

A future test of gravitation using galaxy cluster velocities

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The accelerating expansion of the Universe at recent epochs has called into question the validity of general relativity on cosmological scales. One probe of gravity is a comparison of expansion history of the Universe with the history of structure growth via gravitational instability: general relativity predicts a specific relation between these two observables. Here we show that the mean pairwise streaming velocity of galaxy clusters provides a useful method of constraining this relation. Galaxy cluster velocities can be measured via the kinetic Sunyaev-Zeldovich distortion of the cosmic microwave background radiation. Future surveys can provide large enough catalogs of cluster velocities to discriminate between general relativity and other proposed gravitational theories.

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The most perplexing observation in physics today is the accelerating expansion of the Universe (for a review, see [1]). While such an acceleration can be brought about by a constant energy density of the vacuum, the associated energy scale is a small fraction of an electron volt. This energy scale is not, as far as we know, a natural fundamental scale in physics, and skepticism is warranted about new fundamental physics at room-temperature energy scales which only manifests itself in cosmological phenomena.

The standard hot big bang model of cosmology assumes that the Universe is statistically homogeneous and isotropic, and that its dynamics are determined by general relativity. The Einstein equation describing the evolution of the metric then reduces to the Friedmann equation, which can be written in the form

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{3}(\rho(t) + 3p(t)), \quad (1)$$

where $\rho(t)$ and $p(t)$ are the mean energy density and pressure of the Universe at a time t , and $a(t)$ is the scale factor, giving the ratio of the separation between two objects in the cosmic rest frame at time t to their separation today. The scale factor is the function which describes the expansion history of the Universe, and the Friedmann equation is its dynamical equation. It is clear from Eq. (1) that, if general relativity is correct, we must have $w \equiv p/\rho < -1/3$ for the expansion of the Universe to be speeding up at some particular epoch. Hypothetical stress-energy components obeying this relation have been termed “dark energy.” Current observations show that w today is near -1 .

The only logical alternative to dark energy which can explain the observational data is a modification of general

relativity, so that the dynamics of $a(t)$ are determined by an equation different from Eq. (1). A variety of attempts have been made so far in this direction (see, e.g., [2–4]), although modifying general relativity on cosmological scales while still preserving successes on solar system scales and also matching available cosmological data on structure formation is challenging. Modified gravitation theories also tend to be more difficult to solve than general relativity, so detailed cosmological predictions for a given theory are often lacking.

How can we distinguish between general relativity plus dark energy and modified gravity in a model-independent way? A number of papers have explored the fact that in general relativity, a specific relationship exists between two basic gravitational phenomena in cosmology: the expansion history of the Universe and the growth of cosmic structure [5–9]. For scales well inside the cosmological horizon, the linear-theory growth factor $D(a)$ is determined by the differential equation (see, e.g., [10])

$$\frac{d^2 D}{da^2} + \left(\frac{3}{a} + \frac{1}{H} \frac{dH}{da} \right) \frac{dD}{da} - \frac{3\Omega_m(a)H_0^2}{2a^5 H^2} D = 0, \quad (2)$$

where the Hubble parameter $H(a) \equiv (1/a)(da/dt)$, H_0 is the present value of H , and $\Omega_m(a) \equiv 8\pi G\rho_m(a)/(3H(a)^2)$ is the ratio of the matter density ρ_m to the critical density. (This equation assumes that any energy density components besides matter and radiation have negligible density variations.) The solution to this equation can be described to a very good approximation by

$$\frac{d \ln D}{d \ln a} \simeq \Omega_m(a)^\gamma \quad (3)$$

for a wide range of realistic models [5]. The point behind this useful parametrization is the separation of the effect of expansion history, encapsulated in the function $\Omega_m(a)$, from the growth rate, conveniently described by the single

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exponent γ . For standard cosmological models with dark energy, $\gamma \approx 0.55 + 0.05[1 + w(a = 0.5)]$ (reducing to the familiar $\gamma = 0.6$ for $w = 0$). Sophisticated and general parametrizations of the evolution of the scale factor and structure formation in theories different than general relativity have been constructed [11,12], but Eq. (3) provides a simple, single-parameter relation valid for general relativity that can be observationally tested in principle. While a given modified gravity theory is not guaranteed to have a substantially different value for γ than general relativity with dark energy (see [13] for an example), generically this will be true. Linder and Kahn, for example, calculate that DGP gravity [14] gives $\gamma \approx 0.68$ [15] and give γ for various scalar-tensor theories.

Testing Eq. (3) observationally is not an easy task, however. It requires an observable which measures the growth rate of structure with good precision over a large range of redshifts. Directly measuring galaxy clustering as a function of redshift can in principle give the linear growth factor if the bias factor between galaxy clustering and the underlying mass distribution is understood well enough and if surveys complete to high enough redshift are available. Weak gravitational lensing is a promising route provided challenging measurement and theoretical systematic errors can be controlled [16]. Linder has advocated using redshift-space distortions, which are a measure of the internal velocity dispersions of bound objects like galaxy clusters [17]: up to a bias factor relating the galaxy distribution to the underlying mass distribution, the redshift-space distortion is proportional to the left side of Eq. (3). This technique is promising, but relies on dynamics in the nonlinear regime, and requires spectroscopic redshifts of many galaxies. All of these optical observation methods become increasingly difficult at high redshift.

We advocate a different approach to testing Eq. (3): velocities of galaxy clusters obtained from the kinematic Sunyaev-Zeldovich (kSZ) effect. The microwave background radiation has its blackbody temperature shifted as it passes through a galaxy cluster, with the temperature shift being proportional to the line-of-sight velocity of the cluster and to its total optical depth for Compton scattering of the microwave radiation [18]. For typical masses and velocities of large galaxy clusters, the temperature shift will be on the order of a few micro-Kelvin, on an angular scale of around 1 arc min. The Sunyaev-Zeldovich effect is a powerful probe of cosmology because it is essentially independent of cluster distance. Also, the kSZ effect directly measures velocities with respect to the cosmic rest frame, unlike redshift-based velocity measurements which generally must contend with cosmological redshifts that are much larger than the redshift due to velocities. The kSZ effect, a temperature shift on the order of 1 part in a million of the mean background temperature, has not yet been detected (see [19,20] for upper limits), but a new generation of experiments [21–25] is now making measurements

at these angular scales and has the potential to detect the kSZ effect in clusters.

Here we simply assume that future Sunyaev-Zeldovich surveys will result in a galaxy cluster velocity catalog, with each cluster having its sky position and redshift known exactly and its line-of-sight velocity determined with some characteristic error. From such a catalog, a number of different velocity statistics can be formed which are useful for cosmology. We have previously demonstrated the utility of cluster velocity statistics for constraining properties of dark energy [26,27]. Consider, in particular, the mean pairwise cluster relative velocity [28,29], which can be estimated from only observed line-of-sight velocity components [27]. Using a pair conservation equation, an analytic approximation for the mean relative velocity for two clusters separated by a comoving distance r and at an average scale factor a is [30]

$$v_{ij}(r, a) = -\frac{2}{3}aH(a)\frac{d\ln D}{d\ln a}\frac{r\bar{\xi}(r, a)}{1 + \xi(r, a)}, \quad (4)$$

where $\xi(r, a)$ is the cluster two-point correlation function and $\bar{\xi}(r, a)$ is the correlation function averaged over a sphere of radius r . Both of these correlation functions can be computed from the matter power spectrum and a bias giving the average number of clusters which form in a given overdensity. We assume that this bias is given by the standard Λ CDM cosmology [27], which may not be precisely valid for alternate theories of gravity. However, numerical simulations [31] suggest that deviations in the large-scale bias for alternate theories of gravity are only a few percent, so we expect this assumption to have little effect on our results.

Notice that the amplitude of Eq. (4) is proportional to $d\ln D/d\ln a$, given by Eq. (3). So clearly this galaxy cluster velocity statistic can be used to measure the structure growth index γ . To quantify this assertion, we have computed the constraints on a 5-parameter standard Λ CDM cosmological model from a cluster velocity catalog with a given number of clusters and a given mean velocity error, combined with priors on each parameter expected from the Planck satellite's upcoming measurement of the primary microwave background temperature fluctuations [32], and a measurement of $H_0 = 74.2 \pm 3.6$ km/s/Mpc from the SHOES team [33]. We perform a standard Fisher matrix estimate [34] of the constrained region in the multidimensional parameter space consisting of the amplitude of density fluctuations σ_8 , the power-law index of the primordial density perturbations n , the Hubble parameter today H_0 , the present matter density Ω_m , and the growth index γ . For simplicity we assume the Universe is spatially flat, as indicated by current observations; the fiducial model for the scale factor evolution is standard Λ CDM. Note that cluster velocities are independent of the other standard cosmological parameters, like baryon density or reionization optical depth.

We assume a fiducial cosmological model which is the current best-fit Λ CDM cosmology, and a cluster velocity catalog with 4000 cluster velocities and a given velocity error, corresponding to measuring a velocity for all clusters with masses larger than $2 \times 10^{14} M_{\odot}$ in around 400 square degrees of sky. We compute the likelihood of models with different cosmological parameters by estimating the Poisson error for pairwise cluster velocities in z and r bins for our assumed survey and fiducial cosmological model, and comparing this estimated measurement in each bin with the value of Eq. (4) for a different model. (See [27] for a detailed discussion of estimators and likelihoods.) The resulting constraint on the growth index γ , marginalized over the other parameters, is shown in Fig. 1. The horizontal axis gives the standard error for measuring each cluster line-of-sight velocity and the vertical axis gives $\sigma(\gamma)$, the $1\text{-}\sigma$ standard error on the resulting measurement of γ . We only use pairs of clusters with $r > 20$ Mpc/h in estimating $v_{ij}(r, a)$ to minimize any errors due to nonlinear gravitational effects for nearby cluster pairs: linear theory is quite accurate in this regime.

The assumed cluster mass limit of $2 \times 10^{14} M_{\odot}$ is near the anticipated cluster detection threshold for current Sunyaev-Zeldovich experiments, although they are still a ways from having the sensitivity to measure the smaller kinematic SZ signal. If we measure the cluster velocities with an error of 400 km/s via their kinematic SZ distortion, this will provide a measurement of γ to 0.07, a level which is interesting for discriminating various modified gravity theories from dark energy.

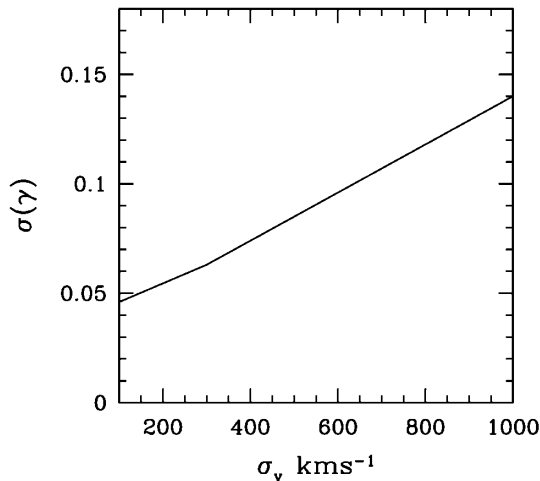


FIG. 1. The $1\text{-}\sigma$ constraint on the growth index γ in a seven-parameter spatially flat cosmological model, constrained by a galaxy cluster velocity survey of 4000 clusters chosen via the Sunyaev-Zeldovich effect, as a function of assumed velocity errors. Prior parameter constraints anticipated from the Planck satellite measurement of the primary microwave background power spectrum [32], plus the current Hubble parameter constraint $H_0 = 74.2 \pm 3.6$ km/s Mpc $^{-1}$ [33] are included.

This simple calculation likely gives a conservative estimate of $\sigma(\gamma)$, for a number of reasons. The error is statistics dominated, so it can be decreased by measuring velocities for clusters in a larger sky region, with $\sigma(\gamma)$ scaling like the inverse square root of the sky area. Measuring velocities more precisely can also somewhat increase the precision in measuring γ , although internal motions of cluster gas provide an astrophysical limit of around 100 km/s to how well cluster velocities can be measured with the kSZ effect [35]. The prior on cosmological parameters used here also does not account for correlations between various parameters constrained by the microwave background power spectrum; a more detailed parameter space investigation will likely result in smaller errors on γ . Including additional velocity statistics or other measures of structure formation also may decrease the error on γ , provided correlations between the various statistics are correctly accounted for. Finally, including γ as an extra parameter generally does not substantially degrade the simultaneous constraints on other cosmological parameters [36], indicating little correlation between the effects of γ and the other parameters.

Our calculation here assumes a linear-theory growth factor when calculating cluster velocities. Since we only use cluster pairs with separations larger than 20 Mpc, this approximation is quite good, and accurately represents the actual variation of the velocity statistic with cosmological parameters. When comparing with real data, more accurate theory predictions can be obtained from large-volume numerical simulations with relatively low resolution. Simulation results will be presented elsewhere.

The Atacama Cosmology Telescope (ACT) and the South Pole Telescope (SPT) are currently mapping large portions of the sky at arc minute resolution in multiple microwave frequency bands. Such measurements will constrain certain linear combinations of gas temperature, line-of-sight gas mass, and line-of-sight gas velocity, depending on frequencies and noise level [37]. The addition of a gas temperature determination from x-ray measurements often greatly improves the precision of cluster velocity determinations. While no cluster peculiar velocities have yet been measured, velocity catalogs for hundreds or thousands of clusters are clearly within reach as the noise level of microwave maps decreases. Correlated foregrounds or backgrounds are a potential problem, which is one rationale for considering a wide range of cluster velocity errors. Source modeling suggests that the range of velocity errors considered here will encompass the actual error range when foregrounds are included [38].

The kinematic SZ effect for galaxy clusters provides a unique window into the growth of structure in the Universe. Like its thermal SZ counterpart, it is essentially independent of cluster distance, so it can probe structure growth over all epochs and over huge volumes. But in contrast to thermal SZ cluster detection to measure the evolution of

cluster number density, cluster velocities derived from the kinematic SZ signal depend only weakly on cluster mass [39] (since the gravitational field causing cluster peculiar velocities provides the same acceleration to all masses), sidestepping systematic uncertainties related to the connection of cluster mass to observed SZ signal. The experimental challenge is daunting: detection of the tiny blackbody kinematic SZ distortion at arc minute resolution in multiple frequency bands, and disentangling this signal from other larger contributions including the thermal SZ

distortion, dust emission, and submillimeter galaxy emission (e.g., [40] for a recent measurement). But progress has been rapid, and the payoff is one of the few reliable methods available for probing the fundamental properties of gravitation on cosmological scales, perhaps shedding light on the accelerating expansion of the Universe.

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