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Neutron electric dipole moment in the gauge-Higgs unification

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We study the neutron electric dipole moment (EDM) in a five-dimensional $SU(3)$ gauge-Higgs unification compactified on $M^4 \times S^1/Z_2$ space-time including a massive fermion. We point out that to realize the CP violation is a nontrivial task in the gauge-Higgs unification scenario and argue how the CP symmetry is broken spontaneously by the vacuum expectation value of the Higgs, the extra space component of the gauge field. We emphasize the importance of the interplay between the vacuum expectation value of the Higgs and the Z_2-odd bulk mass term to get physically the CP violation. We then calculate the one-loop contributions to the neutron EDM as the typical example of the CP violating observable and find that the EDM appears already at the one-loop level, without invoking the three-generation scheme. We then derive a lower bound for the compactification scale, which is around 2.6 TeV, by comparing the contribution due to the nonzero Kaluza-Klein modes with the experimental data.

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I. INTRODUCTION

A gauge-Higgs unification scenario proposed a long time ago [1–4] has attracted recent revived interest as one of the attractive scenarios solving the hierarchy problem without invoking supersymmetry. In this scenario, the Higgs doublet in the standard model is identified with the extra spatial components of the higher dimensional gauge fields. A remarkable feature is that the quantum correction to the Higgs mass is finite and insensitive to the cutoff scale of the theory, in spite of the fact that higher dimensional gauge theories are generally nonrenormalizable. The reason is simply that the Higgs mass-squared term as a local operator is forbidden by the higher dimensional gauge invariance. The radiatively induced finite Higgs mass should be understood as to be described by the Wilson line phase, that is, a nonlocal operator and free from UV divergence. This fact has opened up a new avenue to the solution of the hierarchy problem [5]. Since then, much attention has been paid to the gauge-Higgs unification, and many interesting works have been done from various points of view [6–29].

The finiteness of the Higgs mass has been studied and verified in various models and types of compactification at the one-loop level [30–33][1] and even at the two-loop level [35]. It is natural to ask whether any other finite calculable physical observables exist in the gauge-Higgs unification. In a paper by the present authors [36], we have found a striking fact: We have shown that the anomalous magnetic moment of the fermion in the $(D+1)$-dimensional QED gauge-Higgs unification model compactified on S^1 becomes finite for an arbitrary space-time dimension. The reason is easily understood by relying on an operator analysis. In four-dimensional space-time, a dimension six gauge invariant local operator describes the anomalous magnetic moment:

$$i\bar{\psi}_L \sigma^{\mu\nu}\psi_R F_{\mu\nu}\langle H\rangle. \tag{1.1}$$

However, when included into the scheme of gauge-Higgs unification, the Higgs doublet should be replaced by an extra space component of the higher dimensional gauge field A_y. Then, to preserve the gauge symmetry, A_y should be further replaced by gauge covariant derivative D_y, and the relevant gauge invariant operator becomes

$$i\bar{\Psi}\Gamma^{MN} D_L \Gamma^L \Psi F_{MN}, \tag{1.2}$$

where L, M, and N denote $(D+1)$-dimensional Lorentz indices. The key observation of our argument is that the operator (1.2) vanishes because of the on-shell condition $i\langle D_L\rangle\Gamma^L\Psi = 0$, when D_L is replaced by $\langle D_L\rangle$. As the local operator is forbidden, the anomalous magnetic moment is expected to be free from the UV divergence. The brane localized operator is also forbidden for the following reasoning. The Higgs field is just an extra component of gauge field A_y on the brane, not the covariant derivative D_y. Thus, the fact that the shift symmetry $A_y \to A_y + \text{const}$ is operative even on the brane forbids the brane localized operator relevant for the magnetic moment. We might think of other operators which are obtained by operating $(D_M D^M)^n$ (n: integer) to Ψ in (1.2), but this operator is easily found not to yield an independent operator. In fact, for the operator to be relevant for the magnetic moment, D_M should be replaced by $g\langle A_y\rangle\gamma_5$ and $(D_M D^M)^n$ just reduces to a constant $(g^2\langle H\rangle^\dagger\langle H\rangle)^n$.

From these observations, we confirmed the finiteness of the magnetic moment by an explicit diagrammatical calculation [36]. This is a remarkable specific prediction of the gauge-Higgs unification to be contrasted with the case of the Randall-Sundrum model [37] or the universal extra

[1]For the case of gravity-gauge-Higgs unification, see [34].

dimension scenario [38], in which the magnetic moment of the fermion diverges in the models with more than five space-time dimensions.

Although this result was quite interesting, the above model is too simple to be realistic. In particular, the famous result by Schwinger in ordinary QED could not be reproduced as the contribution of zero modes in the simplified model. Thus, in our subsequent paper [39], we clarified the issue on the cancellation mechanism of ultraviolet (UV) divergences in an improved gauge-Higgs unification model. What we adopted was a $(D+1)$-dimensional $SU(3)$ gauge-Higgs unification model compactified on an orbifold S^1/Z_2 with a massive bulk fermion in a fundamental representation, whose gauge group is large enough to incorporate that of the standard model. The orbifolding is indispensable to obtain chiral theory and to reduce the gauge symmetry to that of the standard model. In order to obtain a realistic Yukawa coupling, we introduced a bulk mass parameter of the fermion, which should have odd Z_2 parity in order to preserve the Z_2 symmetry. The bulk mass causes localization of fermions with different chiralities at different fixed points of the orbifold. Hence, the overlap integral of their mode functions yields an exponentially suppressed Yukawa coupling. In this way, we can freely obtain the light fermion masses, which are otherwise of $\mathcal{O}(M_W)$ in the gauge-Higgs unification scenario, by tuning the bulk mass parameters. We thus have succeeded in recovering Schwinger's result, still keeping the nice feature of the scenario; i.e. the anomalous moment was shown to be finite even in six-dimensional space-time, where other higher dimensional theories such as the universal extra dimension scenario give divergent results. In the most recent paper [40], we also have performed numerical calculations to obtain the contribution of nonzero Kaluza-Klein (KK) modes to the muon anomalous magnetic moment and have derived a useful constraint on the compactification scale by comparing the result with the experimental data.

In this paper, we focus on the CP violation in the gauge-Higgs unification scenario. As the concrete example of the physical observable due to the CP violation, we discuss the neutron electric dipole moment (EDM) whose computation has some similarity to that of the anomalous magnetic moment of fermions. We will work in the same model as in the previous paper [40], i.e. the five-dimensional $SU(3)$ gauge-Higgs unification model compactified on an orbifold S^1/Z_2 with a massive bulk fermion in a fundamental representation.

Let us note that how to break CP symmetry is a nontrivial question in the gauge-Higgs unification scenario, since the Higgs field is nothing but a gauge field to start with and its Yukawa coupling is originally gauge coupling, which is real. As far as the theory itself having CP symmetry, the possible way to break CP is due to the compactification which does not respect the symmetry as in the case of a Calabi-Yau manifold with nontrivial complex structure [41] or by the vacuum expectation value (VEV) of some field which has an odd CP eigenvalue [42]. Both mechanisms may be understood as (a sort of) spontaneous CP violation, since the theory itself preserves the CP symmetry and the way of the compactification is responsible for the determination of the vacuum state. (In fact, the effect of compactification is accompanied by the compactification scale $1/R$, which has a mass dimension, and the corresponding CP violation is "soft.")

In the present model the compactification itself is too simple to break CP, since the orbifold is trivially invariant under a discrete transformation $y \to -y$ (y: extra space coordinate). Thus the possible unique source to break the CP symmetry is expected to be the VEV of the Higgs field, which is the zero mode of A_y, the extra space component of the gauge field.

To see whether this is really the case or not, we argue how the space-time coordinates and each field behave under the P and CP transformations. First, let us note that the EDM is a P- and CP-odd observable, and therefore both P and CP have to be broken to get a nonvanishing EDM. The P and CP transformations in higher dimensional theories need some care. Though we can easily find P and C transformations in a higher dimensional sense, they may not reduce to ordinary four-dimensional P or C transformations when dimensional reduction is performed [41]. In the five-dimensional space-time, however, the spinor is a 4-component one just as in the four-dimensional theory, and P and C transformations may be defined in the ordinary ways.

First the parity transformation is defined for fermions as

$$P\colon\ \Psi \to \gamma^0 \Psi, \tag{1.3}$$

where Ψ denotes the $SU(3)$ triplet fermion. To be precise, the extra space coordinate y turns out to be enforced to change its sign for the kinetic term to be invariant under (1.3), and at first glance it does not seem to correspond to the ordinary four-dimensional P transformation. However, at least the zero-mode fields corresponding to the ordinary particles in the standard model are even functions of y, and the change of the sign is irrelevant for the low-energy effective theory. Let us note that in our model the P symmetry is broken anyway by the orbifolding, no matter whether A_y develops its VEV or not, since the orbifolding is aimed to realize a chiral theory. This may also be known by realizing that the orbifold condition for the fermion

$$\Psi(-y) = \mathcal{P}\gamma^5\Psi(y) \qquad (\mathcal{P} = \mathrm{diag}(+,+,-)) \tag{1.4}$$

is inconsistent with the parity transformation (1.3), since γ^0 does not commute with γ^5.

Next, combining with the C transformation $C\colon \Psi \to i\gamma^2\Psi^*$, we can derive the CP transformation:

$$CP\colon\ \Psi(x^\mu, y) \to i\gamma^0\gamma^2\Psi(x_\mu, y)^*. \tag{1.5}$$

This time, the transformation is consistent with the condition (1.4), since $\gamma^0\gamma^2$ commutes with γ^5. Hence, CP is not violated by the orbifolding. The corresponding transformation properties of the space-time coordinates and the gauge field are fixed so that $\bar{\Psi} i\Gamma^M(\partial_M - igA_M)\Psi$ [$\Gamma^M = (\gamma^\mu, i\gamma^5)$, $A_M = (A_\mu, A_y)$ ($\mu = 0$–3)] is invariant under (1.5). Namely,

$$\begin{aligned} CP\colon x^\mu &\to x_\mu, \qquad y \to y, \\ A_\mu(x^\mu, y) &\to -A^\mu(x_\mu, y)^t, \\ A_y(x^\mu, y) &\to -A_y(x_\mu, y)^t. \end{aligned} \tag{1.6}$$

The Z_2-odd bulk mass term in the Lagrangian $-M\epsilon(y)\bar{\Psi}\Psi$ [$\epsilon(y)$: sign function of y] is also invariant under such a defined CP transformation, as y remains untouched and $\epsilon(y)$ does not change its sign. Let us note that, if the fermions are expanded in terms of the orthonormal set of plane waves as $\Psi(x^\mu, y) = \sum_n e^{i(n/R)y}\Psi^{(n)}(x^\mu)$ with R being the radius of the circle (though real mass eigenstates have different mode functions in the presence of the bulk mass M), the CP transformation necessitates the exchange of the KK modes, $n \leftrightarrow -n$, in addition to the four-dimensional CP transformation for $\Psi^{(n)}(x^\mu)$. Fortunately, this exchange of the KK modes is irrelevant for the zero-mode fermions. Thus the transformation given in (1.5) and (1.6) just reduces to the ordinary four-dimensional CP transformation for zero-mode fields.

We thus realize that A_y has CP eigenvalue -1. Hence, the VEV of A_y is the unique source of the CP violation. As a matter of fact, however, in the case that the Z_2-odd bulk mass term vanishes, the CP violation is found to disappear even for the nonvanishing VEV of A_y. In fact, in this case, we can perform a chiral transformation $\Psi \to e^{i(\pi/4)\gamma^5}\Psi$ such that $i\gamma^5$ disappears from the covariant derivative term $i\bar{\Psi}\Gamma^5 D_5\Psi$, keeping the other parts of the Lagrangian invariant. Now, A_y has a scalar-type coupling with fermions and therefore is now even under the CP transformation:

$$\begin{aligned} CP\colon x^\mu &\to x_\mu, \qquad y \to y, \\ \Psi(x^\mu, y) &\to i\gamma^0\gamma^2\Psi(x_\mu, -y)^*, \\ A_\mu(x^\mu, y) &\to -A^\mu(x_\mu, -y)^t, \\ A_y(x^\mu, y) &\to A_y(x_\mu, -y)^t. \end{aligned} \tag{1.7}$$

The invariance of the action under the CP transformation is easily checked by the use of the change of the integration variable $y \to -y$. Thus the VEV of A_y no longer violates CP. Let us note that in this case the exchange of the KK modes is not needed for fermions.

We thus find that, to break CP physically and to get a nonvanishing EDM, the interplay between the VEV of A_y and the bulk mass M is crucial, and from such a point of view both the VEV and the bulk mass are the cause of the CP violation on an equal footing. The necessity of the interplay will be shown by an explicit calculation of Feynman diagrams later in this paper.

Let us note that the VEV of A_y is needed anyway to get the EDM, since the gauge invariant operator to describe the EDM in the standard model is

$$-\frac{i}{2}\bar{\psi}\sigma^{\mu\nu}\gamma^5 F_{\mu\nu}\langle H\rangle\psi, \tag{1.8}$$

which vanishes when $\langle H\rangle$ and therefore the VEV of A_y vanishes. From the same reasoning to conclude that the anomalous magnetic moment is finite even for six-dimensional space-time, we expect by relying on a similar operator analysis that the EDM is also finite even for 6D theory, though in this paper we work in the 5D space-time.

The purposes of this paper are twofold. One is to confirm that the EDM really appears as a finite calculable observable already at the one-loop level, though the EDM has been shown to appear only at the three-loop level in the standard model [43]. In addition, in our model we introduce only the first generation, and to get the EDM we do not need the three-generation scheme, in clear contrast to the case of the standard model. The other one is to obtain the lower bound on the compactification scale, i.e. the upper bound on the size of the extra space, by comparing the prediction of our model with the experimental data.

In Ref. [42], CP violation in the gauge-Higgs unification scenario has also been discussed. The gauge group that they adopt is $U(1)$, and the extra space is a circle. In our case, the gauge group is $SU(3)$, and the extra space is orbifold so that the model can incorporate the chiral theory of the standard model. The chiral theory clearly violates the P symmetry, in contrast to the case of the $U(1)$ gauge theory discussed in [42]. The introduction of the Z_2-odd bulk mass term is also a new feature of our model. By adopting such a realistic model, we hope that we can derive a realistic prediction for the neutron EDM to be compared with the data, which is expected to have various new ingredients not seen in the prediction in [42], due to the complexity of our model. Notice that the possibility of the recovery of CP symmetry due to the Wilson line phase π at the minimum of the effective potential for A_y pointed out in [42] has no relevance in our model, as we assume the realistic situation where the weak scale, i.e. the VEV of A_y times gauge coupling, is much smaller than the compactification scale $1/R$.[2]

This paper is organized as follows. In the next section, we briefly summarize our model and discuss the mass

[2]In the gauge-Higgs unification, the Higgs mass is expressed by $m_H^2 \simeq m_W^2/(16\pi^2 a^2)$, where m_W is the weak scale. A dimensionless constant a is defined by the $(D+1)$-dimensional gauge coupling and the compactification radius as $g_{D+1}\langle A_y\rangle = g_{D+1}v \equiv a/R$. In order to satisfy a lower bound of Higgs mass $m_H > 114$ GeV, $a \ll 1$ is required. Such a tiny VEV a can be obtained from the potential minimization by tuning matter content appropriately; see, for instance, [16].

eigenvalues and corresponding mode functions of fermions and gauge bosons. In Sec. III, we derive general formulas relevant for the EDM concerning a few types of Feynman diagrams where 4D gauge boson A_μ or 4D scalar A_y are exchanged or self-interaction of the 4D gauge and scalar fields is contributing. The coupling constants in the interaction vertices are left arbitrary there. Then combining with the interaction vertices derived in Ref. [40], we obtain the contribution of each type of Feynman diagram to the EDM. In Sec. IV, we numerically estimate the contribution of nonzero KK modes to the EDM as the function of the compactification scale $1/R$. Comparing with the experimental data, we finally obtain a rather meaningful lower bound for the compactification scale. Section V is devoted to the summary discussion.

II. THE MODEL

Since we employ the same model as that discussed in Ref. [39] to calculate the EDM, we briefly summarize it in this paper. We consider a five-dimensional $SU(3)$ gauge-Higgs unification model compactified on an orbifold S^1/Z_2 with a radius R of S^1. As a matter field, a massive bulk fermion in the fundamental representation of the $SU(3)$ gauge group is introduced. The Lagrangian is given by

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{Tr}(F_{MN}F^{MN}) + \bar{\Psi}(i\not{D} - M\epsilon(y))\Psi, \tag{2.1}$$

where the indices $M, N = 0, 1, 2, 3, 5$, the five-dimensional gamma matrices are $\Gamma^M = (\gamma^\mu, i\gamma^5)$ $(\mu = 0, 1, 2, 3)$,

$$F_{MN} = \partial_M A_N - \partial_N A_M - ig[A_M, A_N], \tag{2.2}$$

$$\not{D} = \Gamma^M(\partial_M - igA_M), \tag{2.3}$$

$$\Psi = (\Psi_1, \Psi_2, \Psi_3)^{\mathrm{T}}, \tag{2.4}$$

and g denotes a gauge coupling constant in the five-dimensional gauge theory. M is a bulk mass of the fermion. $\epsilon(y)$ is a sign function of an extra coordinate y which is necessary to introduce a Z_2 odd bulk mass term.

The periodic boundary condition is imposed along S^1, and Z_2 parity assignments are taken as

$$\begin{aligned} A_\mu(y_i - y) &= \mathcal{P}A_\mu(y_i + y)\mathcal{P}^\dagger, \\ A_y(y_i - y) &= -\mathcal{P}A_y(y_i + y)\mathcal{P}^\dagger, \\ \Psi(y_i - y) &= \mathcal{P}\gamma^5\Psi(y_i + y), \end{aligned} \tag{2.5}$$

where $\mathcal{P} = \mathrm{diag}(+, +, -)$ at fixed points $y_i = 0, \pi R$. By this Z_2 parity assignment, $SU(3)$ is explicitly broken to $SU(2) \times U(1)$. The Higgs scalar field is identified with the off-diagonal block of zero mode $A_y^{(0)}$.

The four-dimensional gauge bosons A_μ and their scalar partners A_y can be expanded in KK modes such that the boundary conditions (2.5) are satisfied:

$$A_{\mu,y}(x, y) = \frac{1}{\sqrt{2\pi R}}A^{(0)}_{\mu,y}(x) + \frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}A^{(n)}_{\mu,y}(x)\cos\left(\frac{n}{R}y\right) \quad \text{(even)}, \tag{2.6}$$

$$A_{\mu,y}(x, y) = \frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}A^{(n)}_{\mu,y}(x)\sin\left(\frac{n}{R}y\right) \quad \text{(odd)}. \tag{2.7}$$

After electroweak symmetry breaking, quadratic terms relevant to the gauge boson mass are diagonalized as

$$\begin{aligned} \mathcal{L}_{\text{mass}} - \frac{1}{2}\int_{-\pi R}^{\pi R} dy[\partial^\mu A^a_\mu - (\partial_y A^a_y - 2m_W f^{6ab}A_{by})]^2 = {}& \sum_{n=1}^{\infty}\Big[\frac{1}{2}M_n^2(B^{(n)}_\mu B^{\mu(n)} + h^{(n)}_\mu h^{\mu(n)}) + \frac{1}{2}(M_n - 2m_W)^2\phi^{(n)}_\mu\phi^{\mu(n)} \\ & + \frac{1}{2}(M_n + 2m_W)^2 Z^{(n)}_\mu Z^{\mu(n)} + (M_n + m_W)^2 W^{+(n)}_\mu W^{-\mu(n)} \\ & + (M_n - m_W)^2 X^{+(n)}_\mu X^{-\mu(n)}\Big] + \frac{1}{2}(2m_W)^2 Z_\mu Z^\mu + m_W^2 W^+_\mu W^{-\mu} \\ & - \sum_{n=1}^{\infty}\Big[\frac{1}{2}M_n^2(B^{(n)}_y B^{(n)}_y + h^{(n)}_y h^{(n)}_y) + \frac{1}{2}(M_n + 2m_W)^2\phi^{(n)}_y\phi^{(n)}_y \\ & + \frac{1}{2}(M_n - 2m_W)^2 Z^{(n)}_y Z^{(n)}_y + (M_n - m_W)^2 W^{+(n)}_y W^{-(n)}_y \\ & + (M_n + m_W)^2 X^{+(n)}_y X^{-(n)}_y\Big] + \frac{1}{2}(2m_W)^2 Z_y Z_y + m_W^2 W^+_y W^-_y, \end{aligned} \tag{2.8}$$

where the gauge-fixing term in the 't Hooft-Feynman gauge is introduced to eliminate the mixing terms between the gauge bosons and the gauge scalar bosons. $M_n = \frac{n}{R}$, and $m_W = 2g\langle A_y^6\rangle = (g/\sqrt{2\pi R})v = g_4 v$. g_4 is a four-dimensional gauge coupling. The KK mass eigenstates and zero-mode mass eigenstates are given by

$$B^{(n)} = \frac{1}{2}(\sqrt{3}A^{3(n)} + A^{8(n)}), \quad h^{(n)} = A^{6(n)}, \quad Z^{(n)} = \frac{1}{\sqrt{2}}\left[\frac{A^{3(n)} - \sqrt{3}A^{8(n)}}{2} - A^{7(n)}\right], \quad \phi^{(n)} = \frac{1}{\sqrt{2}}\left[\frac{A^{3(n)} - \sqrt{3}A^{8(n)}}{2} + A^{7(n)}\right],$$
$$W^{\pm(n)} = \frac{1}{2}[A^{1(n)} + A^{5(n)} \mp i(A^{2(n)} - A^{4(n)})], \quad X^{\pm(n)} = \frac{1}{2}[A^{2(n)} + A^{4(n)} \mp i(-A^{1(n)} + A^{5(n)})], \quad B_\mu = \frac{1}{2}[\sqrt{3}A^3_\mu + A^8_\mu],$$
$$h = A_y^{6(0)}, \quad W^\pm_\mu = \frac{1}{\sqrt{2}}(A^1_\mu \mp iA^2_\mu), \quad X^\pm = \frac{1}{\sqrt{2}}[A^4_y \mp iA^5_y], \quad Z_\mu = \frac{1}{2}(A^3_\mu - \sqrt{3}A^8_\mu), \quad \phi = A^7_y. \tag{2.9}$$

The zero-mode gauge bosons $W^\pm_\mu$, Z_μ, and B_μ correspond to the W boson, Z boson, and photon, respectively, and zero-mode scalar fields $X^\pm$, ϕ, and h correspond to the charged Nambu-Goldstone (NG) boson, neutral NG boson, and Higgs field in the standard model, respectively.

Some comments on this model are in order. First, the predicted Weinberg angle of this model is not realistic: $\sin^2\theta_W = 3/4$ [44]. A possible way to cure the problem is to introduce an extra $U(1)$ or the brane localized gauge kinetic term [10]. Second, the up quark remains massless, and we have no up-type Yukawa coupling. A possible way out of this situation is to introduce second-rank symmetric tensors of $SU(3)$ (**6**-dimensional representation) [16].

On the other hand, we have obtained a quadratic part of the 4D effective Lagrangian of the fermion

$$\mathcal{L} = \sum_{n=1}^{\infty}[\bar{\psi}_1^{(n)}(i\partial\!\!\!/ - m_n)\psi_1^{(n)} + \bar{\psi}_2^{(n)}(i\partial\!\!\!/ - m_n^-)\psi_2^{(n)} + \bar{\psi}_3^{(n)}(i\partial\!\!\!/ - m_n^+)\psi_3^{(n)}] + \bar{d}(i\partial\!\!\!/ - m)d + \bar{u}_L i\partial\!\!\!/ u_L, \tag{2.10}$$

where the mass eigenstates of fermion were obtained as

$$d_L = \Psi_{2L}^{(0)} + \sum_{n=1}^{\infty}\frac{\hat{m}_n}{m_n}\Psi_{3L}^{(n)}, \quad d_R = \Psi_{3R}^{(0)} + \sum_{n=1}^{\infty}(-1)^n\frac{\hat{m}_n}{m_n}\Psi_{3R}^{(n)}, \tag{2.11}$$

$$\psi_{3L}^{(n)} = \frac{1}{\sqrt{2}}\Bigg[\Psi_{2L}^{(n)} + \Psi_{3L}^{(n)} + \frac{M^2}{2m_n^3}m_W(\Psi_{2L}^{(n)} - \Psi_{3L}^{(n)}) - \frac{\hat{m}_n}{m_n}\Psi_{2L}^{(0)} + \sum_{l\neq n}^{\infty}\frac{\tilde{m}_{nl}}{m_n^2 - m_l^2}(m_l\Psi_{3L}^{(l)} - m_n\Psi_{2L}^{(l)})\Bigg], \tag{2.12}$$

$$\psi_{2L}^{(n)} = \frac{1}{\sqrt{2}}\Bigg[\Psi_{2L}^{(n)} - \Psi_{3L}^{(n)} - \frac{M^2}{2m_n^3}m_W(\Psi_{2L}^{(n)} - \Psi_{3L}^{(n)}) + \frac{\hat{m}_n}{m_n}\Psi_{2L}^{(0)} + \sum_{l\neq n}^{\infty}\frac{\tilde{m}_{nl}}{m_n^2 - m_l^2}(m_l\Psi_{3L}^{(l)} + m_n\Psi_{2L}^{(l)})\Bigg], \tag{2.13}$$

$$\psi_{3R}^{(n)} = \frac{1}{\sqrt{2}}\Bigg[\Psi_{2R}^{(n)} + \Psi_{3R}^{(n)} - \frac{M^2}{2m_n^3}m_W(\Psi_{2L}^{(n)} - \Psi_{3L}^{(n)}) - (-1)^n\frac{\hat{m}_n}{m_n}\Psi_{2L}^{(0)} + \sum_{l\neq n}^{\infty}\frac{\tilde{m}_{nl}}{m_n^2 - m_l^2}(m_n\Psi_{3R}^{(l)} - m_l\Psi_{2R}^{(l)})\Bigg], \tag{2.14}$$

$$\psi_{2R}^{(n)} = \frac{1}{\sqrt{2}}\Bigg[\Psi_{2R}^{(n)} - \Psi_{3R}^{(n)} - \frac{M^2}{2m_n^3}m_W(\Psi_{2R}^{(n)} - \Psi_{3R}^{(n)}) + \frac{\hat{m}_n}{m_n}\Psi_{2R}^{(0)} + \sum_{l\neq n}^{\infty}\frac{\tilde{m}_{nl}}{m_n^2 - m_l^2}(m_l\Psi_{3R}^{(l)} + m_n\Psi_{2R}^{(l)})\Bigg], \tag{2.15}$$

$$u_L = \Psi_{1L}^{(0)}, \qquad \psi_1^{(n)} = \Psi_1^{(n)}, \tag{2.16}$$

where

$$m = \frac{2\pi RM}{\sqrt{(1 - e^{-2\pi RM})(e^{2\pi RM} - 1)}}m_W,$$
$$\hat{m}_n = 4\sqrt{\frac{\pi RM}{1 - e^{-2\pi RM}}}\frac{1 - (-1)^n e^{-\pi RM}}{\pi R m_n^3}M_n M m_W,$$
$$\tilde{m}_n = (-1)^n\hat{m}_n,$$
$$\tilde{m}_{nl} = \frac{4nl(1 - (-1)^{n+l})}{\pi R m_n m_l(n^2 - l^2)}(1 - \delta_{nl})m_W M,$$
$$(m_n^\pm)^2 = m_n^2 \pm 2m_W\frac{M_n^2}{m_n}. \tag{2.17}$$

In deriving the above 4D effective Lagrangian of the fermion, the following mode expansions are substituted and integrated out over the fifth coordinate:

$$\Psi(x, y) = \sum_{n=1}^{\infty}\begin{pmatrix}\Psi_{1L}^{(n)}(x)f_L^{(n)}(y) + \Psi_{1R}^{(n)}(x)g^{(n)}(y)\\ \Psi_{2L}^{(n)}(x)f_L^{(n)}(y) + \Psi_{2R}^{(n)}(x)g^{(n)}(y)\\ \Psi_{3L}^{(n)}(x)g^{(n)}(y) + \Psi_{3R}^{(n)}(x)f_R^{(n)}(y)\end{pmatrix} + \begin{pmatrix}\Psi_{1L}^{(0)}(x)f_L^{(0)}(y)\\ \Psi_{2L}^{(0)}(x)f_L^{(0)}(y)\\ \Psi_{3R}^{(0)}(x)f_R^{(0)}(y)\end{pmatrix} \tag{2.18}$$

with the zero-mode wave functions

$$f_L^{(0)} = \sqrt{\frac{M}{1 - e^{-2\pi RM}}} e^{-M|y|}, \qquad f_R^{(0)} = \sqrt{\frac{M}{e^{2\pi RM} - 1}} e^{M|y|} \tag{2.19}$$

and the nonzero KK mode functions

$$f_L^{(n)} = \frac{M_n}{\sqrt{\pi R} m_n} \left[\cos\left(\frac{n}{R} y\right) - \frac{MR}{n} \epsilon(y) \sin\left(\frac{n}{R} y\right) \right], \tag{2.20}$$

$$f_R^{(n)} = \frac{M_n}{\sqrt{\pi R} m_n} \left[\cos\left(\frac{n}{R} y\right) + \frac{MR}{n} \epsilon(y) \sin\left(\frac{n}{R} y\right) \right], \tag{2.21}$$

$$g^{(n)} = \frac{1}{\sqrt{\pi R}} \sin\left(\frac{n}{R} y\right). \tag{2.22}$$

Deriving the vertex functions necessary for calculating the neutron EDM by using the above mass eigenfunctions is straightforward but complicated. We do not repeat here their derivation since the necessary vertex functions are the exact same ones as summarized in Appendix A of our previous paper [40], except that the muon μ should be replaced by the down quark d.

III. CALCULATION OF THE ELECTRIC DIPOLE MOMENT

In this section, we calculate the fermion EDM. Various types of diagrams contributing to the EDM are shown in Figs. 1 and 2. The fermion electric dipole moment is described by dimension 6 operator $-\frac{i}{2} \bar{\psi}_L(p') \sigma^{\nu\rho} \gamma^5 F_{\nu\rho} \langle H \rangle \psi_R(p)$. In general, quantum corrections to the photon vertex $-\frac{e}{3} B_\mu \bar{\psi}(p') \gamma^\mu \psi(p)$ can be written as

$$-i \frac{e}{3} B_\mu \bar{\psi}(p') [\gamma^\mu + \hat{\Gamma}^\mu] \psi(p), \tag{3.1}$$

where B_μ is a photon field,

$$\hat{\Gamma}^\mu = a_\psi \frac{p^\mu + p'^\mu}{2m_\psi} + \frac{d_\psi}{e/3} (p^\mu + p'^\mu) \gamma_5.$$

and a_ψ and d_ψ stand for the anomalous magnetic moment

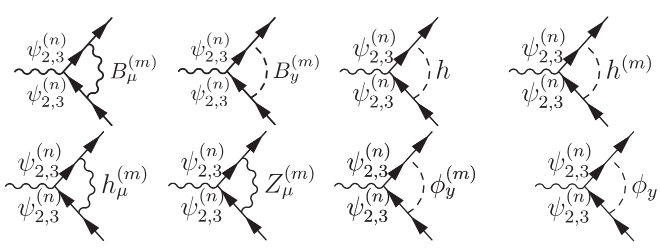


FIG. 1. The diagrams contributing to the EDM at one loop by the neutral current.

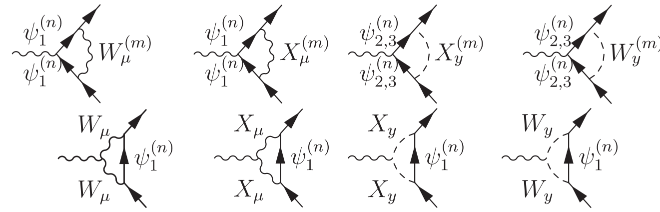


FIG. 2. The diagrams contributing to the EDM at one loop by the charged current.

and the electric dipole moment of ψ, respectively. Since our interest in this paper is in the electric dipole moment, the terms proportional to $\gamma_5(p_\mu + p'_\mu)$ must be extracted.

The diagrams we calculated are shown below.

The upper category of diagrams denotes contributions by the neutral current and the lower one those by the charged current. The calculation is straightforward but lengthy. The detailed calculations are summarized in Appendix B.

As will be seen in Appendix B, all of the standard model diagrams have no contributions to the EDM at one-loop level, which is consistent with the well-known fact that the EDM in the standard model is generated at least at the three-loop level [43].

It is very interesting that the neutron EDM is generated already at one loop as in the case of supersymmetry [45] although it is generated at three loops in the case of the standard model. We can see that the EDM contributions from the neutral current sector are due to the mixing terms between different nonzero KK modes of the down quark, which is proportional to $\tilde{m}_{nl}$. On the other hand, the EDM contributions from the charged current sector are due to the mixing terms between a zero mode and nonzero KK modes of the down quark, which is proportional to $\hat{m}_n$. Since these two mass parameters $\tilde{m}_{nl}$ and $\hat{m}_n$ are proportional to both the bulk mass M and the W-boson mass m_W [see (2.17)], the EDM vanishes if the bulk mass is zero while the Higgs VEV is nonzero, and vice versa. This is consistent with the general discussion described in the introduction on how CP is violated.

IV. NUMERICAL ESTIMATION OF EDM FROM NONZERO KK MODES

We move to a numerical calculation of the neutron electric dipole moment. We expect that the up quark electric dipole moment d_u vanishes in this model, since there is no right-handed up quark and the operator describing the up quark electric dipole moment $\langle H \rangle \bar{u}_{L(R)} \sigma_{\mu\nu} \gamma^5 u_{R(L)} F^{\mu\nu}$ does not exist. Thus, the neutron electric dipole moment d_n in this model is written as follows:

$$d_n = \tfrac{4}{3} d_d - \tfrac{1}{3} d_u = \tfrac{4}{3} d_d. \tag{4.1}$$

To reproduce down quark Yukawa coupling, we must set a bulk mass parameter so as to satisfy the following relation:

$$\frac{m}{m_W} = \frac{2\pi RM}{\sqrt{(1-e^{-2\pi RM})(e^{2\pi RM}-1)}} \sim \frac{4\text{–}8\ \text{MeV}}{80\ \text{GeV}}. \quad (4.2)$$

Thus, we set the bulk mass as $2\pi RM = 25.5$ ($m = 6$ MeV is taken).

Here, only the numerical results are shown. The contributions from the neutral current and the charged current processes to the EDM are denoted by d(N.C.) and d(C.C.), respectively, and are obtained as follows:

$$\begin{aligned} d(\text{N.C.}) &\sim \frac{16}{9} e^3 \left(\frac{MR}{\pi}\right)^4 R^2 m_W (-8.3 \times 10^{-7}), \\ d(\text{C.C.}) &\sim -\frac{2}{9} e^3 \frac{(MR)^3}{\pi^3} R^2 m_W (2.12 \times 10^{-5}) \\ &\quad + \frac{16}{9} e^3 \frac{(MR)^4}{\pi^4} R^2 m_W (8.0 \times 10^{-7}). \end{aligned} \quad (4.3)$$

Combining these results, we obtain the final result on the contribution from nonzero KK mode d(KK) as

$$\begin{aligned} d(\text{KK}) &= d(\text{N.C.}) + d(\text{C.C.}) \\ &\sim -2.3 \times 10^{-23} (R m_W)^2 [e \cdot \text{cm}]. \end{aligned} \quad (4.4)$$

We require that the contribution of KK mode $\frac{4}{3} d$(KK) is less than the experimental upper bound [46],

$$\frac{4}{3} \cdot 2.3 \times 10^{-23} (R m_W)^2 [e \cdot \text{cm}] < 2.9 \times 10^{-26} [e \cdot \text{cm}], \quad (4.5)$$

which gives a lower bound for the compactification scale

$$\frac{1}{R} > 33 m_W \simeq 2.6\ \text{TeV}. \quad (4.6)$$

V. SUMMARY

In this paper we studied the neutron EDM in a five-dimensional $SU(3)$ gauge-Higgs unification compactified on $M^4 \times S^1/Z_2$ space-time including massive fermions belonging to the triplet of $SU(3)$. The smallness of the quark Yukawa coupling is realized by introducing a Z_2-odd bulk mass $M\epsilon(y)$. We pointed out that to realize the CP violation is a nontrivial task in the gauge-Higgs unification scenario where the Yukawa coupling is originally gauge coupling, which is of course real. We identified the transformation properties of each field under P and CP transformations, since to get the nonvanishing EDM both P and CP symmetries have to be broken, though P is broken anyway by the orbifolding. We have found that, since the theory itself is CP symmetric, the unique source of the CP violation in our model is the VEV of the Higgs, the extra space component of the gauge field A_y. In such a sense, CP is broken spontaneously through the Hosotani mechanism [3]. We emphasized that, actually to get physically the CP violating effect, the interplay between the VEV of A_y and the bulk mass M is crucial. In fact, in the hypothetical limit of $M \to 0$, it turned out that by suitable chiral transformation A_y becomes a field with an even CP eigenvalue, whose VEV therefore does not break CP symmetry. From such a point of view both the VEV and the bulk mass are the cause of the CP violation on an equal footing.

We then calculated the one-loop contributions to the neutron electric dipole moment as the typical example of the CP violating observable and found that the EDM appears already at the one-loop level, without invoking the three-generation scheme, in clear contrast to the case of the standard model where the EDM appears only at the three-loop level. The explicit calculation has shown that one-loop contributions from nonzero KK modes to the neutron EDM are generated due to the mixing effects between different nonzero KK modes and between a zero mode and nonzero KK modes. Also, the obtained EDM was proportional to the Higgs VEV and the bulk mass, which was consistent with what we discussed concerning the importance of their interplay to get CP violation.

Furthermore, we could confirm that the standard model contribution to the neutron EDM due to the Kaluza-Klein zero modes vanishes at the one-loop level, as we expect.

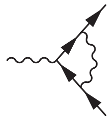
The fact that the EDM appears already at the one-loop level suggests that we may be able to get a rather meaningful lower bound on the compactification scale by a comparison with the data. This turns out to be the case. We could derive a rather meaningful lower bound for the compactification scale, which is around 2.6 TeV, by comparing the contribution due to the nonzero Kaluza-Klein modes with the experimental data.

ACKNOWLEDGMENTS

The work of the authors was supported in part by the Grant-in-Aid for Scientific Research of the Ministry of Education, Science and Culture, No. 18204024 and No. 20025005.

APPENDIX A: GENERAL FORMULAS FOR ELECTRIC DIPOLE MOMENT

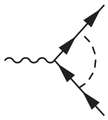
We now derive general formulas for each type of Feynman diagram, leaving the couplings in the interaction vertices arbitrary. First, the gauge boson exchange diagram is given by (a, b, c, and d are generic coupling constants)



$$= \int \frac{d^4k}{(2\pi)^4 i}(aL + bR)\gamma^\nu \frac{-1}{\not{k} + \not{p}' - m_n}(Q_\psi \gamma_\mu)\frac{-1}{\not{k} + \not{p} - m_n}\gamma_\nu(cL + dR)\frac{1}{k^2 - M_G^2}$$

$$\supset -Q_\psi \int \frac{d^4k}{(2\pi)^4 i}\int_0^1 dx \int_0^{1-x} dy \frac{4(ac - bd)(1 - X)m_n}{[k^2 + X(1 - X)m^2 - Xm_n^2 - (1 - X)M_G^2]^3}\gamma_5 P_\mu. \tag{A1}$$

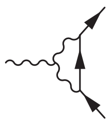
In the second line only the part relevant for the EDM has been extracted. Similarly, the diagram due to the exchange of the scalar partner of gauge boson is given by



$$= \int \frac{d^4k}{(2\pi)^4 i}(aL + bR)\frac{-1}{\not{k} + \not{p}' - m_n}(Q_\psi \gamma_\mu)\frac{-1}{\not{k} + \not{p} - m_n}(cL + dR)\frac{-1}{k^2 - m_G^2}$$

$$\supset -Q_\psi \int \frac{d^4k}{(2\pi)^4 i}\int_0^1 dx \int_0^{1-x} dy \frac{-(ac - bd)Xm_n}{[k^2 - X^2m^2 + Xm^2 - Xm_n^2 - (1 - X)M_G^2]^3}\gamma_5 P^\mu. \tag{A2}$$

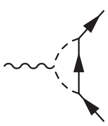
For the diagrams due to the gauge boson self-energy, there are the following three types of diagrams:



$$= \int \frac{d^4k}{(2\pi)^4 i}(aL + bR)\gamma_\rho \frac{-1}{\not{k} - m_n}\gamma_\nu(cL + dR)\frac{1}{(k - p')^2 - M_G^2}\frac{1}{(k - p)^2 - M_G^2}$$

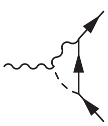
$$\times (-e)[(2p' - p - k)_\rho \eta_{\mu\nu} + (2k - p' - p)_\mu \eta_{\nu\rho} + (2p - p' - k)_\nu \eta_{\mu\rho}]$$

$$\supset e \int \frac{d^4k}{(2\pi)^4 i}\int_0^1 dx \int_0^{1-x} dy \frac{-(ac - bd)[X + 2 + 4(1 - X)]m_n}{[k^2 + X(1 - X)m^2 - XM_G^2 - (1 - X)m_n^2]^3}\gamma_5 P^\mu, \tag{A3}$$



$$= \int \frac{d^4k}{(2\pi)^4 i}(aL + bR)\frac{-1}{\not{k} - m_n}(cL + dR)\frac{-1}{(k - p')^2 - M_G^2}\frac{-1}{(k - p)^2 - M_G^2}e(2k - p' - p)_\mu$$

$$\supset -e \int \frac{d^4k}{(2\pi)^4 i}\int_0^1 dx \int_0^{1-x} dy \frac{-(ac - bd)(1 - X)m_n}{[k^2 + X(1 - X)m^2 - XM_G^2 - (1 - X)m_n^2]^3}\gamma_5 P_\mu, \tag{A4}$$



$$+ (\text{h.c.}) = \int \frac{d^4k}{(2\pi)^4 i}(aL + bR)\gamma^\mu \frac{-1}{\not{k} - m_n}(cL + dR)\frac{1}{(k - p')^2 - M_G^2}\frac{-1}{(p - k)^2 - M_G^2}$$

$$\times (\pm e)M_G + (\text{h.c.})$$

$$= \pm e \int \frac{d^4k}{(2\pi)^4 i}\int_0^1 dx \int_0^{1-x} dy \frac{(ac - bd)(x - y)M_G}{[k^2 + X(1 - X)m^2 - XM_G^2 - (1 - X)m_n^2]^3}\gamma_5 P^\mu$$

$$= 0. \tag{A5}$$

Here M_G, m_n, and m denote masses of the gauge boson, the internal fermion, and the down-type quark, respectively. Q_ψ denotes the electric charge of internal fermion. x and y are Feynman parameters, and $X \equiv x + y$. P_μ is defined as the sum of the external momenta for fermions: $P_\mu \equiv p_\mu + p'_\mu$. In the last diagram of the gauge boson self-energy, the plus (minus) sign corresponds to the diagram where the W_μ (X_μ) boson propagates in the loop, respectively. In all amplitudes, we used the property that the Feynman parameter integral of an odd function of $x - y$ vanishes.

In order to arrive at the above expressions, the numerator of the integrand is calculated as

$$(aL + bR)\gamma_\nu(\not{k} + \not{p}' + m_n)\gamma^\mu(\not{k} + \not{p} + m_n)\gamma^\nu(cL + dR)$$
$$\supset -(ad - bc)\gamma_5(x - y)(1 - X)mP^\mu$$
$$+ 2(ac - bd)m_n\gamma_5(1 - X)P^\mu.$$

In the above calculation, the momentum shift $k \to k - xp' - yp$ is performed, and $\supset$ means that terms relevant for the fermion EDM are extracted. The equation of motion for the external fermion is also utilized:

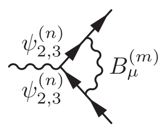
$$\bar{\psi}(p')\gamma_5(x\not{p}' + y\not{p})\psi(p) \to (-x + y)m\bar{\psi}(p')\gamma_5\psi(p). \tag{A6}$$

APPENDIX B: EXPLICIT CALCULATIONS OF EDM

Applying the possible interaction vertices described in Ref. [40] to these formulas derived in Appendix A, we can obtain the amplitudes of the EDM in a straightforward way and list them by classifying into the neutral current sector, charged current sector, and gauge boson self-energy sector. Concerning the mode indices in the amplitude, the summation $\sum_{l,m,n=1}^{\infty}$ should be understood. In our calculation, we adopt approximations $m_W, m \ll 1/R$, and the results are shown at the leading order of $\mathcal{O}(m_W^2)$.

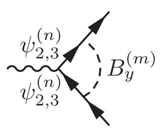
1. Neutral current sector

a. KK mode photon exchange



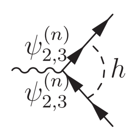
$$= \frac{e}{3}\int\frac{d^4k}{(2\pi)^4 i}\int_0^1 dx\int_0^{1-x} dy\,\frac{2\frac{g_4^2}{3}\frac{M_n^2}{m_n^2}\frac{\pi RM}{1-e^{-2\pi RM}}\left(\frac{I_3}{\pi R}\right)^2\frac{m_l\tilde{m}_{nl}}{m_n^2-m_l^2}(-1)^{n+m}4(1-X)m_n}{[k^2 - Xm_n^2 - (1-X)M_m^2]^3}\gamma_5 P_\mu. \tag{B1}$$

b. KK mode photon partner exchange



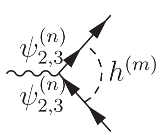
$$= -\frac{e}{3}\int\frac{d^4k}{(2\pi)^4 i}\int_0^1 dx\int_0^{1-x} dy\,\frac{2\frac{g_4^2}{3}\frac{\pi RM}{e^{2\pi RM}-1}\left(\frac{I_4}{\pi R}\right)^2\frac{m_n\tilde{m}_{nl}}{m_n^2-m_l^2}(-1)^{n+m}Xm_n}{[k^2 - Xm_n^2 - (1-X)M_m^2]^3}\gamma_5 P_\mu. \tag{B2}$$

c. Higgs exchange



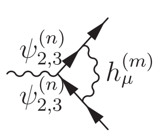
$$= \frac{e}{3}\int\frac{d^4k}{(2\pi)^4 i}\int_0^1 dx\int_0^{1-x} dy\,\frac{g_4^2 I_5^2\frac{m_l\tilde{m}_{nl}}{m_n^2-m_l^2}Xm_n(-1)^n}{[k^2 - Xm_n^2]^3}\gamma_5 P_\mu. \tag{B3}$$

d. KK mode Higgs exchange



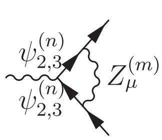
$$= \frac{e}{3}\int\frac{d^4k}{(2\pi)^4 i}\int_0^1 dx\int_0^{1-x} dy\,\frac{2g_4^2 I_6^2\left(\frac{M_n}{\pi R m_n}\right)^2\frac{\pi RM}{e^{2\pi RM}-1}\frac{m_l\tilde{m}_{nl}}{m_n^2-m_l^2}(-1)^{n+m}Xm_n}{[k^2 - Xm_n^2 - (1-X)M_m^2]^3}\gamma_5 P_\mu. \tag{B4}$$

e. KK mode Higgs partner exchange



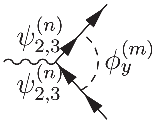
$$= -\frac{e}{3}\int\frac{d^4k}{(2\pi)^4 i}\int_0^1 dx\int_0^{1-x} dy\,\frac{8g_4^2(1-X)m_n\gamma_5 P_\mu}{[k^2 - Xm_n^2 - (1-X)M_m^2]^3} \times\left[\frac{1}{2}\sqrt{\frac{\pi RM}{e^{2\pi RM}-1}}\frac{\hat{m}_l}{m_l}\frac{I_4 I_2}{\sqrt{\pi R}}((-1)^{l+m+n}-1) - \frac{\pi RM}{e^{2\pi RM}-1}\left(\frac{I_4}{\pi R}\right)^2\frac{m_n\tilde{m}_{nl}}{m_n^2-m_l^2}(-1)^{n+m}\right]. \tag{B5}$$

f. KK mode Z boson exchange



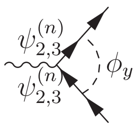
$$= \frac{e}{3}\int\frac{d^4k}{(2\pi)^4 i}\int_0^1 dx\int_0^{1-x} dy\,\frac{g_4^2\frac{\pi RM}{e^{2\pi RM}-1}\frac{m_n\tilde{m}_{nl}}{m_n^2-m_l^2}\left(\frac{I_4}{\pi R}\right)^2(-1)^{n+m}\left(\frac{M_m^2}{m_n^2}+1\right)4(1-X)m_n}{[k^2 - Xm_n^2 - (1-X)M_m^2]^3}\gamma_5 P_\mu. \tag{B6}$$

g. KK mode neutral NG boson ϕ_y exchange



$$= \frac{e}{3}\int\frac{\mathrm{d}^4k}{(2\pi)^4 i}\int_0^1 \mathrm{d}x \int_0^{1-x}\mathrm{d}y\, \frac{g_4^2 X m_n (-1)^{n+m}\gamma_5 P_\mu}{[k^2 - X m_n^2 - (1-X)M_m^2]^3} \times \frac{\pi R M}{\mathrm{e}^{2\pi RM}-1}\left[\left(\frac{I_4}{\pi R}\right)^2 \frac{m_n \tilde{m}_{nl}}{m_n^2 - m_l^2} - \left(\frac{I_6}{\pi R}\right)^2 \left(\frac{M_n}{m_n}\right)^2 \frac{m_l \tilde{m}_{nl}}{m_n^2 - m_l^2}\right]. \tag{B7}$$

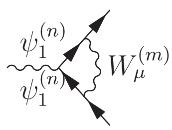
h. Zero-mode neutral NG boson ϕ_y exchange



$$= -\frac{e}{3}\int\frac{\mathrm{d}^4k}{(2\pi)^4 i}\int_0^1 \mathrm{d}x \int_0^{1-x}\mathrm{d}y\, \frac{g_4^2 I_5^2 \frac{m_l \tilde{m}_{nl}}{m_n^2 - m_l^2}(-1)^n X m_n}{[k^2 - X m_n^2]^3}\gamma_5 P_\mu. \tag{B8}$$

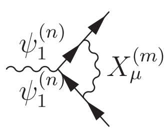
2. Charged current sector

a. KK mode W boson exchange



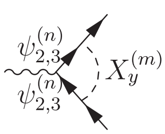
$$= \frac{8}{3}e\int\frac{\mathrm{d}^4k}{(2\pi)^4 i}\int_0^1 \mathrm{d}x \int_0^{1-x}\mathrm{d}y\, \frac{g_4^2 \frac{\hat{m}_l}{m_l}\sqrt{\frac{\pi RM}{\mathrm{e}^{2\pi RM}-1}}\frac{I_4}{\sqrt{\pi R}}\left(\frac{M_m}{m_n}I_1 + I_2\right)(1-X)m_n}{[k^2 - X m_n^2 - (1-X)M_m^2]^3}\gamma_5 P_\mu. \tag{B9}$$

b. KK mode X_μ boson exchange



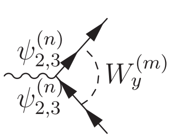
$$= \frac{8}{3}e\int\frac{\mathrm{d}^4k}{(2\pi)^4 i}\int_0^1 \mathrm{d}x \int_0^{1-x}\mathrm{d}y\, \frac{g_4^2 \frac{\hat{m}_l}{m_l}\sqrt{\frac{\pi RM}{\mathrm{e}^{2\pi RM}-1}}\frac{I_4}{\sqrt{\pi R}}\left(\frac{M_m}{m_n}I_1 + I_2\right)(1-X)m_n}{[k^2 - X m_n^2 - (1-X)M_m^2]^3}\gamma_5 P_\mu. \tag{B10}$$

c. KK mode X_y boson exchange



$$= -\frac{2}{3}e\int\frac{\mathrm{d}^4k}{(2\pi)^4 i}\int_0^1 \mathrm{d}x \int_0^{1-x}\mathrm{d}y\, \frac{-g_4^2 \frac{\hat{m}_l}{m_l}\sqrt{\frac{\pi RM}{\mathrm{e}^{2\pi RM}-1}}\left((-1)^{l+m+n}\frac{I_4 I_2}{\sqrt{\pi R}} + \frac{M_n}{m_n}\frac{I_1 I_6}{\sqrt{\pi R}}\right)X m_n}{[k^2 - X m_n^2 - (1-X)M_m^2]^3}\gamma_5 P_\mu. \tag{B11}$$

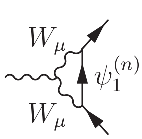
d. KK mode W_y boson exchange



$$= -\frac{2}{3}e\int\frac{\mathrm{d}^4k}{(2\pi)^4 i}\int_0^1 \mathrm{d}x \int_0^{1-x}\mathrm{d}y\, \frac{-g_4^2 \frac{\hat{m}_l}{m_l}\sqrt{\frac{\pi RM}{\mathrm{e}^{2\pi RM}-1}}\left((-1)^{l+m+n}\frac{I_4 I_2}{\sqrt{\pi R}} + \frac{M_n}{m_n}\frac{I_1 I_6}{\sqrt{\pi R}}\right)X m_n}{[k^2 - X m_n^2 - (1-X)M_m^2]^3}\gamma_5 P_\mu. \tag{B12}$$

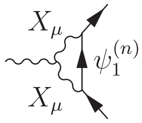
3. Gauge boson self-energy

a. KK mode W_μ boson self-energy diagram



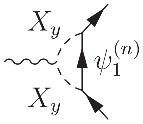
$$= e\int\frac{\mathrm{d}^4k}{(2\pi)^4 i}\int_0^1 \mathrm{d}x \int_0^{1-x}\mathrm{d}y\, \frac{g_4^2 \frac{\hat{m}_l}{m_l}\sqrt{\frac{\pi RM}{\mathrm{e}^{2\pi RM}-1}}\frac{I_4}{\sqrt{\pi R}}\left(\frac{M_m}{m_n}I_1 + I_2\right)(6-3X)m_n}{[k^2 - X M_m^2 - (1-X)m_n^2]^3}\gamma_5 P_\mu. \tag{B13}$$

b. KK mode X_μ boson self-energy diagram



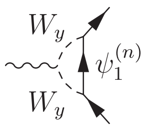
$$= e\int\frac{d^4k}{(2\pi)^4 i}\int_0^1 dx\int_0^{1-x}dy\,\frac{g_4^2\frac{\hat{m}_l}{m_l}\sqrt{\frac{\pi RM}{e^{2\pi RM}-1}}\frac{I_4}{\sqrt{\pi R}}\left(\frac{M_m}{m_n}I_1+I_2\right)(6-3X)m_n}{[k^2-XM_m^2-(1-X)m_n^2]^3}\gamma_5 P_\mu. \quad \text{(B14)}$$

c. KK mode X_y boson self-energy diagram



$$= -e\int\frac{d^4k}{(2\pi)^4 i}\int_0^1 dx\int_0^{1-x}dy\,\frac{-g_4^2\frac{\hat{m}_l}{m_l}\sqrt{\frac{\pi RM}{e^{2\pi RM}-1}}\left((-1)^{l+m+n}\frac{I_4 I_2}{\sqrt{\pi R}}+\frac{M_n}{m_n}\frac{I_1 I_6}{\sqrt{\pi R}}\right)(1-X)m_n}{[k^2-XM_m^2-(1-X)m_n^2]^3}\gamma_5 P_\mu. \quad \text{(B15)}$$

d. KK mode W_y boson self-energy diagram



$$= e\int\frac{d^4k}{(2\pi)^4 i}\int_0^1 dx\int_0^{1-x}dy\,\frac{g_4^2\frac{\hat{m}_l}{m_l}\sqrt{\frac{\pi RM}{e^{2\pi RM}-1}}\left((-1)^{l+m+n}\frac{I_4 I_2}{\sqrt{\pi R}}+\frac{M_n}{m_n}\frac{I_1 I_6}{\sqrt{\pi R}}\right)(1-X)m_n}{[k^2-XM_m^2-(1-X)m_n^2]^3}\gamma_5 P_\mu. \quad \text{(B16)}$$

Various integrals $I_{1\text{–}6}$ are summarized in the following Appendix C.

APPENDIX C: VARIOUS INTEGRALS

Various integrals appearing in the amplitudes are defined as follows:

$$I_1 \equiv \int_{-\pi R}^{\pi R} dy\,\frac{1}{(\sqrt{\pi R})^3}\,S_l(y)C_m(y)S_n(y), \quad \text{(C1)}$$

$$I_2 \equiv \int_{-\pi R}^{\pi R} dy\,\frac{1}{(\sqrt{\pi R})^3}\,\frac{M_n}{m_n}\,S_l(y)S_m(y) \times\left(C_n(y)-\frac{MR}{n}\,\varepsilon(y)S_n(y)\right), \quad \text{(C2)}$$

$$I_3 \equiv \int_{-\pi R}^{\pi R} dy\,e^{-M|y|}C_m(y)\left(C_n(y)-\frac{MR}{n}\,\varepsilon(y)S_n(y)\right), \quad \text{(C3)}$$

$$I_4 \equiv \int_{-\pi R}^{\pi R} dy\,e^{M|y|}S_m(y)S_n(y), \quad \text{(C4)}$$

$$I_5 \equiv \int_{-\pi R}^{\pi R} dy\,\frac{M_n}{\sqrt{\pi R}m_n}\,e^{-M|y|}\sqrt{\frac{M}{1-e^{-2\pi MR}}} \times\left(C_n(y)+\frac{MR}{n}\,\varepsilon(y)S_n(y)\right), \quad \text{(C5)}$$

$$I_6 \equiv \int_{-\pi R}^{\pi R} dy\,C_m(y)\left(C_n(y)-\frac{MR}{n}\,\varepsilon(y)S_n(y)\right), \quad \text{(C6)}$$

with $S_n(y)=\sin(\frac{n}{R}y)$ and $C_n(y)=\cos(\frac{n}{R}y)$.

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