High-energy neutrinos from dark matter particle self-capture within the Sun

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A potential flux of high-energy neutrinos from the annihilation of dark matter particles trapped within the Sun has been exploited to place indirect limits on particle dark matter. In most models, the dark matter interacts weakly, but the possibility of a dark matter particle with a large cross section for elastic scattering on other dark matter particles has been proposed in several contexts. I study the consequences of such dark matter self-interactions for the high-energy neutrino flux from annihilation within the Sun. The self-interaction among dark matter particles may allow dark matter in the halo to be captured within the Sun by scattering off of dark matter particles that have already been captured within the Sun. This effect is not negligible in acceptable and accessible regions of parameter space. Enhancements in the predicted high-energy neutrino flux from the Sun of tens to hundreds of percent can be realized in broad regions of parameter space. Enhancements as large as factors of several hundred may be realized in extreme regions of the viable parameter space. Large enhancements require the dark matter annihilation cross section to be relatively small, $\langle\sigma_{\rm A} v\rangle \lesssim 10^{-27}\,{\rm cm}^3\,{\rm s}^{-1}$. This phenomenology is interesting. First, self-capture is negligible for the Earth, so dark matter self-interactions break the correspondence between the solar and terrestrial neutrino signals. Likewise, the correspondence between indirect and direct detection limits on scattering cross sections on nuclei is broken by the self-interaction. These broken correspondences may evince strong dark matter self-interactions. In some cases, self-capture can lead to observable indirect signals in regions of parameter space where limits from direct detection experiments would indicate that no such signal should be observable.

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I. INTRODUCTION

A great deal of observational evidence indicates that a form of nonrelativistic, nonbaryonic matter constitutes the vast majority of mass in the Universe. The unknown nature of the *dark matter* that binds galaxies and drives cosmic structure formation remains an important problem in cosmology and particle physics. Among dark matter candidates, weakly interacting massive particles (WIMPs), including the lightest superpartners in supersymmetric theories, have received the most attention (for a review, see Ref. [1]). In this paper, I study a potential enhancement in high-energy neutrino fluxes from dark matter annihilations within the Sun in models where a WIMP-like dark matter particle exhibits relatively strong interactions with itself.

Indirect, astrophysical probes of dark matter are an important element of any comprehensive program to identify the dark matter unambiguously. One indirect probe of WIMP dark matter is a potentially detectable flux of high-energy muon neutrinos arising from the annihilation of dark matter particles captured within the Sun [2–12]. A similar signal from within the Earth may also be exploited in this regard [5,6,13], and though the terrestrial signal is typically smaller than the solar signal, it is a valuable cross-check [11]. In fact, these signals have already been brought to bear to limit dark matter elastic scattering cross sections with nucleons at interesting levels [14–18].

This basic scenario is simple. As the Sun moves through the halo of WIMPs, some of the WIMPs scatter elastically off of nuclei within the Sun. Many WIMP-nucleus interactions result in WIMPs moving at speeds lower than the local escape speed relative to the Sun. These particles are captured and for a large region of relevant parameter space they come to thermal equilibrium in the interior of the Sun. Eventually, the buildup of WIMPs within the Sun is limited by the annihilation of these WIMPs producing neutrinos that can escape from the Sun. Annihilation products other than neutrinos interact within the Sun and are not observable at Earth.

Dark matter particles that interact weakly with standard model particles, but exhibit comparably rather strong interactions among themselves, have now been proposed in several different contexts [19–32]. Some bounds on dark matter self-interactions exist [33–37] and observational tests of various scenarios have been proposed [38–42], but a wide range of parameter space remains and will remain viable for $M_{\rm x} \sim$ a few $\times\, 10^2$ GeV dark matter particles with large self-interaction cross sections, $\sigma_{\rm xx} \sim 10^{-24}$ cm^2.

Large cross sections for dark matter particles to scatter elastically off of each other open a new possibility for capture within the Sun. In addition to nuclei, previously captured dark matter particles or dark matter particles otherwise sequestered within the solar interior may serve as additional targets for the capture of halo dark matter particles. I refer to this as dark matter "self-capture" and it is this possibility that I consider in detail in this paper. Previous studies have considered distinct modifications to high-energy neutrino fluxes from the Earth and Sun due to inelastic scattering of dark matter particles against nuclei [43,44].

I begin in Sec. II with a brief sketch of the standard scenario for indirect detection of dark matter via high-energy neutrinos from the solar interior. In Sec. III, I use a simple, order-of-magnitude estimate to show that dark matter self-capture within the Sun may not be negligible for a range of viable and interesting models. On the other hand, self-capture within the Earth is always negligible for models that have not yet been excluded by other means.

In Sec. IV, I give the results of more detailed calculations of the importance of dark matter particle self-capture within the Sun. These results demonstrate that modest enhancements of tens to a hundred percent relative to models in which self-capture is negligible are possible over reasonably broad ranges of interesting parameter space. Significantly larger flux enhancements of up to factors of hundreds are possible in extreme regions of the dark matter parameter space. Throughout, I remain relatively agnostic about the nature of the dark matter and present results as a function of the most directly relevant model parameters, dark matter particle mass M_{x}, dark matter-proton scattering cross section σ_{p}, dark matter self-interaction cross section σ_{xx}, and thermally averaged dark matter annihilation cross section multiplied by relative velocity $\langle \sigma_{\mathrm{A}} v \rangle$. However, I do use the findings of detailed explorations of the parameters available to neutralino dark matter in supersymmetric scenarios as guidance for interesting values of these parameters [45–47].

I summarize my results and conclusions in Sec. V. In particular, I emphasize that flux enhancements from self-capture scenarios may be important for two reasons. First, the solar flux may be significantly altered by self-capture, while the terrestrial flux cannot be. Therefore, the ratio of the solar to terrestrial high-energy neutrino fluxes from dark matter may be markedly different from the standard predictions and this may signify new dark matter interactions. Likewise, the similar correspondence between direct search signals and solar high-energy neutrino fluxes can be broken. In some cases, models that may otherwise be ruled out by direct dark matter searches may produce observable neutrino signals due to the self-capture enhancement. I include in an Appendix the details of the capture rate calculations that I perform, including an example of the capture rates that I compute in the standard scenario of spin-independent capture off of nuclei. This discussion follows the derivations given by Gould [9,11].

II. HIGH-ENERGY NEUTRINOS FROM THE SUN AND DARK MATTER SELF-CAPTURE

In the most well-studied scenarios, captured dark matter particles typically thermalize in the solar interior on a time scale less than the age of the Sun ($\tau_{\odot} \approx 5 \times 10^{9}$ yr) as well as the other time scales in the problem [2–9]. In this case, the time evolution of the number of dark matter particles in the Sun N_{x} follows:

$$\frac{dN_{\mathrm{x}}}{dt} = C_{\mathrm{c}} + C_{\mathrm{s}} N_{\mathrm{x}} - C_{\mathrm{a}} N_{\mathrm{x}}^{2}. \tag{1}$$

The coefficients are the rate of capture of dark matter particles by scattering off of nuclei within the Sun C_{c}, *twice* the rate of annihilation per pair of dark matter particles within the Sun C_{a} (*twice* because each annihilation eliminates two particles), and the rate of capture of dark matter particles by scattering off of other dark matter particles that have already been captured within the Sun C_{s}. The $C_{\mathrm{s}} N_{\mathrm{x}}$ term on the right-hand side of Eq. (1) is the new term that I study in this paper. Capture rates were first computed by Press and Spergel [10] and this calculation was revised, corrected, and greatly expanded upon in an impressive series of papers by Gould [9,11,12]. In principle, evaporation of captured particles from the Sun can also occur, but this is unimportant for masses larger than a few GeV [7,12]. I discuss the specific rates at greater length below and in the Appendix. For the time being, let us focus attention on Eq. (1).

In the standard treatment, self-capture of dark matter particles is ignored ($C_{\mathrm{s}} = 0$). The solution of Eq. (1) for $N_{\mathrm{x}} = 0$ at $t = 0$ is then

$$N_{\mathrm{x}} = \sqrt{\frac{C_{\mathrm{c}}}{C_{\mathrm{a}}}} \tanh(\sqrt{C_{\mathrm{c}} C_{\mathrm{a}}}\, t). \tag{2}$$

There is a time scale for equilibration between dark matter annihilation and dark matter capture, $\tau_{\mathrm{eq}} = 1/\sqrt{C_{\mathrm{c}} C_{\mathrm{a}}}$. For many models of interest, $\tau_{\mathrm{eq}} \ll \tau_{\odot}$ and the solution approaches a steady state,

$$N_{\mathrm{x,eq}} = \sqrt{\frac{C_{\mathrm{c}}}{C_{\mathrm{a}}}}. \tag{3}$$

In this circumstance, the rate of annihilation of captured dark matter particles within the Sun is

$$\Gamma_{\mathrm{a}} = \tfrac{1}{2} C_{\mathrm{a}} (N_{\mathrm{x,eq}})^{2} = \tfrac{1}{2} C_{\mathrm{c}}. \tag{4}$$

The factor of $1/2$ in Eq. (4) arises because there are $N_{\mathrm{x,eq}}^{2}/2$ distinct pairs of particles, while this factor is not present in Eq. (1) because each annihilation eliminates two dark matter particles from the Sun. The annihilation rate Γ_{a} is independent of the annihilation rate coefficient C_{a}. Consequently, the observable flux at Earth is independent of the dark matter mutual annihilation cross section, provided the cross section is large enough that the equilibrium solution [Eq. (3)] obtains. Dark matter particles annihilate upon capture and the flux at the Earth is modulated only by the capture rate C_{c}.

In the first papers to study high-energy neutrinos from dark matter annihilation within the Sun, it was noted that this phenomenology is interesting and useful. First, the flux from annihilations at the rate of Eq. (4) is independent of annihilation cross section, so this is an indirect search method that does not rely on models with relatively large annihilation cross sections. Second, the high-energy neutrino signal from annihilation in the solar interior can be related to other potentially observable signals. I discuss the rate of capture in the Appendix, but it suffices to note that the capture rate of dark matter particles in the Sun should be proportional to the cross section for a dark matter particle to scatter off of a nucleus (σ_N) within the Sun and the local density of dark matter (ρ_x), $C_c \propto \sigma_N \rho_x$. A similar process may operate within the Earth, whereby captured dark matter particles give rise to high-energy neutrinos from the Earth's center and this signal should also grow in proportion to the product $\sigma_N \rho_x$ [5,6,13]. Moreover, signals in direct dark matter search experiments are proportional to this same product as well [1]. As a consequence, indirect detection of dark matter through high-energy neutrinos from the Sun and Earth as well as direct dark matter search experiments may serve to check each other and corroborate any detections.

If self-capture is not ignored this picture may be altered. In particular, the general solution to Eq. (1) with $C_s \geq 0$ is

$$N_{x,s} = \frac{C_c \tanh(t/\zeta)}{\zeta^{-1} - C_s \tanh(t/\zeta)/2}, \quad (5)$$

where

$$\zeta = \frac{1}{\sqrt{C_c C_a + C_s^2/4}}. \quad (6)$$

The steady-state solution at $t \gg \zeta$ is

$$N_{x,eq}^{self} = \frac{C_s}{2C_a} + \sqrt{\frac{C_s^2}{4C_a^2} + \frac{C_c}{C_a}}. \quad (7)$$

Clearly, Eq. (7) reduces to Eq. (3) in the case that $C_s = 0$. Whether or not self-capture of dark matter particles is important can be discerned by comparing the time scales C_s^{-1} and $1/\sqrt{C_c C_a}$, and I will pursue this comparison shortly. Consider the case where $C_s^2 \gg C_c C_a$. In this circumstance, $N_{x,eq}^{self} \simeq C_s/C_a$. The annihilation rate within the Sun is then

$$\Gamma_a = \tfrac{1}{2} C_a (N_{x,eq}^{self})^2 = \tfrac{1}{2} C_s^2 / C_a. \quad (8)$$

The annihilation rate grows in inverse proportion to the rate coefficient C_a when self-capture is possible. As I discuss below, the annihilation rate coefficient is proportional to the annihilation cross section, so models with relatively *low* annihilation cross sections are favored for an indirect neutrino signal from the Sun when self-capture is not negligible.

Figure 1 shows two examples of the evolution of the rate of annihilation of dark matter particles captured within the Sun as a function of time. If self-capture is not negligible, the evolution of the total number of captured dark matter particles may come to be dominated by the second term on the right-hand side of Eq. (1) and N_x may grow exponentially for some time. This exponential growth is eventually terminated by annihilations, so the lower the annihilation cross section, the more exponential growth of N_x is important and the greater the relative flux of neutrinos from the Sun may be (provided the Sun is old enough that the equilibrium solution has been achieved).

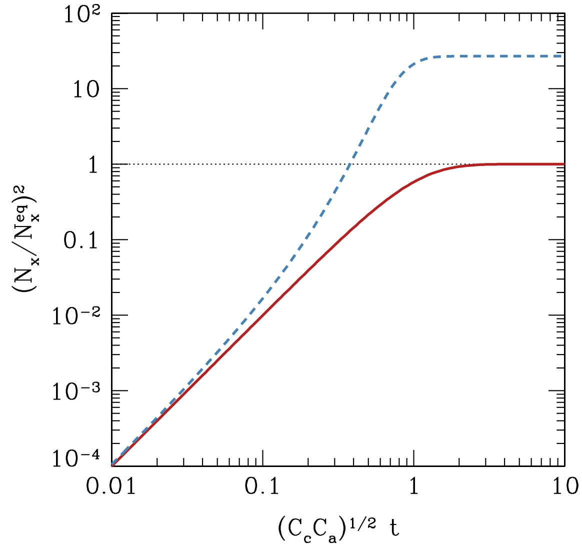

FIG. 1 (color online). The evolution of the annihilation rate of dark matter particles captured within the sun as a function of time. The *solid* line shows evolution to the steady-state solution of Eqs. (3) and (4) in the absence of significant dark matter self-interactions. The *dashed* lines show evolution to the new equilibrium in a model in which self-interactions are important and $C_s = 5\sqrt{C_c C_a}$. Time is shown in units of the standard equilibration time scale $\tau_{eq} = 1/\sqrt{C_c C_a}$ and the squared number of captured dark matter particles is shown relative to the standard equilibrium number $N_{x,eq} = \sqrt{C_c/C_a}$.

III. THE IMPORTANCE OF DARK MATTER SELF-CAPTURE

I now present a simple estimate of the parameter values for which dark matter particle self-capture may be a non-negligible effect. I define the ratio $R_s = C_s^2/C_c C_a$. Comparing to Eqs. (5) and (7), self-capture will be negligible when $R_s \ll 1$ and dominant when $R_s \gg 1$. The task now is to evaluate each of the rate coefficients for a set of parameters describing the interactions of the dark matter particle.

I show in the Appendix that a simple approximation for the self-capture rate is

$$C_s N_x \approx \sqrt{\frac{3}{2}} n_x \sigma_{xx} v_{esc}(R_\odot) \frac{v_{esc}(R_\odot)}{\bar{v}} N_x \langle \hat{\phi}_x \rangle \frac{\mathrm{erf}(\eta)}{\eta}, \quad (9)$$

where n_x is the local number density of dark matter particles in the halo, σ_{xx} is the dark matter elastic scattering cross section, $v_{esc}(R_\odot)$ is the escape speed from the surface of the Sun, $\bar{v}$ is the local three-dimensional velocity dispersion of dark matter particles in the halo, $\langle\hat{\phi}_x\rangle \simeq 5.1$ is a dimensionless average solar potential experienced by captured dark matter particles within the Sun, and $\eta^2 = 3(v_\odot/\bar{v})^2/2$ is the square of a dimensionless speed of the Sun through the Galactic halo. Relation Eq. (9) neglects the recoils of the target dark matter particles; however, I demonstrate in the Appendix that this is a reasonable approximation when escape speeds are significant compared to $v_\odot$ and $\bar{v}$. This condition holds for capture within the Sun.

The full expressions for capture off of nuclei are unwieldy, so I make an effort here to evaluate R_s approximately. To simplify the evaluation of R_s, I temporarily assume that capture of dark matter by scattering off of nuclei is never significantly restricted by the kinematics of the scattering. This is not generally true. In any individual scattering event dark matter particles can lose a fraction of their kinetic energy $\Delta E/E \leq 4M_xM_N/(M_x + M_N)^2$, where M_N is the mass of the target nucleus. Capture within the Sun typically requires $\Delta E/E \gtrsim 1/5$. Therefore, capture can be efficient for dark matter and nucleus masses that differ by a factor of as much as $\sim$20. If the mass of the dark matter particle is sufficiently different from the mass of the nucleus on which it scatters, capture may be kinematically unfavorable.

Neglecting kinematic limitations to capture on nuclei, the capture rate off of a particular nuclear species N is given by a relation analogous to Eq. (9) with $C_sN_x \rightarrow C_c$, $\sigma_{xx} \rightarrow \sigma_N$, $N_x \rightarrow f_N M_\odot/M_N$, and $\langle\hat{\phi}_x\rangle \rightarrow \langle\hat{\phi}_N\rangle$. The quantity σ_N is the scattering cross section of the dark matter particle off of the nucleus N, f_N is the fraction of the solar mass in nucleus N, and $\langle\hat{\phi}_N\rangle$ is the average dimensionless potential experienced by these nuclei. For most nuclei within the Sun, $\langle\hat{\phi}_N\rangle \simeq 3.2$ [11]. Examining Eq. (9), the capture rate off of nuclei in this limit scales with dark matter mass as $C_c \propto M_x^{-1}$. When $M_x \gg M_N$, the kinematic limitation to the capture rate is not negligible, the maximum fractional kinetic energy lost per collision is $\sim 4M_N/M_x$, and the capture rate scales as $C_c \propto M_x^{-2}$.

Assuming that the dark matter particles equilibrate with the solar interior rapidly upon capture, the coefficient C_a is likewise simple to estimate. Let $\epsilon_x(r)$ be the number density of dark matter particles as a function of position in the Sun. The annihilation rate coefficient is then

$$C_a = \frac{4\pi\langle\sigma_A v\rangle}{N_x^2}\int_0^{R_\odot} \epsilon_x^2(r) r^2 dr, \tag{10}$$

and the naive expectation for a nonrelativistic thermal relic dark matter particle is that $\langle\sigma_A v\rangle \sim 10^{-26}\ \mathrm{cm^3\,s^{-1}}$. Under the assumption of a thermal distribution at an effective solar core temperature $T_{\odot,c} = 1.57 \times 10^7$ K, the distribution $\epsilon_x(r) \propto \exp[-M_x\phi(r)/T_{\odot,c}]$, where $\phi(r)$ is the gravitational potential as a function of position within the Sun. Making a further assumption of a constant solar density of $\rho_{\odot,c} = 150\ \mathrm{g/cm^3}$ the integral is straightforward to evaluate. Conventionally, this has been represented in terms of effective volumes,

$$C_a = \langle\sigma_A v\rangle \frac{V_2}{V_1^2}, \tag{11}$$

where

$$V_j = 2.45 \times 10^{27}\left(\frac{100\ \mathrm{GeV}}{jM_x}\right)^{3/2}\ \mathrm{cm^3}. \tag{12}$$

The effective volumes represent $\sim 10^{-6}$ of the total solar volume, so captured particles extend over only $\sim$1% of the solar radius, justifying the constant density approximation [7].

The question arises whether the strong self-interactions among the dark matter particles should alter the assumption of a thermal distribution. A definitive answer to this question requires solving the Boltzmann equation. Such a calculation is extensive and beyond the scope of this paper. It is reasonable to suspect that any modifications will be minor for parameters of interest. In the relevant parameter regime (which will be more clearly delineated below), the increase in the number of captured dark matter particles relative to the standard scenario is modest (less than a factor of $\sim$30), yet collisions among dark matter particles may happen at a rate that is comparable to the rate of collisions between dark matter particles and nuclei. It is a relatively simple matter to use the approximate methods of Ref. [9] to show that particles with $M_x \gtrsim 10$ GeV remain localized well within the solar interior and are not altered by the temperature gradient within the Sun, and that the rate of energy inflow due to capture is slower than the rate of thermalization with the solar interior. Finally, it is also straightforward to show that for masses $\lesssim 10^{12}$ GeV (which far exceeds the unitarity bound for a thermal relic, e.g. [48]), the dark matter density is never sufficiently large for the dark matter to be self-gravitating, so the self-interaction does not lead to rapid collapse via the gravothermal catastrophe (the case of $M_x \gtrsim 10^{12}$ GeV is treated in Ref. [49]). Given these considerations, a standard thermal profile seems a reasonable approximation and I proceed under this assumption.

With expressions for C_c, C_s [Eq. (9)], and C_a [Eq. (11)], all of the pieces are now in place to approximate the ratio $R_s = C_s^2/C_cC_a$. Making the simplifying assumption that capture occurs primarily off of a single type of nucleus, this gives

$$R_s \approx \sqrt{\frac{3}{2}}\frac{\sigma_{xx}^2 v_{esc}(R_\odot)}{\sigma_N\langle\sigma_A v\rangle}\frac{v_{esc}(R_\odot)}{\bar{v}}\frac{\langle\hat{\phi}_x\rangle^2}{\langle\hat{\phi}_N\rangle}\frac{V_1}{V_2}\frac{n_x M_N V_1}{f_N M_\odot}\frac{\mathrm{erf}(\eta)}{\eta}. \tag{13}$$

Evaluating this for the particular case of capture off of oxygen (which is the most important individual element for dark matter capture in the Sun, see Fig. 5 in the Appendix and Ref. [11]), taking $f_{\rm N} = 10^{-2}$ (almost twice the solar oxygen abundance to account for the simplicity of the current estimate), and keeping relevant aspects of the dark matter particle model explicit yields

$$R_{\rm s} \approx 0.4\left(\frac{\sigma_{\rm xx}}{10^{-24}\ {\rm cm}^2}\right)^2\left(\frac{10^{-42}\ {\rm cm}^2}{\sigma_{\rm N}}\right)\left(\frac{10^{-27}\ {\rm cm}^3\,{\rm s}^{-1}}{\langle\sigma_{\rm A} v\rangle}\right) \times\left(\frac{100\ {\rm GeV}}{M_{\rm x}}\right)^{5/2}. \tag{14}$$

The result in Eq. (14) is somewhat startling. Current best bounds on the elastic scattering cross section of dark matter particles off of each other are assumption dependent and approximate, but they indicate that $\sigma_{\rm xx} \lesssim 10^{-23}(M_{\rm x}/100\ {\rm GeV})\,{\rm cm}^2$ [19,33–36]. A slightly less restrictive bound from an analysis of the bullet cluster is probably the least dependent upon particular assumptions [33]. Equation (14) indicates that dark matter particle self-capture within the Sun is not necessarily a negligible effect in acceptable regions of dark matter particle parameter space. Moreover, Eq. (14) neglects the fact that dark matter capture off of nuclei may be kinematically unfavorable, while self-capture can never be kinematically unfavorable because the particles will always have equal mass.

I note in passing that self-capture by the Earth can never be important. As I discuss in the Appendix, collisions between halo dark matter particles and particles already captured within a body may result in the target dark matter particles being ejected from the body upon recoil. The net result in this case is no gain in the number of captured dark matter particles. Ejection by recoil depends upon the ratio of the speed of the particles at infinity to the escape speed from the body. Within the Sun, escape speeds are always significantly higher than the typical relative speeds of dark matter particles at infinity and ejection is only a small correction to the simple solar capture estimate. In the case of the Earth, escape speeds are more than an order of magnitude *lower* than the typical relative speeds of dark matter particles at infinity so almost all collisions within the Earth result in ejection of the target dark matter particle. The ejection of the targets from the Earth introduces the possibility that the halo dark matter particles may scour the Earth of any particles captured through interactions with nuclei. This can be computed in a manner analogous to self-capture, though the sign of the term linear in $N_{\rm x}$ in Eq. (1) would be negative. In the case of the Earth, the ejection rate is small and $R_{\rm s} \lesssim 10^{-3}$ for all parameters of interest. Modifications to the Earth signal are negligible.

In the following section, I show results from a more detailed calculation of the importance of dark matter particle self-capture for high-energy neutrino fluxes observed at the Earth. I use the formulas from Ref. [11] to compute the capture rate of dark matter particles from nuclei as described in the Appendix. These formulas are lengthy and I do not reproduce them in full here, though I give an example of the capture rates that I use in Fig. 5. I use Eq. (A20) to compute the rate coefficient for self-capture of dark matter particles. This relation is derived in the Appendix and includes the reduction in the capture rate due to the potential ejection of target dark matter particles.

The most relevant quantity to compute is the *enhancement* in the neutrino signal due to self-capture of dark matter relative to the neutrino flux expected in the standard calculation. I define the quantity

$$\beta \equiv \frac{N_{\rm x,s}^2}{N_{\rm x}^2}, \tag{15}$$

which is the ratio of the high-energy neutrino flux when self-capture is possible to the high-energy neutrino flux without the possibility of self-capture. My primary results are illustrations of the dependence of β on the parameters $\sigma_{\rm xx}$, $\langle\sigma_{\rm A} v\rangle$, the spin-independent dark matter particle-proton elastic scatting cross section $\sigma_{\rm p}^{\rm SI}$, and $M_{\rm x}$. To evaluate β, I *do not* assume that the equilibrium solutions of Eq. (1) are attained. Rather, I evaluate $N_{\rm x}(t = \tau_\odot)$ and $N_{\rm x,s}(t = \tau_\odot)$ using the general solution of Eq. (5).

I assume that the cross section for dark matter scattering off of nuclei other than hydrogen is given by [1]

$$\sigma_{\rm N}^{\rm SI} = \sigma_{\rm p}^{\rm SI} A^2 \frac{M_{\rm x}^2 M_{\rm N}^2}{(M_{\rm x} + M_{\rm N})^2}\frac{(M_{\rm x} + m_{\rm proton})^2}{M_{\rm x}^2 m_{\rm proton}^2}, \tag{16}$$

where A is the atomic mass number and $m_{\rm proton}$ is the proton mass. Loss of coherence is accounted for in the full formulas through suppression by an exponential form factor [11,12]. In the following examples, I focus primarily on spin-independent interactions. Spin-dependent capture of dark matter off of nuclei occurs only for hydrogen within the Sun and is typically down by roughly two or more orders of magnitude at fixed cross section for high mass ($M_{\rm x} \gtrsim 100$ GeV) dark matter particles. This may be mitigated by the fact that the spin-dependent cross section for scattering off of protons is typically $\sim$1–2 orders of magnitude larger than the spin-independent cross section in viable regions of the constrained minimal supersymmetric standard model parameter space [45]. Moreover, direct search experiments typically use large nuclei with no net spin and exploit the scaling of Eq. (16), so neutrino telescopes [14–16] a very competitive with direct search bounds on a spin-dependent interaction [50–53] and should remain so [54]. In Sec. IV, I show estimates for the spin-independent case as it is more general, including capture off of all nuclei within the Sun, and more interesting for the present purposes in the sense that complementary constraints from direct search experiments are more competitive with indirect methods for spin-independent capture. Including spin-dependent capture would typically add a term that is at most comparable to $C_{\rm c}$ (though this is a model-dependent statement) and I find comparable values of β for spin-dependent capture with spin-dependent cross sections $\sigma_{\rm p}^{\rm SD} \sim 10^2\sigma_{\rm p}^{\rm SI}$.

It is important to set the scale of the signal relative to current and future observations, so I present estimates of absolute fluxes in Sec. IV as well. Annihilations in the Sun lead to a flux of high-energy neutrinos at the Earth. The observable signal at a detector such as IceCube [14,55,56] is a flux of upward-directed high-energy muons induced by scattering of muon neutrinos near the detector. The muon flux at the detector is therefore a relatively complicated product and may be written as

$$\Phi = \frac{\Gamma_{\rm a} n_{\rm T}}{4\pi A_{\oplus}^2} \int_{E_{\rm TH}} dE_\mu \int_{E_{\rm TH}} dE_\nu \int_{E_\mu}^{E_\nu} d\bar{E}_\mu P(\bar{E}_\mu \rightarrow E_\mu, \lambda) \times \frac{d\sigma_{\nu\mu}(E_\nu)}{d\bar{E}_\mu} \sum_{\rm i} P_{\rm osc}({\rm i} \rightarrow \mu) \sum_{\rm f} B_{\rm f} \frac{dN_{\rm i/f}}{dE_\nu}. \tag{17}$$

Individually, the factors in Eq. (17) are relatively simple to interpret. $\Gamma_{\rm a}$ is the annihilation rate in the solar interior, $A_{\oplus}$ is the semimajor axis of the Earth's orbit about the Sun, and $n_{\rm T}$ is the number density of target nuclei near the detector. The quantity $P(\bar{E}_\mu \rightarrow E_\mu, \lambda)$ is the probability for a muon of initial energy $\bar{E}_\mu$ to have final energy E_μ after traversing a path of length λ in the detector material, the differential cross section $d\sigma_{\nu\mu}(E_\nu)/d\bar{E}_\mu$ describes the production of a muon of initial energy $\bar{E}_\mu$ from an incident neutrino of energy E_ν, $P_{\rm osc}({\rm i} \rightarrow \mu)$ is the probability that a neutrino produced as flavor i is a muon neutrino near the detector, $B_{\rm f}$ is the branching ratio to annihilation channel f, and $dN_{\rm i/f}/dE_\nu$ is the differential spectrum of neutrinos of flavor i per unit energy dE_ν produced per f-channel annihilation.

I have evaluated Eq. (17) for an experiment such as IceCube [14,55,56] using the results of the WIMPSIM Monte Carlo simulations [57] as available through the DARKSUSY package [58]. I choose $E_{\rm TH} = 1$ GeV in accord with the common convention for reporting results from neutrino telescopes. An instrument like IceCube observes events above tens of GeV, so sensitivities quoted relative to $E_{\rm TH} = 1$ GeV depend upon an assumed spectrum. I show flux normalizations for two simple choices of branching fraction. I show results for annihilation to W^+W^- gauge bosons only ($B_{\rm W^+W^-} = 1$) as a simple approximation of fluxes that may be produced from annihilation of a typical neutralino and a slightly more optimistic case of annihilation to $\tau^+\tau^-$ only ($B_{\tau^+\tau^-} = 1$). Annihilation to $\tau^+\tau^-$ yields about 3 times higher flux than annihilation to gauge bosons through most of the relevant dark matter particle mass range [54,57].

IV. RESULTS FOR HIGH-ENERGY NEUTRINO FLUX ENHANCEMENTS

I summarize results on the relative importance of a contribution from dark matter particle self-capture to high-energy neutrino fluxes from the Sun in the contour plots of Figs. 2 and 3. There are four parameters of most immediate interest, namely $\sigma_{\rm xx}$, $M_{\rm x}$, $\sigma_{\rm p}^{\rm SI}$, and $\langle\sigma_{\rm A} v\rangle$. Figure 2 displays contours of constant β in the $M_{\rm x}$-$\sigma_{\rm xx}$ plane for several fixed values of $\sigma_{\rm p}^{\rm SI}$ and $\langle\sigma_{\rm A} v\rangle$, while Fig. 3 shows contours of β in the $M_{\rm x}$-$\sigma_{\rm p}^{\rm SI}$ plane for specific choices of $\sigma_{\rm xx}$ and $\langle\sigma_{\rm A} v\rangle$.

Consider Fig. 2, which shows an interesting, general set of results. The shaded regions at the upper left represent values of $\sigma_{\rm xx}$ that are already ruled out [19,33–36]; however, computing these bounds is complex and the position of the boundary in each case remains somewhat controversial [33–35] so this boundary should be regarded as approximate. Notice that boosts in neutrino fluxes of several tens up to 100% can be achieved for quite reasonable parameter values. Significantly larger boosts of up to $\beta \sim 10^3$ can be realized in extreme regions of the viable parameter space.

Beyond this, several additional features of Fig. 2 are worthy of explicit note. Equation (14) indicates that lines of constant β on the $M_{\rm x}$-$\sigma_{\rm xx}$ plane should run as $\sigma_{\rm xx} \propto M_{\rm x}^{5/4}$. In practice, the lines of constant β are somewhat more shallow than this because the approximate form of $C_{\rm c}$ [Eq. (9)] used in the simple estimate of Eq. (14) assumed favorable kinematics for scattering off of nuclei at all dark matter particle masses. In the case of favorable kinematics, $C_{\rm c} \propto M_{\rm x}^{-1}$. However, as the dark matter particle and nucleus masses become less comparable, the capture rate tends to $C_{\rm c} \propto M_{\rm x}^{-2}$ and lines of constant β become shallower, approaching $\sigma_{\rm xx} \propto M_{\rm x}^{3/4}$. In addition, comparing the pair of panels (b) and (c) or (d) and (e) in Fig. 2 it is clear that the scaling $\sigma_{\rm xx}^2 \propto \sigma_{\rm N}\langle\sigma_{\rm A} v\rangle$ from Eq. (14) for fixed β at a particular $M_{\rm x}$ is valid. This is sensible, because these cross sections serve only to scale the rates $C_{\rm s}$, $C_{\rm c}$, and $C_{\rm a}$ (so long as the equilibrium solution is achieved).

Also evident in Fig. 2 is that large enhancements may only be achieved when annihilation cross sections are relatively low $\langle\sigma_{\rm A} v\rangle < 10^{-26}$ cm^3 s^{-1}, where the numerical value is the canonical value for a thermal relic dark matter particle. This can be seen most dramatically in panel (f) where I have taken $\sigma_{\rm p}^{\rm SI} = 10^{-45}$ cm^2 and $\langle\sigma_{\rm A} v\rangle = 10^{-26}$ cm^3 s^{-1} and the enhancements are at most a few percent over viable parameter ranges. Even discounting experimental limitations, it is thought that the intrinsic errors in computing neutrino fluxes from dark matter capture within the Sun should be a few tens of percent [11,12,59–62], so this indicates that such an effect can only be interesting when $\langle\sigma_{\rm A} v\rangle \lesssim$ a few $\times 10^{-27}$ cm^3 s^{-1}. Within the context of scans of restricted regions of supersymmetric parameter space, annihilation cross sections well below this value are achievable [45–47,63], so significantly lower cross sections are attainable in theories with complex particle spectra. Even in exceedingly simple proposals there exists sufficient freedom to set $\sigma_{\rm xx}$ and $\langle\sigma_{\rm A} v\rangle$ apart significantly [22,24]. Consider interaction via exchange of a boson of mass m_V. The perturbative annihilation and scattering cross sections should be related as $\sigma_{\rm A}/\sigma_{\rm xx} \sim (m_V/M_{\rm x})^4$ and both m_V and the cou-

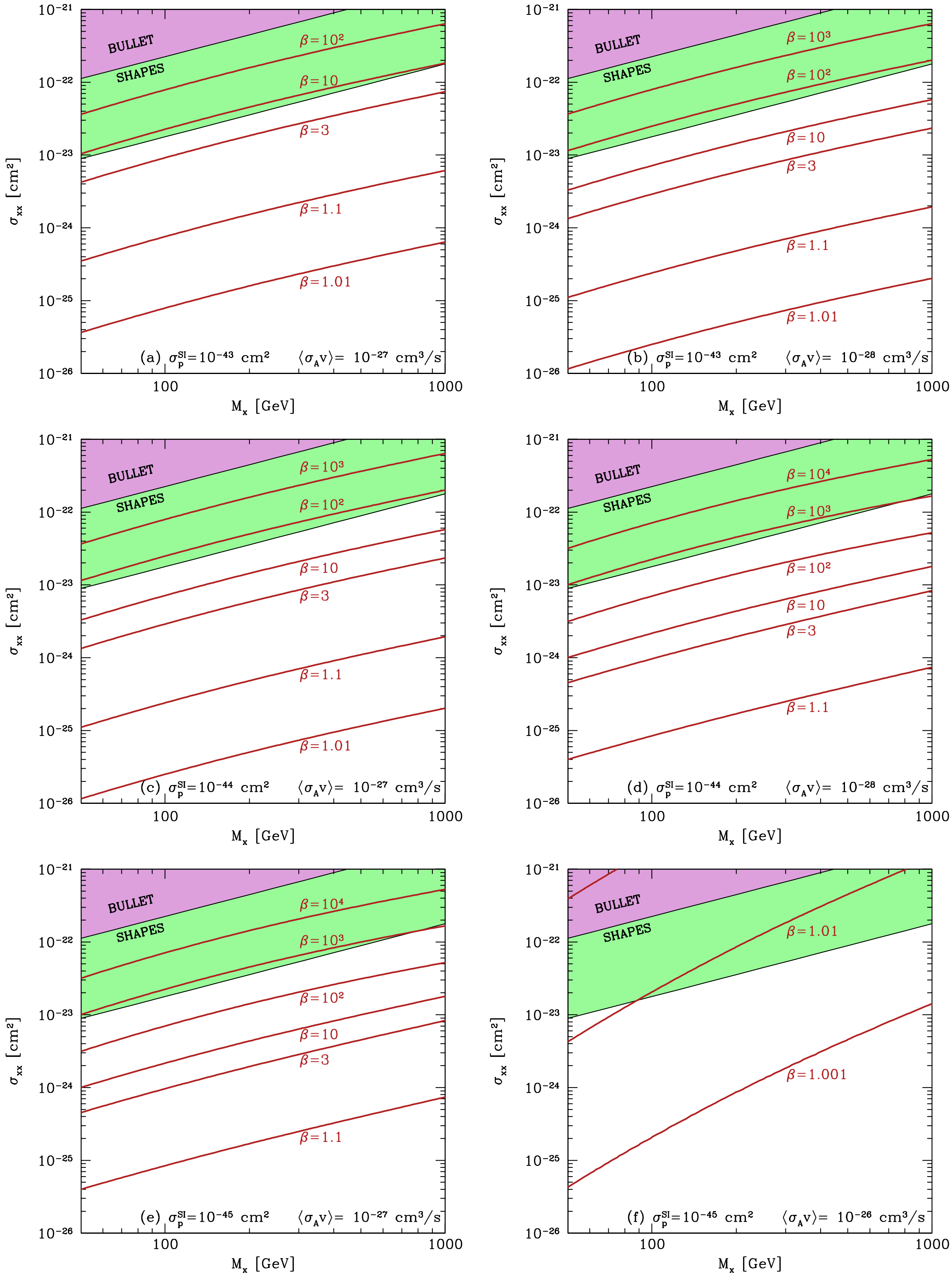

FIG. 2 (color online). Factors of flux enhancement in the M_x-σ_{xx} plane. Each panel shows contours of constant relative flux enhancement in models of self-interacting dark matter. The panels (a)–(f) are labeled with assumed values of σ_p^{SI} and $\langle\sigma_A v\rangle$. The shaded regions at the upper left correspond to parameter values that are disfavored by analysis of either the Bullet Cluster [33] or galaxy cluster shapes [19,34–36].

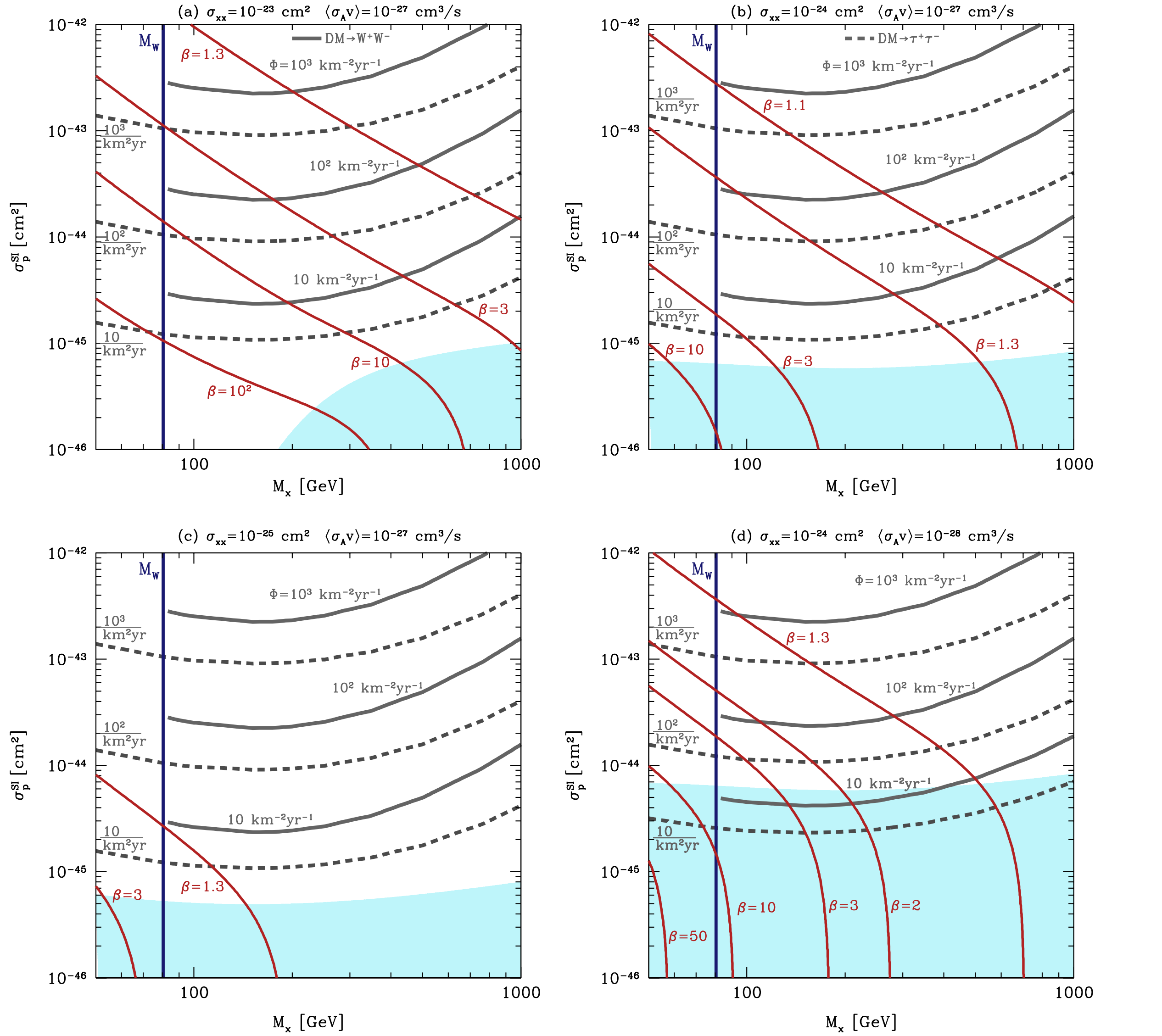

FIG. 3 (color online). Factors of flux enhancement in the M_x-σ_p^{SI} plane. Each panel shows contours of constant relative flux enhancement in models of self-interacting dark matter. The flux enhancement contours are the *red lines with negative slope* labeled by the flux enhancement factor β. Every panel is labeled according to the assumed values of σ_{xx} and $\langle\sigma_A v\rangle$ in each calculation. I show for reference on the background in each plot contours of constant muon flux at a detector on Earth in the case where annihilation happens through W^+W^- (*solid, with cutoff at the W mass*) and the case where annihilation happens through $\tau^+\tau^-$ (*dashed*). These reference flux levels are computed assuming that there is no significant self-capture of dark matter, $C_s = 0$. Shaded regions at the lower ends of these plots correspond to models that are not yet at their equilibrium levels for a Sun of age $\tau_\odot = 5 \times 10^9$ yr. Roughly speaking, current direct dark matter searches constrain the spin-independent dark matter-proton cross section to slightly better than $\sigma_p^{SI} \gtrsim 10^{-43}$ cm² at about 10^2 GeV [50,51].

pling strength remain to be fixed. However, these results do indicate that models of dark matter self-interaction that lead to very large annihilation cross sections (such as the Sommerfeld-enhanced scenarios of significant recent interest, see Refs. [20–23]) will induce little additional neutrino flux due to self-capture.

Figure 3 shows contours of β in the M_x-σ_p^{SI} plane for four choices of σ_{xx} and $\langle\sigma_A v\rangle$ and complements the results in Fig. 2. First, the results of current direct dark matter searches can be compared in this plane. Direct search experiments constrain σ_p^{SI} directly. Current bounds place $\sigma_p^{SI} \gtrsim 10^{-43}$ cm² at $M_x \sim 10^2$ GeV [50–53]. This bound becomes slightly better with decreasing M_x until $M_x \sim 50$ GeV and at higher masses this bound grows $\propto M_x$. The competition is much less severe from spin-dependent searches [51–53], as the indirect limit from the Sun already exceeds the direct search limit by more than an order of magnitude over a wide range of masses [14]. Second, Fig. 3 shows contours of constant absolute muon flux above a threshold of $E_{TH} = 1$ GeV in models where there is no self-interaction in order to set the absolute scale. Two annihilation channels are shown, annihilation into W^+W^- (solid gray) and $\tau^+\tau^-$ (dashed gray). Current experiments are limited to muon fluxes above several hundred per km^2 per year [14,55,56]. Assuming the relatively hard spectra from annihilation to gauge bosons, IceCube with DeepCore extension should optimistically be capable of detecting fluxes down to $\sim$60 km^{-2} yr^{-1} for particle masses above $\sim$200 GeV, with relatively lower sensitivity below this mass [54–56,64,65]. A deep-sea neutrino facility such as the KM3NeT effort [66], building on the ANTARES [67–71], NEMO [72–74], and NESTOR [75,76] work, may achieve comparable or better sensitivities.

Figure 3 also shows regions where the equilibrium solution of Eq. (7) has not yet been attained for a solar age of $\tau_\odot = 5 \times 10^9$ Gyr. I approximate the equilibrium boundary as the contour where the predicted flux is 58% of the value it would be at equilibrium, because $\tanh^2(1) \simeq 0.58$ [see Eq. (3)]. Equilibrium is achieved in the majority of the parameter space corresponding to a potentially detectable signal [54]. However, notice that the contours of constant flux at Earth in scenarios with no self-capture are not the same in each panel [particularly so in panel (d)] because fluxes are no longer determined solely by σ_p^{SI} and M_x for models that are not equilibrated.

Consider panels (b)–(d) of Fig. 3. In these panels, the equilibrium boundary exhibits a very shallow minimum in σ_p^{SI} near M_x of a few hundred GeV. In these panels, the equilibrium boundaries are essentially the same as they would be in the absence of any dark matter self-interaction. The minimum occurs due to the competition between capture and annihilation in the relevant time scale, $\tau_{eq} = 1/\sqrt{C_c C_a}$. The annihilation rate scales with dark matter mass as $C_a \propto M_x^{3/2}$ [see Eqs. (11) and (12)], while at relatively low masses $C_c \propto M_x^{-1}$. For dark matter particle masses greater than several hundred GeV, the capture rate transitions to the regime where kinematic suppression of capture becomes important and $C_c \propto M_x^{-2}$ (I have neglected the orbital effects that also tend to slow thermalization within the Sun for high-mass dark matter candidates [60]).

On the other hand, panel (a) of Fig. 3 shows a distinct feature in the equilibration boundary at $M_x \sim 300$ GeV. The feature is caused by self-capture. At high σ_{xx} and low M_x, self-capture of dark matter is important and can drive rapid equilibration even for very low values of C_c. In the absence of self-capture, the equilibration boundary in panel (a) of Fig. 3 would be relatively flat as a function of M_x as in panels (b)–(d).

The contours of constant β in Fig. 3 show that interesting regions of parameter space can lead to detectable boosts in muon fluxes at Earth of tens of percent to 100%. Somewhat more extreme choices of parameters can lead to boosts of an order of magnitude or more. In particular regions of the parameter space, the dark matter self-interaction can drive a model that would be undetectable or ruled out by direct searches in terms of σ_p^{SI} to be detectable at contemporary or future high-energy neutrino telescopes. This is an interesting possibility, because this implies that the indirect neutrino signal from the Sun would not be related to either direct search results or indirect neutrino signals from the Earth in a straightforward manner. Each contour of β in Fig. 3 exhibits a distinct break as it nears the equilibration boundary. This is because solutions that are well away from equilibrium tend to lie in the linear portion of $N_x(t)$, prior to any significant opportunity for exponential growth (see Fig. 1). As a result, the insight gained from Eq. (14) fails at low cross sections and large enhancement factors require significantly smaller σ_p^{SI} at fixed M_x than one would estimate from the equilibrium assumption.

Of course, it is likely that the effect of flux enhancement due to self-capture is negligible; however, it is useful to know just how large this effect could possibly be. It is simple to make such an estimate and contours of the maximum possible flux enhancement β_{max} are shown in Fig. 4. The "maximum possible flux enhancement" depends upon $\langle\sigma_A v\rangle$ and I compute it as follows. At each

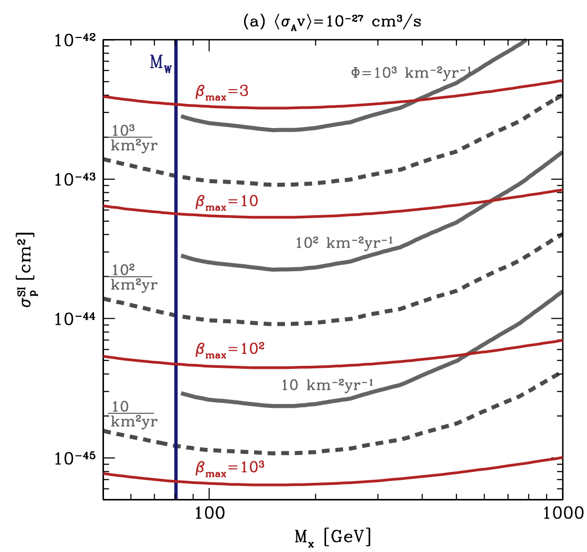

FIG. 4 (color online). Contours of maximum possible flux enhancements given existing bounds on dark matter elastic scattering cross sections. This figure is similar to the figure panels of Fig. 3. However, in this figure, I show contours of β_{max}, the maximum possible flux enhancement at each point in the M_x-σ_p^{SI} plane. I compute this maximal boost at each point by setting σ_{xx} to the maximum allowed value at each value of M_x (see the limits in Fig. 2). The quantity β_{max} is also a function of annihilation cross section and this panel shows β_{max} for $\langle\sigma_A v\rangle = 10^{-27}$ cm^3 s^{-1}. From this figure, it is already clear that certain combinations of parameters may be ruled out with contemporary or forthcoming neutrino telescope data (e.g., Ref. [14]).

value of M_x, I choose the largest value of σ_{xx} that is not already excluded by considerations of large-scale structure (see the contours in Fig. 2). I then compute the flux enhancements at each point in the M_x-σ_p^{SI} plane for a fixed $\langle\sigma_A v\rangle$ ($\langle\sigma_A v\rangle = 10^{-27}$ cm^3 s^{-1} in this case). The enhancement scales approximately as $\sim\langle\sigma_A v\rangle^{-1}$ as given in Eq. (13).

Figure 4 already illustrates that some extreme parameter combinations may be ruled out with contemporary or near-future limits from neutrino telescopes. Notice also that the contours of constant β_{max} are very flat functions of M_x. This is because existing limits on dark matter particle self-interactions scale as $\sigma_{xx} \propto M_x$. This is important for comparison with direct detection experiments which aim to achieve limits on the dark matter-proton scattering cross section on the order of $\sigma_p^{SI} \sim 10^{-44}$ cm^2 in the near future [50–53]. Absent dark matter self-capture, such a limit would indicate that there should be no observable high-energy neutrino flux from the Sun, but self-capture can clearly modify this conclusion.

V. SUMMARY AND DISCUSSION

In this paper I have reconsidered the indirect high-energy neutrino signal from within the Sun in models in which the dark matter particles have significant self-interactions. The influence of self-interactions is that they may allow dark matter particles within the Galactic halo to be captured within the Sun by scattering off of dark matter particles that have already been captured by scattering off of nuclei within the Sun. For sufficiently large dark matter self-interaction cross sections, this can lead to a period during which the rate of capture of dark matter particles by the Sun grows in proportion to the number of dark matter particles already captured by the Sun. The number of dark matter particles within the Sun then grows exponentially until this increase is stopped by efficient annihilation. The net result is that the Sun may contain significantly more dark matter than in models with no dark matter self-interaction and high-energy neutrino signals due to annihilation of these particles may be significantly higher as a result.

In Sec. IV, I showed that mild enhancements of a few tens to one hundred percent are possible over a wide range of viable parameter space that may be probed with contemporary and near-future neutrino telescopes [55,56,67,73–75]. Significantly larger flux enhancements of up to a factor of $\sim10^2$ are possible in more extreme corners of the dark matter parameter space and at flux levels that are not yet within reach of near-term neutrino telescopes. Ten percent enhancements are not particularly interesting at present because intrinsic errors in the flux predictions are at the tens of percent level [11,12,59,60,62], but this situation may improve as experimental advancements drive renewed interest in this signal.

Large flux enhancements require large dark matter self-interaction cross sections $\sigma_{xx} \gtrsim 10^{-24}$ cm^2 and relatively small dark matter mutual annihilation cross sections $\langle\sigma_A v\rangle \lesssim 10^{-27}$ cm^3 s^{-1}. The small annihilation cross section allows the number of dark matter particles within the Sun to grow exponentially for a prolonged period of time, which enables large flux enhancements. More specifically, the neutrino flux enhancement grows approximately as $\propto \sigma_{xx}^2/\sigma_p^{SI}\langle\sigma_A v\rangle$, neglecting the possibility that for some values of these parameters the flux may not reach its equilibrium level for a sun of age $\tau_\odot = 5 \times 10^9$ yr. As a consequence, large enhancements require a disparity between scattering and annihilation cross sections that may be unfamiliar. However, such a disparity is practicable and, in fact, previous proposals of self-interacting dark matter rely on just such relative differences in cross sections in order to produce significant astrophysical effects without annihilating all of the dark matter in the early universe (e.g., Refs. [22,24]).

The high-energy neutrino flux enhancement I compute may have several interesting implications. In Sec. III and in the Appendix, I show that the flux from within the Earth will not be enhanced due to dark matter self-interactions. In the standard picture, the flux from within the Earth can be predicted relative to the solar flux. In viable contemporary models, the Earth signal is often not yet equilibrated and the Sun-to-Earth flux ratio depends upon the dark matter particle mass as well as the capture and annihilation rates. In the self-interacting scenario, the relation between the Sun and Earth neutrino fluxes may no longer hold and significant deviations from any predicted ratio may be a sign of dark matter self-interactions.

Likewise, experiments that undertake direct dark matter searches may exploit indirect detection methods to cross-check limits and/or detections. The correspondence between direct detection experiments and high-energy neutrinos from the Sun is relatively straightforward. Though direct detection rates and high-energy neutrino fluxes depend on somewhat different integrals over the dark matter velocity distribution, in the standard picture they both grow in proportion to the product of the local dark matter density multiplied by the dark matter-nucleon cross section, $\propto \rho_x\sigma_N$. If dark matter exhibits considerable self-interaction, this correspondence is also broken. The neutrino flux from the Sun may be significantly larger than would be predicted based on the limits or detections from direct detection experiments. One extreme possibility is that neutrino fluxes that may seemingly be ruled out by direct searches (based upon limits on σ_p^{SI} and/or σ_p^{SD}) may be realized due to the enhancement from dark matter self-interactions. A broken correspondence between the neutrino fluxes and direct search results may signal dark matter interactions.

The pace of the quest to identify the dark matter is picking up rapidly. Neutrino telescopes play an important role in this endeavor and the indirect limits from existing facilities are already competitive with direct search techniques. The indirect, high-energy neutrino signal from the Sun may serve as a unique probe of new physics confined to the dark sector, and experimental advancements in the near future should shed new light on the properties of the dark matter.

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APPENDIX: CAPTURE AND SELF-CAPTURE OF DARK MATTER PARTICLES IN THE SUN

In the interest of completeness, I give a brief discussion of the capture of self-interacting dark matter particles within the Sun in this Appendix. The treatment here is not original, save for the fact that I consider dark matter particles interacting among themselves, and follows the lucid discussion given in the series of papers by Gould [9,11,12]. I conclude this section with the rate of dark matter particle self-capture. For the results in the main text, I use the full formulas of Ref. [11] to compute dark matter particle capture off of nucleons.

Gould begins by considering capture in an individual spherical shell of the body on which capture is occurring (the Sun in this case) of radius r and local escape speed $v_{\rm esc}(r)$. About this shell, consider a bounding surface of radius R so large that the gravitational field due to the Sun is negligible at R. Let the one-dimensional speed distribution function of dark matter particles at this shell be $f(u)$, where u is the speed at infinity and the integral of $f(u)$ over all speeds gives the number density of dark matter particles. The inward flux of particles of speed u at angle θ relative to radial across the surface at R is then

$$\frac{d\mathcal{F}_{\rm in}}{du d\cos^2(\theta)} = \frac{1}{4} f(u) u. \tag{A1}$$

Changing variables from $\cos^2(\theta)$ to the specific angular momentum $J = Ru\sin(\theta)$, and integrating over the surface area of the sphere at R gives the rate at which dark matter particles enter the surface per unit time, per unit speed, per unit angular momentum,

$$\frac{d\mathcal{R}_{\rm in}}{dudJ^2} = \frac{\pi f(u)}{u}. \tag{A2}$$

Notice that I have written this so that the quantity $\mathcal{R}_{\rm in}$ has dimensions of inverse time.

Take $\Omega(w)$ to be the rate at which a particle with speed w at the shell at r scatters to a speed less than $v_{\rm esc}(r)$. Infalling dark matter particles with speed at R of u that reach the shell at r do so with speed

$$w = \sqrt{u^2 + v_{\rm esc}^2(r)}. \tag{A3}$$

The probability of such a particle to be captured is

$$dP = \frac{\Omega(w)}{w} \frac{2dr}{\sqrt{1 - \frac{J^2}{r^2 w^2}}} \Theta(rw - J), \tag{A4}$$

where the quantity $2dr$ multiplied by the term under the radical is the path length through the shell, dividing by w converts this to the time spent in the shell, $\Theta(x)$ is a step function, and the particular step function above enforces the condition that only particles with $J < rw$ intersect the shell. Multiplying the rate of incoming particles in Eq. (A2) with Eq. (A4), the differential rate of capture within the shell is

$$\frac{dC}{drdudJ^2} = \frac{d\mathcal{R}_{\rm in}}{dudJ^2}\frac{dP}{dr} = \frac{2\pi f(u)}{wu} \frac{\Omega(w)}{\sqrt{1 - \frac{J^2}{r^2 w^2}}} \Theta(rw - J). \tag{A5}$$

The integral over J^2 can be performed leaving the capture rate per unit speed at infinity, per unit shell volume

$$\frac{dC}{dudV} = \frac{f(u)}{u} w\Omega(w), \tag{A6}$$

where I have replaced $4\pi r^2 dr$ with dV. This gives the rate per unit shell volume as an integral over the speed distribution at infinity,

$$\frac{dC}{dV} = \int \frac{f(u)}{u} w\Omega(w) du, \tag{A7}$$

and the task remains to determine $\Omega(w)$, perform the integration over speeds in Eq. (A7), and integrate over the volume of the Sun.

The rate of scattering in the shell is simply $n\sigma w$, with σ the scattering cross section and n the number density of targets. The case of most practical interest is velocity independent and nearly isotropic scattering of infalling dark matter particles against targets that are effectively at rest with respect to the capturing body. In this case, the fractional loss of kinetic energy in a given scattering event is a uniform distribution over the interval

$$0 \le \frac{\Delta E}{E} \le \frac{4M_{\rm x} m}{(M_{\rm x} + m)^2}, \tag{A8}$$

where $M_{\rm x}$ is the mass of the dark matter particle and m is the mass of the particle it scatters off of. The dark matter particle must lose a fraction of its kinetic energy $\Delta E/E > u^2/w^2$ in order to be captured. If the condition

$$4M_{\mathrm{x}}m/(M_{\mathrm{x}}+m)^2 \geq u^2/w^2 \tag{A9}$$

holds, the probability that an individual scattering event leads to capture is

$$p_{\mathrm{cap}} = \frac{v_{\mathrm{esc}}^2(r)}{w^2}\left[1 - \frac{u^2}{v_{\mathrm{esc}}^2(r)}\frac{(M_{\mathrm{x}}-m)^2}{4M_{\mathrm{x}}m}\right]. \tag{A10}$$

Therefore, if Eq. (A9) holds,

$$\Omega(w) = n\sigma v_{\mathrm{esc}}(r)\frac{v_{\mathrm{esc}}(r)}{w}\left[1 - \frac{u^2}{v_{\mathrm{esc}}^2(r)}\frac{(M_{\mathrm{x}}-m)^2}{4M_{\mathrm{x}}m}\right]. \tag{A11}$$

At least one property of Eq. (A11) is familiar. Capture is most efficient when both projectile and target are of the same mass and becomes less efficient as the masses become mismatched.

Combining Eq. (A7) with Eq. (A11) yields the capture rate per shell volume in the Sun,

$$\frac{dC}{dV} = \int n\sigma v_{\mathrm{esc}}^2(r)\frac{f(u)}{u}\left[1 - \frac{u^2}{v_{\mathrm{esc}}^2(r)}\frac{(M_{\mathrm{x}}-m)^2}{4M_{\mathrm{x}}m}\right]du. \tag{A12}$$

Gould has evaluated this expression for the case of a Maxwell-Boltzmann speed distribution including possible form-factor suppression of scattering with large nuclei at high momentum transfer [11,12]. However, the general formulas are rather unwieldy, the integrations are lengthy but straightforward, and presenting them does not add significantly to the insight needed for my purposes. As a result, I will not present the general formulas and will move to a particularly simple special case.

Of particular interest for the present paper is the capture of dark matter particles in the halo by other dark matter particles that have already been captured within the Sun. As a consequence, I will evaluate Eq. (A12) for the special case of $m = M_{\mathrm{x}}$ and for capture by the Sun moving with speed $v_\odot = 220\ \mathrm{kms}^{-1}$ through a Maxwell-Boltzmann distribution of dark matter particles with dispersion $\bar{v} = 270\ \mathrm{km\,s}^{-1}$. The distribution function can then be written

$$f(x) = \frac{2n_{\mathrm{x}}}{\sqrt{\pi}}x^2 e^{-x^2} e^{-\eta^2}\frac{\sinh(2x\eta)}{x\eta}, \tag{A13}$$

in terms of the dimensionless variables $x^2 = 3(u/\bar{v})^2/2$ and $\eta^2 = 3(v_\odot/\bar{v})^2/2$. Integrating over the speed distribution yields

$$\frac{dC}{dV} = \sqrt{\frac{3}{2}}n_{\mathrm{x}}n\sigma v_{\mathrm{esc}}(r)\frac{v_{\mathrm{esc}}(r)}{\bar{v}}\frac{\mathrm{erf}(\eta)}{\eta}. \tag{A14}$$

The total capture rate now requires integrating over the volume of the Sun. This gives

$$C = \sqrt{\frac{3}{2}}n_{\mathrm{x}}\sigma v_{\mathrm{esc}}(\mathrm{R}_\odot)\frac{v_{\mathrm{esc}}(\mathrm{R}_\odot)}{\bar{v}}\frac{\mathrm{erf}(\eta)}{\eta} \times \int_0^{\mathrm{R}_\odot} 4\pi r^2 n\frac{v_{\mathrm{esc}}^2(r)}{v_{\mathrm{esc}}^2(\mathrm{R}_\odot)}dr. \tag{A15}$$

The last integral can be rewritten conveniently by defining a dimensionless potential $\hat{\phi} = v_{\mathrm{esc}}^2(r)/v_{\mathrm{esc}}^2(\mathrm{R}_\odot)$, in which case the last integral is the product of the total number of targets N and the average of $\hat{\phi}$ over all targets within the Sun,

$$C = \sqrt{\frac{3}{2}}n_{\mathrm{x}}\sigma v_{\mathrm{esc}}(\mathrm{R}_\odot)\frac{v_{\mathrm{esc}}(\mathrm{R}_\odot)}{\bar{v}}N\langle\hat{\phi}\rangle\frac{\mathrm{erf}(\eta)}{\eta}. \tag{A16}$$

The numerical factor of $\sqrt{3/2}$ in Eq. (A16) differs from the factor $\sqrt{6/\pi}$ given in Refs. [9,11,12] because Gould defined the error function $\mathrm{erf}(x)$ with an unconventional normalization.

I have assumed that $M_{\mathrm{x}} = m$ to derive Eq. (A16). However, so long as the mass of the target and projectile are not very mismatched, scattering will be likely to lead to capture and Eq. (A16) will be a relatively good approximation for the capture rate. In the case of capture by scattering off of nuclei, the relevant cross section is the elastic scattering cross section off of the nucleus of interest $\sigma = \sigma_{\mathrm{N}}$ and N is the number of such nuclei in the Sun. The total capture rate due to scattering off of all nuclei is the sum of the individual rates for all of the different nuclear species within the Sun.

For dark matter self-capture, the relevant cross section is the elastic scattering cross section of dark matter particles with themselves $\sigma = \sigma_{\mathrm{xx}}$ and $N = N_{\mathrm{x}}$ is the number of dark matter particles already captured within the Sun. Therefore, the dark matter self-capture rate coefficient referred to in the main text can be approximated as

$$C_{\mathrm{s}} = \sqrt{\frac{3}{2}}n_{\mathrm{x}}\sigma_{\mathrm{xx}}v_{\mathrm{esc}}(\mathrm{R}_\odot)\frac{v_{\mathrm{esc}}(\mathrm{R}_\odot)}{\bar{v}}\langle\hat{\phi}_{\mathrm{x}}\rangle\frac{\mathrm{erf}(\eta)}{\eta}. \tag{A17}$$

As discussed in the text, captured dark matter particles typically occupy a very small range of radii within the Sun (typically confined to only a few percent of $\mathrm{R}_\odot$), in which case $\langle\hat{\phi}_{\mathrm{x}}\rangle \simeq 5.1$ [11].

In the case of dark matter particle self-capture, there is one additional complication that must be accounted for that is not relevant for capture off of nuclei. The Sun is optically thin to the propagation of dark matter particles, so a target dark matter particle that receives too much kinetic energy relative to the solar core will be ejected resulting in no net gain of dark matter particles. Therefore, not only must the collision result in an energy exchange of $\Delta E/E \geq u^2/w^2$, but it must be limited to $\Delta E/E \leq v_{\mathrm{esc}}^2(r)/w^2$. This modifies the capture probability per collision (again, taking $m = M_{\mathrm{x}}$) to

$$p_{\mathrm{cap}} = \left(\frac{v_{\mathrm{esc}}^2(r) - u^2}{w^2}\right)\Theta(v_{\mathrm{esc}}(r) - u) \tag{A18}$$

and the capture rate to

$$\Omega(w) = \frac{n\sigma}{w}(v_{\mathrm{esc}}^2(r) - u^2). \tag{A19}$$

When $v_{\rm esc}(r) \gg u$, this modification is relatively minor. This is because in this situation, the incoming dark matter particle must only lose a small fraction of its total energy to be captured and does not necessarily impart enough energy to escape on the target dark matter particle. This is generally the case for the Sun, because escape from the solar interior requires speeds at least 2 times larger than the typical speed at infinity of a dark matter particle. However, the escape speed from the Earth is significantly smaller than the typical speeds of dark matter particles, so collisions within the Earth that lead to capture of the infalling particle will almost always lead to ejection of the target. In fact, most interactions of this kind will lead to both infalling particle and target being unbound from the Earth. An interesting question is to ask whether self-interactions may scour the Earth of captured dark matter particles, but a comparison of the relevant rates along the lines leading to Eq. (14) in Sec. III shows that the removal rate is significantly less than the capture rate for parameters of interest.

This small modification results in a significantly more complex formula for the rate of capture. The calculation follows according to the simple estimate given above. Again, the integrations are lengthy but straightforward, so I will only quote the result. The full rate of capture accounting for the potential recoil and ejection of the target dark matter particles is

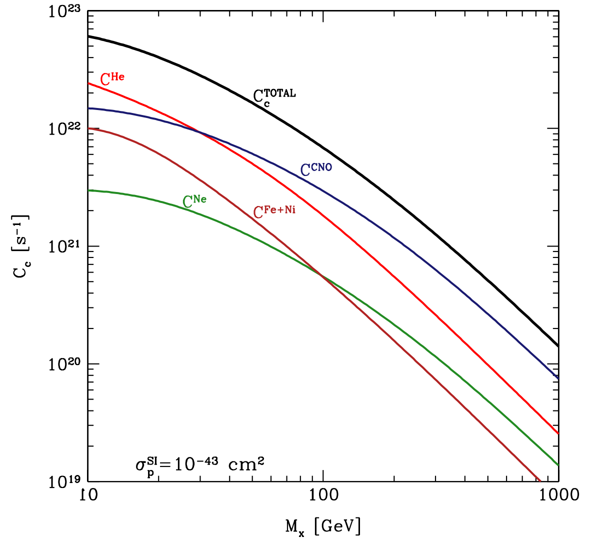

FIG. 5 (color online). Capture rates of weakly interacting dark matter particles used in the calculations in the main text. In this panel, I assume $\sigma_{\rm p}^{\rm SI} = 10^{-43}$ cm^2 and I show capture rates as a function of dark matter particle mass. In addition to the total capture rate, $C_{\rm c}^{\rm TOTAL}$, I also show capture rates off of several elements within the Sun for those elements most important to capture via a scalar interaction. These are He ($C^{\rm He}$), the sum of C, O, and N ($C^{\rm CNO}$, oxygen is the most important of the CNO elements individually), the sum of Fe and Ni ($C^{\rm Fe+Ni}$), and Ne ($C^{\rm Ne}$). At high mass, the capture rate approaches $C_{\rm c} \propto M_{\rm x}^{-2}$ as expected [11].

$$
\begin{aligned}
C_{\rm s} = \sqrt{\frac{3}{2}} n_{\rm x} \sigma_{\rm xx} v_{\rm esc}({\rm R}_\odot) \frac{v_{\rm esc}({\rm R}_\odot)}{\bar v} \eta^{-1} \Bigg(\Bigg[& \langle \hat\phi_{\rm x} \rangle {\rm erf}(\eta) \\
& - \frac{(\langle \hat\phi_{\rm x} {\rm erf}(x_{\rm v} + \eta)\rangle - \langle \hat\phi_{\rm x} {\rm erf}(x_{\rm v} - \eta)\rangle)}{2} \Bigg] \\
& - \frac{2}{3\sqrt{\pi}} \left(\frac{\bar v}{v_{\rm esc}({\rm R}_\odot)} \right)^2 \eta \bigg\{ \frac{\sqrt{\pi}}{2} \eta (2 {\rm erf}(\eta) \\
& - [\langle {\rm erf}(x_{\rm v} + \eta)\rangle - \langle {\rm erf}(x_{\rm v} - \eta)\rangle]) + 2e^{-\eta^2} \\
& - [\langle e^{(x_{\rm v} - \eta)^2} \rangle - \langle e^{(x_{\rm v} + \eta)^2} \rangle] + \frac{2}{\eta} \mathcal{J}(0, \eta) \\
& - \frac{1}{\eta} \langle \mathcal{J}(x_{\rm v} - \eta, x_{\rm v} + \eta) \rangle \bigg\} \Bigg),
\end{aligned} \qquad \text{(A20)}
$$

where $x_{\rm v}^2 = 3(v_{\rm esc}(r)/\bar v)^2/2$, the brackets about a quantity, such as "$\langle q \rangle$" designate the average over all captured dark matter particles of the quantity q, and the integral $\mathcal{J}(s, t) = \int_s^t q^2 e^{-q^2} dq$. The first term in this relation is the simple result from Eq. (A17). The second term in the first set of square braces results from truncating the integral over the speed distribution at $u = v_{\rm esc}(r)$. Typically, $x_{\rm v} \pm \eta > 1$, so this term will be small in comparison to the first term. The terms within the curly braces come from the new piece in the capture rate Eq. (A19). The factor that multiplies the terms in curly braces is typically of order $\sim$0.06 for the Sun. Consequently, the new terms in Eq. (A20) collectively represent relatively small modifications to Eq. (A17). This fits the heuristic understanding that ejection due to recoil will be important only when $v_{\rm esc}(r) \lesssim \bar v \sim v_\odot$.

Though the above sketch of Gould's derivations is instructive for present purposes, the formulas I present here do not suffice to make an adequate estimate of capture by nuclei within the Sun. In all of the detailed results in Sec. IV, I use the full formulas given in Ref. [11] and repeated in the review of Ref. [1]. I take $v_\odot = 220$ km s^{-1}, $\bar v = 270$ km s^{-1}, $\rho_{\rm x} = 0.4$ GeV/cm^3 [77], the solar mass distribution of Ref. [11], and the elemental abundances given in the review of Ref. [78]. I show a specific example of my calculations of the rate of capture of dark matter particles from spin-independent scattering off of nuclei in the Sun with a spin-independent cross section for dark matter-proton scattering of $\sigma_{\rm p}^{\rm SI} = 10^{-43}$ cm^2 in Fig. 5. In addition to the total capture rate, I show also in Fig. 5 contributions to the total capture rate from scattering off of several of the most important nuclei within the Sun. Capture off of hydrogen is down by roughly 2 orders of magnitude throughout most of this range due to the lower cross section relative to heavier nuclei and the unfavorable scattering kinematics for heavy dark matter particles.

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