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Electromagnetic nucleon-to-delta transition in holographic QCD

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We study nucleon-to-delta electromagnetic transition form factors and relations between them within the framework of the holographic dual model of QCD proposed by Sakai and Sugimoto. In this setup, baryons appear as topological solitons of the five-dimensional holographic gauge theory that describes a tower of mesons and their interactions. We find a relativistic extension of the nucleon-delta-vector meson interaction vertices and use these to calculate transition form factors from holographic QCD. We observe that at low momentum transfer, magnetic dipole, electric and Coulomb quadrupole form factors, and their ratios follow the patterns expected in the large N_c limit. Our results at this approximation are in reasonable agreement with experiment.

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I. INTRODUCTION

As is well known, $\Delta(1232)$ resonance is the first excited state of the nucleon and plays an important role in strong interaction physics. Experimentally, Δ's are produced in scattering pions or electron beams off a nucleon target. The $\Delta(1232)$ has isospin $3/2$ and therefore comes in four different charge states: Δ^{++}, Δ^{+}, Δ^{0}, and Δ^{-} with approximately the same mass and width. The spin of the $\Delta(1232)$ is also $3/2$, and it is the lightest known particle with such a spin. It appears that it decays via $\Delta \to N\pi$ with 99% branching ratio [1], and only less than 1% to the total decay width is coming from the electromagnetic (EM) channel ($\Delta \to N\gamma$). This EM $\gamma N\Delta$ transition is predominantly of the magnetic dipole ($M1$) type.

With the lack of complete theoretical control over nonperturbative low energy QCD, at present, one should explore various complementary techniques to gain more understanding on nonperturbative QCD phenomena. The large N_c limit is one such attempt that has been pursued for some time, and it was revived a decade ago by a proposal that in conjunction with the idea of holography, the large N_c QCD in the strong coupling regime may be described by a weakly coupled dual five-dimensional (5D) model, with the additional fifth direction playing the role of the energy scale [2]. Although the precise dual model of large N_c QCD has not been found, several approximate models were proposed applying both so-called "top-down" [3] and "bottom-up" [4] approaches. The study of hadronic form factors in these models of holographic QCD [3–14] allowed us to gain a confidence that the holographic QCD approach can be a useful complementary tool in predicting the low energy behavior of QCD at least in the large N_c limit.

In this work, our goal is to go one step further and investigate the $\gamma^* N \to \Delta$ transition form factors in the framework of holographic QCD. We will work in the top-down model proposed by Sakai and Sugimoto [3]. Baryons in this model have been studied recently [13–20] and our methods are based on some of these developments. In particular, we start our analysis by considering the nonrelativistic nucleon-delta-vector meson vertices found by Park-Yi [17], and for completeness also discuss the related approach by Hashimoto-Sakai-Sugimoto [11].

Using the nonrelativistic result in Ref. [17], we find a relativistic generalization of the nucleon-delta-vector meson vertex which is required for consistent treatment of transition form factors. This in turn is essential for comparison of the model predictions for nucleon-delta transition form factors with experiment. The knowledge of these form factors is proven to be an important and complex check for any model of strong interactions. Thus, we would like to investigate what holographic QCD can tell us about this process and how well its predictions agree with experiment.

Since the virtual photon has three polarizations, the $\gamma^* N \to \Delta$ transition should be in general described by three independent form factors. These three form factors are related to the magnetic dipole ($M1$), electric ($E2$), and Coulomb ($C2$) quadrupole types of transitions. The $\gamma N\Delta$ transition was measured in the pion photoproduction and electroproduction reactions in the Δ-resonance energy region. The $E2$ and $C2$ transitions were found to be relatively small but nonzero at moderate momentum transfers Q^2, with the ratios $R_{\rm EM} = E2/M1$ and $R_{\rm SM} = C2/M1$ being at the level of a few percent. The smallness of these ratios seems to have a purely nonperturbative origin. Indeed, the perturbative QCD studies [21] predict the same strength for $E2$ and $M1$ transitions at asymptotically large Q^2, while

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experimentally, the $E2/M1$ ratio is negative and very close to zero for energies up to $Q^2 \sim 4\ \mathrm{GeV}^2$ [22].

The smallness of the $E2/M1$ ratio is a very well-known prediction of the quark model, where the $N \to \Delta$ transition is described by a spin flip of a quark in the s-wave state, which in the $\gamma N \Delta$ case leads to the $M1$ type of transition. Any d-wave admixture in the wave function of Δ would allow for the $E2$ and $C2$ quadrupole transitions. Therefore, by measuring these transitions, one is able to observe the presence of the d-wave components and, hence, quantify to which extent the nucleon or the Δ wave function deviates from the spherical shape.

Within the nonrelativistic SU(6) quark model it was shown that $E2$ is zero [23] provided that quarks have zero orbital angular momentum. Small values for $E2/M1$ in the region $Q^2 < 4\ \mathrm{GeV}^2$ were obtained in the relativistic quark model, see e.g. Ref. [24]. In the large N_c limit of QCD, it was shown that the $E2/M1$ ratio is of order $\mathcal{O}(1/N_c^2)$ [25] without any assumption about the quark orbital angular momentum or intrinsic deformation of the baryon. We will show below that the same features can be also reproduced from the holographic QCD.

There have been studies on the $\gamma^* N \to \Delta$ transition form factors using other methods, for example, the local quark-hadron duality approach motivated by QCD sum rules [26] and the framework of the light-cone sum rules in [27]. Recent lattice calculations [28] of the $N\Delta$ transition form factors up to $1.5\ \mathrm{GeV}^2$ give small negative values for the ratio $E2/M1$. All these results provide strong evidence that the observed small value of $E2/M1$ has a purely nonperturbative origin. This is why we think that the application of the holographic QCD may shed more light on our understanding of this phenomenon, as it captures both nonperturbative and large N_c features. Some of the reviews describing various aspects of the $\gamma N \Delta$ transition can be found in [29].

The paper is organized as follows. In Sec. II, we briefly review the holographic model proposed by Sakai and Sugimoto [3], and discuss how baryons are described in the model. Instead of going into the details of this construction that are well described in Refs. [3,5], we only outline the five-dimensional effective action that emerges from the model and the field decomposition which are of importance for our further discussions. We then review the primary results in Ref. [17], where nucleons and delta baryons as well as their interactions with the vector mesons are studied within the nonrelativistic formalism. This will be our starting point of the subsequent analysis. In Sec. III, we explore the holographic vector meson dominance feature that emerges from the holographic QCD. In the origin of this lies the observation that there exists a basis for the vector meson fields in which the external electromagnetic field interacts only with vector meson fields, linearly without kinetic mixing. As a result, we end up with a very convenient framework to perform tree-level calculations. In Sec. IV, we outline similar approaches for describing baryons in the Sakai-Sugimoto model (other than Ref. [17]). For completeness, we also study and compare the dependence of some of the main results, when applying these alternative approaches.

In Sec. V, we propose a relativistic generalization for the $N\Delta v^{(n)}$ vertex that is required to obtain relativistic transition form factors. Since our purpose is to perform tree-level calculations, we avoid all problems associated with the higher-spin fermions. Without concerning what is the appropriate 5D relativistic formulation for the spin-3/2 fermions, we simply explore the expectation that after integrating over the holographic fifth direction Z, the theory should effectively become a 4D relativistic theory of spin-3/2 particles, for which Rarita-Schwinger formalism can be implemented. This in turn should reduce to a nonrelativistic theory with $N\Delta v^{(n)}$ vertex that we started with. By considering all possible relativistic operators consistent with 4D symmetries, we eventually find a unique relativistic operator which satisfies these requirements, and write down a 4D relativistic Lagrangian that correctly describes the $N\Delta v^{(n)}$ vertices, up to possible subleading terms of order $\mathcal{O}(1/N_c)$.

In Sec. VI, after giving a general formalism for the interaction vertex and defining the relevant form factors, together with their ratios that are of importance in comparison with experiments, we present our results for the $\gamma N \Delta$ transition form factors from the holographic QCD. We observe that magnetic dipole, electric, and Coulomb quadrupole form factors all depend on a single *holographic form factor*. Similar observation was also made in Ref. [11]. This leads to interesting consequences that are in accord with the expected behavior in the large N_c limit. We also briefly discuss some other observables of interest. We conclude by summarizing our results and pointing out some directions for further development.

II. PRELIMINARIES

We briefly outline the holographic model proposed by Sakai and Sugimoto in Ref. [3] in order to establish our notations and conventions that we are going to use throughout this paper. Although we give a very short description of the original construction, we stress that practical readers can simply start from the resulting 5D gauge theory given in (1) and (2), without referring to the details of the construction. We will also make a few comments about the holographic treatment of baryons and conclude by describing the nonrelativistic nucleon-Δ-vector meson vertex obtained originally in Ref. [17].

A. Model of Sakai and Sugimoto

The Sakai-Sugimoto model is constructed by placing probe N_f $D8$ and $\overline{D8}$-branes in the S^1 compactified N_c $D4$-brane background of type IIA string theory. In the

D-brane picture, the supersymmetry on a $D4$ brane is broken by imposing an antiperiodic boundary conditions on fermions along the S^1 circle. As a result, for energies lower than the compactification scale, we are effectively left with the 4D pure SU(N_c) gauge theory. In accordance with the basics of AdS/CFT [2], in the large N_c limit, the theory describing N_c coincident $D4$ branes is expected to be dual to a supergravity theory in a curved background with flux obtained by solving type IIA supergravity.

Working in the probe approximation, that is when $N_f/N_c \ll 1$, one can ignore the backreaction of $D8$ and $\overline{D8}$ branes embedded in the $D4$-brane background. The introduction of $D8$ and $\overline{D8}$ branes supports the existence of N_f massless flavors of quarks. In this model, quarks of left- and right-handed chiralities appear from $D8 - D4$ and $\overline{D8} - D4$ strings, respectively. This construction aims to reproduce large N_c QCD with N_f massless quarks. The chiral $U(N_f)_L \times U(N_f)_R$ symmetry group of QCD emerges from the $U(N_f)_L \times U(N_f)_R$ gauge theory living on the probe $D8$ and $\overline{D8}$ branes. One expects that the dynamics of this gauge theory, living on the $D8$ and $\overline{D8}$ branes, holographically describes the chiral dynamics of the large N_c QCD with massless quarks.

Geometrically, the chiral $U(N_f)_L \times U(N_f)_R$ symmetry of QCD is spontaneously broken to the diagonal subgroup $U(N_f)_V$ due to the "merging" of these $D8$ and $\overline{D8}$ branes in the background of $D4$ branes. In simple terms, the resulting configuration can be viewed as a stack of N_f $D8$ branes continuously connecting asymptotic regions of the original $D8$ and $\overline{D8}$ branes. The Nambu-Goldstone bosons of the broken chiral symmetry arise from the Wilson line that connects these two asymptotically separated regions on the $D8$ brane. Other modes on the $D8$ branes correspond to a tower of vector and axial-vector mesons, whose interactions among themselves are completely determined from the theory describing $D8$ branes.

The resulting 5D theory on the $D8$ branes is a $U(N_f)$ non-Abelian gauge theory with a Chern-Simons (CS) term in a curved background metric. In addition to the usual 4D Minkowski spacetime coordinates, there is the holographic dimension Z, that ranges from $-\infty$ to $+\infty$. The global $U(N_f)_L$ and $U(N_f)_R$ chiral symmetries of QCD reside at the boundaries $Z = +\infty$ and $Z = -\infty$, respectively. More precisely, gauge fields on these boundaries only couple to the left or right chiral currents of QCD. The action of this 5D gauge theory consists of a gauge-kinetic term, that can be written as

$$S_{D8} = \frac{\pi}{4}\left(\frac{f_\pi}{M_{KK}}\right)^2 \int d^4x dZ \,\mathrm{Tr}\bigg(-\frac{1}{2}(1+Z^2)^{-1/3} F_{\mu\nu}F^{\mu\nu} + M_{KK}^2(1+Z^2)F_{\mu Z}F^{\mu}{}_{Z}\bigg), \qquad (1)$$

and the Chern-Simons term,

$$S_{\mathrm{CS}} = \frac{N_c}{24\pi^2}\int_{R^4\times Z} \mathrm{Tr}\left(AF^2 + \frac{i}{2}A^3F - \frac{1}{10}A^5\right), \qquad (2)$$

where $A = A_\mu dx^\mu + A_z dZ$, and the metric is chosen to be mostly negative. From what follows, we will focus on the case with only two flavors $N_f = 2$ that is sufficient to our present purposes.

The non-Abelian gauge fields $A_M(x^\mu, Z)$ contain all information about the pion, vector, and axial-vector mesons. More precisely, the 5D gauge field A_M can be mode expanded in the $A_Z = 0$ gauge as

$$A_\mu(x, Z) = -\frac{1}{f_\pi}\partial_\mu \pi(x)\psi_0(Z) + \sum_{n\geq 1} B_\mu^{(n)}(x)\psi_n(Z) + \cdots, \qquad (3)$$

where $\psi_0(Z) = \frac{2}{\pi}\arctan(Z)$ [30], and $\{\psi_n(z)\}_{n\geq 1}$ are orthonormal eigenfunctions, satisfying

$$K^{1/3}\partial_Z(K\partial_Z\psi_n) = -\frac{m_n^2}{M_{KK}^2}\psi_n, \qquad \psi_n(Z\to\pm\infty) = 0, \qquad (4)$$

with $K = 1 + Z^2$ and are normalized as

$$\frac{\pi}{4}\left(\frac{f_\pi}{M_{KK}}\right)^2 \int_{-\infty}^{+\infty} dZ K^{-1/3}\psi_n\psi_m = \delta_{mn}. \qquad (5)$$

The 4D fields $\pi(x)$ and $B_\mu^{(n)}(x)$ describe pions and spin-1 mesons. For odd integers $(2n-1)$, $B_\mu^{(2n-1)} \equiv v_\mu^{(n)}$ describe vector mesons, while $B_\mu^{(2n)} \equiv a_\mu^{(n)}$ describe axial-vector mesons [31]. Inserting the field expansion from Eq. (3) into the action (1) and (2), we get a 4D Lagrangian that describes the dynamics of pions and tower of vector and axial-vector mesons. The theory has a unique scale M_{KK}, the popular choice of which is $M_{KK} = 0.949$ GeV, chosen to reproduce the experimental ρ-meson mass.

B. Holographic baryons

Besides mesons, the present holographic model is also able to incorporate baryons, which appear in a way quite similar to the Skyrmion in 4D [32]. In the holographic 5D theory (1), we have topological solitons along the *spatial* Euclidean four dimensions $(\vec{x}, Z)$, whose topological charges are determined by the second Chern number of the gauge potential

$$B = \frac{1}{8\pi^2}\int_{R^3\times Z} \mathrm{Tr}(F \wedge F). \qquad (6)$$

In fact this topological charge is related to the usual Skyrmion number of pions upon 4D interpretation (3), which supports the identification of these solitons as baryons [33].

As the energy of the baryon is minimized at $Z = 0$, due to the specific form of the action (1), it localizes at this position. In fact, the size of this soliton baryon naively tends to shrink to zero by the same reason. From the gauge-kinetic action (1), it follows that the mass of the soliton (M_B) with $B = 1$, which is localized at $Z = 0$ and has zero size would be

$$M_B^0 = 4\pi^2 \frac{\pi}{2}\left(\frac{f_\pi}{M_{KK}}\right)^2 M_{KK}. \tag{7}$$

If the size of the soliton is perturbed by ρ, then the increase in mass is $\delta M_B \sim \rho^2$. This suggests that it would be energetically more favorable for the soliton to stay localized at $Z = 0$ with zero size.

However, the presence of the 5D Chern-Simons term (2) induces an additional electric charge to the topological charge (6), whose electrostatic self-energy ($\sim 1/\rho^2$) compensates the shrinking tendency. As a result, the baryon acquires a stable and finite size [14–16]. Note that the soliton profile should be much like the profile of the instanton in Yang-Mills theory, since the equations that describe this soliton are similar to those of the Euclidean 4D Yang-Mills (YM) theory, except a nontrivial form of the action (1).

The soliton solutions come with a moduli space of zero modes, just like in the case of Skyrmions, and one should quantize along this moduli space. The structure of the moduli space can be easily learned from that of the instantons: we have approximate SO(4) spatial rotations interlocked with the isospin $\mathrm{SU}(2)_I$ in addition to the usual translational moduli. Size is not a moduli due to the nontrivial Z dependence of the metric, but one can also choose to quantize along the size as in Ref. [15]. As a result, one gets nucleons as lowest states of the spectrum, Δ baryons as a next excitation, and other higher excited baryon states.

One of the important lessons we learn from the well-known YM instanton solution is its purely non-Abelian nature in the field profile with the $U(1)$ part being absent. Another lesson is that the profile has a long-ranged tail at large distance r of the form

$$A_m^a \sim -\rho^2 \bar{\eta}_{mn}^a \partial_n \frac{1}{r^2}, \tag{8}$$

where ρ is the stabilized size of the soliton baryon. This can provide important information about the coupling of baryons with the mesons, more precisely with the 5D gauge field $A_M(x, Z = 0)$ at the position of the soliton $Z = 0$. This is essentially in the same vein to the usual pion tail of the Skyrmion solution, whose strength was used by Adkins-Nappi-Witten to obtain the nucleon-pion coupling $g_{\pi NN}$ [32]. Recall that the logic was to replace the *quantized* Skyrmions with a pointlike nucleon field $N(x)$ that has a coupling to the pions with the right strength $g_{\pi NN}$ to source the asymptotic pion tail of the original Skyrmion solution. One subtlety was that the pion tail of the classical Skyrmion solution must be replaced by the expectation value of *quantum* states obtained by semiclassical quantization along the zero modes. Only after that it can be mapped to the profile sourced by the nucleon current $\bar{N}\gamma^\mu\gamma^5 N$ in the *nonrelativistic* limit. Also, the quantization of Skyrmion results in a tower of higher half-integer spin and isospin states as well, with Δ baryons of spin and isospin $3/2$ being a primary example. Through the identical procedure as for nucleons, it was also possible to calculate nucleon-Δ-pion coupling $g_{N\Delta\pi}$ [32].

The same logic may be invoked to find a coupling of *quantized* holographic baryons in 5D to gauge field $A_M(x^\mu, Z)$ that encodes pions and a tower of spin-1 vector mesons via (3). In other words, the coupling of the baryon field to the 5D gauge field A_M must be such to be able to reproduce *quantum state expectation value* of the long-ranged tail (8) of the original soliton solution for the baryon. The necessary semiclassical quantization on the zero modes is essentially identical to that of the Skyrmion, giving us a tower of *nonrelativistic* spectrum of half-integer spin and isospin. By computing expectation values of the long-range tail (8) over these states, one may write down effective local couplings of baryon fields to the field A_M that can reproduce these quantum averaged tails. An important point is that these couplings will involve only the non-Abelian part of A_M as the Abelian U(1) component is absent in the tail (8).

For the lowest spin, isospin $1/2$ states corresponding to a holographic version of the usual nucleons, Refs. [10,14,16] introduced a 5D Dirac spinor $\mathcal{B}$ to write down an effective Lagrangian that encapsulates the above features. The necessary analysis for higher excited baryons was done in Ref. [17]. However, in 5D it seems hard to find a fully relativistic formulation of higher-spin fermions, and the effective fields and Lagrangians in Ref. [17] are only nonrelativistic. In the next subsection, we will summarize this development which serves as a basis of our subsequent analysis. Later, we will show how one can proceed to a necessary relativistic extension of their results, that will be crucial for our purposes.

C. Nonrelativistic treatment

In this section we summarize some of the results of Ref. [17], which will be our starting point in calculating the transition form factors. As we are interested only in $N - \Delta$ transition in this work, the relevant interaction term would involve baryon fields of spin and isospin $1/2$ as well as $3/2$. A convenient *nonrelativistic* notation for spin, isospin $1/2$ states is

$$\mathcal{U}_\alpha^\epsilon, \tag{9}$$

where $\alpha = +, -$ is a two-component nonrelativistic spinor index and $\epsilon = 1, 2$ is the index for the isospin doublet. A similar notation for spin, isospin $3/2$ states is

$$\mathcal{U}_{\alpha_1\alpha_2\alpha_3}^{\epsilon_1\epsilon_2\epsilon_3}, \tag{10}$$

where the three indices α_i's and ϵ_i's must be totally symmetric to be in the $S = I = 3/2$ representation under the spatial rotation and the isospin. These *nonrelativistic* baryon fields are presumed to be localized at the position $Z = 0$, where they minimize their energy. A quantum spread along $Z \neq 0$ can be shown to be a subleading effect in the large N_c limit that holographic QCD is based on. With the above notations, the relevant *nonrelativistic* $N-\Delta$ transition coupling to the 5D gauge field $A_M = A_M^a \frac{\tau_a}{2}$ ($a = 1, 2, 3$) has been obtained to be [17]

$$S_{N\Delta}^{5D} = -k_{1/2}\Big(\tfrac{1}{2}F_{ij}^a \epsilon_{ijk}(\mathcal{U}_\alpha^\epsilon)^*(\sigma_2\sigma_k)^{\beta\beta'}(\tau_2\tau_a)^{ee'}\mathcal{U}_{\beta\beta'\alpha}^{ee'\epsilon} + F_{Zi}^a(\mathcal{U}_\alpha^\epsilon)^*(\sigma_2\sigma_i)^{\beta\beta'}(\tau_2\tau_a)^{ee'}\mathcal{U}_{\beta\beta'\alpha}^{ee'\epsilon}\Big)\Big|_{Z=0}, \quad (11)$$

where

$$k_{1/2} = \frac{\sqrt{3}(N_c + 2)}{4\sqrt{5}} M_{KK}^{-1} = \frac{\sqrt{15}}{4} M_{KK}^{-1}. \quad (12)$$

We have replaced N_c by $N_c + 2$, as was argued in Refs. [14,16,34] to better account for subleading effects. We will discuss this issue in more detail in Sec. IV. Note that the Abelian U(1) part is absent in the above due to the non-Abelian nature of the soliton-baryon profile, as explained before. By inserting the mode expansion (3) of A_M in Eq. (11), one can obtain nucleon-Δ couplings to the pions and a tower of spin-1 (axial) vector mesons.

For our purpose of calculating electromagnetic (EM) form factors, we need to consider vector mesons $B_\mu^{(2n-1)} = v_\mu^{(n)}$ only, as EM does not couple to axial vectors. Because for vector mesons $v_\mu^{(n)}(x^\mu)$, $\psi_{(2n-1)}(Z)$ is even under $Z \to -Z$, it is easily seen that the second term in (11) simply vanishes, and the first piece gives us

$$S_{N\Delta v} = -\frac{1}{2}k_{1/2}\sum_{n\geq 1}\psi_{(2n-1)}(0)(\partial_i v_j^{(n)} - \partial_j v_i^{(n)})^a \epsilon_{ijk}(\mathcal{U}_\alpha^\epsilon)^* \times (\sigma_2\sigma_k)^{\beta\beta'}(\tau_2\tau_a)^{ee'}\mathcal{U}_{\beta\beta'\alpha}^{ee'\epsilon}. \quad (13)$$

This *nonrelativistic* result will be the starting point of our analysis. Notice that due to $F_{ij}^a\epsilon_{ijk}$ structure in (11) this interaction is of "magnetic" type. Since $\psi_{(2n-1)}(Z)$ is an even function for the vector mesons, there is no analogous "electric" type of interaction (unless there is some subleading asymmetric smearing in Z). This observation is in agreement with the fact that the EM transition of $N\Delta$ is predominantly of magnetic ($M1$) dipole type.

It is also instructive to consider $N\Delta\pi$ coupling. In this case, since $\psi_0(Z)$ is an odd function, only the second term survives and as a result [17],

$$S_{N\Delta\pi} = \frac{8k_{1/2}}{\pi f_\pi}(\partial_i\pi^a)(\mathcal{U}_\alpha^\epsilon)^*(\sigma_2\sigma_i)^{\beta\beta'}(\tau_2\tau_a)^{ee'}\mathcal{U}_{\beta\beta'\alpha}^{ee'\epsilon}. \quad (14)$$

The generalization of this interaction to the relativistic case may be important when discussing the photoproduction processes.

III. VECTOR DOMINANCE IN HOLOGRAPHIC QCD

In the Sakai-Sugimoto model, the 5D $U(N_f)$ gauge field A_M ($M = 0, 1, 2, 3, Z$) contains pseudoscalar pions and a tower of vector/axial-vector mesons upon its 4D mode expansion. It can also include external vector potentials that couple to $U(N_f)_L \times U(N_f)_R$ chiral symmetry currents as its non-normalizable modes near $Z \to \pm\infty$ boundaries. In the $A_Z = 0$ gauge (leading order in fields), the mode expansion reads as [3,5]

$$A_\mu(x, Z) = \left(-\frac{1}{f_\pi}\partial_\mu\pi(x) + \mathcal{A}_\mu(x)\right)\psi_0(Z) + \mathcal{V}_\mu(x) + \sum_{n\geq 1} B_\mu^{(n)}(x)\psi_{(n)}(Z) + \cdots, \quad (15)$$

where $\mathcal{V} = \frac{1}{2}(A_L + A_R)$ and $\mathcal{A} = \frac{1}{2}(A_L - A_R)$ are the external vector and axial-vector potentials. By looking at how the model responds to these external potentials, one can study various form factors of chiral symmetry currents. Electroweak form factors of the QCD sector would be of particular interest for applications, see e.g. Ref. [35].

The electromagnetic vector potential A_μ^{EM} can be thought of as an external potential probing the QCD sector by

$$\mathcal{V}_\mu = e\begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3}\end{pmatrix} A_\mu^{\mathrm{EM}}, \qquad \mathcal{A}_\mu = 0. \quad (16)$$

As the axial part $\mathcal{A}_\mu$ is absent, the electromagnetic external potential will couple only to the vector mesons $B_\mu^{(2n-1)} \equiv v_\mu^{(n)}$, which allows one to neglect axial-vector mesons $B_\mu^{(2n)} \equiv a_\mu^{(n)}$ in the above expansion (15). We will see how $\mathcal{V}_\mu$ interacts with the system in the following, neglecting the axial part, which results in a feature quite similar to vector dominance with a tower of vector mesons $v_\mu^{(n)}$.

Keeping only the vector part, and using Eq. (4), we get

$$(1 + Z^2)^{1/3}\partial_Z[(1 + Z^2)\partial_Z\psi_{(2n-1)}] = -\frac{m_{v^n}^2}{M_{KK}^2}\psi_{(2n-1)}, \quad (17)$$

as well as the orthonormality condition:

$$\frac{\pi}{4}\left(\frac{f_\pi}{M_{KK}}\right)^2\int_{-\infty}^{+\infty} dZ(1 + Z^2)^{-1/3}\psi_{(2n-1)}(Z)\psi_{(2m-1)}(Z) = \delta_{nm}. \quad (18)$$

Taking all of these into account, the action (1) for A_M reduces to a 4D action

$$S_{4D} = \int d^4x \Big[\mathrm{Tr}\Big(-\mathcal{F}^{\mathcal{V}}_{\mu\nu} \sum_{n\geq 1} a_{\mathcal{V}v^n} F^{(n)\mu\nu} \Big) + \sum_{n\geq 1} \mathrm{Tr}\Big(-\frac{1}{2} F^{(n)}_{\mu\nu} F^{(n)\mu\nu} + m^2_{v^n} v^{(n)}_\mu v^{(n)\mu} \Big) \Big], \quad (19)$$

where $\mathcal{F}^{\mathcal{V}}_{\mu\nu} = \partial_\mu \mathcal{V}_\nu - \partial_\nu \mathcal{V}_\mu$, $F^{(n)}_{\mu\nu} = \partial_\mu v^{(n)}_\nu - \partial_\nu v^{(n)}_\mu$, and [36]

$$a_{\mathcal{V}v^n} = \frac{\pi}{4} \Big(\frac{f_\pi}{M_{KK}} \Big)^2 \int_{-\infty}^{+\infty} dZ (1 + Z^2)^{-1/3} \psi_{(2n-1)}(Z). \quad (20)$$

The second term in this action describes massive vector mesons, while the first piece represents a kinetic mixing between the external vector potential $\mathcal{V}_\mu$ and the massive vector mesons $v^{(n)}_\mu$. It is more convenient to diagonalize the kinetic terms by shifting the vector meson fields as

$$v^{(n)}_\mu = \tilde{v}^{(n)}_\mu - a_{\mathcal{V}v^n} \mathcal{V}_\mu, \quad (21)$$

which transforms the action to

$$S_{4D} = \int d^4x \sum_{n\geq 1} \mathrm{Tr}\Big(-\frac{1}{2} \tilde{F}^{(n)}_{\mu\nu} \tilde{F}^{(n)\mu\nu} + m^2_{v^n} \tilde{v}^{(n)}_\mu \tilde{v}^{(n)\mu} - 2 m^2_{v^n} a_{\mathcal{V}v^n} \tilde{v}^{(n)}_\mu \mathcal{V}^\mu \Big), \quad (22)$$

in terms of $\tilde{v}^{(n)}_\mu$ fields, up to an additive renormalization of $\mathcal{V}_\mu$ kinetic terms which are divergent anyway. In the $\tilde{v}^{(n)}_\mu$ basis, the mixing to $\mathcal{V}_\mu$ is independent on the momentum transfer, which will make the summation over n of Feynman diagrams more convergent as we will see later. This is the usefulness of this new basis, although any final results must be independent of whether we work in the $v^{(n)}_\mu$ or the $\tilde{v}^{(n)}_\mu$ basis.

Another advantage in using this new basis $\tilde{v}^{(n)}_\mu$ is in a manifest presence of the holographic vector meson dominance feature. Note that the expansion of A_M in (15) including only the vector part becomes in the new basis

$$\begin{aligned} A_\mu(x, Z) &= \mathcal{V}_\mu(x) + \sum_{n\geq 1} v^{(n)}_\mu(x) \psi_{(2n-1)}(Z) \\ &= \Big(1 - \sum_{n\geq 1} a_{\mathcal{V}v^n} \psi_{(2n-1)}(Z) \Big) \mathcal{V}_\mu(x) \\ &\quad + \sum_{n\geq 1} \tilde{v}^{(n)}_\mu(x) \psi_{(2n-1)}(Z). \end{aligned} \quad (23)$$

Using the completeness relation of the eigenfunctions for the vectorlike quantity

$$\frac{\pi}{4} \Big(\frac{f_\pi}{M_{KK}} \Big)^2 \sum_{n\geq 1} (1 + Z^2)^{-1/3} \psi_{(2n-1)}(Z) \psi_{(2n-1)}(Z') = \delta(Z - Z'), \quad (24)$$

and the definition (20) of $a_{\mathcal{V}v^n}$, we can easily derive a *sum rule*:

$$\sum_{n\geq 1} a_{\mathcal{V}v^n} \psi_{(2n-1)}(Z) = 1, \quad (25)$$

which drastically simplifies the expansion

$$A_\mu(x, Z) = \sum_{n\geq 1} \tilde{v}^{(n)}_\mu(x) \psi_{(2n-1)}(Z), \quad (26)$$

without any remnant of the external vector potential $\mathcal{V}_\mu$. In other words, in the $\tilde{v}^{(n)}_\mu$ basis the external vector potential $\mathcal{V}_\mu$ does not interact directly with the system through A_M, but only through momentum-independent mixings with the vector mesons $\tilde{v}^{(n)}_\mu$ via (22). Any interaction of the system with the external vector potential is completely mediated by tree-level exchanges of the massive vector mesons $\tilde{v}^{(n)}_\mu$. Moreover, one can define vector meson decay constants as follows:

$$\langle 0 | J^a_{V\mu}(0) | v^{(n),b} \rangle = g_{v^n} \delta^{ab} \epsilon_\mu, \qquad g_{v^n} \equiv m^2_{v^n} a_{\mathcal{V}v^n}. \quad (27)$$

Therefore, the previous interaction term (13) of $N - \Delta$ with vector mesons can be easily generalized to the case of having an external vector potential $\mathcal{V}_\mu$, by simply replacing $v^{(n)}_\mu$ with $\tilde{v}^{(n)}_\mu$:

$$S_{N\Delta\tilde{v}} = -\frac{1}{2} k_{1/2} \sum_{n\geq 1} \psi_{(2n-1)}(0) (\partial_i \tilde{v}^{(n)}_j - \partial_j \tilde{v}^{(n)}_i)^a \epsilon_{ijk} (\mathcal{U}^\epsilon_\alpha)^* \times (\sigma_2 \sigma_k)^{\beta\beta'} (\tau_2 \tau_a)^{ee'} \mathcal{U}^{ee'\epsilon}_{\beta\beta'\alpha}, \quad (28)$$

without any further change. The electromagnetic $N - \Delta$ transition form factors will be described by tree-level Feynman diagrams, where the external field, $\mathcal{V}_\mu$, couples to transition vertex only through the exchange of vector mesons $\tilde{v}^{(n)}_\mu$.

IV. OTHER APPROACHES

We should point out that there are other similar approaches in the literature, exploring chiral symmetry currents from the soliton solution itself [11–13,15,18,19]. The only difference can be traced back to how we treat the *nonrelativistic* baryon wave function along the Z direction, which was taken to be the δ function in the above, to give the $\psi_{(2n-1)}(0)$ factor. Although this is a right thing to do in a strict large N_c limit sense, Refs. [11,12,15,18,19] went one step further to better approximate the baryon wave functions, which corresponds to including a subleading effect in the large N_c limit. This modification of the baryon wave functions will have its effects only on the factor $\psi_{(2n-1)}(0)$.

In Refs. [11,15], the motion along Z was approximated by harmonic oscillator. For our purposes, it is sufficient to take the ground state wave function:

$$\Psi_B^{n_Z=0}(Z) = \left(\frac{2a}{\pi}\right)^{1/4} e^{-aZ^2}, \tag{29}$$

where $a = \frac{2\pi^3}{\sqrt{6}}(\frac{f_\pi}{M_{KK}})^2 \approx 0.240$. Accordingly, we replace $\psi_{(2n-1)}(0)$ by

$$\langle \psi_{(2n-1)}(Z)\rangle = \frac{\int_{-\infty}^{+\infty} dZ \psi_{(2n-1)}(Z) e^{-2aZ^2}}{\int_{-\infty}^{+\infty} dZ e^{-2aZ^2}}. \tag{30}$$

In fact Refs. [11,15] also treated the size of soliton baryon, ρ, as a quantum mechanical modulus, and any quantity that involves ρ should also be averaged over the resulting wave function on ρ. It can be shown that the coefficient $k_{1/2}$ in (11) is proportional to ρ^2, as the long-ranged tail of the soliton baryon that this term is based on is linear in ρ^2, which can be seen in (8). The resulting quantum average increases ρ^2 and hence $k_{1/2}$ by a factor of

$$\frac{\sqrt{5} + 2\sqrt{5 + N_c^2}}{2N_c} \approx 1.62. \tag{31}$$

Recall that the earlier expression for $k_{1/2} = \frac{\sqrt{3}(N_c+2)}{4\sqrt{5}} M_{KK}^{-1} = \frac{\sqrt{15}}{4} M_{KK}^{-1}$ involves a shift $N_c \to N_c + 2$ that effectively accounts a subleading effect in the large N_c limit [34]. This observation was used in the analysis of Refs. [10,14,16]. This shift corresponds to the increase of $k_{1/2}$ by a factor of $\frac{5}{3} \approx 1.66$ relative to its classical value, which happens to be very close to the above increase by the quantum wave function on ρ. This in fact explains the *fortunate* numerical agreements between Refs. [10,14,16] and Ref. [11]. Although it does not make much difference to replace our previous value $k_{1/2} = \frac{\sqrt{15}}{4} M_{KK}^{-1} \approx 0.968 M_{KK}^{-1}$ with the value from the analysis in Ref. [11],

$$k_{1/2} = 1.62 \times \frac{\sqrt{3} N_c}{4\sqrt{5}} M_{KK}^{-1} \approx 0.941 M_{KK}^{-1}, \tag{32}$$

we will choose the latter for consistency when we discuss the results by the approach of Ref. [11].

As the soliton-baryon size ρ is given by

$$\rho^2 = \frac{N_c}{\pi^3}\sqrt{\frac{3}{10}}\left(\frac{M_{KK}}{f_\pi}\right)^2 \approx (2.364)^2, \tag{33}$$

which is *numerically bigger* than the average size of the quantum wave function in Ref. [11],

$$Z_{\rm av} = \sqrt{\langle Z^2\rangle} = \frac{1}{2}\sqrt{\frac{1}{a}} \approx 1.021, \tag{34}$$

the quantum spread over Z seems numerically subdominant to the initial size effect of the baryons. This seems to be a problem in this approach.

For completeness, we will present both results we obtain using the approaches in Refs. [11,15] and the previous one with the factor $\psi_{(2n-1)}(0)$ based on Ref. [17]. We refer the former as the type II and the latter as the type I model. In summary,

$$\text{type I: } \psi_{(2n-1)}(0)$$
$$\text{type II: } \psi_{(2n-1)}(0) \to \langle \psi_{(2n-1)}(Z)\rangle.$$

As a final comment, the two types of approaches we consider share one common feature that seems to be universal. Because of the identity (25)

$$\sum_{n\geq 1} a_{\mathcal{V} v^n} \psi_{(2n-1)}(Z) = 1, \tag{35}$$

for *any* Z, one can easily see that the *sum rule*

$$\sum_{n\geq 1} a_{\mathcal{V} v^n} \langle \psi_{(2n-1)}(Z)\rangle = 1 \tag{36}$$

holds whatever approximation we use for $\langle \psi_{(2n-1)}(Z)\rangle$. This will give us a universal result for the zero-momentum limit of the form factor, that is a one robust prediction of the model without referring to a specific type of approach.

V. RELATIVISTIC GENERALIZATION

As was discussed earlier, in the holographic model, the external electromagnetic field is carried by the vector mesons. The linear coupling of the electromagnetic potential with these vector mesons is given by the last term in the interaction Lagrangian (22), where the external vector field is expressed through the electromagnetic potential as in Eq. (16). Therefore, the interaction of the electromagnetic field with the hadrons can only occur through the intermediate vector meson exchange. This feature of "holographic vector meson dominance," given by (26), is a relativistic concept, since it emerges from the manifestly relativistic 5D gauge theory. On the other hand, the nucleon-$\Delta - \tilde{v}^{(n)}$ interaction vertex, given by Eq. (28), is *nonrelativistic*. To have a more consistent framework, we have to find a relativistic generalization of this vertex (28). Although, we are interested in low momentum transfers, where the nonperturbative effects are dominant, there is no clear separation of relativistic and nonrelativistic effects. Moreover, for momentum transfers larger than about 2 GeV^2, the relativistic effects may not be negligible. With this generalization in hand, we can find the relativistic nucleon-$\Delta - \tilde{v}^{(n)}$ transition form factors by simply summing over all tree-level Feynman diagrams involving $\tilde{v}_\mu^{(n)}$ meson exchanges.

A difficulty in Ref. [17] for a relativistic formulation was the absence of a relativistic 5D formalism of high spin fermions, notably for spin 3/2 fermions corresponding to a holographic description of Δ baryons. This difficulty might have a chance to be overcome in the future, but there is one point we can make at present without any regard to details of a solution: the resulting 4D theory, after integrating over Z, must be reduced to a 4D relativistic theory for a spin 3/2 field that describes the Δ resonances. Moreover, for consistency, the *nonrelativistic* limit of this 4D theory should precisely reproduce the previous *nonrelativistic* result (28).

In the *nonrelativistic* treatment of Ref. [17], a δ-function localization of the baryon wave functions to $Z = 0$ was assumed, as a leading approximation to the large N_c limit, where the baryons become infinitely heavy. This seems to indicate an intricate intertwining of a relativistic generalization and an inclusion of subleading effects of the large N_c limit: if we take the large N_c limit first, then the baryons should be treated nonrelativistically.

We propose to take a different path instead. We first impose a relativistic formulation before looking at the leading large N_c effects. After integrating over Z, a presumed 4D relativistic theory will be a theory of spin $3/2$ Δ baryons, for which we have a consistent relativistic description in terms of the Rarita-Schwinger formalism. Whatever formalism for a 4D relativistic spin $3/2$ field we would have from the perspective of a presently unknown 5D relativistic formulation, it would be equivalent to the Rarita-Schwinger field at the end. As we take the large N_c limit for the parameters in the resulting relativistic theory, we expect that in the nonrelativistic limit the results must agree with those of Ref. [17]. This expectation relies on the assumption of validity of exchanging the order of the two limits. This way one will be able to fix the parameters in the 4D relativistic theory of Rarita-Schwinger Δ baryons *in the large N_c limit*.

A relativistic generalization of the nucleon-Δ-$\tilde{v}^{(n)}$ vertex (28) of interest in terms of Dirac spinor nucleons and Rarita-Schwinger Δ baryons can be determined within our proposal, which will be the subject of the next sections.

A. Basics of the Rarita-Schwinger spin $\frac{3}{2}$ field

The $\Delta(1232)$ is a spin-$3/2$ resonance. Therefore its spin content can be described in terms of a Rarita-Schwinger (RS) field [37]: $\Psi_\mu^{(\sigma)}$, where μ is the vector and σ the spinor index (the latter index is omitted in the following). Here, we briefly summarize the relevant relativistic formalism for this 4D RS field.

The free and massive RS field obeys the Dirac equation, supplemented with the auxiliary conditions (constraints):

$$(i\not\partial - M)\Psi_\mu = 0, \qquad \partial^\mu \Psi_\mu = 0, \qquad \gamma^\mu \Psi_\mu = 0. \tag{37}$$

The constraints ensure that the number of independent components of the vector-spinor field is reduced to the physical number of spin degrees of freedom. In the interacting theory, the coupling of the RS field must be compatible with the free theory construction in order to preserve the physical spin degrees of freedom, otherwise one ends up with unphysical degrees of freedom with negative-norm states [38,39] or superluminal (acausal) modes [40,41]. The proposals for consistent spin-3/2 couplings were given e.g. in Ref. [42]. Although it can be a subtle issue to discuss consistency of a quantum theory of interacting RS field, we will not be concerned about this here, since the Feynman diagrams, that are required to calculate form factors we are interested in, are tree-level diagrams and involve intermediate vector meson exchanges. In fact, we are only interested in the kinematic description of relativistic spin-$\frac{3}{2}$ particle and its pointlike tree-level interactions with massive vector mesons $\tilde{v}_\mu^{(n)}$, such that these interaction vertices reduce to our previous expression (28) in the nonrelativistic limit.

We will work in the conventions, where the γ matrices are

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}, \qquad \gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{38}$$

Since we are interested in the nonrelativistic limit for the Δ resonances, consider a specific momentum state of $\partial_\mu = -ip_\mu$, which has the following restframe components $p_0 = E = M$ and $p_i = 0$ $(i = 1, 2, 3)$. Then solving equations of motion with constraints, one finds $\Psi_0 = 0$ and

$$\Psi_i = \begin{pmatrix} \mathcal{U}_i \\ 0 \end{pmatrix}, \tag{39}$$

with three two-component spinors $\mathcal{U}_i$ $(i = 1, 2, 3)$ satisfying an important constraint,

$$\sum_{i=1}^{3} \sigma_i \mathcal{U}_i = 0. \tag{40}$$

In fact, $\mathcal{U}_i$ $(i = 1, 2, 3)$ with the constraint (40) is an unconventional way of describing usual nonrelativistic spin $s = \frac{3}{2}$ states of SO(3) rotation group. First note that the independent number of components is indeed $2 \times 3 - 2 = 4$ as in the case of spin $s = 3/2$ representation. It is not difficult to find the similarity transformation between the $\mathcal{U}_i$ representation and the usual representation with $|s = \frac{3}{2}, s_z\rangle$ basis. Writing

$$\mathcal{U}_i = \begin{pmatrix} a_i \\ b_i \end{pmatrix}, \tag{41}$$

with complex numbers a_i and b_i, one finds

$$\begin{aligned}
a_1 &= \sqrt{\tfrac{1}{2}}|\tfrac{3}{2}, \tfrac{3}{2}\rangle + \sqrt{\tfrac{1}{6}}|\tfrac{3}{2}, -\tfrac{1}{2}\rangle, \\
b_1 &= \sqrt{\tfrac{1}{2}}|\tfrac{3}{2}, -\tfrac{3}{2}\rangle + \sqrt{\tfrac{1}{6}}|\tfrac{3}{2}, \tfrac{1}{2}\rangle, \\
a_2 &= i\sqrt{\tfrac{1}{2}}|\tfrac{3}{2}, \tfrac{3}{2}\rangle - i\sqrt{\tfrac{1}{6}}|\tfrac{3}{2}, -\tfrac{1}{2}\rangle, \\
b_2 &= -i\sqrt{\tfrac{1}{2}}|\tfrac{3}{2}, -\tfrac{3}{2}\rangle + i\sqrt{\tfrac{1}{6}}|\tfrac{3}{2}, \tfrac{1}{2}\rangle, \\
a_3 &= -\sqrt{\tfrac{2}{3}}|\tfrac{3}{2}, \tfrac{1}{2}\rangle, \\
b_3 &= \sqrt{\tfrac{2}{3}}|\tfrac{3}{2}, -\tfrac{1}{2}\rangle.
\end{aligned} \tag{42}$$

Recall that in our expression (28), we used yet another form of spin $s = \frac{3}{2}$ representation: a three-indexed objects $\mathcal{U}_{\alpha_1\alpha_2\alpha_3}$ totally symmetric under permutations of $\alpha_i = +, -$. It is easy to relate this representation with the standard $|s = \frac{3}{2}, s_z\rangle$ representation,

$$\begin{aligned} &\mathcal{U}_{+++} = |\tfrac{3}{2}, \tfrac{3}{2}\rangle, \\ &\mathcal{U}_{++-} = \mathcal{U}_{+-+} = \mathcal{U}_{-++} = \sqrt{\tfrac{1}{3}}|\tfrac{3}{2}, \tfrac{1}{2}\rangle, \\ &\mathcal{U}_{--+} = \mathcal{U}_{-+-} = \mathcal{U}_{+--} = -\sqrt{\tfrac{1}{3}}|\tfrac{3}{2}, -\tfrac{1}{2}\rangle, \\ &\mathcal{U}_{---} = -|\tfrac{3}{2}, -\tfrac{3}{2}\rangle, \end{aligned} \tag{43}$$

where sign conventions are chosen simply for later convenience. With the above (42) and (43), one can now easily translate the nonrelativistic limit of the RS field and the three-indexed object $\mathcal{U}_{\alpha_1\alpha_2\alpha_3}$ that was used in (28). One identity that we will need specifically is

$$(\sigma_{[i}\mathcal{U}_{j]})_\alpha = \frac{1}{2\sqrt{2}}\epsilon_{ijk}(\sigma_2\sigma_k)^{\beta\beta'}\mathcal{U}_{\beta\beta'\alpha}, \tag{44}$$

with $[i, j] = \frac{1}{2}(ij - ji)$, that is straightforward to check using (42) and (43).

B. Relativistic $N\Delta\tilde{v}^{(n)}$ vertex

With the gadget in the previous section, we can now find the relativistic $N\Delta\tilde{v}^{(n)}$ vertex, that generalizes the nonrelativistic expression (28). Note that while we have to generalize the space rotation indices (lower indices) into a relativistic Rarita-Schwinger field, we should still keep the isospin indices (upper indices) as they are in (28). To relate to the more conventional notation of Δ baryons, one can simply substitute

$$\begin{aligned} &\mathcal{U}^{111} = \Delta^{++}, \\ &\mathcal{U}^{222} = \Delta^{-}, \\ &\mathcal{U}^{112} = \mathcal{U}^{121} = \mathcal{U}^{211} = \sqrt{\tfrac{1}{3}}\Delta^{+}, \\ &\mathcal{U}^{221} = \mathcal{U}^{212} = \mathcal{U}^{122} = \sqrt{\tfrac{1}{3}}\Delta^{0}. \end{aligned} \tag{45}$$

Since we do not need to modify the isospin structure in (28), we will temporarily omit it, focusing only on the relativistic generalization of the spacetime part.

Up to equations of motion, there are four possible forms of Lorentz invariant coupling between nucleon Dirac spinor N, Rarita-Schwinger Δ-baryon field Ψ_μ, and the nth massive vector mesons $F^{(n)}_{\mu\nu} = \partial_\mu\tilde{v}^{(n)}_\nu - \partial_\nu\tilde{v}^{(n)}_\mu$:

$$\begin{aligned} &(1)\ \bar{N}F^{(n)}_{\mu\nu}\gamma^\mu\Psi^\nu + \text{H.c.} \\ &(2)\ \bar{N}F^{(n)}_{\mu\nu}\gamma^\mu\gamma^5\Psi^\nu + \text{H.c.} \\ &(3)\ \bar{N}F^{(n)}_{\mu\nu}\gamma^{\mu\nu\rho}\Psi_\rho + \text{H.c.} \\ &(4)\ \bar{N}F^{(n)}_{\mu\nu}\gamma^{\mu\nu\rho}\gamma^5\Psi_\rho + \text{H.c.} \end{aligned} \tag{46}$$

However, using the constraint $\gamma^\mu\Psi_\mu = 0$, one can easily show that (3) and (4) are equivalent to (1) and (2).

If we take the nonrelativistic limit as is done in the previous section, the spinors reduce to nonrelativistic two-component spinors as

$$N = \begin{pmatrix} \mathcal{U} \\ 0 \end{pmatrix}, \qquad \Psi_i = \begin{pmatrix} \mathcal{U}_i \\ 0 \end{pmatrix}, \qquad \Psi_0 = 0, \tag{47}$$

and using the explicit form of the γ matrices, one easily checks that (1) does not lead to any nonrelativistic coupling as it couples particles to antiparticles, while (2) becomes

$$-(\mathcal{U}_\alpha)^*(\partial_i\tilde{v}^{(n)}_j - \partial_j\tilde{v}^{(n)}_i)(\sigma_{[i}\mathcal{U}_{j]})_\alpha. \tag{48}$$

From the important identity (44) that we derived in the previous section, this reduces to

$$-\frac{1}{2\sqrt{2}}(\mathcal{U}_\alpha)^*(\partial_i\tilde{v}^{(n)}_j - \partial_j\tilde{v}^{(n)}_i)\epsilon_{ijk}(\sigma_2\sigma_k)^{\beta\beta'}\mathcal{U}_{\beta\beta'\alpha}, \tag{49}$$

which recovers precisely the spacetime index structure of our nonrelativistic coupling (28).

The upshot is that the following relativistic operator,

$$\begin{aligned} S^{\text{rel}}_{N\Delta\tilde{v}} = \frac{\sqrt{30}}{4}M^{-1}_{KK}\sum_{n\geq 1}\psi_{(2n-1)}(0)F^{(n)a}_{\mu\nu}(\tau_2\tau_a)^{ee'} \\ \times \bar{N}^\epsilon\gamma^\mu\gamma^5(\Psi^\nu)^{ee'\epsilon} + \text{H.c.}, \end{aligned} \tag{50}$$

is the correct relativistic form of our nonrelativistic nucleon-Δ-$\tilde{v}^{(n)}$ vertex (28), where $F^{(n)a}_{\mu\nu} \equiv \partial_\mu\tilde{v}^{(n)a}_\nu - \partial_\nu\tilde{v}^{(n)a}_\mu$, and the upper indices a, e, e', and ϵ represent isospin indices. With the help of (45), one can also write the final result in terms of the conventional notation of Δ baryons ($\Delta^{++}, \Delta^+, \Delta^0, \Delta^-$) and the nucleons (p, n):

$$S^{\rm rel}_{N\Delta\tilde{v}} = i\frac{\sqrt{30}}{4}M^{-1}_{KK}\sum_{n\geq 1}\psi_{(2n-1)}(0)F^{(n)+}_{\mu\nu}\left(\sqrt{\frac{2}{3}}\bar{p}\gamma^\mu\gamma^5(\Delta^0)^\nu + \sqrt{2}\bar{n}\gamma^\mu\gamma^5(\Delta^-)^\nu\right)$$
$$- i\frac{\sqrt{30}}{4}M^{-1}_{KK}\sum_{n\geq 1}\psi_{(2n-1)}(0)F^{(n)-}_{\mu\nu}\left(\sqrt{2}\bar{p}\gamma^\mu\gamma^5(\Delta^{++})^\nu + \sqrt{\frac{2}{3}}\bar{n}\gamma^\mu\gamma^5(\Delta^+)^\nu\right)$$
$$+ i\frac{\sqrt{30}}{4}M^{-1}_{KK}\sum_{n\geq 1}\psi_{(2n-1)}(0)F^{(n)0}_{\mu\nu}\left(\sqrt{\frac{4}{3}}\bar{p}\gamma^\mu\gamma^5(\Delta^+)^\nu + \sqrt{\frac{4}{3}}\bar{n}\gamma^\mu\gamma^5(\Delta^0)^\nu\right) + \text{H.c.}, \tag{51}$$

where

$$F^{(n)\pm}_{\mu\nu} = \frac{1}{\sqrt{2}}(F^{(n)1}_{\mu\nu} \mp iF^{(n)2}_{\mu\nu}), \qquad F^{(n)0}_{\mu\nu} = F^{(n)3}_{\mu\nu} \tag{52}$$

are the nth vector meson fields in the EM charge basis, and Δ_μ are the Rarita-Schwinger fields for the Δ baryons. Equation (51) is the final form of the sought-for relativistic couplings between nucleons, Δ baryons, and the massive vector mesons $\tilde{v}^{(n)}_\mu$.

VI. TRANSITION FORM FACTORS

A. Definitions

The $\gamma^* N \to \Delta$ transition is described by the matrix element of the electromagnetic current $J^{\rm EM}_\mu$ between the nucleon state with momentum p and the Δ with momentum p'. It can be written as

$$\langle \Delta(p') \mid J^{\rm EM}_\mu(0) \mid N(p)\rangle = e\bar{\Psi}_\beta(p')\Gamma_{\beta\mu}\gamma_5 N(p), \tag{53}$$

where $N(p)$ and $\Psi_\beta(p')$ describe nucleon and delta, respectively. The conservation of the electromagnetic current implies $q_\beta\Gamma_{\beta\mu} = 0$, where $q = p' - p$ is the photon momentum transfer. For virtual photons ($q^2 \neq 0$), the decomposition of the vertex function can be expressed in terms of three independent scalar form factors $G_i(Q^2)$ with $Q^2 = -q^2$:

$$\Gamma_{\beta\mu} = G_1(Q^2)[q_\beta\gamma_\mu - \not{q}g_{\beta\mu}] + G_2(Q^2)[q_\beta P_\mu - (qP)g_{\beta\mu}] + G_3(Q^2)[q_\beta q_\mu - q^2 g_{\beta\mu}], \tag{54}$$

where $P = (p + p')/2$. Following [43], one can also define the magnetic dipole G_M, electric quadrupole G_E, and Coulomb quadrupole G_C form factors in terms of G_1, G_2, and G_3 as follows:

$$G_M(Q^2) = \frac{m_N}{3(m_N + m_\Delta)}\left[((3m_\Delta + m_N)(m_\Delta + m_N) + Q^2)\frac{G_1(Q^2)}{m_\Delta} + (m^2_\Delta - m^2_N)G_2(Q^2) - 2Q^2G_3(Q^2)\right],$$
$$G_E(Q^2) = \frac{m_N}{3(m_N + m_\Delta)}\left[(m^2_\Delta - m^2_N - Q^2)\frac{G_1(Q^2)}{m_\Delta} + (m^2_\Delta - m^2_N)G_2(Q^2) - 2Q^2G_3(Q^2)\right], \tag{55}$$
$$G_C(Q^2) = \frac{2m_N}{3(m_\Delta + m_N)}\left[2m_\Delta G_1(Q^2) + \frac{1}{2}(3m^2_\Delta + m^2_N + Q^2)G_2(Q^2) + (m^2_\Delta - m^2_N - Q^2)G_3(Q^2)\right].$$

We can also define the ratios $R_{\rm EM}$ and $R_{\rm SM}$ (see e.g. [43–46]) that are often used in the experimental papers:

$$R_{\rm EM}(Q^2) = \frac{E2(Q^2)}{M1(Q^2)} = -\frac{G_E(Q^2)}{G_M(Q^2)}, \qquad R_{\rm SM}(Q^2) = \frac{C2(Q^2)}{M1(Q^2)} = -\sqrt{Q^2 + \frac{(m^2_\Delta - m^2_P - Q^2)^2}{4m^2_\Delta}}\frac{1}{2m_\Delta}\frac{G_C(Q^2)}{G_M(Q^2)}. \tag{56}$$

B. Predictions from holographic QCD

Adding Feynman diagrams that correspond to intermediate vector meson exchanges between the external EM current and the $N\Delta v^{(n)}$ vertex given in Eq. (51) (corresponding to $p \to \Delta^+$ transition, in particular), we will obtain the following result for the form factors:

$$G_1(Q^2) = \sum_{n\geq 1}\frac{g_{v^n}g_{N\Delta v^n}}{Q^2 + m^2_{v^n}}, \qquad G_2(Q^2) = G_3(Q^2) = 0, \tag{57}$$

where $g_{v^n} \equiv m^2_{v^n}a_{\mathcal{V}_{v^n}}$ and

$$g_{N\Delta v^n} = \sqrt{2}\frac{\sqrt{30}}{4M_{KK}}\langle\psi_{(2n-1)}(Z)\rangle. \tag{58}$$

From Table I below, one can observe that for the type I model, the summation in (57) does not converge fast enough, while the type II case is sufficiently convergent.

The reason is that the completeness relation (24) that we used before to derive vector dominance is valid only with integration and not quite true pointwise, similar to the

TABLE I. Various couplings and masses for $M_{KK} = 0.949$ GeV.

n	1	2	3	4	5	6	7	8
$m^2_{v^n}$ (GeV^2)	0.602	2.59	5.94	10.6	16.7	24.0	32.7	42.8
g_{v^n} (GeV^2)	0.164	−0.707	1.615	−2.884	4.508	−6.484	8.869	−11.58
$g_{N\Delta v^n}$ (GeV^{-1}) (I)	14.12	12.88	12.60	12.51	12.46	12.44	12.43	12.42
$g_{N\Delta v^n}$ (GeV^{-1}) (II)	11.84	5.512	0.585	−1.481	−1.101	−0.038	0.407	0.196

Gibbs' phenomenon in Fourier transform theory. In these cases, the following expression,

$$\sum_{n\geq 1}\frac{g_{v^n}g_{N\Delta v^n}}{Q^2+m^2_{v^n}} = \sum_{n\geq 1}\frac{g_{v^n}g_{N\Delta v^n}}{m^2_{v^n}} - \sum_{n\geq 1}\frac{g_{v^n}g_{N\Delta v^n}Q^2}{m^2_{v^n}(Q^2+m^2_{v^n})}$$
$$= \sqrt{2}\frac{\sqrt{30}}{4M_{KK}} - \sum_{n\geq 1}\frac{g_{v^n}g_{N\Delta v^n}Q^2}{m^2_{v^n}(Q^2+m^2_{v^n})}, \qquad (59)$$

should be used instead to have a good convergence at low Q^2, where in the last line we have used the sum rule (25).

Observe, however, that any truncation to a finite number of excited modes (as above) would eventually fail for high Q^2, and a summation over all modes would be required in order to achieve convergence. There is an alternative way of doing this by noting that the form factor is proportional to the Z average of

$$\sum_{n\geq 1}\frac{g_{v^n}\psi_{(2n-1)}(Z)}{Q^2+m^2_{v^n}} \equiv G(Z, Q^2). \qquad (60)$$

Using the completeness relation, one can show that this function satisfies

$$(1+Z^2)^{1/3}\partial_Z[(1+Z^2)\partial_Z G(Z,Q^2)] = \left(\frac{Q^2}{M^2_{KK}}\right)G(Z,Q^2), \qquad (61)$$

with the boundary condition $G(Q^2, Z\to\pm\infty) = 1$. In fact, this is nothing but the bulk-to-boundary propagator (from the $Z\to\pm\infty$ boundary to the bulk Z for the gauge field). It is not difficult to solve this equation numerically for each Q^2. We use this method in the numerical plots later.

Notice that the form factors $G_{2,3}(Q^2)$ are vanishing, since we are working at leading order in N_c, neglecting the subleading effects. In other words, $G_2(Q^2)$ and $G_3(Q^2)$ are expected to be of order $\mathcal{O}(1/N_c)$ in the large N_c limit. The fact that there is only one independent form factor was also observed in Ref. [47], when discussing the form factors of the vector meson in the framework of AdS/QCD, and in Refs. [10,11] for the nucleon form factors in the Sakai-Sugimoto model.

The physically relevant magnetic dipole G_M, electric quadrupole G_E, and Coulomb quadrupole G_C form factors, are predicted from holographic QCD to be

$$G_M(Q^2) = \frac{m_N((3m_\Delta+m_N)(m_\Delta+m_N)+Q^2)}{3m_\Delta(m_N+m_\Delta)}G_1(Q^2),$$
$$G_E(Q^2) = \frac{m_N(m^2_\Delta-m^2_N-Q^2)}{3m_\Delta(m_N+m_\Delta)}G_1(Q^2), \qquad (62)$$
$$G_C(Q^2) = \frac{4m_N m_\Delta}{3(m_\Delta+m_N)}G_1(Q^2).$$

As a result, the ratios take the following form:

$$R_{\mathrm{EM}}(Q^2) = -\frac{(m^2_\Delta-m^2_N-Q^2)}{(3m_\Delta+m_N)(m_\Delta+m_N)+Q^2},$$
$$R_{\mathrm{SM}}(Q^2) = -\sqrt{Q^2+\frac{(m^2_\Delta-m^2_P-Q^2)^2}{4m^2_\Delta}} \qquad (63)$$
$$\times\frac{2m_\Delta}{(3m_\Delta+m_N)(m_\Delta+m_N)+Q^2}.$$

In case when $Q^2=0$, we will have

$$\frac{G_M(0)}{G_1(0)} = \frac{m_N(3m_\Delta+m_N)}{3m_\Delta},$$
$$\frac{G_E(0)}{G_1(0)} = \frac{m_N(m_\Delta-m_N)}{3m_\Delta}, \qquad (64)$$
$$\frac{G_C(0)}{G_1(0)} = \frac{4m_N m_\Delta}{3(m_\Delta+m_N)},$$

$$R_{\mathrm{EM}}(0) = R_{\mathrm{SM}}(0) = -\frac{m_\Delta-m_N}{3m_\Delta+m_N}. \qquad (65)$$

Recalling that baryon masses are of order $\mathcal{O}(N_c)$, while $\delta\equiv m_\Delta-m_N\sim\mathcal{O}(1/N_c)$, we have

$$R_{\mathrm{EM}}(0) = R_{\mathrm{SM}}(0) \simeq -\frac{\delta}{4m_N}\sim-\mathcal{O}\left(\frac{1}{N^2_c}\right). \qquad (66)$$

Although, we work at leading order in N_c, this result is consistent, since holographic QCD predicts $G_{2,3}\sim\mathcal{O}(1/N_c)$, and using Eqs. (55) and (56), one can easily deduce Eq. (66). In agreement with our result, the R_{EM} ratio for the $\gamma N\Delta$ transition was also shown to be of order $1/N^2_c$ in the Ref. [25]. Furthermore, the relation $R_{\mathrm{EM}}(0) = R_{\mathrm{SM}}(0)$ was also observed in Ref. [48] within the large-N_c limit (see also Ref. [49]). These observations provide an additional evidence that the smallness of the $\gamma N\Delta$ R_{EM} ratio is naturally explained in the large N_c limit.

Finally, one can check that, when $Q^2 = 0$,

$$G_1(0) = \sqrt{2}\frac{\sqrt{30}}{4M_{KK}}\sum_{n\geq 1} a_{\mathcal{V}v^n}\langle\psi_{(2n-1)}(Z)\rangle = \frac{\sqrt{15}}{2}\frac{1}{M_{KK}}, \tag{67}$$

where in the last step we used the sum rule from Eq. (25). This result is universal for types I and II models. Therefore, from Eq. (64) it follows that

$$\begin{aligned} G_M(0) &= \frac{\sqrt{15}}{2}\frac{m_N}{M_{KK}}\frac{(3m_\Delta + m_N)}{3m_\Delta} \simeq 2.43\frac{m_N}{M_{KK}}, \\ G_E(0) &= \frac{\sqrt{15}}{2}\frac{1}{M_{KK}}\frac{m_N(m_\Delta - m_N)}{3m_\Delta} \simeq 0.154\frac{m_N}{M_{KK}}, \\ G_C(0) &= \frac{\sqrt{15}}{2}\frac{1}{M_{KK}}\frac{4m_N m_\Delta}{3(m_\Delta + m_N)} \simeq 1.47\frac{m_N}{M_{KK}}. \end{aligned} \tag{68}$$

If we choose $M_{KK} = 0.949$ GeV to fit the ρ-meson mass, and use the experimental nucleon and Δ baryon masses, the above gives us

$$G_M(0) \approx 2.41, \qquad G_E(0) \approx 0.153, \qquad G_C(0) \approx 1.46. \tag{69}$$

However, since we are working in the large N_c limit approximation, for consistency, the terms of order $1/N_c$ have to be dropped, and one should use the same mass for the nucleons and Δ. In this case, we will have $G_E(0) = 0$,

$$G_M(0) = 2G_C(0) = \frac{4}{3}\frac{\sqrt{15}}{2}\frac{m_N}{M_{KK}}. \tag{70}$$

Numerically, we get $G_M(0) = 2G_C(0) \simeq 2.58$ and $R_{\mathrm{EM}} = R_{\mathrm{SM}} = 0$.

We should point out that within the model one finds that the quantized baryon mass is larger than the nucleon mass m_N. In the 5D effective field theory approach that is applied in Refs. [14,16], one finds

$$\frac{m_N}{M_{KK}} \approx 1.98. \tag{71}$$

In order to obtain a better agreement in the baryon sector of the holographic model, different values for either f_π or M_{KK} have to be chosen. The ratio of the classical baryon mass $m_N \approx M_B^0 \sim \mathcal{O}(N_c)$ to M_{KK} scale (7), before quantization is

$$\frac{M_B^0}{M_{KK}} = 2\pi^3\left(\frac{f_\pi}{M_{KK}}\right)^2 \approx 0.59. \tag{72}$$

This is clearly smaller than the ratio considered above, which may signal that the $1/N_c$ expansion and the numerical estimate for the baryon masses are no longer reliable. Since our predictions should be only leading order in N_c, there is no need to fit the parameters of the model (f_π and M_{KK}) to the exact physical results. In particular, if we keep $M_{KK} = 0.949$ GeV, while taking the baryon mass as input from experiment, this may correspond to changing the value for f_π, similar to the case in the Skyrme model [32]. This issue is a problem of model and approximation that is being used. Whatever the resolution of this problem, it should not affect our final results.

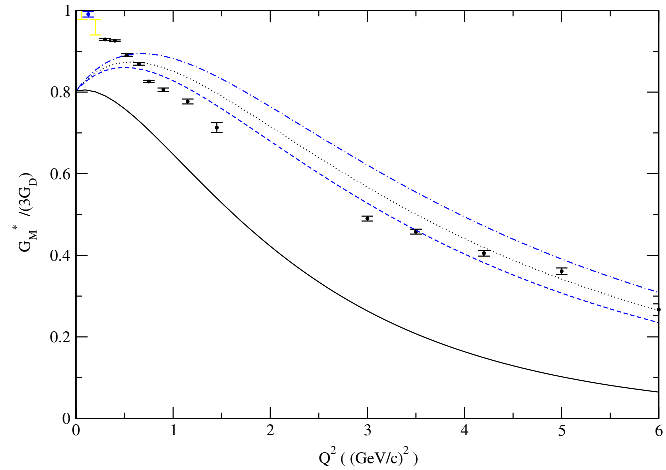

FIG. 1 (color online). The plot of the ratio $G_M^*(Q^2)/(3G_D(Q^2))$ as a function of Q^2, where $G_D(Q^2) = 1/(1 + Q^2/\Lambda^2)^2$ with $\Lambda^2 = 0.71\ (\mathrm{GeV}/c)^2$. The solid and dotted lines are the predictions from the holographic type I and type II models, respectively. The dashed (dash-dotted) curves are from taking the parameter a of Eq. (20) to be 20% larger (smaller) than the value $a \sim 0.240$. The experimental data points are taken from [50].

In Fig. 1 we present a plot for the ratio $G_M^*(Q^2)/(3G_p^D(Q^2))$ that is commonly used in the literature, where $G_p^D(Q^2) = 1/(1 + Q^2/\Lambda_p^2)^2$ is the proton's empirical electric form factor with $\Lambda_p^2 \simeq 0.71\ \mathrm{GeV}^2$, and

$$G_M^*(Q^2) = G_M(Q^2)/\sqrt{1 + \frac{Q^2}{(m_\Delta + m_N)^2}}. \tag{73}$$

The data are taken from the experiments in [50]. The theoretical curves correspond to type I and type II models. One may see that for $Q^2 \geq 0.5\ \mathrm{GeV}^2$ the better agreement with experiment is provided by the type II model. However, both models disagree with experiment (by about 20%) for $Q^2 \leq 0.5\ \mathrm{GeV}^2$. This suggests that, although smearing of baryons is required to get the correct behavior for $Q^2 \geq 0.5\ \mathrm{GeV}^2$, the exact account of $1/N_c$ corrections is required for lower energies.

C. Helicity amplitudes

Equivalently, one can also parametrize the $\gamma^* N\Delta$ transition through the rest frame helicity amplitudes $A_{1/2}$ and $A_{3/2}$ defined in terms of the following matrix elements of the electromagnetic current operator:

$$A_{3/2} \equiv -\frac{e}{\sqrt{2q_\Delta}} \frac{1}{(4M_N M_\Delta)^{1/2}} \times \langle \Delta(\vec{0}, +3/2) | \mathbf{J} \cdot \boldsymbol{\epsilon}_{\lambda=+1} | N(-\vec{q}, +1/2) \rangle,$$
$$A_{1/2} \equiv -\frac{e}{\sqrt{2q_\Delta}} \frac{1}{(4M_N M_\Delta)^{1/2}} \times \langle \Delta(\vec{0}, +1/2) | \mathbf{J} \cdot \boldsymbol{\epsilon}_{\lambda=+1} | N(-\vec{q}, -1/2) \rangle, \tag{74}$$

where the transverse photon polarization vector entering in $A_{1/2}$ and $A_{3/2}$ is given by $\boldsymbol{\epsilon}_{\lambda=+1} = -1/\sqrt{2}(1, i, 0)$, the spin projections are along the z axis (along the virtual photon direction), and q_Δ is the magnitude of the virtual photon three-momentum in the Δ rest frame:

$$q_\Delta \equiv |\mathbf{q}| = \frac{Q_+ Q_-}{2M_\Delta}, \qquad Q_\pm \equiv \sqrt{(M_\Delta \pm M_N)^2 + Q^2}. \tag{75}$$

The helicity amplitudes are functions of the photon virtuality Q^2, and can be expressed in terms of the Jones-Scadron $\gamma^* N\Delta$ form factors as

$$A_{3/2} = -\mathcal{N} \frac{\sqrt{3}}{2}(G_M + G_E) = -\mathcal{N} \frac{\sqrt{3}}{2} G_M + \mathcal{O}\left(\frac{1}{N_c^2}\right),$$
$$A_{1/2} = -\mathcal{N} \frac{1}{2}(G_M - 3G_E) = -\mathcal{N} \frac{1}{2} G_M + \mathcal{O}\left(\frac{1}{N_c^2}\right), \tag{76}$$

where

$$\mathcal{N} \equiv \frac{e}{2}\left(\frac{Q_+ Q_-}{2M_N^3}\right)^{1/2} \frac{(M_N + M_\Delta)}{Q_+}. \tag{77}$$

The above helicity amplitudes are expressed in units $\mathrm{GeV}^{-1/2}$, and reduce at $Q^2 = 0$ to the photocouplings quoted by the Particle Data Group [1]. Experimentally, these helicity amplitudes are extracted from the $M1$, $E2$, and $C2$ multipoles for the $\gamma^* N \to \pi N$ process at the resonance position, i.e. for πN c.m. energy $W = M_\Delta$.

In terms of helicity amplitudes,

$$R_{\mathrm{EM}} = \frac{A_{1/2} - \frac{1}{\sqrt{3}} A_{3/2}}{A_{1/2} + \sqrt{3} A_{3/2}}. \tag{78}$$

Notice that from the Eq. (76) it follows that

$$\frac{A_{3/2}}{A_{1/2}} = \sqrt{3} + \mathcal{O}\left(\frac{1}{N_c^2}\right). \tag{79}$$

This result was also predicted in the Ref. [25] within the framework of the large N_c QCD.

In the case $Q^2 = 0$, we have $Q_\pm = M_\Delta \pm M_N$, and therefore

$$\mathcal{N} = \frac{e}{2}\left(\frac{M_\Delta^2 - M_N^2}{2M_N^3}\right)^{1/2} \simeq 0.094 \ \mathrm{GeV}^{-1/2}. \tag{80}$$

As a result, we will have from holographic QCD

$$A_{1/2} \simeq -121[10^{-3} \ \mathrm{GeV}^{-1/2}],$$
$$A_{3/2} \simeq -209[10^{-3} \ \mathrm{GeV}^{-1/2}], \qquad R_{\mathrm{EM}} \simeq 0\%. \tag{81}$$

The experimental results (MAMI/A2 Collaboration [51] and LEGS Collaboration [52]), taken from the Particle Data Group [1], quote

$$A_{1/2} = -(135 \pm 6)[10^{-3} \ \mathrm{GeV}^{-1/2}],$$
$$A_{3/2} = -(250 \pm 8)[10^{-3} \ \mathrm{GeV}^{-1/2}],$$
$$R_{\mathrm{EM}} = -(2.5 \pm 0.5)\%. \tag{82}$$

From the values of the $\gamma^* N\Delta$ form factors at $Q^2 = 0$, one can extract some interesting static quantities. For the dominant $M1$ transition, one can extract the static $N \to \Delta$ transition magnetic moment $\mu_{N\to\Delta}$ from the value of $G_M(0)$ as [53]

$$\mu_{N\to\Delta} = \sqrt{\frac{M_\Delta}{M_N}} G_M(0), \tag{83}$$

which is expressed in nuclear magnetons $\mu_N \equiv e/(2M_N)$. Furthermore, one can extract a static $N \to \Delta$ quadrupole transition moment $Q_{N\to\Delta}$ as [53]

$$Q_{N\to\Delta} = -6\sqrt{\frac{M_\Delta}{M_N}} \frac{1}{M_N q_\Delta(0)} G_E(0), \tag{84}$$

where $q_\Delta(0)$ is obtained from Eq. (75) for $Q^2 = 0$, as $q_\Delta(0) = (M_\Delta^2 - M_N^2)/2M_\Delta$.

Our results from the holographic QCD framework, with masses of baryons taken from experiments, are

$$G_M(0) \simeq 2.41, \qquad \mu_{N\to\Delta} \simeq 2.76\mu_N,$$
$$Q_{N\to\Delta} \simeq -0.171 \ \mathrm{fm}^2. \tag{85}$$

On the other hand, without taking experimental baryon masses and neglecting terms of order $\mathcal{O}(1/N_c)$, we will get

$$G_M(0) \simeq 2.58, \qquad \mu_{N\to\Delta} \simeq 2.58\mu_N,$$
$$Q_{N\to\Delta} \simeq 0 \ \mathrm{fm}^2. \tag{86}$$

From the experiments, Ref. [54] extracted the values

$$G_M(0) = 3.02 \pm 0.03, \qquad \mu_{p\to\Delta^+} = [3.46 \pm 0.03]\mu_N,$$
$$Q_{p\to\Delta^+} = -(0.0846 \pm 0.0033) \ \mathrm{fm}^2. \tag{87}$$

Taking into account that our results are of only leading order in large N_c, we find about 20% discrepancy with experiments as an indication that the holographic model works consistently. It is an important open problem to systematically improve the large N_c expansion in the holographic QCD.

Transition amplitudes and their ratios were also discussed in the framework of the Skyrme-like models, see e.g. Refs. [55–57]. In particular, Wirzba and Weise [55] performed a modified Skyrme model calculation, at leading order in N_c, where $R_{\rm EM}$ takes values between -2.5% and -6%, depending on the coupling parameters of the stabilizing terms. In [57], Walliser and Holzwarth included rotational corrections, which are of order $1/N_c$, and lead to a quadrupole distortion of the classical soliton solution. Including such corrections, one finds a very good description of the photocouplings and obtains a ratio $R_{\rm EM} = -2.3\%$, consistent with experiment. Similarly, we also expect that quantum corrections, including rotational effects, should improve our results and provide a better agreement with the experimental data.

VII. CONCLUSION

Working in the framework of the holographic dual model of QCD proposed by Sakai and Sugimoto [3,5] with two massless flavors, we consider the electromagnetic $N \to \Delta$ transition form factors at leading order in N_c [58]. By considering a relativistic generalization of the nonrelativistic vertices found in Ref. [17] up to $1/N_c$ ambiguities, we treat the problem in a consistent relativistic way [59]. As a result of holographic computation, we establish that among three independent form factors, only one survives. Besides, the large N_c dependence of transition form factors and their ratios coincide with what was expected in the earlier studies. In particular, the following fact was observed for the ratios: $R_{\rm EM} = R_{\rm SM} \sim \mathcal{O}(1/N_c^2)$.

After employing the approximation where baryons are pointlike, we also consider a simple example, where the baryon wave functions are smeared as a ground state oscillator *à la* Ref. [11]. Although, the value of the $G_M(0)$ form factor remains the same for both models, we seem to get a better agreement with experimental data for energies up to 6 $\rm GeV^2$. This suggests that the finite size effects may indeed improve holographic QCD predictions. We leave the discussion of these effects for further studies. Our most reliable results in this work are the values for the form factors obtained at $Q^2 = 0$.

An interesting direction for further studies includes the possibility for studying transition form factors among various other excited hadron states. This approach can shed more light on photoproduction and electroproduction processes and help us to better understand the nature of baryon excited states.

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Note added.—Simultaneously with our work another article [60] appeared in the arXiv, discussing the same problem but in the framework of the holographic "bottom-up" model. Some of the main results as well as the hierarchies among the different form factors are qualitatively quite independent of the choice of the model.

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[59] The nonrelativistic reduction of relativistic theory is an expansion in powers of the Compton wavelength. In the case of baryon, whose mass is proportional to N_c, the nonrelativistic reduction is an expansion in N_c. In this work, we will only keep terms leading order in N_c, therefore, will not be concerned about the subleading terms that can be included into the action, which greatly simplifies the procedure. On the other hand, for consistency, inclusion of order $1/N_c$ terms should take into account stringy effects which would be difficult to accommodate.
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