Neutrino masses, the cosmological constant, and a stable universe in a Randall-Sundrum scenario

Paramita Dey* and Biswarup Mukhopadhyaya†
Regional Center for Accelerator-based Particle Physics, Harish-Chandra Research Institute, Chhatnag Road, Jhusi, Allahabad—211 019, India

Soumitra SenGupta‡
Department of Theoretical Physics and Center for Theoretical Sciences, Indian Association for the Cultivation of Science, Kolkata—700 032, India
(Received 14 April 2009; published 25 September 2009)

The Randall-Sundrum model of warped geometry in a five-dimensional scenario, aimed at explaining the hierarchy between the Planck and electroweak scales, is intrinsically unstable in its minimal form due to negative tension of the visible brane. A proposed solution to the problem yields a negative cosmological constant in four dimensions. We show that this wrong-sign cosmological constant is restricted to small values, therefore requiring less cancellation from hitherto unknown physics, if bulk neutrinos are postulated to explain the observed neutrino mass pattern. Thus neutrino masses, a stable TeV-brane configuration and new physics in the context of the cosmological constant get rather suggestively connected by the same thread.

DOI: 10.1103/PhysRevD.80.055029 PACS numbers: 11.25.Wx, 12.10.Kt, 14.60.Pq, 95.36.+x

The Randall-Sundrum (RS) scenario provides an elegant explanation of the hierarchy between the Planck (M_P) and electroweak (EW) energy scales [1]. It postulates an extra spacelike compactified dimension, denoted by $y = r_c\phi$, with two 3-branes at $\phi = 0$ and π. The latter contains the physics of the standard model (SM) of elementary particles and is called the ''visible'' brane. The five-dimensional ''bulk'' metric is given by

$$ds^2 = e^{-2kr_c|\phi|}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 d\phi^2, \qquad (1)$$

where k, of the order of the Planck mass, is related to the 5-D cosmological constant, and r_c is the brane-separation. The exponential ''warp'' factor provides the aforementioned hierarchy of mass parameters, once projections on the ''visible'' brane are taken, for $kr_c \simeq 12$ [1]. Since this allows all mass parameters in the 5-D theory, including k and $1/r_c$ to lie in the vicinity of the Planck scale, the solution can be deemed natural.

The entire setup, however, is afflicted with a serious malady, namely, negative tension for the visible brane [2,3]. This makes the brane configuration intrinsically unstable. A recent work [2] has suggested a solution to this problem by showing that one can take other solutions to the warped geometry, where positive brane tension can be achieved. This requires a negative bulk cosmological constant, as in the original RS theory [1], but also generates a nonzero cosmological constant on the visible brane. While in the original Randall-Sundrum model the visible 3-brane is assumed to be flat with zero cosmological constant, the possibility of a more generalized scenario with a nonzero brane cosmological constant was explored in a series of subsequent works [4]. Most of these works were focused in obtaining the effective gravitational theory on an embedded hypersurface in a bulk space-time and addressed the resulting cosmological issues. From a slightly different angle a generalized Randall-Sundrum model with a nonzero brane cosmological constant was proposed by concentrating on a proper resolution of the gauge hierarchy problem along with the possibility of having positive tension on the standard model TeV brane. It has been shown that the effective (3 + 1)- dimensional Einstein's equation on the brane may be obtained with a nonzero brane cosmological constant (either positive or negative) where the TeV brane tension can be positive or negative, depending on the value of the brane cosmological constant. The solutions for the warp factors as well as the brane tensions were derived from the bulk equations for both the de Sitter and anti–de Sitter cases. The modified warp factors which are functions of the induced cosmological constant on the brane tend to the Randall-Sundrum exponential warp factor in the limit when the brane cosmological constant goes to zero. With such a modified warp factor it is easy to obtain a Planck to TeV scale warping (to resolve the gauge hierarchy problem) such that different values of the induced brane cosmological constants correspond to different values of brane-separation modulus r_c, for a given value of the parameter k which is related to the bulk cosmological constant in the five-dimensional anti–de Sitter bulk space-time. This entire spectrum of solutions in the parameter space for both de Sitter (positive brane cosmological constant) and anti–de Sitter (negative brane cosmological constant) regions has been described in [2] for this generalized Randall-Sundrum scenario. It has further been shown that for a

*paramita@mri.ernet.in
†biswarup@mri.ernet.in
‡tpssg@iacs.res.in

wide range of values of negative brane cosmological constant (i.e., anti–de Sitter 3-brane), the brane tension for the TeV brane becomes positive leading to a stable 3-brane configuration for a consistent embedding of the standard model on this brane. It may be recalled that in case of the original RS scenario the visible brane tension was necessarily negative and therefore was intrinsically unstable. In the anti–de Sitter region the magnitude of the "induced" 4-D cosmological constant can be as high as up to 10^{-32} (rendered dimensionless by scaling with M_P^2), which, with a concomitantly adjusted kr_c, can lead to the requisite magnitude for the warp factor [2]. Since the cosmological constant is observationally restricted to a very small value ($\simeq 10^{-124}$), this necessitates some new physics on the 3-brane to cancel the induced negative cosmological constant. Obviously, the smaller this induced brane cosmological constant is in magnitude, the less is the demand on cancellation from new physics, thus making the scenario more favorable. Moreover in this anti–de Sitter region, a large part of the parameter space corresponds to a positive value for the visible brane tension, and the resulting stability makes it even more interesting in studying particle phenomenology.

We show here that a constraint on the magnitude of the induced cosmological constant does indeed follow, once this scenario is used to also generate masses for neutrinos via massive bulk neutrinos [5–8]. While the bulk neutrino mass(es) are destined to be around the Planck scale, a sector of three light neutrinos is generated, which essentially governs the neutrino mass patterns suggested by the solar and atmospheric neutrino deficits [9,10]. This mechanism, including all its variants, has been discussed widely in the original RS setting. Using the solution necessary for a positive brane tension, we find that the constraints from neutrino masses favor relatively small magnitude of the induced cosmological constant. Thus one can relate the apparently disjoint issues of neutrino mass, the cosmological constant and a stable visible brane in warped geometry. To state it from another angle, one strives to ensure that the four-dimensional universe we live in is a stable configuration. In this approach, the conduciveness of such a stable universe to structure formation has a correspondence with tiny neutrino masses with a specific pattern.

At this point we emphasize that although the generalized RS scenario allows us to have a stable brane configuration by rendering the brane tension positive, the problem of modulus (brane-separation parameter r_c) stabilization has to be addressed separately by introducing a bulk scalar field [11] in the light of the modulus stabilization mechanism proposed by Goldberger, Wise and others [12]. In this mechanism it is shown that the bulk scalar field must have a nonzero vacuum expectation value (vev) on the branes in order to achieve modulus stabilization. Since the bulk neutrinos considered here have no vev, they cannot act as stabilizing fields for the brane-separation modulus r_c.

In general, allowing a nonvanishing but negative brane cosmological constant (which has been shown to be a necessary condition for positive brane tension) leads to a warp factor [2,6]

$$e^{-A(\phi)} = \omega \cosh\left(\ln\frac{\omega}{1+\sqrt{1-\omega^2}} + kr_c\phi\right), \qquad (2)$$

where $\omega^2 = -\Omega^2/k^2$ is the absolute value of the dimensionless quantity obtained out of the cosmological constant (Ω) induced on the brane. If one sets $e^{-A} \simeq 10^{-16}$ to ensure the hierarchy between the Planck and EW scales, then one can find two solutions for $kr_c\pi$ for every ω^2. No solution, however, exists for $\omega^2 > 10^{-32}$. One of the solutions, which yields the usual RS value of $kr_c\pi \simeq 36.84$ in the limit of near-vanishing ω, always corresponds to a negative brane tension, and hence is not relevant to our cause. The other solution, leading to positive brane tension, gives increasing values of $kr_c\pi$ as ω^2 decreases, leading to $kr_c\pi \simeq 250.07$ for $\omega^2 \to 10^{-124}$ (see Figures 1 and 2 in Ref. [2]) On the whole, a rather wide region in the $\omega^2 - kr_c\pi$ space is generally allowed, which we set out to constrain from considerations of neutrino masses.

Out of the three neutrino mass eigenstates, the atmospheric and solar neutrino deficits approximately suggest orders of mass-squared separation [9,10] as $\Delta m_{32}^2 \simeq 10^{-20}$–$10^{-21}$ GeV^2 and $\Delta m_{21}^2 \simeq 10^{-22}$–$10^{-23}$ GeV^2, which is consistent with one massless physical state. In general, this allows a normal hierarchy (NH) ($m_3 \gg m_2 \simeq m_1$), an inverted hierarchy (IH) ($m_3 \simeq m_2 \gg m_1$), or degenerate neutrinos (DN) ($m_3 \simeq m_2 \simeq m_1$). In addition, a bilarge mixing pattern has been strongly suggested by the observed data [13].

The RS model allows for an explanation of neutrino masses without introducing additional light neutrino states, by hypothesizing massive bulk neutrinos on the order of the Planck mass [5,7]. One must choose a minimum of two bulk neutrinos ($a = A$, B) for the cancellation of parity anomaly [14]. The Kaluza-Klein (KK) decomposition of such a neutrino is

$$\Psi^a_{L,R}(x^\mu, y) = \sum_n \psi^{a(n)}_{L,R}(x^\mu)\xi^{a(n)}_{L,R}(y) \qquad (3)$$

where $\xi^{a(0)}_L(y)$ can always be chosen to vanish identically (as we require a right-handed zero mode on the visible brane). The function $\xi^{a(0)}_R(y)$ is obtained as

$$\xi^{a(0)}_R(y) = \frac{N^{a(0)}}{\omega^2}\,\mathrm{sech}^2\left(\ln\frac{\omega}{1+\sqrt{1-\omega^2}} + ky\right)e^{-m_a y} \qquad (4)$$

where the normalization constant is obtained from the condition $\int_0^\pi d\phi r_c e^{-3A(\phi)}\xi^{a(0)*}_R\xi^{b(0)}_R = \delta^{ab}$. m_A, m_B should each be at least $0.5k$, so that $\xi^{a(0)}_R$ is suppressed on the visible brane, thus ensuring small neutrino masses. Nonvanishing functions corresponding to both chiralities exist for the higher modes.

The neutrino masses are *prima facie* decided by the effective Yukawa couplings $y_{a\ell}^{(n)}$ (for lepton flavor ℓ) induced on the brane. These are governed by the couplings $Y_{a\ell}$ involving the bulk neutrino fields and the SM lepton and Higgs fields, and the value of $\xi_R^{a(0)}$ at $\phi = \pi$. Once the compact dimension is integrated out, the part of the action containing Yukawa interactions appears as

$$\mathcal{S}_{\text{Yukawa}} = \int d^4x \frac{e^{-3kr_c\pi/2}}{\sqrt{k}} Y_{a\ell}\bar{L}_\ell H \Psi_a(x, \pi), \qquad (5)$$

and thus one has

$$y_{a\ell}^{(n)} = e^{3A(\pi)/2} Y_{a\ell} \xi_R^{a(n)}(\pi)/\sqrt{k}. \qquad (6)$$

Including the KK mass tower $m^{(n)}$, the neutrino mass matrix takes the form

$$\mathcal{M} = \begin{pmatrix} \mathcal{Y}_{3\times2}^0 & \mathcal{Y}_{3\times2}^1 & \cdots \\ 0_{2\times2} & \mathcal{M}_{2\times2}^1 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}, \qquad (7)$$

where

$$\mathcal{Y}_{3\times2}^n = v\begin{pmatrix} y_{Ae}^{(n)} & y_{Be}^{(n)} \\ y_{A\mu}^{(n)} & y_{B\mu}^{(n)} \\ y_{A\tau}^{(n)} & y_{B\tau}^{(n)} \end{pmatrix}, \qquad \mathcal{M}_{2\times2}^n = \begin{pmatrix} m_A^{(n)} & 0 \\ 0 & m_B^{(n)} \end{pmatrix}. \qquad (8)$$

The mass-squared matrix $\mathcal{M}\mathcal{M}^\dagger$, which ultimately decides the mass eigenvalues for the light states, is

$$\mathcal{M}\,\mathcal{M}^\dagger = \begin{pmatrix} \mathcal{F}_{3\times3} & \mathcal{G}_{3\times2}^1 & \cdots \\ \mathcal{G}_{3\times2}^{1T} & (\mathcal{M}_{2\times2}^1)^2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}, \qquad (9)$$

with

$$\mathcal{F}_{3\times3} = v^2\begin{pmatrix} f_{ee} & f_{e\mu} & f_{e\tau} \\ f_{\mu e} & f_{e\mu\mu} & f_{\mu\tau} \\ f_{\tau e} & f_{\tau\mu} & f_{\tau\tau} \end{pmatrix}, \qquad (10)$$

where v is the Higgs vacuum expectation value, $f_{\ell\ell'} = \sum_{n,a} y_{a\ell}^{(n)} y_{a\ell'}^{(n)}$, and

$$\mathcal{G}_{3\times2}^1 = v\begin{pmatrix} y_{Ae}^{(1)}m_A^{(1)} & y_{Be}^{(1)}m_B^{(1)} \\ y_{A\mu}^{(1)}m_A^{(1)} & y_{B\mu}^{(1)}m_B^{(1)} \\ y_{A\tau}^{(1)}m_A^{(1)} & y_{B\tau}^{(1)}m_B^{(1)} \end{pmatrix}. \qquad (11)$$

It is interesting to note that the eigenvalues of the light states, as determined through the diagonalization of $\mathcal{M}\mathcal{M}^\dagger$, are decided only by the block $\mathcal{Y}_{3\times2}^0$ in $\mathcal{M}$. One can easily verify this when the $\mathcal{Y}$'s become c-numbers, i.e., there is just one light neutrino which has a mass term in the top left-hand corner of the mass matrix through coupling with a single right-handed state. In a way, this is reminiscent of those cases in the type II seesaw mechanism [15] where the left-handed Majorana mass prevails. However, this is just an analogy, since the masses here are of Dirac type. It is straightforward to check numerically that the same conclusion holds to a high degree of accuracy for matrix-valued $\mathcal{Y}_{3\times2}$, and the squared masses of the light physical states are obtained just by diagonalizing $\mathcal{Y}_{3\times2}^0\mathcal{Y}_{3\times2}^{0\,\dagger}$. It is also noteworthy that the procedure summarized above leads to the neutrino mass matrix derived in [7], in the limit of $\omega \to 0$, when the solution for negative brane tension is taken (i.e., in the original version of RS).

One should also note that $\mathcal{Y}_{3\times2}^0\mathcal{Y}_{3\times2}^{0\,\dagger}$ *always has one zero eigenvalue*. While this is consistent with the NH and IH scenarios, degenerate neutrinos are disallowed in this approach.

In our numerical study, we randomly vary all the $Y_{a\ell}$ $\ell = (e, \mu, \tau)$, $a = (A, B)$ in the range 0.01–5.0 (this being a "reasonable" range of dimensionless quantities), and $m_{A,B}$ in the range $0.6k - k$, with $k = 10^{19}$ GeV. We demand that, in all cases, $10^{-17} \le e^{-A(\pi)} \le 10^{-15}$. In order to fit the observed neutrino pattern approximately, with a certain degree of tolerance, we have allowed $\Delta m_{32}^2 \simeq 10^{-20}$–$10^{-21}$ GeV^2 and $\Delta m_{21}^2 \simeq 10^{-22}$–$10^{-24}$ GeV^2. The neutrino mixing angles, answering to the observed bilarge pattern, can be reproduced in specific combinations of the $Y_{a\ell}$ included in the scan. Also, it is impossible to have all the $Y_{a\ell}$ degenerate, irrespective of m_A, m_B, since that leads to two zero mass eigenvalues. The other point to note is that the ratio m_A/m_B cannot be widely hierarchical, as that, too, causes one of the columns in $\mathcal{Y}_{3\times2}^0$ to be very small compared to the other due to the exponential factor (see Eqs. (4) and (6)), once more leading to two zero eigenvalues.

The results of our numerical analysis, yielding constraints on the parameter space, are summarized in Figs. 1–3. Figure 1 shows the areas in the $\omega^2 - kr_c\pi$ plane allowed by the NH and IH scenarios in turn. Most remarkably, out of the erstwhile allowed range of ω^2 reaching up to 10^{-32}, only the region corresponding to $\omega^2 \lesssim 10^{-80}$ is allowed in both the NH and IH cases.

While ω^2 can in principle be arbitrarily small, we find, with our range of Yukawa couplings chosen, increasingly fewer solutions as one goes to $\lesssim 10^{-120}$. In any case, for solutions corresponding to positive brane tension, values of ω^2 smaller than this lead to a rather wide hierarchy between k and $1/r_c$, as can be seen from Fig. 1. The absence of solutions for very low values of ω^2 highlights a very important role of neutrinos here: for the positive tension

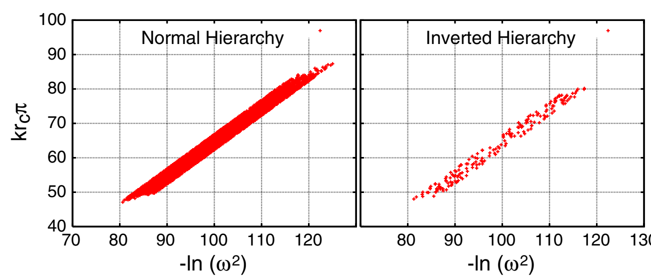

FIG. 1 (color online). Areas in the $\omega - kr_c\pi$ plane for positive tension of visible brane allowed by the NH (left) and IH (right) scenarios of neutrinos.

solution, one cannot consistently generate neutrino masses with arbitrarily small magnitude of the induced cosmological constant. This in turn rules out a large hierarchy between k and $1/r_c$. Phenomenology of the neutrino sector thus renders such a scenario "natural".

Figure 2 shows the ranges of $m_{A,B}$ allowed by our solutions which include the whole range over which we have varied the masses. However, as is evident from Fig. 2(b), the ratio of the two bulk masses is constrained to be in the approximate range 0.9–1.1. The plots are symmetric about $m_A = m_B$, since the labeling of the two bulk neutrinos is arbitrary. This confirms our earlier statement that a wide hierarchy between the two bulk masses is disallowed. Also, there is a depletion of available points around $m_A = m_B$, as one then needs the $Y_{a\ell}$'s to be sufficiently apart.

Finally, Fig. 3 displays the allowed bands of neutrino masses for NH and IH, plotted against ω^2. One mass eigenvalue is zero in each case, due to the nature of the induced mass matrix. It is clear again from these plots that one has the requisite mass patterns for $10^{-120} \lesssim \omega^2 \lesssim 10^{-80}$ only, so long as the solutions for positive brane tension are taken. It is also obvious from this figure, as much as from the previous ones, that one gets much denser scatter plots for NH as compared to IH. This is because one of the eigenvalues is zero from the very structure of the induced mass matrix, and one requires a finer split between the two nonvanishing eigenvalues for IH than for NH. Thus the NH scenario is somewhat more favored than IH, as evinced from our scatter plots.

It is also possible to turn the above conclusions around in the following way. While the requirement of a positive tension brane can lead to values of ω^2 as high as 10^{-32}, observation demands the value to be close to 10^{-124}. One may consider it desirable to keep the generated value of ω^2 at the latter value, unless a separate source to cancel the large induced value can be invoked. If that indeed is the case, and if a mechanism of neutrino mass generation has to be envisioned in terms of bulk sterile neutrinos in a warped geometry, then the bulk masses of these states have to lie in the range of $0.6k$, and a hierarchy of about 80 between k and $1/r_c$ is likely.

In summary, we find that the requirement of fitting the observed neutrino mass pattern in terms of bulk neutrinos imposes stringent constraints on the value of the cosmological constant on the visible brane, if one has to obtain a positive brane tension in an RS theory. More specifically, the magnitude of the induced cosmological constant (inheriting a negative sign) is restricted to be less than 10^{-80} in dimensionless units, thus bringing down its upper limit by 48 orders compared to what is allowed without any reference to neutrinos. The issue of the cosmological constant is admittedly open in most theories beyond the standard model of elementary particles, and some hitherto unknown physics is expected to play a role in its small positive value. In the RS context, our study shows that the necessity of explaining neutrino masses minimizes the required level of fine-tuning to reproduce the observed value of the cosmological constant, presumably through some yet undiscovered feature of the 3-brane that constitutes our visible universe. Alternatively, one obtains constraints on the bulk neutrino masses, and also on the hierarchy between k and $1/r_c$, if there is no other mechanism for explaining the observed value of the cosmological constant. Thus neutrino masses get tantalizingly linked with the issue of the cosmological constant. And remarkably, a positive tension brane, or a stable four-dimensional universe within the RS framework, emerges triumphant while such a link is established.

The work of P. D. and B. M. was partially supported by funding available from the Department of Atomic Energy, Government of India, for the Regional Center for Accelerator-based Particle Physics, Harish-Chandra Research Institute. P. D. and B. M. also acknowledge the hospitality of the Indian Association for the Cultivation of Science, Kolkata, while the work was in progress.

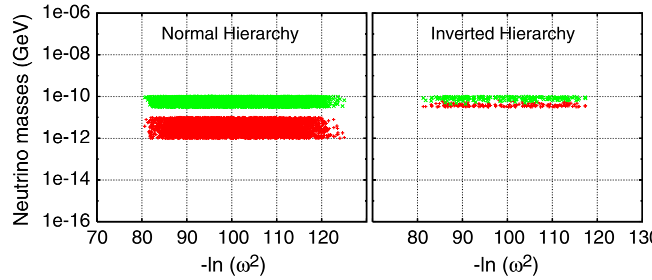

FIG. 3 (color online). Allowed ranges of neutrino masses against $-\ln(\omega^2)$, for NH (left) and IH (right). One mass eigenvalue is always zero in each case.

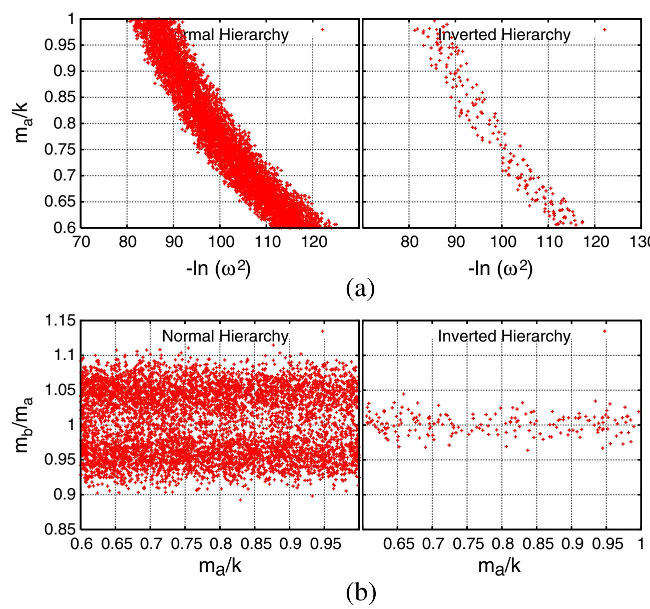

FIG. 2 (color online). The ranges of $m_{A,B}$ allowed by the NH (left panel) and IH (right panel) scenarios *vis-a-vis* $-\ln(\omega^2)$.

[1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); **83**, 4690 (1999).

[2] S. Das, D. Maity, and S. Sengupta, J. High Energy Phys. 05 (2008) 042.

[3] C. Csaki *et al.*, Nucl. Phys. **B584**, 359 (2000); J. W. Chen, M. A. Luty, and E. Ponton, J. High Energy Phys. 09 (2000) 012; M. Sasaki, T. Shiromizu, and K. i. Maeda, Phys. Rev. D **62**, 024008 (2000); J. M. Cline *et al.*, J. High Energy Phys. 06(2003) 048; I. Navarro, J. Cosmol. Astropart. Phys. 09 (2003) 004; J. Garriga and M. Porrati, J. High Energy Phys. 08 (2004) 028; H. P. Nilles, A. Papazoglou, and G. Tasinato, Nucl. Phys. **B677**, 405 (2004); J. Vinet and J. M. Cline, Phys. Rev. D **70**, 083514 (2004); C. P. Burgess *et al.*, J. High Energy Phys. 11 (2004) 069; Y. Aghababaie *et al.*, Nucl. Phys. **B680**, 389 (2004).

[4] J. M. Cline, C. Grojean, and G. Servant, Phys. Rev. Lett. **83**, 4245 (1999); P. Binetruy, C. Deffayet, V. Ellwanger, and D. Langolois, Phys. Lett. B **477**, 285 (2000); T. Shiromizu, K. Maeda, M. Sasaki, Phys. Rev. D **62**, 024012 (2000).

[5] T. Gherghetta and A. Pomarol, Nucl. Phys. **B586**, 141 (2000); H. Davoudiasl, J. L. Hewett, and T. G. Rizzo, Phys. Rev. D **63**, 075004 (2001); T. Gherghetta, Phys. Rev. Lett. **92**, 161601 (2004); G. Moreau and J. I. Silva-Marcos, J. High Energy Phys. 03 (2006) 090.

[6] R. Koley, J. Mitra, and S. SenGupta, Phys. Rev. D **79**, 041902 (2009).

[7] Y. Grossman and M. Neubert, Phys. Lett. B **474**, 361 (2000).

[8] S. Chang *et al.*, Phys. Rev. D **62**, 084025 (2000); B. Bajc and G. Gabadadze, Phys. Lett. B **474**, 282 (2000); I. Oda, Phys. Lett. B **496**, 113 (2000); Jean-Luc Lehners, P. Smyth, and K. S. Stelle, Nucl. Phys. **B790**, 89 (2008).

[9] Q. R. Ahmad *et al.* (SNO Collaboration), Phys. Rev. Lett. **87**, 071301 (2001); Phys. Rev. Lett. **89**, 011301 (2002); **89**, 011302 (2002).

[10] P. I. KrasTeV and A. Y. Smirnov, Phys. Rev. D **65**, 073022 (2002); A. Bandyopadhyay *et al.*, Phys. Lett. B **540**, 14 (2002); M. Maltoni and T. Schwetz, arXiv:0812.3161.

[11] R. Koley, J. Mitra, and S. SenGupta, Europhys. Lett. **85**, 41001 (2009).

[12] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett.**83**, 4922 (1999); C. Csaki, M. Graesser, L. Randall, and J. Terning, Phys. Rev. D **62**, 045015 (2000); J. Lesgourgues, S. Pastor, M. Peloso, and L. Sorbo, Phys. Lett. B **489**, 411 (2000).

[13] P. F. Harrison, D. H. Perkins, and W. G. Scott, Phys. Lett. B **530**, 167 (2002); A. Bandyopadhyay *et al.*, arXiv:0804.4857; C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).

[14] A. N. Redlich, Phys. Rev. Lett. **52**, 18 (1984); C. G. Callan, Jr and J. A. Harvey, Nucl. Phys. **B250**, 427 (1985).

[15] See, for example, R. N. Mohapatra, A. Perez-Lorenzana, and C. A. de Sousa Pires, Phys. Lett. B **474**, 355 (2000); G. Altarelli and F. Feruglio, Springer Tracts Mod. Phys. **190**, 169 (2003); New J. Phys. **6**, 106 (2004); S. Bertolini, M. Frigerio, and M. Malinsky, Phys. Rev. D **70**, 095002 (2004); S. F. King, Rep. Prog. Phys. **67**, 107 (2004); S. Antusch and S. F. King, Nucl. Phys. **B705**, 239 (2005); R. N. Mohapatra and A. Y. Smirnov, Annu. Rev. Nucl. Part. Sci. **56**, 569 (2006); R. N. Mohapatra *et al.*, Rep. Prog. Phys. **70**, 1757 (2007).