy the WMAP bounds and the PBH constraints. However, for $N = 60$ integral values of p and q do not lead to the formation of PBHs, while for $N = 100$, the parameter set $\{p, q\} = \{2, 3\}$ does.

B. The running mass model

In this analysis we considered the parameter ranges in (3.4), for which we find that this model generates a large range of N values. In Fig. 6 we plot the allowed parameter space for $10 \leq N \leq 110$ and $N = 54 \pm 7$.

From Fig. 7 we find that in order to avoid the overproduction of primordial black holes, $n > 1$ would be ruled out for $N = 45$, and this bound is strengthened to ruling out $n > 0.95$ for $N = 60$. These bounds are slightly stronger than those found by [67] who rule out $n \gtrsim 1.1$ for $N = 45$.

However, there is a discrepancy between our spectral index contour lines and [67], which we found was resolved by evolving our system an extra ~ 7 e-folds. Via a process of elimination, we think this may be due to the fact that

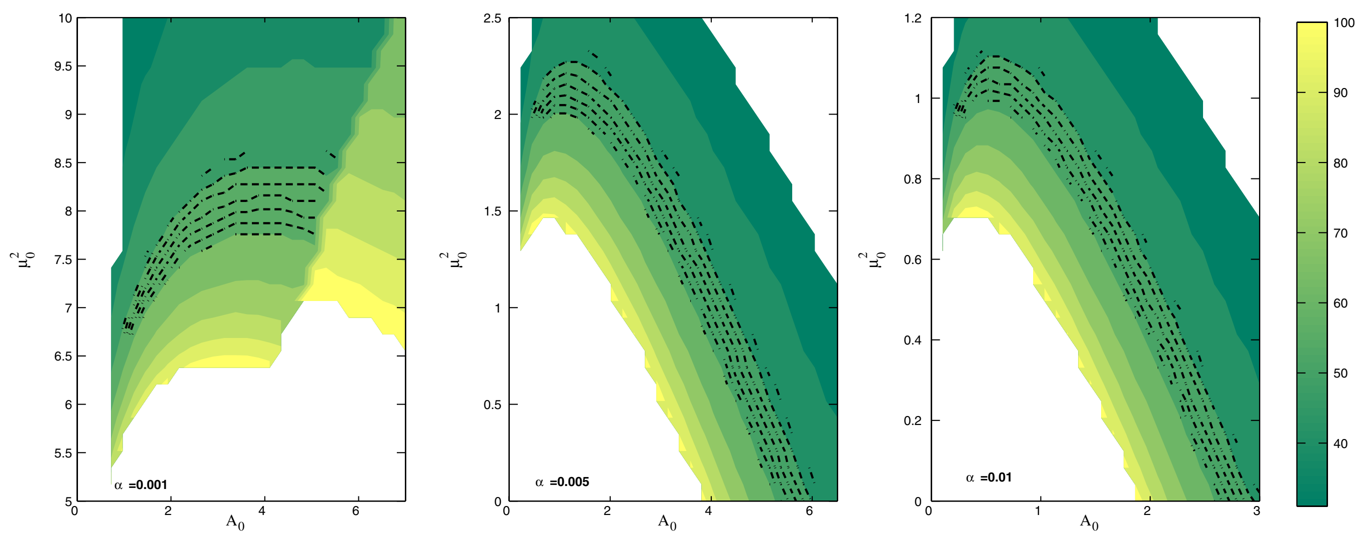

FIG. 6 (color online). Contour plots of the number of e-folds produced in the running mass model for three values of the gauge coupling $\alpha = [0.001, 0.005, 0.01]$. We found that $\alpha = 0.1$ did not satisfy the WMAP bounds on n_s and n'. We have filtered out the allowed parameter space for $10 \leq N \leq 110$, and colored it in shades of green. The dashed regions in each plot correspond to the more "reasonable" bound $N = 54 \pm 7$.

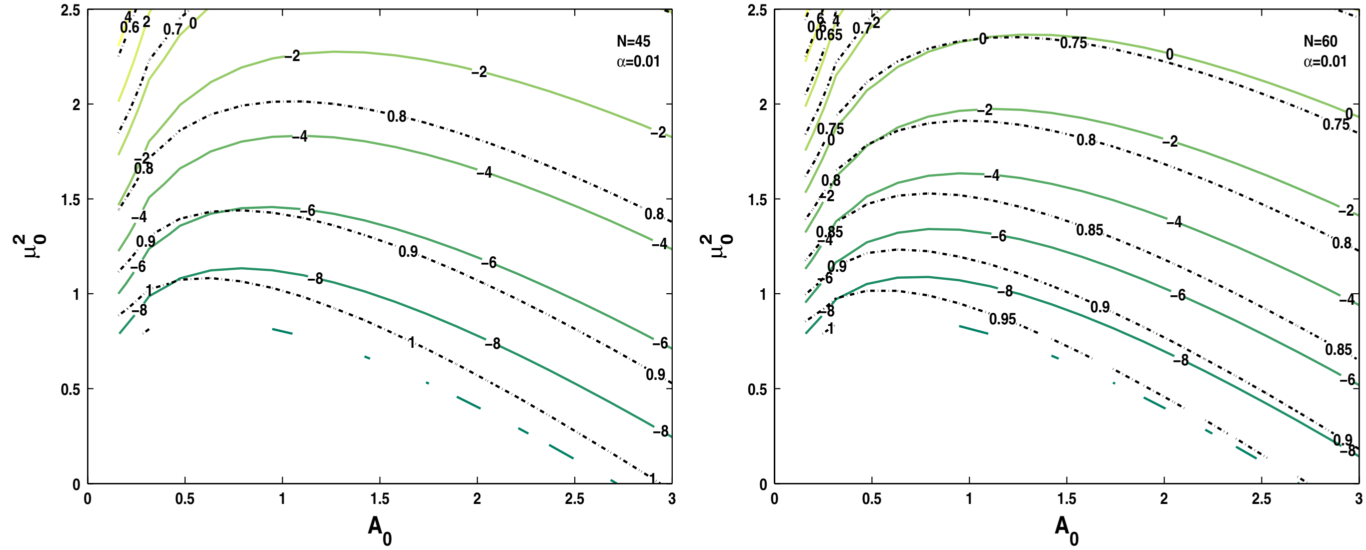

FIG. 7 (color online). In these plots we fix N and α, plotting contour lines of the spectral index (dashed line) and $\log(\mathcal{B})$ (solid line). Note that the contour lines do not exactly match Ref. [67], an anomaly that we discuss in the text. Parameter space below $\log(\mathcal{B}) \sim -8$ is excluded.

Ref. [67] solved the background equations numerically without resorting to the slow-roll approximation, while using the extended slow-roll formalism of [68] to solve for the perturbations.

Either way, as our method underestimates the allowed parameter range for each n, then using Ref. [67] method would strengthen our conclusions i.e. our bounds are conservative.

V. DISCUSSION AND CONCLUSIONS

In this paper we utilized the spectrum on scales $\theta \lesssim 0.3°$, corresponding to the end of inflation, to further the field of inflation model discrimination. The spectrum on these scales has yet to measured, but future CMB surveys such as the PLANCK mission may constrain its value. Astrophysical phenomena determines the upper bound to be $\mathcal{P}_{\zeta_e} \sim 10^{-4}$, corresponding to the criteria for the formation of primordial black holes (PBHs). In terms of the ϵ slow-roll parameter, this means that the value of ϵ at the end of inflation must be much smaller than its value at horizon exit (1.6). The models of inflation that satisfy this condition must therefore exhibit the unique property of having a flatter slope towards the end of inflation. So far, we only know of generic hilltop models of inflation that fulfil this criteria.

We have investigated whether these generic models of hilltop inflation would lead to the production of primordial black holes with a spectrum $\mathcal{P}_{\zeta_e} \sim 10^{-4}$. We found that within the range of parameters allowed by the latest WMAP data, the hilltop model (2.1) would lead to the formation of PBHs without violating astrophysical bounds for $p < 2.5$ and $q \lesssim 3$ if $N > 60$, and for $p \sim 2$, $2 < q \ll 3$ if $N > 40$. Integral values of p and q, which have some theoretical motivation, only lead to PBH formation within the bound (1.6) for $p = 2$ and $q = 3$, with $60 \ll N < 100$. In all cases it seems that near maximal running is required. If, however we were to allow $N \gg 110$ the range of parameters that would lead to PBH formation would be extended.

The allowed parameter range for the production of primordial black holes with $\mathcal{P}_{\zeta_e} \lesssim 10^{-4}$ in the running mass model is again dependent on N as can be seen from Fig. 6. We find that for $\alpha = 0.01$ and $\mu_0^2 \gtrsim 1.1$, black holes *could* form after $N < 47$ e-folds, and therefore before what can be considered a "reasonable" end to inflation. This is problematic on two counts, if we assume that the PBHs formed prior to the end of inflation, then this could lead to the overclosure of the universe (cf. [69]). On the same note, we know that on CMB scales the spectrum is too small to support PBH production. On the second count, assuming that the formation of the PBHs coincided with the end of inflation, then the arguments we presented in Sec. I A apply. Thus, using N as a discriminator we rule out $A_0 > 3$ and $\mu_0^2 > 1.1$ for $\alpha = 0.01$, $A_0 > 6$ and $\mu_0^2 > 2.4$ for $\alpha = 0.005$, and $A_0 > 5$ and $\mu_0^2 > 8.75$ for $\alpha = 0.001$. As we mentioned in the text $\alpha = 0.1$ is ruled out on WMAP consistency grounds.

This model has also been analyzed by [70], in which the authors use neutrino and γ- ray background data to constrain the PBH mass spectrum, which determines the spectral index on small scales $k \sim 15\ \mathrm{Mpc}^{-1}$. Then assuming that the running mass model is correct they reconstruct the power spectrum, finding that on these small scales the spectrum is highly sensitive to the running of the spectral index. Combining these two pieces of information they get bounds on n_s and n_s', which turn out to be inclusive of the

WMAP limits on these parameters. That is, by using a different approach to ours [70] conclude that the running mass model is consistent with WMAP while avoiding PBH overproduction, congruously with our findings.

Finally, we note that our generic results are consistent with the findings of [71]. They tackle the question of PBH production utilizing the arising constraints to derive bounds on the cosmological parameters, and conclude that the PBH constraint is "strongly" dependent on N and the spectrum at the end of inflation. Characteristics exhibited by our specific models.

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